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The Rate Control Index for Traffic Flow

by Michael Ball, Robert Hoffman

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The Rate Control Index for Traffic Flow

Robert L. Hoffman*, Michael O. Ball†

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* Metron Inc., Scientific Consultants
11911 Freedom Drive, Suite 800
Reston VA, 20190-5602

† R.H. Smith School of Business and Institute for Systems Research
University of Maryland
College Park, MD 20742

Abstract: The objective of Air Traffic Flow Management is to maintain safe and efficient use of airspace and airports by regulating the flow of traffic. In this paper, we introduce a single-valued metric for post-operatively rating the performance of achieved traffic flow against targeted traffic flow. We provide variations on the metric, one of which factors out stochastic conditions upon which a plan is formulated, and show how these improve on current traffic control analysis techniques. The core of the metric is intuitive and simple, yet leads to an interesting optimization problem that can be efficiently solved via dynamic programming. Numerical results of the metric are given as well as a sample of the type of analysis that should follow a low rating by the metric. Although this metric was originally developed to rate the performance of Ground Delay Programs, it is equally applicable to any setting in which the flow of discrete objects such as vehicles is controlled and later evaluated.

1 Introduction

Air traffic flow managers maintain safe and efficient use of airspace resources by redistributing flights both in space and time in accordance with available capacities. The flow of traffic through airspace sectors and through airports is regulated to ensure that airspace components do not become overloaded and that throughput is maintained. At times of excessive demand or reduced capacity, special air traffic initiatives, such as ground delay programs (GDPs), are put into effect.

The increase in air traffic in the United States over the last 20 years has necessitated more frequent use of traffic flow management initiatives. For instance, in 1998, there were 187 ground delay programs (GDPs) run at San Francisco airport alone. As a result, there has been considerable interest both in the aviation and research communities regarding traffic flow management and its analysis. The issues of deepest concern have been the efficient use of airport landing resources and the equitable distribution of arrival slots among competing airlines. See reference [5] for further treatment of this topic. Substantial efforts have been under way since the mid 1990's to revamp the manner in which traffic flow initiatives are planned and executed. Most notable is the joint industry-FAA Collaborative Decision Making project (CDM), which has made major changes to GDP procedures. See reference [9] for details on ground delay program enhancements.

One can see the need to assess the quality of traffic control actions. In this paper, we provide practical solutions for three major aspects of post-operative, traffic flow analysis.

The first aspect is the basic need for a simple way of contrasting aggregate traffic flow with desired traffic flow. We introduce the rate control index (RCI), which gives a single performance value to the flow of traffic into an airport or sector of space for a fixed time horizon. Unlike more traditional methods for comparing traffic distributions, the rate control index bears an intuitive relation to the events that have lead to the deviation from the plan. In essence, the metric tracks the aggregate flight movements that have caused realized traffic flow to differ from planned traffic flow. Although the basic concept behind the metric is intuitive and simple, the normalization of the metric leads to an interesting optimization problem.

The second aspect is the need to factor out from post analysis major stochastic factors upon which a control action is based. Every traffic flow initiative is based on forecasts of demand and resource capacities, which are often not realized because they are dependent upon highly stochastic conditions. Air traffic demand

predictions, such as the number of arrivals to an airport, are vulnerable to airline operational deviations while capacity predictions for airspace sectors, runways, etc., are highly subject to weather conditions. Direct measurement of traffic control actions is not always the best way to judge the performance of a new program or initiative; naive or ill-chosen metrics are heavily influenced by the quality of the forecasts upon which a plan of action was based. For instance, in a GDP, the objective is to deliver a specified number of aircraft to the airport during a fixed time horizon. A common metric for program evaluation is landings-per-hour. If runway conditions turn out to be more severe than previously forecasted, then this metric will (correctly) reveal that the realized landing rate does not match the desired landing rate. To a large degree, the program performance was beyond the control of participating parties and yet is being judged (in part) by the forecast upon which it was based. For situations such as this, we will show how to model traffic flow into an airport independently of the ability of the airport to land aircraft. Our metric provides for the inclusion or exclusion of these types of stochastic factors.

The third aspect of traffic flow analysis we address is the need for a metric that tracks control actions on a nominal basis, thus acting as a counter-balance, or cross check, to an aggregate anlaysis. Directives are given to individual flights (altering flight paths, arrival times, etc.) in order to affect the aggregate flow of traffic. Final success of these efforts is typically measured based on aggregate metering of traffic. For instance, traffic flow managers may want to deliver 30 flights per hour to region of space, and they may obtain 30 flights per hour, but were these the 30 flights that were intended? If so, one might choose to ignore this issue and happily accept the results. But it's important to know when the aggregate flow is being achieved for the wrong reason. In particular, this helps explain why, at other times, the desired aggregate flow is not achieved.

We demonstrate a nominal version of the RCI metric, which meters traffic flow according to which flights are in the flow and at which time. This gives information that is complementary to and compatible with the aggregate form of RCI and is very useful for revealing the underlying stochastic processes that can disrupt traffic flow or fortuitously cancel each other to produce proper traffic flow.

Section 2 of this paper introduces the core of the RCI metric; Section 3 covers in detail the mathematical underlying the metric; Section 4 demonstrates the use of the metric through Ground Delay Program analysis. Lastly, Section 5 provides some closing comments.

2 The Core of the RCI Metric

The perspective taken in this paper is one of post-analysis. We view a traffic flow initiative as a plan of action which is to be later analyzed for effectiveness. We assume that there is a planning time horizon, during which the flow of traffic into some region is to be regulated. Usually, the goal of regulation is to reduce the rate of flow, but there could be instances in which throughput is the primary goal and so the rate is to be increased. The rate control index (RCI) meters the controlled flow of any uniform set of objects through space but we will develop the metric in the context of its original motivation, an air traffic control ground delay program (GDP).

We assume that a traffic flow manager has set a goal for each time period $t = 0, 1, ..., T$ of a time horizon, meaning the number of vehicles that should be delivered to an airport (or more generally, pass through a region of airspace). Once the time horizon has passed, the actual number of flights is recorded. This establishes two distributions of flights: the planned distribution $P = [p_0, p_1, ...p_T]$ and the realized distribution $R = [r_0, r_1, ...r_T]$, where p_t and r_t are the planned and realized number of flights during time period t, respectively. The question is how to weigh the realized distribution against the planned distribution. Ideally, we would like a single-valued metric that will lend itself toward trend analyses. An obvious idea here is to form a distribution of errors

$$E = P - R = [p_0 - r_0,\ p_1 - r_1, ...,\ p_T - r_T]$$

by vector subtraction, then to apply a standard variance technique, such averaging over the time periods or summing the squares of the deviations. There are two reasons why this might not be the best approach. One is that it develops unintuitive (or meaningless) units that are difficult to translate into tangible quantities or cost assessments. Secondly, consider the scenario in which 30 flights are planned to arrive in each of two consecutive hours. Suppose that one flight scheduled for the first hour arrives late and spills over into the second hour arrival count, i.e., $P = [30, 30]$ and $R = [29, 31]$. Standard variance techniques would report an average error of one flight per hour, for each of the two hours. While this is statistically correct, note that the traffic flow "error" was that one flight migrated one hour, and yet this error is recorded twice, once in the first hour and once in the second.

The alternative measure of variance that we propose is to record the aggregate flight movement, or drift, that lead to the deviation of R from P. This is the same as the minimum amount of flight movement that would be necessary to revert R

to P. In the case $R = [29, 31]$ and $P = [30, 30]$, one would have to move one flight one hour earlier in time to transform R back into P. The final error, then, is tallied as (minus) one flight-hour. More generally, given two finite distributions over the same time horizon, P and R, we define the difference $P - R$ to be the least amount of flight movement (through time) that is necessary to convert R into P. This can be computed by a greedy algorithm that sweeps through increasing time periods, $t = 0, 1, 2, ...$, moving however many flights are necessary to achieve the desired distribution locally. The simplistic example that follows demonstrates the computation of this distribution difference and the intuition behind the remaining computation of the rate control index. Subsequent sections of this paper will cover the more complex aspects of the mathematics.

Example: Suppose that a 4-hour GDP is planned and that the planned arrival acceptance rate (PAAR) for each of those 4 hours is 30 flights. Then the planned distribution is $P = [30, 30, 30, 30]$. Further suppose that the actual number of flights that arrived at the airport (or the airport terminal space) is given by $R = [27, 32, 35, 24]$. Then the rate control index for R (relative to P), denoted $RCI(P, R)$, is computed via the following two-part calculation.

Part 1: Compute the difference $P - R$. This is the minimum amount of flight movement that is necessary to turn R into P. We sweep left to right through R (increasing t), moving however many flights (say f_{t+1}) are necessary from r_t to r_{t+1} to achieve $r'_t = r_t - f_{t+1} = p_t$. Counting left-hand movements as negative and right-hand movements as positive, we must move 3 flights from hour 1 to hour 0 ($f_1 = -3$), move 1 flight from hour 2 to hour 1 ($f_2 = -1$), and move 4 flights from hour 2 to hour 3 ($f_3 = 4$). See Figure 1.

So far, R has been transformed into the distribution, $R' = [30, 30, 30, 28]$. Note that this is 2 flights short of the desired distribution, because $\sum p_t - \sum r_t = 120 - 118 = 2$. We create a "slush fund" of 2 flights at the end of R to compensate for the lack of conservation of flights. Equivalently, we could have started with an augmented distribution $R' = [27, 32, 35, 24, 2]$. Similarly, we extend P to a five-hour distribution, $P' = [30, 30, 30, 30, 0]$. To complete the example, we move 2 flights from hour 4 to hour 3 ($f_4 = -2$). The summary of flight-movements is given in Table 1.

Now we can compute the cost of transforming R into P. Let c^- be the average cost (say, in dollars per hour) of delaying a flight for one unit of time and let c^+ be the average cost (say, in dollars per hour) of a flight arriving early by one unit of time. Then we break the flight-movements into left-hand movements

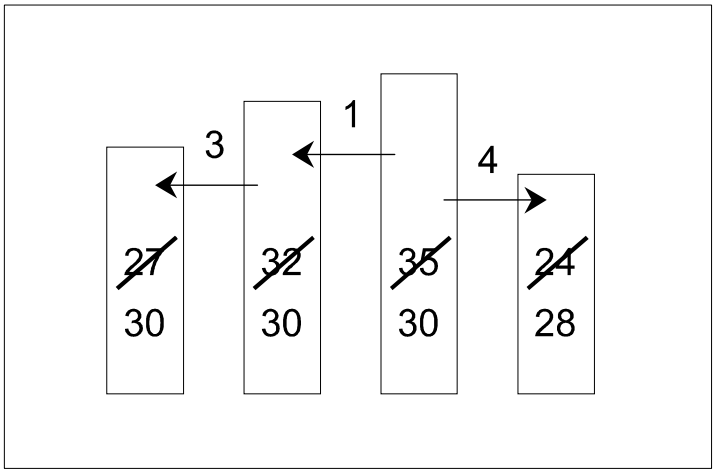

Figure 1: Flight movements necessary to transform R into P

Movement	**Notation**	**Flight-hours**
Hr 0 $\longrightarrow$ Hr 1	f_1	-3
Hr 1 $\longrightarrow$ Hr 2	f_2	-1
Hr 2 $\longrightarrow$ Hr 3	f_3	4
Hr 3 $\longrightarrow$ Hr 4	f_4	-2

Table 1: Summary of Flight Movements through Time

$M^- = |-3| + |-1| + |-2| = 6$ and right-hand movements $M^+ = 4$, corresponding to tardiness and earliness, respectively. The final cost is given by

$$c^- M^- + c^+ M^+ = 6c^- + 4c^+ \ .$$

In this example, we opt to set $c^- = c^+ = 1.0$ to obtain pure units of $4 + 6 = 10$ flight-hours. In other words, R was off from P by 10 flight-hours. Intuitively, this means that in order to turn R into P, one would have to do the work equivalent to moving 1 flight 10 hours, or 2 flights 5 hours, etc. One variation on the metric is to retain the positive and negative sums to show the breakdown of this total.

Part 2: Normalize the distribution error $P' - R'$. To compare GDPs of differing lengths and number of flights, we normalize by dividing the distribution error by the cost of the worst-case scenario. This means we must find the five-hour redistribution W of the 120 flights in PAAR with the highest cost difference, $P - W$. In general, this involves solving a max-min problem, which is covered in Section 3. For now, we take as a given that $W = [0, 0, 0, 0, 120]$ with a cost of

$$P' - W = (|-4| + |-3| + |-2| + |-1|) \times 30 = (10) \times 30 = 300 \, \text{flight-hours}.$$

(W corresponds to the scenario in which all the flights land in the final hour.)Thus, the rate control index we assign to the distribution R is

$$\frac{10 \text{ flight-hours}}{300 \text{ flight-hours}} = 0.03\overline{3}.$$

Since we have kept pure cost parameters of 1.0, this is a pure ratio that indicates the error of the realized distribution. In order to make the index more palatable, we phrase the performance of the realized distribution in terms of what was achieved rather than what was not achieved. Subtracting from 1.0, we obtain a final rate control index (RCI) of

$$RCI \ (P', R) = 1.0 - .03\overline{33} = 0.9\overline{66}$$

If preferred, this final index can be transformed into a percentage, 96.67%. The interpretation of the index is that the realized distribution achieved 96.67% of the intended (planned) distribution. This number can then be used for comparing GDP performance on different days and lends itself nicely to trend analyses (see Section 4). Values below 90% should be investigated for causality.

In practice, strategic traffic flow management plans such as a GDP are often revised several times before or during execution, in accordance with changing

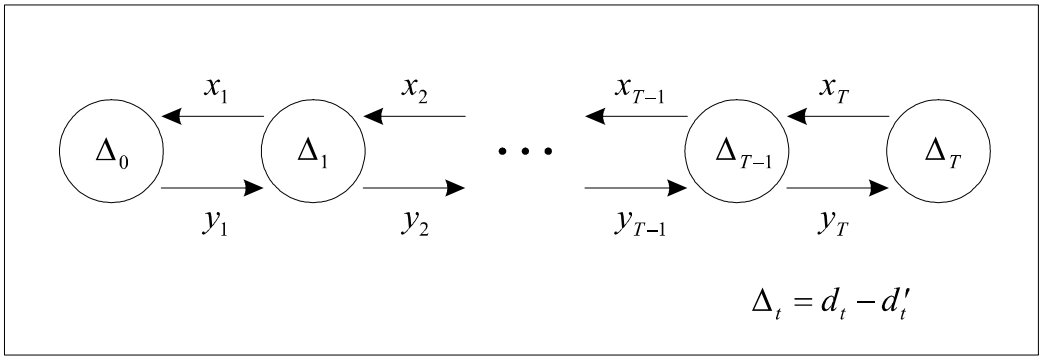

Figure 2: Network flow representation of a redistribution

demand and capacity, thus changing the planned distribution. The rate control index can accommodate such revisions by setting p_t to be the latest target value for time period t that was set prior to time period t. This way, the target value p_t can be changed as many times as one wished but the program will ultimately be evaluated on the final target value.

3 The Mathematics of RCI

3.1 The Difference of Two Distributions

We define a (finite) *distribution* $D = (d_0, d_1, ..., d_T)$ to be any $(T+1)$-tuple of non-negative numbers. D is said to have *order* $(T+1,\ S)$, where $S = \sum_{t=0}^{T} d_t$ is the 'mass' of D. Consider the transformation of D into another distribution, $D' = (d'_0, d'_1, ..., d'_T)$, of the same order. This transformation can be characterized in terms of non-negative flow variables x_t and y_t that satisfy

$$\begin{aligned} &d_t - d'_t = x_t + y_{t+1} - x_{t+1} - y_t, \text{ for } t = 1, 2, ..., T-1 \\ &d_0 - d'_0 = y_1 - x_1 \\ &d_T - d'_T = x_T - y_T \end{aligned} \tag{1}$$

as illustrated in Figure 2.

We wish to establish a systematic means for comparing two distributions, D and D', of the same order. The motivating example of Section 2 suggests that this should be the minimum cost of transforming one distribution into the other in

the context of the network flow in Figure 2. So, we establish two cost parameters, $c^+ \geq 0$ and $c^- \geq 0$, which represent the cost of moving a unit of mass within distribution D one unit to the left and to the right, respectively. Thus, if we let $X = (x_1, x_2, , , .x_T)$ and $Y = (y_1, y_2, ..., y_T)$, the cost of the transformation of D into D' is given by the minimization of the following objective function against appropriate conservation of flow equations.

$$C(X,Y) = c^- \sum_{t=0}^{T} x_t + c^+ \sum_{t=0}^{T} y_t \tag{2}$$

It is not hard to show (see reference [4]) that a feasible solution (X,Y) to such a network flow problem is optimal if and only if it has the acyclic property, meaning if $x_t > 0$, then $y_t = 0$, and if $y_t > 0$, then $x_t = 0$. (See reference [2] for a treatment of network flow problems and their properties.) This serves as justification of the greedy algorithm used in the example in Section 2 to find the minimum cost flow. In light of the acyclic property, in comuting a minimum cost soution, it is only necessary to keep track of the *net* flow between nodes of Figure 2. Let f_t be the net flow from node $t-1$ to t, that is, $f_t = y_t - x_t$. Then we can represent the transformation of D into D' by a T-tuple, $F = (f_1, f_2, ..., f_T)$, which can be computed using the following greedy algorithm.

GREEDY
Set $f_1 = d_0 - d'_0$
For $t = 2$ to T :
    set $f_t = d_{t-1} - d'_{t-1} + f_{t-1}$

Alternatively, we can compute F directly from

$$f_t = \sum_{s=1}^{t} (d_{s-1} - d'_{s-1}) \text{ for } t = 1, 2, ..., T. \tag{3}$$

We call F a *redistribution vector* (relative to D) and denote it's action by $F : D \rightarrow D'$. F is unique for a given pair, D and D'. (See Figure 3). Later, we will see that there are several advantages to this alternative representation of the transformation, beyond uniqueness and the elimination of needless variables. Conversely, given any $D = (d_0, d, ..., d_T)$ and $F = (f_1, f_2, ..., f_T)$ that satisfy Equation (3) for some (nonnegative) distribution $D' = (d'_0, d'_1, ..., d'_T)$, then F is a redistribution of D into D'.

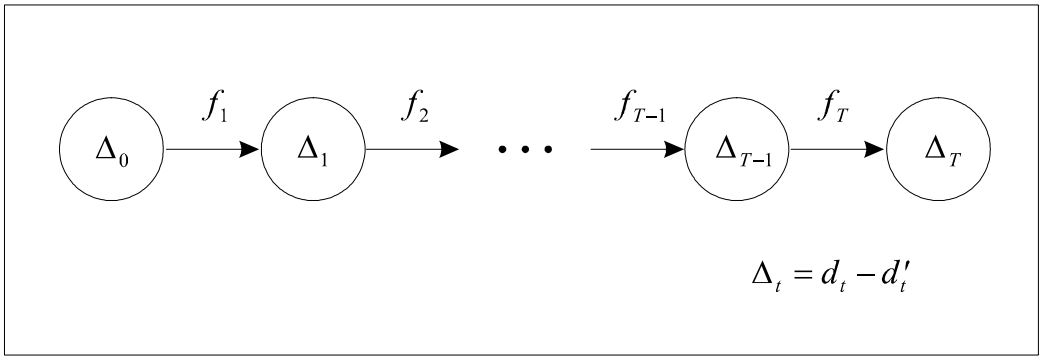

Figure 3: Transformation of distribution D into distribution D' by vector F

Given a vector F the transformation cost can be written as,

$$C(F) = \sum_{t=1}^{T} \left[c^+ \max\{f_t, 0\} + c^- \max\{-f_t, 0\} \right]. \tag{4}$$

Intuitively, when $c^+ = c^- = 1.0$, $C(F)$ is the amount of 'work' done to transfrom D into D'.

For the purposes of comparing two distributions D and D' of the same order, we define their difference to be the cost of transforming D' into D. That is,

$$D - D' = C(F), \tag{5}$$

where $F : D' \rightarrow D$ and the cost function C is fixed à priori. Note that this difference is not, in general, commutative. Since we allow that $c^+ \neq c^-$, a situation may arise in which $D - D' \neq D' - D$. That is, the cost of transforming D into D' may be not be equal to the cost of transforming D' back into D. However, once $F : D' \rightarrow D$ is known with corresponding cost $C(F) = ac^+ + bc^-$, then the reverse transformation, $G : D \rightarrow D'$, given by $G = (-f_1, -f_2, ..., -f_T)$, has cost $C(G) = bc^+ + ac^-$. In this respect, the direction or transformation established by (5) is arbitrary and consistency of application is all that matters.

We use this distribution difference as the raw (unnormalized) score of the rate control index (RCI). Given a planned distribution of arrivals over time, $P = (p_0, p_2, ..., p_T)$, and a realized distribution, $R = (r_0, r_2, ..., r_T)$, we define their difference to be $P - R$, as set by (5). In practice, there are complicating factors when the realized distribution does not have the same order as the planned distribution

(some vehicles may arrive beyond the considered time horizon or not arrive at all, e.g., flight cancellations). These spill-overs and cancellations can be handled by making appropriate extensions of the two distributions so that they again have the same order, as in the motivating example of Section 2. Care should be taken, however, that the effects on the cost function make intuitive sense in the context of the overall problem.

3.2 Normalization of RCI

3.2.1 The Worst-case Scenario

In the motivating example of Section 2, we saw that the normalization step of the RCI computation requires that we find the redistribution W of a fixed distribution D with the worst-case (highest) value, $D - W$. Consequently, we must consider what types of distributions could realistically be formed from D. For instance, could the planned distribution $D = (2, 4, 6)$ be transformed into either of the realized distributions $D' = (0, 0, 12)$, or $D' = (12, 0, 0)$? In theory, yes, but in the context of arriving vehicles, these represent the case in which all vehicles arrive in the last and first time periods, respectively. In the air traffic management case and possibly in many other applications, for each vehicle, there is an earliest arrival time (based on its scheduled arrival time) but no practical latest arrival time. This means that D' is at least conceivable but that D'' is effectively impossible.

To model earliest arrival times, we fix a *leftward bounding distribution* $B = \{b_0, b_{1,}, ..., b_T\}$ of order $(T + 1, S)$ and consider the family of (re)distributions of B defined via

$$\Omega^B = \left\{ D' \mid \text{there is a nonnegative redistribution vector } F : B \to D' \right\}.$$

Then if $D' \in \Omega^B$, D' can be obtained from B by making strictly rightward shifts in B. For example, if $D = (0, 3, 3)$ and $B = (1, 2, 3)$, then $D' = (0, 0, 6)$ is in Ω^B via $F = (1, 3)$, while, in contrast, $D'' = (3, 0, 3)$ is not in Ω^B , because $G : B \to D''$ is uniquely determined by $G = (-2, 0)$, which contains a negative entry. We assume that B has been properly constructed from the original distribution D so that $D \in \Omega^B$. Note that given any D' of the same order as B, $D' \in \Omega^B$ if and only if

$$\sum_{s=0}^{t} d'_s \leq \sum_{s=0}^{t} b_s \text{for all } t, \tag{6}$$

which provides a membership criteria for Ω^B.

Recall that our approach to the normalization of the RCI metric is to divide the difference of two distributions, $D - D'$, by $D - W$, where W is the worst case (highest cost) scenario. Then, given D, we seek the solution to

$$\max\left\{C(F) \mid F : W \to D \text{ and } W \in \Omega^B\right\}. \tag{7}$$

Next, we show how to solve (7) by dynamic programming. Had we retained the network flow description, finding a solution to (7) would require us to find the maximum value over a family of linear programming problems. This is complicated by the presence of the side constraint $W \in \Omega^B$. This max-min problem can be solved by dynamic programming.

3.2.2 A Dynamic Programming Solution

We wish to write a recursive relation for finding a solution to (7). Fix a distribution $D = (d_0, d_1, ..., d_T)$ of order $(T+1, S)$, where $S = \sum_{t=0}^{T} d_t$. Let $B = (b_0, b_1, ..., b_T)$ be a bounding distribution for D and let $D' = (d'_0, d'_1, ..., d'_T)$ be any distribution of order $(T+1, S)$. For any x, let

$$cst(x) = c^+ \max\{x, 0\} + c^- \max\{-x, 0\}.$$

Then the corresponding cost of F, given below, is consistent with (4).

$$C(F) = \sum_{t=1}^{T} cst(f_t)$$

For any distribution $E = (e_0, e_1, ..., e_T)$, we establish the following partial sum notation:

$$E_t = \sum_{s=0}^{t-1} e_s \text{ for } t = 1, 2, ..., T.$$

Thus, for $F : W \to D$, using (3), we can now write the t^{th} component of $F = (f_1, f_2, ..., f_T)$ as $f_t = W_t - D_t$. For a fixed t with $0 \le t \le T$, we define the following truncated vectors.

$$F^t = (f_1, f_2, ..., f_t),\ W^t = (w_1, w_2, ..., w_t), \text{ and } D^t = (d_1, d_2, ..., d_t)$$

We now define the sub-problem for our dynamic programming recursion as

$$Cmax(t, \beta) = \max_{W^t F^t}\left\{C\left(F^t\right) \mid f_s = W_s - D_s \text{ and } W_s \le B_s \text{for } s \le t; W_t = \beta;\ W^t \ge 0\right\}.$$

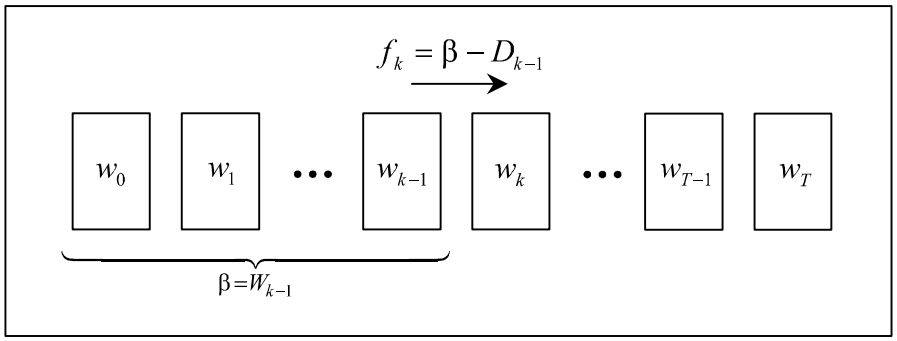

Figure 4: Finding the highest cost redistribution W when the partial sum W_{k-1} is set to β.

where $Cmax\,(0,\beta) = 0$ for all β. It might not appear that W_s is restricted to be less than or equal to $S = \sum d_t$; this is captured by the $W_s \leq B_s$ restriction since B_T must equal S. In essence, $Cmax\,(t,\beta)$ is the cost of transforming (the most costly) W^t into D^t when we have set the partial sum W_t equal to β (see Figure 4). Then, $Cmax\,(t,\beta)$ can be found recursively via

$$Cmax\,(t,\beta) = \begin{cases} 0, \text{if } t = 0 \\ -\infty, \text{ for } \beta > B_t \\ \max\limits_{b=0,1,\ldots,\beta} \{Cmax\,(t-1,b) + cst\,(f_t) \;\mid f_t = \beta - D_t\}, \text{else.} \end{cases} \tag{8}$$

and the maximal cost that we seek in (7) is given by $Cmax\,(T,S)$. The correctness of (8) in the case of $t = 0$ follows from the definition of $Cmax\,(\;)$ and in the case of $\beta > B_t$ because values of $W_t > B_t$ are not feasible. For the general case, note that by fixing $W_t = \beta$, f_t (and its cost) are fixed. To insure the non-negativity of w_t , the possible W_{t-1} over which the max is taken must be less than or equal to β. This recursion might seem simple, even trivial, since f_t does not vary in the max operation. The complexity of this problem lies within the restrictions imposed by the B_t vector which limits the values to which the max operation is applied. In fact, without, the B_t restrictions, extreme or trivial solutions would always result.

Equation (8) can be readily used to construct a forward recursion dynamic programming algorithm. It is clear that the running time is at most $O(TS^2)$ since there are at most $O(TS)$ $Cmax\,(\;)$ values to be computed and the computation of each one requires at most $O(S)$ comparisons.

4 Ground Delay Program Analysis

In this section, we demonstrate the usefulness of the RCI metric in the context of ground delay programs.

4.1 Modeling Air Traffic Flow into an Airport

A Ground Delay Program (GDP) is an FAA initiative to reduce the flow of aircraft into an airport. A GDP is implemented whenever it is predicted that the arrival demand at an airport will exceed the arrival capacity for a significant period of time. In essence, flights bound for a single airport are held at their origin airports in lieu of anticipated airborne holding. This prevents the airport the from being inundated with the hazardous and unwanted airborne holding that would result at the destination airport if flights were allowed to depart on schedule. Most GDPs are prompted by adverse weather conditions that can dramatically reduce the airport acceptance rate (AAR). Other causes are runway construction and special airport operations. GDPs are planned several hours in advance and can run for periods as long as 12 hours. (For more background on ground delay programs and the ground holding problem, see references [3], [10], [11], [12], [13], [14], [15], [16], and [17].)

Since the primary purpose of a GDP is to control the rate of flow of aircraft into an airport, the typical metric for evaluating the performance of a GDP is to measure the actual landings per hour ($LAND_t$ where t varies over the discretized time intervals over which the GDP operated) against the planned landings per hour ($PAAR_t$). Although this is often taken to be the "bottom line" in a GDP, it is in fact a hybrid analysis that blurs the appropriateness of the plan with the execution of the plan. The appropriateness of the plan is largely determined (in hindsight) by the matching of the GDP parameters with the resulting airport conditions (such as the timing of a weather front) the severity of runway conditions, the expected demand, and so on. Since these conditions are hard to predict with accuracy, there needs to be a mechanism to analyze the success with which the plan was *executed*, independently of the appropriateness of the plan and the forecasts upon which it was based. (See reference [3] for treatment of stochastic airport capacity.)

The simple solution is to meter the traffic as it enters the terminal space of the airport, rather than on the runways. The problem is that the terminal area of an airport is not so easily defined and, even if it were, the time at which a flight enters the terminal space is generally not recorded or such data is hard to

obtain. The data that is readily obtained is from the enhanced traffic management system (ETMS) is the (runway) departure time of each flight f, the arrival time of f denoted by $ARTA_f$, and a sequence of ETAs (estimated time of arrival), as f moves along its flight path.

One solution is to define a radius of geographical distance (or time) about the airport and declare that once a flight has passed over this boundary, it has entered the terminal airspace of the airport. This should roughly correspond to the point at which flights are ordinarily put into a state of airborne holding. (We recognize that at some airports, airborne holding of certain flights can take place far out from the terminal space of the airport.) The objective is to meter the traffic at this point, then use the RCI metric to compare the traffic flow against the desired traffic flow. The metric will tell us how successfully the GDP delivered flights to the airport, independently of the airport status (capacity) and any airborne that may have taken place.

We assume the existence of a model that will estimate, post facto, for each flight f the amount of airborne holding that it incurred, which we denote ABH_f (see [6] for one such model). From this, we can deduce that f was in a state of airborne holding at a given time $t < ARTA_f$ if and only if $ABH_f \geq ARTA_f - t$.

For a set of contiguous time intervals $t = 1, 2, ..., T$, let DEL_t be the number of flights that is delivered to the airport during time interval t, meaning, arrived at the border of the terminal space (but not necessarily landed). Let ABH_t be the number of flights that are in a state of airborne holding at the end of interval t. Let $LAND_t$ be the number of flights that land during time period t. If we view the airport as a closed network, then we have the following elementary relationships (see Figure 5).

$$DEL_t = (ABH_t - ABH_{t-1}) + LAND_t$$

After a GDP, three distributions can be assembled: $PAAR, DEL$, and $LAND$. One would want to measure $RCI\,(PAAR, LAND)$ and $RCI\,(PAAR, DEL)$, as in the following case study.

4.2 The Aggregate Version of RCI

Figure 6 shows a graphical analysis of the performance of a ground delay program (GDP) conducted at San Francisco airport (SFO) on March 5, 1998. The planned acceptance rate of flights (the PAAR distribution) was set at 32 flights per hour

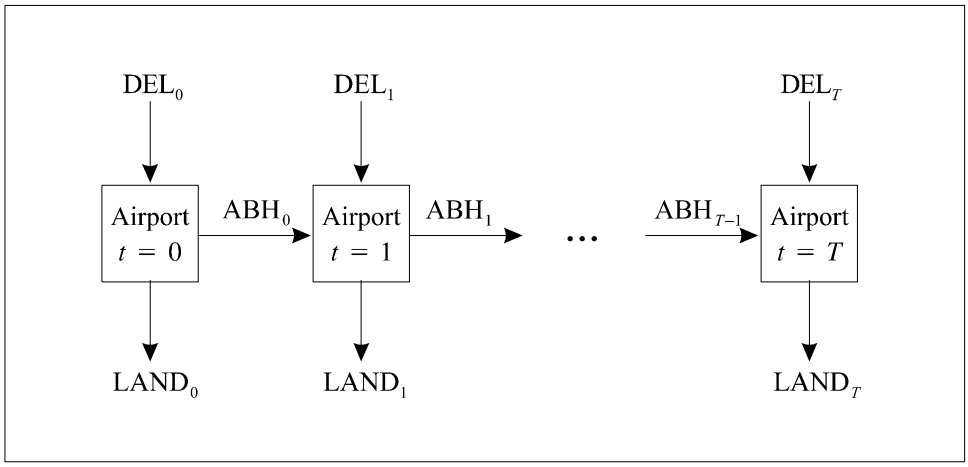

Figure 5: Model of an airport as a closed system

for each of the six hours of the GDP. The RCI value assigned to this GDP was 88.24%. This is below the 1998 RCI average for SFO, approximately 92.0%, it would be a likely candidate for further analysis. The RCI value of 88.24% was computed based on PAAR versus DEL, the distribution of "flights delivered" to the terminal airspace of the airport (but not necessarily landed), indicated by the asterisks. Also shown are the distributions LAND, flights landed at the airport (indicated by solid dots) and the ABH, the size of the airborne holding queue at the end of each hour (indicated by triangles).

A quick glance at Figure 6 shows why the RCI(PAAR, DEL) was not closer to the optimal value, 100%. There were too many flights delivered to the airport early in the program: in the first hour, 41 flights were delivered when only 32 were intended. One possible explanation for this is that the GDP was implemented too late and some of these 41 flights were already airborne when the program was planned, hence, they could not have been held on the ground. Some of these flights may have been assigned ground holds but departed too early, but this is a less likely explanation. Moreover, there were drastically too few flights delivered in the 2100z hour: 16 flights compared to the desired 32. Some of the flights intended to arrive in hour 2100z may have arrived in the 2000z hour. Note the airborne holding queue that resulted when the glut of flights arrived in the 2000z hour. It took most of the rest of the program for this queue to dissipate.

One can see that the LAND distribution more closely follows PAAR than does

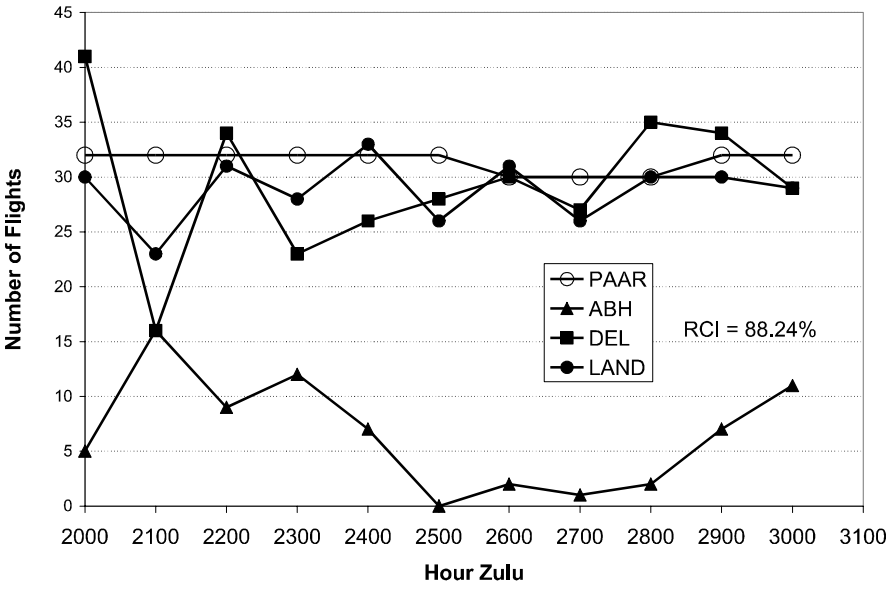

Figure 6: Analysis of ground delay performance at San Francisco Airport, March 3, 1998

DEL. Thus, the value of RCI(PAAR, LAND) (not computed) would be slightly better than RCI(PAAR, DEL). This is not uncommon: some of the arrival flow is smoothed out by airborne holding, resulting in a smoother distribution.

An interesting feature of this GDP was that in the 2100z hour, even with the pressure of a substantial airborne holding queue, the airport was able to land only 23 flights, instead of the forecasted number, 32. Airport tower traffic counts confirm that in the 2100z hour, the controllers at the airport favored departures.

4.3 The Nominal Version of RCI

In this section, we demonstrate the use of a nominal version of the rate control index. For each flight f, we compute the amount of arrival delay, M_f via

$$M_f = |\text{actual arrival time } - \text{ planned arrival time}|\,.$$

The unnormalized RCI score is $\sum_f M(f)$, which represents the amount of flight movement through time that would be necessary to restore all flights to their planned arrival time. This number is normalized by dividing by the cost of the worst-case scenario, $\sum_f W(f)$, where $W(f)$ is the most arrival delay that could have occurred for flight f (i.e., the farthest time period from its planned period of arrival). The final value is subtracted from 1.0 and multiplied by 100%.

Each point on the scatter plot in Figure 7 is an ordered pair, $\left(RCI^{Nom}, RCI^{Agg}\right)$, for a GDP run at SFO during the period January 1 to October 28, 1998. Consider the point $(64, 92)$ for July 10. The value $RCI^{Agg} = 92\%$ indicates that, in the aggregate, the distribution of landed flights closely matched the desired distribution. However, the low value of RCI^{Nom} reveals that, too often, the flights that landed in a given time interval were not the flights that were intended to land in that time interval. Some of the flights arrived earlier than planned while others arrived too late. On the whole, for any given time interval, the number of flights that migrated out of a time interval was almost equal to the number of flights that migrated into that time interval, hence, the aggregate numbers of flights were preserved. The stochastic processes canceled each other out and there was a great deal of 'luck' involved in achieving the high value of $RCI^{Agg} = 92\%$.

In general, points in the upper right quadrant of Figure 7 correspond to days in which the aggregate distribution of flights was achieved and most of the flights arrived in their expected time periods. These are the best-run programs - the goal of each program being $(100\%, 100\%)$. Given two points (x_1, y_1) and (x_2, y_2), with $x_1 < x_2$ and $y_1 = y_2$, we can say that they had the same level of aggregate success

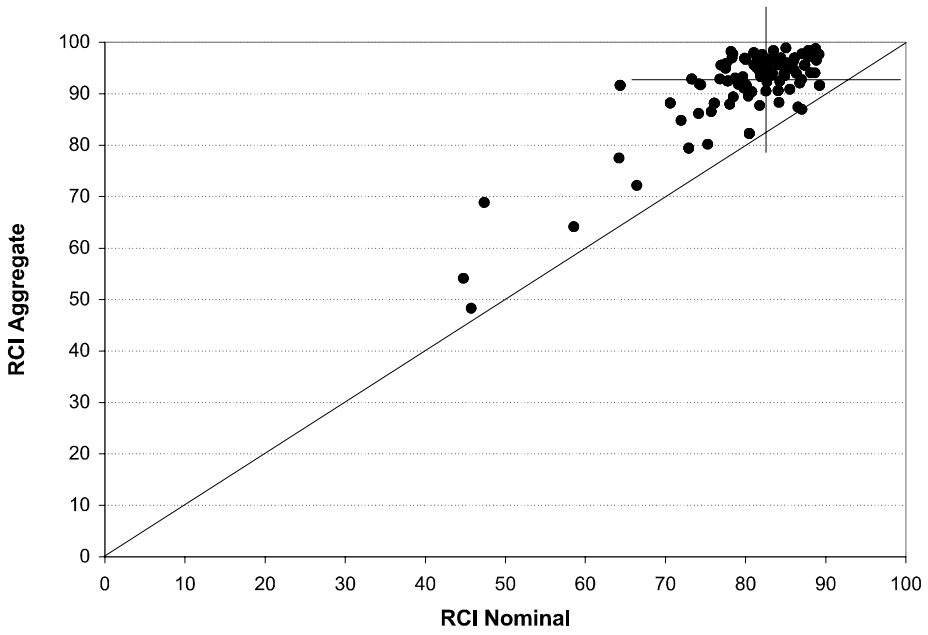

Figure 7: Nominal and aggregate RCI values for Ground Delay Programs at SFO, January - October, 1998

but that program first program involved more 'luck' while the second program involved more 'skill'.

The center of mass of the squares lies at about $(82, 92)$, marked by the cross. This indicates that the ground delay performance at SFO is generally quite good. In general, RCI^{Nom} is about 10% less than RCI^{Agg}. Note that all of the points lie above the 45-degree line. This is to be expected since as RCI^{Nom} increases, more flights are arriving in their planned arrival periods and so the aggregate distribution of flights is more likely to match the planned distribution, which also increases the value of RCI^{AGG}. Note that when $RCI^{Nom} = 100\%$, the only way for RCI^{Agg} to fall below 100% is for there to be arrivals that were not anticipated by the GDP.

5 Closing Remarks

We have introduced a new metric for the evaluation of planned versus realized traffic flow for a region of space or an airport. In its most general form, the metric generalizes to a comparison of two finite distributions, hence, has the potential for use in any area of traffic management in which vehicular movement through time is regulated. This represents a substantial improvement over the standard techniques for comparing two finite distributions. The metric naturally lends itself to intuitive interpretation (when decomposed into left and right-handed movements) and to cost evaluation. The development of an aggregate and nominal version of the metric captures the two crucial aspects of traffic flow management: *how many* flights flowed through a region of space and *which* flights flowed through the region. Also, we have shown how to apply the metric to factor out the effects of inaccurate forecasts from performance analysis of a traffic flow initiative.

We showed how to compensate for controlled flights that arrive outside the planned time horizon by augmenting the realized distribution. The intuitive justification for this was that the cost added is assessed at a rate commensurate with the wasted capacity due to these flight cancellations. However, there doesn't seem to be an analogous compensation for the case in which unanticipated flights arrive within the planned time horizon. For consistency with flight shortages, the logical adjustment for these pop-up flights would be to augment the planned distribution. Unfortunately, the added cost would be assessed at a rate commensurate with the movement of flights backward in time, for which there is no intuitive obvious justification. In practice, pop-up flights comprise a small enough percentage of the

total flights that their effect on the RCI metric is minimal when they are ignored. However, future development of the metric should incorporate pop-up flights.

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