WPS 0384

Policy, Research and External Affairs

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International Commodity Markets

International Economics Department
The World Bank
April 1990
WPS 384

The Rational Expectations Hypothesis in Models of Primary Commodity Prices

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The standard linear model fails to account for primary commodity price movements in any significant area, so it is important to do more empirical work to learn to which commodities this nonlinear model applies.

Policy, Research, and External Affairs

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International Commodity Markets

This paper — a product of the International Commodity Markets Division, International Economics Department — is part of a larger effort in PRE to understand the short- and long-run behavior of primary commodity prices and the implications of movements in these prices for the developing countries. Copies are available free from the World Bank, 1818 H Street NW, Washington DC 20433. Please contact Aban Daruwala, room S7-040, extension 33713 (64 pages).

Muth's Rational Expectations Hypothesis (REH) revolutionized economic theory and modeling on price formation in a simple agricultural market. Gilbert studied the results of the few econometric models of primary commodity markets that have incorporated the REH.

In the medium to long term, primary commodity prices are determined by the intersection of the commodity's consumption (demand) and production (supply) curves, but in the short term stockholding tends to even out price movements.

In a commodity price model, it is useful to distinguish between application of the REH to the "physical" production and consumption relationships and its application to how intertemporal stockholding affects short-term price determination. In practice, most econometric work has concentrated on the implications of the REH for stock and price relationships.

The standard speculative stock demand model (the one Muth originally used) relates stockholding to expected capital gains. One can estimate this relationship directly or can obtain the implied solved price equation which related the current price to its lagged value and to a specific function of current and future values of the exogenous variables and disturbances in the production and consumption equations. Whatever the precise specification adopted, the model performs poorly.

Why? Actual stock data for primary commodities apparently do not relate mainly (or even substantially) to speculative stockholdings. If stocks are specified as the dependent variable, one needs to model transactions and precautionary stockholdings as well as speculative stockholdings.

But the quality and character of the stock data cannot explain why, in estimating solved price models, investigators have failed to find the predicted dependence of the current price on expected future supply and demand movements.

Perhaps the simple equations adopted by the commodity modeler cannot reflect the markets looking ahead in the manner implied by the REH, but even so one should be able to find the negative reaction of primary commodity prices to rises in interest rates implied by the REH. The almost universal failure of modelers to find this effect suggests that the model is incorrect.

The standard Muth stockholding model derives its simplicity from ignoring the non-negativity constraint on stocks. This results in linear solved stock and price equations, allows explicit solution of price, and permits use of standard econometric methods — but can produce distortions.

Recent work based on Gustafson's contribution investigates commodity stock behavior under the REH with the non-negativity constraint imposed on stocks. This model implies weaker forward-looking behavior and price responses to interest rate changes than those implied by the linear model. The possibility of stockout clearly implies that the price will respond in a nonlinear manner to supply and demand disturbances. But the REH implies that this nonlinearity will be fairly smooth since even if enough stocks are available currently, the fact that one could face stockout eventually influences current behavior.

This nonlinear REH model seems to provide a good explanation for sugar prices, for which there is clear evidence of nonlinearity in price responses. But it is inappropriate for the aluminum industry, for which there is no evidence of nonlinearity and in which speculative stockholding is not important. More work in this area should be a high priority.

Produced at the PRE Dissemination Center

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by
Christopher L. Gilbert

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*This paper is the final report of research undertaken for the International Commodity Markets Division of the World Bank. I am grateful to B. J. Choe, Ron Duncan, Montague Lord, and an anonymous commentator for comments on the initial draft. Address for correspondence: Department of Economics, Queen Mary and Westfield College, Mile End Road, London E1 4NS, England.

A. Introduction

1. The purpose of this paper is to examine the implications of the rational expectations hypothesis for the econometric modeling of primary commodity markets. This examination is undertaken in the specific context of the work of the International Commodities Division of the World Bank which is required to produce price forecasts for all major primary commodities over a long term (1-5 years) horizon on an annual basis. In the interest of brevity I ignore other possible implications of rational expectations, e.g., for the possibility of bubble-like behavior, which either do not relate or have not year been related to econometric modeling in these markets.

2. It is generally recognized that an understanding of expectations formation is crucial to modeling price formation in primary commodity markets. One reason for this importance is that primary commodity production typically exhibits long lags. This is most obviously true in mining, where a new mine will have a lead time of 7-10 years; and in tree crops, where trees only become productive 3-5 years after planting. Investment decisions in these industries therefore depend upon expectations of market conditions likely to prevail many years ahead; and the same factors may also operate to a more limited extent on the demand side.

3. However, this observation is also true of manufacturing industry. What distinguishes primary industries from manufacturing in this respect is their competitive structure (see, e.g., Labys, 1980) which, as the consequence of short-run inelasticity of both demand and supply, results in substantial price volatility. Consequently, a primary commodity-producing enteprise will need to form expectations of the prices it is likely to obtain whereas, by contrast, a manufacturing enterprise will focus on likely levels of future demand.

4. The second reason for the importance of expectations formation in understanding

primary commodity markets relates to the role of stocks (see Ghosh *et al.*, 1987, ch.2). Price variability, in conjunction with the high degree of product homogeneity, provides incentives for third parties to carry stocks forward in periods of low prices in the expectation of higher prices in the future. This activity raises prices in periods of excess supply and tends to lower prices in periods of excess demand, thereby providing a degree of automatic price stabilization (Samuelson, 1957; Wright and Williams, 1982, 1984). It is facilitated by active trading of many of the most important primary commodities on futures markets. By taking an offsetting futures position the stockholder is able to substantially reduce the riskiness of the combined (stock plus futures) transaction and this will imply an increased willingness to carry stock (Gilbert, 1989c).

5. For both of these reasons, current supply and demand (including demand for stocks) of a primary commodity will depend both on expected prices and on prices previously expected to prevail in the current period. A higher expected future price will raise current stock demand and thereby raise the current price. A higher expectation in the past of today's price will have tended to raise current supply and will depress the current price.

B. The Rational Expectations Hypothesis

6. Expectations formation in primary markets has traditionally been modeled as 'adaptive'. In adaptive expectations the agent updates his previous expectation by a proportion of the current forecasting error. Thus, if the current price p_t exceeds last period's expectation $p^e_{t|t-1}$, he will modify his expectation for next period by anticipating a price somewhere between his previous expectation and the current price. Thus

$$(1) \qquad p^e_{t+1|t} = a.p^e_{t|t-1} + (1-a).p_t$$

This procedure may be rationalized as providing the optimal forecast if the price p follows a first order integrated moving average (IMA(1,1)) process with moving average coefficient −a (Muth, 1960):

(2) $$\Delta p_t = \varepsilon_t - a\varepsilon_{t-1}$$

However, there is no reason to suppose that this process has any great generality and adaptive forecasts can therefore be systematically beaten (Pesaran, 1987, pp.19–21). Recently, therefore, economists have tended to focus on a more general approach known as rational expectations.

7. The rational expectations hypothesis (REH) states that in the formation of expectations about future economic conditions economic agents use all the information available to them efficiently. The implication is that their expectations may be modeled as the mathematical expectations of the relevant variables conditional upon the information set available to those agents (see e.g., Begg, 1982 or Pesaran, 1987). In the simple case in which all agents have the same information set I_t we may write the market expectation in period t $p^e_{t+k|t}$ of the price p_{t+k} k periods ahead as

(3) $$p^e_{t+k|t} = E(p_{t+k}|I_t) .$$

8. Some commentators have viewed the REH as too strong. Most objections fall into one of three classes (see Pesaran, 1987, chs. 3 and 4):

i) Agents lack the required information to act in this way;

ii) It is too costly for agents to act in this way; and

iii) Agents do not know (or are obliged to learn about) the underlying structural model and so are unable to form rational expectations in the manner proposed by Muth (1961).

9. It is apparent that the first two objections are equivalent and that they pose the important question of the extent to which agents will be both able and willing to acquire relevant information. Different individuals will have different information sets and will thus form different expectations (Pesaran, 1987, ch.4). Thus, for individual j we will have $p_j{}^e{}_{t+k|t} = E_t(p_{t+k}|I_{jt})$ where I_{jt} is j's information set in period t. However, if the commodity is traded on an active futures market the futures price (in this case the k period future) will to a greater or lesser extent aggregate all the relevant information in each individuals information set. Provided that agents observe the futures price it may be 'as if' they observe the entire set of information available on the market, and so a common expectation, equal to the futures price adjusted for any systematic bias, will result (Bray, 1981, 1985).

10. The premise of the third objection considered in paragraph 8 is obviously correct, but the conclusion does not follow. Rather. one can model agents as forming expectations using a 'black box' model (for example, a VAR or a Kalman filter model) rather than a full structural model (Sims, 1980; Engle and Watson, 1987; Harvey, 1987; Granger and Newbold, 1989, pp.297–302). In principle this should result in a model which is at worst inefficient. In practice it may be difficult to find a model which is sufficiently general to be unrestrictive while at the same time providing some structure for the estimates, particularly if one wishes to allow for the possibility of non-stationarity (drift in the mean or the variance of the process) which may be important in commodity prices.

11. In this paper I follow the consensus approach of regarding the REH as a sensible general framework within which to discuss price formation in this and other markets; but at the same time acknowledge that if the hypothesis is applied using simplistic assumptions either about the information available to agents or about the form of the stochastic process followed by the commodity price this may give rise to misleading implications.

12. The recent literature on commodity market modeling has focussed on three quite separate sets of implications of the REH for commodity market behavior. These are

i) The REH imposes testable cross-equation restrictions on the distributed lags on the price variables appearing in the production and consumption equations.

ii) The REH provides a framework within which one can analyze commodity market reactions to news and to other anticipated future developments.

iii) The REH has implications about the character of commodity price cycles, and in particular about departures from linearity in the response of prices to supply and demand shocks.

I discuss (i) in connection with the expectational specification of the production and consumption equations in section C, (ii) in connection with specification of the price equation in section D, and (iii) in section F.

C. Expectational Specification of the Production and Consumption Equations

C1. Cross Equation Restrictions

13. Consider a model in which (aggregate) consumption C and production Q of the commodity both depend linearly on the lagged expectation of the current price in addition to the current price itself. One has

$$C_t = \alpha_0 + \alpha_1 p_t + \alpha_2 p^e_{t|t-1} + \alpha_3' x_t \tag{4}$$

and

$$Q_t = \beta_0 + \beta_1 p_t + \beta_2 p^e_{t|t-1} + \beta_3' x_t \tag{5}$$

where x_t is a vector of other explanatory variables (possibly including lags). Now suppose that the relevant part of agents' information sets consist entirely of the price series; i.e. $I_t = \{p_t, p_{t-1}, \ldots\}$. The mathematical expectation of p_t given I_{t-1} is just the regression of p_t on I_{t-1}. Hence

$$\begin{aligned} p^e_{t|t-1} &= \pi_0 + \pi_1 p_{t-1} + \pi_2 p_{t-2} + \ldots + \pi_k p_{t-k} \\ &= \pi_0 + \pi(L) p_{t-1} \end{aligned} \tag{6}$$

where L is the lag operator and lags greater than k are omitted as negligibly small. Substitution of this expression into the consumption and production equations gives

$$C_t = (\alpha_0 + \alpha_2 \pi_0) + \alpha_1 p_t + \alpha_2 \pi(L) p_{t-1} + \alpha_3' x_t \tag{7}$$

and

(8) $$Q_t = (\beta_0 + \beta_2\pi_0) + \beta_1 p_t + \beta_2\pi(L)p_{t-1} + \beta_3'x_t$$

Write the distributed lag in the consumption equation as a(L) and that in the production equation as b(L). Then

(9) $$a(L) = \alpha_1 + \alpha_2\pi_1 L + \alpha_2\pi_2 L^2 + \alpha_2\pi_3 L^3 + \ldots + \alpha_2\pi_k L^k$$

and

(10) $$b(L) = \beta_1 + \beta_2\pi_1 L + \beta_2\pi_2 L^2 + \beta_2\pi_3 L^3 + \ldots + \beta_2\pi_k L^k$$

Hence for $i > 1$,

(11) $$a_i/a_1 = b_i/b_1 = \pi_i/\pi_1$$

Equation (11) gives a total of 2(k-1) restrictions (compare Wallis, 1980).

14. This set of restrictions arises because the structural consumption and production equations do not contain the lagged prices p_{t-1}, p_{t-2} etc. Hence, any apparent effect of these lagged prices on Q_t and C_t must be attributable to the indirect effect of the lagged prices through the price expectation $p^e_{t|t-1}$. But since both producers and consumers are, by hypothesis, rational, and since, again by hypothesis, they possess the same information, the relative informativeness of p_{t-1}, p_{t-2} etc. must be the same in each equation (the absolute informativeness will depend on the sizes of the price coefficients α_2 and β_2).

15. The most straightforward way of imposing these 2(k-1) restrictions is to estimate the price autoregression by single equation methods and then to impose these coefficients by

linearly restricting a_2, a_3 etc. and b_2, b_3 etc. to the required ratios. The restrictions may be tested by calculating the likelihood ratio of the restricted estimates relative to the same equations estimated without imposition of the restrictions. Write the estimated standard error of the unrestricted consumption equation as S_c and that of the restricted consumption equation as s_c; similarly write the standard errors of the production equations as S_q and s_q; and write the unrestricted and restricted error covariances respectively as RS_cS_q and rs_cs_q. Then the log-likelihood of the unrestricted estimates is proportional to $-(S_c^2 - 2RS_cS_q + S_q^2)$ and that of the restricted estimates is proportional to $-(s_c^2 - 2rs_cs_q + s_q^2)$. Hence

$$(12) \qquad 2[(s_c^2 - 2rs_cs_q + s_q^2) - (S_c^2 - 2RS_cS_q + S_q^2] \sim \chi^2_{2(k-1)}$$

This test readily generalizes to desegregated models in which price expectations enter a greater number of equations. Both the restricted estimates and likelihood ratio tests may be computed very easily using standard regression packages (RATS, TSP etc.).

16. A difficulty with the two step procedure outlined in paragraph 15 is that the estimated coefficient standard errors and t values in the restricted equations must be interpreted as being conditional upon the restrictions holding precisely (Pagan, 1984, 1986). An alternative estimation procedure, which circumvents this difficutly is to jointly estimate the production, consumption and price equations as a system using either Three Stage Least Squares (TSLS, available in both RATS and TSP) or Full Information Maximum Likelihood (FIML) and to impose these restrictions on the estimates. Most packages will generate a test statistic automatically.

17. In practice, it seems likely that attempts to impose cross equation restrictions on price distributed lags in commodity market models are likely to result in rejections. There are two reasons why it may not be possible to impose restrictions of this sort. First, if

lagged prices enter the production and consumption equations in addition to lagged price expectations, the REH may not restrict the distributed lags. If, for example, the consumption and production equations contain the lagged price level p_{t-1}, the number of restrictions implied by the REH is reduced to 2(k-2). If the price process is well described by a second order autoregression, as seems likely, then k=2 and there are no restrictions that can be imposed (compare Pesaran, 1987, ch.6). If the investigator fails to recognize that the lagged prices do enter the structural equation, imposition of the 2(k-1) = 2 REH restrictions is likely to give a rejection. In this case, however, the rejection may be explained by model misspecification (omission of the lagged prices) and not failure of rational expectations.

18. It is difficult to think of convincing reasons why lagged prices, as distinct from lagged price expectations, should enter commodity production and consumption equations. At the same time, I have already emphasized the long lags that arise particularly in commodity production. Typically, therefore, one should expect commodity production to depend on expectations formed at a number of different periods in the past relating to the current price - i.e. to $p^e_{t|t-1}$, $p^e_{t|t-2}$, $p^e_{t|t-3}$ etc.. Furthermore, adjustment costs make it expensive to alter production sharply from one period to the next, and this will imply that current production also depends on expectations like $p^e_{t+1|t-2}$ and $p^e_{t-1|t-2}$ (Nickell, 1978). This suggests that the price expectation $\beta_2 p^e_{t|t-1}$ should be replaced by a term of the form $\Sigma\Sigma\gamma_{jk}p^e_{t+j|t-k}$. But it is obvious that substitution of the price process into the production equation under this modification will imply no restrictions whatsoever.

C2. Backward Representations

19. Recognition of the complexity of the distributed lags in commodity production has motivated most investigators to ignore the fact that production equations should in principle be specified in terms of expected prices and to simply estimate unrestricted distributed lags in terms of actual prices (see e.g. Fisher *et al.*, 1972; Chhabra *et al.*, 1981; Chung and Ukpong, 1981; Ghosh *et al.*, 1987, ch.4; Gilbert and Palaskas, 1990). This procedure may be provided with a more sophisticated justification in terms of estimation of the 'backward' representation of a forward-looking process. We have seen that solution of the price autoregression into the production and consumption equations results in equations which are specified entirely in terms of lagged variables - see equations (7) and (8). Unrestricted estimation of these equations gives the backward representation, whereas imposition of the restrictions (11) across the production, consumption and price equations gives the forward representation. So long as one is concerned only with forecasting, and not with identification of structural parameters, it would appear to be immaterial whether one estimates the forward or the backward representation. Estimation of the forward representation should result in greater efficiency but would generalise any misspecification bias across all three equations. This is the familiar single equation-systems estimation trade-off.

20. The question of the equivalence of forward and backward representations of expectational processes has been discussed in the recent applied macroeconomics literature (Cuthbertson, 1988; Hendry, 1988; Favero, 1989). If there is some structural change affecting, in our case, the price process while leaving the structural production and consumption equations unaffected, then the forward representation of the model would be structurally constant while the backward representation would fail structural constancy tests (see Harvey, 1981, for discussion of these tests). Alternatively, one might find that the backward representation is constant while the forward representation is non-constant. This

would imply that it is not possible to rationalize the backward representation in terms of the supposedly structural forward looking production and consumption equations.

C3. The Error Correction Specification

21. If, for whatever reason, one does adopt a backward representation for the production and consumption equations in a commodity market model, there are strong arguments for using the 'error correction' specification. This specification is generally associated with the work of David Hendry, particularly in relation to the UK consumption function (Davidson *et al.*, 1978) and is associated in his work with the 'general-to-simple' modeling strategy (Gilbert, 1986, 1989a). This approach to modeling commodity production and consumption equations has been argued most strongly by Lord (1988).

22. Consider a general distributed lag relationship linking two variables x and y:

$$(13) \qquad y_t = \alpha_0 + \sum_{i=1}^{k} \alpha_{1i} y_{t-i} + \sum_{j=0}^{k} \alpha_{2j} x_{t-j}$$

We may write equation (13) equivalently as

$$(14) \qquad \Delta y_t = \beta_0 + \sum_{i=1}^{k-1} \beta_{1i} \Delta y_{t-i} + \sum_{j=0}^{k-1} \beta_{2j} \Delta x_{t-j} + \beta_3 y_{t-k} + \beta_4 x_{t-k}$$

The general-to-simple strategy is to look for data-acceptable simplifications of the distributed lags $\beta_1(L)$ and $\beta_2(L)$. This search may be aided by orthogonalization of the distributed lags in terms of higher order differences ($\Delta^2 y_{t-1}$, $\Delta^3 y_{t-1}$, etc.). However, if one is to maintain the error correction structure of the resulting model one must retain the two lagged level variables y_{t-k} and x_{t-k}. A possible simplification of (14) might be

(15) $$\Delta y_t = \gamma_o + \gamma_1 \Delta x_t + \gamma_2 \Delta^2 x_t - \gamma_3 (y_{t-k} - \tau_4 x_{t-k})$$

23. Equation (15) implies a (static) equilibrium relationship between x and y of

(16) $$y^* = \gamma_0 / \gamma_3 + \gamma_4 x$$

(set $\Delta x_t = \Delta y_t = 0$ in equation (15)). The 'error correction' property arises from the fact that if y_{t-k} is above its equilibrium value y^*, Δy_t will be lower than would otherwise be the case; and *vice versa* if y_{t-k} is below y^*. Much of the appeal of this specification derives from the way that the long run equilibrium solution is embedded in the short run dynamic adjustment equation. However, it is now recognized that the validity of this approach depends on whether or not the data identifies an equilibrium relationship between the variables in question. Technically, this is the question of whether the variables entering the error correction term are 'cointegrated'.

C4. Cointegration

24. Most variables considered by economists are 'non-stationary' in the technical sense that their means and variances alter over time. This is obviously true of trending variables. First or second differencing will, however, usually reduce these variables to stationarity. Thus, although a price in level terms will typically be non-stationary, the first difference of its logarithm or the second difference of its level is likely to be stationary. The number of differencing operations required to give stationarity defines the order of integration of the variable. Suppose, in the example considered in paragraph 22, y represents production of a commodity, x represents the commodity price and both x and y are I(1) (i.e. their first differences are stationary). Then any arbitrary linear

combination of x and y, such as y - γ_3x, will in general also be I(1). However, if there is an equilibrium relationship between x and y, there must exist a linear combination of the two variables which is stationary, i.e. I(0). This result, which is known as the 'Granger representation theorem' (Engle and Granger, 1987), follows from the fact that if there is an equilibrium relationship, x and y cannot in the long run diverge by more than a small amount. Consequently, there must be some mechanism pulling the two variables back together. The variables are then said to be cointegrated, and Engle and Granger also showed that it is always possible to give an error correction representation to relationships linking cointegrated variables.

25. If the error correction specification is to be adopted, this should be preceded by tests for cointegration. The tests most usually employed for this purpose are the Augmented Dickey-Fuller (ADF) and Durbin-Watson tests - see Engle and Granger (1987) for discussion. For an example of commodity market econometrics which adopts this approach see Gilbert (1989b).

D. Estimation of Structural Expectational Commodity Price Equations

D1. Traditional Price Models

26. The area of commodity market modeling which has been most substantially affected by the rational expectations 'revolution' is the modeling of the commodity price relationship. The traditional approach to modeling commodity prices has been to relate either the price level, or its first difference, to the level of commodity stocks (sometimes deflated by consumption), or the first difference of this variable (see, for example, Fisher *et al.*, 1972; Labys, 1973; Burger and Smit, 1988; Manger, 1988). Note that in a closed

system, the change in stocks is the current supply-demand imbalance. Thus one has an equation of the form

(17) $$\Delta p_t = \alpha_0 - \alpha_1 \Delta s_t = \alpha_0 - \alpha_1 (Q_t - C_t)$$

(The equation is likely to contain additional terms relating to inflation, exchange rate changes etc. but this does not affect the basic principle). The REH suggests that models of this sort may be inadequate in two contexts:

i) In forecasting: agents may have information on or may be able for other reasons to predict future production and consumption levels and hence stock movements; this will have implications for the current price.

ii) In policy analysis: if one wishes to analyze the effects on commodity prices of different demand management or price control policies, one should take into account the fact that agents will anticipate these policies, and that this will have implications for the current price.

D2. Speculative Stockholding Models

27. The extent to which commodity prices in different periods are linked depends on the extent to which a discrepancy between current and expected future prices results in additional stockholding. The incentive to carry an additional unit of stock is given by

(18) $$p^e_{t+1|t} - (1+r_t+\delta)p_t + c(s_t)$$

where r_t is the interest rate, δ is the rate of stock depreciation and $c(s_t)$ is the marginal convenience yield on stock. The convenience yield function c(.) is generally taken as having negative first derivative ($c'<0$) and to be asymptotic to zero (implying $c''>0$). It represents the yield resulting from any transactions or precautionary demand for stock (see Ghosh *et al.*, 1987, pp.27-29) and is required to explain the observation that stock is frequently carried even when this activity incurs a financial penalty. However, Wright and Williams (1989) have argued that apparent positive levels of convenience yield arise as an aggregation error, and that at a sufficiently disaggregated level convenience yield may be neglected. Acceptance of this argument has the merit of simplifying the discussion so I adopt it in what follows.

28. There are two classes of stockholding model. The first assumes risk neutrality which implies that additional stocks will be carried so long as this gives rise to a positive return. The equilibrium condition in these models is therefore that either stocks are zero and the incentive to carry stocks is non-positive, or stocks are positive and the incentive to carry additional stocks is zero. This may be written as a Kuhn-Tucker inequality (at least one of the two expressions must hold as an equality):

$$p_t \geqslant (1-\delta)p^e_{t+1|t}/(1+r_t) \ : \ s_t \geqslant 0 \tag{19}$$

The non-negativity restriction on stocks is therefore crucial to this model. For stockholding models in this tradition see Gustafson (1958a,b), Gardner (1979), Newbery and Stiglitz (1982) and Wright and Williams (1982). The price implications of these models are discussed in Gilbert (1985) and Miranda and Helmberger (1988). More recent work, reported in Deaton and Laroque (1989), is reviewed in section F.

29. The alternative approach is to assume risk aversion. In this case the stockholding equation is given as

$$s_t = s_0 + \alpha(p^e_{t+1|t} - (1+r_t+\delta)p_t) \tag{20}$$

where $\alpha = 1/A\sigma^2_p$, σ^2_p is the price variance and A is the market coefficient of absolute risk aversion. It is convenient in this model to ignore the non-negativity constraint on stocks since this results in a linear price model. Ignoring this constraint may be justified if there is a sufficiently large level of non-speculative stocks s_0 to absorb negative speculative stockholdings. We will return to this question in section E below. This approach underlies the famous Muth (1961) model, and also the developments of that model in Pesaran (1987), Currie *et al.* (1988), Ramanujam and Vines (1989), and Gilbert and Palaskas (1990). Gilbert (1989c) analyzes a model which combines the non-negativity constraint on stocks with risk aversion - see section F.

30. As indicated above, the major advantage of the Muth risk neutral model is linearity. Forecasting with nonlinear models poses few problems, but policy analysis using control methods, as in Ghosh *et al.* (1987) is greatly facilitated by linearity. On the other hand, it is possible to argue that a linear price relationship will never be able to fully explain the volatility of prices on primary commodity markets - see section F. In the remainder of this section I concentrate on analysis of the linear Muth model.

D3. Structural Estimation of the Muth Model

31. The Muth model may be estimated by either structural or reduced form methods. The structural approach involves regressing the level of stocks s_t on the expected speculative gain $p^e_{t+1|t} - (1+r_t+\delta)p_t$. The obvious difficulty is that the price expectation $p^e_{t+1|t}$ is not observed. Three possible approaches are represented in the literature:

i) Substitute a forward or futures price for the unobserved expected price;

ii) Generate the expected prices as the fitted values of a time series (ARIMA) model for the price; and

iii) Substitute the actual price for the unobserved expected price and estimate by Instrumental Variables (IV) to control for the resulting measurement error.

32. If the data relate to an individual commodity and if that commodity is traded on a futures market it would be natural to replace the expected price by the futures price f_t. The analysis is simplified by imposing a value for the stock depreciation rate δ, and for simplicity we take a value of zero (realistic for metals). The implied model is then

$$(21) \qquad s_t = \alpha_0 + \alpha_1(f_t - (1+r_t)p_t)$$

or its logarithmic near-equivalent

$$(22) \qquad s_t = \alpha_0 + \alpha_1(\ln f_t - \ln p_t - r_t)$$

(since $\ln(1+r_t) \approx r_t$). This is the natural approach to estimating speculative stock demand functions but is only available for commodities for which a suitable forward or futures price is available. Although many of the commodities which are most important in international trade are quoted on an exchange, there is generally only substantial liquidity in contracts with maturities from three to six months. A price from an illiquid contract will not provide a reliable indicator of the market's expectations since it is not possible for a trader with divergent views to buy or sell more than a small amount at that maturity without moving the price against himself. Consequently, it is only sensible to use forward or futures prices as measures of expected prices if one is modeling at a monthly or quarterly frequency.

33. The second procedure discussed in paragraph 31 is that followed by Lord (1988).

Using annual data, he estimated three-equation (production, consumption and stock demand) mini-models for seven primary commodities (maize, cocoa, coffee, copper, cotton, soybeans and sugar). He specified stock demand as an error correction on production or consumption of the commodity and the expected commodity price (not the expected capital gain) with this expectation given as the forecast from an ARIMA model fitted to the price process. There are three difficulties with this approach:

i) Solution of the production, consumption and stock demand equations into the market clearing identity entails a reduced form for the commodity price and this will differ from the ARIMA model used to generate the price expectations. This is not obviously consistent with the REH and at most, therefore, the ARIMA model can be said to give an approximation to rational expectations.

ii) A particular implication of this inconsistency is that if there is a structural change in either the production or consumption equations, this will be reflected in the reduced form but not in the price ARIMA. The Lord procedure is therefore vulnerable to the Lucas (1976) critique.

iii) The use of regressors constructed from previous regressions (e.g. the predictions from an ARIMA model) in a least squares regression gives rise to biased coefficient standard errors and t statistics (Pagan, 1984, 1986). (This is the same point as that made in paragraph 16 in connection with estimated production and consumption equations). One way of looking at this in the current context is to note that the Lord approach may be rationalized as a Two Stage Least Squares (TSLS) estimator (see paragraph 34); however, Lord does not make the second stage corrections to the coefficient standard errors.

34. The REH implies that the expected price $p^e_{t+1|t}$ will differ from the realized price by

an unforecastable innovation

(23) $$p_{t+1} = p^e_{t+1|t} + \epsilon_{t+1}$$

where $E(\epsilon_{t+1}|I_t) = 0$. This suggests substitution of p_{t+1} for $p^e_{t+1|t}$ together with estimation by Instrumental Variables (IV) to give consistent estimates despite the measurement error arising from this substitution (McCallum, 1976). Hence in this case one performs IV estimation of

(24) $$s_t = \alpha_0 + \alpha_1(p_{t+1} - (1+r_t)p_t)$$

or

(25) $$s_t = \alpha_0 + \alpha_1(\ln p_{t+1} - \ln p_t - r_t)$$

In practice, it is likely to be difficult to find good instruments for $p^e_{t+1|t}$. Changes in commodity prices over short periods are dominated by random and unforecastable components. One requires instruments which are uncorrelated with these innovations but are correlated with the systematic components of the price changes. The candidates for these instruments are lagged values of the exogenous variables appearing in the model; or alternatively, but less efficiently, lagged prices. Indeed, use of lagged prices as instruments would be the IV equivalent of the Lord procedure interpreted as TSLS and this implies that the asymptotic efficiency gain of the general IV procedure over Lord's approach is through use of a fuller set of instruments. The practical difficulty is that, in view of the large proportion of the variance of the actual price change accounted for by the price innovation, one will require a very large sample for the desirable asymptotic properties of IV estimation to become evident. This problem will be less acute with the Lord procedure which implicitly uses a smaller number of instruments and suggests that his approach may in practice give greater efficiency in small samples.

35. This discussion strongly suggests that use of the futures price is very much to be preferred where this is possible. (This is not to say, however, that the resulting estimates will be satisfactory since the model itself may be misconceived - see paragraph 41). In cases where there is no futures price, and this will include the majority of annual commodity market modeling exercises, the Lord (1988) approach is probably to be preferred on efficiency grounds to the more standard IV approach involving susbstitution of the actual price change for the anticipated change.

36. Gilbert and Palaskas (1990) have highlighted a general difficulty in the estimation of speculative stock demand functions. They show that on actual commodity market data stock levels and the actual price changes, measured as $\ln p_{t+1} - \ln p_t - r_t$, are of different orders of integration (stocks are I(1) and the price changes are I(0)). The same is almost certainly true of the difference between the futures and the spot price (the 'basis' in futures terminology) measured by $\ln f_t - \ln p_t - r_t$. It is not possible to explain an I(1) variable as a linear function of an I(0) variable since the characteristics of the two are quite different. This implies that actual stock data cannot be explained using the speculative stock demand approach alone. At the very least it is necessary to augment this model by terms which can account for transactions and precautionary stocks which may reasonably be supposed to be non-stationary (and hence at least I(1)). See, for example, Lord (1988) who also uses the (I(1) or I(2)) expected price level in place of the I(0) difference between the expected and the current price.

D4. The Ramanujam and Vines Model

37. Ramanujam and Vines (1989) use a variant of the speculative stockholding model discussed in paragraph 32 which (i) introduces adjustment costs in association with commodity stockholdings and (ii) inverts to make the price the dependent variable.

Minimization of the discounted costs of stockholding in the presence of adjustment costs gives a familiar second order difference equation for stocks in terms of the current expected speculative gain. The standard procedure would be to factorize this second order equation to give a forward lead and a backward lag. Inversion of the lead would then give a forward looking partial adjustment stock demand function

$$(26) \qquad s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 \sum_{i=0}^{\infty} \gamma^i (\ln p^e_{t+i|t} - \ln p^e_{t+i-1|t} - r^e_{t+i-1|t})$$

Equation (26) could be estimated by IV provided (and this seems implausible) that enough instruments were available. In view of the non-stationarity of stock levels one would expect to find β_1 near unity, and this might be taken to suggest that the adjustment cost hypothesis is confirmed. That conclusion would be erroneous since the high coefficient would only reflect the different orders of integration of the two variables under consideration.

38. Ramanujam and Vines prefer to estimate the Euler equation rather than its solved version. This involves estimating an equation of the form

$$(27) \qquad \ln p_{t+1} - \ln p_t - r_t = \delta_0 + \delta_1 s_{t+1} + \delta_2 s_t + \delta_3 s_{t-1}$$

subject to a nonlinear restriction on the δ coefficients. Estimation is by IV to take into account the measurement error resulting from the substitution of the realization s_{t+1} for its planned level $s^e_{t+1|t}$ and the endogeneity of the current stock level s_t. For certain commodity groups (the data consist of commodity price indices) they estimate the same equation in terms of first differences. The results are not particularly impressive and the suspicion remains that the estimated coefficients reflect only the need to difference the stock variable in order to obtain stationarity.

D5. Why are Structural Stockholding Models Unsatisfactory?

39. The discussion in the previous paragraphs makes it clear that structural estimation of the Muth model has not been very successful. There appear to be two reasons for this. First, actual commodity market stock data do not correspond very closely to the theoretical speculative stockholding concept (see also Trivedi, 1990). Stocks are also held for transactions and precautionary reasons. Moreover, stock data are frequently incomplete - it is well-known that published commodity market stock series fail to satisfy market clearing identities. Where this is the case it is likely that speculative stockholdings, other than those held on commodity exchanges, will often be particularly poorly covered.

40. The second reason for the unsatisfactory performance of the structural models relates to the fact that if futures prices are unavailable, the anticipated speculative gain is not observed. Furthermore, we have seen that IV estimation of equations in which the actual gain is substituted for the anticipated gain tends to give poor results because of the lack of good instruments for the anticipated component of the actual gain.

41. It seems likely that even in cases in which the anticipated gain is observed via the futures price that the structural approach will be relatively uninformative. This is because to the extent that stock is carried forward in periods of excess commodity supply, this will drive the expected future price down and pull the current cash price up, thereby reducing the anticipated gain. Indeed, in the extreme case of risk neutrality we have seen that whenever positive stocks are held the expected capital gain net of interest costs and depreciation is equal to zero. In that circumstance the expected price $p^e_{t+1|t}$ (or the futures price f_t) conveys no information additional to that in the spot price. This is not to deny that expected future market conditions are reflected in the futures price, but rather to assert that, in this case, they are equally reflected in the cash price. In the case of risk neutrality, therefore, the futures price only conveys information additional to

that in the spot price when stocks are zero; but that information is then clearly not useful in explaining the (zero) level of stocks. The same tendency will exist if stockholders are risk averse although there will now be some information content in the anticipated capital gain. But the higher the propensity to carry stocks for any anticipated speculative gain the less informative will be that anticipated gain about future market conditions. For commodities traded on active futures markets, hedging significantially reduces the riskiness of stockholding (Gilbert, 1989c). This implies that the structural stockholding model is likely only to be useful for commodities where stock levels are typically low.

42. Inversion of the stock demand function to make the price the dependent variable, as in Ramanujam and Vines (1989) and, implicitly, Trivedi (1990) does not overcome these problems. To the extent that stocks are inadequately measured, this merely translates a problem of poor fit into a problem of measurement error. The stock coefficient in the estimated price equation will typically be biased towards zero, and hence the complete model (production, consumption, stocks) will tend to underforecast price movements. By contrast, if the model is estimated with stock as the dependent variable, as in Lord (1988), the price change coefficient in the stock demand equation will tend to be biased towards zero and consequently when the relationship is implicitly inverted in solving for the market clearing price, the implied coefficient on the stock level will be too high and the model may over-forecast price movements.

E. Estimation of the Solved Muth Model

E1. The Pesaran-Trivedi Approach

43. The discussion in the preceeding paragraphs suggests that it is desirable to look for an alternative to structural estimation in modeling primary commodity prices. The most widely used alternative is to estimate the 'solved' commodity price equation. The procedure here is to embed the stockholding equation in a set of commodity supply and demand equations and to derive what we may loosely refer to as the reduced form for the commodity price. This approach is followed by Ghosh *et al.* (1987), Gilbert and Palaskas (1990) and Trivedi (1990). A conceptual difficulty in analyzing solved forward looking RE models is that in general the solution of these models may be given a number of analytically equivalent representations even when (and this is not guaranteed) the actual solution is unique (see Pesaran, 1987, ch.5). The fact that different investigators have adopted different specifications for their commdity price equations does not therefore imply that these specifications are inconsistent, although they may be. At the same time, although alternative representations of the same solution are algebraically equivalent, it is quite possible that one of these representations may provide a more satisfactory basis for empirical modeling than the others.

44. It will generally be most useful to consider the so-called 'forward' solution of the model and this is the approach adopted by Pesaran (1987) and Trivedi. The Gilbert and Palaskas model is also a variant of the forward solution approach. Pesaran and Trivedi show that the forward solution of a standard but perhaps restrictively simple model may be written as

$$(28) \qquad \ln(p_t/\bar{p}) = \mu_1 \ln(p_{t-1}/\bar{p}) + b \sum_{i=0}^{\infty} \mu_2^{-i} (v^e_{t+i|t} - c v^e_{t+i|t-1})$$

where v_t is a particular linear combination of the disturbances and the exogenous variables in the production and consumption equations. Equation (28) may be thought of as a partial adjustment (coefficient μ_1) towards a temporary equilibrium price p^*_t

$$(29) \qquad \ln(p^*_t/\bar{p}) = b \sum_{i=0}^{\infty} \mu_2^{-i}(v^e_{t+i|t} - cv^e_{t+i|t-1})$$

which depends on a discounted sum (discount factor μ_2^{-1}) of expectational quasi-differences (quasi-difference coefficient c) of current and lagged expectations of the factors shifting the demand and supply functions.

45. To illustrate consider the following simplified version of the model considered in Trivedi (1990):

Production:

$$(30) \qquad Q_t = \gamma_0 + \gamma_1 \ln(p^e_{t|t-1}/\bar{p}) + \gamma_2 x_{1t} + u_{1t}$$

Consumption:

$$(31) \qquad C_t = \beta_0 - \beta_1 \ln(p_t/\bar{p}) + \beta_2 x_{2t} + u_{2t}$$

Stock demand:

$$(32) \qquad s_t = \alpha_0 + \alpha_1(\ln p^e_{t+1|t} - \ln p_t)$$

Market clearing:

$$(33) \qquad s_t = s_{t-1} + Q_t - C_t$$

In this model the composite variable v_t, defined in connection with equation (28), is given as

$$(34) \qquad v_t = [(\beta_2 x_{2t} + u_{2t}) - (\gamma_2 x_{1t} + u_{1t})]/\beta_1$$

Suppose the disturbances u_{1t} and u_{2t} are serially independent, but that the exogenous variables x_{1t} and x_{2t} may each be expressed as first order autoregressions

(35) $$x_{1t} = \delta_1 x_{1,t-1} + \epsilon_{1t}$$

and

(36) $$x_{2t} = \delta_2 x_{2,t-1} + \epsilon_{2t}$$

where ϵ_{1t} and ϵ_{2t} are independent and serially independent. Then

(37) $$x_{1,t+i|t} = \delta_1{}^i x_{1t}$$

and

(38) $$x_{2,t+i|t} = \delta_2{}^i x_{2t}$$

allowing us to write the expected value of the composite term v_t as

(39) $$v_{t+i|t} = [\beta_2 \delta_1{}^i x_{1t} - \gamma_2 \delta_2{}^i x_{2t}]/\beta_1$$

In this model μ_1 and μ_2 (equal here to $1/\mu_1$) are the roots of the quadratic

(40) $$\alpha_1 \mu^2 + (2\alpha_1 + \beta_1 + \gamma_1)\mu + \alpha_1 = 0$$

and the expectational quasi-difference coefficient c is given as

(41) $$c = (1 + \gamma_1/\alpha_1)\mu_1 = g\mu_1 \text{ say.}$$

46. If one possessed estimates of all three structural equations (30-32, production, consumption, stock demand) it would be possible to infer both the values of $v^e{}_{t+i|t}$ $(i=0,1,\ldots)$ for each t, and also the coefficient of the price adjustment equation. This

would be exactly analagous to calculating the reduced form of a standard linear model by solving from the estimates of the structural coefficients using the formula $\Pi = -B^{-1}\Gamma$. For two reasons, however, one both cannot and would not wish to follow this approach. First, we have seen that structural estimation of the speculative stock demand equation tends to be unreliable; and second, the supposedly structural production and consumption equations used to derive this price adjustment equation are too simple to be credible. It is therefore more sensible to use this price equation as a guide in specifying an estimable equation than as a precise implication of the REH. If that approach is followed the estimated price equation will substitute for the speculative stock demand equation.

47. Trivedi recommends replacing the forward lead $\Sigma\mu_2^{-i}(v^e_{t+i|t} - cv^e_{t+i|t-1})$ in equation (28) by short distributed lags of the exogenous variables. In the simple example employed in pararaph 41, in which there are only two exogenous variables both of which follow AR1 processes, the sum $\Sigma\mu_2^{-i}v^e_{t+i|t}$ depends only on x_{1t} and x_{2t} while the sum $\Sigma\mu_2^{-i}v^e_{t+i|t-1}$ depends only on $x_{1,t-1}$ and $x_{2,t-1}$. That substitution implies an estimating equation

$$(42) \qquad \ln(p_t/\bar{p}) = \mu_1 \ln(p_{t-1}/\bar{p}) + \theta_{11}x_{it} + \theta_{12}x_{1,t-1} + \theta_{21}x_{2t} + \theta_{22}x_{2,t-1}$$

48. Trivedi also considers an alternative representation of the forward solution in which the current price depends on the current stock level s_t and the forward lead on the v_t expectations:

$$(43) \qquad \ln(p_t/\bar{p}) = a_1 s_t + a_2 \sum_{i=0}^{\infty} \mu_2^i \, v^e_{t+i|t}$$

He refers to this equation as 'structural' although it is not clear in what way it is either more or less structural than equation (28). Since the two representations are formally equivalent, choice between them depends on whether or not one has available satisfactory stock data rather than on a structural form–reduced form dichotomy. The corresponding estimable equation from this specification is

$$(44)\qquad \ln(p_t/\bar{p}) = a_1 s_t + \varphi_{11} x_{1t} + \varphi_{21} x_{2t}$$

(where the stock level is engogenous). Equation (44) forms the basis for equations estimated in that paper.

49. Although the estimable equations (42) and (44) are derived within an RE commodity market model and hence are compatible with the REH, it is nevertheless the case that they fail to give a satisfactory representation of the implications of the REH for commodity market behavior. First, it is not possible to use these equations to incorporate any advance information either on movements of the exogenous variables (e.g. policy announcements) or on the equation residuals (so called 'conjunctural' analysis – see Keating, 1985). Second, the specifications are vulnerable to the Lucas (1976) critique since a change in the processes governing the exogenous variables x_1 or x_2 would alter the θ coefficients in equation (42) or the φ coefficients in equation (44).

50. These observations prompt a search for a procedure intermediate between inference of the price adjustment equation from the structural equations and unrestricted estimation of an equation the specification of which is suggested by the rigorously derived model. Direct substitution of the processes for x_1 and x_2 into the price adjustment equation (28) gives

$$(45)\qquad \ln(p_t/\bar{p}) = \mu_1 \ln(p_{t-1}/\bar{p}) + \frac{b\gamma_2}{\beta_1(1-\mu_1\delta_1)}\left[x_{1t} - g x_{1,t-1}\right]$$
$$- \frac{b\beta_2}{\beta_1(1-\mu_1\delta_2)}\left[x_{2t} - g x_{2,t-1}\right]$$

If the parameters β_1, β_2 and γ_2 are imposed from the production and consumption equations (30, 31) and δ_1 and δ_2 from the exogenous variable autoregressions (35, 36), only three parameters remain to be estimated (μ_1, b and g). Estimation by nonlinear least squares (NLLS) is straightforward and can be implemented in most standard regression packages. This equation therefore uses the RE commodity price theory to impose two restrictions on equation (28). Unfortunately, it does not appear possible to obtain a similarly restricted version of equation (43).

E2. The Gilbert-Palaskas Model

51. The underlying philosophy of the approach adopted by Ghosh *et al.* (1987) and Gilbert and Palaskas (1990) is to attempt to obtain a natural generalization of the traditional 'myopic' commodity price equation (17) to allow for rational expectations. In the traditional approach the change in the commodity price is related to the current market imbalance $Q_t - C_t$, equal to the change in stock Δs_t. This suggests that in a forward looking model the current change in price should be related to a discounted sum of present and future market imbalances. Two questions arise:

i) At what rate should expected future imbalances be discounted? and

ii) Should one model expected future imbalances or the innovations in these imbalances (i.e. the unanticipated components of the imbalances) as affecting the commodity price?

52. The fundamental equation in the Gilbert and Palaskas model is

$$(46) \qquad \ln p_t = a + \mu_1(\ln p_{t-1} + r_{t-1}) - b(1-\mu_1)\sum_{i=0}^{\infty}\mu_1^i(\zeta_{t+i|t} - \lambda\zeta_{t+i|t-1})$$
$$- \mu_1 \sum^{\infty} \mu_1^i(r_{t+i|t} - \lambda r_{t+i|t-1})$$

$i=0$

where ζ_t is the systematic component of the current supply-demand imbalance (i.e. that part of Q_t-C_t which is independent to the commodity price). It is immediately apparent that the model is very similar to that derived by Trivedi (1990) and discussed in paragraphs 44-50. Indeed, the parameter μ_1 which governs the rate at which future market imbalances are discounted is defined as the smaller root of the same quadratic equation (40). Note that as α approaches infinity, i.e. the market approaches risk neutrality, both μ_1 and λ approach unity and the equation tends to the pure innovation relationship

$$(47) \qquad \ln(p_t/\bar{p}) - \ln(p_{t-1}/\bar{p}) - r_{t-1} = a - b\sum_{i=0}^{\infty}(\zeta_{t+i|t} - \zeta_{t+i|t-1}) - c\sum_{i=0}^{\infty}(r_{t+i|t} - r_{t+i|t-1})$$

On the other hand, as α becomes very small one obtains the static price equation

$$(48) \qquad \ln(p_t/\bar{p}) = a - b\zeta_t$$

This model therefore reflects the intuition that the greater the degree of stockholding, the greater the extent that anticipated future conditions affect the current market price.

53. As is the case with the model analyzed by Trivedi, there is an alternative way of expressing the Gilbert-Palaskas model which conditions on the level of stocks. Gilbert and Palaskas also consider that variant of their model which may be written as

$$(49) \qquad \ln p_t = a - b(1-\mu_1)\left[s_{t-1} + \sum_{i=0}^{\infty}\mu_1^i\zeta_{t+i|t}\right] - \mu_1\sum_{i=0}^{\infty}\mu_1^i r_{t+i|t}$$

As previously, choice between the two equations (46 and 49) depends on whether or not the stock series is sufficiently comprehensive and reliable to be useful.

54. Estimation of the Gilbert–Palaskas price adjustment equations (46, 49) is complicated, but not notably more complicated than estimation of the Pesaran–Trivedi adjustment equation. It is necessary to generate for each sample period t a sequence of expected market imbalances ζ_t, $\zeta_{t+1|t}$, $\zeta_{t+2|t}$, etc. In principle, these sequences should be infinite, but in practice it is probably satisfactory to curtail them at a relatively short horizon – Gilbert and Palaskas looked 10 periods ahead and Ghosh *et al.* (1987) used an even shorter horizon. The expected imbalances may be obtained by taking the estimated model with the price equation deleted, and running it forward say 10 periods from each starting date in the sample. In the run for period t every price dated t+1 or later is replaced by the sample mean price $\bar{p}$. The estimated supply–demand imbalances $\zeta_{t+i|t}$ are then estimated as forecast production in period t+i less forecast consumption in period t+i in the run starting at date t. These estimated imbalances are then treated as data in the commodity price adjustment equation which can be estimated by NLLS. It is apparent that estimation of and forecasting with the Gilbert–Palaskas model requires estimated equations for the exogenous variables in the production and consumption equations in the same way as does the Pesaran–Trivedi approach.

55. The same procedure is followed in forecasting out of sample. An initial set of forecasts is made for the expected market imbalances with the price set at its sample mean value, and the price in the current and each succeeding period is then forecast using these expected imbalances as data. Note however that there is no need in this case to calculate a new set of expected imbalances for each successive period since under the REH $E_t x_{t+i|t+j} = x_{t+i|t}$.

56. A difficulty with estimating equations in which some of the variables are calculated from preliminary regressions is that the sampling error associated with the estimated

coefficients used in these constructions introduces measurement error into the constructed variables (Pagan, 1984, 1986; Pesaran, 1987, ch.7) which will in princple bias the estimated coefficient standard errors and prevent correct inference. In principle, the non-sphericality introduced by this measurement error may be overcome by joint estimation of the price adjustment and production and consumption equations by maximum likelihood (ML). The cost of this approach is that it allows any misspecification in the production and consumption equations to affect the estimates of the price adjustment equation. One therefore runs the danger of creating bias and inconsistency in the coefficient estimates in the vain attempt to eliminate bias in the coefficient standard errors. In practice, Gilbert and Palaskas found that the NLLS estimates were preferable to the ML estimates.

E3. Comparison of the Pesaran-Trivedi and Gilbert-Palaskas Models

57. The essential difference between the Pesaran-Trivedi and Gilbert-Palaskas models relates to their treatment of the exogenous variables. Pesaran and Trivedi express the RE price solution in terms of the current and expected future values of the exogenous variables x_1 and x_2 and the disturbances u_1 and u_2 in the production and consumption equations. In the Gilbert-Palaskas model these variables are replaced by current and future expectations of the market supply-demand imbalance $Q - C$. Because these variables are endogenous it is necessary to correct them for price changes, but linearity makes this trivial. Within the simple model considered in paragraph 41, the price-independent component of this imbalance is given as

$$(50) \qquad \zeta_t = (\gamma_0 - \beta_0) + (\gamma_2 x_{1t} - \beta_2 x_{2t}) + (u_{1t} - u_{2t})$$

However, it follows immediately that

$$(51) \qquad \zeta_t = (\gamma_0 - \beta_0) + \beta_1 v_t$$

where v_t is the composite term defined in equation (28). Hence, within the context of this simple model the Gilbert–Palaskas solution is identical to that obtained by Pesaran and Trivedi.

58. More generally, the major advantage of the Gilbert–Palaskas solution over that suggested by Pesaran and Trivedi is that it is much more conservative of degrees of freedom. This advantage is important in considering models with a larger number of exogenous variables than the two permitted in the Trivedi (1990) model. In a disaggregated model, which distinguishes production and consumption over a number of different geographical areas (or alternatively product types or end–uses), there will typically be a comparably large number of exogenous variables and it will not in general be feasible or desirable to incorporate all of these in the estimated price equation. This was the situation encountered in Ghosh *et al.* (1987). The Gilbert–Palaskas model restricts all of these variables to enter the price equation with their weights in the production and consumption equations. Hence these variables will all be reflected in a single forward distributed lead irrespective of their number.

59. An additional merit of the Gilbert–Palaskas approach, already indicated above, is that by relating the current price to current and future market imbalances, it generalizes the traditional models in which the current price adjustment is related simply to the current market imbalance. A useful way of looking at this generalization is to note that the appropriate state variable to which the price reacts is an appropriately discounted sum of current and future imbalances and not just the current imbalance. This is also implicit in the restricted version of the simple Pesaran–Trivedi model but the algebraic structure of the estimating equations do not immediately suggest this intuition.

60. It is also possible to argue that the Gilbert-Palaskas model is more robust to the Lucas (1976) critique than is the Pesaran-Trivedi model. A change in the process generating one of the exogenous variables will not affect the parameterization of the commodity price adjustment equation expressed in terms of current and future supply-demand imbalances, although it will be necessary to take this change into account in forecasting those imbalances. Furthermore, if the modeler has direct information on likely future supply-demand positions (a situation which is by no means impossible in the mining industry), this information may be directly incorporated into the price forecasts by overriding the forecast supply-demand imbalances for those periods covered by this information.

61. In the foregoing I have compared the Gilbert-Palaskas model with the simplified version of the Pesaran-Trivedi model which arises from embedding the speculative stock demand function in the simple supply-demand model outlined in paragraph 41. The Pesaran-Trivedi model is however somewhat more general than this. In particular, Trivedi (1990) augments this simple model by including a transactions demand for stocks related to expected consumption in the next period. Trivedi's stock demand equation therefore becomes

$$s_t = \alpha_0 + \alpha_1(\ln p^e_{t+1|t} - \ln p_t) + \alpha_2 C^e_{t+1|t} \tag{52}$$

As a consequence of this complication, the exogenous variables x_1 and x_2 influencing production and consumption respectively no longer affect the price with the same relative weights with which they affect the market imbalance. Hence this model is somewhat more genral than the Gilbert-Palaskas model. I noted in paragrpah 36 the difference between the orders of integration of the stocks and anticipated capital gains and this does suggest that a large part of actual stockholdings cannot be explained solely by the speculative motive and that one should, as a consequence, also consider the transactions and precautionary motives. It is possible that as more and better stock data become available allowing detailed modeling of transactions and precautionary stocks it will be possible to use a model of this form to isolate the speculative components of stock demand.

E4. Interest Rate Effects in the Gilbert and Palaskas Model

62. A second respect in which the Gilbert and Palaskas model differs from the model in Trivedi (1990) is in the treatment of interest rates. The Pesaran–Trivedi model allows a constant interest rate, but since this enters as a parameter it does not generalize to variable rates. The semi-logarithmic functional specification adopted by Ghosh *et al.* (1987) and Gilbert and Palaskas, by contrast, allows the interest rate to enter as a separate state variable. It has been suggested, see e.g. Currie *et al.* (1988), that interest rate effects on primary commodity prices provide an important route by which monetary policy in the developed world affects LDCs. The inclusion of interest rates in the commodity price model in a non-parametric manner is therefore of some interest in order to quantify the importance of this effect.

63. Note that interest rates enter the Gilbert–Palaskas price adjustment equation (46) in two distinct ways. First, there is the positive effect of the interest rate level. This Hotelling (1931) effect requires that commodity prices rise with the rate of interest, but the effect is moderated in relation to the degree of market risk aversion. Second, the commodity price is seen as responding negatively to a change in the interest rate. This is a standard asset market effect. A higher interest rate requires, through the level effect, that the price rise faster; if there were no fall in price, this would result in a price higher at every subsequent point in time, which would create excess supply of the commodity. Hence the price must jump down in order that it can subsequently rise faster. A difficulty with standard models which simply enter current and lagged interest

rates into commodity price equations is that they are likely to confuse these two effects.

E5. How General are Procedures Based on Solved RE Models?

64. A difficulty with both the Pesaran-Trivedi and the Gilbert-Palaskas approaches is that in order to obtain the RE solution for the commodity price it is necessary to embed the speculative stock demand function in a simple supply-demand model. In the case of Gilbert and Palaskas, that model must have two restrictive features:

i) the quantity equations must be linear in the quantity variables (production, consumption, stock demand) and linear in either the price variables (Pesaran-Trivedi) or in the logarithms of the price variables (Gilbert-Palaskas); and

ii) the model cannot permit any partial adjustment dynamics in the quantities and allows at most a one period lag on the expected price.

65. The semi-logarithmic functional specification adopted by Gilbert and Palaskas has the attractive feature of maintaining linearity across identities while at the same time allowing linear decomposition of relative prices. Ghosh *et al.* (1983) report that it receives empirical support. In any case the fit on commodity production and consumption equations is seldom so good that a precise functional specification is very strongly indicated by the data. The more worrying feature of the Pesaran-Trivedi and Gilbert-Palaskas approaches is therefore the restrictiveness of the dynamic specification permitted in the production and consumption equations.

66. It is possible to acknowledge the restrictiveness of the assumptions required to derive the Gilbert-Palaskas model but to defend the model on the argument that embedding the

speculative stock demand function in a general supply-demand model may be expected to give rise under rational expectations to a reduced form representation with the same general structure. However, the introduction of longer lags will give rise to a high order difference equation which will not permit analytic solution; and other (perhaps more appealing) functional specifications will result either in a nonlinear price model (this is the case if the production and consumption equations are logarithmic), in which case the reduced form is only defined implicitly, or in a less tractable linear model (as with production and consumption equations which are linear in the level of the price).

67. The generality of this approach arises from the fact that in a linear model, any variable, and therefore in particular the commodity price, may be expressed as an appropriately weighted sum of the past, present and futures disturbances on all the equations. If we follow conventional practice and suppose that the stockholding equation is exact this implies that the commodity price may be expressed as a sum of the disturbances on the production and consumption equations. In the Gilbert-Palaskas model, z_t is just the difference between the (sum of the) production disturbance(s) and (the sum of the) consumption disturbance(s). The model requires that the weights associated with the expected values of these future disturbances should decline exponentially; and that the weights associated with past disturbances, which are reflected in the autoregressive term, should have the same pattern. More general supply and demand models will result in more complicated weighting patterns, but the reduced form commodity price equation will always have this general structure. There is thus no reason to suppose that a regression strategy which proceeds by progressive relaxation of this equation will result in a particularly restrictive equation. This is the strategy adopted in Gilbert and Palaskas.

E6. How Well Do Forward-Looking Models Perform?

68. A number of econometric analyses of commodity price formation undertaken over the past few years have adopted the forward-looking approach discussed in the previous paragraphs. In general the results have not been encouraging but there are some interesting pointers which may be useful in planning future work in this area. I shall discuss three studies which attempt to use the Muth framework, or some variant of this, in analyzing commodity price formation. These are Ghosh *et al.* (1987), Gilbert and Palaskas (1990) and Trivedi (1990).

69. Ghosh *et al.* (1987) estimated a detailed quarterly model of the world copper market and compared a price adjustment equation which contained only the current supply-demand balance with a similarly specified equation containing the expected supply-demand balance for up to seven quarters ahead. They found weak evidence that it is useful to include the supply-demand balance for one and two quarters ahead in the equation, but that further leads are not helpful. Ghosh *et al.* suggested three possible reasons for this:

i) The markets may only look two quarters ahead;

ii) Although the markets do look further than two quarters ahead, the simple auxiliary equations used to forecast the exogenous variables do not adequately reflect these expectations more than two quarters ahead;

iii) Although the markets do in principle look further than two quarters ahead these expectations have very little information content since the markets have very little long distance information.

70. Gilbert and Palaskas (1990) estimated three equation 'mini-models' for six commodities (cocoa, coffee, copper, natural rubber, sugar and tin) using annual data. They estimated both the price adjustment specification (46) and the specification which utilizes lagged stock data (49). For three of the commodities (copper, natural rubber and sugar) the latter specification gave a superior fit, but for two of these commodities there was no evidence of any forward-looking behavior. Only in the case of copper was there clear evidence of forward-looking behavior, confirming the results obtained by Ghosh *et al.* (1987). For the remaining three commodities the model did not appear to give an adequate description of the sample data. As Wallis (1990) noted in his comment this may be because the Gilbert and Palaskas model ignores the major interventions by international stabilization agencies in the coffee and tin markets. Indeed, of the six commodities considered by Gilbert and Palaskas, only copper was completely free from intervention over their sample period.

71. A second reason for the greater evidence of forward-looking behavior in the Ghosh *et al.* (1987) study may derive from the fact that their model was quarterly and incorporated substantial institutional detail. In particular Ghosh *et al.* assumed that the market correctly anticipated both the timing and duration of labor disputes in the US copper industry (these strikes at the end of three year labor contracts resulted in major shortages in periods of tight demand), and the timing and extent of releases of metal from the US strategic stockpile (the GSA stockpile). By contrast, the Gilbert-Palaskas mini-models were insufficiently detailed to permit incorporation of this sort of information.

72. Trivedi (1990) estimated equations of the form (44) for tea, cocoa, coconut oil and palm oil all estimated on samples of annual data by Instrumental Variables. Only in the case of cocoa was he able to find strong evidence of a robust price-sensitive inventory relationship. Furthermore, as Trivedi himself noted, it is unclear that the φ coefficients on the exogenous variables can be interpreted as confirming the presence of

forward-looking behavior.

73. These studies also provide some evidence on the extent of interest rate effects on commodity prices. As indicated above, these effects are central to the transmission of monetary effects from the developed to the developing countries in certain North-South models - see in particular Currie *et al.* (1988). In an early study, Heal and Barrow (1980) claimed to find strong evidence of interest rate effects on metals prices. They concluded that "There is no question that the results reported here appear to confirm, at least in general terms, ... [the] view ... that resource price movements should be related to returns on other assets ...". However, both Trivedi and Gilbert and Palaskas find only very weak evidence for interest rate effects on commodity prices; and in Ramanujam *et al.* interest rate effects are only obtained by restriction of the interest rate level coefficient to be equal to that of the lagged price level. Gilbert and Palaskas conclude "... academic commentaries which suggest that interest rates link developed country monetary policy to developing country terms of trade may be guilty of giving an excessive role to an influence whose effect is of a low order of magnitude".

74. If any conclusion can be drawn from these three studies it is that attempting to incorporate forward-looking behavior in a commodity market model is only sensible within a model which incorporates sufficient institutional detail to allow the modeler to reflect the information actually available to the market. It seems possible that this will often imply modeling at a higher than annual data frequency since markets will generally not possess information going very far ahead. Current experience suggests that there is little to be gained apart from theoretical elegance in attempting to incorporate forward-looking behavior in very simple 'academic' aggregate market clearing models where the only implications of the REH are the restrictions on the distributed lags of the exogenous variables in the commodity price equation.

F. Stockout and Nonlinearity of the Price Relationship

F1. The Gustafson Model

75. An alternative tradition in modeling commodity prices, anticipated in paragraph 28, stems from the Kuhn–Tucker condition (19) which states that either the commodity price rises with the rate of interest or stocks are zero. In the latter case there is no connection between the market in successive periods and the price can rise at a lower rate or even fall. That behavior is characteristic of many agricultural commodities (e.g. potatoes) which exhibit rising prices through the harvest year but where the price falls as the new crop becomes available. For commodities for which this provides a good characterization, the Muth model analyzed in sections C and D would only hold within the harvest year, while for the analysis of price movements between harvest years one would naturally use the static Marshallian equilibrium model.

76. Samuelson (1957) analyzed a somewhat more complicated case in which the stock would typically be consumed within the harvest year but in the event of an abnormally good harvest there would be a carryover into the next year. In this model, the carryover decision is endogenous. This model also provided the basis for the model analyzed by Gustafson (1958a,b) who derived, under the assumption of risk neutrality, the optimal carryover decision (see also Gardner, 1979, Newbery and Stiglitz, 1982, Wright and Williams, 1982, 1984, and Gilbert, 1988). The price implications of a certainty equivalence version of the Gustafson model were analyzed by Gilbert (1985) who showed that in the situation that stock is carried, locally (i.e so long as the storage horizon was not revised) the commodity price would follow a martingale process, and so price changes would be unforecastable, but that once stockout occured the price could fall in a manner which could be anticipated but which would not allow speculative profits to be made.

77. The Gustafason model as analyzed in Gustafson (1958a,b), Gardner (1979), Newbery and Stiglitz (1982) and Wright and Williams (1982, 1984) has two important implications for commodity price adjustment. These are

i) Storage depends only on the carrying cost (the interest rate in simple models) and on total availability of the commodity where total availability is equal to the stock carried over from the past plus current supply taken to be insensitive to the current price.

ii) The marginal storage propensity, which is zero until availability reaches a critical value, then rises monotonically with availability.

The implications of these results for price adjustment are:

i) The commodity price can only be a function of total availability and the interest rate; and

ii) Since any given supply shock ϵ will be transmitted to the price as $(1-s')\epsilon$, where s' is the marginal storage propensity, the same shock will have much greater price impact in a tight market (i.e. low availability) than in a weak market.

This second implication, due initially to Hillman *et al.* (1975) and Gardner (1979), is important since it means that commodity price adjustment equations should be nonlinear. The Muth model, by contrast, implies a constant marginal storage propensity and hence a linear response of price to supply shocks.

78. This feature is clearly evident in the development of the Gustafson model analyzed in Gilbert (1985). Here the assumption of semi-logarithmic production and consumption

equations implies that the initial marginal storage propensity is one half, rising to two thirds, then three quarters etc.. The implication is that the price response to any supply (or in this model also demand) shock will fall to the proportions one half, one third, one quarter etc. of the tight market response as availability increases. But as Wright and Williams (1982) show, and Gilbert (1988) confirms, the presence of lagged price responses tends to increase the marginal storage propensity.

79. The Gilbert (1985) model also generalizes the availability concept to allow for predictable movements of production and consumption in future periods. If it is expected that positive stock will be held for h periods, generalized availability a_t is defined as

$$(53) \qquad a_t = s_{t-1} + \zeta_t - \frac{1}{h-1} \sum_{i=1}^{h} \zeta^e_{t+i|t}$$

where ζ_t is the price independent component of the supply-demand imbalance defined in connection with equation (46). It is a feature of this model that, so long as positive stocks are held, intertemporal demand or supply shifts have no impact on the commodity price. One would expect this feature to disappear within a more general framework.

F2. The Deaton and Laroque Model

80. In an important contribution, Deaton and Laroque (1989) have developed methods for obtaining the price function without assuming certainty equivalence, and Gilbert (1989c) generalizes their approach to allow risk aversion. Deaton and Laroque analyze a model in which production Q_t is randomly distributed about its mean level and does not exhibit any price response, consumption C_t is given by the general monotonic demand function $C_t = P^{-1}(p_t)$, there is a constant interest rate r, and the rate of stock deterioration is δ. In this model total availability a_t is given as $Q_t + (1-\delta)s_{t-1}$. They prove that this model possesses a stationary rational expectations equilibrium in which storage and hence the

commodity price will both be functions of the single state variable, availability. Write the storage function as $s_t = g(a_t)$ and the price function as $p_t = f(a_t)$. The functions f(.) and g(.) are defined implicitly by the equations

(54) $$f(a) = \max[P(a),\ \beta(1-\delta)E_Q f(Q + (1-\delta)g(a))]$$

where $\beta = 1/(1+r)$ and

(55) $$g(a) = \max[0,\ a - P^{-1}(f(a))]$$

The interpretation of these functions is as follows. If current availability a_t is low, storage is zero (the first arm of (55)), and the price clears the market at $P(a_t)$. If availability is higher, the expected price in period t+1 is given as $Ef(a_{t+1}\,|\,t)$. But next period's availability a_{t+1} is simply next period's production Q_{t+1} plus that part of current storage which has not depreciated. The second arm of (54) gives the discounted value of this expected price. The second arm of (55) states that storage is equal to current availability less current consumption which is given as a function of current price.

81. The price function given by Deaton and Laroque results from solving equations (54) and (55) to obtain an implicit representation for f(.) as

(56) $$f(a) = \max[P(a),\ \beta(1-\delta)E_Q f(Q+(1-\delta)(a-P^{-1}(f(a))))]$$

There is an analagous representation for the storage function g(.) (not given explictly by Deaton and Laroque) as

(57) $$g(a) = \max[0,\ a - P^{-1}(\beta(1-\delta)E_Q(Q+g(a)-g(Q+g(a))))]$$

Deaton and Laroque compute the price function f(.) using numerical integration techniques by searching for a fixed point in function space for equation (56); while Gilbert (1989c) uses the same method to obtain the storage function g(.) using an equation very similar to (57).

82. Gilbert (1989c) generalizes the Deaton and Laroque procedure to accommodate risk aversion. The storage function is now generalized from equation (20) to

$$(58) \qquad s_t = \max[0,\ \alpha_t((1-\delta)p^e_{t+1|t} - (1+r_t)p_t/\bar{p}]$$

or, using a logarithmic approximation and setting δ to equal zero,

$$(59) \qquad s_t = \max[0,\ \alpha_t(\ln p^e_{t+1|t} - \ln p_t - r_t)]$$

where α_t is given as

$$(60) \qquad \alpha_t = \frac{1}{A\ E_t(\ln p_{t+1} - E_t \ln p_{t+1})^2}$$

and A is the market coefficient of absolute risk aversion. The stationary rational expectations equilibrium associated with this storage condition in the Deaton and Laroque model is characterized by three equations: the price function f(a), storage function g(a) and volatility function v(a) which gives the variance of the next period's price (the expression in the denominator of (59)) as a function of current availability. The three equations are

$$(61) \qquad f(a) = P(a - g(a))$$

which gives the current price as a function of current availability less current storage,

$$(62) \qquad g(a) = \max\big[0,\ E_Q(f(Q+g(a))) - f(a) - r\big]/Av(a)$$

which is simply equation (59) and

$$(63) \qquad v(a) = E_Q\big[f(Q+g(a)) - E_Q f(Q+g(a))\big]^2$$

which is equation (60). These equations may again be solved by numerical integration methods, although the requirement to compute the conditional volatility (63) implies that the computational burden is now considerably greater than in the risk neutral case considered by Deaton and Laroque.

83. The price functions f(a) computed by Deaton and Laroque and Gilbert are smooth and convex. Indeed, Deaton and Laroque prove that convexity of the inverse demand function P(.) implies convexity of the price function f(a). The consequence is that a supply shock will have a significantly smaller effect on the commodity price when availability is high (i.e. when a large volume of stocks is carried) than when availability is low (i.e., when stocks are near or equal to zero). This confirms the result in Gilbert (1985), but demonstrates, as anticipated, that dropping certainty equivalence smooths out the piecewise linear price function generated by the Gustafson model.

84. The paradigmatic commodity which underlies the Deaton and Laroque model is an annual agricultural crop with a well-defined harvest period. For a commodity of this sort it is reasonable to adopt the textbook caricature in which production depends only on expected prices but is subject to substantial unplanned variation due to weather conditions. The converse case is that of a continuously produced metal where short run price variability originates almost entirely in shifts of the demand function which is highly

inelastic in the short term but where there is considerably greater flexibility in response in production. For a metal of this sort the Deaton–Laroque concept of availability should be replaced by net demand, defined as consumption less inherited stocks and the inverse supply function substitutes for the inverse demand function. The analysis is the same from then on.

F3. Implications for Econometric Modelling

85. As noted throughout this discussion, the major implication of the Gustafson–Deaton and Laroque approach is the nonlinearity of the price response function. By contrast, almost all applied work on commodity market modeling has adopted a linear framework. It is arguable, however, that only by using nonlinear functions can one hope to explain the enormous price movements that occur from time to time in commodity markets. The period 1973–74 is most notable in recent history. If the price response in subsequent periods of substantial excess supply had matched the response to the relatively modest excess demands in that period, commodity prices in the nineteen eighties would have been even lower than those observed. A linear model which explains the eighties cannot explain the mid-seventies, and *vice versa*.

86. This nonlinearity is apparent from simple differencing of the price function f(a) defined in (56) or (61). For ease of comparison with the models I have considered earlier I take equation (61) and make the interest rate explicit as

$$\ln p_t = f(a_t, r_t) \tag{64}$$

with $a_t = s_{t-1} + \zeta_t$ and where ζ_t is defined in equation (46). Write the derivatives of f(.,.) with respect to a and r respectively as f_a and f_r. Noting that

$$s_t = s_{t-1} + (Q_t - C_t) = s_{t-1} + \zeta_t + (e+\varepsilon)\ln p_t \tag{65}$$

where e and ε are respectively the contemporaneous price elasticities of consumption and production, one obtains

$$\Delta \ln p_t = \frac{f_a}{1 + (e+\varepsilon)f_a} (Q_t - C_t) + \frac{f_r}{1 + (e+\varepsilon)f_a} \Delta r_t \tag{66}$$

This is a straightforward generalization of the traditional model (17) (note that both f_a and f_r will be negative). However, $f_a = 1 - g_a$ falls (in absolute value) as availability increases giving the required nonlinear response. It is obviously likely that f_r is non-constant and the Gilbert (1985) results suggest that this coefficient should be increasing in availability.

87. This model has only two state variables (availability and the interest rate). By contrast, the Muth model considered in sections D and E gives the price adjustment as a function also of future expected market imbalances, or, equivalently, of expected future values of the exogenous variables in the model. We have noted in the context of the Gustafson model that Gilbert (1985) generalizes the availability variable to take into account future expected imbalances (equation (53)), and a similar generalization is required in the Deaton and Laroque model if it is to accommodate forward-looking behavior. This must be a priority for future research in this area.

88. It is worth briefly noting the implications of the Gilbert (1985) and Deaton and Laroque models for mean reversion in commodity prices. In general, there is no reason why real commodity prices should rise or fall over time unless there are structural shifts in the supply or demand functions. This suggests that real commodity prices should be stationary, or I(0) (see paragraph 24). This implies, *via* the Granger representation

theorem (Engle and Granger, 1987) that in a linear model there must exist an error correction mechanism (see section C3) bringing real commodity prices back to their long term level. Indeed, Ghosh *et al*. (1987) adopted this specification in modeling the copper price. The mean reversion process implied by the nonlinear Gilbert (1985) and Deaton and Laroque models is, however, somewhat different. So long as positive stocks are held, the commodity price varies in a random walk manner about a trend rising at the rate of interest (i.e. it follows a martingale process with positive drift). Over time this drift will take the price above its long-term mean to an increasing extent. There will, however, always be the possibility of a sufficiently large negative shock (i.e., a fall in production or rise in consumption) as to result in stockout. That shock will result in a further and possibly sharp rise in the price but will also break the connection between the current price and the price in succeeding periods. Hence, after stockout there will be a new price path, also drifting upwards at the rate of interest, but starting at a lower level. The implication is that if one correctly models the nonlinearity in the commodity price response function an error correction mechanism will be superfluous; but that in a linear approximation to the nonlinear model it may be necessary.

F4. Empirical Evidence on Nonlinearities in Commodity Price Responses

89. There has as yet been very little empirical work directed at the issue of nonlinearity of the price response function in econometric commodity market models, and none hitherto directed specifically at the implications of the Deaton-Laroque model. In paragraphs 90-92 I look respectively at an agricultural crop commodity, sugar, and in paragraphs 93-96 at a metal, aluminum. In the case of sugar, the Deaton-Laroque model appears to give a very good approximation to the market process and there is clear evidence of nonlinearity while in aluminum the Deaton-Laroque model is unhelpful and there is no evidence of departures from nonlinearity.

90. Figure 1 plots the International Sugar Agreement (ISA) free market sugar price (c/lb) over the period 1967–87, deflated by the US producer prices index (all items, 1980=1.00) against availability of sugar defined as production plus lagged inventories as a percentage of trend consumption. Although the scatter does not define a very precise relationship, there is clear evidence from the plot of non–linearity. This remains true if the price is logged. The major outlier (marked) from this pattern is the observation for 1975 when the price was falling fast through the year despite low availability. This may be accounted for consistently with the RE hypothesis in terms of anticipations of the sharp decline in consumption which was to occur in 1976.

91. The best fitting linear relationship over this period is

$$(66)\quad \ln(PISA_t/USPP_t) = \underset{(1.661)}{8.045} + \underset{(0.102)}{0.310}\ln(PISA_{t-1}/USPP_{t-1}) - \underset{(0.469)}{1.735}\ln EXR_t$$

$$- \underset{(0.009)}{0.014}t - \underset{(1.218)}{6.128}AVAIL_t$$

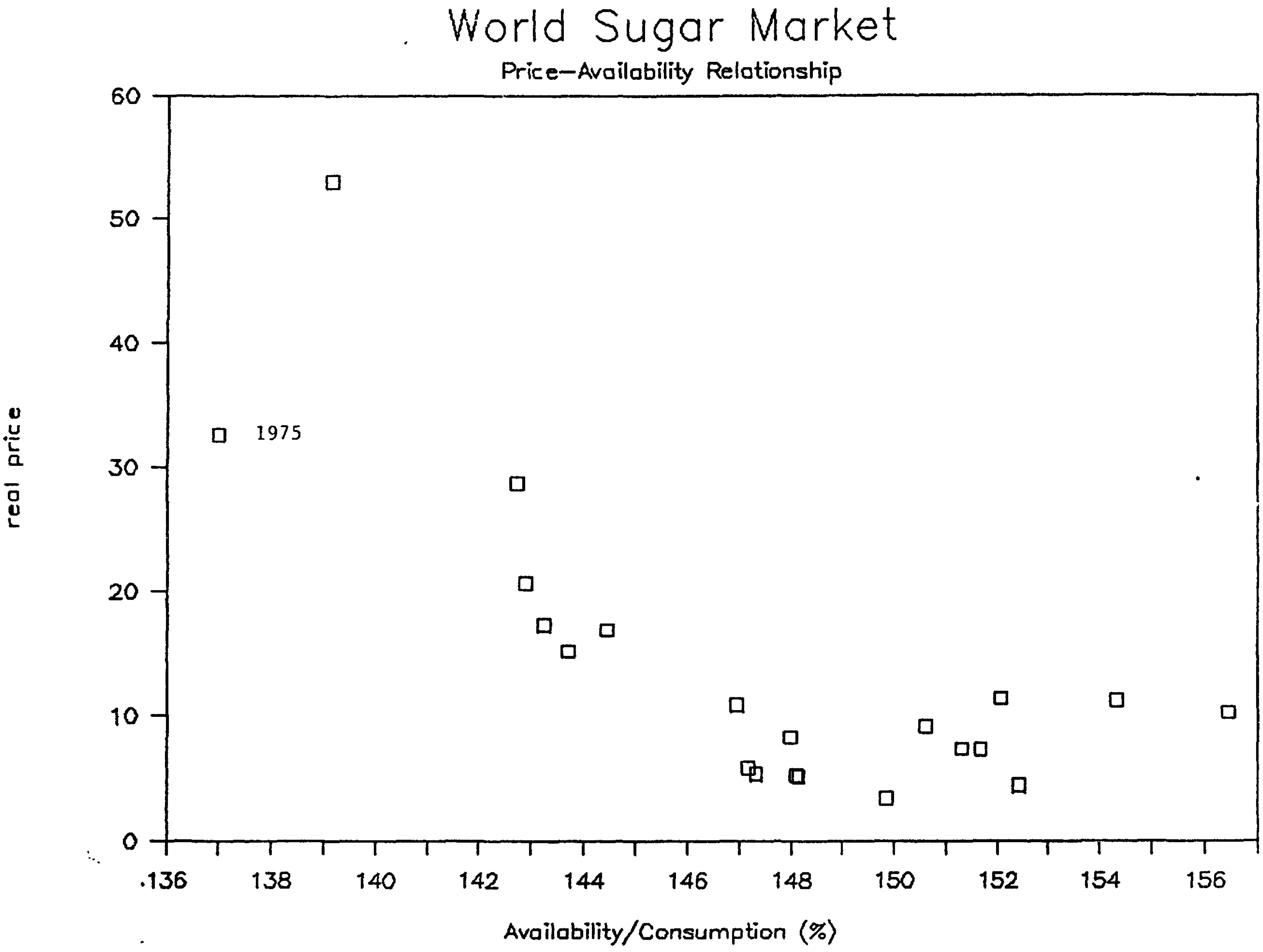
$R^2 = 0.904$ DW = 2.18 s.e. = 0.246

LM test for serial correlation: $F_{3,13} = 1.26$

where PISA is the ISA free market sugar price, USPP is the US producers price index (all items), EXR is a GDP–weighted index of US dollar exchange rates (see Gilbert, 1989b) and the availability variable AVAIL is defined as

$$AVAIL_t = (PROD_t + STOCKS_{t-1})/CONS^*_t$$

FIGURE 1


World Sugar Market
Price—Availability Relationship
real price
60
50
40
30
20
10
0
1975
136
138
140
142
144
146
148
150
152
154
156
Availability/Consumption (%)

where PROD is non-socialist world sugar production, STOCKS are total non-socialist world sugar stocks, and $CONS^*_t$ is the fitted value from an exponential trend fitted to non-socialist world sugar consumption over the same period. Coefficient standard errors are given in parentheses. Addition of the lagged stock variable (relative to trend consumption) gives a t-value of 0.11 confirming the restriction that production and lagged stocks should have the same effect on price. The absolute value of the coefficient on the dollar exchange rate variable is substantially in excess of unity indicating excess response here – see Gilbert (1989b) for a possible explanation. The time trend is included to account for possible fall in production costs but is not significant in this specification. I was unable to find any role for an interest rate.

92. The wideness of the scatter in Figure 1 indicates that the nonlinearity in this relationship is not clearly defined and attempts to fit a hyperbloic function were not successful. However the nonlinearity does become more clear by fitting a spline function:

$$(67)\quad \ln(PISA_t/USPP_t) = \underset{(3.469)}{12.266} + \underset{(0.104)}{0.257}\ln(PISA_{t-1}/USPP_{t-1}) - \underset{(0.459)}{1.606}\ln EXR_t$$

$$- \underset{(0.009)}{0.016}t - \underset{(2.833)}{2.211}AVAIL^+_t - \underset{(2.330)}{9.185}AVAIL^-_t$$

$R^2 = 0.917$ $\quad$ DW = 2.44 $\quad$ s.e. = 0.237

LM test for serial correlation: $F_{3,12} = 1.11$

where $AVAIL^+ = \max(AVAIL, 1.4725)$ and $AVAIL^- = \min(AVAIL, 1.4725)$ so that $AVAIL^+ + AVAIL^- = AVAIL$ (1.4725 is the sample mean of AVAIL). A t-test of equation (66) against equation (67) (i.e. the test that the coefficients on $AVAIL^+$ and $AVAIL^-$ are equal) gives the value of 1.52, so the evidence for a nonlinearity is inconclusive, but equation (67) indicates that conditional upon a kink at the mean value for availability the price response to marginal increase in availability at high values of availability is insignificantly different from zero while at low values this response is clearly negative. If a dummy for 1975 is included in the two equations the t-test on equality of the two AVAIL coefficients rises to 2.78. This is in line with the visual evidence from Figure 1 where the 1975 observation is the clear outlier from the nonlinear pattern.

93. Metals industries are characterized by demand which is subject to quite sharp shocks in conjunction with relatively slowly moving supply functions. Short to medium term demand elasticities tend to be low since metals consumers, who are typically fabricators of finished or semi-fininished goods, are committed to particular production processes. Although primary supply is not generally responsive to the metals price in the very short term, there is considerable flexibility in the medium term (periods of a year and up) so long as production is less than its capacity level. By comparison with agricultural commodities, producing companies (private or parastatal) are fairly large and this in conjunction with product quality (impurity) differences implies that these companies can exercise a certain degree of monopoly power, but in most industries this does not extend to the ability to set prices. Typically, producers have annual or longer contracts with consumers for supply of specified quantities at (to be determined) free market prices. Production is therefore largely to order with only residual quantities delivered to or purchased from the free market, which nevertheless determines the price of the contractual qunatities. This suggests a model in which production is geared to expected consumption. For a discussion of this form of market arrangement see Ghosh *et al.* (1987, pp.60-66).

94. These features are evident in the aluminum industry. Over the 40 year period 1948-87, the coefficient of variation of the annual change in aluminum production was 7.2% while that for aluminum consumption was 10.3%. By contrast, in sugar over the period 1967-87, the coefficient of variation of the change in production was 4.0% against

2.4% for consumption. Furthermore, much of the variation in aluminum production may be accounted for by reference to changes in expected consumption. We may see this by considering the three variable two lag VAR obtained by regressing the log of aluminum production and consumption respectively on two lags of production, consumption and the deflated, exchange rate adjusted aluminum price (see paragraph 95). An F-test for the exclusion of the lagged consumption variables from the production VAR gave the highly significant value of $F_{2,22} = 5.88$ while that for the exclusion of the lagged production variables from the consumption VAR gave an insignificant value of $F_{2,22} = 2.23$. This test confirms that at a least a certain proportion of the variation in aluminum production may be explained by lagged consumption levels, and this is most easily rationalized in terms of changes in expected consumption.

95. The most satisfactory equation for the aluminum price over the sample 1960-87 is

$$\begin{aligned} (68)\quad \ln(ALP_t{*}EXR_t/USPP_t) = {} & \underset{(0.071)}{-0.043} - \underset{(0.003)}{0.010t} + \underset{(0.216)}{0.512}(\ln CONS_t - E_{t-1}\ln CONS_t) \\ & - \underset{(0.362)}{1.545}\ln(PROD_{t-1}/CONS_{t-1}) \end{aligned}$$

$R^2 = 0.569$ DW = 1.81 s.e. = 0.105

LM test for serial correlation: $F_{3,21} = 2.16$

where ALP is the LME settlement price from 1978 and prior to that the *Metal Bulletin* 'certain other transactions' indicator price, both in c/lb, USPP is the US producer prices index (all items), EXR is the GDP-weighted exchange rate index referred to in paragraph 91, PROD is non-socialist world production of primary aluminum and CONS is non-socialist world consumption of primary aluminum (source: Metallgesellschaft, *Metal Statistics*). The lagged expectation of current consumption, used to define the consumption innovation $\ln CONS_t - E_{t-1}\ln CONS_t$, was generated by recursive estimation of the equation

$$\ln CONS_t = \beta_o + \beta_1 t$$

This equation may be rationalized by noting that the price expected by producers in period t-1 to hold in period t will be given by the intersection of the expected demand curve with the known and only slowly changing industry supply curve. However, the price will differ from this expected price as actual demand exceeds or falls short of expected demand since the short run supply response of the primary sector is very low and the excess or deficit is met by changes in stocks. Unfortunately we do not have access to a long comprehensive stock series and cumulation of excess supplies is unreliable since it also cumulates systematic inaccuracies in the data. That stocks are important is however indicated by the significance of the lagged production consumption ratio since

$$\ln(PROD_t/CONS_t) \simeq \Delta STOCK_t/CONS_t$$

If stocks are high, then the expected price will be lower than that given by the intersection of the expected demand and supply curves, and *vice versa* if stocks are low. However, further lags of this variable are insignificant. We suspect that these stocks are in general held by producers rather than speculators.

96. There is no clear evidence that a nonlinear framework is inadequate here. Replacement of equation (68) by a spline function defined analagously with equation (67) by splitting both the consumption innovation and lagged production-consumption ratios at their mean values allowed acceptance of the linear equation with an F valaue of $F_{2,22}$ =

1.08. Although the absence of adequate stock data obliges caution, there does not seem to be strong evidence that speculative stockholding is very important in the aluminum industry, and hence none of the models analyzed in this paper are clearly relevant. Rather price is determined by demand shifting backwards and forward along a near linear supply surve (see Anthony Bird Associates, 1989, for evidence on historic cost curves) but with unexpected demand movements resulting in sharp price changes due to short run primary supply inelasticity. Stocks are typically held by producers whose decisions are based on the relative costliness of adjusting production schedules and holding stock rather than on a speculative basis and they therefore tend to displace current production rather than act directly on prices.

97. The Deaton-Laroque model appears well-suited to the agricultural crop commodity (sugar) but unhelpful in relation to the continuously produced metal (aluminum). However, in rejecting that model for the metals industry, we do not find any evidence that speculative stockholding models which ignore the non-negativity constraint on stocks perform better. On the contrary, the evidence suggests that in these industries it is misconceived to view price formation in terms of speculative stockholding behavior which would appear to be typically a short term phnemonenon. Instead, aluminum prices appear to be largely explicable in terms of traditional supply-demand models.

G. Future Research

98. This survey has been somewhat pessimistic in its assessment of the usefulness of rational expectations methods in modeling primary commodity markets, but at the same time some hopeful directions have emerged. On the negative side I draw the following conclusions:

i) The length and complexity of the distributed lags in the production of primary commodities implies that there is little to be gained from attempting to impose restrictions on these lag distributions in the way suggested by the REH. It will probably continue to be preferable to use unrestricted and apparently *ad hoc* distributed lags of actual prices in these relationships.

ii) There is also little to be gained from attempting to build structural speculative stock demand models and then solving for price *via* the market clearing identity. This is both because stock data are usually of poor quality, and because the speculative demand theory is too partial as an explanation of these stock series for the resulting implied price models to be of value.

iii) The large amount of work undertaken over recent years which utilizes the Muth price model has produced singularly little by way of achievement. In particular, there is little evidence that fit is improved by attempting to model market expectations of future supply and demand conditions; and a great deal of evidence that the interest rate effects implied by this class of models are exaggerated.

iv) It may anyway be misconceived to see speculative stockholding as the central determinant of intertemporal pricing in non-agricultural markets where stocks are predominantly held by producers and when an element of monopolistic behavior is evident.

On the positive side, I conclude

v) Recent work which takes explicit account of the non-negativity constraint on stocks has provided a superior theoretical framework than that previously available within which commodity price theory may be developed. Furthermore, there does appear to be some empirical support for the implication that the price effect of supply and demand shocks depends in a nonlinear manner on the overall state of the market (as measured by availability).

99. In the light of the above I suggest that the priorities for future research on commodity prices should

i) increased attention to modeling transactions and precautionary stock demands as better stock data becomes available;

ii) the development of further nonlinear commodity price models explicitly based on the Deaton and Laroque (1989) stationary rational expectations equilibrium price functions;

iii) the incorporation into these models of additional and more complicated state variables reflecting market expectations of future conditions as in developments of the Muth model; and

iv) urgent examination of the appropriateness of the Deaton–Laroque framework for the modelling of metals markets and markets for tree crop commodities.

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