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The radius-dependence of velocity dispersion in elliptical galaxies

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Summary. The equations of stellar hydrodynamics are used to derive the radial variation of velocity dispersion in galaxies whose mass-to-light ratios are constant and whose brightness profiles obey the $r^{1/4}$-law. One finds that the projected central velocity dispersion in such a system should be about 40 per cent lower than the peak velocity dispersion. The observability and physical interpretation of this phenomenon is discussed. There is some evidence that a similar effect has been observed in the structure of rich clusters of galaxies.

1 Introduction

Our knowledge of the nature of giant elliptical galaxies has recently been deepened in two important respects: measurements of the radial variation of velocity dispersion within giant elliptical galaxies are now available (Sargent *et al.* 1978; Schechter & Gunn 1979; Davies 1979) and the quality of the published photometry of these galaxies has improved markedly (King 1978; Carter 1978, 1979; de Vaucouleurs & Capaccioli 1979; Schweizer 1979). This note is concerned with the connection between these two types of observation. In particular I draw attention to a rather remarkable consequence for velocity dispersion measurements of two recent studies of the central brightness distributions of elliptical galaxies.

A very straightforward connection between velocity dispersion and mass density is furnished by the equations of stellar hydrodynamics (e.g. Oort 1965). These equations apply quite generally to all types of equilibrium, near-collisionless stellar system, irrespective of the complexities regarding non-classical integrals of stellar motion that have arisen recently in regard to measurements of rotation velocities in elliptical galaxies (for a review see Binney 1979). Therefore one may hope to gain useful insight into the structure, and perhaps origin, of elliptical galaxies by applying the equations of stellar hydrodynamics to observations.

In the next section, I first discuss the velocity dispersion structure that should be associated with the rather centrally-peaked brightness profiles that two recent studies (de Vaucouleurs & Capaccioli 1979; Schweizer 1979) have suggested may more nearly represent the small-scale structure of many giant elliptical galaxies than the near-isothermal models of King (1966), Wilson (1975) and others. One finds that the equations of stellar hydrodynamics predict that there should be a local minimum in the velocity dispersion at the centre of a sharply centrally-concentrated galaxy. In Section 3 I clarify the significance of this phenomenon with the aid of a family of simple models and discuss the possibility of observing central minima in the velocity dispersions of galaxies. Section 4 sums up and discusses the applicability of these ideas to the case of rich clusters of galaxies.

2 The de Vaucouleurs model

For the sake of definiteness, I shall in the following be primarily concerned with the giant E1 galaxy NGC 3379, which has been recently studied photometrically by de Vaucouleurs & Capaccioli (1979) and spectroscopically by Sargent *et al.* (1978). De Vaucouleurs & Capaccioli find that, if proper allowance is made for the effects of seeing, NGC 3379 has a brightness profile which is at least as peaked as that obtained by extrapolating de Vaucouleurs' $r^{1/4}$-law (which fits the observations of NGC 3379 over a range of more than 10 mag) right in to the centre of the galaxy. In some respects this conclusion is surprising. Indeed the $r^{1/4}$-law is itself the most strongly peaked of all the laws commonly used to describe the surface brightnesses of elliptical galaxies; the Hubble (1930) and King (1966) profiles predict respectively finite and zero central gradients of surface brightness in a linear–linear plot of surface brightness against radius, whereas, in these coordinates, de Vaucouleurs' law envisages an infinite central surface-brightness gradient.

However, several considerations indicate that serious consideration must be given to the possibility that the brightness of NGC 3379 does continue to rise steeply near its centre: (i) If the star density does not rise near the centre of NGC 3379 as rapidly as does the luminosity, much of the central light will derive from a point source and therefore not from the same stellar population that emits most of the galaxy's light. The absorption line-strength, γ, might consequently be expected to show a minimum near the centre. No minimum was observed by Sargent *et al.* (1978), however. (ii) The case of NGC 3379 is not an isolated one. King (1978) has already remarked that many of the giant ellipticals he has studied photometrically show larger central brightness gradients than would be expected if they were well approximated by lowered isothermal models. Further, Schweizer (1979) has recently argued that nearly all King's galaxies have unresolved cores; according to Schweizer, King's profiles are best understood as seeing-broadened de Vaucouleurs profiles. As Schweizer emphasizes, a particular attraction of this interpretation of King's data is that it brings the central surface brightnesses of distant ellipticals into line with those of Local Group galaxies; if the cores of King's galaxies are resolved, their central surface brightnesses are appreciably lower than that of, for example, M31 as observed by Stratoscope II (Light, Danielson & Schwarzschild 1974). Therefore it seems probable that the star density does rise strongly towards the centre of NGC 3379, and that one has to ask what radial variation of velocity dispersion is to be expected.

In the interests of simplicity, let me assume that the true (seeing-corrected) brightness profile of NGC 3379 obeys the de Vaucouleurs law at all radii, and that the mass-to-light ratio is constant everywhere. The image of NGC 3379 is nearly circular, so it is reasonable to attempt to model the galaxy as a spherical system. Standard considerations then enable one to derive the space-density of stars by deprojection of the surface brightness. The accuracy of this procedure may be checked by numerically projecting the derived space-density back on to the plane of the sky. I find that the agreement is good to better than 1 per cent.

Although I assume that NGC 3379 is spherically-symmetric, it would not be in order to presuppose that the velocity dispersion is everywhere isotropic. The strongest assumption which can be fairly safely made is that the velocity dispersion is the same in all directions perpendicular to a given radial vector. If $\sigma_r(r)$ denotes the velocity dispersion along radial vectors and $\sigma_t(r)$ the dispersion in the perpendicular directions, one then has that the general equations of stellar hydrodynamics, which may be written

$$\frac{\partial}{\partial x_j}(\rho\overline{v_i v_j}) = -\rho\frac{\partial\Phi}{\partial x_i}, \tag{1}$$

reduce to the single equation

$$\frac{d}{dr}(\rho\sigma_r^2) + \frac{2\beta}{r}\rho\sigma_r^2 = -\rho\frac{d\Phi}{dr}. \tag{2}$$

Here $\rho(r)$ is the mass-density, Φ is the gravitational potential of the system and

$$\beta = 1 - (\sigma_t/\sigma_r)^2 \tag{3}$$

describes the degree of velocity anisotropy in the system. In a galaxy whose density profile flattens near the centre, one would expect β to decrease inwards because even if most of the stars of the system were on highly radial orbits, there would still be a substantial tangential component of $\boldsymbol{\sigma}$ in the central constant-density region in which most orbits would be at perigalacticon. If, however, the flattening of the density profile were only slight, as for example in the case of a galaxy whose projected profile obeyed the de Vaucouleurs law, the inwards decrease of β might also be only modest. Therefore it is worthwhile to investigate the radial variation of σ_r under two hypotheses; (A) β = constant, and (B) $\beta = r^2/(r_c^2 + r^2)$, which is the form of β which results from Eddington's form of distribution function, $f(\mathbf{x}, \mathbf{v}) = K \exp - (\alpha E + \gamma J^2)$, where E is the stellar energy per unit mass and J^2 the square of total stellar angular momentum per unit mass; with this notation $r_c^2 = (\alpha/2\gamma)$. Equation (2) may be immediately reduced to a quadrature under either of these hypotheses, to yield

$$\sigma_r^2(r) = G\left\{\int_r^\infty \rho M(r)\, r^{(2\beta-2)}\, dr\right\}\Big/(r^{2\beta}\rho) \tag{4A}$$

or

$$\sigma_r^2(r) = G\left\{\int_r^\infty \rho M(r)[(r_c/r)^2 + 1]\, dr\right\}\Big/[(r_0^2 + r^2)\,\rho], \tag{4B}$$

for cases (A) and (B) respectively. Here Poisson's equation has been used to eliminate Φ in favour of $M(r)$, the mass interior to r.

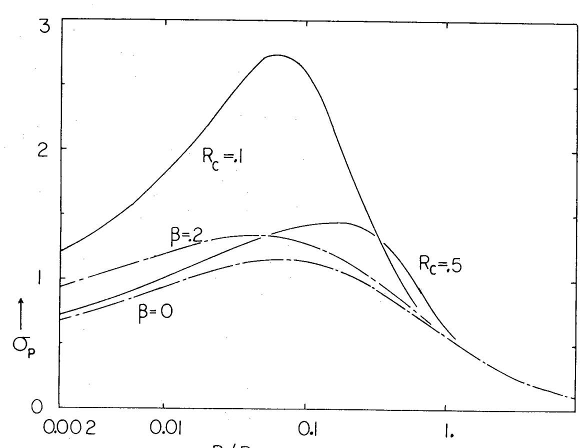

Figure 1. Projected line-of-sight velocity dispersion versus radius for de Vaucouleurs models. The broken curves have been obtained from equation (4A) with the indicated values of the velocity–anisotropy parameter β. The full curves are derived from equation (4B) for the variable-anisotropy model and are labelled by the corresponding values of the isotropy-radius r_c.

Fig. 1 shows the radial variation of projected σ that is derived by means of equations (4) from the de Vaucouleurs profile by taking (i) $\beta = 0$ or 0.2 under case (A), and (ii) $r_c = 0.5$ or 0.1 under case (B). The curves for $\beta = 0$ and $\beta = 0.2$ peak at $r = 0.07r_e$ and $r = 0.05r_e$ respectively (r_e is the effective radius, interior to which the $r^{1/4}$-law places half the galaxy's light). The curves obtained under case (B) peak at rather larger radii; $r = 0.2r_e$ and $r = 0.06r_e$ respectively. The depth of the central depression in σ is in every case greater than 40 per cent of the peak dispersion.

3 Significance of the σ-profiles

The significance of the peaks in the σ-curves of Fig. 1 must depend on the answers to three questions: (i) Is the non-monotonic behaviour of $\sigma(r)$ caused by some pathology of the particular model assumed here or a rather general property of a whole class of realistic models? (ii) If the $\sigma(r)$-profile of NGC 3379 really does look like those shown in Fig. 1, would this have been detected by Sargent *et al.* (1978)? (iii) If the observations of Sargent *et al.* lacked sufficient resolution to detect a central minimum in $\sigma(r)$, are adequately precise observations in principle possible? Let me discuss these points in turn.

3.1 GENERALITY

Consideration of a class of simple models will serve to show why the profiles of Fig. 1 have central minima. As King (1972) has remarked, a system whose mass-density $\rho(r)$ obeys the formula

$$\rho(r) = \rho_0/(1 + [r/a]^2)^{3/2} \tag{5}$$

has a density and velocity-dispersion structure near its centre that is similar to that of a King lowered isothermal sphere (King 1966). The dashed curves in Fig. 2 show such profiles. The numbered curves in Fig. 2 show the projected density and velocity dispersion profiles of models whose mass-densities are superpositions of two components of the type specified

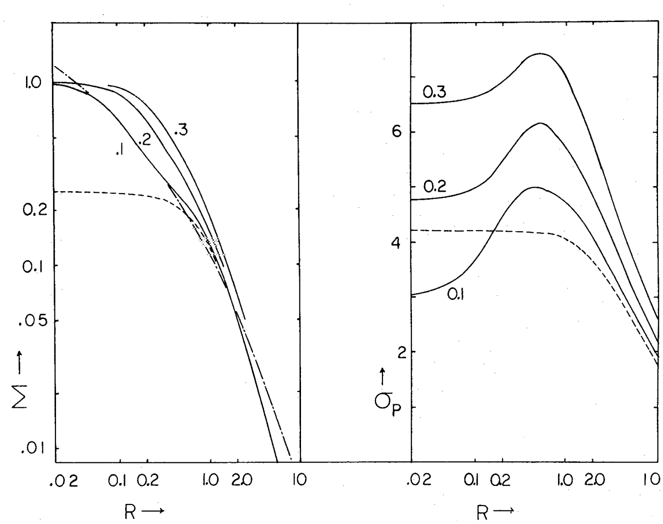

Figure 2. Projected surface density, Σ, and velocity dispersion σ_p, profiles of isotropic, two-component models. The dashed lines show the Σ- and σ_p-profiles of the standard component defined by equation (5) with $\rho_0 = a = 1$. The full curves are for models having additional components with the indicated values of a and $\rho_0 = 3/a$. The dashed–dot line in the left-hand panel shows the Σ-profile of a de Vaucouleurs model having $r_e = 17.9$.

by equation (5). One component in each case has $\rho_0 = a = 1$ (the 'standard' component) and corresponds to the dashed curves. The secondary components have $a \neq 1$ and $\rho_0 = 3/a$. With this choice of ρ_0 exactly three-quarters of the central light of any model comes from the secondary component. The curves in Fig. 2 are labelled by the corresponding a-values.

One sees that all the two-component models have $\sigma(r)$-profiles which peak near $r = 0.6$. The depth of the central depressions in the σ-profiles increases as the a-value of the secondary component is reduced, because, while the fraction of the central light which is attributable to the secondary component remains constant, the central velocity-dispersion, σ_c, which would be associated with the secondary component if it were isolated, obeys $\sigma_c^2 \sim \rho_0 a^2$, or $\sigma_c^2 \sim a$ when account is taken of the a-dependence of ρ_0. The left-hand panel of Fig. 2 indicates, however, that as the a-value of the secondary component shrinks towards $a = 0.1$, the projected density of the entire model becomes well-approximable for $r > 0.003\, r_e$ by the de Vaucouleurs law.

Thus it is clear that the failure of the de Vaucouleurs model to flatten off interior to $r = 0.1\, r_e$ is directly responsible for the central depression in the $\sigma(r)$-profile one derives from it. Indeed the central brightness peak of the de Vaucouleurs model may be interpreted as being due to the presence at the centre of the galaxy of a cool, tightly-bound sub-system of stars. But note that not every centrally-peaked brightness distribution will be associated with a central depression in σ; a two-component model in which more than 90 per cent of the central light was contributed by a secondary component having $a = 0.1$ would have a centrally-peaked σ-profile. Also if the projected density profile were to rise near the centre as r^{-1}, the velocity dispersion would be constant near the centre because the system would then resemble the infinite central-density isothermal sphere. However, neither of these models would be approximable by the de Vaucouleurs law.

3.2 OBSERVABILITY

De Vaucouleurs & Capaccioli estimate that the effective radius, r_e, of NGC 3379 is $r_e = 54$ arcsec, from which it follows that in the isotropic case σ should peak near $r = 3.8$ arcsec. Fig. 3 shows the projected velocity dispersions one obtains by convolving three of the appropriately-scaled models of Fig. 1 with a Gaussian point-spread function, $f(r) \sim \exp - (r^2/2\delta^2)$, having $\delta = 0.7$ arcsec. From this figure it is apparent that in the most

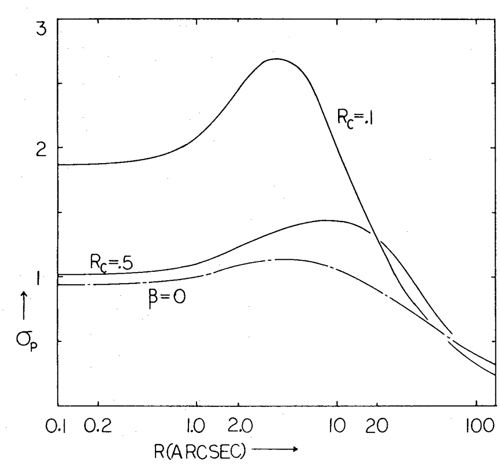

Figure 3. Plots of projected line-of-sight velocity dispersion scaled to the effective radius of NGC 3379. The models of Fig. 1 have been convolved with a Gaussian point-spread function corresponding to 0.7 arcsec seeing.

pessimistic case, observations at 0.7 arcsec resolution would require less than 15 per cent internal accuracy to detect the predicted central depression in σ. The measurements of Sargent *et al.* are subject to only 10 per cent standard errors. However, these measurements have spatial resolution of 2.4 arcsec, which is not sufficient for any effect to be detectable with 10 per cent error bars. Nevertheless, it is clear from Fig. 3 that precise measurements made in good seeing should detect the predicted central depression in σ if it actually exists in galaxies like NGC 3379.

3.3 INTERPRETATION IN TERMS OF DISTRIBUTION FUNCTIONS

The foregoing considerations have not required any assumption regarding the nature of the phase-space distribution function, $f(\mathbf{x}, \mathbf{v})$ of the systems. It is, however, interesting to enquire whether anything could be concluded about the distribution function of galaxies if the latter were found to have velocity-dispersion minima near their centres.

The simplest assumption one can make regarding the distribution function of a spherical system is that it depends on the phase-space coordinates only through the energy, $E = \frac{1}{2}v^2 + \Phi$, of the constituent particles. Even in more complex models, as when f depends on E and the total angular momentum J, for example, the radial density structure is principally determined by the energy dependence of f (Michie 1964). Therefore, let me write $f = f(E)$ and $\sigma^2 = v^2$, so that (E_m being such that $f(E) = 0$ for $E > E_m$)

$$\overline{v^2} = \int_{\Phi}^{E_m} [2(E-\Phi)]^{3/2} f(E)\, dE \bigg/ \int_{\Phi}^{E_m} [2(E-\Phi)]^{1/2} f(E)\, dE. \tag{6}$$

Differentiating this with respect to Φ, one obtains

$$d\overline{v^2}/d\Phi = -3 + \overline{v^2} \times \overline{v^{-2}}. \tag{7}$$

Φ is a monotonically increasing function of r, so $d\overline{v^{-2}}/dr > 0 \Rightarrow d\overline{v^2}/d\Phi > 0 \Rightarrow \overline{v^2} \times \overline{v^{-2}} > 3$; i.e. the necessary and sufficient condition for the velocity dispersion to increase outwards is that there should be relatively large populations of both fast- and slow-moving stars, compared to those of intermediate velocity. The two-component model discussed above satisfies this condition because it has a bi-modal velocity distribution. However, bi-modality is not necessary for the velocity dispersion to increase outwards, as may be seen by considering the distribution function

$$\begin{aligned} f(E) &= \exp\, [\![-k\sqrt{2[E-\Phi(0)]}]\!] && 0 < 2[E-\Phi(0)] < 1 \\ f(E) &= 0 && 1 < 2[E-\Phi(0)]. \end{aligned} \tag{8}$$

For then, $\overline{v^2} \times \overline{v^{-2}} > 3$ for small r and $k > 2.7$. In fact the relation $\overline{v^2} \times \overline{v^{-2}} = 3$ appears to be a rather special property of the Gaussian distribution of velocities, so that in a general collisionless system there is no fundamental reason why $d\overline{v^2}/d\Phi$ should be negative.

4 Conclusions

Evidence is accumulating that the star-densities of giant elliptical galaxies continue to rise very close to the centres. If, as has been suggested by de Vaucouleurs & Capaccioli (1979) and by Schweizer (1979), the brightness rises approximately according to de Vaucouleurs' $r^{1/4}$-law, one might expect to be able to observe a 40–60 per cent drop in the velocity dispersion near the nucleus. Models exhibiting a wide range of velocity-dispersion structure

– ones with isotropic dispersion or with constant or variable velocity-anisotropy – all display central velocity-dispersion depressions. The radius at which the peak velocity dispersion occurs does depend somewhat on the assumed form of velocity anisotropy, but the value $r = 0.07r_e$ of the isotropic model may be considered typical. For the nearby giant elliptical galaxy NGC 3379 this radius corresponds to 3.8 arcsec. In 0.7 arcsec seeing the apparent velocity dispersion of NGC 3379 would, in the isotropic case, peak at 4.5 arcsec and then drop at the centre to 82 per cent of its peak value.

Although the prediction of a central depression in the velocity dispersion is not sensitive to ones assumptions regarding velocity anisotropy, simple two-component models of elliptical galaxies make it clear that the prediction is sensitive to the assumption of constant mass-to-light ratio. The de Vaucouleurs model can be approximated by a system of two superposed nearly isothermal spheres, such that the innermost sphere has one-tenth the scale-length and three times the central surface brightness of the outer component. If the mass-to-light ratio of the central component were more than three times that of the outer component, it would have the higher velocity dispersion, and the overall velocity dispersion would rise towards the centre. It is to this type of mass-to-light ratio variation that Sargent *et al.* have recently appealed to account for an apparent rise in σ near the centre of M87. The present investigation shows that a galaxy which, like M87, has a centrally-peaked brightness profile, would have a centrally-peaked mass-to-light ratio even if σ were constant near the centre.

It would seem that no inactive elliptical galaxy has been studied at a spatial and spectral resolution at which the predicted depression in σ could have been detected. However, an observational test of this prediction for nearby elliptical galaxies seems to be feasible and indeed constitutes an important task for the future.

The requirement that the velocity dispersion increase outwards at certain radii does not impose any very remarkable constraint on galaxian stellar distribution-functions; one requires merely that both the high- and low-velocity stellar groups be more populous (relative to the middle range) than is implied by a Gaussian distribution of velocities.

Quintana (1979) has recently shown that the surface density of the Coma cluster is still rising at one-third of a best-fit isothermal core–radius on account of a 'narrow density peak within 0.1 Mpc of NGC 4874'. The considerations of this paper indicate that this phenomenon should be associated with a central depression in the cluster velocity dispersion. Struble (1979) has collected the available velocity-dispersion data for Coma and several other clusters, and finds that the velocity dispersion in Coma indeed shows a central minimum, although the statistical significance of the result is only marginal. However, many clusters do appear to possess tight central groupings and it seems probable that these will be associated with central depressions in the velocity dispersion. If this is indeed the case and if a similar effect is discovered in giant ellipticals, this similarity of structure will have important consequences for theories of the origins of the central density peaks in the two kinds of system.

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