

*THE RELATION BETWEEN RESPONSE RATES
AND REINFORCEMENT RATES IN A
MULTIPLE SCHEDULE*¹

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In a multiple schedule, exteroceptive stimuli change when the reinforcement schedule is changed. Each performance in a multiple schedule may be considered concurrent with other behavior. Accordingly, two variable-interval schedules of reinforcement were arranged in a multiple schedule, and a third, common variable-interval schedule was programmed concurrently with each of the first two. A quantitative statement was derived that relates as a ratio the response rates for the first two (multiple) variable-interval schedules. The value of the ratio depends on the rates of reinforcement provided by those schedules and the reinforcement rate provided by the common variable-interval schedule. The following implications of the expression were evaluated in an experiment with pigeons: (a) if the reinforcement rates for the multiple variable-interval schedules are equal, then the ratio of response rates is unity at all reinforcement rates of the common schedule; (b) if the reinforcement rates for the multiple schedules are unequal, then the ratio of response rates increases as the reinforcement rate provided by the common schedule increases; (c) the limit of the ratio is equal to the ratio of the reinforcement rates. Satisfactory confirmation was obtained for the first two implications, but the third was left in doubt.

Some recent research suggests quantitative statements that relate the rate at which a pigeon pecks a key to the rate at which key-pecking is reinforced on variable-interval schedules of reinforcement. A brief summary of that research establishes the background for the present work.

A. Catania and Reynolds (1968): These experimenters studied the rate of key-pecking maintained by variable-interval (VI) schedules of reinforcement that provided from 8.4 to 300 reinforcements per hour (rft/hr). The VI schedules were arithmetic, *i.e.*, they were formed by random ordering from progressions of interreinforcement intervals in which the successive intervals differed by an additive constant. The evidence obtained with six pigeons suggested a monotonically increasing, negatively accelerated relationship between the rate of key-pecking and the rate of reinforcement. Although individual differences among the pigeons were considerable, data

averaged over the six pigeons were reasonably fitted over the range studied by the power function:

$$R = Kr^{0.2}, \quad (1)$$

where R represents response rate, r represents reinforcement rate, and K is a constant depending on the units of measurement.

B. Herrnstein (1961): This experimenter studied concurrent VI schedules of reinforcement. A VI schedule was assigned to each of the two keys in the experimental chamber; the schedules were independent in the sense that the availability of reinforcement for responding on one of the keys was not affected by the availability of reinforcement for responding on the other. A reinforcement assigned by a particular VI schedule could be produced only by responding on the appropriate key, however. Under the restriction that the reinforcement rate for the two keys combined be 40 rft/hr, Herrnstein varied the reinforcement rate for one of the keys relative to the overall reinforcement rate. He found that the response rate on the key in question relative to the overall response rate approximated the relative reinforcement rate for that key, *i.e.*,

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$$\frac{R_1}{R_1 + R_2} = \frac{r_1}{r_1 + r_2}, \quad (2)$$

where R and r have the same designations as in equation (1), and the subscripts identify the two keys. The quantities $(R_1 + R_2)$ and $(r_1 + r_2)$ represent the overall, *i.e.*, the two keys combined, rates of responding and reinforcement, respectively.

C. Catania (1963): Suppose that the response rate developed with a given reinforcement rate is the same whether the reinforcement rate in question is applied to a single response key, as for equation (1), or is distributed between two response keys. Herrnstein's experiment could provide a direct test of that supposition, inasmuch as he distributed 40 rft/hr between the two keys in several ways, including zero rft/hr (extinction) on one of the keys in conjunction with the total of 40 rft/hr on the other. Catania (1963, Fig. 2, p. 256) analyzed Herrnstein's data, some data obtained by Findley (1958) and his own data to show that the supposition was in fact approximately correct. He generalized equation (1) above to the form:

$$R_1 + R_2 = K(r_1 + r_2)^{0.2}, \quad (3)$$

where R , r , and K have the designations enumerated above, and the subscripts again identify the two keys. Employing equations (2) and (3), Catania derived and experimentally verified the expression:

$$R_1 = \frac{Kr_1}{(r_1 + r_2)^{0.8}}. \quad (4)$$

Equation (4) states that the rate of responding, R_1 , on one of the keys of a concurrent pair is a joint function of the reinforcement rate for that key, r_1 , and the reinforcement rate for the other key, r_2 , but is independent of the response rate on the other key: R_2 does not appear in the equation.

The above considerations have dealt with VI schedules programmed singly and also concurrently. The purpose of the following discussion is to generalize those considerations to VI schedules that are programmed as components of a multiple schedule; specifically, we are concerned with the derivation of a quantitative statement that expresses a relationship between two VI response rates developed in a multiple schedule as a function of the reinforcement rates provided by those VI sched-

ules. The rationale suggesting the derivation is illustrated in Fig. 1.

Concurrent VI schedules of reinforcement are shown in the upper panel of the figure. Each of the VI schedules A and B is assigned to a key, and the two keys are concurrently available to the pigeon. The two keys may be transilluminated by light of the same color or by light of different colors.

The middle panel of Fig. 1 shows a multiple VI VI schedule of reinforcement, in which the two VI schedules, now 1 and 2, are assigned to a key successively, and each schedule is associated with a distinct key color. When one of the schedules is assigned to the key, the other is removed. Interreinforcement intervals for either schedule are timed only when that

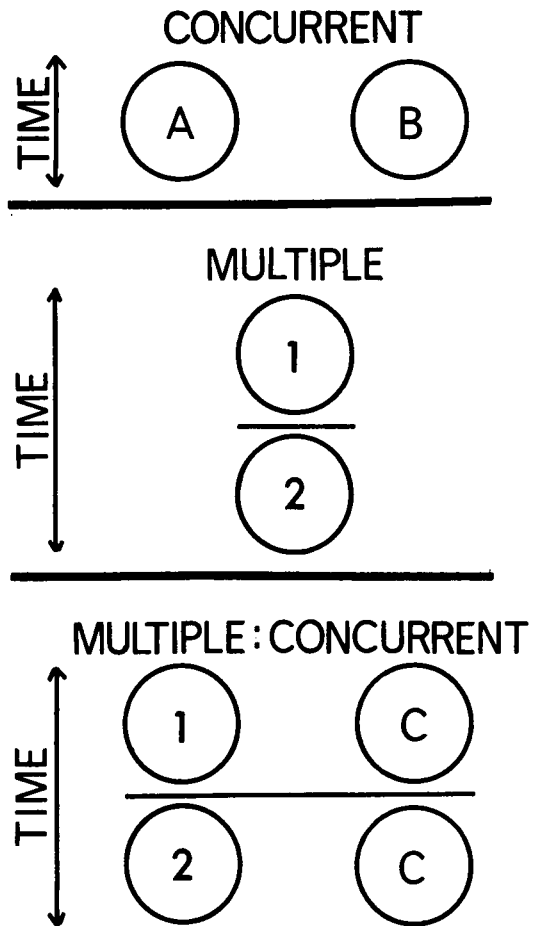


Fig. 1. Schematic representation of concurrent schedules of reinforcement (upper panel), multiple schedule (center panel), and a two-component multiple schedule in which each component is a pair of concurrent schedules (bottom panel). Schedule C is a variable-interval schedule common to each of the pairs.

schedule is assigned to the response key. The duration of time during which a schedule is assigned to the key is ordinarily controlled by the experimenter.

For heuristic reasons, it is interesting to view each of the VI performances developed on the multiple schedule as concurrent with other behavior. The performance developed on VI schedule 1 is concurrent with some behavior X, and the performance on schedule 2 is concurrent with some behavior Y. Behaviors X and Y are hypothetical and not identified in a simple multiple schedule. When they are realized by identifying them with VI schedules of reinforcement, the same VI schedule for both X and Y is perhaps the simplest case, the scheme shown in the bottom panel of Fig. 1 is obtained. The multiple schedule shown in the bottom panel consists of two pairs of concurrent VI schedules with schedule C common to the two concurrent pairs. Schedules 1 and 2, which follow from the simple multiple VI VI schedule considered above, may be the same or different.

The following symbols are used in the derivation that follows and throughout the remainder of this paper:

- R_1 : the response rate on the first VI schedule.
- $R_{C,1}$: the response rate on the common VI schedule when it is concurrent with the first VI schedule.
- $R_1 + R_{C,1}$: the overall response rate for the concurrent pair that includes the first VI schedule.
- R_2 : the response rate on the second VI schedule.
- $R_{C,2}$: the response rate on the common VI schedule when it is concurrent with the second VI schedule.
- $R_2 + R_{C,2}$: the overall response rate for the concurrent pair that includes the second VI schedule.
- r_1 : the reinforcement rate provided by the first VI schedule.
- r_2 : the reinforcement rate provided by the second VI schedule.
- r_C : the reinforcement rate provided by the common VI schedule.
- $r_1 + r_C$: the overall reinforcement rate for the concurrent pair that includes the first VI schedule.
- $r_2 + r_C$: the overall reinforcement rate for

the concurrent pair that includes the second VI schedule.

Consider the concurrent schedules 1 and C. Equation (2) requires that:

$$\frac{R_1}{R_1 + R_{C,1}} = \frac{r_1}{r_1 + r_C},$$

which may be solved for r_C :

$$r_C = \frac{r_1 R_{C,1}}{R_1}.$$

Similarly, for the concurrent schedules 2 and C,

$$r_C = \frac{r_2 R_{C,2}}{R_2};$$

hence,

$$\frac{r_1 R_{C,1}}{R_1} = \frac{r_2 R_{C,2}}{R_2}.$$

Rearranging terms:

$$\frac{R_1}{R_2} = \frac{r_1 R_{C,1}}{r_2 R_{C,2}}.$$

The terms $R_{C,1}$ and $R_{C,2}$ may be replaced by their equivalents according to equation (4), *i.e.*,

$$R_{C,1} = \frac{K r_C}{(r_1 + r_C)^{0.8}} \text{ and } R_{C,2} = \frac{K r_C}{(r_2 + r_C)^{0.8}}.$$

Simplifying and rearranging terms after the substitutions:

$$\frac{R_1}{R_2} = \frac{r_1}{r_2} \times \frac{(r_2 + r_C)^{0.8}}{(r_1 + r_C)^{0.8}}. \tag{5}$$

Equation (5) states that the ratio of the response rates developed on the first and second VI schedules is a function of the reinforcement rates provided by those schedules in conjunction with the reinforcement rate provided by the common VI schedule. Further, equation (5) has the following properties:

- (a) If r_1 and r_2 are equal, then the ratio of response rates R_1/R_2 is unity at all values of r_C .
- (b) If r_1 and r_2 are not equal, and $r_1 > r_2$, then the ratio of response rates R_1/R_2 is smallest when r_C equals zero, and the ratio increases monotonically as r_C increases.
- (c) The limit of R_1/R_2 as r_C increases is the ratio of reinforcement rates r_1/r_2 .

The experiment described below was designed to provide data bearing on the three properties of equation (5) that were just enumerated.

METHOD

Subjects and Apparatus

Three male Silver King pigeons that had served in a brief earlier version of the present research, after an earlier experiment on stimulus generalization, were maintained at their reduced body weights. The stable experimental weights were 90%, 87%, and 90% of the free-feeding weights originally determined (about a year and a half earlier) upon receipt from the supplier, but 82%, 83%, and 81% of the stable free-feeding weights determined 30 days after the present experiment was completed.

Daily experimental sessions were each terminated after 60 reinforcements; a reinforcement consisted of 3- to 5-sec (depending upon the bird) access to mixed grain. The pigeons were never fed outside of the experimental chamber; water was available in the home cage but not in the chamber.

The apparatus consisted of a Lehigh Valley Electronics pigeon box equipped with two Gerbrands response keys mounted symmetrically above the grain hopper 2.5 in. apart, center to center, and 8.5 in. above the floor of the box. About 15 g of force was required to operate each key; every operation produced a click from a relay armature (Potter and Brumfield, Type KRP) mounted behind but not on the front wall. Routine programming equipment was employed in conjunction with two cumulative recorders and electromagnetic counters.

General Procedure

Since the pigeons had been used in a previous experiment, with only several weeks intervening before the start of the present research, a single session of continuous reinforcement was followed during the next session by the final procedure. The basic procedure was a two-component multiple schedule in which each component consisted of two concurrent (arithmetic) VI schedules of reinforcement. Experimental sessions consisted of repeated exposures to each component of the multiple schedule in an alternating sequence. An ex-

posure to a component of the multiple schedule lasted for 15 min.

Each of the two response keys could be transilluminated with light of four colors. Each color was associated with the assignment of a particular VI schedule to a key; sometimes that assignment was to the left key and sometimes to the right. In one component of the multiple schedule, the first VI schedule was assigned to Key 1, which was transilluminated with red light (red key), while the common VI schedule was assigned to Key C, a blue key. The other component of the multiple schedule consisted of the second VI schedule assigned to Key 2, which was a green key, and the common VI schedule again assigned to Key C, which now was yellow. Thus, although a single sequence of interreinforcement intervals provided the common VI schedule for both components of the multiple schedule, a different key-color was associated with the common schedule during each component. Different sequences of interreinforcement intervals were employed for the first and second VI schedules, and at a given time, only one or the other sequence was operative depending upon which component of the multiple schedule was in effect. When a schedule assigned a reinforcement, the next interval in that sequence was not timed until the end of the feeder cycle for the assigned reinforcement.

Reinforcement assigned by the common VI schedule but not delivered during the component of the multiple schedule in which it was assigned could be produced during the other (next) component of the multiple schedule; reinforcements assigned by the first and second VI schedules, however, could be delivered only during the appropriate component of the multiple schedule. A reinforcement so assigned, but not delivered during the component in which it was assigned, was held until the next time that component was operative during the same experimental session. Such a reinforcement was not preserved until the next daily session in the event that the session in which it was programmed terminated before the appropriate component of the multiple schedule was reinstated.

A changeover delay (COD) was programmed during both components of the multiple schedule. The changeover delay (Herrnstein, 1961; Catania, 1966) was a period of time in which a response could not be reinforced after the

organism changed from responding on one key to responding on the other. The first response on a key after a changeover initiated the delay period. The first response to occur after the delay expired could be reinforced if a reinforcement had been assigned for the key in question before the changeover or during the delay interval. A changeover that occurred during a delay interval started the delay interval anew. The changeover delay was programmed with respect to the keys as physical entities, rather than as representing particular VI schedules at particular times; the changeover delay was programmed continuously throughout the session and independently of the transitions from one component of the multiple schedule to the other.

Specific Procedure

Table 1 summarizes the specific procedure, which is described in detail below. The changeover delay was set at 4 sec and maintained throughout the remainder of the experiment, with one exception noted below. The common VI schedule was assigned to the left and right response keys in an alternating sequence from one exposure to the next of a given component of the multiple schedule. The component of the multiple schedule with which each session began was alternated from session to session. At each transition from one component of the multiple schedule to the next, at the end of each 15-min period, the key-lights were extinguished for 5 sec, and the

VI programmers and recording functions were halted.

The experiment consisted of two sequences of experimental sessions. In the first sequence, designed to examine the first property of equation (5), the reinforcement rates for the first and second VI schedules were intended to be equal; in the second sequence, designed to examine the second property of equation (5), the reinforcement rate for the first VI schedule was greater than for the second. Within each sequence, the reinforcement rate provided by the common VI schedule was varied as the experimental variable. The nominal values of the reinforcement rates employed in the two parts of the experiment were:

i. The first and second VI schedules were intended each to provide 20 rft/hr, and the common VI schedule was intended to provide, in succession, 20 rft/hr (31 sessions), 0 rft/hr (25 sessions), and 60 rft/hr (45 sessions).

ii. The first VI schedule was intended to provide 40 rft/hr, and the second VI schedule was intended to provide 10 rft/hr. The common VI schedule was again intended to provide, in succession, 20 rft/hr (25 sessions), 0 rft/hr (30 sessions), and 60 rft/hr (29 sessions). For reasons to be discussed later, a final 16 sessions employed the following nominal reinforcement rates: the first VI, 40 rft/hr; the second VI, 11.5 rft/hr; the common VI, 60 rft/hr. In addition, the COD was reduced to 2 sec during the final 16 sessions.

Table 1

A summary of the several experimental conditions. Each row identifies the two components of a multiple schedule; each component consists of a concurrent pair of VI schedules, specified as reinforcements per hr (nominal). The duration of exposure to each component of a multiple schedule was 15 min; the keys were darkened and inoperative for 5 sec between components. The changeover delay was 4 sec, except during the last condition (ii row 4) when it was reduced to 2 sec.

		First Multiple Component		Second Multiple Component		
		1st VI	Comm VI	2nd VI	Comm VI	
		Key 1	Key C	Key 2	Key C	
		Red	Blue	Green	Yellow	Sessions
i	1.	Conc 20 rft/hr	20 rft/hr	Conc 20 rft/hr	20 rft/hr	31
	2.	Conc 20 rft/hr	0 rft/hr	Conc 20 rft/hr	0 rft/hr	25
	3.	Conc 20 rft/hr	60 rft/hr	Conc 20 rft/hr	60 rft/hr	45
ii	1.	Conc 40 rft/hr	20 rft/hr	Conc 10 rft/hr	20 rft/hr	25
	2.	Conc 40 rft/hr	0 rft/hr	Conc 10 rft/hr	0 rft/hr	30
	3.	Conc 40 rft/hr	60 rft/hr	Conc 10 rft/hr	60 rft/hr	29
	4.	Conc 40 rft/hr	60 rft/hr	Conc 11.5 rft/hr	60 rft/hr	16

RESULTS

The data collected each day consisted of the time during which each component of the multiple schedule was available to the bird, the numbers of responses emitted on the several keys, and the numbers of reinforcements produced by responding on those keys. Change-over frequencies were recorded also, but those data are not reported.

The data collected during the final six sessions for each experimental condition were pooled, and the following calculations were performed. Rates of responding: the total number of responses emitted on each key of a concurrent pair was divided by the total time the two keys were concurrently available for responding. The results were expressed as *responses per min*, and the sum of the rates of responding for the two keys of a concurrent pair represents the overall rate of responding for the concurrent VI schedules in question. Rates of reinforcement: the total number of reinforcements produced by responding on each key of a concurrent pair was divided by the total time the two keys were concurrently available for responding. The results were expressed as *reinforcements per hr*, and the sum of the reinforcement rates for the two keys of a concurrent pair represents the overall rate of reinforcement for the concurrent VI schedules in question.

The results of the calculations are presented in Table 2. The symbols R and r, along with the several subscripts, are the same as defined above immediately preceding the derivation of equation (5). The first half of the table is for the first part of the experiment during which the first and second VI schedules were each programmed to provide nominal reinforcement rates of 20 rft/hr. The three columns for each bird represent the data obtained for the three nominal reinforcement rates programmed for the common VI schedule, *i.e.*, 0, 20, and 60 rft/hr.

The second half of the table shows data obtained during the second part of the experiment during which the nominal reinforcement rates programmed for the first and second VI schedules were 40 and 10 rft/hr, respectively. The columns labeled 60* are for the special condition during which the nominal reinforcement rate for the second VI schedule was increased from 10 to 11.5 reinforcements per

hour and the COD was reduced from 4 to 2 sec.

All of the figures, computations, and tables presented below were prepared from the data contained in Table 2.

The results are presented in terms of three topics: first, whether or not certain assumptions made in the derivation of equation (5) are justified by the data; second, how R_1/R_2 varied as a function of the reinforcement rate provided by the common VI schedule; third, the deviations of actual reinforcement rates from nominal reinforcement rates and some implications of those deviations for the outcome of the experiment.

A. Some Basic Assumptions

The derivation of equation (5) involved three major assumptions that may be evaluated from the data in Table 2. The following analyses view the experiment as consisting of 12 pairs of concurrent schedules. That view is simple and adequate for our purposes, although it neglects interactions that might have occurred between the components of the several multiple schedules.

1. The derivation assumed that for each pair of concurrent VI schedules of reinforcement, the rate of responding on one of the keys relative to the overall rate of responding approximated the rate of reinforcement for responding on that key relative to the overall rate of reinforcement; that is, it was assumed that equation (2) was true for each pair of concurrent VI schedules programmed.

Figures 2 and 3 provide evidence bearing on that assumption. Figure 2 shows the results of the first part of the experiment, and Fig. 3 shows the results of the second part. Each of the panels labeled B1, B2, and B3 shows the data for an individual bird. Each point in the figures shows the results for an individual pair of concurrent VI schedules; the filled points are for the concurrent pairs that included the first VI schedule (Key 1), and the unfilled points are for the concurrent pairs that included the second VI schedule (Key 2). The diagonal line in each panel represents the locus of all points that exactly satisfy equation (2), and the proximity of the data points to those lines indicates the degree to which the relative rates of responding on the first and second VI schedules approximated the respective relative rates of reinforcement. The

Table 2

Rates of responding and reinforcement for each VI schedule. Overall rates of responding and reinforcement for each pair of concurrent VI schedules are shown. The symbols are defined in the text.

i. Key 1 = Key 2 = 20 rft/hr (Nominal)												
Bird	B1			B2			B3					
	0	20	60	0	20	60	0	20	60			
<i>Rft/hr</i> Key C (Nominal)												
<i>Rate of Responding (Resp/min)</i>												
R ₁	42.6	42.7	27.8	24.0	35.5	22.0	48.5	34.2	23.7			
R _{C.1}	1.6	38.9	66.8	0.8	43.6	44.7	1.4	27.3	47.3			
R ₁ +R _{C.1}	44.2	81.6	94.6	24.8	79.1	66.7	49.9	61.5	71.0			
R ₂	35.0	40.5	30.0	34.1	34.7	23.0	54.8	30.5	19.7			
R _{C.2}	1.3	40.9	67.2	1.3	32.5	38.6	2.9	29.5	52.1			
R ₂ +R _{C.2}	36.3	81.4	97.2	35.4	67.2	61.6	57.7	60.0	71.8			
<i>Rate of Reinforcement (Rft/hr)</i>												
r ₁	19.0	19.9	19.3	20.5	19.8	18.9	20.3	20.7	17.7			
r _C	0	18.5	56.2	0	18.9	53.8	0	20.5	56.5			
r ₁ +r _C	19.0	38.4	75.5	20.5	38.7	72.7	20.3	41.2	74.2			
r ₂	19.5	20.6	17.7	19.2	19.1	17.9	20.0	19.6	18.4			
r _C	0	20.6	56.6	0	20.9	56.7	0	19.4	58.6			
r ₂ +r _C	19.5	41.2	74.3	19.2	40.0	74.6	20.0	39.0	77.0			
ii. Key 1 = 40, Key 2 = 10 rft/hr (Nominal)												
Bird	B1				B2				B3			
	0	20	60	60*	0	20	60	60*	0	20	60	60*
<i>Rft/hr</i> Key C (Nominal)												
<i>Rate of Responding (Resp/min)</i>												
R ₁	66.2	58.4	51.6	48.5	30.7	35.7	26.4	24.2	85.5	52.7	36.2	38.8
R _{C.1}	0.9	39.3	60.3	58.6	0.0	25.8	32.0	28.8	0.2	27.1	36.2	47.2
R ₁ +R _{C.1}	67.1	97.7	111.9	107.1	30.7	61.5	58.4	53.0	85.7	79.8	72.4	86.0
R ₂	33.4	27.9	14.8	19.5	32.1	25.1	9.9	9.3	53.1	25.2	11.0	14.9
R _{C.2}	2.3	51.3	67.2	62.1	0.4	29.9	36.7	36.1	4.7	45.4	54.8	59.0
R ₂ +R _{C.2}	35.7	79.2	82.0	81.6	32.5	55.0	46.6	45.4	57.8	70.6	65.8	73.9
<i>Rate of Reinforcement (Rft/hr)</i>												
r ₁	41.6	38.0	37.8	39.3	39.2	37.7	35.4	38.6	39.4	39.5	38.3	37.3
r _C	0	18.5	55.7	59.5	0	17.3	51.7	56.5	0	19.1	52.6	53.9
r ₁ +r _C	41.6	56.5	93.5	98.8	39.2	55.0	87.1	95.1	39.4	58.6	90.9	91.2
r ₂	10.3	9.0	7.6	12.5	10.3	10.4	6.5	11.1	10.0	10.3	7.8	10.0
r _C	0	20.0	58.4	54.0	0	20.6	58.9	54.8	0	19.3	57.8	59.2
r ₂ +r _C	10.3	29.0	66.0	66.5	10.3	31.0	65.4	65.9	10.0	29.6	65.6	69.2

*The special condition in which the nominal reinforcement rate programmed for Key 2 was increased to 11.5 rft/hr and the COD reduced to 2 sec.

The symbols and subscripts in the left-hand column are defined in the text.

lower right panel (AVE) in each figure consists of data points determined by averaging the corresponding points for the three birds.

2. Equation (2) was solved for r_C (the reinforcement rate provided by the common VI schedule) for both concurrent pairs of a multiple schedule, and the results of those solutions were equated in order to derive the expression for R₁/R₂. That step in the derivation assumed, therefore, that the reinforce-

ment rate provided by the common VI schedule was the same for both components of each multiple schedule that was examined. Evidence bearing on that assumption is presented in Table 3. The upper half of the table is for the first part of the experiment, and the lower half is for the second part. The several columns for each bird represent the nominal reinforcement rates assigned by the common VI schedule. The entries in the rows *Diff* repre-

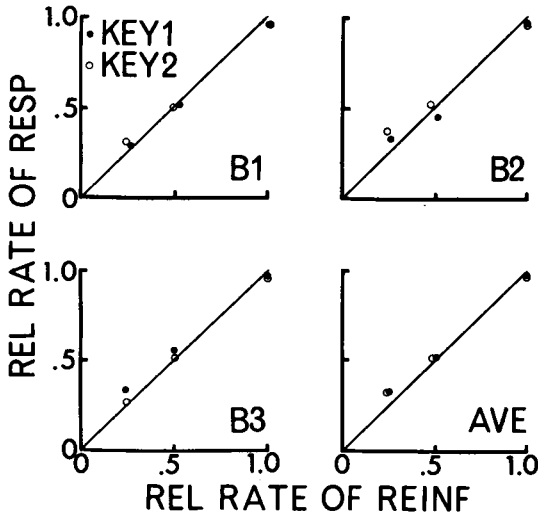


Fig. 2. The relative rates of responding on Key 1 (filled circles) and on Key 2 (unfilled circles) are plotted against the relative rates of reinforcement for responding on those keys. The common key was concurrent with Key 1 and with Key 2 in each instance. The data are from the first part of the experiment, and the panel labeled AVE shows averaged data.

sent the differences (direction neglected) between the actual reinforcement rates provided by the common VI schedule in the two components of each multiple schedule. The *Diff* entries were obtained from the r_c data in Table 2.

The largest *Diff* in the first part of the experiment was 2.9 rft/hr (Bird B2); the largest *Diff* in the second part of the experiment was 7.2 rft/hr (Bird B2). In both instances, those

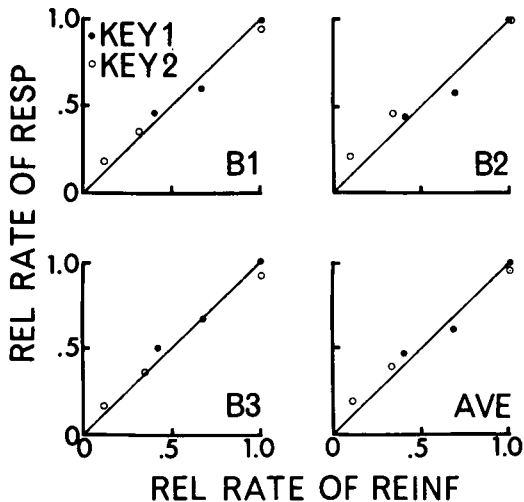


Fig. 3. This figure is similar to Fig. 2 and shows data obtained during the second part of the experiment.

differences were observed when the common VI schedule provided a nominal 60 rft/hr. The entries in the rows % *Diff* express each difference as a percentage of the smaller of the two reinforcement rates from which it was calculated. The largest % *Diff* in the first part of the experiment was 11.4% (Bird B1); the largest % *Diff* in the second part of the experiment was 19.1%, and the next largest % *Diff* was 13.9% (both, Bird B2).

3. A third assumption in the derivation of equation (5) involved the use of equation (4) to eliminate the terms $R_{c,1}$ and $R_{c,2}$ from the right-hand side of the final expression. As had been noted, equation (4) was derived by Catania (1963) from equations (2) and (3). The bearing of the data on equation (3) is shown in Fig. 4.

The overall response rate (Table 2) for each

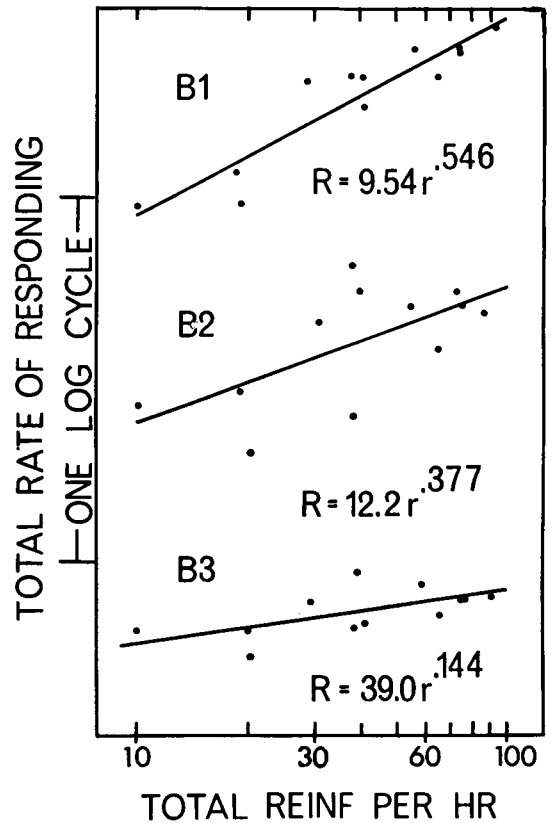


Fig. 4. The overall or total rate of responding is plotted against the overall or total rate of reinforcement for each concurrent pair of variable-interval schedules programmed during the experiment. The coordinates are logarithmic; the best fitting straight line and corresponding power function are shown for each bird.

Table 3

Differences and percentage differences between the two observed reinforcement rates for the common VI schedule within each multiple schedule programmed during the experiment.

i. Key 1 = Key 2 = 20 rft/hr (Nominal)											
Bird	B1			B2			B3				
Rft/hr Key C (Nominal)	0	20	60	0	20	60	0	20	60		
Diff	0	2.1	0.4	0	2.0	2.9	0	1.1	2.1		
% Diff	**	11.4%	0.7%	**	10.6%	5.4%	**	5.7%	3.7%		

ii. Key 1 = 40, Key 2 = 10 rft/hr (Nominal)												
Bird	B1				B2				B3			
Rft/hr Key C (Nominal)	0	20	60	60*	0	20	60	60*	0	20	60	60*
Diff	0	1.5	2.7	5.5	0	3.3	7.2	1.7	0	0.2	5.2	5.3
% Diff	**	8.1%	4.9%	10.2%	**	19.1%	13.9%	3.1%	**	1.0%	9.9%	9.8%

*The special condition in which the nominal reinforcement rate programmed for Key 2 was increased to 11.5 rft/hr and the COD reduced to 2 sec.

**This % Diff is not defined.

concurrent pair of VI schedules has been plotted against the corresponding overall reinforcement rate. The coordinates in Fig. 4 are logarithmic, since a power function such as equation (3) plots as a straight line with logarithmic coordinates. The best fitting straight line, calculated by the method of least squares (Lewis, 1966, pp. 15-19), is shown for each bird, in addition to the corresponding power function. The data points for Bird B2 are widely dispersed around the best fitting line; the dispersions for Birds B1 and B3 are smaller.

B. The Main Results

The main outcome of the experiment is shown in Fig. 5, which includes individual (B1, B2, B3) and averaged (AVE) data for the three birds. The reinforcement rate provided by the common VI schedule is plotted on the horizontal axis, and the ratio of the response rates R_1/R_2 for the first and second VI schedules is plotted on the vertical axis.

1. If the first and second VI schedules provide reinforcements at the same rate, then equation (5) states that the ratio of their response rates R_1/R_2 should equal unity at all values of r_C , the reinforcement rate provided by the common VI schedule. That expected

result is represented by the unbroken horizontal line at $R_1/R_2 = 1.0$ in each quadrant of Fig. 5. The observed values of R_1/R_2 are plotted above the nominal values of r_C , i.e., 0, 20, and 60 rft/hr, and those data points are connected by dashed lines.

For Birds B1 and B3, the empirical functions approximated 1.0 at the three values of r_C . Opposite trends are evident for those birds; the function decreased slightly for Bird B1 and increased slightly for Bird B3. The ratio R_1/R_2 was slightly below 0.75 at $r_C = 0$ rft/hr for Bird B2, but rose and remained virtually at 1.0 when r_C was programmed for the nominal values of 20 and 60 rft/hr. The averaged function is close to 1.0 at all three values of r_C .

2. If the reinforcement rate for the first VI schedule is greater than for the second, then equation (5) states that the ratio of their response rates R_1/R_2 increases as r_C increases. The unbroken line that appears as an increasing, concave downward curve in each quadrant of Fig. 5 connects the theoretical values of R_1/R_2 computed from the nominal reinforcement rates employed in the second part of the experiment, i.e., $r_1 = 40$, $r_2 = 10$, $r_C = 0$, 20, 60 rft/hr. The dashed lines connect the obtained values of R_1/R_2 .

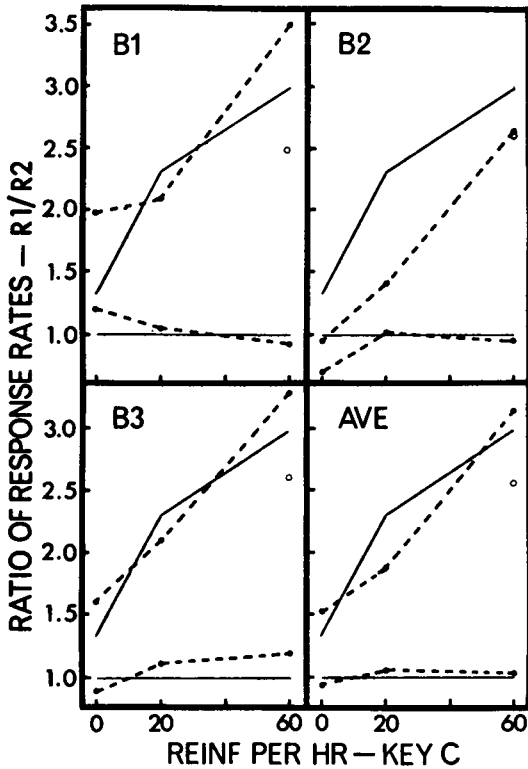


Fig. 5. The ratio of response rates R_1/R_2 for the first and second variable-interval schedules is plotted against the nominal values of the reinforcement rate provided by the common variable-interval schedule. The continuous, horizontal lines show the expected values of R_1/R_2 during the first part of the experiment, and the continuous, concave downward lines show the expected values of the ratio during the second part of the experiment. The broken lines connect the filled circles, which show the obtained values of the ratio. The unfilled circle in each panel shows the redetermined value of the ratio during the special condition denoted in the text by 60*. The panel labeled AVE shows averaged data.

The curves for Birds B2 and B3 are similar in shape; the former curve, however, is uniformly displaced below the latter, which approximates the theoretical function fairly closely. The initial point at $r_c = 0$ rft/hr in the curve for Bird B1 is high relative to the theoretical function; the points at $r_c = 20$ and 60 rft/hr are closer to the theoretical function and similar to the corresponding points for Bird B3. The largest difference between the averaged and theoretical curves is at $r_c = 20$ rft/hr.

The isolated, unfilled points at $r_c = 60$ rft/hr show the values of R_1/R_2 obtained during the special condition during which r_2 was in-

creased to a nominal 11.5 rft/hr and the COD reduced to 2 sec. For Birds B1 and B3, the redetermined points are considerably below the original points; for Bird B2, there is little difference between the original point and the redetermined point.

3. With the reinforcement rates just considered, equation (5) predicts a negatively accelerated increase in R_1/R_2 as r_c increases indefinitely. The limit of the increase is given by r_1/r_2 in equation (5). The data in Fig. 5 bear on those predictions. In contrast with the prediction from equation (5), the functions for the three birds are positively rather than negatively accelerated. If the redetermination of R_1/R_2 at $r_c = 60$ rft/hr is considered rather than the original determination, only the curve for Bird B3 becomes negatively accelerated. Further, the data permit no reasonable conclusion with respect to the likely asymptote of the increase in R_1/R_2 had r_c been increased further.

C. Some Final Implications of the Data

Table 2 shows that the actual reinforcement rates for the several VI schedules of reinforcement differed from their nominal values. Some implications of those deviations for the two right-hand terms of equation (5) are shown in Table 4. The table is organized into two parts, which pertain to the first and second parts of the experiment, respectively. Within each part of the table, the left-hand side is concerned with the term r_1/r_2 , and the right-hand side is concerned with the term $(r_2 + r_c)^{0.8}/(r_1 + r_c)^{0.8}$.

Consider the first part of the experiment in which the first and second VI schedules were intended each to provide 20 rft/hr. The theoretical values of the ratio r_1/r_2 are each 1.00 for the three nominal values of r_c . The observed values of r_1/r_2 differ from the theoretical values in no systematic way and by no more than 0.09, corresponding to a 9% deviation from the theoretical value. The theoretical values of the term on the right side of the table are also 1.00 for all nominal values of r_c . The largest difference between a theoretical and obtained value is 0.06, corresponding to a 6% deviation.

In the second part of the experiment an essentially similar result was obtained in the case of the term $(r_2 + r_c)^{0.8}/(r_1 + r_c)^{0.8}$. The theoretical values of that term, computed with

Table 4

Theoretical and observed values of the two terms on the right side of equation (5). Differences and percentage differences between the theoretical and observed values are shown.

<i>i. Key 1 = Key 2 = 20 rft/hr (Nominal)</i>								
Bird and Rft/hr Key C (Nominal)	r_1/r_2				$(r_1 + r_c)^{0.8}/(r_1 + r_c)^{0.8}$			
	Theoret- ical	Ob- served	Diff	% Diff	Theoret- ical	Ob- served	Diff	% Diff
0	1.00	0.97	-0.03	3%	1.00	1.03	+0.03	3%
B1 20	1.00	0.97	-0.03	3%	1.00	1.06	+0.06	6%
60	1.00	1.09	+0.09	9%	1.00	0.99	-0.01	1%
0	1.00	1.07	+0.07	7%	1.00	0.95	-0.05	5%
B2 20	1.00	1.04	+0.04	4%	1.00	1.03	+0.03	3%
60	1.00	1.06	+0.06	6%	1.00	1.02	+0.02	2%
0	1.00	1.02	+0.02	2%	1.00	0.99	-0.01	1%
B3 20	1.00	1.06	+0.06	6%	1.00	0.95	-0.05	5%
60	1.00	0.96	-0.04	4%	1.00	1.03	+0.03	3%
<i>ii. Key 1 = 40, Key 2 = 10 rft/hr (Nominal)</i>								
0	4.00	4.04	+0.04	1%	0.330	0.327	-0.003	1%
B1 20	4.00	4.22	+0.22	6%	0.574	0.588	+0.014	2%
60	4.00	4.97	+0.97	24%	0.750	0.756	+0.006	1%
60*		3.14	-0.86	22%		0.726	-0.024	3%
0	4.00	3.81	-0.19	5%	0.330	0.345	+0.015	5%
B2 20	4.00	3.62	-0.38	10%	0.574	0.635	+0.061	11%
60	4.00	5.45	+1.45	36%	0.750	0.798	+0.048	6%
60*		3.48	-0.52	13%		0.746	-0.004	1%
0	4.00	3.94	-0.06	2%	0.330	0.334	+0.004	1%
B3 20	4.00	3.84	-0.16	4%	0.574	0.587	+0.013	2%
60	4.00	4.91	+0.91	23%	0.750	0.765	+0.015	2%
60*		3.73	-0.27	7%		0.803	+0.053	7%

*The special condition in which the nominal reinforcement rate programmed for Key 2 was increased to 11.5 rft/hr and the COD reduced to 2 sec.

the nominal reinforcement rates employed during the second part of the experiment, are 0.330, 0.574, and 0.750 for the three nominal reinforcement rates assigned the common VI schedule. The observed reinforcement rates, when substituted into that term, yield values that differ from the theoretical values by no more than 0.061 or 11% of the corresponding theoretical value. Although small, the deviations tend to be positive.

The theoretical values of the term r_1/r_2 in the second part of the experiment are each 4.00 for the three nominal values of the reinforcement rate for the common VI schedule. The observed values of r_1/r_2 differ in no systematic way and by no more than 0.38 or 10% of the theoretical value only for the 0 and 20 rft/hr values of the reinforcement rate for the common VI schedule. When the common VI schedule was programmed to deliver a nominal 60 rft/hr, the observed values of r_1/r_2

were consistently larger than the theoretical value of 4.00. The smallest deviation was 0.91 or 23% of the theoretical value (Bird B3).

Because of the consistent and relatively large increase in the observed values of r_1/r_2 above 4.00 at a nominal 60 rft/hr for the common VI schedule, the special condition was instated at the end of the experiment. The observed values of r_1/r_2 and $(r_1 + r_c)^{0.8}/(r_1 + r_c)^{0.8}$ for the special condition are shown in Table 4 in the rows denoted by 60*.

The largest deviation from 0.750 (the right side of the table) was 0.053, corresponding to a 7% deviation from the theoretical value (Bird B3). The largest deviation in the observed value of r_1/r_2 was 0.86 or 22% of the theoretical value 4.00 (Bird B1). The next largest deviation was 0.52 or 13% (Bird B2). More important, the direction of the deviation for each bird was negative rather than positive, as was the case for the original deter-

mination at 60 rft/hr. Finally, the value of r_1/r_2 for each bird during condition 60* was smaller than the value of r_1/r_2 during either of the earlier conditions corresponding to 0 and 20 rft/hr for the common VI schedule.

DISCUSSION

A simple method to generate equation (5) is to form the ratio between the equivalents of R_1 and R_2 given by equation (4). The derivation displayed was more involved, but it followed reasonably from the considerations discussed in connection with Fig. 1; more important, it made explicit several of the basic assumptions or conditions that affect the validity of equation (5). The evidence for those assumptions has been presented.

Figures 2 and 3 show a strong correlation between the relative rates of responding and reinforcement during both parts of the experiment, as required by equation (2). All of the data points, the filled ones for the concurrent schedules involving the first VI schedule and the unfilled ones for the concurrent schedules involving the second VI schedule, cluster closely about the diagonal lines that represent the perfect correlation. Herrnstein (1961) noted a maximum deviation in his experiment of about 0.08 between the relative rates of responding and reinforcement. If 0.08 is adopted as a reference, then the 36 independent points plotted in Fig. 2 and 3 include only four that represent more extreme deviations (0.13, once; 0.11, twice; 0.09, once), three that represent deviations equal to 0.08, and 29 that represent deviations smaller than 0.08. Of the four deviations larger than 0.08, three were for Bird B2. Although the concurrent VI schedules of the present experiment were programmed in pairs in two-component multiple schedules, the relative rates of responding and reinforcement were closely matched in spite of interactions that might have developed between the components of individual multiple schedules.

A second assumption concerned the equivalence of the two values of r_c , the reinforcement rate for the common VI schedule, within each multiple schedule examined. The calculations relating to that assumption were presented in Table 3; they consisted of simple differences between the two observed values of r_c for each

multiple schedule in addition to percentage differences as defined.

Only when the common VI schedule was intended to provide 0 rft/hr could the reinforcement rate be controlled precisely. The differences in Table 3 for all of the conditions showing 0 rft/hr for r_c are, therefore, equal to exactly zero. Precise control over r_c was impossible when 20 and 60 rft/hr were intended, and differences larger than zero between the two values of r_c were probable. Evaluating those differences against the equivalence assumption requires a criterion that is necessarily arbitrary. A percentage difference of 10% appears to be a defensible criterion, since two values of r_c that differ by about 2 rft/hr against 20 or 6 rft/hr against 60 do not control rates of responding that differ markedly when programmed as isolated VI schedules of reinforcement. (Employing the power functions shown in Fig. 4, for example, the response rates calculated for 18 and 20 rft/hr and for 54 and 60 rft/hr do not differ in either case for any of the birds by as much as 10% of the smaller response rate.) Only the percentage difference of 19.1% for Bird B2 during the second part of the experiment clearly fails when judged against the 10% criterion; the 13.9% difference, also for Bird B2 during the second part of the experiment, is notable. The differences for Birds B1 and B3 during the second part of the experiment are acceptable, as are all of the differences observed for the three birds during the first part of the experiment.

The final assumption for which evidence was presented concerned equation (3), the power function relating the overall or total rates of responding and reinforcement for concurrent VI schedules of reinforcement. As noted, the data points in Fig. 4 for Bird B2 are widely dispersed about the best-fitting straight line, and it would be hazardous to conclude anything about the shape of the function for that bird. The dispersions for Birds B1 and B3 are smaller, and a power function is not an unreasonable conclusion for those birds. However, we have not performed statistical tests to determine whether a function of some other shape would fit the data more parsimoniously than does a power function, since the differences are likely to be small. Note, however, that if corresponding data points for the three birds are compared,

they tend to be related to the best-fitting straight lines in the same fashion. The data points at (about) 10 rft/hr, for example, are for all three birds above the lines, while the points at (about) 20 rft/hr are uniformly below the lines. The systematic distribution of the data points about the best-fitting lines cannot be dismissed, since it might imply that the power function is a poor choice or, more likely, that there were important interactions between the components of the several multiple schedules. Because of the latter possibility, the present experiment cannot be considered a sensitive (fair) test of the power assumption.

A final consideration concerning the power functions shown in Fig. 4: only the function for Bird B3 has an exponent approximating the value 0.2, which appears in equations (1) and (3). The exponent 0.546 for Bird B1 is notably different. Catania's (1963) analyses of the data for several birds yielded exponents that appeared more uniform in magnitude, although his most extreme example (Findley's Bird #5) was an exponent that seems to be larger than the 0.546 obtained in this experiment.

In conclusion, the data do not clearly support the assumptions expressed by equation (3); nor, however, do they clearly refute those assumptions, particularly the form of the function relating the overall rates of responding and reinforcement. If a power function can be assumed from the data shown in Fig. 4, it is not likely to have the exponent 0.2, however. When the exponents shown in Fig. 4 were substituted for 0.2 in the derivation of equation (5), the theoretical functions shown in Fig. 5 differed among the three birds; however, there was no improvement in predictive value.

There are no criteria that stipulate whether or not a particular assumption has been met unequivocally, and some reservations have been expressed. It appears, however, that the data are reasonably consistent with the several demands. The remainder of the text is concerned with the main outcome of the experiment.

That the ratio of response rates R_1/R_2 approximated unity at all of the reinforcement rates assigned the common VI schedule during the first part of the experiment was shown in Fig. 5, and that result requires no further

comment. The increase in the ratio R_1/R_2 during the second part of the experiment requires interpretation.

Consider the right side of equation (5). As r_C increases, the ratio R_1/R_2 increases toward the limit r_1/r_2 because the term $(r_2 + r_C)^{0.8} / (r_1 + r_C)^{0.8}$ approaches unity. In an experiment in which the several reinforcement rates are maintained precisely at the intended values, the increases in R_1/R_2 may be ascribed to the expected changes in the latter term, since the value of r_1/r_2 is independent of r_C . The calculations for the second part of the experiment shown in Table 4 indicated that the value of r_1/r_2 was approximately constant only for the nominal reinforcement rates 0 and 20 rft/hr assigned r_C . In fact, for two of the birds, B2 and B3, the value of r_1/r_2 decreased slightly when r_C was increased from 0 to 20 rft/hr, and the correlated increase in R_1/R_2 must have resulted, therefore, from the effect of r_C on the value of the term $(r_2 + r_C)^{0.8} / (r_1 + r_C)^{0.8}$.

The marked increase in R_1/R_2 for all of the birds when r_C was increased to 60 rft/hr is ambiguous, since, for all birds, r_1/r_2 was shown to increase substantially above the values observed when r_C was programmed for 20 rft/hr. The special condition, denoted by 60* in the tables, was instated at the end of the experiment in order specifically to obtain a second estimate of R_1/R_2 , with r_C programmed for a nominal 60 rft/hr, but with the resulting value of r_1/r_2 no larger than observed previously when r_C was intended to deliver 20 rft/hr. As noted, the second estimates of r_1/r_2 were for all of the birds smaller than the initial estimates; in fact, they were smaller than the values of r_1/r_2 observed when r_C was programmed for 20 rft/hr. In spite of that overcompensation, the redetermined ratios of response rates R_1/R_2 were larger than the ratios of response rates obtained when r_C was programmed for 20 rft/hr, a fact that removes the ambiguity and implies that the value of r_C determines the size of the ratio R_1/R_2 in the manner required by equation (5).

It is concluded that the results of the present experiment provide empirical support for the first and second properties of equation (5) that were stated immediately after its derivation. Other properties of equation (5) state the shape of the function relating R_1/R_2 to r_C and the limit of the increase in R_1/R_2 as r_C

is increased indefinitely. The data permitted no reasonable statement in regard to those properties of equation (5), and further comment seems unwarranted in the absence of additional data.

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