THE RADIATION RESISTANCE OF RESONANT TRANSMISSION LINES

Presented in Partial Fulfillment of the Requirements for the Degree of MASTER OF ENGINEERING

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PREFACE

It is here fitting to thank all those to whom I am indebted, both for technical help and for financial assistance. I would like to thank Dr. R.A. Chipman for his technical assistance and his interest in the project, while acting as director of the research. His assistance has in many ways improved the technical knowledge of the author, and his interest in the undertaking made problems less difficult.

I would also like to extend my note of thanks to the National Research Council of Canada for financial assistance provided in the form of a summer grant.

A note of thanks is also due to my colleague Mr. J.E. Boucher, for his assistance in building the apparatus used in this investigation.

Furthermore, may I also thank Mrs. R. Harris and my wife for providing their stenographic ability in order that this treatise may be presented.

INTRODUCTION

The problem of radiation losses from high frequency parallel-wire transmission lines was first realized as early as 1900, and has become a matter of controversy ever since. Many writers have investigated the problem from a theoretical standpoint, but very little has been done in the way of measuring these losses experimentally. At low frequencies radiation losses are negligible, but they become appreciable at high frequencies thus affecting the design of parallel-wire lines for use in the higher portion of the frequency spectrum. In order that the controversy may be wholly or at least partially settled, and to correct existing data, this treatise has therefore been prepared.

This paper describes the measurement of radiation resistance from resonant parallel-wire lines in the frequency range 300 M_c/sec. to 1200 M_c/sec. To determine the dependence of radiation losses on line length, lines varying in length from one-quarter wavelength to four wavelengths, with open and short-circuited terminations have been investigated. The results obtained agree with theory in some ways whereas they disagree in others.

Radiation losses are similar in some respects to ohmic losses and as such are undesirable in many engineering applications. Methods of decreasing these losses have been devised, and the amount by which these methods remedy the losses is determined. If it ever becomes necessary to increase the radiation losses, the methods by which this may be attained also are investigated.

I PAST HISTORY

Ever since it was discovered that the power lost in an antenna is considerably more than the ohmic losses of this antenna, engineers have attributed this additional power loss to radiation. Ohm's law was well known at this time and the ohmic or heat losses in an electrical circuit were defined as the square of the current flowing in the circuit times the resistance of this circuit. Radiation losses are somewhat similar to ohmic losses, as a result a radiation resistance has been defined for any circuit which may lose power through the electromagnetic field associated with it. In order to have a radiation resistance similar to the resistance expressed by Ohm's law it had generally been agreed that radiation resistance should be thought of as, the power lost in watts divided by the square of the current at a point where this current has a maximum value.

At present there exist two methods of obtaining the power radiated from a resonant circuit. The first is the method of integrating the Poynting vector over an infinite sphere with the radiating element at the center, while the second is the induced e.m.f. method of Brillouin(1). Both methods are discussed in detail in a book published by Stratton(2) in 1941 and yield the same result for the same case. Bechmann(3) showed in 1931 that the two methods are equivalent.

The problem of losses in transmission lines due to radiation was suspected over half a century ago and it was Mie(4) who first applied

Maxwell's equations to the parallel-wire transmission line. In 1900, Mie had stated that there was no power lost to the surrounding space from a long parallel-wire line. He said that this is the case irrespective of the spacing between the two wires or the frequency. This statement seemed to satisfy engineers for a decade, however, the question was once more approached by Steinmetz(5) in 1909 and again in 1919.

Steinmetz considered a parallel-wire line with a separation between the two wires which was small compared with a quarter-wavelength. By assuming a cosine function for the current and voltage in each conductor, he went on to find the magnetic field at a small distance from the transmission line. This magnetic field then induces a voltage which contains two terms, a sine term and a cosine term, in the two conductors. The sine component, he says, is the e.m.f. of self-induction which gives rise to the inductance, while the cosine term is in phase with the current and gives rise to what he called the magnetic radiation resistance. In this way Steinmetz found a distributed radiation resistance proportional to the separation of the wires and to the square of the frequency.

Shortly after this Carson(6) became interested in the problem and published two articles, the first in 1921 and again in 1924. He said that the plane transverse waves propagating along the two wires would not remain plane when a termination was encountered and, therefore, radiation losses would occur due to the terminations. By using a Maxwell-Lorentz method Carson obtained an expression for these losses for the case of a line terminated in its characteristic impedance. In order that his expression may be compared with the results obtained by later writers it has been changed into a suitable form and is given in Table I Page . For this case, Carson defines his radiation resistance as the power lost divided by the square of the r.m.s. current.

In his second paper Carson started from Maxwell's equations and by determining the appropriate solution corresponding to the geometry of the system, he obtained a solution involving two distinct types of current and voltage waves. The first he called "principal wave", or the wave of ordinary engineering theory. The second, which he called "complementary waves" consists of an infinite number of waves which individually satisfy Maxwell's equations. He says that the principal wave is not attenuated by radiation whereas the complementary waves are actually the ones which may add or subtract energy from the principal wave. The principal wave is only affected by radiation at points where a discontinuity exists along the line, whereas the complementary waves are so highly attenuated that they are not easily detectable by ordinary means, but they do play an important part in the radiation field. Carson also states that the radiation from a system can be calculated to a good approximation, without explicit recognition of the complementary waves. At great distances from the wires, however, the electromagnetic field of the complementary current waves is, in general, large compared with that of the principal wave and almost entirely determines the direction and character of energy flow. Nevertheless, the radiation field and flow of energy out of the system can be completely calculated by means of the retarded vector potential, in terms of the currents in the wires. This is the method which Carson had used when he calculated the radiation losses from a transmission line in his previous article.

Just before Carson had written his second article, Manneback[7] had presented the first complete analysis of a transmission line which was open-circuited at both ends and an integral number of half-wavelengths long. Manneback used the method of integrating the Poynting vector over a very large sphere with the transmission line at the center, subject to the condition that the separation between the two conductors was very much less than the wavelength. The expression he obtains for radiation resistance is also given in Table I. From this theory he concludes that there is a negligible amount of loss in a transmission line due to radiation, and that even at a frequency of 3 megacycles the radiation loss is only about $\frac{1}{3600}$ of the total power lost by heat. This is quite true at this frequency, however, at frequencies greater than 1000 megacycles this ratio becomes much greater.

After Manneback's analysis the question may have been considered temporarily settled for another decade had almost gone by before other writers had considered the problem. In 1929, however, Pistalkors[8] was probably the first to use the induced e.m.f. method of making calculations of radiation resistance of antennas involving more than one wire. He considered the case of two parallel wires of equal length, separated by a constant distance, and any multiple of a half-wavelength long. Although his calculations were made assuming that the currents in the two wires was in phase, his results could easily be converted to cover the case of the two wires being excited in phase opposition. When the necessary changes are made, and if it is assumed that the separation between the two wires is small compared with the wavelength the result obtained is in agreement with that of Manneback.

Carter(9) was the next writer to consider the problem. He considered the same case as Manneback, and obtained the expression shown in Table I. This equation is subject to the following limitations:

(a) the line is an integral number of half-wavelengths long,

(b) the conductors have vanishingly small radii,

(c) a sinusoidal current distribution exists on the lines.

If we again introduce the limitation that the separation between the two conductors is small compared to the wavelength and consider the length as an integral number of half-wavelengths, this equation then becomes exactly the same as that of Manneback.

About the same time that Carter had considered the problem Sterba and Feldman(10) also published an article, in which they had considered three or four cases of the transmission line. By postulating the current distribution on the transmission line, calculating the electromagnetic fields and from the fields, the associated radiation by means of Poynting's theorem, they derived equations similar to the one Carter had obtained. These equations simplify considerably when it is assumed that the spacing between the lines is small compared to the wavelength, and if only resonant sections of lines are used. Thus for a line terminated in its characteristic impedance and carrying a non-attenuated travelling wave they show that the radiation resistance is given by:

$$R_\lambda = 160\,\pi^2\left(\frac{d}{\lambda}\right)^2$$

where d is the separation between the lines, and λ is the wavelength. For a line bearing a standing wave and having current maxima at both ends (short-circuited line) the radiation resistance is only three-quarters of what it is for a line terminated in its characteristic impedance. Sterba and Feldman consider two more cases, the first is a line which has a current minimum at one end and a current maximum at the other end; while the second is a line having current maximas at both ends.

The result for the line bearing current maxima at both ends agrees with that obtained by Manneback in 1923, and later by Carter. It is interesting to note however, that all these results are independent on the length of the line but are only functions of the line spacing and the wavelength. Another striking feature is that the radiation resistance of the line terminated in its characteristic impedance is the sum of the radiation resistance for an open-circuited line plus that for a short circuited line.

A year later, Whitmer(11) published an article on "Radiation Resistance of Concentric Conductor Transmission Lines" in which he devotes a page or two to the problem of the open parallel-wire line. In his method Whitmer first finds the power radiated from wire A due to the current flowing in it, and the power radiated from wire A due to the current flowing in wire B. This is also done for wire B. Since the currents in the two wires are 180 degrees out of phase with each other, the total power radiated will be twice the difference between the power radiated by self induction and the power radiated by mutual induction. This analysis leads to the same result obtained by Carter in 1932.

One of the most recent writers who analyzed the parallel-wire transmission line was L.E. Reukema(12). In a paper published in 1937, Reukema shows that radiation resistance is actually of dominant importance in determining the selectivity factor Q and the input impedance Z_s for both parallel wire and concentric conductor lines at high radio frequencies. This changes the optimum design of the line, whether used as a low-loss inductive or capacity reactance or to give high selectivity or high impedance as a resonant line. Design equations for maximum selectivity and maximum impedance are given in this paper, and he shows that the optimum spacing to wire size for a parallel wire line now becomes 6.186, whereas this ratio was 3.6 if radiation resistance is neglected. It is found that Q and Z_s for optimum design are both inversely proportional to the cube root of the frequency, whereas previous analysis showed both increasing as the square of the frequency.

By defining radiation resistance in the accepted manner, Reukema derives expressions for the radiation resistance of parallel-wire lines with various terminations. Only the end results are given in this article and it is difficult to say what method he may have used to arrive at these results. It would appear that the expressions obtained are dependent on line length, however, this becomes vanishingly small when lengths greater than two half-wavelengths are considered. The result he gets for an open circuit line seems to be in agreement with that of previous writers.

The most recent attack on the problem was made by King(13) in 1945. He shows that the radiation resistance for a line shorted at both ends and an even number of half-wavelengths in length is $120\left(\frac{\pi d}{\lambda}\right)^2$ ohms, including the effect of the terminations. This is the same result obtained by previous writers but for the case of a line open-circuited at both ends. King also seems to think that the radiation resistance of the same shorted line, but an odd number of half-wavelengths in length is proportional to $\left(\frac{\pi d}{\lambda}\right)^4$ instead of $\left(\frac{\pi d}{\lambda}\right)^2$ as previous writers have obtained.

Table 1 is a complete summary of the works of most of the authors, who have analyzed the problem. Practically all the authors agree that the radiation resistance of a line, open-circuited at both ends, is given by

$$R_r = 120 \left(\frac{\pi d}{\lambda}\right)^2 .$$

Only four writers have considered the case of a line terminated in a short circuit at both ends. The results of these four authors differ in each case and it is probable that all four are in error.

Author	Cases Considered	Results Obtained	Comments
Mie.	Endless parallel-wire lines.	Concluded that no radiation existed from the line.	
Steinmetz	Long parallel-wire lines.	$R_\lambda = \frac{8\pi^2 f^2 d}{c} \times 10^{-9}$ ohms where d= wire separation λ= wavelength c= 3×10^{10} cm./sec.	Said that this radiation resistance is evenly distributed along the line and can be considered a new line parameter.
Carson	Parallel-wire line terminated in its characteristic impedance.	$R_\lambda = 240\left(\frac{\pi d}{\lambda}\right)^2\left(1 - \frac{\sin 4\pi L/\lambda}{4\pi L/\lambda}\right)$ where d= wire spacing λ= wavelength L= length of line.	The bracket in the expression is equal to unity when lengths greater than a few wavelengths are considered. Does not include the effect of the shorting bars.
Manneback	Parallel-wire lines open-circuited at both ends and an even number of half-wavelengths long.	$R_\lambda = 120\,\pi^2\left(\frac{d}{\lambda}\right)^2$ where d and are defined above.	Concludes that radiation losses are negligible compared to the heat losses of the line.
Pistolkors	Parallel-wire lines open-circuited at both ends and an even number of half-wavelengths long.	$R_\lambda = 120\,\pi^2\left(\frac{d}{\lambda}\right)^2$	Actually considers the case of the pair of wires excited in phase, however, making the necessary changes yields the same result as Manneback obtained.
Carter	Considered the same case as Pistolkors. Assumes that the wires are excited in phase. Changing the results to suit the transmission line case, his method gives the following expression.	$R_\lambda = 60\left[E + \log_e 2mL - C_i(2mL) + 2C_i(md) - C_i\, m\left(\sqrt{d^2+L^2} + L\right) - C_i\, m\left(\sqrt{d^2+L^2} - L\right)\right]$ where E= 0.5772(Euler's Constant), m = $2\pi/\lambda$, and Ci is the cosine integral function and can be expressed as an infinite series.	When L = $2\pi/\lambda$ and $(d/\lambda) << 1$ are introduced in this equation, and if the cosine integral is expressed in the form of an infinite series, then the expression becomes $R_\lambda = 120\,\pi^2\left(\frac{d}{\lambda}\right)^2$
Sterba and Feldman	1. Parallel - wire line terminated in its characteristic impedance. 2. Line open - circuited at both ends. 3. Line short - circuited at both ends.	1. $R_\lambda = 160\,\pi^2\left(\frac{d}{\lambda}\right)^2$ 2. $R_\lambda = 120\,\pi^2\left(\frac{d}{\lambda}\right)^2$ 3. $R_\lambda = 40\,\pi^2\left(\frac{d}{\lambda}\right)^2$	These expressions originate from an expression similar to the one obtained by Carter. The expression for the line shorted at both ends, does not include the effect of the shorting bars.
Whitmer	Case of the parallel - wire line open - circuited at both ends.	$R_\lambda = 120\,\pi^2\left(\frac{d}{\lambda}\right)^2$	This expression again owes its origin to an expression similar to that obtained by Carter.
Reukema	1. Line open - circuited at one end and fed at the other end. 2. Line short - circuited at one end and fed at the other end.	1. $R_\lambda = 120\left(\frac{\pi d}{\lambda}\right)^2\left[1 + \left(\frac{1}{3} + \frac{1}{(\beta x)^2}\right)\sin^2\beta x\right]$ 2. $R_\lambda = \left(\frac{\pi d}{\lambda}\right)^2\left\{80 + 120\left[\frac{1}{3} - \frac{2}{(\beta x)^2} + \left(\frac{1}{3} + \frac{1}{(\beta x)^2}\right)\sin^2\beta x + \frac{1}{(\beta x)^3}\sin 2\beta x\right]\right\}$ where β = $2\pi/\lambda$ and x = length of line.	These expressions seem to be dependent on line length, but if lines longer than one half - wavelength are considered, and if the lengths are multiples of a half - wavelength, then the results reduce to those obtained by Sterba and Feldman.
King	Lines shorted at both ends and an even number of half - wavelengths long.	$R_\lambda = 120\left(\frac{\pi d}{\lambda}\right)^2$	This result includes the effect of the shorting bars.

Table 1 - A summary of the works of several authors.

II A COMPARISON WITH THE WORK OF HOY AND CARR

In the spring of 1950, Hoy[14] and Carr[15] each had presented a treatise in partial fulfillment for the degree of Master of Engineering at McGill University. Hoy had studied the effect of radiation on the properties of resonant short-circuited parallel-wire lines, while Carr had done the same for open-circuited. Both had used the Chipman method inaugurated by Dr. Chipman[16] in 1939. Since the Chipman method is very convenient for measurements of this type, it was also used in the present thesis.

It was suspected that the work of Hoy and Carr was considerably in error for several reasons and it was decided that their work should be repeated and checked. The main reasons why their work was repeated are ennumerated and discussed in the following few pages.

1. One of the chief reasons for the repetition is that a detector, amplifier and recording meter far superior to the detector system used before was employed. Hoy's and Carr's detector system consisted mainly of a small detector coupling loop which detected the resonant current at the infinite plane end of the line, a thermocouple, a chopper amplifier and an Esterline Angus 0-5 milliamper recording meter. Since it was necessary to stay on the linear portion of the thermocouple characteristic curve a chopper amplifier had to be used to amplify the D.C. output to a point where it could be measured by the recording meter. This chopper amplifier was not linear and there was evidence of distortion for the higher gain positions. This, therefore, was probably the unit which gave the largest error in the final results. The chart coordinates of the Esterline Angus, recording meter are not rectangular therefore it was not immediately evident

whether the resonance curve was distorted in any way. The detector system used this time consisted of the same detector coupling loop and thermocouple, however, the output from the thermocouple was fed directly into a Leeds and Northrop Speedomax recording meter. This meter has an amplifier enclosed in the meter itself and the measuring circuit is of the potentiometer type. It has a 0-200 microvolt scale with a chart that uses rectangular coordinates, thus making it possible to notice any distortion in the resonance curve immediately.

2. Nearly all of the theory on radiation resistance from parallel-wire lines assumes that the lines have vanishingly small radii in comparison with the wavelength. The two-wire line on which Hoy and Carr had made their measurements had a diameter of three-eighths of an inch thereby disregarding this assumption.

3. The shorting bar that was used to terminate the short-circuited line was improper. Theory states that the shorting bar should be of the same material, size and shape as a small section of the line itself. Hoy had used a brass shorting bar 3/4 x 3/4 x 1 3/4 inches in dimension. In addition to this, two screws were placed at the ends of the shorting bar in order that it could be clamped rigidly to the line. This made the overall length of the shorting bar slightly more than two inches, whereas the spacing between the wires was only one inch. Technically speaking this would make the radiation resistance of the shorting bar appreciably different than that given by $80\left(\frac{\pi d}{\lambda}\right)^2$, where d is taken as the separation between the two wires of the line.

4. As the transmission line is driven through resonance the input impedance to the transmission line is constantly changed. This impedance

change may or may not be reflected back to the oscillator through the power input loop. In order that the oscillator may see the same impedance regardless of the length of the transmission line, it is common practice to use a substantial length of high loss cable between the oscillator and the power input loop. Hoy and Carr had used a type of high loss cable to decouple the oscillator, but the length used was not sufficient to prevent possible changes in the oscillator conditions.

5. When a transmission line is excited by a power input coupling loop two possible modes of excitation may exist on the line. The first is the transmission-line mode which has currents flowing in opposite directions in the two wires, the other is the antenna mode in which case the current in the two wires flows in the same direction. If the line is to be used as a transmission line then only the first mode is desirable. This is usually achieved by placing the power input loop in such a position that a maximum of the transmission-line mode and a minimum of the antenna mode will exist on the line. The amount of the antenna mode present can be determined by taking a resonance curve when only one wire of the two-wire line is tested. Unfortunately the work of Hoy and Carr did not offer any check data on the magnitude of excitation of the antenna mode. The presence of the undesirable antenna mode would give considerable error in the measurements if it were present to any great extent. This is due to the fact that the selectivity of the antenna mode is a great deal lower, and its resonance curve a great deal wider than that of the transmission-line mode. Thus the presence of a small amount of the antenna mode would have the effect of raising the zero level of the transmission-line mode resonance curve.

6. The presence of the power input and detector loops in the

measuring apparatus introduces additional losses in the system. In order that the losses due to these two coupling loops could be determined, Hoy had taken measurements of the shielded short-circuited line, but this was not done in the proper manner. A true shielded shorted pair also has a shorting plane at the end of the two-wire line thereby providing a path for the current from the transmission line to the shield. Hoy's shielded shorted-line was not a true shielded shorted-pair since no means of conducting the current from the termination to the shield was provided. This would give peculiar end effects and would probably make the losses due to the coupling loops substantially higher.

In considering these possible sources of error, it was decided that the experiment should be repeated, both to attempt to reduce the margin of error and to evaluate the effects of these faults.

III DESCRIPTION OF APPARATUS

(a) General

A complete block diagram of the apparatus is given in Figure 1. Power is fed from the A.C. mains through a constant voltage transformer to the high voltage power supply. This high voltage power supply then supplies the necessary voltage and current for the converted radar transmitter, which in turn feeds the resonant transmission line with the required high frequency energy. The coupled output from the two-wire line is then converted to D.C. by the thermocouple and the D.C. is fed to the Leeds and Northrup patentiometer type recording meter.

A photograph of the apparatus is seen in Figure 2. The synchronous line drive motor with the speed reducing gear is seen in the lower left hand corner of the photograph. The brass rod connecting the speed reducing gear to the micrometer line drive projects vertically from the speed reducing gear. The lower portion of the transmission line is visible just to the right of the brass rod. Proceeding from left to right for the remainder of the picture, we have; the selenium rectifier, which supplies the necessary D.C. energy for the D.C. blower motor of the oscillator, the high frequency radar oscillator with its associated meters placed on top and the high voltage power supply necessary for the operation of the oscillator. The constant voltage transformer placed between the A.C. mains and the high voltage power supply is just visible over the top of the two meters sitting atop the oscillator. Unfortunately it was impossible to get the Leeds and Northrup recording meter in the same photograph, however, the leads leading to the recording meter can be seen crossing the photograph at an angle of forty five degrees. Some of the vertical supports of the structure

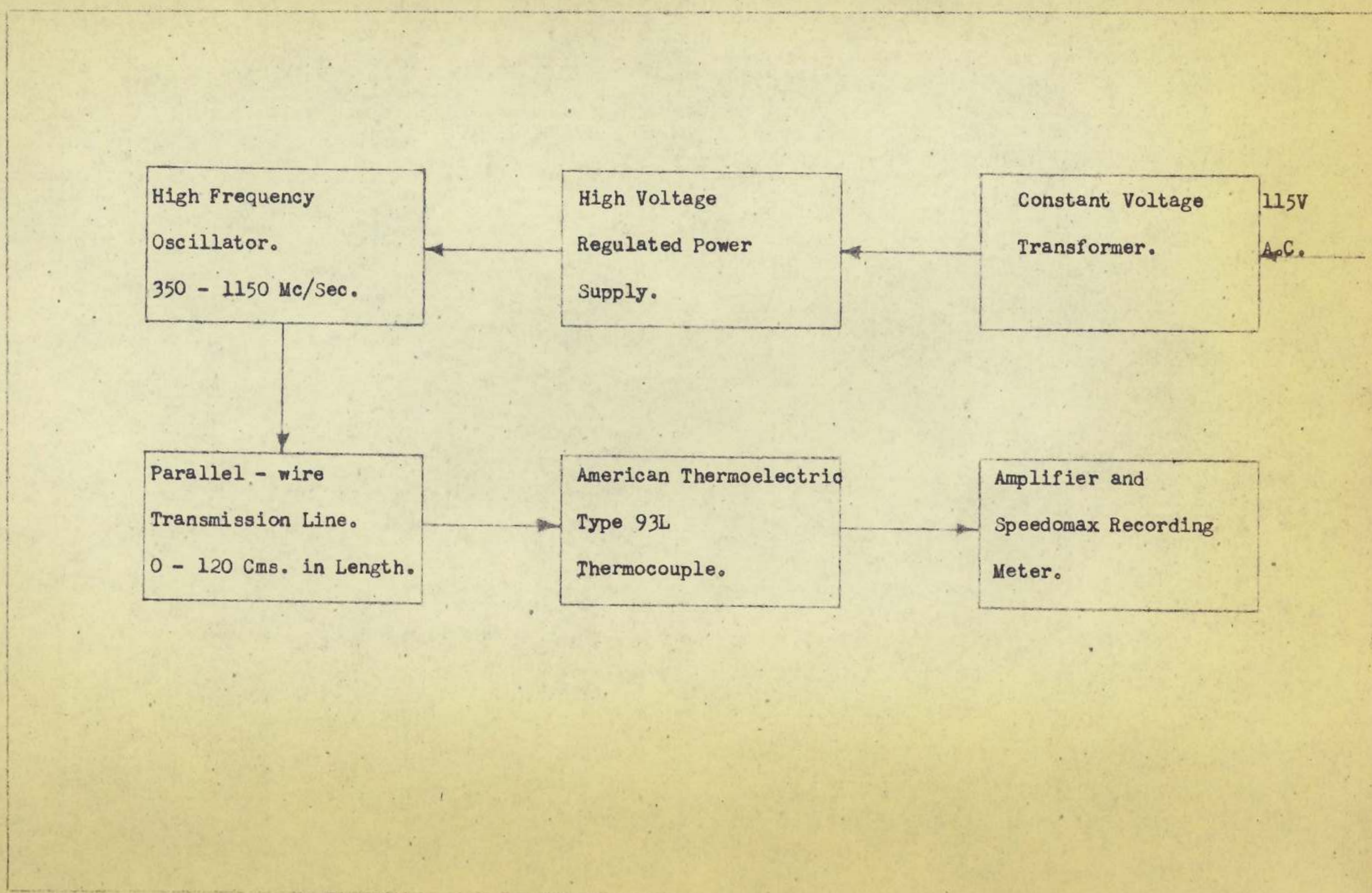


Figure 1 - Block diagram of the Apparatus.

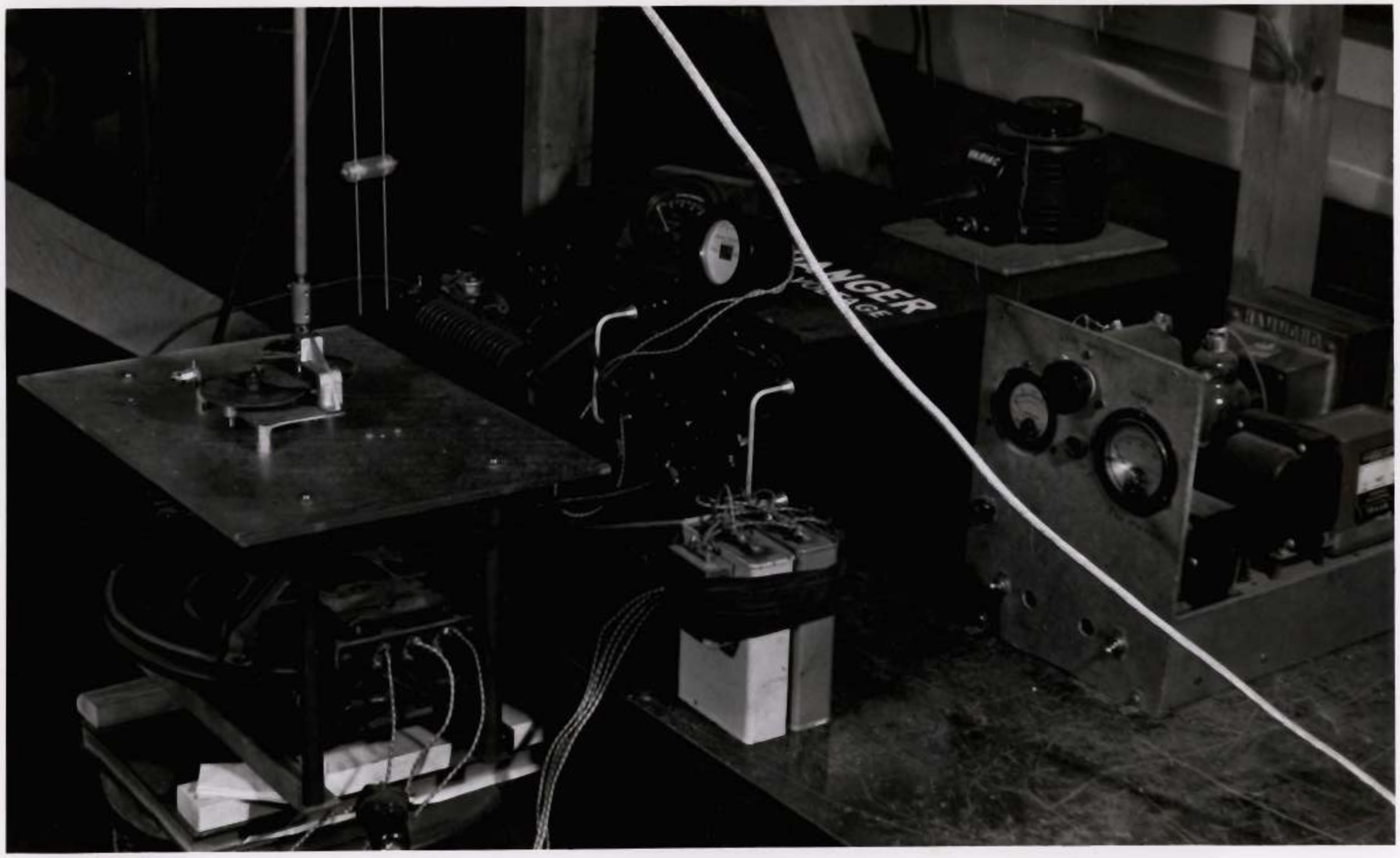


Figure 2.

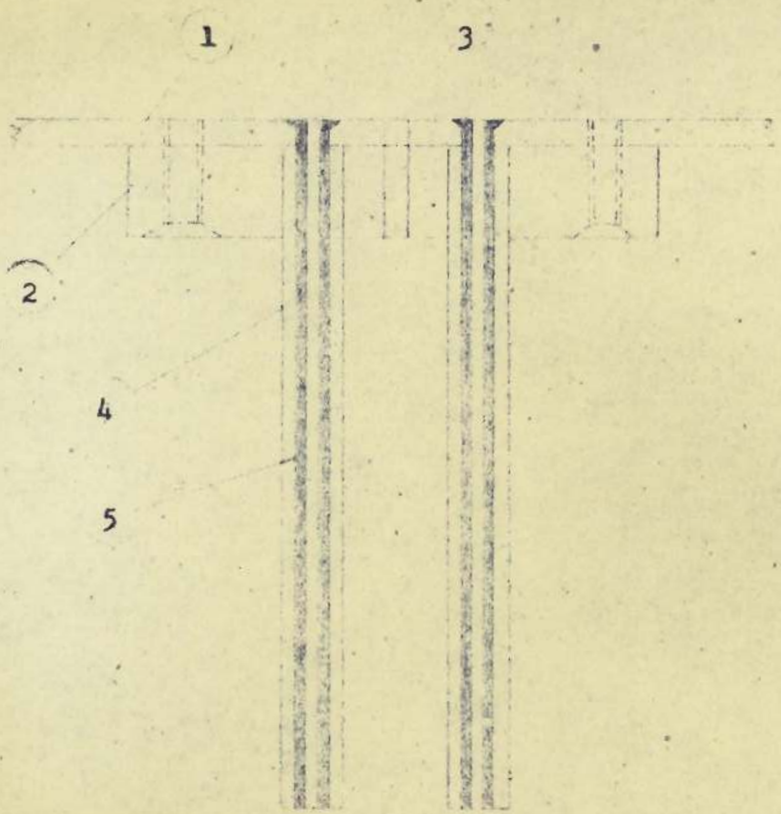
Photograph of most of the apparatus used during operation. All this apparatus is located below the wood supporting structure and the aluminum plane.

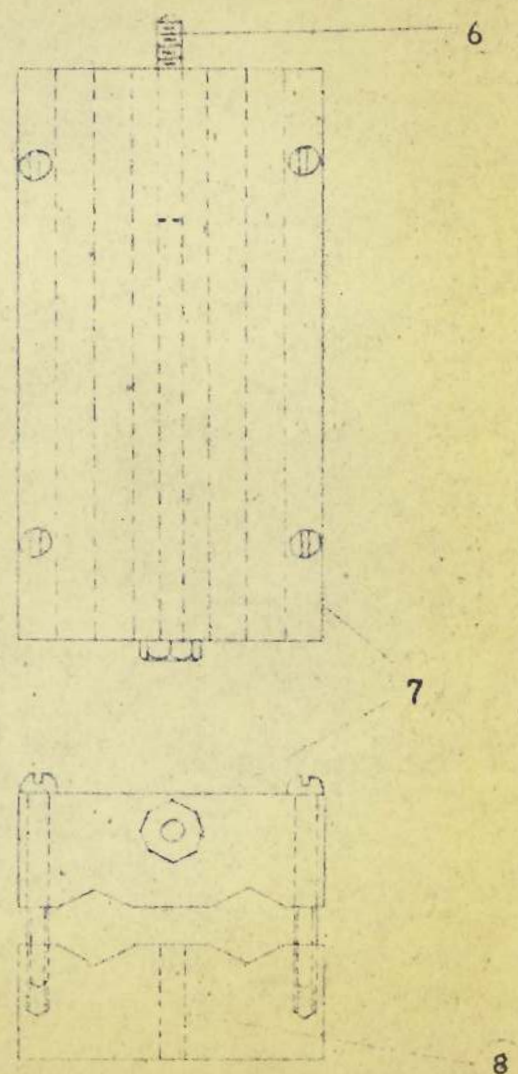
supporting the overhead aluminum plane can be seen in the background of the photograph.

(b) The Transmission Line and Plane Assembly

A drawing of the aluminum plane and transmission line assembly can be seen in Figure 3. The plane consisted of two aluminum plates, one-eighth of an inch thick, eight feet long and three feet wide. These two aluminum plates were fastened side by side on to the supporting structure, thus forming a plane eight feet by six feet. This plane is considered to be an infinite plane and has a coefficient of reflection of unity. The transmission line was composed of two parallel silver rods, 0.101 inches in diameter and spaced one inch apart. It extended above the plane approximately at the center of the infinite plane. The line was sufficiently long so that lengths from 0 to 120 centimeters above the plane could easily be obtained. In order to ensure good contact between the aluminum plane and the silver two-wire line, silver tubing of 0.101 inches inside diameter was placed inside hollow brass tubing of three-eighths inches outside diameter. The silver tubing extended slightly above the plane and was then flattened to fit the countersunk holes in the plane. This is shown in Figure 3. The brass tubing was then clamped in the V-grooves of the line guides and the line guides were in turn fastened to the circular brass block marked 2 in Figure 3.

Referring now to Figure 4 it is seen that the transmission line is clamped to the carriage of the micrometer drive. Since metal screws had a tendency to damage the silver rods slightly, hardwood screws were therefore used to clamp the line firmly in the polystyrene block, which in turn was clamped to the micrometer carriage. The micrometer had a total vertical





1 - 6 x 8 feet Aluminum Plane.
2 - Circular Brass Block 1 inch thick.
3 & 8 - Slots cut for positioning of Coupling Loops.
4 - 3/8 inch outside dia. Brass Tubing.
5 - 0.101 inches inside dia. Silver Tubing.
6 - Bolt for fastening Line Guides to Brass Block 2.
7 - Transmission Line Guides with V - grooves.

Figure 3 - Aluminum Plane and Line Assembly Details.

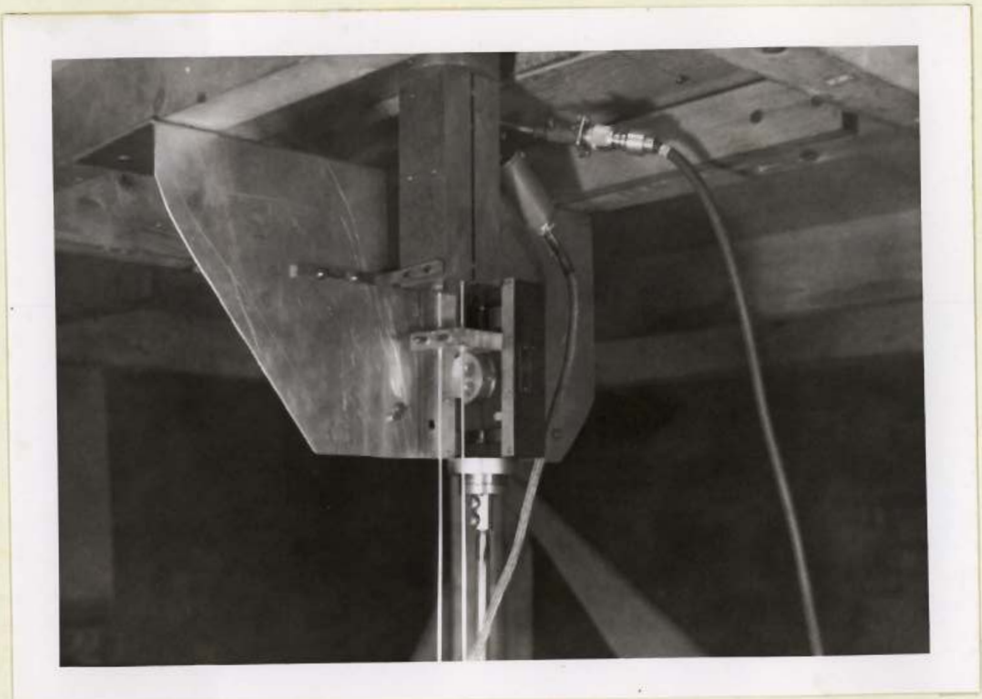


Figure 4.

Closeup of the micrometer line drive and V-groove line guides.

Figure 5.

Closeup of the power input and detector coupling loops.

motion of five centimeters and so it was necessary to first find the approximate resonant length, then clamp the line by means of the wood screws and make fine adjustments with the micrometer carriage. The micrometer scale could be accurately read to 0.001 centimeters, while the resonant length, lo, could only be measured to an accuracy of 0.1 centimeters. The two limits of accuracy are approximately the same because the error in the measurement of the resonant length is considered for lengths much greater than five centimeters in most cases. Attached to the micrometer and extending to the lower edge of the photograph is the brass drive shaft, which is indirectly connected to the line drive motor through a speed reducing gear.

The power input connector can be seen just to the right and close to the top of the line guides. The power is brought in by means of a RG/21U high loss cable, and is coupled to the transmission-line through a small loop. As can be seen from Figure 5, the position of this coupling loop is on a line at right angles to the line joining the centers of the two conductors and about three-quarters of an inch away from each conductor. The detector coupling loop is directly in the center of the two conductors and projects only slightly above the aluminum plane. In order that no direct coupling between the power input loop and the detector loop can exist, it was necessary to elevate the power input loop about three-quarters of an inch above the plane. The resonant current detected by the detector loop is fed to a thermocouple, which is encased in a brass shield and is visible directly below the power input connector in Figure 4.

Since the transmission line was not sufficiently rigid when lengths longer than about twenty centimeters were used, it was necessary to support the two wires in some manner. Figure 6 is a drawing of how this

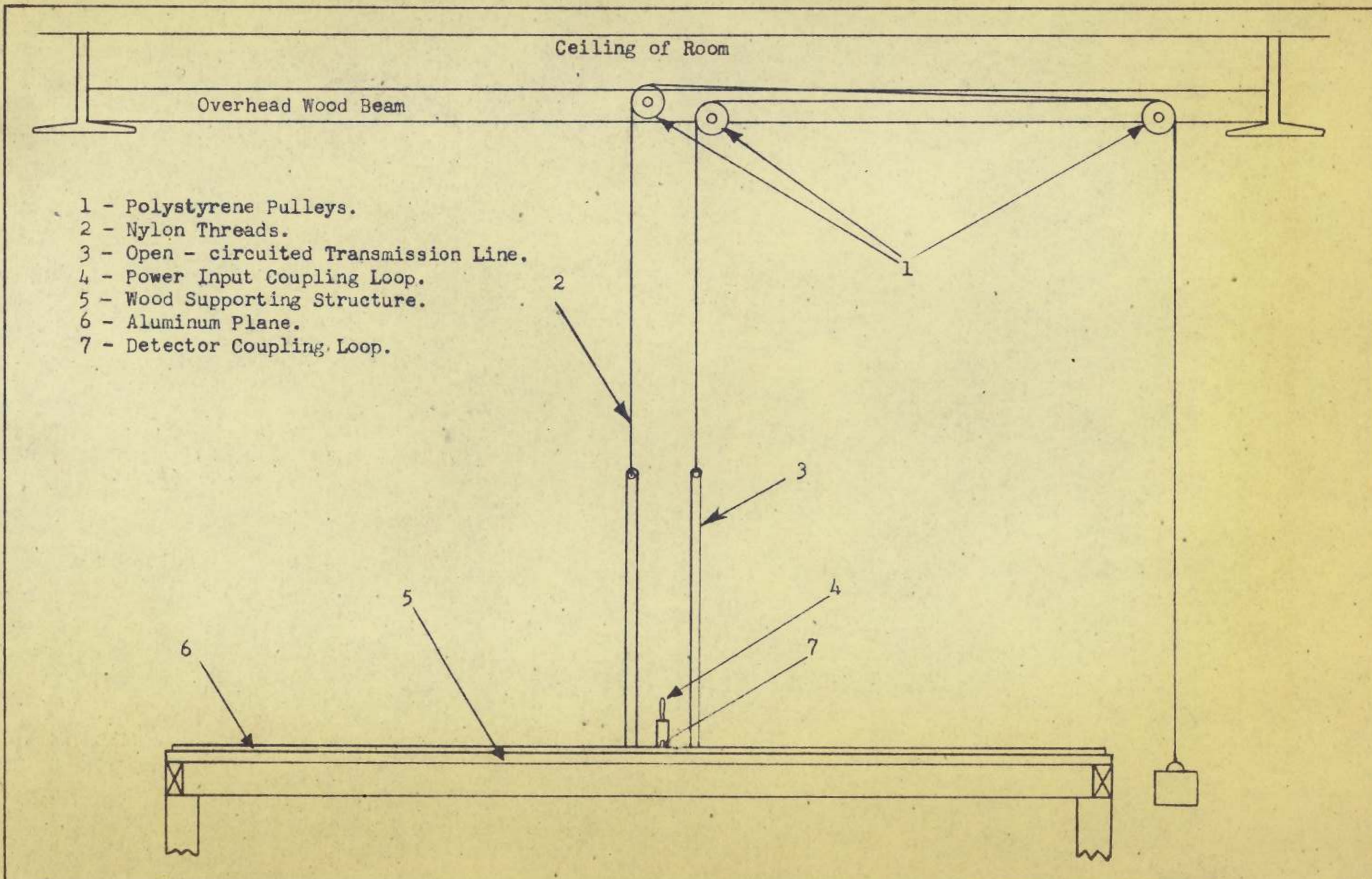


Figure 6 - Method of supporting the Transmission Line.

was achieved. A wood beam was fastened between two I beams, which were part of the ceiling in the room containing the apparatus. Three polystyrene pulleys were then attached to the wood beam as shown in the diagram. The two wires of the open-circuited line were supported by means of two nylon threads which passed over these pulleys and the threads were counter-weighted at the other end. By placing sufficient weight at the other end of the nylon thread the lines could thus be maintained steady and parallel. Only one nylon thread was necessary for the short-circuited line, and it was attached to the center of the shorting bar.

(c) The Line Drive Motor and Speed Reducing Gear

Since it was necessary to have synchronism between the motion of the recording meter chart and that of the transmission line, a motor of the same type as that of the chart drive had to be used. For this reason a phonograph synchronous motor was selected to drive the micrometer drum. The turn-table of the phonograph motor was removed and a speed reducing gear was mechanically coupled to the drive shaft as is shown in Figure 7. The synchronous speed of the motor was 78.2 r.p.m., and the speed of the drive shaft was reduced to 1.072 r.p.m. by means of the reduction gear supported above the motor. This, therefore, meant that the micrometer drum revolved 1.072 turns every minute. One revolution of the micrometer drum corresponds to a total vertical motion of 1 millimeter in the transmission line, thus the length of the line was increased 1.072 millimeters every minute. Most of the resonance curves were taken with a chart speed of 2 inches per minute signifying that 1 inch of paper corresponded to 0.536 millimeters of line length.

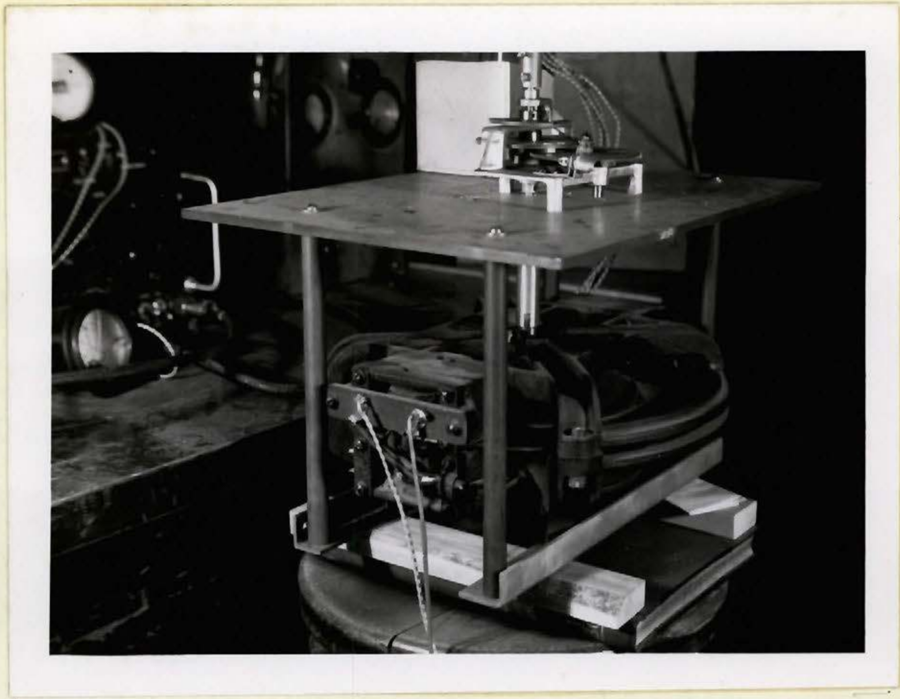


Figure 7.

Photograph of the synchronous line drive motor with the speed reducing gear.

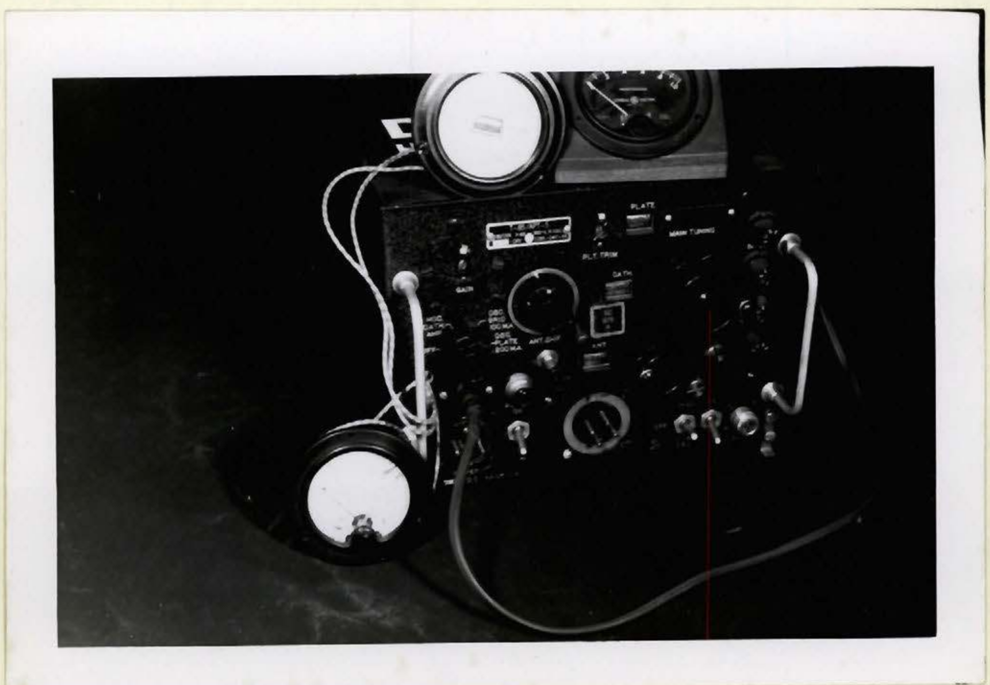


Figure 8.

Oscillator and its associated meters.

(d) The Oscillator

A photograph of the Radar Noise Transmitter used as the oscillator can be seen in Figure 8. This oscillator was a converted T-85/APT-5, with the removal of the photoelectric noise source and video amplifiers comprising the major change. The chief connections on the set are: the oscillator output, the grid current meter jack and the power input connector. The controls used during operation were: the antenna loading control and counter, the main tuning control and counter, the oscillator bias control, the antenna shift control and the power input switches. All these controls and connections are plainly visible in the photograph. The oscillator utilized a 3C22 lighthouse tube with plate and cathode lines for tuning.

The main tuning control was adjusted so that the plate and cathode lines could be tuned separately or both at the same time. Position of the plunger in the cathode line determines the mode of operation. For a given setting of the plate plunger, a setting of the cathode plunger will cause operation at a frequency for which the plate tank is 1/4 wave long, this being the first mode. By shifting the cathode plunger in the same direction, oscillation will again occur at a frequency such that the plate tank is 3/4 wave long; this being the third mode. The antenna shift control is set at the "pull" position for frequencies above 900 M_c/sec., and the oscillator bias control adjusts the plate and grid current of the lighthouse tube. Cooling for the 3C22 lighthouse tube is supplied by a D.C. blower motor which obtains its power through a selenium rectifier built for this purpose. The filament of the lighthouse tube is supplied through the original transformers of the set, while the high voltage of the plate is supplied from a high voltage regulated power supply. The best operating range of the oscillator was found to be between 350-1150 M_c/sec.

The frequency was measured by two different units. The first was a General Radio Corporation Type 720-A Superheterodyne Frequency meter, whereas the second was a type TS-69A/AP Resonant Cavity Frequency Meter. Both units are visible in Figure 9, and the G.R. Superheterodyne Frequency meter appears at the left of the photograph while the other is at the right of the picture. At no time did the measured frequency differ by more than 2 megacycles between the readings given by each meter. This indicated that the frequency was measured to an accuracy of one half of one percent.

(e) High Voltage Power Supply

The high voltage necessary for the plate of the lighthouse tube was supplied by a regulated power supply capable of supplying 1000 volts at 150 milliamperes. The voltage was regulated by a standard electronic voltage regulator, thus giving a constant voltage for any one setting of the control variac. In order to further assure negligible voltage fluctuations, a constant voltage transformer was interposed between the A.C. mains and the regulated power supply. A photograph of the power supply may be observed in Figure 10.

(f) Power Input and Detector Loops

The unit by which the power was fed to the transmission line is depicted in Figure 11. Power was conducted from the oscillator by means of a high loss cable which was terminated in a small coupling loop at the end. The coaxial high loss cable was a RG/21U type with an attenuation of 46 db per 100 feet at 1000 M_c/sec. The power level to the transmission line was controlled through the use of various lengths of high loss cable. The shortest length of high loss cable used was 20 feet, indicating that sufficient

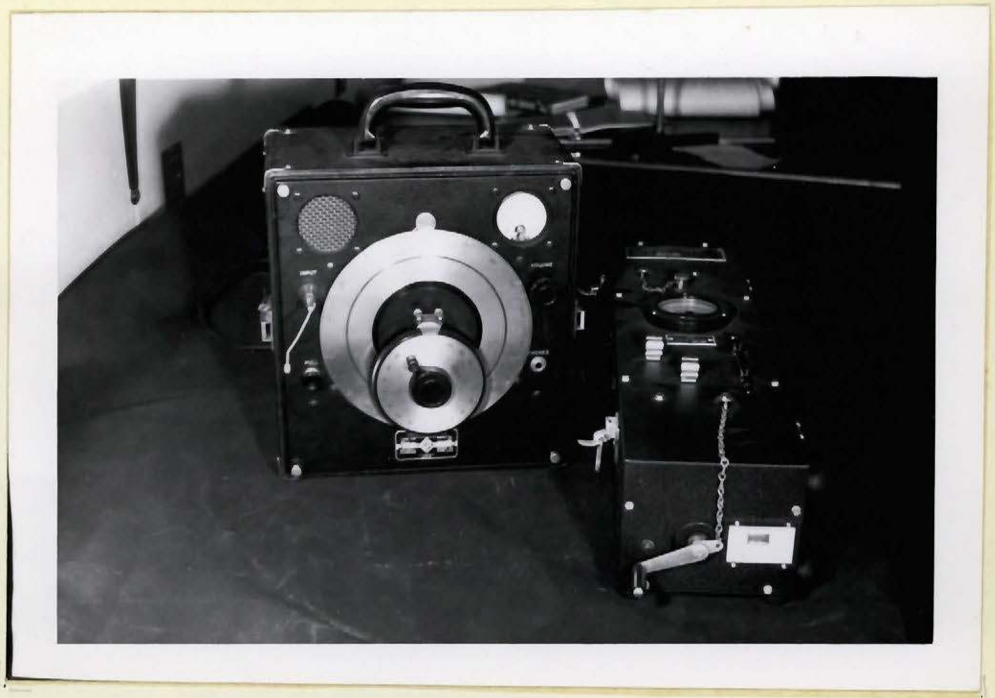


Figure 9.

The two frequency measuring units.

Figure 10.

High voltage regulated power supply.

decoupling between the oscillator and the two-wire line existed at all times. Sufficient lengths of high loss cable were cut so that any length between 5 and 375 feet could be attained by joining several pieces together.

The thermocouple and detector coupling loop can be observed in Figure 12. As can be seen in the photograph, the thermocouple was encased in a brass shield so that only the small coupling loop at the end of the shield picked up the energy from the transmission line. The thermocouple itself was an American Thermoelectric Co. Ultra-High Frequency Thermocouple, Type 93L. Its heater resistance was 600 ohms whereas its couple resistance was 12 ohms. Since this thermocouple is a square low detector, the half power points of the resonance curve would therefore appear half way between the peak and the zero level of the voltage resonance curve. A shielded pair was used to conduct the D.C. from the thermocouple to the Speedomax recording meter.

(g) The Speedomax Recording Meter

The measuring circuit of this recorder is fundamentally a potentiometer, with filtering and damping circuit and amplifier interposed between the thermocouple and the measuring circuit. The D.C. off-balance between the thermocouple and the measuring circuit is first passed through a filter to eliminate any A.C. pick-up in the thermocouple leads and is then fed through a synchronous vibrator type converter to an input transformer. The output of this transformer is then passed through three stages of amplification to a power tube which in turn controls a two phase motor that drives the measuring circuit slide wire. One phase of this motor is connected across the line, whereas the second phase, or control winding, is energized by the amplifier. The phase relationship between the two windings of this

Figure 11.

Power input coupling loop.

Figure 12.

Detector coupling loop and thermocouple.

motor is such that the motor will run in either direction, depending upon the polarity of the off-balance D.C. between the thermocouple and the measuring circuit. To prevent overshoot or coasting of the motor, the same circuit used for filtering is also used for damping purposes. Photographs of the recording meter are given in Figures 13 and 14, while a complete circuit diagram appears in Figure 15.

The recorder measures the voltage across the output terminals of the thermocouple and has a full scale reading of 0-200 microvolts. The chart drive motor is a synchronous induction motor and gears are available to give paper speeds of 20, 10, 5, 2, 1 and 1/2 inches per minute. The fact that both the line drive motor and the chart drive motor are synchronous motors makes it possible to obtain a constant relationship between the motion of the transmission line and that of the chart on which the readings are recorded.

Returning to Figure 13 it is observed that the output of the thermocouple is brought in to the recording meter through the input connector at the lower left, whereas the A.C. power necessary to operate the meter is brought in through the connector at the lower right of the wood panel. The center connector visible in Figure 13 was used as the source of A.C. power for the synchronous line drive motor. Thus both the line drive motor and the recording meter could be set operating by means of switches located on the panel mounting of the recording meter. Careful inspection will reveal that a resonance curve has just been obtained on the paper chart. Figure 14 shows some of the internal parts of the recording meter. The circular disc at the center of the photograph contains the potentiometer slide wire and the base of the amplifier section can be seen at the bottom of the outer case.

Figure 13.

Speedomax Polentiometer Type Recording Meter.

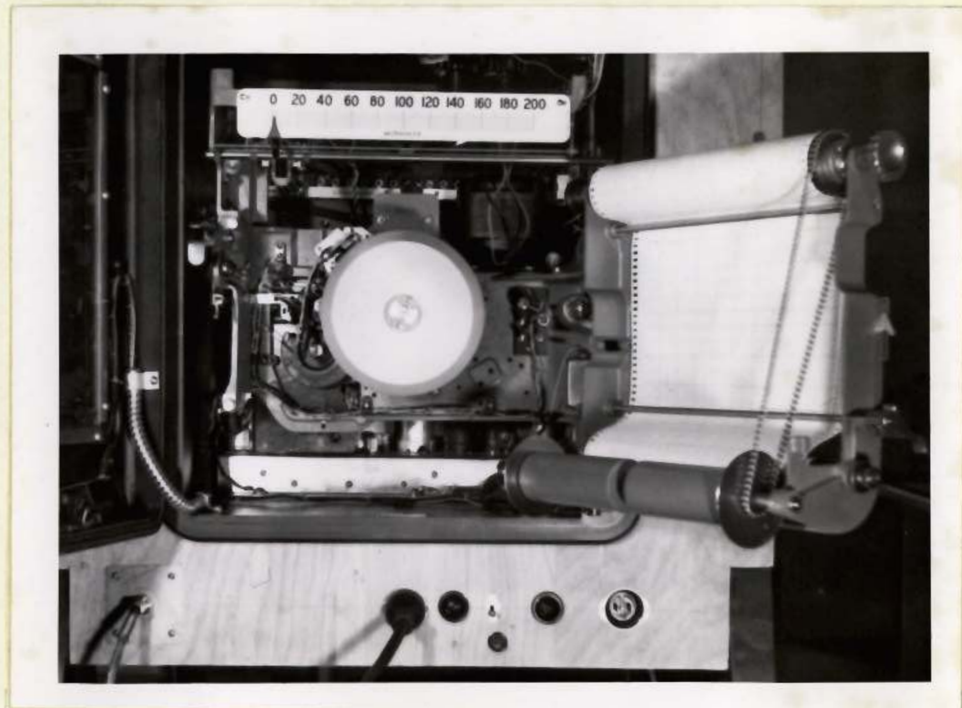


Figure 14.

Some of the working parts of the Speedomax Recording Meter.

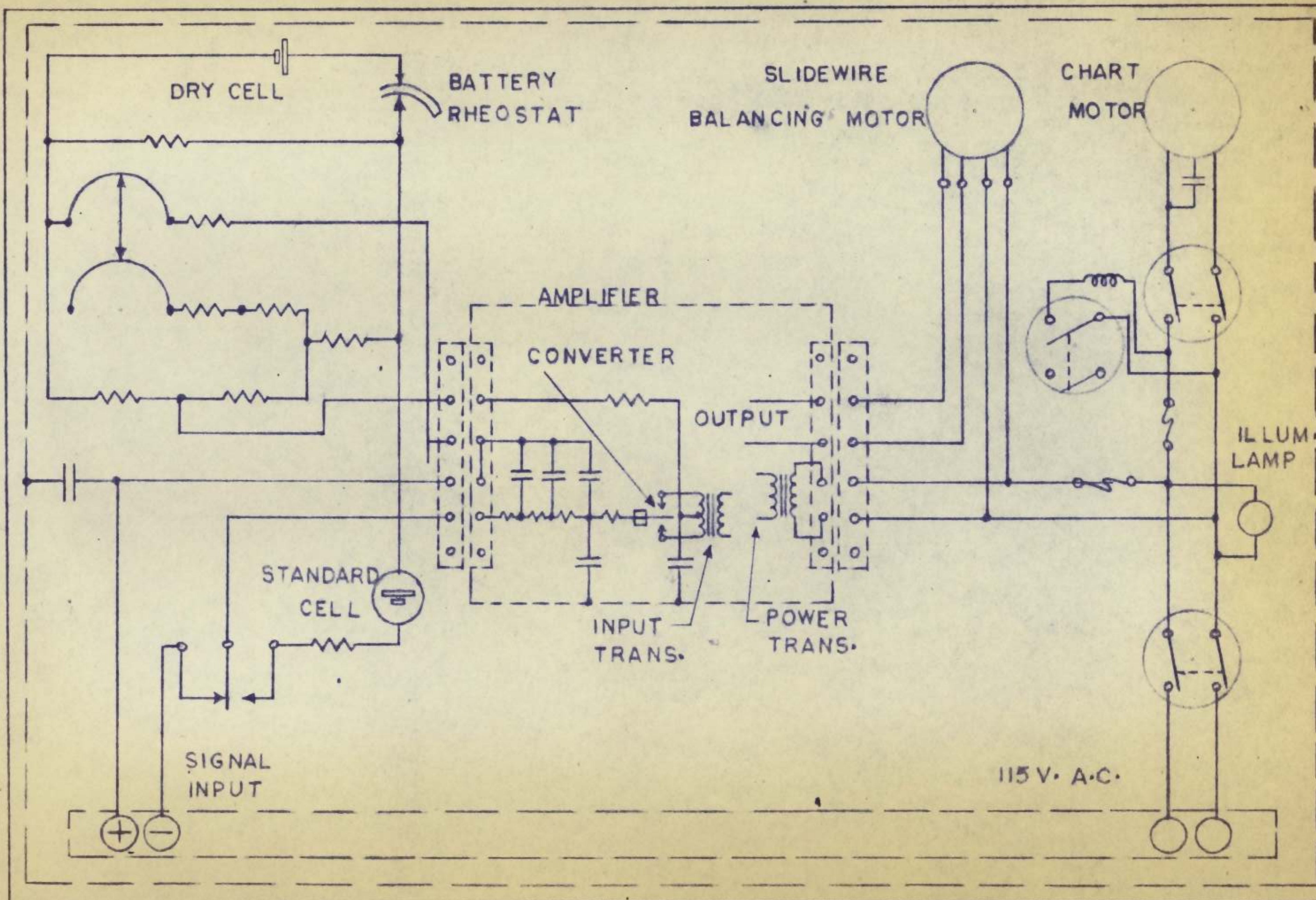


Figure 15 - Circuit diagram of the Speedomax Recording Meter.

IV MEASUREMENTS AND RESULTS

(a) Theory of Measurements.

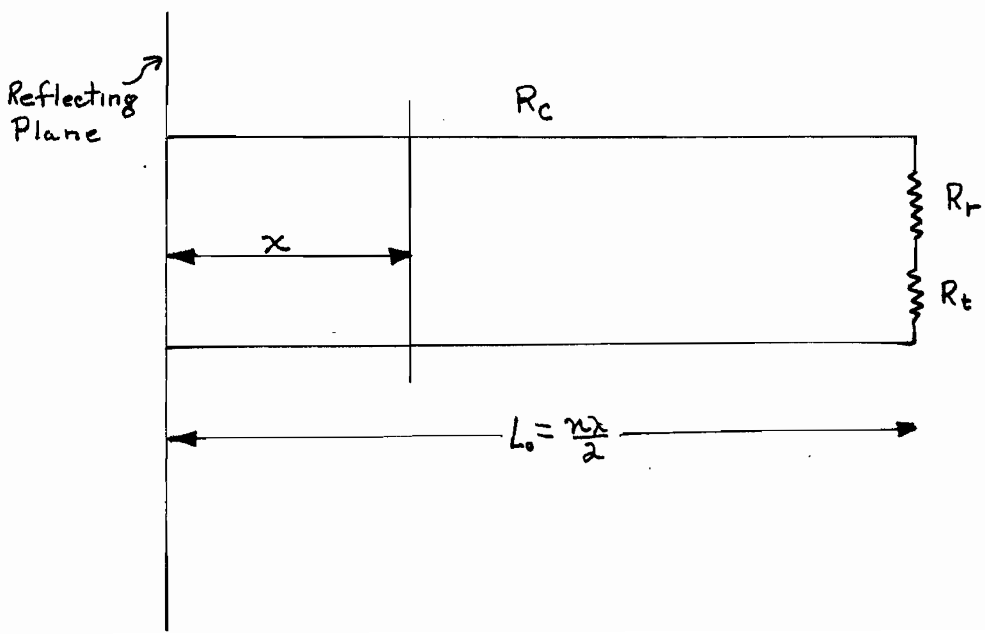
The selectivity factor Q of any resonant circuit is defined as,

$$Q = 2\pi f \frac{\text{Peak energy stored in the circuit}}{\text{Average power lost in the circuit}}, \qquad (1)$$

where f is the frequency in cycles per second. Consider now the case of the parallel-wire shorted transmission line as is shown in the figure below. Let us assume that any losses due to radiation and the effects of the power input and detector loops, will appear as series terminal resistances R_r and R_t respectively. Also consider the case when the line is above an infinite perfectly conducting plane. This plane is considered to have a coefficient of reflection of unity. The transmission line also has ohmic resistance R_c, which is distributed along the length of the line. Since this is a shorted line, the current at any point on the line is then given by

$$i = I_o \sin\omega t \cos \frac{2\pi}{\lambda} x \qquad (2)$$

where $\omega = 2\pi f$, I_o is the peak current, t is time, λ is wavelength and x is the distance from the infinite plane.



Consider now the peak energy stored in the magnetic field. The inductance of a small differential length of line is Ldx, where L is the inductance per unit length of the two wire line. The peak energy stored in this differential element of length becomes;

$$dW_s = \frac{1}{2}(L\,dx)\,i^2 . \qquad (3)$$

Substituting equation (2) into equation (3), and disregarding the $\sin\omega t$ factor since only peak energy is considered, we obtain upon integration over the entire length

$$W_s = \frac{1}{2}L\int_0^{\frac{n\lambda}{2}}\left[I_0 \cos\frac{2\pi}{\lambda}x\right]^2 dx = \frac{1}{8}n\lambda I_0^2 L \qquad (4)$$

where n is an integer 0, 1, 2 etc.

Now consider the time average power lost in the circuit. The factor $I_0 \sin\omega t$ now will become $\frac{I_0}{\sqrt{2}}$ since a time average is being considered. The power lost is given by;

$$P_L = \int_0^{\frac{n\lambda}{2}} R_c\left[\frac{I_0}{\sqrt{2}}\cos\frac{2\pi}{\lambda}x\right]^2 dx + \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r + \left(\frac{I_0}{\sqrt{2}}\right)^2 R_t \qquad (5)$$

Upon integration expression (5) becomes

$$P_L = \frac{1}{8}n\lambda R_c I_0^2 + \frac{I_0^2}{2}R_r + \frac{I_0^2}{2}R_t \qquad (6)$$

Substituting equations (4) and (6) into equation (1) our expression for Q becomes;

$$Q = 2\pi f\left[\frac{\frac{1}{8}n\lambda L}{\frac{1}{8}n\lambda R_c + \frac{R_r}{2} + \frac{R_t}{2}}\right] \qquad (7)$$

If we now use the fact that the resonant length of the line L_0 is equal to $n\lambda/2$, $\beta = 2\pi/\lambda$ and $\lambda = c/f$ we obtain,

$$Q = \frac{1}{2}\left[\frac{\beta L_0 L c}{\frac{R_c L_0}{2} + R_r + R_t}\right] \qquad (8)$$

where c is the velocity of light and is a constant. By inserting the equations $c = \frac{1}{\sqrt{LC}}$ and $Z_0 = \sqrt{\frac{L}{C}}$,

where C is the capacity per unit length of line and Z_0 is the characteristic impedance of the line, into equation (8) we obtain,

$$Q = \frac{1}{2}\left[\frac{\beta L_o Z_o}{\frac{R_c L_o}{2} + R_r + R_t}\right] \tag{9}$$

This will actually be the measured Q of the unshielded line. Let us therefore, rewrite equation (9) as,

$$Q_{unsh} = \frac{1}{2}\left[\frac{\beta Z_o L_o}{\frac{R_c L_o}{2} + R_r + R_t}\right] \tag{10}$$

Thus if R_c is calculated and R_t and Q are measured, then the radiation resistance will be given by;

$$R_r = \frac{1}{2}\left[\frac{\beta Z_o L_o}{Q_{unsh}} - R_c L_o - 2 R_t\right] \tag{11}$$

The terminal resistance R_t is due to the effect of the two coupling loops on the circuit. If R_r can be eliminated from the equation in some manner, then the losses will only be due to the ohmic resistance R_c and the effects of the power input and detector coupling loops. R_r can be eliminated by making Q measurements on the same line, but with a shield around it so that it cannot radiate. Let us designate the measured selectivity factor of this case as the Q of the shielded line (i.e. Q_{sh}). It mut be remembered, however, that the ohmic resistance and the characteristic impedance of the shielded line will also be different than those of the unshielded line. Let us designate tham as;

R_{c_1} = ohmic resistance of shielded line

Z_{o_1} = characteristic impedance of shielded line.

The selectivity factor of the shielded line as given by equation (9) then becomes;

$$Q_{sh} = \frac{1}{2}\left[\frac{\beta L_o Z_{o_1}}{R_{c_1} L_o/2 + R_t}\right] \tag{12}$$

Rearranging equation (12) we then obtain the series terminal resistance R_t, which was assumed to be due to the losses in the power input and detector loops.

$$R_t = \frac{1}{2}\left[\frac{\beta Z_{0_1} L_0}{Q_{sh}} - R_{c_1} L_0\right] \tag{13}$$

It can be seen from the above analysis that by making two measurements on the same line, (i.e. one on the unshielded line and one on the shielded line) it is possible to calculate the radiation resistance of the unshielded line by means of equations (11) and (13).

(b) Calibration of the Apparatus.

After the apparatus had been assembled it was necessary to make certain tests to determine whether the apparatus was supplying true readings. The causes of faulty readings were ennumerated and each suspected fault was checked.

(1) One of the first tests made, was to determine if the oscillator contained harmonics beside the fundametal frequency desired. To check this the oscillator was turned on and a small antenna was attached to the output of the oscillator. The frequency of the oscillator was measured with the General Radio Corporation superheterodyne frequency meter, which had a frequency range of 100 to 210 Mc/Sec. It was therefore necessary to locate the position of the successive beats on the frequency meter to obtain the frequency output of the oscillator. It was found that for frequencies between 350 and 1150 Mc/Sec. only the fundamental frequency was present. For frequencies lower than 350 and higher than 1150 Mc/Sec., varying amounts of harmonics was detected. This was later reaffirmed when a frequency of 300 Mc/Sec. was attempted. This is therefore, the chief reason why all the measurements were done in the frequency range 350 to 1150 Mc/Sec.

2. The second test made was to determine the relation between the chart speed and the vertical motion of the transmission line. Both the chart speed and the speed of the line drive motor were timed over intervals of fifteen minutes and each time it was discovered that a linear relation existed between the two. It was discovered that with a chart speed of two inches per minute, one inch of paper corresponded to 0.536 millimeters of line length.

3. It was next necessary to determine whether the thermocouple was operating on the linear portion of its characteristic curve. A curve of the operating traits of the thermocouple was drawn and is shown in Figure 16. The graph indicated that the curve is linear up to one milliampere output current, thus indicating that output currents of 1.0 milliamperes or less could be drawn from the thermocouple while still remaining on the linear portion of the curve. In order to obtain readings less than full scale on the recording meter, voltages of less than 200 microvolts have to be measured. This means that the voltage across the output terminals of the thermocouple should be 200 microvolts or less. Since the couple resistance of the thermocouple is 12 ohms, therefore a current of only 16.67 microamps is necessary to give a full scale deflection on the Speedomax recording meter. This is far below the maximum value which can be drawn from the thermocouple and therefore operation on the linear portion of the thermocouple square law characteristic was assured.

4. After the preliminary checks had been completed it was necessary to determine if the resonance curves drawn by the Leeds and

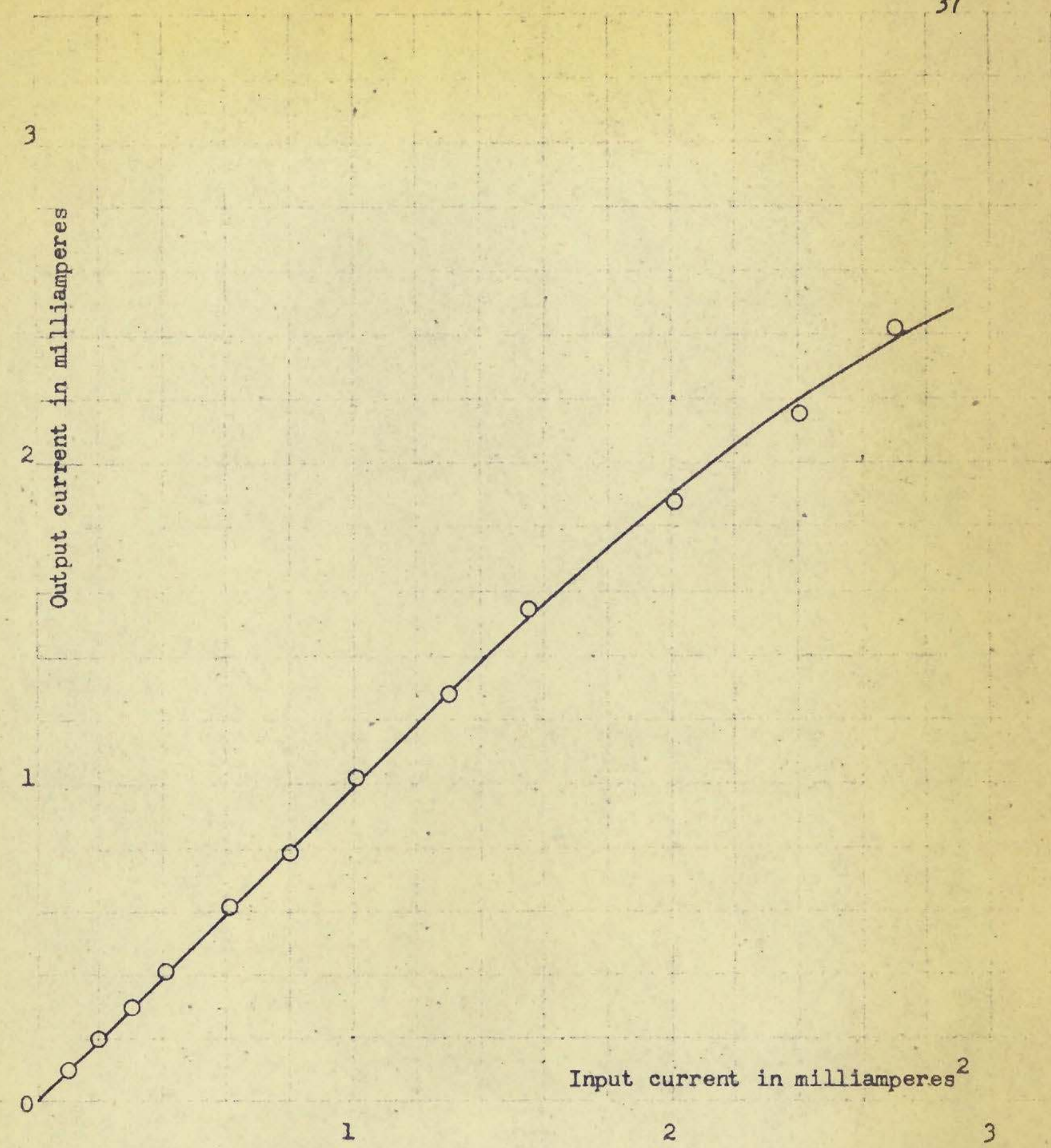


Figure 16

Calibration curve to determine the square - law characteristics of the Thermoelectric Thermocouple.

Northrup Speedomax recording meter were true resonance curves. The method by which this was done is shown in Figure 17. It is known that a voltage resonance curve can be plotted into a straight line if $\sqrt{(V_{max.}/V)^2 - 1}$ is plotted against frequency, where $V_{max.}$ is the voltage at the peak and V is the voltage at any point on the resonance curve. Since the resonance curves were obtained by varying the length of the transmission line, Figure 17 therefore has line length as the absicssa. The resonance curve analyzed was that of a 2/2 wave resonant shorted transmission line at a frequency of 1100 Mc/Sec. The plot of Figure 17 departs very little from a straight line, thereby indicating that the voltage resonance curve taken was a true resonance curve.

(c) Method of Measurement.

In order to determine the variation of radiation resistance with frequency, it was decided to take measurements at five different frequencies covering the full frequency range of the oscillator. At each frequency setting the measurements were to be taken on various lengths of transmission line for both the open and short-circuited lines. It was found that the transmission line was sufficiently long so that lengths from one quarter wavelength up to and including sixteen quarter wavelengths could easily be obtained. In order to determine the losses introduced by the two coupling loops themselves, it was necessary to repeat all the measurements with a brass cylinder shielding the line.

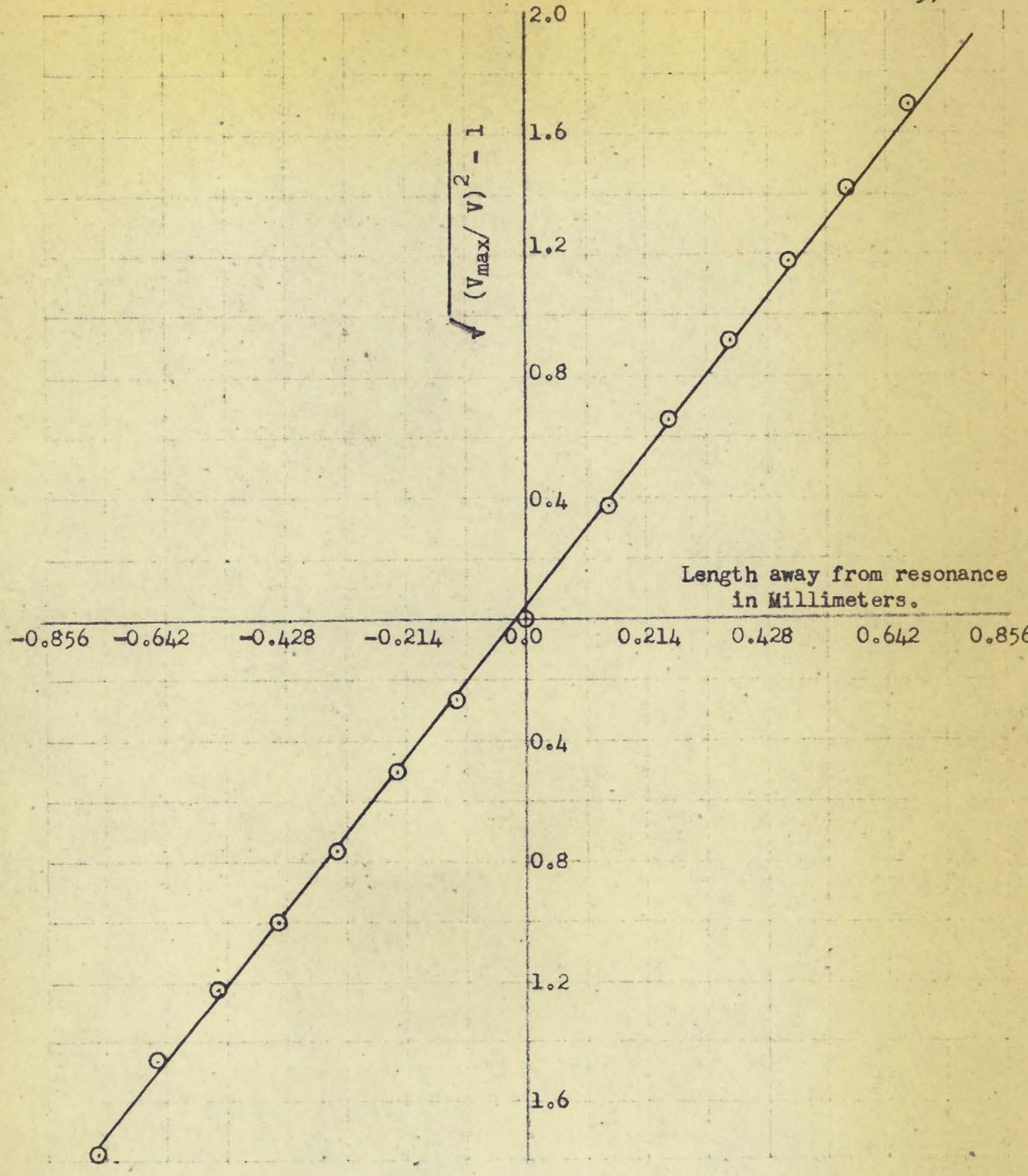


Figure 17.

This graph was plotted to determine whether the resonance curves traced by the meter were true resonance curves. The graph indicates that the resonance curve of the 2/2 wavelength shorted line at a frequency of 1100 Mc/Sec., was a true resonance curve. The resonance curve had a measured Q of 192 and was similar to all the other curves taken.

Once the apparatus was in operation the procedure was reduced to simple and methodical labor. The oscillator was first adjusted to provide power at the required frequency, and the power was fed to the line by means of a high loss cable terminating in the power input coupling loop. As mentioned before, the approximate position of the resonance curve was first found by allowing the line to slide freely in the polystyrene block connecting the two wire line to the micrometer drive. Fine adjustments were made by means of the micrometer drive. When the position of the resonance curve was located, it was frequently necessary to change the amount of high loss cable between the oscillator and the power input loop. By doing this, the amount of power fed to the transmission line was controlled, and the amplitude of the resonance curve could therefore be adjusted. The transmission line was then set at a length slightly below resonance and the reading of the micrometer was taken and marked on the paper chart.

Since the synchronous line drive motor and the recording meter could both be controlled from switches located on the mounting panel of the recording meter, it was easy to control both at the same time. Both were switched on simultaneously and were again switched off when a complete resonance curve had been obtained. The final reading of the micrometer was again recorded on the chart and the length of the line above the aluminum plane was measured with a meter stick. The true resonant length of the transmission line consequently could be obtained by measuring the amount of paper covered between the peak of the resonance curve to the point on the chart where the apparatus was stopped. This measurement was converted to a corresponding increment of transmission line length, and was then subtracted from the measured length of the line in order to obtain the true resonant length.

Typical resonance curves are shown in Figures 18, 19, 20, and 21. Each curve shows a characteristic marker pulse when the apparatus was turned on and again when the apparatus was turned off. This is just another feature of the apparatus so that the points at which the apparatus was started and stopped could be marked automatically. Each marker pulse corresponds to an appropriate reading of the micrometer. This micrometer reading is marked immediately above its corresponding marker pulse. Figures 18 and 19 are resonance curves taken on the open-circuited line when the line was 11/4 wavelengths long. Both resonance curves are taken at a frequency of 696 Mc/Sec., and the only difference between the two is that Figure 18 is that of the unshielded case, while Figure 19 that of the shielded case. The selectivity factor Q is calculated from the expression;

$$Q = \frac{L_0}{2\Delta L} ,$$

where L_0 is the true resonant length in cms., and the $2\Delta L$ is the width of the resonance curve at the half power points, measured in the same units. Due to the fact that the thermocouple detector is a square-law detector, the half power points appear half way between the peak and zero levels of the voltage resonance curve. It is interesting to note that in practically all the resonance curves taken, the zero level appears at the point where the recording meter

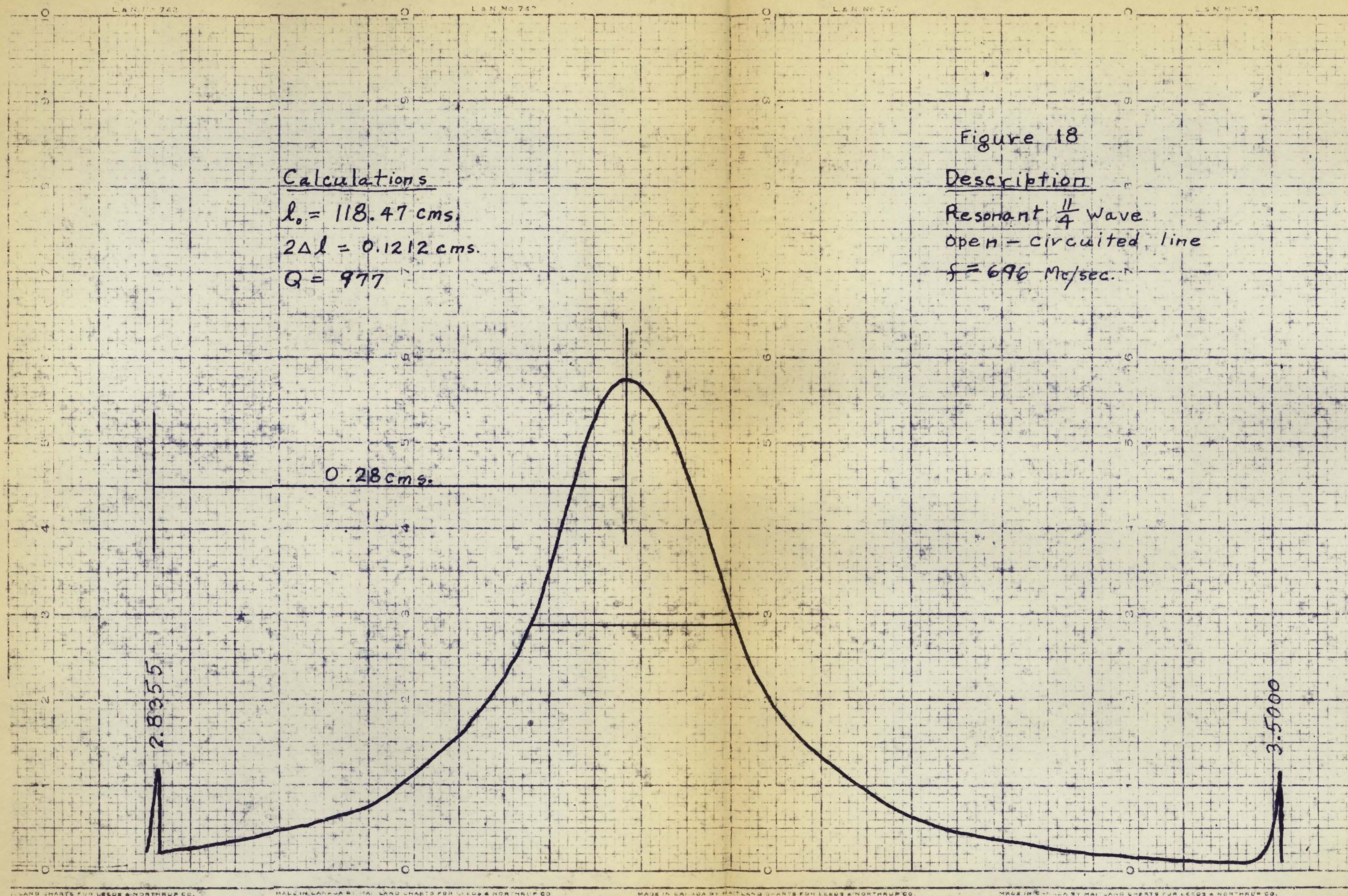


Figure 18
Calculations
l_0 = 118.47 cms.
$2\Delta l$ = 0.1212 cms.
Q = 977
Description
Resonant $\frac{11}{4}$ wave
open - circuited line
f = 696 Mc/sec.
0.28 cms.
2.8355
3.5000

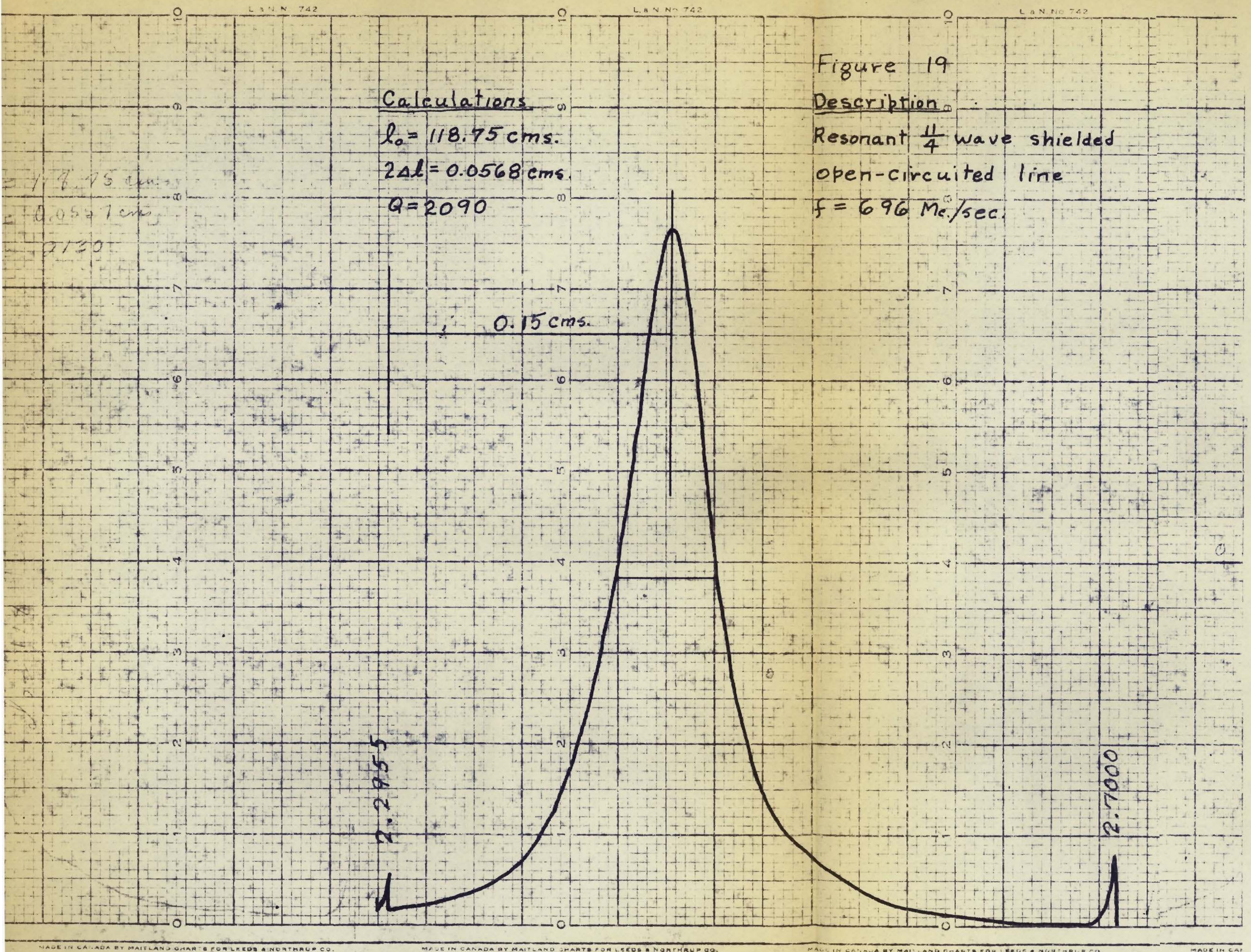


Figure 19
Description
Resonant $\frac{1}{4}$ wave shielded
open-circuited line
f = 696 Mc./sec.
Calculations
l_o = 118.75 cms.
$2\Delta l$ = 0.0568 cms.
Q = 2090
0.15 cms.
2.2955
2.7000

reads zero voltage (i.e. at the bottom of the recording meter chart).

Figures 20 and 21 are examples of resonance curves taken on a short-circuited line, 6/2 wavelengths in length, at a frequency of 918 Mc/Sec. Again the only difference between the two curves is that Figure 20, is the case of the unshielded shorted line whereas Figure 21 that of the shielded case. It can be noted that the width at the half power points of the unshielded resonance curves is of the order of two to three times as great as the width of the shielded curves. This therefore indicates that the losses due to radiation are appreciable thereby lowering the selectivity factor Q of the resonant line. At least two resonance curves were taken at each setting and an average Q was obtained. The average Q value was used when the calculations of radiation resistance were done.

As mentioned in Section 11 a circular conducting plate was placed at the top of the shielded shorted line, thereby providing a path for the current from the transmission line to the shield. This conducting plate was constructed in such a manner so it could easily slide inside the brass shield. In order to further assure that the two wires would not bend in any way while a resonance curve was in progress, sufficient counterweights were placed at the far end of the nylon thread to keep the line freely moving during operation. A diagram of the shielded short-circuited line can be seen in Figure 21A. The brass shield was a circular brass pipe, four inches in diameter and about 140 cms. in length. This arrangement can thus be considered a true shielded shorted pair.

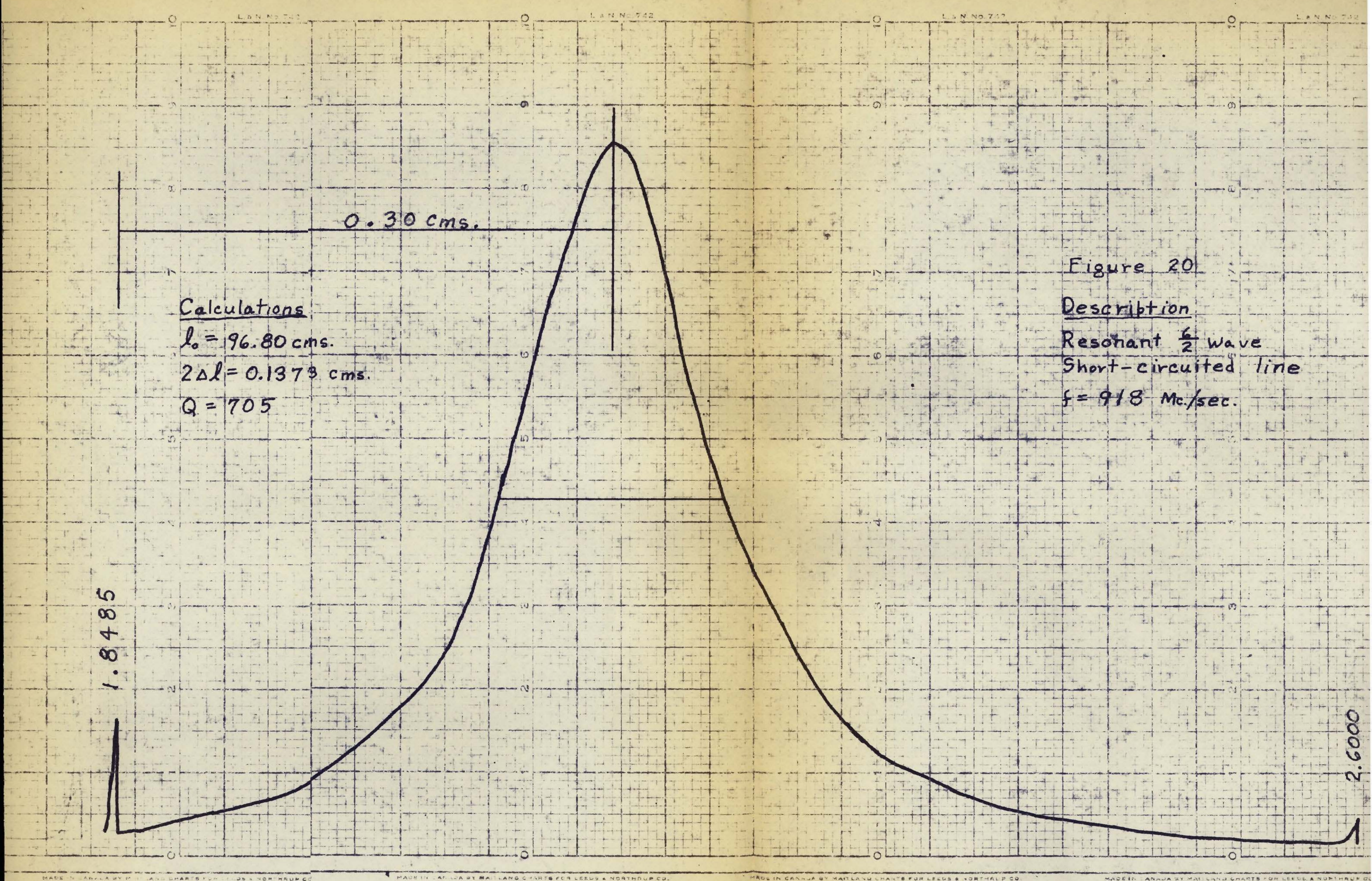

0.30 cms.
Calculations
l_0 = 96.80 cms.
$2\Delta l$ = 0.1373 cms.
Q = 705
1.8485
Figure 20
Description
Resonant $\frac{6}{2}$ wave
Short-circuited line
f = 918 Mc./sec.
2.6000

Figure 21

Calculations

$l_o = 97.76$ cms.

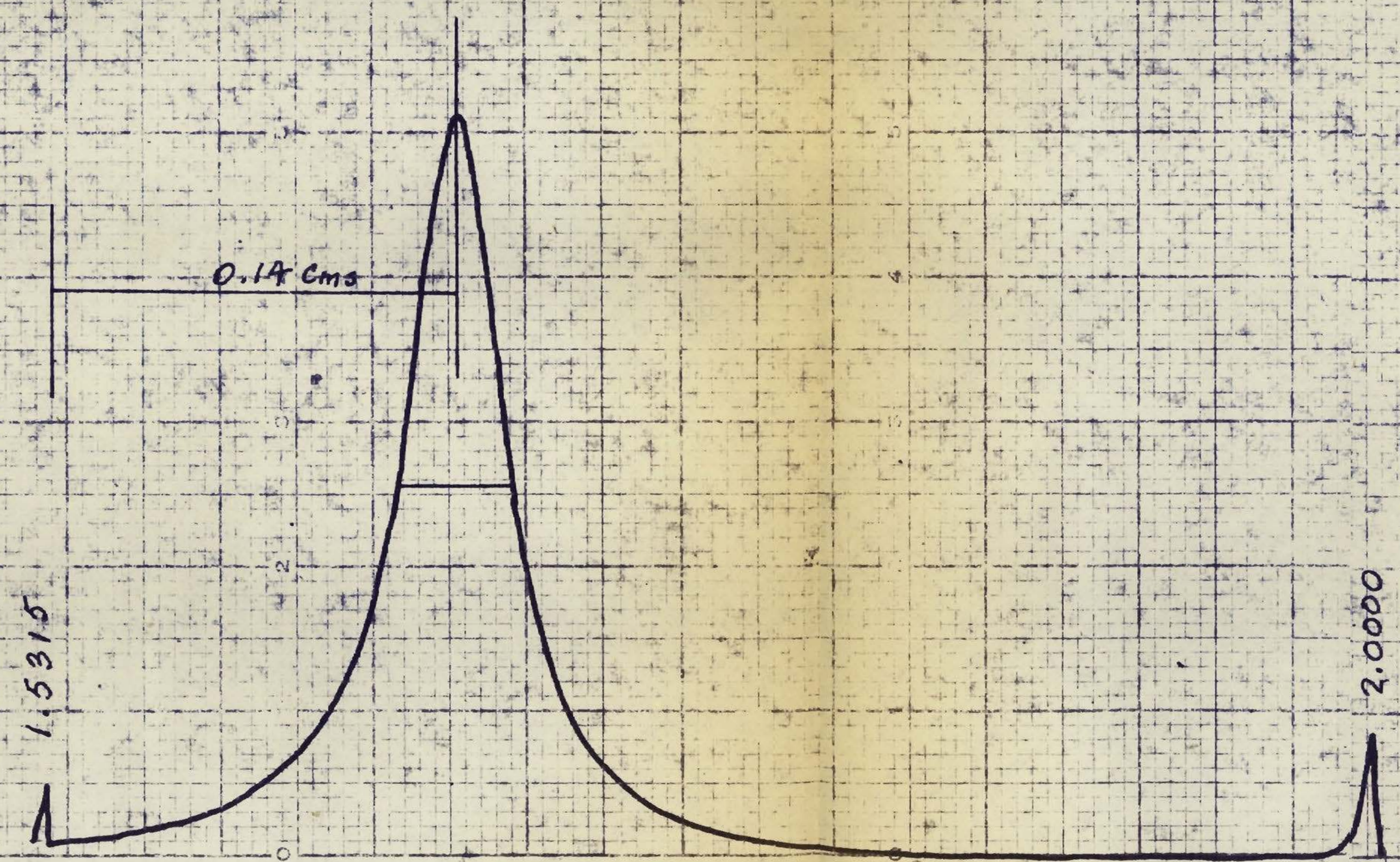
$2\Delta l = 0.0408$ cms.

$Q = 2400$

Description

Resonant $\frac{12}{4}$ wave shielded short-circuited line.

$f = 918$ Mc/sec.



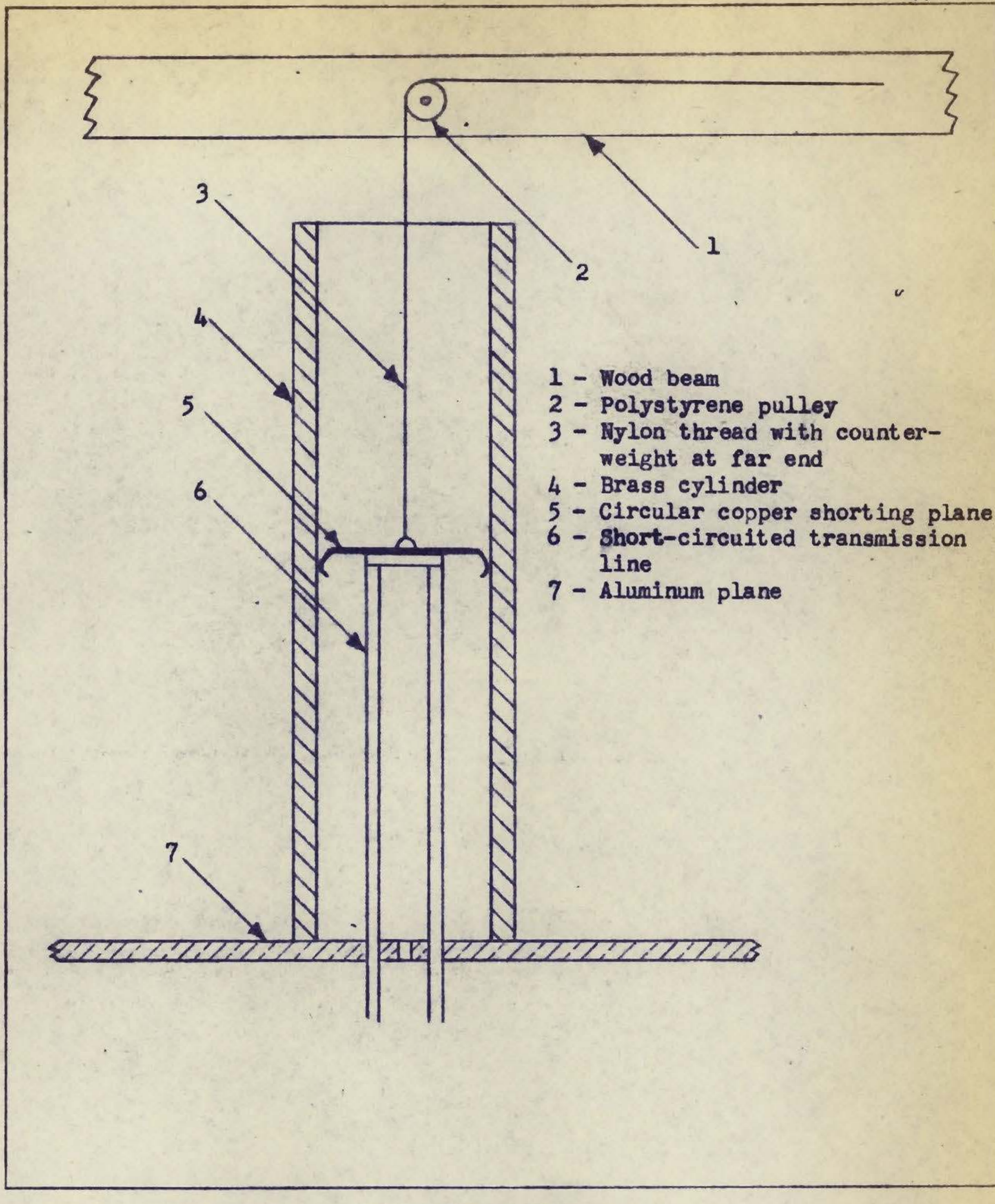


Figure 21A

Details of the shielded shorted pair. The shorting plane at the end of the line was constructed so as to move freely inside the brass shield and maintain good electrical contact with the brass shield.

(d) Results.

By using the theory given in Section lV, Part (a), the necessary calculations had been made, and the results are presented in two forms. The first form consists of a table and a number of graphs showing the measured Q of the transmission line for lines varying in length from one-quarter wavelengths to sixteen-quarter wavelengths. This is done at five different frequencies in the range 350 to 1100 Mc/Sec. The second form consists of the actual calculation of radiation resistance and presentation in graphical form.

The selectivity factor Q has been calculated from the various resonance curves obtained, and the results are given in Table ll. Table ll indicates that at least two readings of the Q were obtained at each line setting and both values are given in the table. The measured resonant length of the line for each corresponding setting of the line is also given. This table gives the results for both the open and short-circuited terminations, and it can be seen that the Q decreases with frequency. This can be seen more clearly if we proceeded horizontally across the table for any one length setting of the line. If frequency is held constant, the selectivity factor generally shows an increase with increasing length. This can be observed if we proceed vertically down the table at a constant frequency. In some cases the general trend is not obeyed, however if we consider only the odd or only the even quarter wavelengths then a general increase is quite evident.

In order to show the increase in measured Q with the

Frequency in Mc/Sec. / Line Length in Quarter - Wavelengths.	398.4		550		696		918		1100	
	Q values	l_o Cms.	Q values	l_o Cms.	Q values	l_o Cms.	Q values	l_o Cms.	Q values	l_o Cms.
1	357 371	18.09	202 202	12.99	125 128	10.27	69 69	7.61	57 57	6.47
2	548 538	37.00	343	26.50	251 249	20.99				
3	627 630	55.70	482 500	40.26	317 319	31.90	204 205	23.90	142	20.27
4	853 898	74.57	637 659	53.70	455 451	42.32	278 281	31.87	195 192	27.19
5	896 928	92.84	598 598	67.34	547 537	53.75	313 306	40.27	252 246	34.38
6	1080 1095	112.32	837 837	81.04	655 640	63.84	400 408	48.11	311 311	41.01
7			777 758	94.30	742 735	75.30	526 527	56.72	336 326	48.38
8			1035 1021	108.31	810 810	85.54	543 535	64.33	367 373	55.00
9			960 960	121.13	840 869	96.90	675 643	73.10	410 427	62.19
10					942 940	106.97	580 638	80.71	496 498	68.91
11					977 944	118.48	715 720	89.34	455 467	76.13
12							710 705	96.96	534 542	82.91
13							807 828	105.45	501 497	90.03
14							796 800	113.31	658 661	96.83
15									593 576	104.31
16									680 680	110.72

Table 11 - The measured resonant length and selectivity or Q of open and short -circuited lines.

number of quarter wavelengths of resonant line more clearly, two graphs have been plotted. Figure 22 is a plot of measured Q values against the number of quarter wavelengths of resonant short-circuited line. This has been done for each of the five frequencies at which the measurements had been taken. The plot clearly indicates the increase in Q with line length. It is also clearly seen that the curve of the Q values at a frequency of 398.4 Mc/Sec. is the uppermost curve, while the curve at 1100 Mc/Sec. is the lowest on the graph. This again indicates that the measured Q of a transmission line decreases with frequency if the length of the line in wavelengths is held constant. This is easily explained by the fact that radiation losses are becoming more excessive and increase as the square of the frequency, thereby lowering the Q of the line. Unfortunately, the variation of the measured Q with constant line length in centimeters had not been investigated, however an indication of this may be obtained by comparing the Q values at two different frequencies and position where the measured line length is practically the same. For example the measured resonant length of the 12/4 shorted line at 918 Mc/Sec. is 96.96 centimeters, while that for a 14/4 shorted line at 1100 Mc/Sec. is 96.83 centimeters. These two measured resonant lengths are practically the same and it is found that there is a drop in the Q value between the setting at 918 Mc/Sec. and that at 1100 Mc/Sec. This may therefore indicate that the selectivity factor Q would drop slightly if frequency were changed, while the actual physical length of the line remained the same.

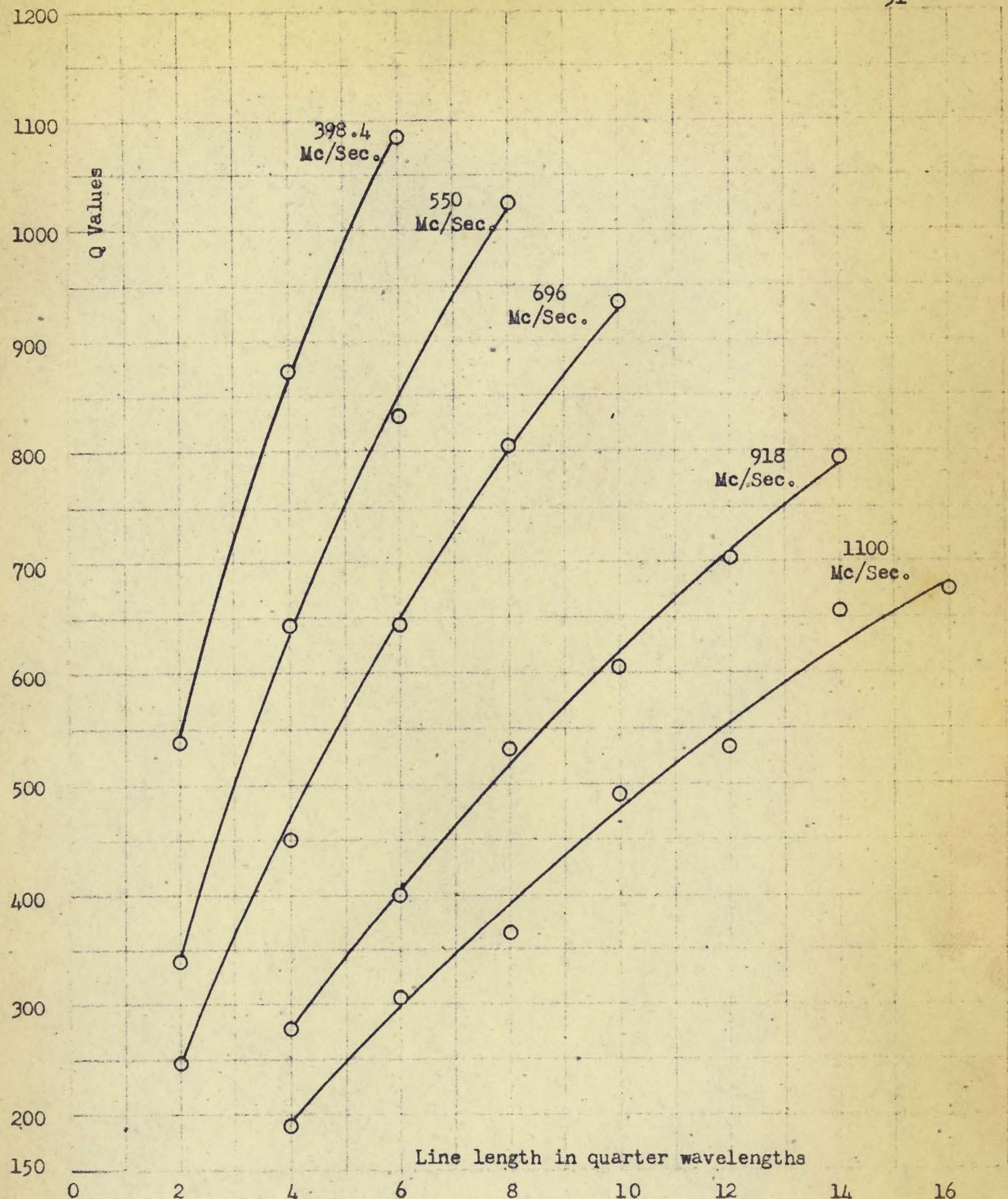


Figure 22
The selectivity or Q of the unshielded short - circuited line as a function of the number of quarter wavelengths.

Figure 23 is a plot of the measured Q values against the number of quarter wavelengths of resonant line for the open-circuited line. Most of the points noted above are also evident from this plot. It is also interesting to note that all the curves would probably start from the origin of the graph if measurements below a quarter wavelength could be obtained.

The calculated radiation resistance of the transmission line was obtained from equations 11 and 13, as given in the theory of Section lV, Part (a). The method used to obtain the final result is shown in Table 111. The constants of the transmission line were calculated and are shown in Appendix 1. The amount of terminal resistance R_t was first calculated by using equation 13. This was done for each of the different shielded line settings obtained at that particular frequency. Since the losses due to the coupling loops is independent of line length but only dependent on frequency, then R_t should be a constant if frequency is held constant. For this reason, the various values of R_t obtained at constant frequency were averaged, and this average together with the ohmic resistance of the line was subtracted from the apparent radiation resistance in order to obtain a true radiation resistance. Table 111 shows the calculated values of radiation resistance. obtained by varying the length of the line at a constant frequency of 918 Mc/Sec. The calculated values of radiation resistance are all practically the same, and therefore it is conclusively proved that radiation resistance is not a function of line length. Tables similar to Table 111 were prepared for each different frequency at which measurements were taken and the values of radiation

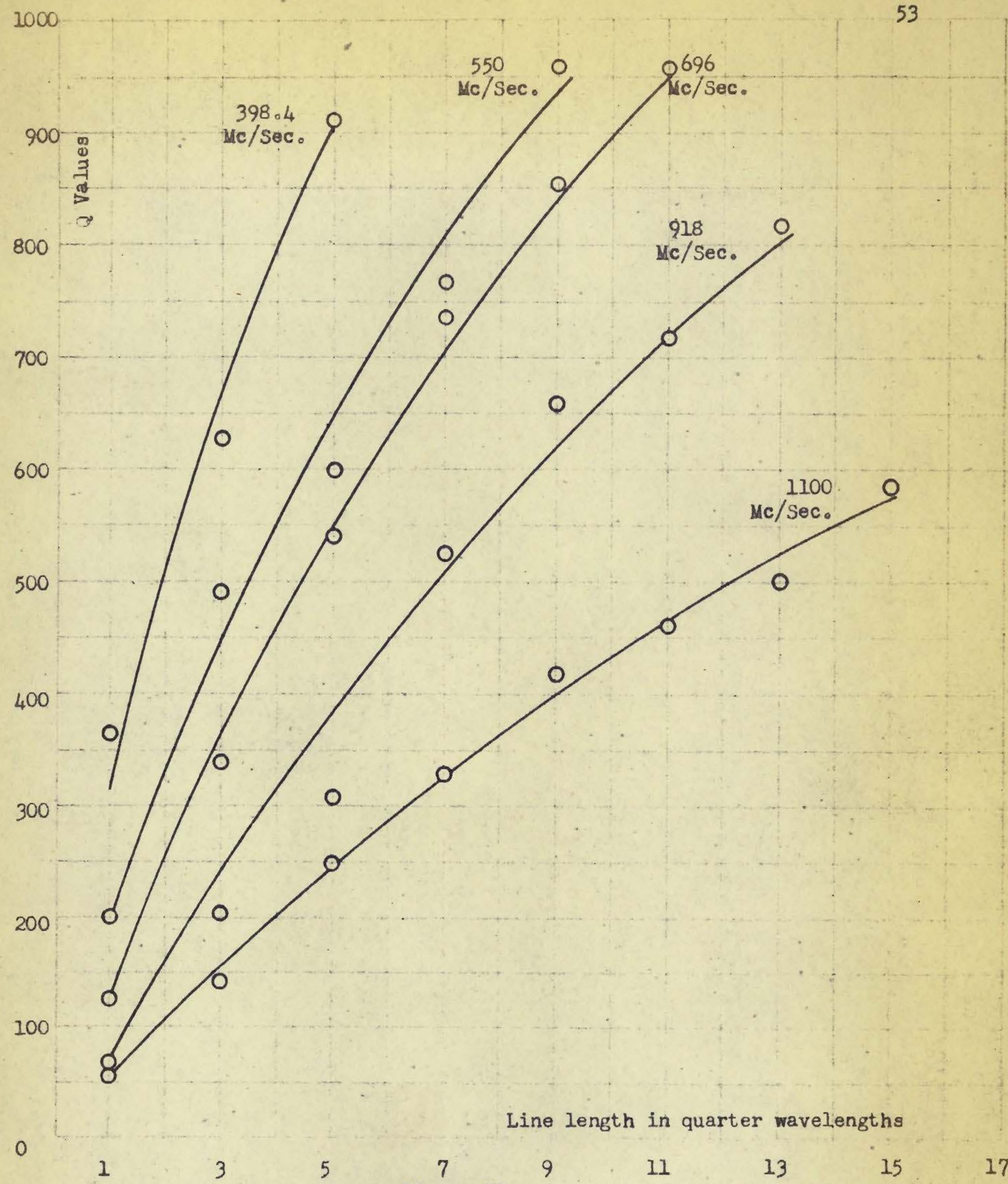


Figure 23
The selectivity or Q of the unshielded open - circuited line as a function of the number of quarter wavelengths.

Number of quarter wave-lengths	4	6	8	10	12	14	16	
Average Q_{sh}	1500	1880	2150	2250	2640	2730	2800	
L_o	27.19	41.01	55.00	68.91	82.91	96.83	110.72	
$R_{c_1}L_o$	0.590	0.890	1.193	1.494	1.798	2.100	2.400	
$\frac{\beta Z_{o_1} L_o}{Q_{sh}}$	1.405	1.690	1.985	2.375	2.435	2.750	3.060	
$R_t = \frac{1}{2}\left[\beta Z_{o_1} L_o/Q_{sh} - R_{c_1}L_o\right]$	0.408	0.400	0.396	0.441	0.319	0.325	0.330	Average $R_t = 0.374$
Average Q_{unsh}	194	311	370	497	538	660	680	
$R_c L_o$	0.554	0.836	1.122	1.405	1.690	1.975	2.255	
$\frac{\beta Z_o L_o}{Q_{unsh}}$	11.54	10.87	12.25	11.43	12.71	12.10	13.42	
$R_r = \frac{1}{2}\left[\beta Z_o L_o/Q_{unsh} - R_c L_o - 2R_t\right]$	5.119	4.643	5.190	4.639	5.136	4.689	5.159	Average $R_r = 4.93$

Table 111.

Sample calculation of the radiation resistance of the shorted line at 1100 Mc/Sec. Constants at 1100 Mc/Sec.: $R_c = 2.04$ ohms/meter, $\beta = \frac{2\pi}{\lambda} = 23$, $Z_o = 358$ ohms, $Z_{o1} = 337$ ohms, $R_{c1} = 2.17$ ohms/meter. All other calculations at the various frequencies for both the open and the shorted-line were done in similar tabular form. In each case an average value of R_t and R_r was obtained.

resistance were averaged in each case.

Figure 24 is a plot of the radiation resistance of the short-circuited line as a function of frequency. Each point represents the average of at least three values obtained at that frequency. For purposes of comparing the results with those given by the theory of Sterba and Feldman, it was necessary to double the measured values. Sterba and Feldman considered a transmission line shorted at both ends and radiating into the whole of space, while the experimental setup in the present work consisted of only half the antenna radiating in only half the space. This is the reason why the measured results had to be doubled in order that they may be compared with a line similar to the one of Sterba and Feldman.

The broken line in Figure 24 represents the radiation resistance of a line shorted at both ends and radiating in the half of space. This is given by the equation $R_r = 100(\pi d/\lambda)^2$. The difference between the two plots is actually within the 5% limit of accuracy discussed in the following section. These results show that the radiation resistance of a line shorted at both ends and an even number of half-wavelengths long is given by the equation; $R_r = 200(\pi d/\lambda)^2$. Eighty percent of this is due to the radiation resistance of the two shorting bars at the ends of the line, and twenty percent is due to the transmission line itself. The radiation resistance of two shorting bars is given by $160(\pi d/\lambda)^2$, if they are sufficiently far apart so that the currents in each have no effect on one another.

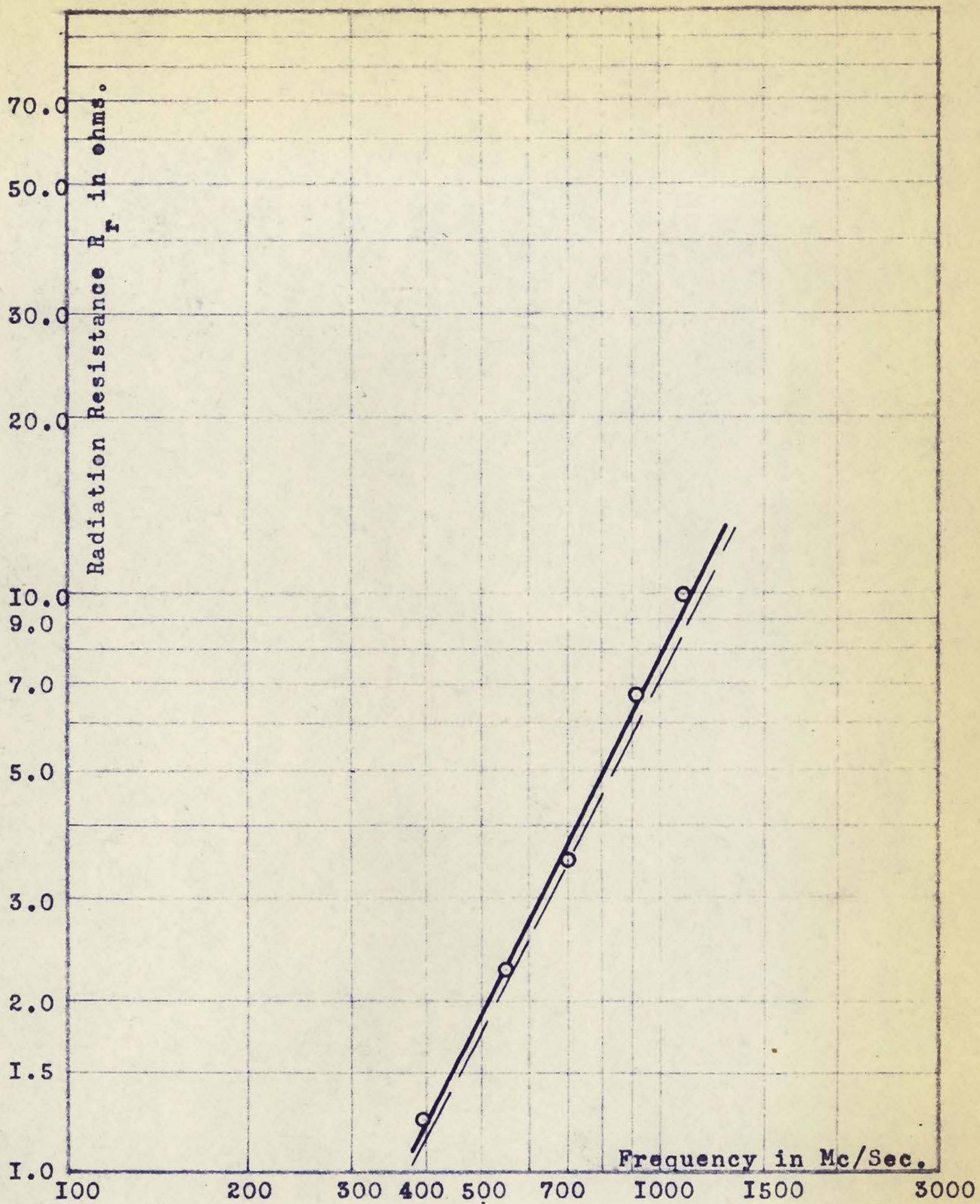


Figure 24.

The measured radiation resistance of the shorted line. Each point represents an average of at least three measured values at that frequency. The broken line is the plot of $100(\pi d/\lambda)^2$, and is the same as Sterba and Feldman's results for a shorted line including the effect of the shorting bar.

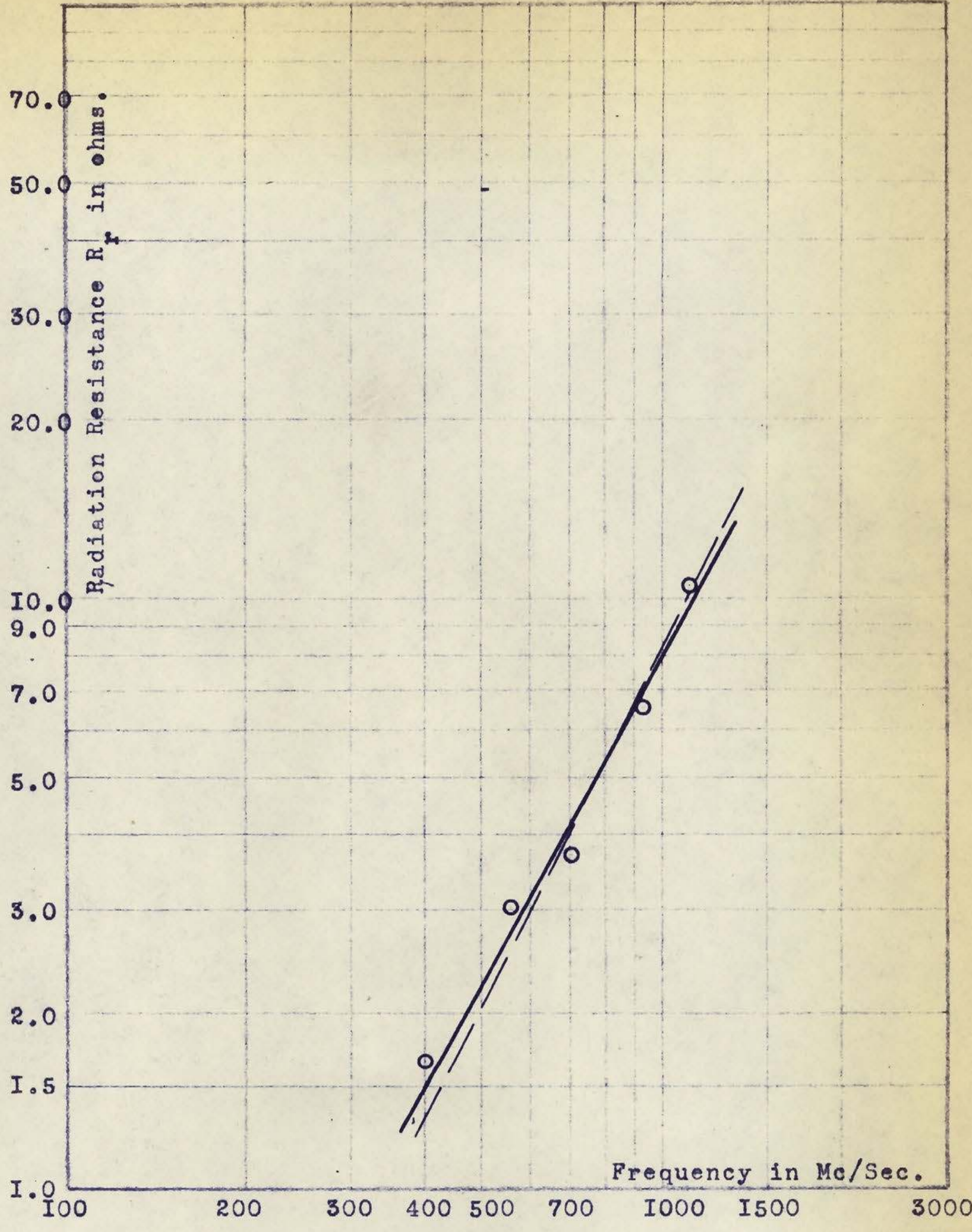


Figure 25.

The measured radiation resistance of the open-circuited line. Again each point represents an average of at least three measured values at that frequency. The broken lin is the plot of $120(\pi d/\lambda)^2$, and is the same as Sterba and Feldman's results for an open - circuited line open at both ends.

Figure 25 is a plot of the radiation resistance of the open-circuited line as a function of frequency. Here again each point represents an average of at least three values obtained at that frequency. As in Figure 24, the measured results have been doubled so that the final results may be compared with the open-circuited line considered by Sterba and Feldman. The radiation resistance of a line open-circuited at both ends is given by the equation; $R_r = 120(\pi d/\lambda)^2$, and the broken line in Figure 25 is a plot of this equation.

The plot of Figure 25 indicates that the difference between the theoretical and experimental curves is again very small, and that this difference probably falls within the limit of accuracy of the measurements. It is also clear that the measured radiation resistance of an open-circuited line agrees with the results of Sterba and Feldman.

(e) Conclusions.

(1) Since the calculations show that the radiation resistance at a single frequency does not vary with the length of the line, it is therefore quite definite that radiation resistance of transmission lines is not a function of line length. This is in accordance with the results obtained by most of the authors who have attempted the problem.

(2) Figure 24 clearly shows that the measured radiation resistance of a shorted parallel wire line is in good agreement with the theoretical results of Sterba and Feldman. It is found that most of the measured radiation resistance is due to the terminating shorting bar, and only about 20% is due to the line itself.

(3) Figure 25 shows that the measured radiation resistance of an open-circuited line, open at both ends, is also in agreement with the theoretical result of Sterba and Feldman for this particular case. It is therefore conclusively prooved that the radiation resistance of a line open at both ends is given by $120(\pi d/\lambda)^2$.

Referring back to Table I on page 9, it is found that the experimental results for the shorted line agree with the theoretical results of Sterba and Feldman, and Reukema. The results of King seem to be in error. The experimental result of the open line is found to be in agreement with, Manneback, Pistolkors, Carter, Sterba and Feldman, Whitmer and Reukema.

(f) Accuracy of Measurements.

An accurate estimation of the maximum error that can be expected is provided if a complete analysis is done on those reading which gave lowest Q values. Table II indicates that the lowest Q value was obtained when the line was 1/4 wavelength long and frequency was 1100 Mc/Sec. The measured resonant length at this point is 6.47 centimeters, therefore this will obviously be the point with the greatest experimental error.

Let us assume that an error of ± 0.05 in. is possible in measuring the width of the resonance curve at the half power points, and that the measured resonant length can be measured to ± 0.1 cms. The chart width of the resonance curve for the case mentioned above is 1.1 in. This means that the error in the measurement of $2\Delta L$ is approximately 4.5%. Similarly the percetage or in the measured resonant length will be about 1.5%. The shielded Q of the line was obtained from $Q = L_0/2\Delta L$, therefore the maximum error in the unshielded Q of the transmission line is the sum of the two percentage errors obtained above. At least two measurements of the Q of the line were obtained at each setting and the average of these two was used for purposes of calculating the radiation resistance. This therefore means that the maximum possible percentage error in the average value of the unshielded line Q is $6/\sqrt{2}$, or about 4.2%.

Since the width at the half power points of the resonance curve for the shielded line is only about 1/3 to 1/2 as large as that of the unshielded line, the maximum possible error in the measured Q value of the shielded line is obviously larger than that for the unshielded line. The error in the average Q values of the shielded line is therefore of the order of 10%. Average shielded Q values were used to calculate the effective series terminal resistance due to losses in the coupling loops, and therefore an error of approximately 10% exists in each of the calculated R_t values. Since at least three independent values of R_t were averaged to obtain an average R_t value at constant frequency, then the error in the average R_t value is reduced to approximately 5.8%.

By considering the errors in; the unshielded Q of the line, the measured resonant length L_o and the average value of R_t, and by proper substitution in the equation for radiation resistance, it is found that each calculated value of radiation resistance has an error of approximately 7.6%. Again at least three values of radiation resistance were averaged to obtain the average radiation resistance at a single frequency. The maximum possible error in the average value of radiation resistance at a single frequency is therefore reduced to approximately 4.4%. It was found that frequency could be measured accurately to a half of one percent, consequently the estimated accuracy would change very little if this source of error was included in the calculations.

In addition to the errors introduced by the methods of measurement, errors due to slight variations in the separation between the two wires, and errors due to the possibility of the antenna mode of excitation were also present. Errors due to the first of these causes was minimized by taking more than one measurement at the same setting and averaging the final results. The errors due to the possibility of the antenna mode are discussed in the following few pages and it is found that these are also negligible.

The above analysis indicates that the resulting measurements of radiation resistance of both the open and short-circuited transmission lines are in error by not more than five percent.

(g) Effects of the Antenna Mode.

When a transmission line is excited by a small coupling loop, two modes of excitation are possible. The first and desirable mode is the transmission-line mode, while the second is the antenna mode. Since it is highly desirable that the detector should only pick up the transmission-line mode, the detector was placed on the line joining the two wires and exactly half way between them. Figure 25A is a drawing of the current distribution in the aluminum plane when the line is excited in either of the two modes of excitation. It is seen that the current paths of the transmission-line mode start from

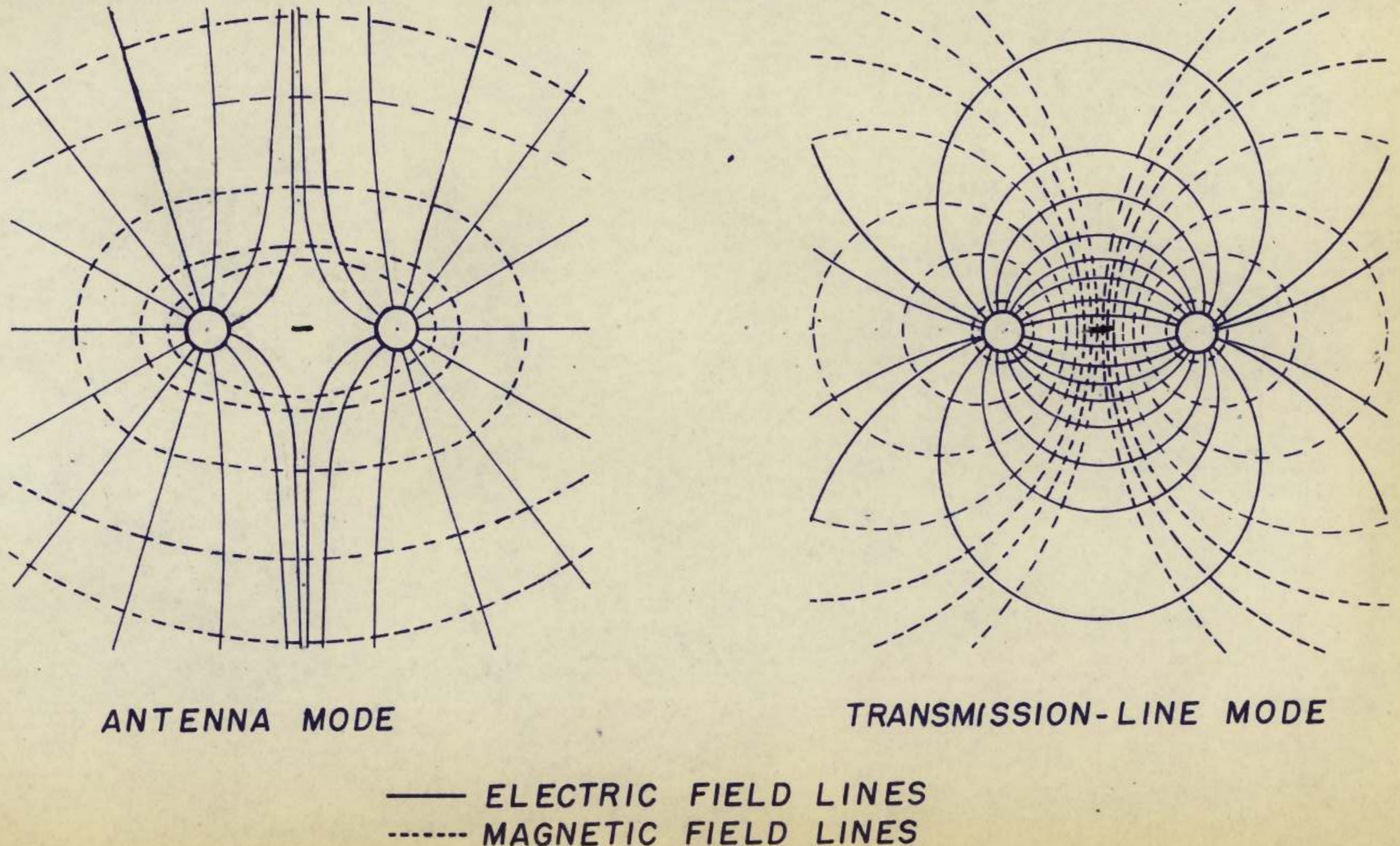


Figure 25A.

The possible electromagnetic field patterns of the two-wire line. The current pattern in the aluminum ground plane is the same as the electric field lines.

one conductor and go to the other conductor. This is due to the fact that the currents in the two conductors are exactly 180° out of phase, and the infinite plane acts as a path for the current between the two conductors.

If only the antenna mode is excited then the currents in the two wires flows in the same direction and no current can flow between the two wires through the infinite plane. This is clearly shown in Figure 25A. Very little or no power of the antenna mode can be detected by the detector coupling loop if the loop is placed in the position indicated in Figure 25A.

In order that the magnitude of Q for the antenna mode may be compared with the measured Q of the transmission-line mode, an approximate Q value for an antenna, 1/4 wavelength long at a frequency of 696 Mc/Sec. was obtained from a set of curves published by Brown and Woodward[22] in 1942. It turns out that the selectivity factor Q for the antenna is approximately 3.5, while the measured Q of an unshielded transmission line of the same length and at the same frequency is 127. It is therefore quite evident that if the resonance curve for the line excited in the transmission-line mode were superimposed on the resonance curve of the line excited in the antenna mode, the resonance curve of the antenna mode would be so wide that it would only change the zero level of the resonance curve of the transmission-line mode. The amount of antenna mode present can therefore be estimated by the position of the zero levels on the resonance curve. It was found that practically every resonance curve taken had its zero level at the point where the recording meter read zero voltage. This therefore indicates that practically none of the antenna mode was detected by the detector coupling loop.

In order to check the calculated selectivity factor Q of the antenna mode it was decided to obtain a resonance curve when only one wire of the two wire line was present. It was found that no resonance curve was detected even when a maximum amount of power was fed to the power input coupling loop. This clearly gives an indication that the selectivity factor of the antenna mode was extremely low. Since the two coupling loops were placed in such a position that a maximum of the transmission-line mode and a minimum of the antenna mode could exist on the line it was therefore concluded that the effects of the antenna mode were negligible.

V METHODS OF DECREASING THE RADIATION LOSSES IN TRANSMISSION LINE CIRCUITS.

In the light of the results obtained for the open and short-circuited transmission lines, it seems that radiation from the lines is caused by sudden changes in the electromagnetic field pattern existing around the transmission line. Abrupt changes in the electromagnetic field pattern will therefore occur at points on the transmission line where some type of discontinuity may exist. These discontinuities may consist of any of the following: bends, tapers, terminations, and any other objects near the transmission line that will affect the field pattern. Some of these discontinuities will undoubtedly cause serious changes in the field pattern of the line, whereas others may not change the pattern to any great extent. A sharp change in the field pattern will mean that more power will be lost to the surrounding space and less will be reflected back to the source.

It is known that flat planes approximately six wavelengths in diameter behave very much like perfectly reflecting planes, and therefore flat plane terminations may conceivably reduce the radiation losses of resonant transmission line. This type of termination may not change the field pattern of the line to any great extent and may result in a transmission line circuit superior for such purposes as highly selective circuits.

Another means of reducing the amount of radiation losses is by making the discontinuity a gradual one. It is quite definite that a sharp bend in a transmission line will result in more power loss than if the bend were a gradual one. Thus the shorting bar of the short-circuited line could be replaced by bringing the ends of the two wires together. This then becomes a tapered line, and the sharp discontinuity of the shorting bar is distributed over the whole length of the line. Since the discontinuity of the tapered line is a distributed one, it was believed that the radiation losses of this type of line may be reduced, consequently the selectivity factor Q of the tapered line would increase.

With the problem of reducing radiation losses in mind, it was decided that some of these possibilities should be investigated. The two methods mentioned above as well as one or two others, were investigated and the results obtained are presented in graphical or tabular form.

(a) Transmission Line with Partial Shield.

In order to determine the effect that a metal ring or partial shield may have on the electromagnetic field pattern of the transmission line, two measurements were taken with a small circular tin shield surrounding the line. The first measurement was taken when a circular tin shield 3 3/4 inches in diameter and 3 inches long was placed around the line. It was placed on the aluminum plane and surrounded the base of a short-circuited transmission line 4/4 wave-lengths in length. A photograph of the line and partial shield is shown in Figure 26.

The Q of the 4/4 wave resonant shorted line without the partial shield was first measured and found to be approximately 454.

Figure 26.

Parallel-wire transmission line with partial shield.

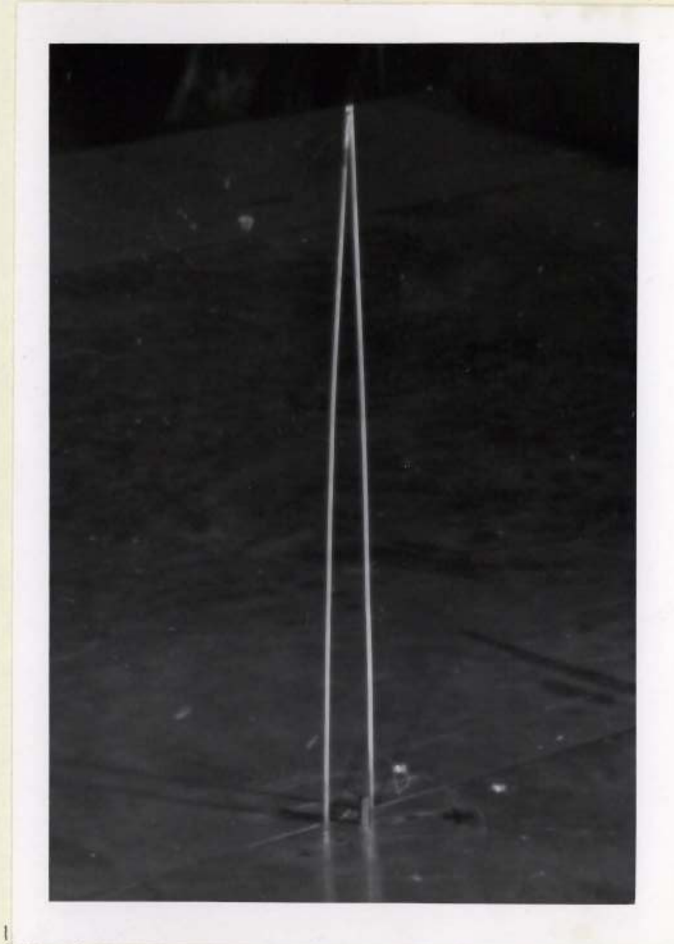


Figure 27.

The tapered line.

The metal shield was then placed in position and the measured Q was found to be about 247. A second partial shield of the same diameter but only 3/4 inches in length was then placed on the aluminum plane and the measured Q of this system was found to be approximately 388.

The results of this investigation indicates that the radiation losses of a transmission line with partial shield are approximately twice as great as the losses for the resonant line alone. The partial shield obviously changes the electromagnetic field pattern in such a manner that a considerable amount of power is lost to the surrounding space. The fact that the selectivity factor Q increases when the length of the partial shield is reduced shows that radiation losses have been decreased. Nevertheless the fact that the Q is still less than that for the shorted line alone, indicates that distortion of the electromagnetic field is still there.

Just what would happen if the length of this partial shield were held but diameter were allowed to increase or decrease has not been investigated. The effect of metal rings of large diameter compared with the wavelength would obviously have little effect on the field pattern near the transmission line, since the metal portions of the shield would be a considerable distance away from the line itself.

(b) The Tapered Transmission Line.

In 1937 Reukema[(12)] published an article on transmission lines at very high radio frequencies. In this article he had stated that the selectivity factor Q of a short-circuited line could be substantially increased if the ends of the two wires were brought together to give the necessary short circuit. Reukema undoubtedly had the idea that the portion of the radiation losses due to the shorting bar could be eliminated and this would greatly increase the Q of the line.

In order to determine if the losses of a shorted line could be reduced by tapering the line, a number of measurements were taken on a line which had its ends connected together. The ends of the two wires of the transmission line were soldered together with silver solder and the line was then placed through the aluminum ground plane from above. Thus the separation between the two wires remained one inch at the aluminum plane but decreased to zero at the end above the plane. A photograph of the tapered line as it projected above the aluminum ground plane may be seen in Figure 27. Measurements were taken at four different frequencies, and at each frequency three to five different resonant lengths were investigated.

The results of this partial investigation are given in Figure 28 and in Table lV. Figure 28 is a plot of the measured Q values as a function of the number of quarter wavelengths of resonant tapered line. The plot is very similar to that of Figure 22, the case of the shorted line. Comparing the results given in Figure 28 with those of Figure 22, shows that the Q of the tapered line is approximately 20% greater than that of the shorted line, when the frequency is 398.4 Mc/Sec. At a frequency of 1100 Mc/Sec., however, the Q of the tapered line becomes almost twice as great as the corresponding Q's of the shorted line. This is obviously explained by the

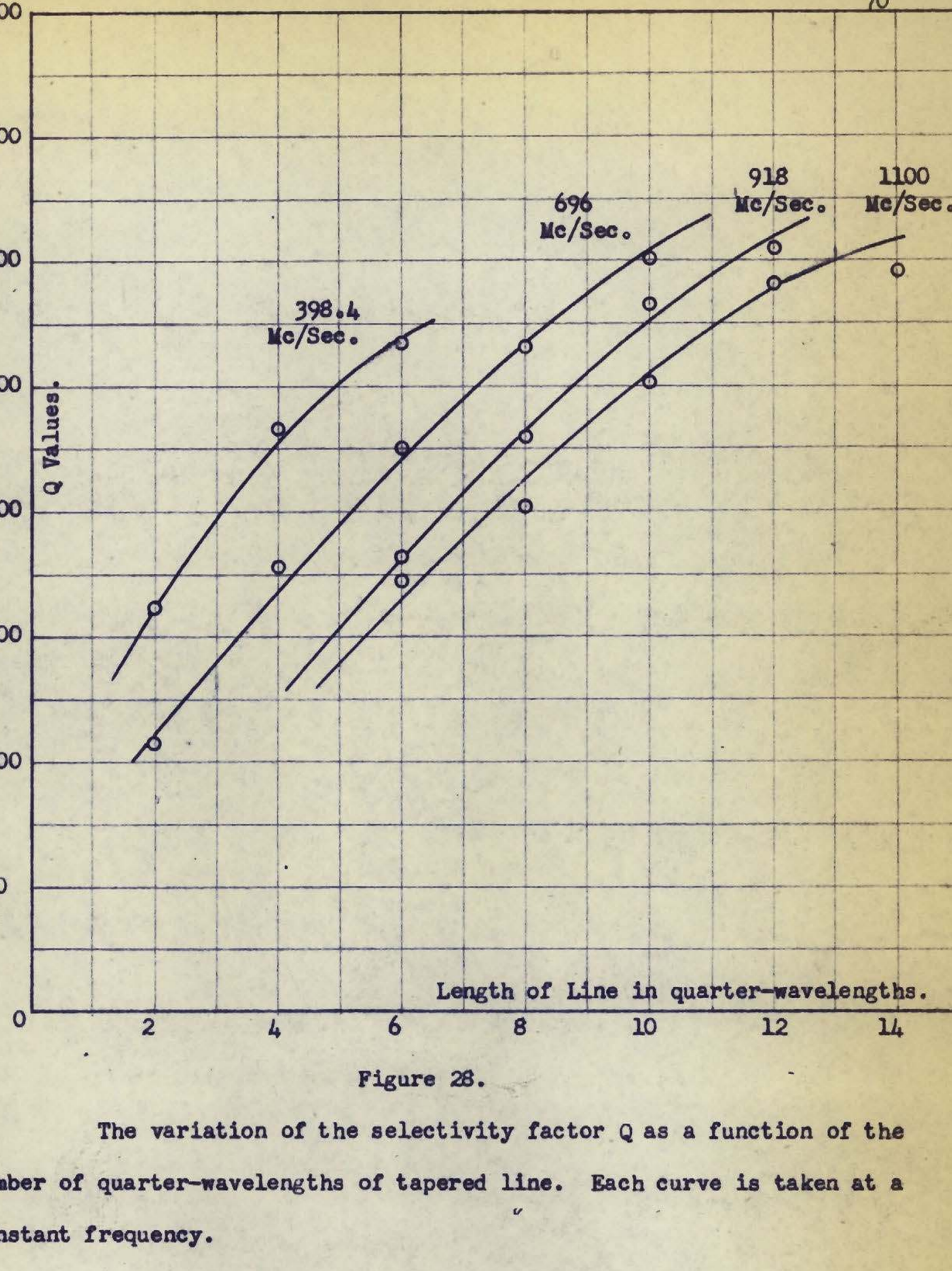


Figure 28.

The variation of the selectivity factor Q as a function of the number of quarter-wavelengths of tapered line. Each curve is taken at a constant frequency.

Line Length in Quarter Wavelengths \ Frequency Mc/Sec.	398.4		696		918		1100	
	R_r ohms	L_o cms.	R_r ohms	L_o cms.	R_r ohms	L_o cms.	R_r ohms	L_o cms.
2	.49	41.68	.93	24.63				
4	.56	78.93	.96	46.48				
6	.69	116.18	1.06	68.02	1.49	51.23	1.71	42.90
8			1.10	89.62	1.43	67.58	1.96	57.78
10			1.14	111.23	1.30	83.76	1.82	71.67
12					1.43	99.99	1.77	85.51
14							2.06	99.36

Table lV.

The calculated radiation resistance and measured resonant length of the tapered line.

fact that radiation resistance is a function of the square of the frequency, and therefore a small change in the radiation losses at the lower frequencies would be increased four fold if the frequency were doubled. Since the ohmic losses are dependent only on the square root of the frequency, while the radiation losses are dependent on the square of the frequency, then it is quite obvious that reduction in radiation losses would have a greater effect on the Q than reductions in the ohmic losses of the line.

The general trend of the Q curves for the tapered line are very similar to those of the shorted line. Here, just as in Figure 22, it is seen that the Q of the tapered line decreases with increasing frequency, and increases with the number of quarter wavelengths of resonant line.

Table lV shows the variation of the radiation resistance of the tapered line with frequency and the length of the line in quarter wavelengths. These calculations were made assuming that the characteristic impedance of the tapered line is the same as that for a uniform parallel wire line. This is obviously not the case, but the calculations were made purely for purposes of comparing the results with those of the shorted line. It was also assumed that the shielded Q values of the tapered line were the same as the corresponding shielded Q values of the shorted line. By making these assumptions the calculations then became similar to the ones shown in Table lll.

Table lV indicates that the calculated radiation resistance of the tapered line is considerably less than that for the shorted line. For example the radiation resistance of the tapered line at a frequency of 918 Mc/Sec. and a resonant length of eight quarter wavelengths is 1.43 ohms, while the radiation resistance of the shorted line for exactly the same conditions is 3.12 ohms. This therefore, shows that the radiation losses of the tapered line are only about 40% of the radiation losses from the short-circuited line. This is probably explained by the fact that since the discontinuity is a gradual one, the electromagnetic field does not differ much from the ordinary plane wave field of a parallel-wire line.

Table lV also gives an indication that the radiation resistance of a tapered line may also be a function of the length of the line. This phenomenon is particularly evident at a frequency of 398.4 Mc/Sec. and again at a frequency of 696 Mc/Sec. It is seen that at 696 Mc/Sec. the radiation resistance of the tapered line is 0.93 ohms when a line two quarter-wavelengths long is used, and increases to 1.14 ohms when the line is ten quarter-wavelengths. This dependency on line length is also due to the fact that the discontinuity is distributed over the whole length of the line, and therefore radiation should occur from the whole line instead of from the termination alone.

Comparing the measured resonant lengths as given in Table lV with those of the even quarter-wavelength lines of Table ll, shows that the measured resonant length of the tapered line is substantially greater than the corresponding measured resonant lengths of the shorted line. In fact the values given in Table lV are even greater than the free space wavelength at that particular frequency. For example the free space wavelength at a frequency of 398.4 Mc/Sec. is 75.4 cms., whereas the measured resonant length of a four quarter-wavelength tapered line is 78.93 cms. This is an extremely interesting phenomenon and is probably due to the fact that the inductance and capacity of the tapered line are not distributed constants but vary with the length of line. Since the velocity of propagation is defined as $1/\sqrt{LC}$, where L is the inductance per unit length and C the capacity per unit length, then this will also vary with the length of the tapered line. It is probably this change in the velocity of propagation that causes this interesting effect.

(c) Effects of Flat Plane Terminations.

Figure 30 is a photograph of a parallel-wire transmission line with a flat circular copper plane termination. In order to obtain sufficient data on the behavior of such terminations, both the case of the tapered line with flat plane terminations, and the parallel-wire line with flat plane termination were investigated. The size of the plane was varied from about two inches in diameter to approximately eighteen inches in diameter. The flat circular planes were constructed from sheet copper approximately one thirty-second of an inch thick, and they were soldered to the ends of the transmission line with an acid core type solder. A more durable type of cord was used to support the transmission line and terminating plane in a vertical position. This was necessary since the fine nylon thread could not support a copper plane eighteen inches in diameter. Figure 30 also indicates the method of supporting the line with the plane termination.

Measurements on the parallel-wire line with plane termination were made at two different resonant lengths. The procedure was the same when the tapered line with plane termination was done. The results obtained were put in graphical form so that the effect of the plane on the selectivity factor Q could be seen more easily.

The results of the parallel-wire line with flat plane termination are given in Figure 29. Here the measured Q of the system is plotted against the diameter of the shorting plane in wavelengths. The measurements were made at a frequency of 1100 Mc/Sec. and two different resonant lengths were investigated so that two curves could be plotted for the same system. Figure 29 shows that the curves start from values which are the same as those for a shorted line, decrease to a minimum at a plane diameter of about 0.34 wavelengths, increases to a maximum at a plane diameter of 1.4 wavelengths, and begin to fall again. This pattern seems to be somewhat similar to a Fresnel intensity pattern that is obtained in optics from a zoning plate. This effect is most likely due to the peculiar edge effects that are part of the circular plane. It is clearly seen that the Q of the 6/4 wave line approaches the shielded Q value when the size of the plane is approximately 1.4 wavelengths in diameter. This phenomenon would obviously prove very advantageous when high Q transmission line circuits are desired. The point at which the measured Q drops to a minimum, is quite obviously the point where that particular plane diameter has the greatest effect on the field pattern of the line. Thus a plane 0.34 wavelengths in diameter acts more like

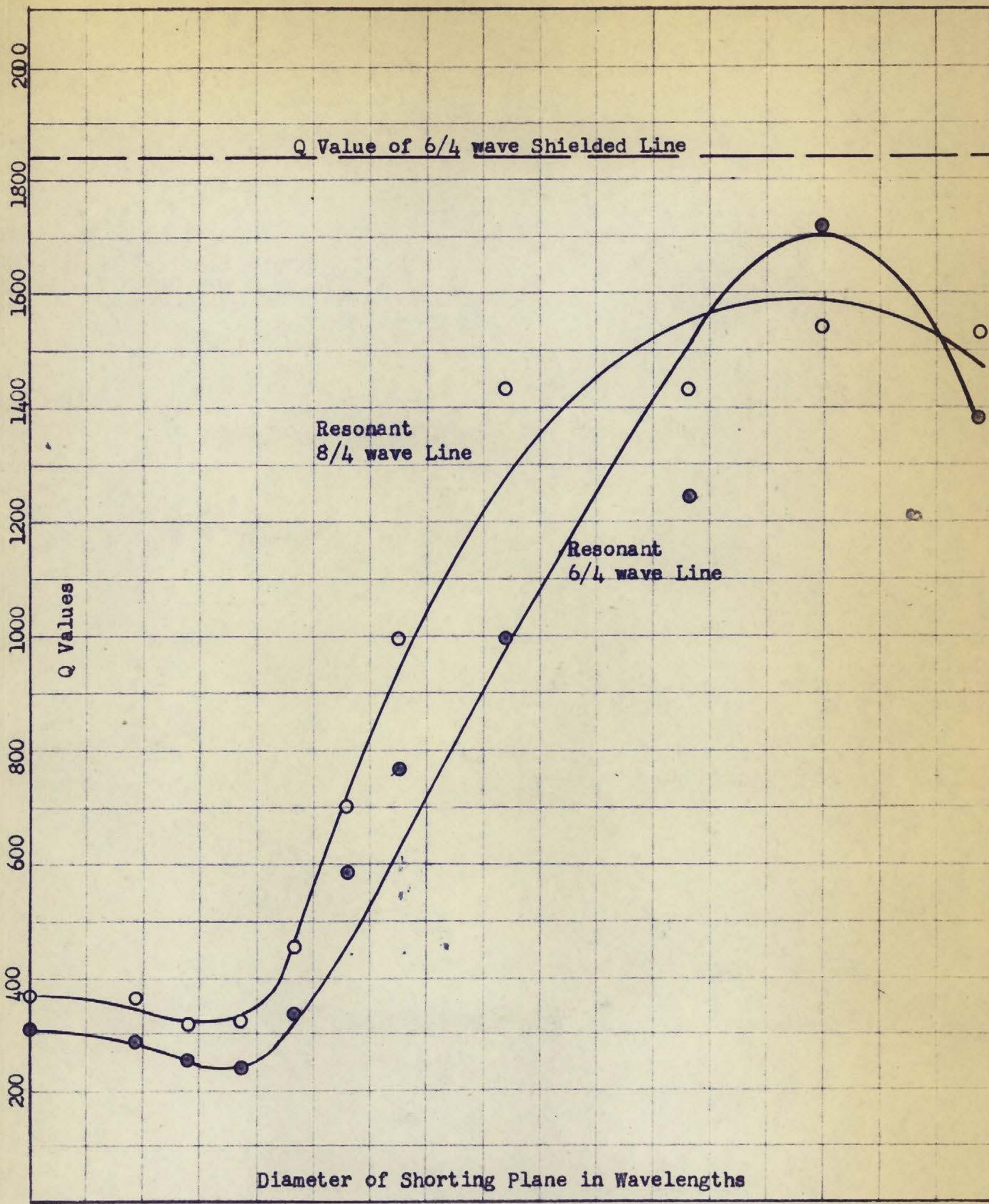


Figure 29.

The selectivity factor Q of the shorted line, with circular copper plane termination, as a function of the diameter in wavelengths of the copper shorting plane. The measurements were taken at 1100 Mc/Sec.

Figure 30.

Transmission line with circular flat plate termination.

a radiating current element instead of a reflecting plane.

In view of the results obtained from the parallel-wire line with flat plane termination, it was suspected that a flat plate termination may further increase the selectivity factor Q. For this reason therefore, it was decided to repeat the measurements of the plane terminated line, on a tapered line with the same circular planes placed at its end.

The results of this investigation are given in Figure 31. Figure 31 indicates that the results are similar to those of Figure 29, however the peak of the curve is not quite as high as it was before. Further comparison shows that the Fresnel characteristic is somewhat more pronounced for the case of tapered line and plane. The position of the first minimum for the tapered line and plane occurs at a plane diameter which is essentially the same as that for the parallel-wire line. The position of the first maxima is obviously quite different. For example the first maximum in Figure 31 occurs at a plane diameter of 0.8 wavelengths whereas in Figure 29 the first maximum appears at a plane diameter of 1.4 wavelengths. The differences between the results of the tapered line with plane and those of the parallel-wire line with plane, is probably due to the fact that the fields of the tapered line are already distorted by the tapering on the line.

Unfortunately it was impossible to use flat planes greater than about 1.6 wavelengths in diameter. This would have meant a drastic change in the method of supporting the transmission line and time did not permit this undertaking.

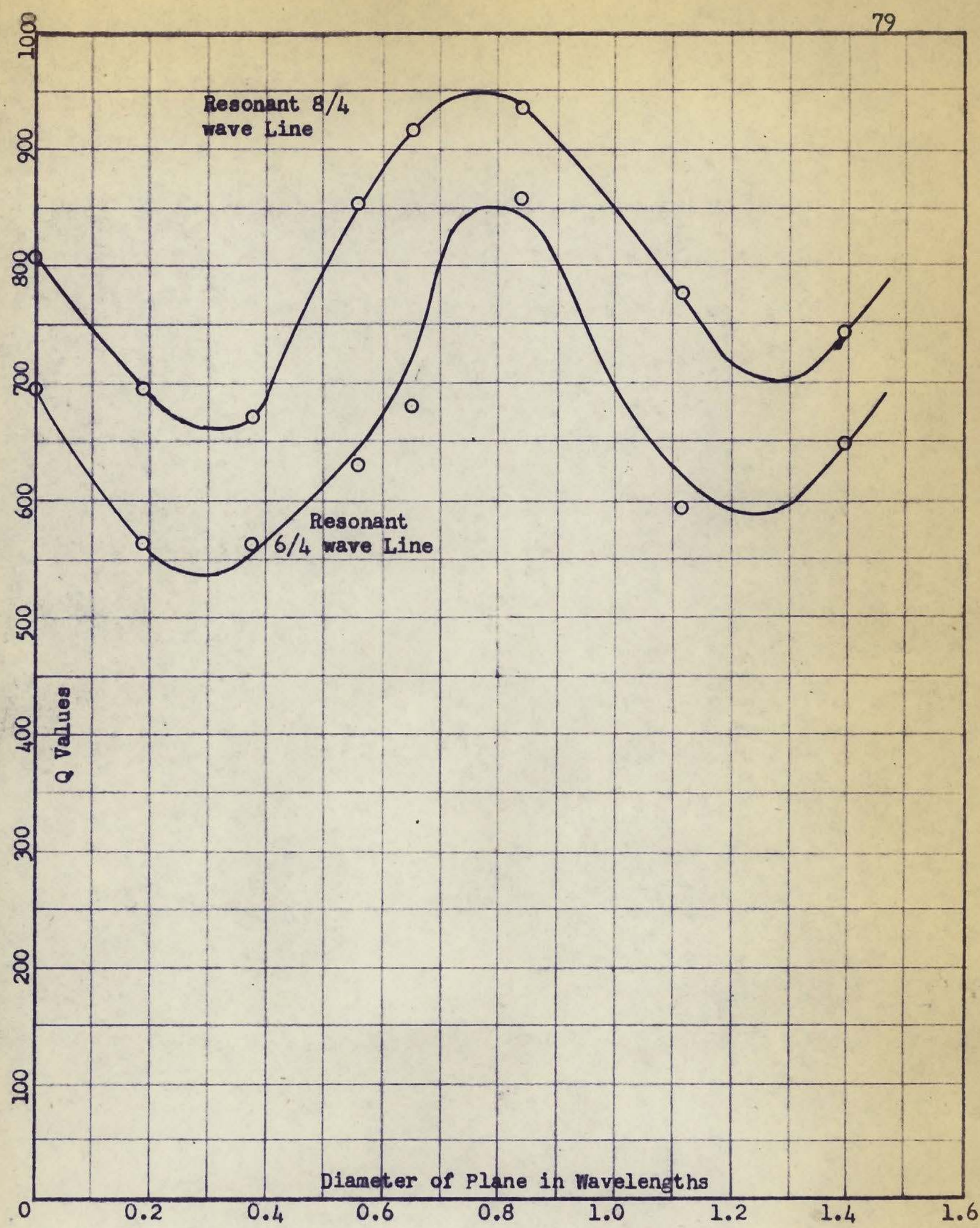


Figure 31.

The selectivity factor Q of the tapered line, with circular copper plane termination, as a function of the diameter in wavelengths of the terminating plane. The measurements were taken at a frequency of 1100 Mc/Sec.

(d) Conclusions.

(1) A circular metal ring or partial shield greatly increases the radiation losses of a resonant two-wire line.

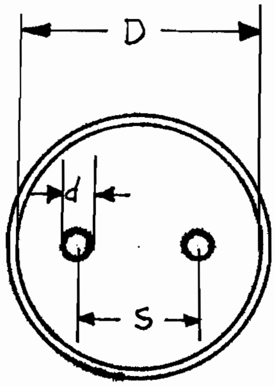
(2) The selectivity factor Q of a shorted line is substantially increased if the short circuit is achieved by bringing the ends of the wires together to form a tapered line. This consequently reduces the radiation losses of the line.

(3) The selectivity factor Q of a shorted transmission line is greatly increased if a circular flat plane 1.4 wavelengths in diameter replaces the convential shorting bar. Planes approximately 0.34 wavelengths in diameter decrease the Q of the shorted line.

APPENDIX 1.

Constants of the transmission line.

(a) Shielded Line.



2 is the silver two wire line and 3 is the brass shield.

Ohmic Resistance $R_{c_1} = \frac{2R_{s_2}}{\pi d}\left[1 + \frac{1+2P^2}{4P^4}\left(1 - 4q^2\right)\right] + \frac{8R_{s_3}}{\pi D} q^2 \left[1 + q^2 - \frac{1+4P^2}{8P^4}\right]$

ohms/meter, where $P = \frac{S}{d}$ and $q = \frac{S}{D}$.

Also $R_{s_2} = 2.52 \times 10^{-7}\sqrt{f}$, $R_{s_3} = 5.01 \times 10^{-7}\sqrt{f}$,

and f = frequency in cycles per second.

Characteristic Impedance $Z_{o_1} = 120\left\{\log_e\left[2P\left(\frac{1-q^2}{1+q^2}\right)\right] - \frac{1+4P^2}{16P^4}\left(1 - 4q^2\right)\right\}$ ohms

The attenuation constant is given by $\alpha_1 = R_{c_1} / 2Z_{o_1}$ nepers/meter.

(b) For Unshielded Line.

$R_c = \frac{7.88\sqrt{f}}{d/2} \times 10^{-6}$ ohms/meter, where d/2 is in centimeters.

$Z_o = 276 \log_{10} \frac{2S}{d}$ ohms.

$\alpha = \frac{R_c}{2Z_o}$ nepers/meter.

The theoretical Q of the two wire line is given by;

$Q = \frac{\beta}{2\alpha}$ where $\beta = \frac{2\pi}{\lambda}$ and λ is the wavelength in meters.

Most of these equations are taken from Ramo and Whinnery[17] and from King, Mimno and Wing[18].

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