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ABSTRACT

The Bethe-Bloch theory of stopping, including shell corrections and the density-effect correction, is used to calculate theoretical ranges for standard Ilford G.5 emulsion. Only the mean ionization potential is an adjustable parameter. Making corrections for the K shells of all emulsion atoms except hydrogen, and also correcting for the L shells of iodine, silver, and bromine, one can obtain a fit to the measured ranges for protons of 1 to 700 Mev. The mean ionization potential found is 331 ± 6 ev, and I/Z has a value 12.1 $\pm$ 0.2 ev as an average for all emulsion elements except hydrogen. At low velocities the hydrogenlike atomic model used in the shell-correction calculation appears to overaccentuate the shell effects. A better agreement with the experimental data for proton energies below 40 Mev is obtained semiempirically. A range table for emulsion of standard density is given. A calculated range table for water, useful for adjusting ranges measured under nonstandard density conditions, is also included.

THE RANGE-ENERGY RELATION IN EMULSION
Part 2.   THE THEORETICAL RANGE*

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Introduction

In Table I of Part I we have listed our measured particle ranges with
their associated statistical errors.[1] It is now proposed to draw on theoretical
considerations as well as on other experimental data to obtain the best continuous
functional relationship between the particle energy and its range in emulsion.
The same notation as in Part I is used throughout.   The appropriately corrected
Bethe-Bloch theory of stopping contains only one adjustable parameter, the
mean ionization potential.[2] By means of the theory we use all the experimental
points to improve our knowledge of the range for each particular velocity.
The theoretical relation provides a means of interpolating between the experi-
mental points, and of extrapolating to energies beyond those for which ranges
have been measured.

Characterization of the Stopping Medium

Nuclear track emulsion is a complex substance.   It consists of heavy
inorganic crystals a few tenths of a micron in diameter suspended in an organic
medium.   The matrix material, which is largely gelatin, readily absorbs water,
and the density of the emulsion varies with its water content, and also locally
as the statistical ratio of heavy to light components fluctuates.   The rate at which
charged particle loses energy in the heavy component varies with velocity in
a different way from that in the light component.

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Commission.

We assume the average composition[3] of emulsion having a density of 3.815 g/cm³ to be that given in Table I. This "standard" density of emulsion is arbitrary, but it has been chosen near that normally found for Ilford G.5 emulsion in equilibrium at a relative humidity of 60%.[3]

Stopping Theory

For particles of low velocity, Niels Bohr[4] has theoretically deduced Geiger's empirical law that the ranges increase with the cube of the particle velocity. This suggests plotting the low ranges in emulsion against the 3/2 power of the kinetic energy. In Fig. 1 the data are seen to define straight lines. It should be noted, however, that finite intercepts are obtained if the lines are extended to zero energy. The intercept consists of (a) the term B_z, which allows for the range extension caused by capture of electrons at low particle velocities,[5] and (b) the overestimate of the ranges commonly introduced when one measures a saturated track between the extremities of the first and last grains. The measured length of the track of a particle cannot, of course, be less than one grain diameter. Very short ranges are meaningless because of the granular two-component structure of emulsion. Adjusting the ranges to standard emulsion, we obtain the following range-energy relation for $\tau < 1$ Mev:

$$\lambda = 0.0006 + 0.00138 \, \tau^{3/2} \text{cm.} \tag{1}$$

The Bethe-Bloch theory is usually employed to calculate the rate of energy loss of a fast charged particle penetrating matter. The uncorrected theory is valid, however, only in the velocity interval extending from a point well above the velocity of the fastest electron in the stopping material to a point near the minimum of ionization. In this interval the stopping behavior of an element, Z_i, is characterized by the electron density $N_i Z_i$ and a "mean ionization potential" I_i. (N_i is the number of atoms per cubic centimeter. According to the Bethe-Bloch theory the mean space rate of energy loss, Q_i, of a particle carrying z electronic units of charge is given by

Table I. Composition of "standard" emulsion

Element	Z_i	g/cm^3	N_i ($\times 10^{-20}$)	$N_i Z_i$ ($\times 10^{-22}$)
Ag	47	1.8088	101.01	47.476
Br	35	1.3319	100.41	35.143
I	53	0.0119	0.565	0.299
C	6	0.2757	138.30	8.298
N	7	0.0737	31.68	2.218
S	16	0.0072	1.353	0.216
H	1	0.0538	321.56	3.216
O	8	0.2522	94.97	7.597

Totals: 3.815 g/cm^3; 1.0446 $\times 10^{24}$ electrons/cm^3

$$\vartheta_i = \frac{2\pi z^2 e^4}{mc^2 \beta^2} (N_i Z_i) \left[\ln\left(\frac{2mc^2 \beta^2 \gamma^2 t}{I_i^2}\right) - 2\beta^2\right] ergs/cm . \tag{2}$$

Here $t \equiv \dfrac{(\gamma^2 - 1)\mu c^2}{(\mu/2m + m/2\mu + \gamma)}$ is the maximum energy transfer possible to

an (assumed stationary) electron of mass m by the particle of mass μ, velocity βc, and kinetic energy $\mu c^2 (\gamma - 1)$. In the approximation to which the theory is usually carried, the particle-structure-dependent effects as well as the spin and sign-of-charge effects in the stopping cross section are neglected. If the stopping particle is heavy compared with an electron, and γ is small compared with this mass ratio, one obtains a simpler formula which does not contain the particle mass. This is

$$\vartheta_i = \frac{4\pi z^2 r_0^2}{\beta^2} N_i Z_i \left[\ln\left(\frac{2mc^2}{I_i} \beta^2 \gamma^2\right) - \beta^2\right] mc^2/cm \tag{3}$$

where $r_0 = e^2/mc^2$.

Even when a medium (such as emulsion) that contains several different kinds of atoms is used as the stopping material, the form of the energy-loss equation remains unchanged:

$$\mathcal{I} = \frac{4\pi z^2 r_0^2 n}{\beta^2} \left[\ln \left(\frac{2mc^2}{I} \beta^2 \gamma^2 \right) - \beta^2 \right] mc^2/cm . \tag{4}$$

Here n, the total number of electrons per cubic centimeter, is given by $n = \sum_i N_i Z_i$, and the mean ionization potential, I, is found from

$$n \ln I = \sum_i N_i Z_i \ln I_i .$$

For heavy atoms, described by the Fermi-Thomas model, F. Bloch found that I_i/Z_i is a constant, K, with a value of about one Rydberg.[8] This is a useful concept despite the fact that small variations from constancy doubtless occur as the electronic shell structure is built up, and the precise value of the constant has not proved easy to measure.

On the other hand, there is little reason to doubt that for sufficiently high—but not too high—velocities an expression of the form of Eq. (4) is capable of giving the rate of energy loss with good accuracy.

When the particle velocity is lowered so that it is comparable to the velocities of some of the electrons in the stopping atoms, Eq. (4) becomes progressively less valid. The tightly bound electrons are perturbed only adiabatically by the moving particle, and become ineffective in stopping. The behavior of the logarithm at low velocities is also wrong. K- and L-shell corrections have been calculated by Walske[9,10] which extend the usability of the theory toward lower velocities, but when applied to the heavy-component elements of emulsion, the corrections are large for proton energies of even several Mev. This part of the range-energy curve is perhaps the most used, and the lack of an exact theory in this region is a serious misfortune. At low velocities other effects also set in. The stopping of a positive particle is influenced by the neutralization of its charge by electrons that it captures,[5] and for heavy particles at low velocities, nuclear collisions become important.

At very high velocities the unlimited rise of the ionization implied by
Eq. (4) is curtailed by the polarizability of the medium. This effect has been
most thoroughly evaluated by Sternheimer.[11]

For a particular element one may, in principle, write an exact expression
for $\mathcal{J}_i$ by including an additive term, which is usually small:

$$\mathcal{J}_i = \frac{4\pi z^2 r_0^2}{\beta^2} N_i Z_i \left[\ln\left(\frac{2mc^2}{I_i} \beta^2 \gamma^2\right) - \beta^2 - \frac{C_i(\beta)}{Z_i} \right] mc^2/cm \quad (5)$$

Here C_i is the sum of all the individual corrections for the element in question.
The corresponding expression for emulsion is

$$t = \mathcal{J}/z^2 = \frac{4\pi r_0 n}{\beta^2} \left[\ln\left(\frac{2mc^2}{I} \beta^2 \gamma^2\right) - \beta^2 - C(\beta) \right] mc^2/cm \quad (6)$$

In this form I is considered a fixed constant. It has a characteristic value
which may be determined at moderately high energies where $C(\beta)$ approaches
zero. At very high velocities $C(\beta)$ is interpreted as one half of Sternheimer's
δ, and at low velocities

$$C(\beta) = \frac{\Sigma_i N_i C_i}{n} \quad . \quad (7)$$

The _effective_ mean ionization potential deviates from the constant I by an
approximate percentage $100\, C(\beta)$.

Evaluation of $C(\beta)$

By utilization of the curves of Walske,[9,10] the shell correction contribution
to $C(\beta)$ have been evaluated. For the _ith_ element we have

$$C_i = (C_K + C_L + \dots)_i . \quad (8)$$

K-shell corrections were made for all the elements in emulsion except
hydrogen, and L-shell corrections also were included for iodine, silver, and
bromine. With these additions the usefulness of the Bethe-Bloch theory is

extended down to about 1 Mev for protons, but it is difficult to estimate the possible errors remaining from the uncorrected shells and from the approximations in the atomic model used. Since Walske's calculation is non-relativistic, the curves do not behave correctly at high velocities. Following a suggestion of Dr. Walske, we have used $\beta\gamma$ to replace β wherever it appears in his formulas.

Sternheimer[11] has given an empirical formula for δ which estimates the density-effect correction at high velocities. The constants in the formula depend on the mean ionization potential of the stopping material. From our measurements (Part 1) at velocities at which the density effect is negligible, we have found $I = 331$ ev. For this value the density correction is

$$C = \delta/2 = 2.303 \log_{10} \beta\gamma - 2.66 + 0.103 (3-\log_{10}\beta\gamma)^{2.97} . \tag{9}$$

The shell corrections are already vanishingly small at the high velocities at which the density effect must be taken into consideration, so that two different functions for C are applicable--one at low velocities and the other at high. Numerical values of $C(\beta)$ are listed in Table II.

Evaluation of the Mean Ionization Potential

By integrating $\lambda = \lambda_1 + \displaystyle\int_{\tau_1}^{\tau} \frac{d\tau}{b}$ one may predict the range for an energy τ in an interval, where (with an assumed mean ionization potential) b may be calculated from Eq. (6). Each range measurement in Part 1 provides an almost independent estimate of the mean ionization potential. The best value of the mean ionization potential may be derived in this way from the measured points. Our range measurements are in satisfactory agreement with the adjusted dry-emulsion measurements of earlier experiments for $\tau < 5$ Mev. As a most probable value we take $\lambda_1 = 176.5$ microns for $\tau_1 = 5$ Mev. There is also good agreement between measurements of the $\pi-\mu$ decay range. Here we take $\tau = 36.55$ Mev and $\lambda = 5345 \pm 22$ microns. From these points we obtain a mean ionization potential of 330 ± 7 ev. The highest-energy point was obtained with good accuracy, and it is the least influenced by shell-correction effects.

Table II. C(β) Calculated from the theories of Walske and Sternheimer

τ	C	τ	C	τ	C
1.2	-0.081	5.0	0.072	200	0.016
1.4	-0.059	5.4	0.075	260	0.011
1.6	-0.040	6.0	0.079	300	0.009
1.8	-0.023	7.0	0.084	400	0.006
2.0	-0.009	8.0	0.085	500	0.005
2.2	+0.003	9.0	0.086	700	0.004
2.4	0.014	10.0	0.088	1,000	0.002
2.6	0.023	12.0	0.087	1,200	0.004
2.8	0.031	16.0	0.083	1,400	0.009
3.0	0.038	20.0	0.078	1,600	0.020
3.2	0.043	24.0	0.073	1,800	0.033
3.4	0.048	28.0	0.067	2,000	0.046
3.6	0.053	30.0	0.065	3,000	0.115
3.8	0.057	40.0	0.055	4,000	0.184
4.0	0.060	50.0	0.048	5,000	0.250
4.2	0.063	70.0	0.038	10,000	0.524
4.4	0.065	100.0	0.030	20,000	0.902
4.6	0.067	140.0	0.023	30,000	1.167
4.8	0.070				

From the 5-Mev point and this measurement we derive a mean ionization potential of 332 ± 13 ev. The other measured points give values both above and below these values, and the over-all weighted average, considering only statistical errors, is I = 327 ± 4 ev. In view of the several corrections that were found necessary, however, small systematic errors may remain in some of the experimental points. The possible influences of inexact corrections to the various points have been reviewed, and I prefer to quote 331 ± 6 ev for the mean ionization potential of emulsion. The sensitivity of the range to the mean ionization potential is shown in Fig. 2 which can be used to estimate the uncertainty in the range curve arising from the remaining uncertainty in the mean ionization potential.

If one assumes that I_i/Z_i is a constant, K, we may evaluate it, knowing the emulsion composition. For I = 331 ± 6 ev K is 12.25 ± 0.22 ev. Actually the hydrogen of the emulsion should probably be treated separately. If the mean ionization potential attributed[11] to hydrogen is 17.6 ev, we obtain K = 12.1 ± 0.2 ev for the remaining elements of the emulsion. Since only about 18% of the emulsion electrons are in the light atoms of the C, N, O group, this value of K should be substantially that applying to the heavy elements of the Ag, Br, I group.

The value K = 12.1 ev is in fairly good agreement with the results of Burkig and McKenzie[12] and Bichsel, Mozley, and Aron,[13] while Caldwell[14] has proposed an even higher I/Z ratio. It is not, however, in accord with the lower ratios of Mather and Segrè.[15]

Range Calculations

A theoretical range-energy relation for standard emulsion was calculated by using the table of C(β), and the following formulas

(a) for $\tau < 1$ Mev,
$$\lambda = 0.0006 + 0.00138 \, \tau^{3/2} \text{ cm};$$

(b) for $\tau > 1$ Mev,
$$\lambda = 0.00144 + \int_{1}^{\tau} \frac{d\tau}{\iota} \text{ cm},$$

$$\iota = \frac{0.5326}{\beta^2} \left[\ln (3088 \, \beta^2 \gamma^2) - \beta^2 - C \right] \text{ Mev/cm}.$$

In order that ranges in emulsion of other than standard density may be corrected, the range in water has also been calculated. The K-shell correction for oxygen and the density effect have been included. This was done by finding an empirical function to fit $C_K(\beta)$. The formulas are:

(a) for $\tau > 0.1$ Mev,

$$\lambda_w = 0.00011 + \int_{.1}^{T} \frac{d\tau}{b_w},$$

$$b_w = \frac{0.170}{\beta^2} \left[\ln (1.38 \times 10^{-4} \beta^2 \gamma^2) - \beta^2 - C \right] \text{Mev/cm},$$

with C given by

$$C \approx \frac{1}{880 \beta^2} \exp \left(-\frac{1}{227 \beta^2} \right) \quad \text{(at low velocities)};$$

(b) for $\beta > 0.86$, we have

$$C \approx \ln \beta \gamma - 1.735 + 2.75 \times 10^{-2} (4.606 - \ln \beta \gamma)^{2.69}.$$

Table III is the calculated range-energy relation for water. In Fig. 3 the theoretically calculated ranges in emulsion are compared with the measurements. The agreement on the whole is remarkably good. Of the 14 experimental points, just the expected number, 4, depart from the theoretical curve by more than one standard deviation. Also there is no definite monotonic trend with particle velocity. The deviations at 5.447 Mev and at 21.21 Mev, however, are too large to be compatible with the measurement errors, and it is believed that the hydrogen-atomic model used by Walske for the shell corrections causes C at low velocities to vary too abruptly with particle velocity. In fact, by using a constant value of 0.057 for C (corresponding to an effective mean ionization potential of 350 ev), one obtains a virtually perfect fit to the experimental data in the interval 5 to 40 Mev.

Table III. Calculated range-energy relation for water

τ (Mev)	λ_w (cm)	τ (Mev)	λ_w (cm)	τ (Mev)	λ_w (cm)	τ (Mev)	λ_w
0.2	2.3×10^{-4}	6.8	620.2×10^{-4}	64	3.480×10^{0}	500	117.2×1
0.4	5.8	7.6	755.3	68	3.880	600	155.1
0.6	10.5	8.4	902.1	72	4.299	700	195.4
0.8	16.3	9.2	1061	76	4.737	800	237.5
1.0	23.4	10.0	1230	80	5.192	900	281.1
1.2	31.4	12	1704	84	5.666	1000	325.9
1.4	40.6	14	2246	88	6.156	1200	418.3
1.6	50.7	16	2855	92	6.664	1400	513.4
1.8	61.9	18	3529	96	7.189	1600	610.3
2.0	74.0	20	4266	100	7.730	1800	708.5
2.4	101.0	24	5927	120	10.68	2000	807.4
2.8	131.6	28	7829	140	14.00	4000	1805
3.2	165.7	32	9965	160	17.68	6000	2785
3.6	203.3	36	1.233×10^{0}	180	21.69	8000	3745
4.0	244.2	40	1.491	200	26.00	10000	4686
4.4	288.5	44	1.771	240	35.47	14000	6527
4.8	335.9	48	2.073	280	45.95	18000	8325
5.2	386.6	52	2.394	320	57.33	22000	10092
5.6	440.4	56	2.736	360	69.50	26000	11834
6.0	497.3	60	3.098	400	82.38	30000	13556

The Range-Energy Relation

At all high velocities the theoretical range curve is the best available, but below about 40 Mev the calculated shell corrections are imperfect and a semiempirical determination of the range-energy relation is preferred. Adjusting the Geiger-Bohr relation to emulsion of standard density, we use $\lambda = 0.00006 + 0.00138 \, \tau^{3/2}$ for the initial part of the curve. Vigneron[16] has utilized the best of the old data including those from Rotblat[6] and Cüer and Jung[7] to prepare his range table. The data for proton energies up to about 5 Mev were derived from thin layers of evacuated Ilford emulsion. The appropriate density was certainly near 4.02 g/cm^3, and we have assumed this density in adjusting the empirical data compiled by Vigneron to standard conditions. Between 1 and 5 Mev this curve is in excellent agreement with our measurements, and one may reasonably adopt it as a standard range-energy relation. Above 5 Mev the data quoted by Vigneron correspond to progressively lower emulsion densities, and one cannot make a simple density adjustment. One can, however, assume that the ratios of our measured ranges to those computed from the uncorrected Bethe-Bloch theory define a slowly varying function of the particle velocity, and thus can obtain a smooth semiempirical range curve. This has been done in the interval 5 Mev $< \tau <$ 40 Mev. Above 40 Mev the range has been determined solely from the theoretical. The range calculation has been terminated at 35,000 Mev for a number of reasons, particularly because the particle mass begins to enter ι in an explicit nonnegligible way. The compilation of ranges that results from these considerations is given as Table IV and the comparison between the table and the measurements of Part 1 are given in Table V.

Recent measurements by Friedlander, Keefe and Menon[17] at 87.4, 117.9 and 146.5 Mev are in a velocity interval where we have made no measurements. Their ranges are on the average one percent lower then those of Table IV. Their energy calibration was made by determining the ranges of the same particle groups in aluminum, the range curve for which was derived from the work of Bloembergen and van Heerden.[18] The ratio of emulsion ranges to aluminum ranges must vary smoothly and monotonically in this region, but for the energy points cited their observed ratios are 1.215, 1.178, and 1.194 respectively. Evidently uncertainties of 1% or 2% are

Table IV. The range-energy relation for standard emulsion.
The best results in use of this table are obtained if the range
measurement techniques and the corrections are the same as those applied
in the experimental part of this work.

T (Mev)	λ (cm)	T (Mev)	λ (cm)	T (Mev)	λ (cm)	T (Mev)	λ (cm)
0.2	1.8×10^{-4}	20.0	1858×10^{-4}	300	20.14×10^{0}	1000	124.2×10^{0}
0.4	4.1	22.5	2283	320	22.37	1200	158.7
0.6	7.0	25.0	2744	340	24.67	1400	194.1
0.8	10.5	27.5	3243	360	27.04	1600	229.9
1.0	14.4	30.0	3777	380	29.48	1800	266.1
1.2	18.7	32.5	4347	400	31.98	2000	302.4
1.4	23.4	35.0	4952	420	34.53	2200	338.8
1.6	28.6	37.5	5591	440	37.14	2400	375.3
1.8	34.3	40.0	6264	460	39.81	2600	411.7
2.0	40.4	42.5	6970	480	42.52	2800	448.0
2.5	57.4	45	7709	500	45.28	3000	484.2
3.0	76.7	50	9275	520	48.08	3200	520.4
3.5	98.3	55	1.097×10^{0}	540	50.93	3400	556.4
4.0	122.3	60	1.278	560	53.81	3600	592.3
4.5	148.4	65	1.471	580	56.73	3800	628.1
5.0	176.5	70	1.675	600	59.69	4000	663.7
5.5	206.5	75	1.891	620	62.68	4200	699.2
6.0	238.4	80	2.117	640	65.71	4400	734.6
6.5	272.4	85	2.353	660	68.76	4600	769.9
7.0	308.2	90	2.600	680	71.84	4800	805.0
7.5	346.0	100	3.124	700	74.96	5000	840.0
8.0	385.7	110	3.686	720	78.09	6000	1013
8.5	427.2	120	4.286	740	81.26	7000	1184
9.0	470.6	130	4.923	760	84.44	8000	1352
9.5	515.7	140	5.594	780	87.65	9000	1518
10	562.6	150	6.298	800	90.88	10000	1682
11	662.8	160	7.034	820	94.13	11000	1844
12	769.6	170	7.800	840	97.40	12000	2005
13	882.5	180	8.596	860	100.7	13000	2164
14	1002	190	9.421	880	104.0	14000	2323
15	1129	200	10.27	900	107.3	15000	2479
16	1262	220	12.06	920	110.7	20000	3249
17	1402	240	13.95	940	114.0	25000	4000
18	1548	260	15.92	960	117.4	30000	4735
19	1700	280	17.99	980	120.8	35000	5459

Table V. The tabulated ranges compared with the measurements.

T (Mev)	λ measured (cm)	λ table (cm)
1.295	$20.7 \pm 0.2 \times 10^{-4}$	20.9×10^{-4}
2.421	$53.9 \pm 0.6 \times 10^{-4}$	54.7×10^{-4}
2.45	$55.67 \pm 0.34 \times 10^{-4}$	55.7×10^{-4}
5.00	$175.1 \pm 2.0 \times 10^{-4}$	176.5×10^{-4}
5.477	$204.6 \pm 0.64 \times 10^{-4}$	205.1×10^{-4}
5.477	$205.5 \pm 1.1 \times 10^{-4}$	205.1×10^{-4}
10.00	$562.7 \pm 2.6 \times 10^{-4}$	562.6×10^{-4}
13.96	$988.3 \pm 7.4 \times 10^{-4}$	997.2×10^{-4}
21.21	$2056 \pm 5 \times 10^{-4}$	2064×10^{-4}
36.55	$5345 \pm 22 \times 10^{-4}$	5348×10^{-4}
200	10.31 ± 0.07	10.27
340	24.74 ± 0.10	24.67
540	51.15 ± 0.45	50.93
700	74.97 ± 0.36	74.96

present in the measurements by Friedlander et al. The actual ratios of their ranges to those derived from Table IV are 1.007, 0.982, 0.990. It is noticeable that these ratios rise and fall with their emulsion/aluminum ratios.

Acknowledgments

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ranges as great as those of Mather and Segrè will usually have suffered at least one diffraction scattering interaction. Their correction allowed only for the Coulomb scattering of a noninteracting particle. The data of Mather and Segrè give evidence of the presence of an additional scattering effect, and they themselves remarked that they were unable to explain the shapes of their ionization curves. It may also be worth pointing out in this connection that successive diffraction scattering events are not independent, as one usually assumes in plural- and multiple-scattering theory. Correlations will exist between successive scattering events when the particle can be polarized. The resultant scattering angles will then be even larger than estimated from the simple theory.
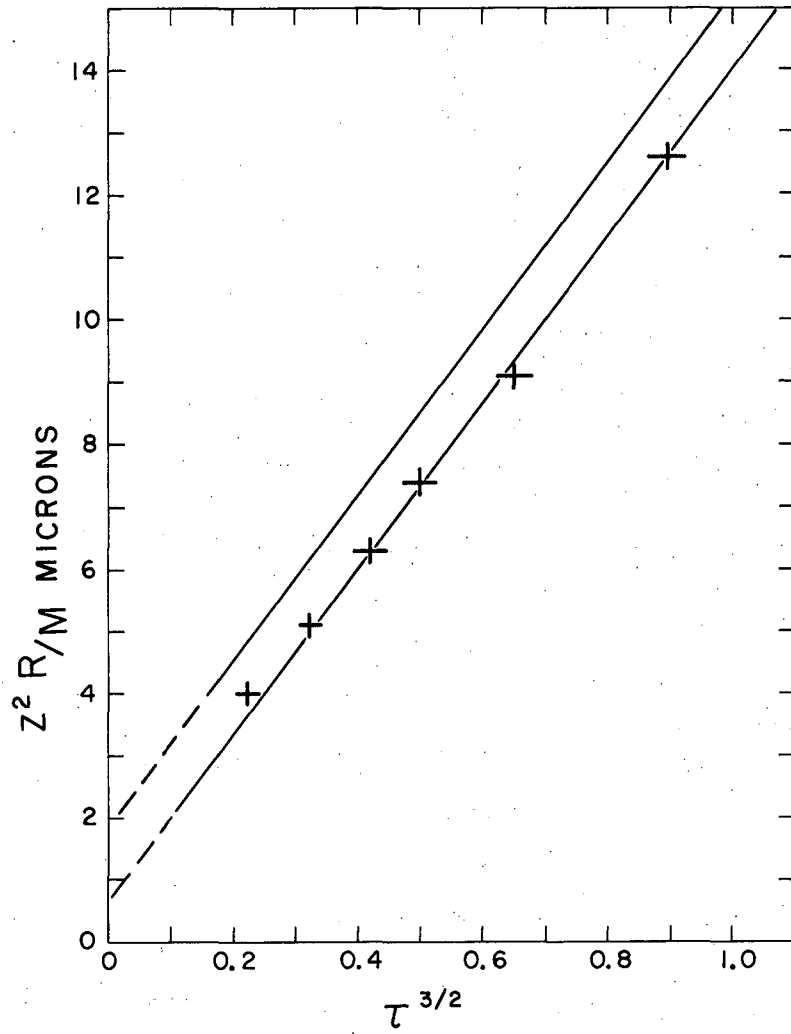
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Legends

Fig. 1. The range-energy data for $\tau < 1$ Mev. The straight lines are found when the (presumably smoothed) ranges from Rotblat[6] are plotted against $\tau^{3/2}$. The upper line is for alpha particles and the lower line is for protons. The dotted extensions of the lines give intercepts of 1.9μ and 0.74μ, respectively. The crosses shown are experimental points for protons found by Cüer and Jung[7] with the errors indicated. The data are for vacuum-desiccated emulsion.

Fig. 2. Graph of $\dfrac{R}{I} \cdot \dfrac{\Delta I}{\Delta R}$, the percent increase in mean ionization potential to bring about a 1 percent increase in emulsion range. βc is the particle velocity.

Fig. 3. The deviation of the measured ranges from the purely theoretical range curve. βc is the particle velocity.

MU-12941

PERCENT INCREASE IN MEAN IONIZATION POTENTIAL TO BRING ABOUT A ONE PERCENT INCREASE IN EMULSION RANGE

MU-12942

COMPARISON OF MEASURED POINTS WITH
THEORETICAL CURVE

MU-12943