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The Recursive Fitting of Multivariate Complex Subset ARX Models

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Abstract

In this paper we propose an innovative recursive learning algorithm to sequentially estimate multivariate complex subset autoregressive models with exogenous variables (VARX models), including full-order models. This paper suggests the use of the recursive fitting of multivariate complex subset ARX models in conjunction with order selection criteria to select an 'optimal' multivariate complex subset ARX model. The recursive procedure can be embedded in a tree algorithm. We fit the necessary models associated with the bottom stage, and then recursively fit models which include more variables, until finally we fit recursively the full order ARX model with maximum lags P and Q.

Keywords: Recursion; Learning Algorithm; Multivariate Complex Subset Autoregressive modelling with Exogenous Variables.

1. INTRODUCTION

Mathematical model researchers for applied time-series systems are often concerned that the coefficients of their established models may not be constant over time, but vary when the models are disturbed by changes arising from outside factors. This concern has motivated researchers to develop sequential estimation algorithms that allow users to update subset time series models at consecutive time instants. This approach will allow for the coefficients to slowly evolve, and can show evolutionary changes detected in model structures. Hannan and Deistler [1] proposed a recursive estimation of an autoregressive (AR) model. Chen *et al* [2] suggested recursive updating procedure for the learning process of a multi-layer neural network.

The aim of this paper is to provide an algorithm for the recursive fitting of multivariate complex subset ARX models, including full-order models. The algorithm is developed for the selection of an optimal complex subset ARX model by employing model selection criteria. The ultimate goal of this research is to investigate an efficient procedure for selecting the optimal multivariate complex subset ARX model subject to possible zero or absence entries in each existing coefficient matrix. It is unwise to neglect possible zero constraints on the complex coefficient matrices of the optimal complex subset ARX model selected, whether these constraints represent the absence of a full complex matrix or perhaps simply a part of a complex one.

An algorithm for the recursive fitting of vector real subset ARX models has been presented in [3]. In most of the engineering literature [4,5], systems possessing multiple inputs and multiple outputs are referred to as vector systems where each input represents an input channel and each output represents an output channel. Mittnik [6] suggested procedures for estimating an internally balanced state space representation of VARX models. This work enriches model building techniques by using model reduction concepts. In time series modelling, subset models [7,8] are often employed, especially when the data exhibits some form of periodic behaviour, perhaps with a range of different natural periods in terms of hours, days, months, and years, in applications involving weather, humidity, and temperature recordings, and short-term and long-term electricity load data. Thus many researchers have drawn attention to vector subset time series system analysis. When the impulse-response matrix needs only an imaginary part, i.e. by imposing the constraint that the real part is null, the model becomes a vector imaginary subset model. When the impulse-response matrix is only real, the model will be a vector real subset model. In this paper, we propose procedures for selecting the optimal multivariate complex subset ARX model by using the proposed recursions in conjunction with model selection criteria tailored to conform to multivariate complex ARX schemes.

As stated in [3], the proposed recursions provide a computational procedure which can be conventionally embedded in an inverse tree algorithm. The structure of the tree algorithm provides great benefit in implementing software on a multi-c.p.u. computing machine, such as a supercomputer. Thus the proposed recursive algorithm is superior to non-recursive algorithms.

2. THE RECURSIVE ESTIMATION OF THE MULTIVARIATE COMPLEX SUBSET ARX MODELS

Let y(t) and x(t) be jointly stationary, zero mean multivariate complex stochastic processes. The dimension of y(t) is m and the dimension of x(t) is n. We consider the ARX(p,q) model with the deleted lags $i_1, i_2,..., i_s$ for y(t) and deleted lags $j_1,j_2,...,j_r$ for x(t), so that a model is of the form

$$\sum_{i=0}^{p} A_i^*(I_s)y(t-i) + \sum_{j=0}^{q} B_j^*(J_r)x(t-j) = \varepsilon(t)^A, \{ A_0^*(I_s) = I,\ A_i^*(I_s) = 0, \text{ as } i\varepsilon I_s,\ B_j^*(J_r)=0, \text{ as } j\varepsilon J_r \} \quad (1)$$

where I_s represents an integer set with elements $i_1,i_2,...,i_s$, $1 \le i_1 \le i_2... \le i_s \le p-1$, and J_r represents an integer set with elements $j_1,j_2,...,j_r$, $0 \le j_1 \le j_2... \le j_r \le q-1$. $\varepsilon(t)^A$ is a $m\times 1$ zero mean stationary complex disturbance process which is uncorrelated with any variables included in (1) except y(t).

By using the orthogonality principle [9], the estimates of parameters $A_{ip}^{*q}(I_s)$ and $B_{jp}^{*q}(J_r)$ of the fitted ARX (p,q) model are solutions of the following normal equations:

$$\sum_{i=0}^{p} A_{ip}^{*q}(I_s)\mu_{k-i} + \sum_{j=0}^{q} B_{jp}^{*q}(J_r)\gamma_{k-j} = 0 \qquad k = 1,2,...,p;\ k \notin I_s\ ,$$

$$\sum_{i=0}^{p} A_{ip}^{*q}(I_s)\gamma_{i-l}^{\tau} + \sum_{j=0}^{q} B_{jp}^{*q}(J_r)\nu_{l-j} = 0 \qquad l = 0,1,...,q;\ l \notin J_r,$$

and we have $V_{pq}^{A}(I_s, J_r) = \sum_{i=0}^{p} A_{ip}^{*q}(I_s)\mu_{-i} + \sum_{j=0}^{q} B_{j}^{*q}(J_r)\gamma_{-j}$, and

$$\Delta_{pq}^{A}(I_s, J_r) = \sum_{i=0}^{p} A_{ip}^{*q}(I_s)\mu_{p-i} + \sum_{j=0}^{q} B_{jp}^{*q}(J_r)\gamma_{p-j}\ . \tag{2}$$

where τ denotes the conjugate transpose, $\mu_k, \nu_k,$ and γ_k are the sample estimates of $E\{y(t)y^{\tau}(t-k)\}$, $E(x(t)x^{\tau}(t-k)\}$, and $E\{x(t)y^{\tau}(t-k)\}$ respectively. $\mu_k = \mu_{-k}^{\tau}$ and $\nu_k = \nu_{-k}^{\tau}$. In addition, $V_{pq}^{A}(I_s, J_r)$ is the estimate of the power matrix and $\Delta_{pq}^{A}(I_s, J_r)$ is the estimate of the cross-covariance matrix between $\varepsilon(t)^A$ and y(t-p-1). Note that the orthogonality principle has been adopted to estimate the coefficient matrices in (1) and y(t-p-1) is indeed not a variable included in (1), thus $\Delta_{pq}^{A}(I_s, J_r)$ is not null [10].

Following [3], we consider an ARX (p-1,q) model of the form

$$\sum_{i=0}^{p-1} A_i^{*}(I_s)y(t-i) + \sum_{j=0}^{q} B_j^{*}(J_r)x(t-j) = \varepsilon(t)^A\ .$$

The analogous normal equations, the associated estimated power and cross-covariance matrices are the same as (2) with the exception that p is replaced by p-1.

Now we need to introduce another ARX(p-1,q) model of the form

$$\sum_{i=1}^{p} E^{*}_{p-i}(I_s) y(t+p-i) + \sum_{j=0}^{q} F^{*}_{q-j}(J_r) x(t+p-j) = \varepsilon(t)^{E} \quad \{E^{*}_{0} = I,\ E^{*}_{p-i}(I_s) = 0 \text{ as } i \,\varepsilon\, I_s$$

$$F^{*}_{q-j}(J_r) = 0 \text{ as } j \,\varepsilon\, J_r$$

where $\varepsilon(t)^{E}$ is a $m\times 1$ zero mean disturbance process. The highest subscript for E is p-1, and for F is q. We refer to this ARX model as an EF type model. Similarly, by using the orthogonality principle, we can obtain the analogous normal equations. After solving the estimates of parameters $E^{*}_{p-i}(I_s)$ and $F^{*}_{q-j}(J_r)$, we get

$$V^{E}_{p-1\,q}(I_s, J_r) = \sum_{i=1}^{p} E^{*\,q}_{p-i\,p-1}(I_s)\mu_{p-i} + \sum_{j=0}^{q} F^{*\,q}_{q-j\,p-1}(J_r)\gamma_{p-j},$$

and

$$\Delta^{E}_{p-1\,q}(I_s, J_r) = \sum_{i=1}^{p} E^{*\,q}_{p-i\,p-1}(I_s)\mu_{-i} + \sum_{j=0}^{q} F^{*\,q}_{q-j\,p-1}(J_r)\gamma_{-j} \ . \tag{3}$$

Thus the following (p-1,q) to (p,q) recursions are available,

$$A^{*q}_{i\,p}(I_s) = A^{*q}_{i\,p-1}(I_s) + A^{*q}_{p\,p}(I_s)\, E^{*\,q}_{p-i\,p-1}(I_s) \quad i = 1,\ldots,p-1 \tag{4}$$

$$B^{*q}_{j\,p}(J_r) = B^{*q}_{j\,p-1}(J_r) + A^{*q}_{p\,p}(I_s)\, F^{*\,q}_{q-j\,p-1}(J_r) \quad j = 0,\ldots,q \tag{5}$$

$$A^{*q}_{p\,p}(I_s) = -\,\Delta^{A}_{p-1\,q}(I_s, J_r) \,/\, V^{E}_{p-1\,q}(I_s, J_r) \tag{6}$$

$$V^{A}_{p\,q}(I_s, J_r) = V^{A}_{p-1\,q}(I_s, J_r) + A^{*q}_{p\,p}(I_s)\, \Delta^{E}_{p-1\,q}(I_s, J_r) \tag{7}$$

$$\Delta^{A}_{p-1\,q}(I_s, J_r) = \Delta^{E\,\tau}_{p-1\,q}(I_s, J_r) \ . \tag{8}$$

In the special case, where the consecutive coefficient matrices $A^{*}{}_{p-k}$ for the lags of y(t-p+k), k = 1,...,a (a≤p-1) of the AB type ARX(p-1,q) model are missing, the estimated coefficient matrices are null, i.e. $A^{*\,q}_{p-k\,p-1}(I_s)$ =0, k=1,2,...,a, and then the corresponding coefficient matrices and V^{A} from the AB type (p-a-1,q) model are sufficient to continue the recursive estimations.

To develop the recursions for the EF type ARX(p,q) model of the form

$$\sum_{i=0}^{p} E^*_{p-i}(I_s)y(t+p-i) + \sum_{j=0}^{q} F^*_{q-j}(J_r)x(t+p-j+1) = \varepsilon(t)^E \ , \tag{9}$$

we introduce two models: a GH type ARX (p-1,q) model of the form

$$\sum_{i=0}^{p-1} G^*_i(I_s)y(t-i-1) + \sum_{j=0}^{q} H^*_j(J_r)x(t-j) = \varepsilon(t)^H \tag{10}$$

where $\varepsilon(t)^H$ is a $n\times1$ zero mean disturbance process, with

$$E\{\varepsilon(t)^H \varepsilon(t)^{H^\tau}\} = V^H(I_s, J_r)$$

and

$$E\{\varepsilon(t)^H y^\tau(t-p-1)\} = \delta^G(I_s, J_r) \ ;$$

an $\tilde{E}\tilde{F}$ type ARX (p,q-1) model of the form

$$\sum_{i=0}^{p} \tilde{E}^*_{p-i}(I_s)y(t+p-i) + \sum_{j=1}^{q} \tilde{F}^*_{q-j}(J_r)x(t+p-j+1) = \varepsilon(t)^{\tilde{E}} \ \{\tilde{E}^*_0(I_s) = I, \tilde{E}^*_{p-i}(I_s) = 0 \text{ as } i \,\varepsilon\, I_s \ \tilde{F}^*_{q-j}(J_r) = 0 \text{ as } j \,\varepsilon\, J_r\}, \tag{11}$$

where $\varepsilon(t)^{\tilde{E}}$ is an $m\times1$ zero mean disturbance process, with

$$E\{\varepsilon(t)^{\tilde{E}} \varepsilon(t)^{\tilde{E}^\tau}\} = \tilde{V}^E(I_s, J_r) \ ,$$

$$E\{\varepsilon(t)^{\tilde{E}} x^\tau(t+p+1)\} = \tilde{\eta}^F(I_s, J_r) \ ,$$

and $$E\{\varepsilon(t)^{\tilde{E}} x^\tau(t+p-q)\} = \tilde{\Delta}^F(I_s, J_r).$$

Then we have the following recursive equations

$$
\begin{aligned}
E^{*\,q}_{p-i\,p}(I_s) &= \tilde{E}^{*\ q-1}_{p-i\,p}(I_s) + F^{*q}_{q\,p}(J_r)G^{*q}_{i\,p-1}(I_s) \quad i = 0,1,\ldots,p-1 \\
F^{*\,q}_{q-j\,p}(J_r) &= \tilde{F}^{*\ q-1}_{q-j\,p}(J_r) + F^{*q}_{q\,p}(J_r)H^{*q}_{j\,p-1}(J_r) \quad j = 1,2,\ldots,q \\
F^{*q}_{q\,p}(J_r) &= -\tilde{\eta}^{F}_{p\,q-1}(I_s,J_r) \,/\, V^{H}_{p-1\,q}(I_s,J_r) \\
V^{E}_{p\,q}(I_s,J_r) &= \tilde{V}^{E}_{p\,q-1}(I_s,J_r) + F^{*q}_{q\,p}(J_r)\eta^{G}_{p-1\,q}(I_s,J_r) \\
\eta^{G}_{p-1\,q}(I_s,J_r) &= \tilde{\eta}^{F\,\tau}_{p\,q-1}(I_s,J_r).
\end{aligned}
$$

Again, if the consecutive coefficient matrices $G^{*}_{\ p-k}$ for the lags of y(t-p+k-1), k = 1,2,...,b (b≤p-1) of the GH type ARX(p-1,q) model are missing, this GH type model is equivalent to a GH type (p-b-1,q) model.

Also note that the EF type ARX (p,q) model of (9) is equivalent to the $\tilde{E}\tilde{F}$ type ARX (p,q-1) model of (11), i.e. as F_q of (9) is missing, we may substitute an $\tilde{E}\tilde{F}$ type model from (11) for the EF type model of (9).

Next, for the recursions to estimate the GH type ARX (p,q) model, we also need the information from the GH type ARX (p-1,q) model of the form (10) and from the $\tilde{E}\tilde{F}$ type ARX (p,q-1) model of the form (11). Thus, the following recursions are obtained:

$$
\begin{aligned}
G^{*q}_{i\,p}(I_s) &= G^{*q}_{i\,p-1}(I_s) + G^{*q}_{p\,p}(I_s)\tilde{E}^{*\ q-1}_{p-i\,p}(I_s) \quad i = 0,1,\ldots,p-1 \\
H^{*q}_{j\,p}(J_r) &= H^{*q}_{j\,p-1}(J_r) + G^{*q}_{p\,p}(I_s)\tilde{F}^{*\ q-1}_{q-j\,p}(J_r) \quad j = 1,2,\ldots,q \\
G^{*q}_{p\,p}(I_s) &= -\eta^{G}_{p-1\,q}(I_s,J_r) \,/\, \tilde{V}^{E}_{p\,q-1}(I_s,J_r) \\
V^{H}_{p\,q}(I_s,J_r) &= V^{H}_{p-1\,q}(I_s,J_r) + G^{*q}_{p\,p}(I_s)\tilde{\eta}^{F}_{p\,q-1}(I_s,J_r)\ .
\end{aligned}
$$

To consider the recursive estimation of the $\tilde{E}\tilde{F}$ type ARX (p,q-1) model, (11), we need to introduce a CD type ARX (p-1,q) model of the form

$$\sum_{i=0}^{p-1} C^{*}_{p-1-i}(I_s) y(t+q-i) + \sum_{j=1}^{q+1} D^{*}_{q+1-j}(J_r) x(t+q-j+1) = \varepsilon(t)^{D} \{ D^{*}_{0}(J_r) = I,$$
$$D^{*}_{q-j+1}(J_r) = 0, \text{ as } j \,\varepsilon\, J_r,\ C^{*}_{p-i-1}(I_s) = 0 \text{ as } i \,\varepsilon\, I_s\}, \tag{12}$$

where $\varepsilon(t)^{D}$ is a $n \times 1$ zero mean disturbance process, with

$$E\{\varepsilon(t)^{D} \varepsilon(t)^{D\,\tau}\} = V^{D}(I_s, J_r),$$

and

$$E\{\varepsilon(t)^{D} y^{\tau}(t+q-p)\} = \Delta^{C}(I_s, J_r).$$

Now rewrite an AB type ARX (p-1,q) model of (2) in the form

$$\sum_{i=0}^{p-1} A^{*}_{i}(I_s) y(t-i) + \sum_{j=1}^{q+1} B^{*}_{j-1}(J_r) x(t-j+1) = \varepsilon(t)^{A},$$

and recall an EF type ARX(p-1,q) model of the form

$$\sum_{i=1}^{p} E^{*}_{p-i}(I_s) y(t+p-i) + \sum_{j=1}^{q+1} F^{*}_{q+1-j}(J_r) x(t+p-j+1) = \varepsilon(t)^{E}\ .$$

We can derive the following formulae:

$$\tilde{E}^{*\ q-1}_{p-i\,p}(I_s) = E^{*\ q}_{p-i\,p-1}(I_s) - \tilde{F}^{*q-1}_{0\,p}(J_r) C^{*\ q}_{p-i-1\,p-1}(I_s) + \tilde{E}^{*q-1}_{p\,p}(I_s)\, A^{*q}_{i\,p-1}(I_s)$$
$$i = 1, 2, \ldots, p-1$$
$$\tilde{F}^{*\ q-1}_{q-j\,p}(J_r) = F^{*\ q}_{q-j+1\,p-1}(J_r) - \tilde{F}^{*q}_{0\,p}(J_r) D^{*\ q}_{q-j+1\,p-1}(J_r)$$
$$+ \tilde{E}^{*q}_{p\,p}(I_s) B^{*\ q}_{j-1\,p-1}(J_r) \quad j = 1, 2, \ldots, q-1$$
$$\tilde{V}^{E}_{p\,q-1}(I_s, J_r) = V^{E}_{p-1\,q}(I_s, J_r) - \Delta^{E}_{p-1\,q}(I_s, J_r)[V^{A}_{p-1\,q}(I_s, J_r)]^{-1} \Delta^{A}_{p-1\,q}(I_s, J_r)$$
$$+ \tilde{\Delta}^{F}_{p\,q-1}(I_s, J_r)[V^{D}_{p-1\,q}(I_S, J_R)]^{-1} \Delta^{C}_{p-1\,q}(I_s, J_r)$$
$$\tilde{F}^{*q-1}_{0\,p}(J_r) = -\tilde{\Delta}^{F}_{p\,q-1}(I_s, J_r) / V^{D}_{p-1\,q}(I_s, J_r)$$
$$\tilde{E}^{*q-1}_{p\,p}(I_s) = -\Delta^{E}_{p-1\,q}(I_s, J_r) \,/\, V^{A}_{p-1\,q}(I_s, J_r)$$
$$\Delta^{C}_{p-1\,q}(I_s, J_r) = \tilde{\Delta}^{F\,\tau}_{p\,q-1}(I_s, J_r) \qquad \Delta^{A}_{p-1\,q}(I_s, J_r) = \Delta^{E\,\tau}_{p-1,q}(I_s, J_r)\ .$$

Also note that if both $\tilde{E}^*_p(I_s)$ and $\tilde{F}^*_0(J_r)$ of the $\tilde{E}\tilde{F}$ type ARX (p,q-1) model of (11) are missing, this $\tilde{E}\tilde{F}$ type model is equivalent to an EF type ARX (p-1,q-1) model.

The recursions for the CD type ARX (p,q) model of the form

$$\sum_{i=0}^{p} C^*_{p-i}(I_s)y(t+q-i) + \sum_{j=0}^{q} D^*_{q-j}(J_r)x(t+q-j) = \varepsilon(t)^D,$$

arise from rewriting a CD type ARX (p-1,q) model from (12) so that we have

$$\sum_{i=0}^{p-1} C^*_{p-1-i}(I_s)y(t+q-i) + \sum_{j=0}^{q} D^*_{q-j}(J_r)x(t+q-j) = \varepsilon(t)^D.$$

In addition, we need an $\tilde{E}\tilde{F}$ type ARX(p,q-1) model of the form

$$\sum_{i=0}^{p} \tilde{E}^*_{p-i}(I_s)y(t+p-i) + \sum_{j=0}^{q-1} \tilde{F}^*_{q-1-j}(J_r)x(t+p-j) = \varepsilon(t)^{\tilde{E}},$$

to develop the following recursive formulae:

$$\begin{aligned}
C^{*\,q}_{p-i\,p}(I_s) &= C^{*\,q}_{p-i-1\,p-1}(I_s) + C^{*q}_{0\,p}(I_s)\tilde{E}^{*\ \ q-1}_{p-i\,p}(I_s) \quad i = 0,1,\dots,p-1 \\
D^{*\,q}_{q-j\,p}(J_r) &= D^{*\,q}_{q-j\,p-1}(J_r) + C^{*q}_{0\,p}(I_s)\tilde{F}^{*\ \ q-1}_{q-j-1\,p}(J_r) \quad j = 0,1,\dots,q-1 \\
C^{*q}_{0\,p}(I_s) &= -\Delta^{C}_{p-1\,q}(I_s,J_r)/\tilde{V}^{E}_{p\,q-1}(I_s,J_r) \\
V^{D}_{p\,q}(I_s,J_r) &= V^{D}_{p-1,q}(I_s,J_r) + C^{*q}_{0\,p}(I_s)\tilde{\Delta}^{F}_{p\,q-1}(I_s,J_r).
\end{aligned}$$

Therefore a chain of subset ARX model recursions is available and forms a complete cycle. In summary, we describe a (p-1,q) to (p,q) subset ARX recursion algorithm as follows:

1. Compute $\Delta^{A}_{p-1\,q}(I_s,J_r), \Delta^{C}_{p-1\,q}(I_s,J_r)$ and $\eta^{G}_{p-1\,q}(I_s,J_r)$,

2. Compute $A^{*q}_{p\,p}(I_s,J_r), C^{*q}_{0\,p}(I_s,J_r), F^{*q}_{q\,p}(I_s,J_r), G^{*q}_{p\,p}(I_s,J_r), \tilde{F}^{*q}_{0\,p}(I_s,J_r), \tilde{E}^{*q}_{p\,p}(I_s,J_r)$,

3. Compute $V^{A}_{pq}(I_s, J_r), V^{D}_{pq}(I_s, J_r), V^{E}_{pq}(I_s, J_r), V^{H}_{pq}(I_s, J_r), \tilde{V}^{E}_{p\,q\text{-}1}(I_s, J_r)$,

4. Compute

$$\begin{aligned}
&\tilde{E}^{*\ q\text{-}1}_{p\text{-}i\,p}(I_s), i=0,\dots,p-1,\ \tilde{F}^{*\ q\text{-}1}_{q\text{-}j\,p}(J_r), j=1,\dots,q,\\
&A^{*q}_{i\,p}(I_s), i=1,\dots,p-1,\ B^{*q}_{j\,p}(J_r), j=0,\dots,q,\\
&C^{*\ q}_{p\text{-}i\,p}(I_s), i=0,\dots,p-1,\ D^{*\ q}_{q\text{-}j\,p}(J_r), j=0,\dots,q-1,\\
&E^{*\ q}_{p\text{-}i\,p}(I_s), i=0,\dots,p-1,\ F^{*\ q}_{q\text{-}j\,p}(J_r), j=1,\dots,q,
\end{aligned}$$

and
$G^{*q}_{i\,p}(I_s),\ i=0,\dots,p-1,\ H^{*q}_{j\,p}(J_r),\ j=1,\dots,q$.

In deriving the (p,q-1) to (p,q) recursive formulae, we introduce a $\tilde{C}\tilde{D}$ type ARX (p-1,q) model of the form

$$\sum_{i=1}^{p}\tilde{C}^{*}_{p\text{-}i}(I_s)y(t+q-i)+\sum_{j=0}^{q}\tilde{D}^{*}_{q\text{-}j}(J_r)x(t+q-j)=\varepsilon(t)^{\tilde{D}}\ \{\tilde{D}^{*}_{0}(J_r)=0, \tilde{D}^{*}_{q\text{-}j}(J_r)=0 \text{ as } j\,\varepsilon\, J_r\ \tilde{C}^{*}_{p\text{-}i}(I_s)=0 \text{ as } i\,\varepsilon\, I_s\},\tag{13}$$

where $\varepsilon(t)^{D}$ is a $n\times 1$ zero mean disturbance process.

Use the analogous relations for deriving the (p-1,q) to (p,q) recursive relations, we can have the (p,q-1) to (p,q) recursions, which can also be obtained by applying the (p-1,q) to (p,q) recursions the following exchange:

$i\leftrightarrow j, I_s\leftrightarrow J_r, \Delta\leftrightarrow\eta, p\leftrightarrow q, A\leftrightarrow H, C\leftrightarrow F, E\leftrightarrow D, G\leftrightarrow B.$

Note that $i\leftrightarrow j$ means every i will be replaced by a j, and every j will also be replaced by an i.

So far, we have constructed ascending recursions, where complex (p,q) order ARX models associated with the k-th stage are estimated with information from (p-1,q) order or (p,q-1) order complex ARX models available at the (k-1)-th stage. This structure provides great benefit in working within a parallel computing environment. In fitting all complex subset ARX models up to lag P and lag Q for y(t) and x(t) respectively, the recursive computational procedure can be embedded in an inverse tree algorithm. The root of the tree represents the complex full order (P,Q) ARX model at the top stage of the tree and the complex ARX models with only one y vector variable and one x vector variable make up the bottom stage. Further p and q denote the order of the fitted scheme, p = 1,2,..,P, and q = 0,1,...,Q. We fit the necessary models associated with the bottom stage, and then recursively fit complex subset ARX models which include more variables, moving to higher stages, until finally we fit recursively the complex full order ARX model with maximum lags P and Q. At each stage, the (p,q-1) to (p,q) recursions are performed if possible, and of course the (p-1,q) to (p,q) recursions are introduced when the k-th stage complex ARX model includes only one x variable, i.e. the (p,q-1) to (p,q) recursion cannot be utilised. Of course an alternative is to perform the (p,q-1) to (p,q) recursions and to follow with all necessary (p-1,q) to (p,q) recursions. The interested reader is referred to [3]. The ascending recursions can be alternatively written in the descending format, which, for simplicity, are not presented.

Further, by imposing the constraint that all real matrices are null matrices, the resulting ascending recursions become the recursions for fitting vector imaginary subset ARX models. Similarly, by constraining all imaginary matrices to null matrices, the ascending recursions for fitting vector real subset ARX models can be derived, which have been presented in [3].

The proposed procedures for selecting the optimal multivariate complex subset ARX model are then summarised in the following two steps:

Step 1: Minimise Akaike's AIC to select the best complex full order AB type ARX model from all complex full order ARX models with the order of y from 1,...,K and the order of x from 0,1,...,L, where K > P and L > Q. Schwarz's BIC is not used, because we are aware of the BIC's parsimonious propensity [3]. We employ AIC to avoid missing any relevant parameters. The AIC has the following form:

$$\mathrm{AIC} = \log|\hat{V}_A| + [2/N]\,S,$$

with N, the sample size, S, the number of independent parameters, and $\hat{V}_A$, the estimated power matrix. Please note that, in this case, each existing coefficient matrix A^*_i has $2m^2$ independent parameters, and each existing B^*_j has 2mn parameters.

Step 2: After the maximum lags P and Q are selected, we then obtain the optimal complex subset ARX model by using the proposed recursions for fitting complex ARX models in conjunction with the BIC criterion. The criterion has the form:

$$\mathrm{BIC} = \log|\hat{V}_A| + [\log N/N]\,S,$$

and the selected model has the minimum value of BIC.

However, if the natural process can be fully described by an imaginary impulse-response matrix, the optimal model would be a vector imaginary subset model. Thus we also need to search for the optimal imaginary subset ARX model by repeating the two steps above with the constraint that all real coefficient matrices are null. In this case, the ascending recursions for imaginary subset ARX models will be used, and each existing coefficient matrix A^*_i has only m^2 independent parameters, and each existing B^*_j has only mn parameters.

Analogously, if the natural process can be fully described by a real impulse-response matrix, the optimal model would be a vector real subset model and could be obtained from the recursions defined in [3].

Subsequently, we use the BIC criterion to evaluate the optimal complex subset ARX model, the optimal imaginary subset ARX model, and the optimal real subset ARX model to select the optimal subset ARX model.

After the optimal subset ARX model is selected, it is suggested that every independent parameter in each existing coefficient matrix be treated as a variable, then extend the tree pruning method developed in [11] in conjunction with the BIC criterion to select the overall optimal subset ARX model with zero constraints. If the true optimal subset model is a vector subset ARX model with some real coefficient matrices and some imaginary coefficient matrices, then the selected optimal subset model would still be a complex subset ARX model. To establish such a model as "optimal", the tree pruning method would have to be used after the proposed recursions. At present a detailed study for evaluating this tree pruning algorithm for selecting the overall optimal subset ARX model with zero constraints is being carried out by the authors, but excluded in the scope of this paper.

Please note that the proposed procedure for selecting the optimal multivariate complex subset ARX model is different from [3]. The reasons are as follows:

By imposing the constraint that all real coefficient matrices are null, the recursive fitting formulae for complex subset models can be reduced to the recursive formulae for imaginary subset models. However, this cannot be achieved by imposing any constraint on the recursive fitting formulae for real subset models. Moreover, the question as to whether the natural process is complex or imaginary in nature cannot be assessed using only the recursive fitting formulae for real subset models to analyse any time series system.

Further, the number of independent parameters is an important part of the selection criteria for evaluating the optimal model. In this analysis, any existing complex coefficient matrix of y(t-i), i=1,..,p has $2m^2$ independent parameters, and any existing complex coefficient matrix of x(t-j), j=0,..,q has 2mn independent parameters., whereas a real or an imaginary coefficient matrix attached to y(t-i) has only m^2 independent parameters, and a real or an imaginary coefficient matrix of x(t-j) has only mn independent parameters. However, in [3], neither complex nor imaginary coefficient matrices can be handled.

3. CONCLUSION

In the previous sections, we have given an effective recursive algorithm for fitting multivariate complex subset ARX models. The algorithm widens the possible use of the recursive method, and leads to a straightforward and neat analysis for a variety of signal processing and control theory applications. This algorithm is applicable to full-order model cases, allows users to update optimal multivariate complex subset ARXs at consecutive time instants, and can show evolutionary changes detected in multivariate complex subset ARX structures. This new approach is particularly useful in complex relationships where the relevant time series have been generated from structures subject to evolutionary changes in their environment. Further investigations will be undertaken to apply the proposed algorithms to the relevant fields (see Chen *et al* [2], O'Neill *et al* [12]).

REFERENCE

[1] E.J., Hannan and M. Deistler, *The statistical theory of linear systems*, John Wiley and Sons, New York, (1988).

[2] A. Chen, J. Penm and R.D. Terrell, An evolutionary recursive algorithm in selecting statistical subset neural network/VDL filtering. *Journal of Applied Mathematics and Decision Sciences,* (2006), Article ID 46592, 1-12.

[3] J. Penm, J.H.C. Penm and R.D. Terrell, The recursive fitting of subset VARX models, *Journal of Time Series Analysis*. 14, 6, (1993), 603-619.

[4] E.A. Robinson, *Multichannel Time Series Analysis with Digital Computer Programs*. San Francisco: Holden-Day, (1978).

[5] D. Manolakis, N. Kalouptsidis, and G. Carayannis, Fast algorithms for optimum lag multichannel Wiener filters, in *Proc. IEEE Int. Symp. Circuits Syst.*, Rome, Italy, (1982), 362-365.

[6] S. Mittnik, Multivariate time series analysis with vector state space models, *Computers and Mathematics with Applications*, 17, 819, (1989), 1189-1201.

[7] C.W.J. Granger and P. Newbold, *Forecasting Economic Time Series*, Second Edition, New York: Academic Press, (1986).

[8] V. Haggan and O.B. Oyetunji, On the selection of subset autoregressive time series Models, *J. Time Series Anal.* 5, 2, (1984), 103-113.

[9] B.D.O. Anderson and J.B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ: Prentice-Hall, (1979).

[10] P.Whittle, On the fitting of multivariate autoregressions and the approximate factorization of the spectral density matrix, *Biometrika* 50, (1963), 129-134.

[11] J. Penm and R.D.Terrell, Multivariate subset autoregressive modelling with zero constraints for detecting causality, *Journal of Econometrics*, 3, (1984), 311-330.

[12] O'Neill, T.J., Penm, J. and Terrell, R.D., The sequential estimation of subset vector autoregressive modelling with a forgetting factor and an intercept variable using the exact window case. *International Journal of Theoretical and Applied Finance* 7, 8, (2004), 1-17.

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