The relationship between the Dang Van criterion and the traditional bulk fatigue criteria

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Abstract: A formal relationship is established between the traditional bulk fatigue criteria such as the Goodman rule and the Sines criterion, and the recent crack initiation criterion of Dang Van. The constants implied by the Dang Van procedure may be formally connected to the fatigue limit and mean stress dependence given by the Goodman rule, under conditions of uniaxial loading. The biaxial fatigue criterion of Sines is also compared with the Dang Van procedure. The similarity in the approaches may be further extended to permit finite initiation times to be estimated by the Dang Van criterion when the fatigue limit is exceeded.

Keywords: crack initiation, the Dang Van criterion, the Goodman rule, fatigue, mean stress effects, *R*-ratio effects, the Sines criterion

NOTATION

a	real constant
A	real constant
A_i	amplitude of temporal variation in the macroscopic stress component Σ_i
b	real constant
B	real constant
$I_{1_{\text{stat}}}$	first invariant of the static component of the stress tensor
J_2	second invariant of the deviatoric stress tensor
$J_{2_{\text{alt}}}$	second invariant of the alternating component of the deviatoric stress tensor
R	ratio defined as the minimum stress to the maximum stress
s_i	microscopic deviatoric stress principal component i
$\boldsymbol{s}$	microscopic deviatoric stress tensor
S_i	macroscopic deviatoric stress principal component i
$\boldsymbol{S}$	macroscopic deviatoric stress tensor
t	time
α	real constant
β	real constant
μ	real constant
$\boldsymbol{\rho}^*$	time-independent residual stress tensor
$\overline{\sigma}$	hydrostatic pressure
σ_*	real constant
σ_{a}	alternating stress
σ_i	principal stress i in the context of the Lode parameter
σ_{m}	mean (static) stress
σ_{UTS}	ultimate tensile strength
$\boldsymbol{\sigma}$	microscopic stress tensor
$\overline{\sigma}_{\text{stat}}$	hydrostatic pressure of the static stress component
σ_0	real constant
σ_{Y}	yield stress
Σ_i	macroscopic principal stress i
$\Sigma_{i_{\text{alt}}}$	alternating macroscopic principal stress i
$\Sigma_1^{\max}, \Sigma_1^{\min}$	maximum and minimum values respectively of the macroscopic stress in the uniaxial case
$\boldsymbol{\Sigma}$	macroscopic stress tensor
$\tau_{\max}$	maximum shear stress
$\tau_{\max_{\text{alt}}}$	maximum shear stress of the alternating component

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1 INTRODUCTION

The traditional 'bulk' approaches to fatigue attempt to correlate the fatigue strength of a material with the stress state present at the critical point, where a crack initiates and grows. An experimentally determined relationship is found between the pre-existing stress present, and the number of cycles of loading experienced, up to the point of failure. This approach, leading to the so-called *S–N* diagrams, was started by Wöhler in the 1850s, and modest developments were made over the subsequent 75 years. The approach is still used today in some industries, but the rise of crack propagation studies, and the powerful connection established between crack tip stress intensity factor range and propagation rate, meant that design against fatigue switched to this approach in the last quarter of the twentieth century. Now, work has progressed further; increasingly strong materials have brought higher working stresses and, with these, relatively high rates of crack propagation so that, once a crack has initiated, the number of cycles to failure is rather short in many safety-critical applications. This has meant that a crack *initiation* criterion is now regarded as an essential development. Research in this field was originally focused on the short-crack regime, in an attempt to extrapolate fracture mechanics ideas backwards [**1**, **2**]. This technique is helpful, but the alternative approach is to consider some combination of pre-existing stresses at the critical point, and to decide whether they are sufficient to cause a crack to nucleate. There are the critical plane approaches, such as the Fatemi–Socie [**3**] and Smith–Watson–Topper [**4**] methods, and a variation on these themes, due to Dang Van *et al.* [**5**]. The Dang Van criterion has a more physically based underpinning and is based on the idea that plasticity at the grain level may be present, even when the bulk macroscopic stress is elastic, because of the localization effect of individual grains. Dang Van *et al.* noted that shakedown can occur at the grain level and stated that a crack will initiate only when, in the steady state, plasticity still exists at the grain level. Conversely, no crack will initiate if shakedown occurs at the grain level.

2 THE GOODMAN RULE AND THE DANG VAN CRITERION: SIMILARITIES AND DIFFERENCES

In an investigation carried out on the experiments available to calibrate the Dang Van criterion [**6**], it was noted that a behaviour similar to that described by the Goodman rule is predicted; for the component to have an infinite life, as the R ratio increases, the allowable stress range decreases. The Goodman rule is of course empirical but nevertheless well established; it has been used widely and with great success since its appearance in the 1930s and indeed still features prominently in fatigue applications. Variations on the Goodman theme are available, e.g. those by Soderberg* or Gerber† [**7**]. The present paper demonstrates the equivalence of the Dang Van criterion and the Goodman rule. The implied relationship between the Dang Van and the Goodman parameters is examined, in an attempt to link the two and thus to provide a passage from one to the other. The issue of separating initiation life from total life should be noted here. Dang Van *et al.* dealt only with crack initiation times in high-cycle fatigue applications, where the number of cycles to cause propagation to failure is often negligible compared with the number of cycles needed to complete initiation, and hence the use of the criterion for total life is valid. The Goodman rule, on the other hand, deals with total life and hence does not give specific information about the two phases of fatigue damage. To give a simple example, if there were two round bars each having the same surface stress, one subjected to oscillatory bending and the other to fluctuating bulk tension, a straightforward application of the Goodman rule would predict the same life to failure. It is clear that, in reality, although the initiation times for the two problems will be similar, the propagation times for the two problems will be very different, and this would not be reflected in a 'total life' approach.

A second observation concerns multiaxiality. The Goodman rule is uniaxial, but there exists a criterion given by Sines and Ohgi [**8**] concerned with two and possibly three dimensions. This criterion was historically one of the first multiaxial fatigue criteria to follow the Goodman rule. The great advantage of the Dang Van *et al.* approach is that it is equally suited to uniaxial and complex multiaxial stress states. A link between the Sines criterion and the Dang Van criterion therefore not only would further demonstrate the equivalence of the two approaches as seen in the uniaxial case but also would enable simplifications in both. It would also provide the opportunity to test the critical quantities for crack initiation assumed by Dang Van *et al.* (maximum shear stress and hydrostatic pressure) against data such as that presented in reference [**8**]. At first sight there appears to be a discrepancy: Sines and Ohgi used the von Mises stress as the controlling parameter, whereas Dang Van *et al.* used the maximum shear stress. Further investigation presented below, however, indicates that the two quantities give very similar results.

The third observation concerns finite life predictions. The Dang Van criterion only deals with the boundary between infinite and finite life. The Goodman rule and the Sines criterion, however, are applicable to any finite life. The mapping, therefore, between the two provides a way of extending the Dang Van *et al.* approach to predict specific cyclic lives as well, thus broadening the use of the criterion [**9**].

3 RELATIONSHIP BETWEEN THE DANG VAN CRITERION AND THE GOODMAN RULE: UNIAXIAL STRESS STATE

In their approach, Dang Van *et al.* avoided the difficult task of attempting to find the stress localization tensor

* The Soderberg line is a straight line identical with that of Goodman apart from the use of σ_Y instead of σ_{UTS}.

† Gerber used a segment of a parabola which has the same end points as the Goodman line.

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or the individual yield properties of the grain in order to investigate shakedown at the grain level and instead calibrated their criterion using simple experiments [5]. Starting from the bulk elastic stress state, they found the residual stresses which minimize the tendency to yield at the point experiencing the severest state of stress (pseudo-shakedown calculation) and then compared the properties of this resultant stress state with an experimentally obtained fatigue limit. There is experimental evidence that the crack initiation environment is sensitive to both the range of shear stress τ_{max} experienced, at the key point, and the hydrostatic stress $\bar{\sigma}$ present. They therefore assumed that a crack will initiate when some linear combination of these quantities *measured at the grain level* reaches a critical value. The optimized microscopic (grain level) load path at the critical point is thus traced in τ_{max}–$\bar{\sigma}$ space and then checked against the fatigue line which has been calibrated using simple experiments. The form of the fatigue limit assumed is

$$\tau_{max} + a\bar{\sigma} = b, \qquad a, b \in \mathbb{R} \tag{1}$$

Here, it is argued that 'no initiation' on the Dang Van criterion corresponds to infinite life on a classical bulk criterion, as it is assumed that a crack which initiates when there are only moderate stress gradients present will propagate to failure. At its simplest, therefore, no crack initiation means that the stress state lies below the fatigue limit, if one exists, on a traditional criterion.

As part of the investigation carried out in reference [6], the effect of varying the R ratio (where $R = \Sigma_1^{min}/\Sigma_1^{max}$) of the loading was examined. It was discovered that, as the R ratio increases, the allowable stress range has to be reduced in order for the load path to remain within the fatigue limit. This is shown schematically in Fig. 1 for a cyclic variation in a uniaxial stress state, where the locus traced by the load path in τ_{max}–$\bar{\sigma}$ space is a set of V shapes. The overall size of the V shapes is controlled by the stress amplitude, whereas the position of their vertex on the $\bar{\sigma}$ axis is determined by the R ratio. As the R ratio increases, the stress amplitude has to decrease in order for the right-hand side tip of the V to fall on the fatigue line. A similar behaviour has long been observed and quantified using the Goodman rule, according to which the alternating stress σ_a of the oscillation has to decrease as the mean stress σ_m increases. Figure 2 presents the Goodman diagram. The σ_m-axis intercept is customarily chosen to be equal to either the yield stress σ_Y of the material (Soderberg line) or the ultimate tensile strength σ_{UTS} (Goodman line). It can, therefore, be seen that both the Dang Van and Goodman approaches clearly predict similar behaviours. The following analysis provides a simple means of deducing the parameters of one of the two criteria if those of the other are known.

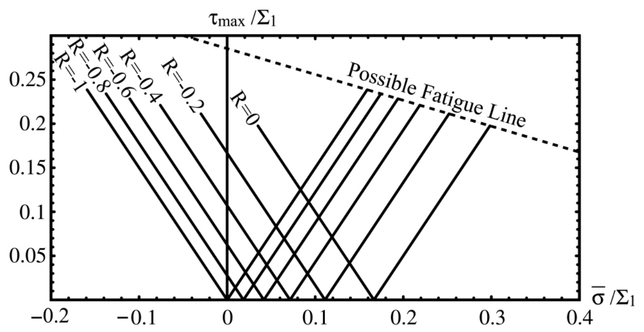

Fig. 1 Mean stress effect in the Dang Van *et al.* space

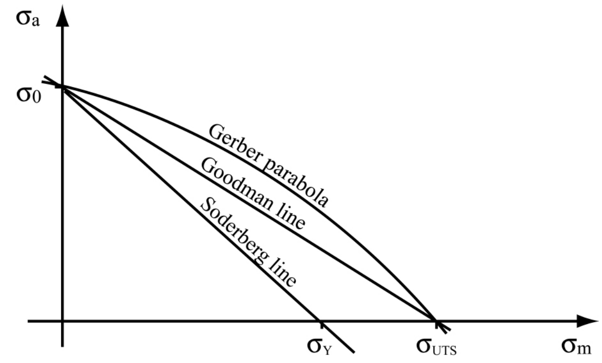

Fig. 2 The Goodman line and variations

3.1 The Dang Van and the Goodman parameters

Consider the problem of a uniaxial bulk stress state of the form $[\Sigma_1, 0, 0]$ fluctuating between Σ_1^{min} and Σ_1^{max}, with R as defined above. The Dang Van *et al.* analysis begins by considering the passage from the macroscopic (bulk) to the microscopic (grain level) stress state. A sinusoidal temporal variation is assumed:

$$\Sigma_1 = \frac{A_1}{2}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \tag{2}$$

The first stage of the passage is to find a time-independent residual stress state $\boldsymbol{\rho}^*$, which when superimposed on the macroscopic stress state $\boldsymbol{\Sigma}$, minimizes the tendency of the grain to yield. The resultant stress state is termed the microscopic stress state $\boldsymbol{\sigma}$:

$$\boldsymbol{\sigma} = \boldsymbol{\Sigma} + \boldsymbol{\rho}^* \tag{3}$$

Only the deviatoric components need be considered, as only they affect yield. The hydrostatic components are hence preserved, and the residual stress field may be considered to be purely deviatoric:

$$\boldsymbol{\sigma} = \boldsymbol{\Sigma} + \mathrm{dev}(\boldsymbol{\rho}^*) \tag{4}$$

As has been explained in more detail in reference [5], the determination of the optimal residual stress state is achieved by finding the centre of the smallest hypersphere in six-dimensional *deviatoric* stress space $(s_{11}/\sqrt{2},\ s_{22}/\sqrt{2},\ s_{33}/\sqrt{2},\ s_{12},\ s_{13},\ s_{23})$, which completely encompasses the load path. The load path, being straight in this case, is simply a diameter of the hypersphere. The hydrostatic pressure is given by

$$\overline{\sigma}(t) = \frac{A_1}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \tag{5}$$

and so the macroscopic deviatoric stresses are

$$\begin{aligned} S_1 &= \frac{A_1}{3}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \\ S_2 = S_3 &= -\frac{A_1}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \end{aligned} \tag{6}$$

The residual stresses are therefore given by

$$\mathrm{dev}(\boldsymbol{\rho}^*) = \left[-\frac{A_1}{3}\frac{1+R}{1-R}, \frac{A_1}{6}\frac{1+R}{1-R}, \frac{A_1}{6}\frac{1+R}{1-R}\right] \tag{7}$$

The microscopic deviatoric components now are

$$\begin{aligned} s_1 &= \frac{A_1}{3}\ \sin(\omega t) \\ s_2 = s_3 &= -\frac{A_1}{6}\ \sin(\omega t) \end{aligned} \tag{8}$$

and hence the resulting maximum shear stress and hydrostatic pressure are

$$\begin{aligned} \overline{\sigma}(t) &= \frac{A_1}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \\ \tau_{\max}(t) &= \frac{1}{2}\,\mathrm{Tresca}\ (\boldsymbol{s}) = \left|\frac{A_1}{4}\ \sin(\omega t)\right| \end{aligned} \tag{9}$$

which define parametrically the V shapes depicted in Fig. 1. The resulting maximum shear stress and hydrostatic pressure at the peak (right-hand side tip of the V) of an oscillation of amplitude Σ and stress ratio R are therefore

$$\begin{aligned} \overline{\sigma} &= \frac{A_1(1+R)}{6(1-R)} \\ \tau_{\max} &= \frac{|A_1|}{4} \end{aligned} \tag{10}$$

The fatigue limit according to Dang Van *et al.* is defined by the straight line

$$\tau_{\max} + a\overline{\sigma} = b \tag{11}$$

while the Goodman line is given by

$$\sigma_a = \sigma_0\left(1 - \frac{\sigma_m}{\sigma_*}\right) \tag{12}$$

The value of σ_* can be set to σ_Y or σ_{UTS} (or to an intermediate value; see the earlier footnote) according to the demands of the design. The relationships between the alternating and mean stresses, and the stress range and R ratio are simply

$$\begin{aligned} \sigma_a &= \frac{A_1}{2} \\ \sigma_m &= \frac{A_1(1+R)}{2(1-R)} \end{aligned} \tag{13}$$

Now, from equations (10), (11) and (13) the relationship between the alternating and mean stresses implied by the Dang Van criterion may be found:

$$\sigma_a = \frac{6b}{3+2a}\left(1 - \frac{\sigma_m}{3b/a}\right) \tag{14}$$

This means that in order for the Dang Van criterion and the Goodman rule to coincide, the necessary relationships between the constants are

$$\begin{aligned} \sigma_0 &= \frac{6b}{3+2a} \\ \sigma_* &= \frac{3b}{a} \end{aligned} \tag{15}$$

The solution to the converse problem can also be found; i.e. solve for the Dang Van fatigue line constants in terms of the Goodman parameters. From equations (10), (12) and (13), the fatigue line is obtained as

$$\tau_{\max} + \frac{3\sigma_0}{2(\sigma_* - \sigma_0)}\overline{\sigma} = \frac{\sigma_*\sigma_0}{2(\sigma_* - \sigma_0)} \tag{16}$$

and the constants as

$$\begin{aligned} a &= \frac{3\sigma_0}{2(\sigma_* - \sigma_0)} \\ b &= \frac{\sigma_*\sigma_0}{2(\sigma_* - \sigma_0)} \end{aligned} \tag{17}$$

4 RELATIONSHIP BETWEEN THE DANG VAN CRITERION AND THE SINES CRITERION: BIAXIAL STRESS STATE

The case of a biaxial stress state of the form $[\Sigma_1, \Sigma_2, 0]$ where the principal directions do not rotate is now considered. In general, sinusoidal variations in the principal stress components, each with its own amplitude, frequency, R ratio and phase difference, would need to be considered. This case would cover biaxial tension–compression and combined tension–torsion. For the purposes of the Sines criterion, however, no phase difference between the two components is considered, the frequencies of oscillation are equal and the R ratios are equal too. The formulation needed is thus

$$\Sigma_1 = \frac{A_1}{2}\left[\frac{1+R}{1-R} + \sin(\omega t)\right]$$
$$\Sigma_2 = \frac{A_2}{2}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \tag{18}$$

4.1 The Dang Van analysis

As before, the analysis begins by finding the macroscopic deviatoric stress components. The hydrostatic pressure is given by

$$\overline{\sigma}(t) = \frac{A_1 + A_2}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \tag{19}$$

and hence the macroscopic deviatoric stresses by

$$S_1 = \frac{A_1}{3}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] - \frac{A_2}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right]$$
$$S_2 = -\frac{A_1}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] + \frac{A_2}{3}\left[\frac{1+R}{1-R} + \sin(\omega t)\right]$$
$$S_3 = -\frac{A_1}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] - \frac{A_2}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right] \tag{20}$$

The residual stresses are therefore given by

$$\text{dev}\ (\boldsymbol{\rho}^*) = \left[\left(-\frac{A_1}{3} + \frac{A_2}{6}\right)\frac{1+R}{1-R}, \left(\frac{A_1}{6} - \frac{A_2}{3}\right)\frac{1+R}{1-R}, \frac{A_1 + A_2}{6}\,\frac{1+R}{1-R}\right] \tag{21}$$

The microscopic deviatoric components now are

$$s_1 = \left(\frac{A_1}{3} - \frac{A_2}{6}\right)\sin(\omega t)$$
$$s_2 = \left(-\frac{A_1}{6} + \frac{A_2}{3}\right)\sin(\omega t)$$
$$s_3 = -\frac{A_1 + A_2}{6}\sin(\omega t) \tag{22}$$

and hence the resulting maximum shear stress and hydrostatic pressure are

$$\overline{\sigma}(t) = \frac{A_1 + A_2}{6}\left[\frac{1+R}{1-R} + \sin(\omega t)\right]$$
$$\tau_{\max}(t) = \tfrac{1}{2}\max(|s_1 - s_2|, |s_1 - s_3|, |s_2 - s_3|)$$
$$= \max\left(\frac{|A_1 - A_2|}{4}, \frac{|A_1|}{4}, \frac{|A_2|}{4}\right)|\sin(\omega t)| \tag{23}$$

In $\tau_{\max}-\overline{\sigma}$ space this represents a set of V shapes, the position and size of each V depending on the stress amplitudes and R ratio. As in the uniaxial case, the intersection of the right-hand side extremities of these V shapes with the fatigue limit provides the relationship between allowable stresses and R ratio. The fatigue limit is given by

$$\tau_{\max} + a\overline{\sigma} = b \tag{24}$$

and so its intersections with the extremities are given by setting $\sin(\omega t) = 1$ in equation (23) above:

$$\max\left(\frac{|A_1 - A_2|}{4}, \frac{|A_1|}{4}, \frac{|A_2|}{4}\right) + a\frac{A_1 + A_2}{6}\left(\frac{1+R}{1-R} + 1\right) = b \tag{25}$$

4.2 The Sines criterion

The Sines criterion is directly concerned with the relationship between stress amplitudes and R ratio. Stresses are split into their alternating and static components, and the proposed relationship for infinite life in conventional notation is [**8**]

$$\sqrt{J_{2_{\text{alt}}}} \leqslant A - BI_{1_{\text{stat}}}, \qquad A, B \in \mathbb{R} \tag{26}$$

where $J_{2_{\text{alt}}}$ and $I_{1_{\text{stat}}}$ are the second deviatoric stress invariant and hydrostatic pressure over the entire load path respectively. A first obvious difference between the two criteria is the use of J_2 instead of $\tau_{\max}$; in the Dang Van *et al.* case a linear relationship between $\tau_{\max}$ and hydrostatic pressure is assumed, whereas in the Sines case a linear relationship is similarly assumed between $\sqrt{J_2}$ and hydrostatic pressure. It can be demonstrated, however, that the use of either of these two quantities gives rise to very similar results. This is best done by making use of the Lode parameter, which was originally used to demonstrate the difference between the Tresca and the von Mises yield criteria.

The parameter is defined as follows:

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \tag{27}$$

where σ_1, σ_2 and σ_3 are the principal stresses in order of magnitude. The parameter is based on the effect of the intermediate principal stress and thus takes values from -1 to 1 as σ_2 varies from the smallest principal stress σ_3 to the largest σ_1. By assuming a linear relationship between $\tau_{\max}$ and hydrostatic pressure as in the Dang Van *et al.* case,

$$\tau_{\max} + a\overline{\sigma} = b \tag{28}$$

the parameter may be used to find the corresponding relationship between $\sqrt{J_2}$ and hydrostatic pressure:

$$\sqrt{J_2} = \frac{1}{\sqrt{3}}\sqrt{3b^2 - 6ab\overline{\sigma} + 3a^2\overline{\sigma}^2 + b^2\mu^2 - 2ab\mu^2\overline{\sigma} + a^2\mu^2\overline{\sigma}^2} \tag{29}$$

By choosing a wide range of values for the constants a and b, it can be seen from Fig. 3 that the curve obtained by plotting $\sqrt{J_2}$ against $\overline{\sigma}$ is remarkably close to the straight line defined by equation (28). This demonstrates that the use of $\sqrt{J_2}$ and $\tau_{\max}$ in the context of the Sines criterion can be interchanged without any significant difference. In order, therefore, to compare the Sines criterion with the Dang Van criterion, the fatigue limit predicted by the former will be written as

$$\tau_{\max_{\text{alt}}} + \alpha\overline{\sigma}_{\text{stat}} = \beta, \qquad \alpha, \beta \in \mathbb{R} \tag{30}$$

For the biaxial loading case [equation (18)], this becomes

$$\begin{aligned} &\frac{1}{2}\max(|\Sigma_{1_{\text{alt}}} - \Sigma_{2_{\text{alt}}}|, |\Sigma_{1_{\text{alt}}}|, |\Sigma_{2_{\text{alt}}}|) + \alpha\frac{A_1 + A_2}{6}\frac{1+R}{1-R} = \beta \\ &\Rightarrow \max\left(\frac{|A_1 - A_2|}{4}, \frac{|A_1|}{4}, \frac{|A_2|}{4}\right) + \alpha\frac{A_1 + A_2}{6}\frac{1+R}{1-R} = \beta \end{aligned} \tag{31}$$

Comparing equations (31) and (25), it is noted that they are indeed very similar, the only difference being that Sines and Ohgi used the static hydrostatic component, whereas Dang Van *et al.* employed the total value. This gives rise to an additional factor of 1 in $(1+R)/(1-R)+1$ in the Dang Van *et al.* case; that aside, the criteria are identical. This factor will cause greater discrepancies when $(1+R)/(1-R)$ is comparable in magnitude with 1, i.e. when R tends to -1, which represents the fully reversing case.

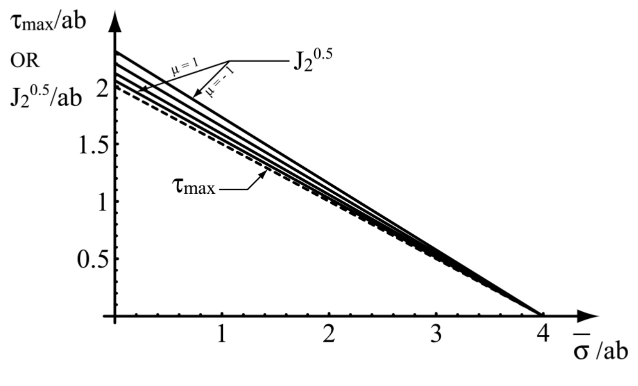

Fig. 3 Plots of the fatigue limit in terms of maximum shear stress and the von Mises stress versus hydrostatic pressure

5 CONCLUSIONS

Brief overviews of the Dang Van criterion, the Goodman rule and the Sines criterion have been given. The similarities and differences between them have been summarized, and each one's applicability to fatigue problems briefly discussed. The Dang Van criterion for uniaxial tensile–compressive loading has been compared with the Goodman rule, and the predictions found to be identical. The relationship between the constants for the two approaches has been found. The Dang Van criterion has also been compared with the Sines criterion for biaxial loading. It has been found that the two approaches are very similar in nature, although some discrepancy between the two may be observed at or near fully reversing conditions.

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