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The Relationship between Interest Rates and Metal Price Movements

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1. INTRODUCTION

In recent years there have been many analyses of the rate of resource depletion, both with a view to defining an optimal depletion rate (as in Dasgupta and Heal (1974), Heal (1975)) and also with a view to analysing the depletion rate that one might expect to result from market forces (as in Dasgupta (1973), Solow (1974), Stiglitz (1974)). It is easily established (see Heal (1975), Solow (1974)) that a necessary condition for a finite stock of an exhaustible resource to be allocated efficiently over time is that the price, net of extraction costs, should rise at a rate equal to the rate of return on other assets. And, not surprisingly, competitive markets will under certain circumstances realize this condition. In particular, if owners of the resource regard it as a capital asset constituting an element of their portfolio, then they will hold it just as long as the return that it gives them (the rate of increase of the net price) is no less than the returns available elsewhere. Equilibrium in the asset market will then imply the realization of the necessary condition mentioned earlier.

This simple but convincing theorizing clearly implies that if resource markets are functioning efficiently, there will be a strong association between the rates of change of resource prices and the rates of return on other assets. In particular, as certain commodities (for example, copper, tin, lead and zinc) are exhaustible resources, the theory would predict that in an efficient allocation the rates of change of their prices would be related to rates of return on other assets. Our aim in this paper is to construct and test a series of models of resource markets whose demand and supply functions incorporate the idea that an exhaustible resource is an asset whose rate of price appreciation is a factor determining holding decisions, and which explicitly recognize the possibility of arbitrage between resource markets and markets for other capital assets.

The conclusions we reach are very tentative, but suggest that the matter is considerably more complex than simple equilibrium theory would suggest. In particular, the returns to other assets do appear to be important determinants of resource price movements, but it seems to be changes in these returns, rather than their level, that have the greatest influence. There are a variety of possible explanations of this, and we try to discriminate between these in the latter part of the paper.

2. AN ARBITRAGE MODEL

In this section we present a particular model of price determination in a resource market. It is a model in which traders are assumed to have the option of switching funds between this market and capital markets, and do this on the basis of expectations about relative rates of return in those markets. Here we shall work with very simple functional forms, though later in the paper we consider the properties of a more general class of models of which this is a simple, but interesting and typical, example. One of the advantages of working with the simple model is that it enables us to analyse clearly the effects of different assumptions about the methods of expectation formation and about stochastic specification on the lag and error structures of the reduced form. It turns out that a precise analysis of these lag and error structures is of great importance in interpreting the results.

The model considered has a fairly obvious structure. It is supposed that the resource price always adjusts so that supply and demand are equated. If p is the current price and p' a weighted average of past prices, and likewise y is current income and y' a weighted average of past incomes, supply is just taken to depend on p' and y': $S(p', y')$. The rationale for including p' is that supply responds to price changes with a lag: y' is included in case the level of economic activity affects investment in the extension of extractive and refining capacity directly, rather than via the price of the output.

The demand function is more complex and contains two distinct elements: one is a log-linear function of price and income, and this is multiplied by a term which depends on the ratio of the expected rate of capital gain from the resource to the expected rate of capital gain attainable on other assets.

$$D = p^{\eta(p)} y^{\eta(y)'} \left[\frac{\tilde{p}/p}{\tilde{O}/O} \right]^{a_1}$$

where $\eta(p)$ and $\eta(y)'$ are of course price and income elasticities, $\tilde{p}$ is the resource price expected to rule at some future date, O is the price of some other asset, and $\tilde{O}$ is again the price this is expected to exchange for at the same future date. The motivation underlying this functional form is clear: demand consists of a "normal" or "user" element depending in the obvious way on price and income, and this is scaled up or down according to whether or not the resource is expected to be a good investment in the near future. Thus if its price is expected to rise at a rate in excess of those of other assets, demand is increased, and *vice versa*. The multiplicative term is introducing an element of arbitrage between resource and capital markets into the model. One of our aims is to assess the importance of this effect. Obviously, realization of the efficiency conditions mentioned in the introduction would require very effective arbitrage.

An alternative interpretation of the demand function may be worth mentioning. This is that traders and speculators are distinct agents in the market, with trader demand depending on $p^{\eta(p)} y^{\eta(y)'}$ and speculator demand conditioned by $(\tilde{p}/p)/(\tilde{O}/O)$, but with a multiplicative rather than additive interaction. This has the implication that, given a set of expectations about rates of return, speculators are more willing to enter a market, the greater is the level of regular or user demand in that market.

Taking the demand and supply functions together, market clearing implies that

$$S(p', y') = p^{\eta(p)} y^{\eta(y)'} \left[\frac{\tilde{p}/p}{\tilde{O}/O} \right]^{a_1} \varepsilon_1(t) \qquad \ldots(1)$$

where $\varepsilon_1(t)$ is a lognormally distributed serially independent error process. An obvious response to such an equation is to enquire why the term in anticipated returns appears only on the right-hand side: why should suppliers not also modify their behaviour according to expected price changes? The answer is clearly that one can imagine a term identical to that in square brackets appearing on the LHS, raised perhaps to a power b_1. But it is then abundantly clear that a_1 and b_1 could not both be estimated: we therefore imagine the multiplicative terms of this type concentrated on the RHS with a_1 the net exponent.

Differentiating (1) logarithmically w.r.t. time and using the following notation:

$$\dot{p}/p = r_c, \qquad \dot{\tilde{p}}/\tilde{p} = \tilde{r}_c, \qquad \dot{O}/O = r, \qquad \dot{\tilde{O}}/\tilde{O} = \tilde{r}, \qquad \dot{y}/y = g$$

we have

$$\dot{S}/S = a_1(\tilde{r}_c - \tilde{r}) - a_1(r_c - r) + \eta(p)r_c + \eta(y)'g + \frac{d}{dt}\log \varepsilon_1. \qquad ...(2)$$

In order to make further progress, it is necessary to specify how the anticipated values $\tilde{r}_c$ and $\tilde{r}$ are formed. Consider first the term $\dot{\tilde{p}}/\tilde{p}$. Introducing the time argument explicitly, this might be written $\dot{\tilde{p}}(t+h)/\tilde{p}(t+h)$ where $t+h$ is the time to which expectations formed at t refer. Using this notation, the term $\dot{\tilde{p}}/\tilde{p}$ can be written

$$\dot{\tilde{p}}/\tilde{p} = \lim_{\Delta t \to 0} \left\{ \frac{\tilde{p}(t+h+\Delta t) - \tilde{p}(t+h)}{\Delta t \cdot \tilde{p}(t+h)} \right\}.$$

As it is not unreasonable to assume that speculators' expectations in commodity markets are of a very short-term type, we shall also let h tend to zero, and define

$$\dot{\tilde{p}}/\tilde{p} = \lim_{\Delta t \to 0} \lim_{h \to 0} \left\{ \frac{\tilde{p}(t+\Delta t+h) - \tilde{p}(t+h)}{\Delta t \cdot \tilde{p}(t+h)} \right\}.$$

Clearly $\lim_{h \to 0} \tilde{p}(t+h) = \tilde{p}(t)$, and it is obvious to assume that $\tilde{p}(t) = p(t)$. Hence

$$\dot{\tilde{p}}/\tilde{p} = \lim_{\Delta t \to 0} \left\{ \frac{\tilde{p}(t+\Delta t) - p(t)}{\Delta t \cdot p(t)} \right\}.$$

Now, a reasonable first-order approximation to $\tilde{p}(t+\Delta t)$ is clearly

$$\tilde{p}(t+\Delta t) = p(t) + p(t)r_c^e(t)\Delta t$$

where $r_c^e(t)$ is the expected rate of price change at time t, so that

$$\frac{\tilde{p}(t+\Delta t) - p(t)}{\Delta t \cdot p(t)} = r_c^e(t).$$

Although it is reasonable to assume that the current price level $p(t)$ can be observed accurately, one would clearly not wish to make this assumption about its current rate of change $r_c(t)$: an approximation to this has to be built up from past observations, and it is assumed that an agent's best approximation to $r_c(t)$ is given by the distributed lag form $a_2 r_c(t)/(D+a_2)$, where D is the differential operator. Hence in (2) we can make the substitution

$$\tilde{r}_c = a_2 r_c/(D+a_2) + \varepsilon_2(t) \qquad ...(3)$$

where $\varepsilon_2(t)$ is a white noise error process, and by similar arguments one can justify the assumption that

$$\tilde{r} = a_3 r/(D+a_3) + \varepsilon_3(t). \qquad ...(3')$$

In order to make (2) operational, it is necessary to specify the form of the supply function. This is assumed to take the very simple form $S(p', y') = p'^{a_4} y'^{\eta(y)''}$, with p' and y' defined by the lag processes

$$p'(t) = \frac{3\lambda p(t)}{(D+\lambda)^3}, \qquad y'(t) = \frac{\mu y(t)}{D+\mu}. \qquad ...(4)$$

Substituting from (3), (3') and (4) into (2) yields the following second-order differential equation, which contains only observable variables:

$$\ddot{r}_c(a_1 + a_4 - \eta\{p\}) + \dot{r}_c(a_1 a_3 + a_2 a_4 + a_3 a_4 - \eta\{p\}a_2 - \eta\{p\}a_3) + r_c(a_2 a_3 a_4 - \eta\{p\}a_2 a_3)$$
$$= \ddot{r}a_1 + \dot{r}a_1 a_2 + \ddot{g}\eta\{y\} + \dot{g}(a_2 + a_3)\eta\{y\} + g a_2 a_3 \eta\{y\} + \varepsilon. \qquad ...(5)$$

In this equation, $\eta\{y\}=\eta\{y\}'-\eta\{y\}''$ and is a net income elasticity. It could thus be zero even though the income variable exerted a significant influence on both sides of the equation. The error process ε will exhibit third-order serial correlation, and if one believes the stochastic specifications (1), (3) and (3') this will be of the moving-average type. But in fact if we operate with a short time-period, as will be the case, the assumption that the errors in (1), (3) and (3') are uncorrelated is unreasonable. They are likely to exhibit substantial positive serial correlation, of the sort that will lead to a mixed autoregressive-moving average error process in (5).

Rather than estimate the differential equation (5), we have chosen to estimate a difference equation approximation to it. There is a growing literature on the estimation of stochastic differential equations and of discrete forms of these, and the merits of different approaches have been discussed *inter alia* by Sargan (1974) and Phillips (1974). The transformation applied to (5) is one discussed by these authors, and seems to have desirable properties: it is

$$\dot{x}(t)=x(t+1)-x(t)$$
$$\ddot{x}(t)=x(t+2)-2x(t+1)+x(t)$$
$$x(t)=\tfrac{1}{2}(x\{t+1\}+x\{t\}).$$

Applying this transformation, (5) becomes

$$r_c(t)=A_1r_c(t-1)+A_2r_c(t-2)+A_3r(t)+A_4r(t-1)+A_5r(t-2)+A_6g(t)+A_7g(t-1)$$
$$+A_8g(t-2)+\varepsilon(t) \qquad \text{...(6)}$$

where the arguments denote values of variables in particular time-periods. The coefficients in (6) are related to the original parameters by the formulae

$$A_1=2-a_3-a_2a_4/\theta+\eta(p)a_2/\theta-a_2a_3a_4/2\theta+\eta(p)a_2a_3/2\theta$$
$$A_2=-1+a_3+a_2a_4/\theta-\eta(p)a_2/\theta-a_2a_3a_4/\theta+\eta(p)a_2a_3/\theta$$
$$A_3=a_1/\theta$$
$$A_4=(a_1a_2-2a_1)/\theta$$
$$A_5=(a_1-a_1a_2)/\theta$$
$$A_6=\eta(y)/\theta$$
$$A_7=-2\eta(y)/\theta+\eta(y)a_2/\theta+\eta(y)a_3/\theta+\eta(y)a_2a_3/2\theta$$
$$A_8=\eta(y)/\theta-\eta(y)a_3/\theta+\eta(y)a_2a_3/2\theta-\eta(y)a_2/\theta$$

where $\theta=a_1+a_4-\eta(p)$.

Obviously estimating (6) is not entirely straightforward: the equation contains lagged endogenous variables, groups of variables which will be collinear, autocorrelated errors, and has coefficients which are complex non-linear functions of the parameters of the original model. It is also true that some parameters are under and some over-identified. We have in fact used two different approaches to estimating (6). The first estimates the coefficients A_1 to A_8 without any attempt to impose on them the restrictions implicit in the formulae relating them to the parameters. The estimation method, a member of the class of generalized instrumental variable estimators (GIVE), was developed by Hendry (1974) on the basis of work by Sargan (1958), and produces asymptotically efficient, normally distributed and consistent estimates of the coefficients of an equation with lagged endogenous variables and an autoregressive error process. Of course, the error process in (6) is probably not purely autoregressive, but is a mixed autoregressive-moving average process, but Monte Carlo studies by Hendry and Trivedi (1972) suggest that the biases produced in approximating a moving average process by an autoregressive one of similar order are not large. Indeed, subsequent analytical results due to Hendry (1975) confirm that in some simple cases the biases in the coefficients are unimportant, and that if the true error process is mixed autoregressive-moving average, then a pure autoregressive process is a very good approximation.

Fortunately one of the important constraints implicit in the coefficient-parameter relationships has a very simple form and is easily tested against the unconstrained estimates: it is that

$$A_3 + A_4 + A_5 = 0.$$

Obviously this can be tested by seeing whether the sum of the interest rate coefficients is significantly different from zero. The constraint is in fact satisfied to a very high degree of accuracy–a very interesting finding whose implications are considered in some detail below.

This summing to zero property results from the characteristics of the differential equation (5) and the method by which it is transformed to a difference equation. Suppose we have an equation in continuous time as follows:

$$y = A_1\dot{x} + A_2\ddot{x} + A_3\dddot{x} \qquad \ldots(7)$$

in which the level of x does not appear (as r does not in (5)), and which we wish to transform into discrete time form using the approximations given above. Then this yields

$$y = B_1x(t) + B_2x(t+1) + B_3x(t+2) + B_4x(t+3) \qquad \ldots(8)$$

where

$$\textstyle\sum_{i=1}^{4} B_i = -A_1 + A_2 - A_3 + A_1 - 2A_2 + 3A_3 + A_2 - 3A_3 + A_3 = 0. \qquad \ldots(9)$$

This property turns out to be true for whatever length of lags we include because x is still not in the original equation. If x were to be included, then the B coefficients would sum to A_0, the coefficient of x in the original formulation. Of course the B coefficients in equation (8) can take on any values, so that if they do sum to zero the implication is that x is not a relevant explanatory variable in (7). In terms of our theory, this implies that the rate of interest should not appear in equation (5) and therefore that the price of other assets is not a relevant variable in the demand function, which is therefore asymmetric, since it does include the price of the resource.

Another implication of this property is that we can experiment with different lengths of lag in the expectations equations and still examine whether or not the demand curve contains the price of other assets, rather than, or as well as, the expected return to these, as one of its arguments.

The second approach to estimating the model is to estimate the parameters of the original system directly, which means estimating (6) subject to non-linear constraints on the coefficients. The results of this exercise are presented in the appendix.

3. DATA AND ESTIMATION

The data covers the period from July 1965 to June 1977, giving 144 observations on each of the monthly variables. This covers a period of fairly stable prices up to the middle of 1973. After this, and up to the end of the period studied, prices rose more sharply and were more volatile. The same can be said to be true, though to a lesser extent, for interest rates. Up to about early 1973, rates were reasonably stable. Mid 1972 saw a low point in the level of interest rates, but from this they rose rapidly, and from then to the end of the period of study the fluctuations were larger and more frequent. There does appear to be some evidence that towards mid-1977 resource prices have been settling down a little, but this conclusion is drawn only from a casual inspection of the last few observations of the data.

The resource price data used (for copper, lead, tin and zinc) is the three-month forward price quoted on the London Metal Exchange (L.M.E.). A variety of prices are available (spot, settlement and forward) and the forward price was chosen because the forward market yields the greatest volume of trading (in fact the results are not particularly sensitive to the series chosen). The volume of trade carried out on the L.M.E. has declined over the years (as a proportion of total trade) and it might be argued that its price is not particularly representative any more. Although it is true that many deals are now directly between companies and are carried out on a "producer price" basis, this price is based upon the quoted L.M.E. price. It is also true that the supply and demand curves net of extra market trade will establish the same price as would have been established if all trade had gone through the market.

The interest rate used is the return to maturity of a 91-day UK Treasury Bill and the growth rate variable is based upon the OECD Index of Industrial Production. This series will almost certainly be measured to a lesser degree of accuracy than the others. The monthly changes in output are likely to be of the same order of magnitude as its measurement error, though the fact that they are positively serially correlated may mean that the first differenced series is measured more accurately than the original output series.

All of the estimated equations were run in money terms, for a number of reasons. Previous investigation had shown that the results were not particularly sensitive to whether the equations were run in real or money terms. Secondly, it proved difficult to find a satisfactory price deflator for the series. Thirdly, it appears that on theoretical grounds it does not matter whether one uses real price changes and real interest rates, or their money equivalents, since the deflator has the same effect on both sides of the equation.

All of the results were obtained using the GIVE (Generalized Instrumental Variable Estimator) econometric programme (for details see Hendry (1973)). This enables one to allow for a variety of error structures in the estimated equation. All results except those for resource interaction were obtained using OLS techniques, corrected for the error structure. The estimates headed A assume that the errors are NID and is therefore a straightforward OLS calculation. The restricted transformed equation (RTF) estimates the equation with any desired autoregressive scheme (in our case, second order) incorporated. The unrestricted transformed equation (URTF) estimates an unrestricted (second order) autoregressive transform of the structural form. A comparison of the residual sums of squares of the A and URTF indicate whether the errors are in fact NID. If not, this could be due to either an autoregressive scheme being required or a different lag structure or explanatory variables needing to be included. The results of this comparison are shown in the tables as the F-statistic under the URTF column. If this is significantly in excess of zero then something more than the structural form of the equation is required. The form of autoregressive restriction can be tested by a comparison of the URTF and RTF equations. The value of this test statistic is given in the tables as a χ^2-statistic in the RTF column. If its value is not significantly greater than zero then the correct autoregressive process has been found.

4. RESULTS

Tables I–IV show the results of estimating equation (6) for copper, lead, tin and zinc respectively. The estimated equation is slightly different from that formulated for the following reasons. Only the current and not the lagged values of the growth rate are included since in earlier work the latter were never found to be significant at the 95 per cent confidence level. It can be seen from the tables that the current value of the growth rate is not a significant variable either, and is included for informational purposes only. A longer lag on the interest rate variable is also included because in earlier regressions, when only $r(-1)$ and $r(-2)$ (see "Interpretation of Results" section for complete notation) were included, as in the original model, the F test indicated that a different lag structure and/or autoregressive scheme was required. The combination of the longer interest rate lag and second-order autocorrelation (different autoregressive schemes were tried) appeared to give the best results and so these are reported here. A longer lag structure on the interest rate is of course a corollary of a longer lag structure on the expectations formation equations.

The results for the four different resources show similar patterns though there are some important differences. In general, the best results are obtained from zinc, and the worst are provided by copper. Looking first at the value of the R^2 statistic, this ranges from around 0·2 (copper) to 0·5 (zinc). These are not very high values but it must be remembered that the data is in first difference form where high values of R^2 are not common. The values of the F and χ^2 parameters indicate that the dynamic specification of the equation appears correct. The case of lead is, however, the exception here, where the significance of χ^2 and the non-significance of α (the autoregressive parameter) indicate that a different order of autoregressive disturbance is probably required. The other three resources have a non-significant value of χ^2 and therefore the restriction imposed in estimating the RTF equation is valid. All of the resources exhibit second-order serial correlation.

As already mentioned, the coefficient on the growth variable never attains significance. This could be due to one of two factors. Either the rate of growth is not a relevant variable in the determination of demand or supply, or that it has similar effects on both sides of the market and therefore cancels itself out as a determinant of price changes.

The coefficient on the lagged dependent variable is significant for all resources and is always positive, (usually around 0·5) indicating that we have not uncovered the simple correlation due to $(p_t - p_{t-1}) - 1$ and $(p_{t-1} - p_{t-2}) - 1$ having a common variable in p_{t-1}. The twice lagged dependent variable is in general not significant, the exception being the case of copper.

It is the coefficients of the interest rate variables which are of most interest. The model predicts that these coefficients should sum to zero, and this property is strongly confirmed by the empirical results. For all resources, the sum of the interest rate coefficients was found not to be significantly different from zero at the 95 per cent confidence level. This result was obtained by running similar regressions where the coefficients were constrained to sum to zero, and then comparing the error sums of squares of the constrained and unconstrained regressions using an F-test. In no case was a significant F-statistic found. (The results of these calculations are not shown).

The sums of the interest rate coefficients are shown in the table as "$\sum r$" and the values can be seen to be extremely small when compared to their constituent parts, the coefficients themselves. The insignificance of the sum is clearly not due to the insignificance of the individual coefficients, except perhaps in the case of copper. The other three each have a significant coefficient on the current interest rate variable, and lead and zinc also have a significant coefficient on the interest rate lagged once, but of opposite sign to that on the interest rate. Other interest rate coefficients are also significant (lead, $r(-3)$, tin, $r(-3)$, zinc, $r(-3)$, according to the RTF form of the equations) though no particularly strong pattern emerges.

It thus seems to be confirmed that the "adding up" property holds for all of the resources. This means that the equation can be written with the rate of change of the interest rate as the independent variable. As confirmation of this, the equation was also estimated with the change of the interest rate as the independent variable explicitly. Tables V–VIII report these results.

TABLE I

Copper forward

Variable	A	URTF	RTF
$r_c(-1)$	0·436*	0·519*	0·503*
$r_c(-2)$	−0·176*	−0·328*	0·066
g	−0·020	−0·024	0·021
r	0·018	0·016	0·015
$r(-1)$	−0·018	−0·015	−0·013
$r(-2)$	−0·002	−0·004	−0·009
$r(-3)$	0·018	0·017	0·017
$r(-4)$	−0·013	0·012	−0·001
$r(-5)$	−0·003	−0·008	−0·009
$r_c(-3)$		0·301*	
$r_c(-4)$		−0·124	
$g(-2)$		0·089	
$r(-6)$		0·017	
$r(-7)$		−0·013	
R^2	0·212	0·294	
S	0·067	0·065	0·066
$\sum r$	0·00040	0·00013	0·00009
F		2·89*	
α		−0·42	−0·38*
χ^2			7·17

Interpretation of Results:
Variable names:

$r_c(\cdot)$: is the dependent variable lagged, the length of lag being given in brackets. In all regressions the dependent variable is the proportional rate of change of the commodity price, i.e. $r_c(0)=(p_t-p_{t-1})/p_{t-1}$, where p_t is the price of the commodity in period t.
g: proportional rate of growth of putput.
$r(\cdot)$: interest rate and lagged values thereof, the length of lag being indicated by the figure in brackets.
$\Delta r(\cdot)$: change in the interest rate and its lagged values, i.e.

$$\Delta r = r - r(-1)$$

$$\Delta r(-1) = r(-1) - r(-2)$$

$r'_c(\cdot)$: rate of change of resource price other than the dependent variable, e.g. the rate of change of tin prices in an equation where $r_c(\cdot)$ relates to copper.

Other output
Equation types: (see text)

A: OLS estimate assuming NID errors.
URTF: Unrestricted transformed equation.
RTF: Restricted transformed equation.

Informative statistics
*: indicates that the coefficient or parameter is statistically significant at the 95 per cent confidence level.
R^2: R^2.
S: Standard error of the regression.
$\sum r$: sum of the interest rate coefficients.
F: a test of significance of the additional parameters in the URTF equation. Thus if the F-statistic is significantly different from zero, we are right to include the additional variables, i.e. that there is some sort of autoregressive structure, or that there should be a different lag structure to the equation.
α: the value of the autoregressive parameter.
χ^2: a test of the validity of the autoregressive restriction in the RTF equation. If the value is significant, then the results obtained from the RTF equation are significantly different from what would be obtained if the autoregressive restriction were not imposed, and hence the restriction is invalid.

TABLE II

Lead forward

Variable	A	URTF	RTF
$r_c(-1)$	0·510*	0·501*	0·514*
$r_c(-2)$	−0·039	−0·065	−0·008
g	−0·179	−0·201	−0·177
r	0·017*	0·020*	0·017*
$r(-1)$	−0·025*	−0·028*	−0·025*
$r(-2)$	0·002	0·004	0·001
$r(-3)$	0·027*	0·025*	0·029*
$r(-4)$	−0·017	−0·018	−0·019
$r(-5)$	0·003	0·024*	0·003
$r_c(-3)$		0·038	
$r_c(-4)$		−0·172	
$g(-2)$		−0·037	
$r(-6)$		−0·029*	
$r(-7)$		0·003	
R^2	0·311	0·405	
S	0·049	0·046	0·049
$\sum r$	0·00067	0·00084	0·00059
F		3·90*	
α		−0·02	−0·065
χ^2			19·86*

TABLE III

Tin forward

Variable	A	URTF	RTF
$r_c(-1)$	0·360*	0·394*	0·352*
$r_c(-2)$	0·122	0·021	0·300*
g	−0·056	0·004	−0·023
r	0·013*	0·013*	0·013*
$r(-1)$	−0·011	−0·012	−0·010
$r(-2)$	−0·007	−0·003	−0·009
$r(-3)$	0·016*	0·014	0·020*
$r(-4)$	−0·014	−0·018*	−0·016*
$r(-5)$	0·002	0·021*	0·003
$r_c(-3)$		0·092	
$r_c(-4)$		0·019	
$g(-2)$		0·085	
$r(-6)$		−0·019*	
$r(-7)$		0·004	
R^2	0·300	0·359	
S	0·033	0·032	0·033
$\sum r$	0·00083	0·00063	0·00052
F		2·30*	
α		−0·22	−0·26*
χ^2			8·50

The results are very much as expected and are very similar to the previous ones. The R^2 statistics take on very similar values, as do the coefficients on the lagged dependent variables and the growth rate. The coefficient of Δr (i.e. $r\{t\}-r\{t-1\}$) is significantly different from zero for all resources except copper, and the coefficient of $\Delta r(-3)$ (i.e. $r\{t-3\}-r\{t-4\}$) is usually significant also. In addition, the successive coefficients on the interest rate change variables are very nearly equal to the cumulative sum of the

TABLE IV

Zinc forward

Variable	A	URTF	RTF
$r_c(-1)$	0·564*	0·586*	0·593*
$r_c(-2)$	−0·078	−0·183	0·061
g	−0·146	−0·093	−0·111
r	0·031*	0·028*	0·030*
$r(-1)$	−0·044*	−0·041*	−0·043*
$r(-2)$	0·008	0·012	0·004
$r(-3)$	0·027*	0·017	0·029*
$r(-4)$	−0·006	0·002	−0·008
$r(-5)$	0·015*	−0·010	−0·012
$r_c(-3)$		0·210*	
$r_c(-4)$		0·002	
$g(-2)$		0·132	
$r(-6)$		−0·010	
$r(-7)$		0·007	
R^2	0·453	0·493	
S	0·048	0·047	0·048
$\sum r$	0·00035	0·00019	0·00015
F		1·93	
α		−0·22	−0·25*
χ^2			7·00

TABLE V

Copper forward: interest rate changes as the independent variable

Variable	A	URTF	RTF
$r_c(-1)$	0·437*	0·518*	0·506*
$r_c(-2)$	−0·181*	−0·329*	0·042
g	−0·006	−0·017	0·030
Δr	0·018	0·016	0·015
$\Delta r(-1)$	−0·0003	0·001	0·001
$\Delta r(-2)$	−0·002	−0·002	−0·006
$\Delta r(-3)$	0·016	0·015	0·011
$\Delta r(-4)$	0·004	0·004	0·005
$r_c(-3)$		0·304*	
$r_c(-4)$		−0·126	
$g(-2)$		0·096	
$\Delta r(-5)$		−0·003	
$\Delta r(-6)$		0·008	
R^2	0·215	0·286	
S	0·067	0·065	0·065
F		2·47*	
α		−0·32	−0·355*
χ^2			5·64

coefficients on the interest rate variables, a property which only holds if the "adding up" property holds.

To illustrate this, we have as our two estimating equations:

$$r_c(t) = A_1 r_c(t-1) + A_2 r_c(t-2) + A_3 g + A_4 r(t) + A_5 r(t-1) + A_6 r(t-2) + A_7 r(t-3) + A_8 r(t-4) + A_9 r(t-5)$$

and

$$r_c(t) = B_1 r_c(t-1) + B_2 r_c(t-2) + B_3 g + B_4 \Delta r(t) + B_5 \Delta r(t-1) + B_6 \Delta r(t-2) + B_7 \Delta r(t-3) + B_8 \Delta r(t-4).$$

TABLE VI

Lead forward: interest rate changes as the independent variable

Variable	A	URTF	RTF
$r_c(-1)$	0·525*	0·541*	0·529*
$r_c(-2)$	−0·024	−0·066	0·019
g	−0·175	−0·181	−0·170
Δr	0·017*	0·019*	0·017*
$\Delta r(-1)$	−0·008	−0·010	−0·008
$\Delta r(-2)$	−0·006	−0·004	−0·007
$\Delta r(-3)$	0·021*	0·020*	0·024*
$\Delta r(-4)$	0·004	0·002	0·003
$r_c(-3)$		0·058	
$r_c(-4)$		−0·165	
$g(-2)$		−0·018	
$\Delta r(-5)$		0·028*	
$\Delta r(-6)$		−0·009	
R^2	0·304	0·402	
S	0·049	0·047	0·049
F		4·06*	
α		−0·12	−0·08
χ^2			20·40*

TABLE VII

Tin forward: interest rate changes as the independent variable

Variable	A	URTF	RTF
$r_c(-1)$	0·403*	0·418*	0·378*
$r_c(-2)$	0·164	0·036	0·353*
g	−0·051	0·016	−0·006
Δr	0·013*	0·012*	0·012*
$\Delta r(-1)$	0·002	0·001	0·002
$\Delta r(-2)$	−0·005	−0·002	−0·007
$\Delta r(-3)$	0·012*	0·012*	0·014*
$\Delta r(-4)$	−0·002	−0·007	−0·003
$r_c(-3)$		0·111	
$r_c(-4)$		0·045	
$g(-2)$		0·096	
$\Delta r(-5)$		0·015*	
$\Delta r(-6)$		−0·004	
R^2	0·273	0·346	
S	0·034	0·032	0·033
F		2·74*	
α		−0·32	−0·306*
χ^2			8·50

Assuming $\sum_{i=4}^{9} A_i = 0$ and since $\Delta r(t) = r(t) - r(t-1)$, etc., we have the following relationships among the coefficients:

$$B_4 = A_4,$$

$$B_5 = A_4 + A_5,$$

$$B_6 = A_4 + A_5 + A_6,$$

$$B_7 = A_4 + A_5 + A_6 + A_7,$$

$$B_8 = A_4 + A_5 + A_6 + A_7 + A_8,$$

TABLE VIII

Zinc forward: interest rate changes as the independent variable

Variable	A	URTF	RTF
$r_c(-1)$	0·569*	0·589*	0·594*
$r_c(-2)$	−0·071	−0·167	0·072
g	−0·143	−0·088	−0·107
Δr	0·030*	0·027*	0·029*
$\Delta r(-1)$	−0·014	−0·014*	−0·013
$\Delta r(-2)$	−0·005	−0·002	−0·009
$\Delta r(-3)$	0·022*	0·014	0·020*
$\Delta r(-4)$	0·016*	0·012	0·012
$r_c(-3)$		0·217*	
$r_c(-4)$		−0·002	
$g(-2)$		0·141	
$\Delta r(-5)$		0·003	
$\Delta r(-6)$		0·013	
R^2	0·451	0·498	
S	0·048	0·047	0·048
F		2·33	
α		−0·22	−0·26*
χ^2			8·61

and

$$B_8 = -A_9.$$

If we look at the case of zinc we find this property holds almost perfectly: (Tables IV and VIII).

i	B_i	$\sum_{j=4}^{4+i} A_j$
4	0·029	0·030
5	−0·013	−0·013
6	−0·009	−0·009
7	0·020	0·020
8	0·012	0·012

and $B_8 = -A_9 = 0{\cdot}012$. Similar results hold for the other resources.

It therefore appears that interest rate changes and not interest rates themselves are the relevant independent variables, and hence that a constant interest rate implies more or less constant resource prices.

We also briefly examined the model with inter-action between different resource markets as well as between resource and capital markets. This involved incorporating the price of tin in the demand for copper equation, on the assumption that price movements observed in one market might influence expectations in the other, thus including the proportional rate of increase of the tin price (and its lagged values) on the RHS of equation (6). These results are presented in Table IX. They are not particularly encouraging as the inclusion of the tin price variables removes a lot of the explanatory power of the lagged values of the copper price variable, a somewhat counter-intuitive result. We have not followed up this approach therefore, but we do not rule out its possibilities, especially with

TABLE IX

Showing the effects of resource price interaction: copper and tin

Variable	A	URTF	RTF
$r_c(-1)$	0·259	0·305	−0·095
$r_c(-2)$	−0·205*	−0·250	−0·527*
g	0·008	−0·024	−0·030
r	0·010	0·008	0·005
$r(-1)$	−0·012	−0·011	−0·005
$r(-2)$	0·000	−0·004	−0·006
$r(-3)$	0·008	0·011	0·012
$r(-4)$	−0·001	0·006	−0·004
$r(-5)$	−0·005	−0·028	−0·004
r'_c	0·755*	0·820*	0·948*
$r'_c(-1)$	0·036	0·023	0·259
$r'_c(-2)$	0·209	0·356	0·823*
$r_c(-3)$		0·207	
$r_c(-4)$		0·055	
$g(-2)$		0·089	
$r(-6)$		0·024	
$r(-7)$		−0·006	
$r'_c(-3)$		−0·244	
$r'_c(-4)$		−0·490	
S	0·062	0·058	0·064
χ^2	23·50*	1·60	14·03
$\sum r$	−0·00117	−0·00042	−0·00265
α			0·466*

some of the other resources, since copper does not appear to perform well in this type of model.

5. ALTERNATIVE EXPLANATIONS

There are in fact a number of possible reasons why we might expect to find changes in interest rates to be relevant explanatory variables, rather than their levels, and we examine these alternatives in more detail here.

First we show that the connection between resource price movements and interest rate changes can be obtained from any demand and supply equation of a certain form. Suppose we have an equilibrium model of the following form:

$$S(p, r_c, r) = D(p, r_c, r)$$

where p, r_c and r are as previously defined. Differentiating with respect to time, we have:

$$\frac{\partial S}{\partial p}\dot{p} + \frac{\partial S}{\partial r_c}\dot{r}_c + \frac{\partial S}{\partial r}\dot{r} = \frac{\partial D}{\partial p}\dot{p} + \frac{\partial D}{\partial r_c}\dot{r}_c + \frac{\partial D}{\partial r}\dot{r}.$$

Dividing through by p and rearranging:

$$r_c\left(\frac{\partial S}{\partial p} - \frac{\partial D}{\partial p}\right) + \frac{\dot{r}_c}{p}\left(\frac{\partial S}{\partial r_c} - \frac{\partial D}{\partial r_c}\right) + \frac{\dot{r}}{p}\left(\frac{\partial S}{\partial r} - \frac{\partial D}{\partial r}\right) = 0$$

$$\Rightarrow r_c\left(\frac{\partial S}{\partial p}\cdot\frac{p}{S}\cdot - \frac{\partial D}{\partial p}\cdot\frac{p}{S}\right)\frac{S}{p} + \dot{r}_c\left(\frac{\partial S}{\partial r_c}\frac{r_c}{S} - \frac{\partial D}{\partial r_c}\cdot\frac{r_c}{S}\right)\frac{S}{r_c p} + \dot{r}\left(\frac{\partial S}{\partial r}\cdot\frac{r}{S} - \frac{\partial D}{\partial r}\cdot\frac{r}{S}\right)\frac{S}{rp} = 0.$$

Writing $\eta_\beta^\propto$ for the (assumed constant) elasticity of $\propto$ w.r.t. β, we have:

$$r_c(\eta_p^S - \eta_p^D) + \frac{\dot{r}_c}{r_c}(\eta_{r_c}^S - \eta_{r_c}^D) = \frac{\dot{r}}{r}(\eta_r^D - \eta_r^S).$$

We thus have a relationship between the rate of resource price change and proportional rate of change of the interest rate. This indicates that our earlier model is a particular example of a more general class of models incorporating resource prices and relative rates of return in their demand and supply equations. Note that we could include other parameters, such as income, into the model without altering the basic structure. Note also that in this model we cannot determine the direction of effect of changes in the interest rate upon changes in resource prices since we do not know the size of the relevant elasticities.

An alternative way of accounting for the relevance of interest rate changes is to assume that investors are concerned more with the possibility of capital gains on their investment rather than with the interest that would accrue. Suppose for example, that the investor has the opportunity of purchasing at time t a non-interest-bearing bond which will be redeemed at T for a price $B(T)$. The price at t will then be determined according to:

$$\frac{B(T)-B(t)}{B(t)} = (T-t)\cdot r. \qquad \text{...(10)}$$

Where r is the one period interest rate and therefore has to be multiplied by $(T-t)$, the life of the bond (for the sake of simplicity we have ignored the effect of compounding, which should not be very important on a 3 month bond, the type to which our results relate).

$$\frac{B(T)}{B(t)} = (T-t)\cdot r + 1$$

$$\frac{\dot{B}(t)}{B(t)} = \frac{r}{1+(T-t)\cdot r} - \frac{(T-t)\cdot \dot{r}}{1+(T-t)\cdot r}. \qquad \text{...(11)}$$

Thus, if the investor is comparing capital gains on natural resources to those on this other asset, this will establish a relationship between resource price movements and interest rate changes. As the bond approaches maturity, i.e. $t \to T$, the capital gain on the bond approaches the rate of interest. Note that in this formulation we can determine the direction of influence of $\dot{r}$, assuming that the rates of return on the two assets are forced into equality. Alternatively, we could explicitly include equation (11) in the model by incorporating it into equation (2). Thus instead of having $\dot{O}/O = r$ we would have $\dot{O}/O = \dot{B}/B$ (and $\dot{\tilde{O}}/\tilde{O} = \dot{\tilde{B}}/\tilde{B}$). This would obviously result in a very complex estimating equation instead of equation (5). However, we can see that such an equation, though complex in the parameters, would include a term incorporating r. As we have already seen, the empirical results strongly suggest that r should not be included in an equation such as (5) so on these grounds we do not favour this class of explanation.

Another possible explanation of why we find interest rates changes to be important is that this is the result of some statistical artefact. An argument might run along the following lines. Suppose we write our estimating equation in terms of the means of the variables, i.e.

$$\bar{r}_c(t) = A_1\bar{r}_c(t-1) + A_2\bar{r}_c(t-2) + A_3\bar{g}(t) + A_4\bar{r}(t) + A_5\bar{r}(t-1) + A_6\bar{r}(t-2) + A_7\bar{r}(t-3) + A_8\bar{r}(t-4) + A_9\bar{r}(t-5) + \bar{\varepsilon}_t.$$

Suppose further that the average rate of increase of resource prices has been approximately zero. The coefficient on the growth rate variable has also been found to be insignificantly different from zero. Hence the sum of the interest rates (multiplied by their respective coefficients) must equal zero. If the means of the interest rate variables are approximately equal, this implies that the sum of the coefficients must necessarily equal zero.

There are a number of reasons why this does not seem an adequate explanation of our results. As Table X shows, the average rate of increase of resource prices over the period has been significantly non-zero in all cases except copper (and copper has in general performed less well in our tests than the other resources). We have also tried running the same equations but including a constant to see if the same result obtains. This means that the interest rate coefficients no longer have to sum to zero even if $\bar{r}_c(t)=\bar{r}_c(t-1)=\bar{r}_c(t-2)=A_3=0$ and the interest rate means are sufficiently close together. The results showed that the interest rate coefficients still summed to zero in all four cases, and the constant was not significantly different from zero in any case.

TABLE X

Average rates of change of resource prices: July 1965–June 1977

Estimates of $r_c=(\dot{p}/p)$ per cent

Resource	Monthly	Annual equivalent
Copper	0·4	4·9
Lead	1·3*	16·8
Tin	2·2*	29·8
Zinc	1·3*	16·8

* Indicates significantly different from zero at the 95 per cent confidence level.

Even if it were in fact correct that $\bar{r}_c$ were equal to zero, this would have important implications. If $\bar{r}_c=0$, there must be some economic mechanism which is generating data with this property, and which clearly does not conform to simple equilibrium theories. The present asset-market equilibrium type models predict that $\tilde{r}_c=\bar{r}$, while this line of argument denies any connection between resource price movements and interest rates (in either levels or changes form) and must therefore imply some sort of long-run disequilibrium in asset markets. For the above reasons, then, we tend to reject the "statistical artefact" line of argument.

The most unsatisfactory part of our model is the implication that if interest rates are constant then the rate of change of resource prices must be zero, a conclusion which contradicts the simple asset market equilibrium arguments. One way out of this dilemma is to posit that there should be a feedback mechanism between resource prices and interest rates. Hendry has shown that a model incorporating a feedback mechanism between dependent and explanatory variables can, under certain circumstances, be characterized by an equation involving differences in the explanatory variables alone (see Hendry (1978)). It could be, therefore, that our results are due to mis-specification of the estimating equation. However, it is extremely difficult to obtain an estimating equation involving interest rate levels and some form of error correcting mechanism from our original arbitrage model. We therefore consider the following model:

$\tilde{r}_c=\tilde{r}$ (i.e. resource prices are expected to grow at the rate of interest) ...(12)

with expectations formed by:

$$\tilde{r}_c=\frac{a_1^2 r_c}{(D+a_1)^2} \qquad \ldots(13)$$

and

$$\tilde{r} = \frac{a_2^2 r}{(D+a_2)^2}. \qquad \text{...(14)}$$

Substituting (13) and (14) into (12) yields

$$a_1^2 \ddot{r}_c + 2a_1^2 a_2 \dot{r}_c + a_1^2 a_2^2 r_c = a_2^2 \ddot{r} + 2a_1 a_2^2 \dot{r} + a_1^2 a_2^2 r. \qquad \text{...(15)}$$

Transforming into discrete time one obtains

$$r_c(t) = \left(\frac{2a_1^2 - 2a_1^2 a_2}{a_1^2}\right) r_c(t-1) + \left(\frac{2a_1^2 a_2 - a_1^2}{a_1^2}\right) r_c(t-2) + \frac{a_2^2}{a_1^2} r(t) + \left(\frac{-a_2^2 + 2a_1 a_2^2 - a_2^2}{a_1^2}\right) r(t-1)$$
$$+ \left(\frac{a_2^2 - 2a_1 a_2^2}{a_1^2}\right) r(t-2) + \left(\frac{a_1^2 a_2^2}{2a_1^2}\right)(r(t-1) - r_c(t-1)) + \left(\frac{a_1^2 a_2^2}{2a_1^2}\right)(r(t-2) - r_c(t-2)). \qquad \text{...(16)}$$

We have thus obtained an equation similar to the one from the arbitrage model except that we now have an error correcting mechanism through which the difference between r and r_c feeds back to r_c (the income variable, previously found to be unimportant, is also absent). The coefficients on $r(t)$, $r(t-1)$ and $r(t-2)$ again sum to zero, so the equation may again be written in terms of interest rate differences, as follows:

$$r_c(t) = \left(\frac{2a_1^2 - 2a_1^2 a_2}{a_1^2}\right) r_c(t-1) + \left(\frac{2a_1^2 a_2 - a_1^2}{a_1^2}\right) r_c(t-2) + \left(\frac{a_2^2}{a_1^2}\right)(r(t) - r(t-1))$$
$$+ \left(\frac{2a_1 a_2^2 - a_2^2}{a_1^2}\right)(r(t-1) - r(t-2)) + \left(\frac{a_1^2 a_2^2}{2a_1^2}\right)(r(t-1) - r_c(t-1))$$
$$+ \left(\frac{a_1^2 a_2^2}{2a_1^2}\right)(r(t-2) - r_c(t-2)). \qquad \text{...(17)}$$

Various forms of this equation were tested, incorporating various different lag structures. For comparability with earlier results we had up to five lags on the interest rate variable, and for simplicity we included only one error correcting mechanism (e.c.m.) at a time. Thus the actual estimating equation turned out to be:

$$r_c(t) = \textstyle\sum_{i=1, i\neq j}^{2} A_i r_c(t-i) + A_3 g + \sum_{i=0, i\neq j}^{5} A_{i+4} r(t-i)$$
$$+ A_{10}(r_c(t-j) - r(t-j)), \qquad j = 1, \ldots, 4. \qquad \text{...(18)}$$

The interesting aspects of this equation are again the properties of the interest rate coefficients. We tested so-called "short" and "long" hypotheses. The former hypothesizes that $\sum_{i=0, i\neq j}^{5} A_{i+4} = 0$, the latter that $\sum_{i=0}^{5} A_{i=4} = 0$. The implications are as follows. If the short hypothesis is accepted, then equation (18) may be written in terms of interest rate differences and an e.c.m. If the long hypothesis is accepted then it can be written either in terms of interest rate differences alone (with no e.c.m.) or in terms of interest rate levels and an e.c.m. Acceptance of neither hypothesis implies that the equation is as written above, equation (18).

From the point of view of the basic theory, this is a slightly more satisfying model in that it is possible to obtain interest rate differences as explanatory variables but to have a determinate long run rate of price change through the effect of the e.c.m.

The results obtained from running these equations are somewhat ambiguous in so far as no uniform pattern of behaviour emerges. Imposing the restriction that an e.c.m. is present proves valid in the cases of the second and (for some resources) fourth order lags. Tables XI–XIV show the results of including a second order lag restriction. Comparison with Tables I–IV (where no such restriction is included) shows many similarities: sizes and signs of coefficients, levels of significance, R^2-statistics, etc. The problem is that although it is valid to impose a second order lag restriction it is not a very strong restriction, since the coefficients of $r_c(-2)$ and $r(-2)$ individually are generally insignificant. Hence equations (10)–(13) show evidence of only a very weak e.c.m. at work, if any.

Evidence for the long and short hypotheses is also somewhat ambiguous. Both of the hypotheses are valid for copper, lead and zinc. For tin, both hypotheses are rejected. There is very little therefore that one can say with any certainty. Experimentation with alternative lag structures might provide better results, though we have found the second order lag restriction to be most promising. The data and methods used do not allow us to discriminate between the alternative models and hypotheses put forward. However, it does appear that a model of resource price movements including only interest rate levels must be rejected as inadequate, and that a better model would involve interest rate differences (with or without an e.c.m.) or interest rate levels and an e.c.m.

TABLE XI

Copper forward: error correction mechanism incorporated

Variable	A	URTF	RTF
$r_c(-1)$	0·438*	0·519*	0·505*
$(r_c(-2)-r(-2))$	−0·177*	0·354	0·057
g	−0·024	−0·024	0·011
r	1·795*	1·648	1·408
$r(-1)$	−1·989	−1·507	−1·651
$r(-2)$	—	—	—
$r(-3)$	1·630	1·746	1·383
$r(-4)$	−1·260	−1·287	−1·352
$r(-5)$	−0·314	−0·772	−0·719
$r_c(-2)$		−0·682	
$r_c(-3)$		0·301*	
$(r_c(4)-r(-4))$		−0·124	
$g(-2)$		0·089	
$r(-6)$		1·685	
$r(-7)$		−1·271	
R^2	0·212	0·294	
S	0·067	0·065	0·066
$\sum r^*$	−0·138	0·242	0·069
$\sum r^{**}$	0·039	0·012	0·012
F		2·42*	
$\propto$		−0·32	−0·37*
χ^2			7·58

6. CONCLUSIONS

The research reported here is based on a premise underlying most theoretical analyses of markets for exhaustible resources, namely that such resources are best viewed as assets

TABLE XII

Lead forward: error correction mechanism incorporated

Variable	A	URTF	RTF
$r_c(-1)$	0·508*	0·501*	0·512*
$(r_c(-2)-r(-2))$	−0·038	−0·398	−0·006
g	−0·178	−0·201	−0·176
r	1·730*	1·970*	1·737*
$r(-1)$	−2·412*	−2·816*	−2·446*
$r(-2)$	—	—	—
$r(-3)$	2·786*	2·497*	2·997*
$r(-4)$	−1·736	−1·956	−1·927
$r(-5)$	−0·341	2·393*	−0·307
$r_c(-2)$		0·332	
$r_c(-3)$		0·038	
$(r_c(-4)-r(-4))$		−0·172*	
$g(-2)$		−0·037	
$r(-6)$		−2·906*	
$r(-7)$		0·327	
R^2	0·311	0·405	
S	0·049	0·047	0·049
$\sum r^*$	0·027	−0·491	0·054
$\sum r^{**}$	0·065	0·079	0·060
F		3·25*	
$\propto$		−0·02	−0·066
χ^2			19·87*

TABLE XIII

Tin forward: error correction mechanism incorporated

Variable	A	URTF	RTF
$r_c(-1)$	0·368*	0·379*	0·353*
$(r_c(-2)-r(-2))$	0·110	0·339	0·292*
g	−0·063	0·001	−0·032
r	1·344*	1·294*	1·233*
$r(-1)$	−1·382*	−1·156	−1·294*
$r(-2)$	—	—	—
$r(-3)$	1·332*	1·464*	1·748*
$r(-4)$	−1·380	−1·798*	−1·665*
$r(-5)$	0·280	2·024*	0·323
$r_c(-2)$		−0·318	
$r_c(-3)$		0·071	
$(r_c(-4)-r(-4))$		0·034	
$g(-2)$		0·081	
$r(-6)$		−1·449*	
$r(-7)$		0·059	
R^2	0·293	0·353	
S	0·033	0·032	0·033
$\sum r^*$	0·194	0·438	0·345
$\sum r^{**}$	0·084	0·065	0·053
F		1·91	
$\propto$		−0·22	−0·27*
χ^2			8·79

TABLE XIV

Zinc forward: error correction mechanism incorporated

Variable	A	URTF	RTF
$r_c(-1)$	0·546*	0·586*	0·581*
$(r_c(-2)-r(-2))$	−0·064	−1·168	0·072
g	−0·140	−0·093	−0·106
r	3·023*	2·754*	2·967*
$r(-1)$	−3·965*	−4·144*	−4·058*
$r(-2)$	—	—	—
$r(-3)$	3·071*	1·730	3·162*
$r(-4)$	−0·570	−0·245	−0·712
$r(-5)$	−1·588*	−0·999	−1·272
$r_c(-2)$		0·984	
$r_c(-3)$		0·210*	
$(r_c(-4)-r(-4))$		0·002	
$g(-2)$		0·131	
$r(-6)$		−0·992	
$r(-7)$		0·749	
R^2	0·45	0·49	
S	0·048	0·047	0·048
$\sum r^*$	−0·029	−1·147	0·087
$\sum r^{**}$	0·035	0·019	0·015
F		1·69	
$\propto$		−0·22	−0·25*
χ^2			7·21

whose yields are their rates of price appreciation. As immediate corollary of such a view is that decisions on whether to acquire or dispose of such assets will be influenced by expectations about the rates of return obtainable on them relative to those obtainable on other assets. In particular, one would expect that in a perfectly-informed market in equilibrium, those rates of return would be forced into equality, and that in general the relationship between them would be an important determinant of behaviour in the market.

There is no question that the results reported here appear to confirm, at least in very general terms, such a view of the world. This view suggests that resource price movements should be related to returns on other assets, and the existence of such a relationship is clearly supported by the data. The precise details of the relationship are certainly not those suggested by theories of markets with perfect information in equilibrium. However, this should not of itself be surprising: traders in the markets studied clearly do not have access to perfect information about the future, but have rather to base their decisions on expectations which must in essence be based on past observations. The models analysed here suggest that in such situations, the relationships that will be established between resource price movements and the returns on other assets are more complex than those that emerge from the full-information equilibrium models of Hotelling and his successors, and their suggestions appear to receive corroboration.

APPENDIX ON NON-LINEAR ESTIMATION

As we noted in Section 3, the coefficients A_i of the reduced form (6) are complex non-linear functions of the parameters of the original model, and this implies, *inter alia*, that there must be certain relationships between these coefficients. Fortunately one of these relationships, from an economic point of view the most interesting one, has a very simple form, and so in the main part of the text we have proceeded by estimating the reduced form without placing restrictions on the coefficients, and then investigating subsequently whether the coefficients so estimated satisfy this particular adding-up restriction. This they have always done, to a high degree of accuracy. However, there is clearly an alternative approach, which is to use non-linear estimation techniques to estimate the parameters of the structural form directly, and we now report briefly on the results of this. In fact some of the parameters of the original model are under identified, and instead of being able to estimate all six parameters $\eta(p)$, $\eta(y)$, a_1, a_2, a_3, a_4 it is possible only to estimate

$$\theta_1 = a_1/(a_1 + a_4 - \eta\{p\})$$
$$\theta_2 = a_2$$
$$\theta_3 = a_3$$
$$\theta_4 = \eta(y)/(a_1 + a_4 - \eta\{p\}).$$

Hence the two lag coefficients of the expectation formation equations can be estimated directly, but the remaining parameters can only be identified in combinations. A non-linear autoregressive maximum likelihood procedure was used: its theoretical basis is to be found in Hendry (1971)), who also developed the programme, GENRAM.

The results of estimates for copper, lead and zinc over the period December 1965 to December 1973 are given below:

	Copper	Lead	Zinc
$\theta_1 = \dfrac{a_1}{a_1 + a_4 - \eta(p)}$	0·028	0·018	0·044
$\theta_2 = a_2$	0·716	0·888	1·068
$\theta_3 = a_3$	0·549	0·425	0·377
$\theta_4 = \dfrac{\eta(y)}{a_1 + a_4 - \eta(p)}$	0·040	0·021	−0·109

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