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THE RADIO-FREQUENCY QUADRUPOLE LINEAR ACCELERATOR*

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Summary

The radio-frequency quadrupole (RFQ) is a new linear accelerator concept in which rf electric fields are used to focus, bunch, and accelerate the beam. Because the RFQ can provide strong focusing at low velocities, it can capture a high-current dc ion beam from a low-voltage source and accelerate it to an energy of 1 MeV/nucleon within a distance of a few meters. A recent experimental test at the Los Alamos Scientific Laboratory (LASL) has confirmed the expected performance of this structure and has stimulated interest in a wide variety of applications. We review the general properties of the RFQ, and present examples of applications of this new accelerator.

Introduction

Two years ago LASL's Accelerator Technology Division initiated a program to investigate the properties of the RFQ accelerating structure, a concept originally proposed by Kapchinskii and Teplyakov.[1] The operation of an RFQ system in the USSR has been described previously.[2] Our development of this structure, the first outside the USSR, has culminated in a successful experimental test[3] of the RFQ principle. We conclude that the RFQ is an attractive ion accelerator for many applications in the energy range below about 2 MeV/nucleon. Papers have been presented that have described the general properties of the RFQ,[4] the RFQ beam dynamics,[5,6] and the RFQ resonator.[7] Applications of the RFQ as an injector into conventional drift-tube linacs have been presented for 1) high-energy accelerator facilities,[4] 2) a high-intensity neutron generator,[8] 3) a pion generator for cancer therapy,[9] 4) a heavy ion accelerator concept,[10] and 5) heavy ion induced fusion.[11] However, the RFQ need not be regarded exclusively as an injector for conventional drift-tube linacs. For certain ion beam applications in the 1-MeV/nucleon range, the RFQ itself may be an attractive small accelerator. In this paper we describe the properties of the RFQ and we present some examples of RFQ designs both as 1-MeV/nucleon accelerators, and as injectors for drift-tube linacs in an accelerator system.

General Properties of the RFQ

The RFQ uses rf transverse electric fields to focus ions traveling along its axial region. Figure 1 is a schematic section of the RFQ resonator. It operates in a modified TE_{210} mode in which the currents flow transversely to the z-axis. The current flow results in + - + - polarities on the pole tips at a certain time, thus producing a quadrupole focusing, or defocusing, force in a given transverse plane. One-half cycle later these forces reverse sign to produce an overall strong focusing effect. The focusing force at a given time is spatially continuous along the z-axis. If the pole tips have a constant radius, only a radial focusing force is present. In Fig. 1 the pole tips have a sinusoidal-like variation in radius. Between the x-plane and y-plane pole tips this variation in radius is shifted by $\beta\lambda/2$. This pole-tip modulation produces longitudinal accelerating fields in addition to the transverse focusing field.

In conventional drift-tube linacs the focusing force, which is obtained from magnetic quadrupoles within the drift tubes, becomes too small at low velocities to confine the beam, especially at high beam currents. The RFQ can operate at a velocity below that of conventional linacs because the focusing is obtained from the velocity-independent electric force. The operation of a linear accelerator at low velocities permits adiabatic bunching of the dc input beam within the linac structure, resulting in high capture and transmission efficiencies (>90%). Adiabatic bunching is not restricted to low energy in principle, but its application at high energies becomes too costly in length. If the bunching and accelerating functions are combined so that the adiabatic bunching is done while the beam is simultaneously accelerated, the space-charge limit can be raised significantly, compared with the conventional approach in which the beam is first bunched before injection into a linac accelerator. Furthermore, the high capture efficiencies obtained with the RFQ make efficient use of the ion source output beam.

The strong focusing from the electric field, together with the combined-function adiabatic bunching, results in a high space-charge limit, even though the dc injection voltage may be as low as 30 to 50 kV. An output ion beam energy in the 1-MeV/nucleon range is typically obtained within a few meters.

In comparison with a dc accelerator at the same beam current and output energy, the RFQ frequently

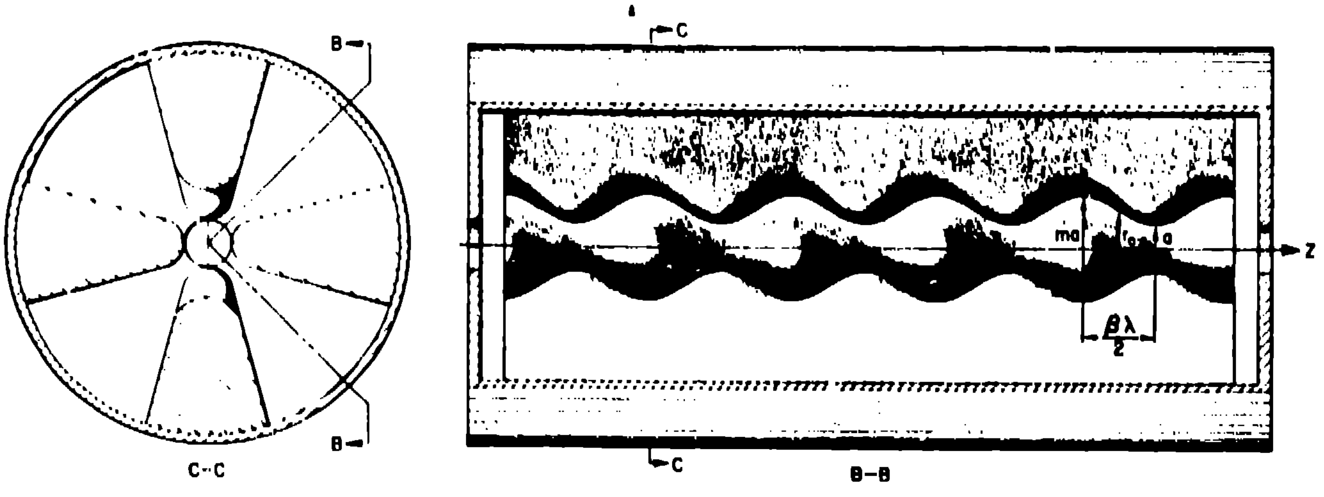

Fig. 1. Schematic Drawing of an RFQ Resonator.

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offers the advantage of a much reduced size. The strong focusing forces in the RFQ allow large beam currents with good output beam quality; the beam-current capacity of the RFQ appears to be greater than most dc accelerators. The RFQ offers greater flexibility than dc accelerators in the use of ion sources. A physically large and complex ion source for either positive or negative charge can be used as an injector. For convenience in operation of the ion-source, it is possible to apply a bias dc voltage to the RFQ, so that the ion source is at ground potential. Also, because the rf amplitude is the only physical parameter to be adjusted, the RFQ is relatively simple to operate as compared with most other accelerators. Some disadvantages of the RFQ are (1) the RFQ is probably less flexible than dc accelerators for output energy variation and (2) for energies greater than about 2 MeV/nucleon the average accelerating field is less than that for a conventional linac. However, extending the operation of the RFQ to higher energies does not degrade the beam performance, and for low duty factor applications can result in less total power consumption than for a conventional linac with dc focusing magnets.

Thus, an RFQ can be designed to accept an intense low-velocity unbunched ion beam from a low-voltage dc injector; then it will provide radial focusing, bunching and acceleration to about 1 MeV/nucleon in a few meters. In applying the RFQ in a specific case, it is possible to optimize its characteristics in a flexible way to obtain the desired compromise between transmission efficiency, emittance growth, beam current capacity, overall length, and power dissipation.

Techniques to provide rf power to RFQ systems have been described by Potter, et al.[7] A four-vane resonator similar to Fig. 1 can be used for frequencies greater than about 50 MHz, where the outer diameter is about 1.2 m. With lower frequencies, a Wideröe resonant system could be used; or with very low frequencies suitable for heavy ions, an externally resonant structure[12] may become useful. A multiple beam channel RFQ has been suggested by Swenson.[12] If the current capacity of a single RFQ channel is not sufficient, a multichannel RFQ array and the funneling technique[12] could be used to increase the total current to higher values.

Beam Dynamics Analysis and Pole-Tip Design

As a basis for beam dynamics simulation calculations and for construction of RFQ pole tips we use the lowest-order potential function. Figure 2 summarizes our procedure. Electric fields obtained from the potential function were used to construct transformations for the beam dynamics simulation program PARMTEQ.[5],[6] This program, which includes space-charge effects, analyzes specific RFQ designs to obtain transmission efficiency, radial emittance growth, and radial and longitudinal characteristics of the output beam. From an appropriate isopotential surface, we obtain the radial variation of the pole tips and the variation of the radius of curvature in the transverse plane, both as a function of z. Figure 1 shows the unit cell of length $\beta\lambda/2$, through which the radius varies from ma to a. In the center of the unit cell, the pole tips have quadrupolar symmetry and the radius r_0. The pole tips are constructed by generating a data file on magnetic tape that is used as input to a numerically controlled milling machine.

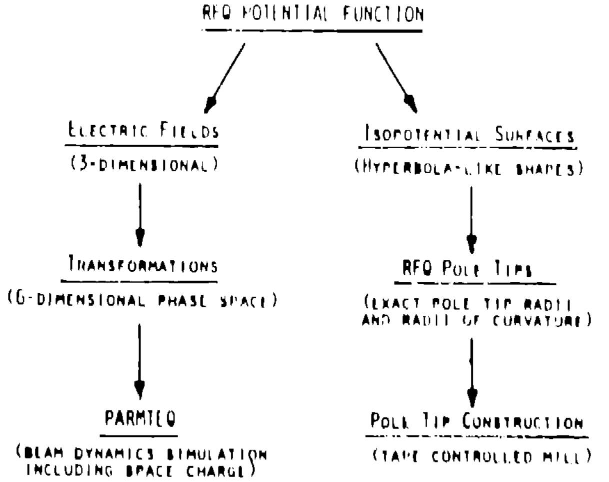


Fig. 2. Beam Dynamics Simulation and Pole Tip Geometry

Beam Dynamics Design Procedures for RFQ Systems

The methods used to synthesize RFQ systems to meet specific objectives have been described[6] previously. Figure 3 shows how the design procedures can be divided into four separate sections. The unbunched beam from the dc injector first goes into the radial matching section, where the radial focusing strength is increased from a low initial value to a higher final value in $\sim 5\beta\lambda$. This allows the beam to adapt itself adiabatically to the time-varying focusing forces. The next section is the shaper where we start to ramp both ϕ_s and E_0, the average longitudinal field. The phase angle ϕ_s starts at -90^0 and increases linearly with z.

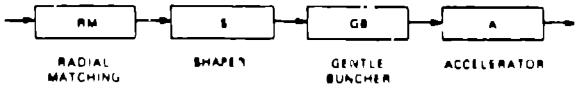


Fig. 3. RFQ Functional Block Diagram.

E_0 starts at zero and is slowly turned on by increasing the amplitude of the pole-tip modulation. If the parameters are chosen correctly, we can "shape" the particle distribution at the output of the shaper to occupy the desired level in the phase stable region of the bucket. Next is the gentle buncher where two conditions are imposed, as suggested by Kapchinskii. The average z-length of the bunch is held constant and the small amplitude longitudinal frequency is held at a fixed value. This allows the final value of ϕ_s to be attained at the end of the gentle buncher and preserves the particle distribution throughout. In the accelerator section, the ions are accelerated to their final energy at a constant value of ϕ_s.

A design constraint is imposed that makes the average radius of the pole tips r_0 constant, except in the radial matching section. This keeps the distributed capacitance nearly constant for easier resonator tuning. Also, the quantity V/r_0 is held constant, where V is the intervane voltage, allowing control of the peak surface field, which occurs near the point where the poles have minimum separation. These constraints are consistent with a constant voltage along the vanes and with a constant value of the focusing strength B.

Properties of the RFQ Four-Vane Resonator

The RFQ resonator can be understood in terms of the modes of a waveguide with uniform four-fold symmetrical cross section. The vane-tip potentials are assumed to be constant, independent of the z-coordinate. For heavy capacitive loading by the vanes, two families of modes are necessary for a reasonable approximation of the field distribution, the TE_{21n} and the TE_{11n} modes. The TM_{lmn} designation refers to the RFQ modes analogous to the modes of a right circular cylinder. The mode with uniform vane potentials and quadrupolar fields is the TE_{210}, which is the RFQ operating mode. To couple power into the RFQ without disturbing the field distribution, a coaxial manifold is used.[7] The manifold is a TEM-mode resonator with angled coupling slots into each RFQ quadrant at each manifold magnetic field maximum.

Radio-Frequency Quadrupole Experimental Test

Beginning in February 1980, we conducted a full-scale experimental test[5] of the RFQ principle. The test linac was designed to accept protons from a 100-keV dc injector and to focus, bunch, and accelerate to a final energy of 640 keV. The RFQ test resonator vanes had a length of 1.1 m and an o.d. of 0.15 m. Radio-frequency power was furnished by a 425-MHz klystron that was coupled to the resonator through a coaxial manifold and coupling slots. The beam dynamics design for this test and the calculated performance have been described previously.[6]

The reliability and the simplicity of operation of the test linac were outstanding. After an initial conditioning period, the RFQ operated continuously, accelerating a 20-mA beam current for more than a month at a duty factor of 0.1%. Also, the test has shown that the RFQ is tolerant of both constructional errors and errors in the rf resonator tuning.

Applications of the RFQ

The great flexibility inherent in the RFQ concept suggests its use in a wide variety of accelerator systems. Recently we have made preliminary beam dynamics designs for a large number of cases where the requirements were widely different. We will illustrate these by describing four systems. Each example will be discussed briefly, and most of the details will be presented in tabular form. The currents listed in the tables are the microscopic averaged electrical current and thus correspond to the average current for 100% duty factor. The current limit listed was calculated from the formulas given by Wangler.[13] The emittances are normalized values and are to be multiplied by π to obtain the ellipse area in cm-mrad units. The quantity E_S is the assumed maximum surface electric field; L is the cumulative RFQ length from the input. The diameter of the RFQ resonant cavity is inversely proportional to frequency and has a value of about 32 cm at [illegible]00 MHz.

RFQ for Nuclear Physics

An RFQ-based accelerator for a nuclear physics research facility is presented in the first example. The Physics Division at LASL has expressed an interest[14] in a low-energy accelerator in which two modes of operation are possible; one for q/A = 1/4 beams and the other for q/A = 1/2, where q is the charge state and A is the mass number. The RFQ design is determined by the requirement for q/A = 1/4 acceleration. The same accelerator then provides even better performance for all larger values of q/A, including q/A = 1/2.

Table 1 gives the design parameters and summarizes the results for q/A = 1/4. All beam currents listed should be interpreted as electrical current. The nominal electrical current of 10 mA is contingent upon the capability of the ion sources to provide this amount. An injection voltage of 50 kV is required for q/A = 1/4 and a final energy of 1 MeV/nucleon is reached in a distance of 4.24 m. For q/A = 1/4 a PIG source generates ion beams[15] up to at least neon at beam currents of a few tenths of an electrical milliampere. A more complex and expensive EBIS source[16] could extend the operation to heavier ions through uranium. Figure 4 shows the results of analyzing this RFQ with PARMTEQ; beam characteristics are plotted versus cell number. In Fig. [illegible](a), the x- and y-projections of 360 particles are plotted at the point in each cell where the beam is circular. The upper and lower dotted lines give the bore dimensions. The top of Fig. 4(b) shows the bunching of 360 particles initially distributed uniformly in phase (unbunched). The ordinate is the particle phase minus the synchronous phase. At the bottom, the particle energy minus the synchronous energy is plotted. In the phase and energy plots, the dotted lines give the location of the zero space-charge separatrix. The full width at half maximum energy spread of the output beam is 5 keV/nucleon, which gives $\Delta W/W = 0.005$. This energy spread could be reduced if necessary by using a debuncher cavity after the RFQ. The addition of some independently phased cavities after the RFQ would allow energy variation both above and below the nominal 1-MeV/nucleon output value.

Table 2 gives the results for q/A = 1/2 operation at a nominal current of 25 mA. The injection voltage for q/A = 1/2 is 25 kV and the final energy is still 1 MeV/nucleon. In this mode of operation the RFQ could accelerate a high-intensity deuteron beam as a possible low-energy neutron generator. Another possibility is to use a commercially available positive polarized deuteron source[17] to accelerate polarized deuterons with beam currents as high as a tenth of a milliampere or more. The use of a low-frequency buncher between the ion source and the RFQ could provide one means of generating short beam pulses that would allow time-of-flight experiments for polarized neutron-induced reactions. High-intensity polarized protons are also obtainable from such an ion source and could be accelerated in the same RFQ to an energy of 1 Mev.

200 MHz RFQ for Proton Linac

Many high- or medium-energy facilities have a low-energy accelerator system that uses a 200-MHz proton drift-tube linac. The conventional injection

Table 1

RFQ for Nuclear Physics; q/A = 1/4 Beams

Ions: He^{+}, C^{3+}, O^{4+}, Ne^{5+}

Frequency: 200 MHz

Nominal Current: 10 mA

Current Limit: 30 mA

PARMTEQ RESULTS

Input Current: 10 mA

Output Current: 9 mA

Emittance

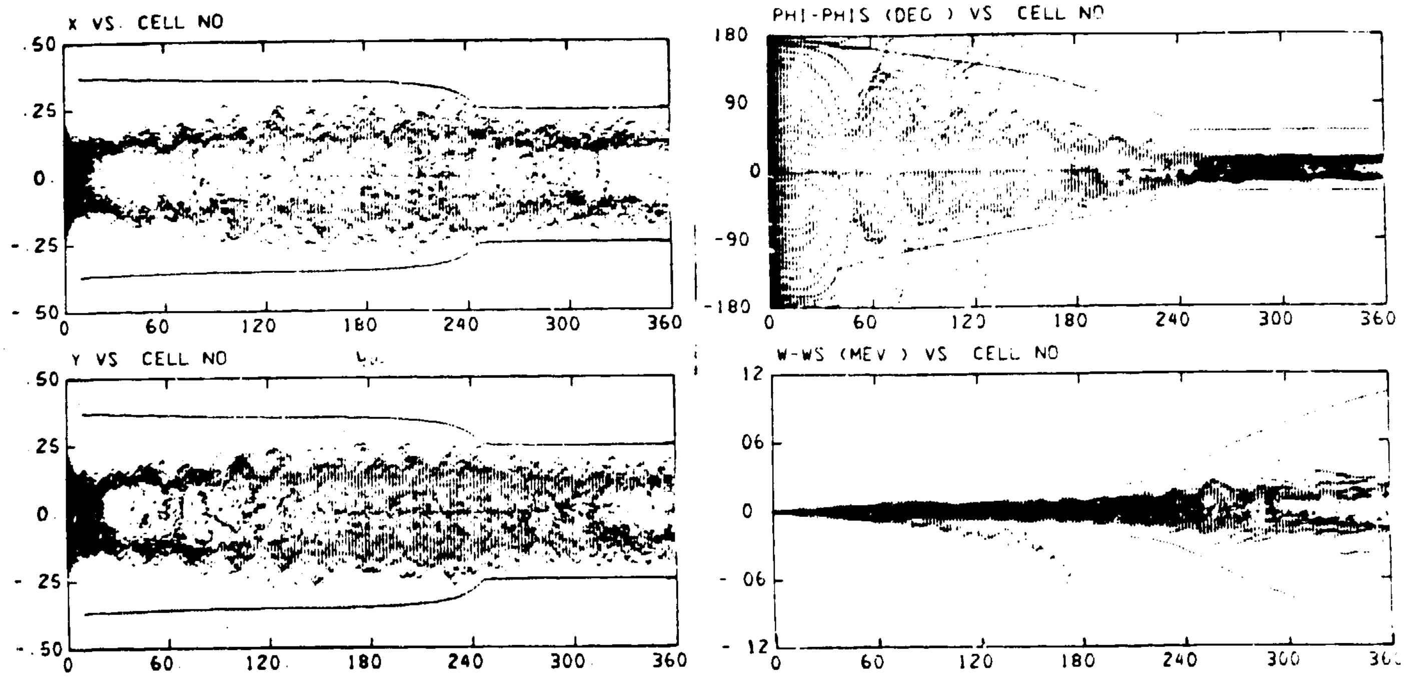
Input (90%): 0.035

Output (90%): 0.041

Input (RMS): 0.0083

Output (RMS): 0.0092

		RM	S	GB	A
W(MeV/A)	0.0125	0.0125	0.027	0.17	1.0
E_s(MV/m)	5.1	28.7	28.7	23.7	28.7
m	1.00	1.00	1.10	1.90	1.90
a(cm)	1.53	0.37	0.35	0.25	0.25
r_0(cm)	1.53	0.37	0.37	0.37	0.37
ϕ_s(deg)		-90	-61	-26	-26
E_0(MV/m)	0	0	0.57	2.69	1.26
V(kV)	78	78	78	78	78
B	0.20	3.43	3.43	3.43	3.43
L(m)	0	0.04	0.71	1.40	4.24



(a) Fig. 4. PARMTEQ Simulation Results. (b)

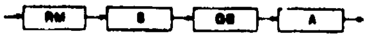
Table 2

RFQ for Nuclear Physics; q/A = 1/2 Beams

	Ions: d^+, He^{++},...							
	Frequency:	200 MHz	W(MeV/A)	0.0125	0.0125	0.027	0.17	1.0
	Nominal Current:	25 mA	E_s(MV/m)	5.1	28.7	28.7	28.7	28.7
	Current Limit:	99 mA	m	1.00	1.00	1.10	1.90	1.90
	PARMTEQ RESULTS		a(cm)	1.53	0.37	0.35	0.25	0.25
	Input Current:	25 mA	r_o(cm)	1.53	0.37	0.37	0.37	0.37
	Output Current:	23 mA	ϕ_s(deg)		-90	-76	-63	-63
Emittance	Input (90%):	0.035	E_o(MV/m)	0	0	0.57	2.69	1.26
	Output (90%):	0.049	V(kV)	78	78	78	78	78
	Input (RMS):	0.0083	B	0.40	6.86	6.86	6.86	6.86
	Output (RMS):	0.0109	L(m)	0	0.04	0.71	1.40	4.24

Table 3

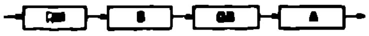
RFQ for 200 MHz



	Ion: Proton							
	Frequency:	200 MHz	W(MeV)	0.025	0.025	0.030	0.34	0.75
	Nominal Current:	60 mA	E_s(MV/m)	5.7	29.5	29.5	29.5	29.5
	Current Limit:	107 mA	m	1.00	1.00	1.15	2.46	2.46
	PARMTEQ RESULTS		a(cm)	3.68	0.53	0.50	0.30	0.29
	Input Current:	60 mA	r_o(cm)	3.68	0.53	0.53	0.53	0.53
	Output Current:	59 mA	ϕ_s(deg)	–	-90	-83	-30	-30
Emittance	Input (90%):	0.042	E_o(MV/m)	0	0	0.63	3.91	2.66
	Output (90%):	0.066	V(kV)	113	113	113	113	113
	Input (RMS):	0.010	B	0.2	9.68	9.68	9.68	9.68
	Output (RMS):	0.015	L(cm)	0	5.5	31	73	93

Table 4

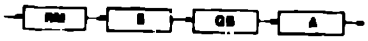
High Current Proton RFQ



	Ion: Proton						
	Frequency: 100 MHz	W(MeV)	0.10	0.10	0.115	1.26	2.00
	Nominal Current: 300 mA	E_0(MV/m)	3.4	22.9	22.9	22.9	22.9
	Current Limit: 500 mA	m	1.00	1.00	1.11	2.50	2.50
	PARMTEQ RESULTS	a(cm)	12.1	1.83	1.74	1.00	0.98
	Input Current: 320 mA	r_0(cm)	12.1	1.83	1.83	1.83	1.83
	Output Current: 315 mA	ϕ_s(deg)		-90	-83	-30	-30
Emittance	Input (90%): 0.35	E_0(MV/m)	0	0	0.44	2.64	2.16
	Output (90%): 0.43	V(kV)	306	306	306	306	306
	Input (RMS): 0.083	B	0.20	8.8	8.8	8.8	8.8
	Output (RMS): 0.104	L(m)	0	0.22	1.30	3.55	4.09

Table 5

RFQ for Plasma Diagnostics



	Ion: $^6Li^-$						
	Frequency: 50 MHz	W(MeV)	0.10	0.10	0.11	1.11	6.00
	Nominal Current: 100 mA	E_s(MV/m)	3.6	18.0	18.0	18.0	18.0
	Current Limit: 171 mA	m	1.00	1.00	1.10	1.78	1.78
	PARMTEQ RESULTS	a(cm)	8.47	1.68	1.61	1.20	1.17
	Input Current: 110 mA	r_0(cm)	8.47	1.68	1.68	1.68	1.68
	Output Current: 102 mA	ϕ_s(deg)		-90	-83	-30	-30
Emittance	Input (90%): 0.21	E_0(MV/m)	0	0	0.27	1.67	0.82
	Output (90%): 0.24	V(kV)	223	223	223	223	223
	Input (RMS): 0.050	B	0.2	5.03	5.03	5.03	5.03
	Output (RMS): 0.056	L(m)	0	0.18	1.37	4.87	11.54

system for this linac consists of a 750-kV Cockcroft-Walton generator, followed by systems for both bunching and low-energy beam transport. This example illustrates how the RFQ could replace the conventional injection system. The RFQ, shown in Table 3, bunches and accelerates a 60-mA dc beam of either H^+ or H^- from 25 keV to 750 keV in a length of only 93 cm.

High-Current Proton RFQ

An example of a proton RFQ that accelerates 300 mA of beam current to 2 MeV is given in Table 4. It operates at 100 MHz and uses a dc injection voltage of 100 kV. Such an RFQ might be suitably applied as an injector to a drift-tube linac for a proton spallation-type neutron generator such as is under consideration at the Chalk River Nuclear Laboratory.[18]

RFQ for Plasma Diagnostics

Table 5 gives the design parameters for an RFQ to accelerate 100 mA of ^{6}Li to 1 MeV/nucleon. If the output beam is stripped of one electron to make a neutral atom, it could be injected into a magnetically confined plasma. The double-charge exchange reaction of the beam with helium ions in the plasma provides the basis for a technique[19] to measure a velocity spectrum of the helium.

For this application, operation at a few discrete energies may be all that is necessary. As was discussed previously, a continuous energy variation could be obtained by adding independently phased cavities after the RFQ. For a nonresonant RFQ configuration, another possibility for continuously changing the energy is to change the frequency. A change in frequency would affect both the injection and final energies. Also, with separate drives to separate sections of the RFQ, a number of discrete energies could be produced by simply lowering the excitation of one or more downstream sections to about 80% of the design excitation. This would drop the particles out of the acceleration process but still would contain them radially in the quadrupole fields.

Acknowledgments

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