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The Relationships Among Working Memory, Math Anxiety, and Performance

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Individuals with high math anxiety demonstrated smaller working memory spans, especially when assessed with a computation-based span task. This reduced working memory capacity led to a pronounced increase in reaction time and errors when mental addition was performed concurrently with a memory load task. The effects of the reduction also generalized to a working memory-intensive transformation task. Overall, the results demonstrated that an individual difference variable, math anxiety, affects on-line performance in math-related tasks and that this effect is a transitory disruption of working memory. The authors consider a possible mechanism underlying this effect—disruption of central executive processes—and suggest that individual difference variables like math anxiety deserve greater empirical attention, especially on assessments of working memory capacity and functioning.

> Affect is the least investigated aspect of human problem solving, yet it is probably the aspect most often mentioned as deserving further investigation. (Mandler, 1989, p. 3)

In this article, we continue a program of research that examines the possible cognitive consequences and correlates of mathematics anxiety. As discussed elsewhere (Ashcraft & Faust, 1994; Ashcraft, Kirk, & Hopko, 1998), this work attempts to integrate two rather independent lines of research that have coexisted since the early 1970s. The first concerns studies of math anxiety per se, beginning with the important article by Richardson and Suinn (1972) and largely conducted within the psychometric tradition. The second is the study of mathematical cognition itself, focusing on the underlying mental representations and processes used in arithmetic and mathematics performance, work stemming principally from Groen and Parkman's (1972) classic article.

It is surprising yet apparently true that up until this integrative research was begun, no one had considered whether math anxiety had any *on-line effect* on an individual's math performance, that is, an effect on underlying cognitive processes as the individual performs a math task. To be sure, the literature contains many reports of the general negative effects that math anxiety has on math performance and achievement (see the thorough meta-analysis by Hembree, 1990). For example, individuals with high math anxiety take fewer math courses, earn lower grades in the classes they do take, and demonstrate lower math achievement and aptitude than their counterparts with low math anxiety. However useful this information is, it does not address the underlying cognitive processes involved in doing math, for example, mental processes that access the memory representations of mathematical knowledge. The same is largely true of the work reported in McLeod and Adams (1989). Its focus on relatively slow problem-solving tasks, especially when evaluated in classroom settings, precludes a fine-grained examination of mental representations and processes. Thus, the general focus of our research is to examine performance in standard cognitive frameworks and on-line tasks. We hope to examine the influence that math anxiety exerts on mathematical cognition and to identify the processing components that are so influenced.

Math Anxiety and Performance

Across several initial studies, we have found substantial evidence for performance differences as a function of math anxiety. These differences typically are not observed on the basic whole-number facts of simple addition or multiplication (e.g., $7 + 9$, 6×8) but are prominent when somewhat more difficult arithmetic problems are tested. In particular, Ashcraft and Faust (1994; also Faust, Ashcraft, & Fleck, 1996) have shown that high-math-anxiety participants have particular difficulty on two-column addition problems (e.g., $27 + 18$), owing largely to the carry operation. When such problems were answered correctly, the time estimate for the embedded carry operation was nearly three times as long for high-anxiety participants as it was for low-anxiety participants (Faust et al., 1996). Thus, high-math-anxiety participants showed slower, more effortful processing on a procedural aspect of performance, performing the carry operation (for suggestive evidence on math affect and procedural performance in a numerical estimation task, see LeFevre, Greenham, & Waheed, 1993). Furthermore, their higher error rates on these problems, often showing classic speed–accuracy tradeoffs when confronted with relatively difficult arithmetic, indicated a willingness to sacrifice accuracy on especially difficult trials, either to avoid having to deal with the stimulus problem or merely to speed the experimental session along.

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Math Competence

A rival interpretation, of course, is that high-anxiety participants are simply less competent in math, unable to perform the necessary calculations at the same level of accuracy as low-anxiety individuals. The literature documents that there is indeed a significant relationship between math anxiety and math competence or achievement, a correlation of −.31 in Hembree's (1990) meta-analysis. If the correlation holds across all levels of problem difficulty, then competence and math anxiety are completely confounded, and performance differences cannot be uniquely attributed to either factor.

Results reported elsewhere, however, suggest that there is not a complete confounding of math anxiety and math competence. Faust et al. (1996), for instance, showed equivalent performance across math-anxiety groups to simple one- and two-column addition and multiplication problems when those problems were tested in an untimed, pencil-and-paper format. It is important to note that the larger of these problems had shown math-anxiety effects in laboratory tasks, suggesting strongly that the on-line anxiety reaction had compromised participants' ability to demonstrate their basic competence. Ashcraft and Kirk (1998) examined math competence and math anxiety more thoroughly in a study that administered a standardized math achievement test. Simple whole-number arithmetic problems showed no math anxiety effects at all, whereas accuracy for the higher math-anxiety groups did decline more on the later test lines at which more difficult arithmetic (e.g., mixed fractions) and mathematics problems (e.g., factoring) were tested.

Finally, Hembree (1990) noted an interesting outcome in his meta-analysis on math anxiety. Reports on the most effective treatment interventions for math anxiety, behavioral and cognitive–behavioral approaches, also presented evidence of posttreatment increases in math achievement or competence scores to levels "approaching the level of students with low mathematics anxiety" (p. 43). Because the treatments did not involve instruction or practice in mathematics, it is quite improbable that treatment itself improved individuals' math competence. Instead, it seems very likely that the low pretreatment achievement scores of high-math-anxiety individuals were depressed by math anxiety during the assessment itself and that relief from math anxiety then permitted a more accurate assessment of math achievement and competence.

On the basis of such evidence, it would appear that lower math competence cannot be offered as a simple, wholesale explanation for all the performance decrements associated with high math anxiety (see Ashcraft & Kirk, 1998, for a full discussion of this argument). Instead, these performance decrements seem to call for an explanation involving on-line cognitive processing.

Math Anxiety and Working Memory

For several reasons, we suspected that an assessment of the capacity and functioning of working memory (Baddeley, 1986; Baddeley & Hitch, 1974) might provide a useful explanation of results such as Faust et al.'s (1996) carry effect. A growing body of evidence now attests to the centrality of working memory to processes such as reading and reading comprehension (Just & Carpenter, 1992), reasoning (Baddeley & Hitch, 1974; Jonides, 1995), and retrieval from long-term memory (Conway & Engle, 1994; Rosen & Engle, 1997; for a collection of articles, see Richardson et al., 1996, and the contributions to this special issue). The various components of these mental processes are often attributed to one or another of the three major subcomponents—the *central executive,* the *auditory rehearsal loop,* or the *visuo-spatial sketchpad*—in Baddeley's (1986, 1992) well-known model.

There is a supportive although not extensive literature on the role of working memory in mathematical cognition as well. Since Hitch's (1978) early article on multistep arithmetic problem solving, there have been several reports on the critical role of working memory in math performance. As an example, Geary and Widaman (1992) demonstrated that working memory capacity was closely related to skill in arithmetic problem solving and, in particular, to the speed of both fact retrieval and execution of the carry operation. In both cases, the higher the capacity of working memory, the faster were the component processes (see also Ashcraft, 1992, 1995; Lemaire, Abdi, & Fayol, 1996; Widaman, Geary, Cormier, & Little, 1989). So, for example, executing the carry operation is thought to be controlled by working memory, thus placing significant demands on the capacity of the working memory system (Ashcraft, Copeland, Vavro, & Falk, 1999; Hitch, 1978; Logie, Gilhooly, & Wynn, 1994).

Accordingly, we hypothesized that a major contributor to the performance deficits found for high-math-anxiety participants involves working memory. In particular, such deficits are predicted to stem from that portion of working memory, presumably the central executive, that applies the various procedures of arithmetic during problem solving (Ashcraft et al., 1999; but cf. Butterworth, Cipolotti, & Warrington, 1996; see Darke, 1988, and Sorg & Whitney, 1992, for additional evidence concerning anxiety and working-memory processes).

More generally, Eysenck and Calvo (1992) have proposed an overall model of the anxiety-to-performance relationship in cognitive tasks, which is called the *processing efficiency theory.* Their most relevant prediction for the present topic is that performance deficits due to generalized anxiety will be prominent in exactly those tasks that tap the limited capacity of working memory.[1] In their theory, the intrusive thoughts and worry characteristic of high anxiety are thought to compete with the ongoing cognitive task for the limited processing resources of working memory. The result of such competition is either a slowing of performance or a decline in accuracy—in other words, lower cognitive efficiency. Because high-anxiety individuals must expend greater cognitive effort to attain the same level of performance achieved by low-anxiety individuals, processing efficiency is lower for high-anxiety individuals.

Most of Eysenck's work (see Eysenck, 1992, for an integrative summary) is based on results with either generalized anxiety disorder individuals or individuals who exhibit high trait anxiety. For the present studies, we extended Eysenck's predictions to math anxiety. If this extension is valid, then an assessment of working memory capacity and functioning should reveal differences as a

[1] Eysenck (1992) discusses a whole range of anxiety-related phenomena, for instance, increased physiological arousal, selective attention, and distractibility. For the tasks under consideration here, however, the consequences of anxiety that affect working memory processes are the most relevant.

function of math anxiety, especially on tasks that require intensive processing within working memory. We do not test the specifics of Eysenck and Calvo's (1992) prediction here, which states that it is intrusive thoughts and worry (in this case, about math) that detract from available working memory capacity. Instead, we assess the more general prediction that math anxiety disrupts working memory processing when the cognitive task involves arithmetic or math-related processes. In this sense, our prediction is not appreciably different from simpler models of attentional or working memory disruption, for instance, Kahneman's (1973) prediction that stress will disrupt processing that depends on attentional (working memory) factors.

Experiment 1

Experiment 1 evaluated the hypothesized relationship between math anxiety and working memory capacity. This assessment was embedded in a broad-based assessment of possible relationships among these and other factors, especially math computational skill and math attitudes. For a full report on the attitude and computational skill assessments, see Ashcraft and Kirk (1998).

Method

Participants

A total of 66 participants, recruited from lower level undergraduate psychology classes, received course credit for participating. After completing informed consent procedures, they were administered the math-anxiety and working memory tests described below (along with the computational skill and attitudes assessments) and then were debriefed and excused. Sessions lasted approximately 90 min.

Instruments

Participants responded to a short (10-item) information sheet, which asked various demographic and math background questions. They also received a test of working memory capacity and a math-anxiety questionnaire, described below.

Demographic information. The one-page information sheet asked for age, gender, year in school, and ethnic group, as well as self-reported number of math courses and grades for both high school and college. We also asked for subjective ratings, on a 1–5 scale (1 = *not at all*), regarding how much participants enjoyed math and how math-anxious they were.

Math anxiety. The sMARS (short Mathematics Anxiety Rating Scale; Alexander & Martray, 1989) is a 25-item version of the most widely used measure of this construct, the 98-item MARS (Richardson & Suinn, 1972). The sMARS assesses an individual's level of apprehension and anxiety about math on a 1–5 Likert scale, asking for participants' responses about how anxious they would be made by various settings and experiences (e.g., "Taking the math section of a standardized test"). Fleck, Sloan, Ashcraft, Slane, and Strakowski (1998) tested several shortened forms of the MARS and found the 25-item sMARS to yield a very high correlation with participants' overall MARS scores ($r = .96$) and acceptable test–retest reliability ($r = .746$ at a 2-week retest interval).

Working memory capacity. Salthouse and Babcock's (1990) listening span (L-span) and computation span (C-span) tasks assessed participants' working memory capacity by requiring them to store increasing numbers of words or digits in working memory while processing simple verbal or arithmetic tasks. In the L-span task, participants hear a number of simple sentences, one by one, and must answer a simple question about the current sentence before hearing the next (e.g., "Last fall the farmers had a good harvest. When?"; "The children in the car wanted to stop for ice cream. Where were the children?"). At the end of the set, the participant must then recall the final word in each of the presented sentences (e.g., harvest, ice cream), in serial order. Three trials are presented at each span length, with testing continuing until the participant fails to respond correctly to at least two of these trials (note that each sentence or problem in the block must also be answered correctly). For the C-span test, simple arithmetic problems replace the sentences (e.g., 5 + 2 = ?, 9 − 6 = ?). Participants give the answer to each problem (7, 3), one by one, and then must recall the last number (2, 6) in each of the several problems within that trial, in order. Thus the span tasks require both on-line processing for sentence comprehension or problem solution simultaneous with storage and maintenance of information in working memory for serial recall.

Procedure

Groups ranging in size from 7 to 24 participants were tested in a group setting. After the informed consent procedure, participants completed the demographic sheet and then were given the four categories of tests (including the two categories reported in Ashcraft & Kirk, 1998), sequenced randomly for each session. To ensure comparability of sessions, the span task stimuli and instructions were presented on a tape recording.

Results

Demographic Data

Table 1 presents summary statistics on the eight demographic characteristics of the sample. For clarity, the high school and college grades variables are reported on the standard 4-point scale (i.e., A = 4.0, etc.), as is class year (i.e., freshman, sophomore, etc.). Note that the *n* is reduced on college grades because 15 of the participants had either not yet enrolled in or not yet completed a college math course. The means and standard deviations for the self-reported enjoyment and math-anxiety questions are also included in Table 1. The values in this table suggest that the sample

Table 1
Summary Values, Sample Means, and Standard Deviations for Demographic Variables for Experiments 1–3

Variable	Experiment 1 (N = 66)	Experiment 2 (N = 45)	Experiment 3 (N = 45)
Gender (M/F)	33/33	15/30	10/35
Age	22.6 (4.57)	24.4 (6.11)	25.3 (7.68)
Class year	2.09 (1.10)	2.04 (1.44)	2.87 (1.07)
No. of high school courses	3.66 (0.96)	3.28 (0.83)	3.62 (1.15)
High school math grades	2.60 (0.83)	2.53 (0.92)	2.73 (0.89)
No. of college courses	2.28 (1.97)	1.68 (1.37)	2.84 (1.85)
College math grades[a]	2.78 (0.78)	1.85 (1.42)	2.53 (1.12)
Rated math anxiety[b]	2.89 (1.28)	2.47 (1.39)	3.31 (1.44)
Rated enjoyment of math[b]	3.00 (1.21)	2.50 (1.34)	2.78 (1.36)
Ethnic group (% of total sample)			
African American	18	22	13
Asian-Pacific	5	7	7
Caucasian	68	67	76
Hispanic	8	2	2

Note. Standard deviations in parentheses. M = male; F = female.
[a] $n = 51$ in Experiment 1; $n = 40$ in Experiment 2; $n = 45$ in Experiment 3.
[b] 1–5 scale, 1 = *not at all.*

of 66 participants was a relatively conventional undergraduate sample for an urban university.[2]

Mathematics Anxiety

Columns 1 and 2 of Table 2 present the overall sample means on the sMARS, the demographic variables that showed significant correlations with math anxiety, and the two working memory assessments. The mean sMARS score of 36.3 ($SD = 16.3$) was slightly higher than the normative values in Richardson and Suinn (1972) but very close to the values reported by Fleck et al. (1998), a mean of about 35.0, with a standard deviation of approximately 16.0. The very small difference in sMARS scores between men (35.6) and women (37.1) was nonsignificant ($F < 1.0$).

Column 3 of Table 2 presents the correlations between sMARS and the remaining variables, and Columns 4–8 present group means and analysis of variance (ANOVA) results on these variables, using math-anxiety group (low, medium, high) as a between-subjects variable.[3] Cutoff scores for categorizing participants into the three anxiety groups were determined empirically by the overall sample mean and standard deviation, such that low-math-anxiety scores were at least 1 standard deviation below the overall sample mean on the sMARS, and high-math-anxiety scores were at least 1 standard deviation above the mean. Scores for the medium-math-anxiety group fell in the 1 standard deviation range centered on the sample mean, from 0.5 standard deviation below to 0.5 standard deviation above the mean. Twelve, 23, and 15 participants had scores that placed them in the low-, medium-, and high-math-anxiety groups, respectively—50 of the original 66 participants.

Grouping participants by their level of math anxiety yielded one significant group difference on the demographic variables, concerning number of high school math courses taken, and two differences that approached significance, on high school grades and self-rated math anxiety (1–5 scale). Note that the full correlations between sMARS and these three demographic variables were all significant.

Math Anxiety and Working Memory Span

As shown at the bottom of Table 2, individuals at higher levels of math anxiety showed significantly lower working memory capacity scores than those at lower anxiety levels. To see if the decline in capacity differed as a function of type of span task, we conducted a two-way ANOVA on the span scores. The analysis revealed significant main effects for anxiety group, $F(2, 47) = 11.22$, $MSE = 1.23$, and for type of span task, $F(1, 47) = 10.17$, $MSE = 0.77$, both $ps < .01$. The F value for the interaction, however, was nonsignificant, $F(2, 47) = 1.71$, computed $p = .193$.[4]

The result, taken at face value, indicates that individuals at higher levels of math anxiety have a reduced working memory capacity when tested with either a computation-based or language-based span task. This runs counter to the prediction that a decline in span scores due to math anxiety should be math specific and should not be apparent in nonmath testing situations. Because of the overlap between the two span tasks (e.g., the full sample correlation between the two span tasks was .38), we conducted several multiple regression analyses to partial out the contributions of one span task to examine the math-anxiety effect on the other span task.

In brief, anxiety group was more predictive of an individual's C-span score than the individual's own L-span score was. The original correlation between C-span and math anxiety (−.44) remained significant and essentially unchanged (−.40), even after the common variance with L-span was statistically partialed out. In contrast, the original correlation between L-span and math anxiety (−.36) became nonsignificant (−.22) when the common variance with C-span was statistically removed. These results suggest that the decline in computation-based working memory span was clearly related to math anxiety, above and beyond the similarities in performance between computation- and verbal-based span tasks. Variations in verbal-based span, however, were less clearly related to math anxiety (see also Dark & Benbow, 1990, 1991), although the two original correlations did not differ significantly from each other ($z = -.64$). We postpone further discussion of this issue until Experiment 3, in which these relationships are tested again.

Discussion

The correlational results of this study were straightforward and replicated several of the global effects of math anxiety reported by Hembree (1990); high-math-anxiety individuals enroll in fewer math courses and earn lower grades in the math classes they do take. The most important new finding was that working memory capacity was negatively associated with math anxiety. Given the importance of working memory functioning to a variety of cognitive and intellectual tasks (e.g., Just & Carpenter, 1992; Logie et al., 1994), it becomes genuinely important to explore the relationship between working memory and math anxiety more fully.

This is especially the case given the positive relationship reported elsewhere between working memory and math performance (Ashcraft & Kirk, 1998; Geary & Widaman, 1992; Hitch, 1978). In particular, the possibility exists that the lower working-memory capacity that seems characteristic of high-math-anxiety individuals may be at least partially responsible for the performance decrements commonly found with math anxiety. It may be, further, that this reduced working-memory capacity is an on-line effect, one that disrupts information processing in arithmetic and math tasks. The argument here is that those aspects of math performance that rely especially on working memory will be the aspects most affected by math anxiety. If this disruption can be demonstrated on a rather straightforward arithmetic task, one on which competence differences can be ruled out, then this would be evidence of an on-line math-anxiety reaction. More generally, we would expect to find such evidence whenever the math task places heavy demands on the capacity of working memory. The following two experiments were designed to follow up on this prediction.

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[2] Unless otherwise noted, all findings reported as significant here achieved at least the .05 significance level.

[3] The full correlation matrix for Experiment 1 is available upon request.

[4] We note in passing that the superiority of our participants on the C-span task ($M = 3.4$ vs. 2.8 for L-span) is reversed from the data in Salthouse and Babcock (1990; in that report, $M = 2.87$ for the C-span task and 3.37 for the L-span task), possibly owing to our group presentation method or the lengthy test sessions.

Table 2
Means (and Standard Deviations), Correlation Coefficients, and Math-Anxiety Group Means for Experiment 1

Variable	M	r with sMARS	Anxiety group: Low	Med.	High	$F(2, 47)$	$p<$
sMARS	36.3 (16.3)	1.0	12.3	37.1	57.4	255.20	.001
Demographics							
No. of high school courses	3.66 (0.96)	−.28	4.08	3.56	3.53	3.72	.05
High school math grades	2.60 (0.83)	−.29	3.0	2.50	2.40	2.13	.13
Rated math anxiety	2.89 (1.28)	.42	2.25	3.17	3.20	2.60	.08
Working-memory capacity							
L-span	2.80 (1.01)	−.36	3.7	2.4	2.6	7.83	.01
C-span	3.44 (1.16)	−.44	4.3	3.4	2.8	7.33	.01

Note. $r_{crit}(66) = .254$ at $p = .05$; $r_{crit}(66) = .330$ at $p = .01$. sMARS = short Mathematics Anxiety Rating Scale; Med. = medium; L-span = listening-span task; C-span = computation-span task; crit = critical.

Experiment 2

We explored the hypothesis that math anxiety disrupts working memory by using an on-line mental addition task, in which participants see the problem and are timed as they produce its answer verbally. Problem difficulty ranged from basic addition facts (e.g., 4 + 3) up through two-column additions with carrying (e.g., 47 + 18). Importantly, this range of problems corresponds to difficulty levels that showed no math anxiety effects in math achievement testing (Ashcraft & Kirk, 1998). Performance differences here should therefore be attributable to the on-line disruption of math processing rather than to preexisting group differences in competence and skill.

Mental addition was embedded here within a dual task. The reasoning behind the dual-task procedure (e.g., Baddeley, 1986) is that a demanding secondary task will compete with the primary task for working memory capacity, to the extent that each task requires the working memory system for successful performance. As the tasks increase in difficulty, their combination begins to overload the working memory system, with the interference visible as an increase in either reaction times (RTs) or errors. The relevant manipulations in the primary task were difficulty of the addition problem and whether the problem involved carrying from the units to the tens column; few, if any, other independent variables are more reliably associated with increasing difficulty in arithmetic processing than these (e.g., Ashcraft, 1992; Widaman et al., 1989). The secondary task here was a memory load task, holding either two or six random letters in working memory for later recall. The intent was to determine the point at which available working memory capacity becomes insufficient to maintain normal speed and accuracy on the two tasks.

Results with this combination of tasks (Ashcraft et al., 1999) have indicated a strong interaction between task (control vs. dual task) and the problem difficulty factors. In those results, the more difficult problems, especially those with carrying, showed increased solution latencies and dramatically higher error rates under high-memory-load conditions. This effect was interpreted to mean that more difficult addition problems rely significantly on working memory because performance to them was compromised when the system was also taxed with the memory load task.

We go one step further here in predicting that this interference *pattern should* be exaggerated at the higher levels of math anxiety. Participants at higher levels of math anxiety show lower working-memory capacity when processing numbers, as demonstrated in Experiment 1. Thus, the interference experienced under our dual-task procedure should be even more pronounced at higher levels of math anxiety. Indeed, the on-line math-anxiety effect may be conceptualized as functionally similar to a secondary task in that it drains working-memory capacity that otherwise would be available for task-relevant processing, as suggested earlier. In a real sense, high-math-anxiety participants may face a triple-task situation, with arithmetic and memory load performance further compromised by the on-line anxiety effect.

Method

Participants

Undergraduates in lower level psychology courses volunteered their participation in return for extra credit. We continued testing until the three math-anxiety groups had 15 participants each; the sMARS cutoffs for group membership were the same as in Experiment 1. The additional participants who were tested had sMARS scores that fell outside of the critical ranges or, toward the end of testing, had a score that placed them in an already-filled group. During testing, 2 participants were dropped (and replaced) because they were unable to trigger the voice relay reliably, as were 8 who were unable to achieve an overall accuracy rate of 75% on letter recall. Table 1 presents summary values for this sample.

Apparatus

The demographic sheet and the sMARS were in pencil-and-paper format. The experimental task was instrumented on ordinary laboratory personal computers, using the Micro Experimental Laboratory software package (MEL Professional 2.01, 1995; Schneider, 1988). The software presented all task instructions on the screen and recorded RTs automatically. At the end of each trial, the experimenter entered the participant's response to the arithmetic problem, and the software scored responses for accuracy. Spoken letter recall was recorded for later scoring.

Tasks

In the full dual-task condition, there were three main events per trial, each event separated from the next by a blank screen. The three events were (a) presenting the set of letters to be held in working memory, with the interval terminated by the participant's key press (Event 1), (b) presenting the addition problem to be solved, with the interval terminated by the participant's spoken response (Event 2), and (c) prompting for recall of the letters, with the interval terminated by the experimenter after recall was recorded (Event 3). Participants were encouraged (although not required) to read the letter set aloud (Event 1) before moving on to the arithmetic problem. The RT interval began when the addition problem appeared on the screen (Event 2) and was terminated when the participant stated the answer to the problem. This was followed by the prompted recall of the letter set (Event 3). Thus, on each trial, participants (usually) read the letter set aloud, spoke the answer to the addition problem, then recalled the letter set orally. After the experimenter recorded letter recall and keyed in the addition answer, a "Ready" signal appeared, followed by the beginning of the next trial.

The appropriate controls for this dual task involve testing both the primary (math only) and secondary (letter recall only) tasks alone. To guard against the possibility that vocalizing per se in the dual task might cause some interference, both control conditions held overall vocalizations constant. That is, in the math-only task, participants still named the letter set aloud when it was presented (Event 1), and after solving the math problem, they named the letters when the prompt appeared (Event 3). To eliminate the load on working memory, the letters were shown on the screen during the prompt, and participants merely read them aloud. Likewise, during the letter-recall-only task, the answer to the addition problem was shown along with the problem on the screen (Event 2), and the participants merely read the answer aloud prior to recalling the letter set at the recall prompt.

Stimuli

Sixty addition problems were evenly divided among the six stimulus conditions, the result of the 3 × 2 manipulation of problem size and carry. The three levels of problem size were basic fact, medium, and large, with these labels denoting, respectively, problems with single-digit operands (e.g., 4 + 3), with a double- and a single-digit operand (e.g., 15 + 2), and with two double-digit operands (e.g., 23 + 11). Half of the problems in each set were no carry problems (the examples just given), and half involved a carry (e.g., 7 + 9, 16 + 8, 25 + 17). The set of 60 problems was then permuted twice, yielding three comparable sets of problems to be used in the three task conditions. In one permutation, one of the two problem operands was randomly selected and changed by ±1 or 2; in the other permutation, the other operand was changed in the same way. Thus, the three different sets of addition problems were comparable in difficulty, yet involved no stimulus repetition across tasks. All problems had either 1- or 2-digit sums; the largest problem was 45 + 31.

Letter sets for the secondary task were composed of either two or six randomly selected consonants, selected with approximately equal frequencies across the entire experiment. Letter sets were modified as needed to avoid meaningful combinations, combinations that repeated a letter, and combinations that contained any pair of letters in alphabetical sequence. The letter combinations were also permuted, by changing one letter in each combination (subject to the same restrictions), to yield the sets necessary for the three tasks. As was the case with the three sets of addition problems, the letter combinations were comparable across tasks while not involving any exact repetition.

Procedure

Participants were tested individually in sessions that averaged 75 min. They were told the general purpose of the experiment, gave their informed consent, and then were given a short practice set that presented and tested each of the three tasks. They then proceeded to the experiment itself, with order of the three tasks counterbalanced across participants. The detailed purpose of the study was explained at the end of the session. Previous testing with this task (Ashcraft et al., 1999) indicated that the full set of 60 trials on the letter-recall-only task needlessly prolonged the session and was unnecessary for a reliable estimate of memory load performance. Thus, the letter-only task was reduced from 60 to 36 trials, with 3 trials in each experimental condition, versus the 5 per condition in math-only task and the dual task.

Results

All ANOVAs reported here used the same mixed design: math anxiety (low, medium, high) as a between-subjects variable and the remaining variables as within-subjects factors. As stated, the factors of problem size (basic fact, medium, and large) and carry status (no carry, carry) manipulated the difficulty of the addition problems, and the memory load condition (two- versus six-letter loads) manipulated difficulty in the secondary task. Thus, the within-subjects factors formed a 3 × 2 × 2 design, with a total of 12 stimulus conditions and five trials per condition.

Letter Recall Only

The number of correctly recalled letters was analyzed in the letter-recall-only task as a function of the three within-subjects independent variables (though, of course, no genuine arithmetic processing was necessary in this task). The only effect that achieved conventional significance was the main effect of memory load, $F(1, 42) = 76.68$, $MSE = 0.12$, $p < .001$, in which recall of the 2-letter sets averaged 100% correct, versus 96% correct recall of the 6-letter sets. The results indicated no relationship between math anxiety and letter recall in this control task; mean correct recall across the anxiety groups was 5.89, 5.87, and 5.81 letters (out of 6 possible; note that for all analyses, recall on 2-letter trials was converted to the same 6-point scale). The three-way interaction of anxiety, problem size, and memory load approached significance (computed $p = .059$) but only because of a strong ceiling effect in the 2-letter conditions (from 99% to 100% correct on 2-letter trials).

The letter-only results indicate that even the heavier memory loads were well within participants' basic working memory capacities. Furthermore, with the nonnumerical stimuli in this task, there were no apparent differences in working-memory capacity across the three anxiety groups. As such, anxiety-related effects in the math-only and dual-task conditions cannot plausibly be attributed to a general deficiency in working memory capacity for the higher anxiety groups. Note, finally, that the virtually perfect performance on the lighter memory loads suggests minimal involvement of working memory for storing and recalling two letters.

Math Only

Both RTs and errors to the addition problems were analyzed with the mixed ANOVA design described above. Of the original 2,700 possible RTs (60 for each of 45 participants), 0.6% were identified as outliers, 2% were spoiled owing to difficulties with the voice relay (e.g., triggering by an extraneous noise), and 4% of trials were answered incorrectly; RTs from these trials were excluded from the analyses.

Latency Results

Problem size and the carry status of the problem, as well as the interaction of these two variables, had strong effects on RTs; for the interaction, $F(2, 84) = 25.77$, $MSE = 201{,}473$, $p < .001$. The diverging pattern of this interaction, shown with accompanying error rates in Figure 1, replicates very closely the pattern reported in Ashcraft et al. (1999). There was a nonsignificant (computed $p = .10$) tendency for the low-anxiety group ($M = 1{,}957$ ms) to be marginally slower than the medium group ($M = 1{,}825$ ms) and for the high-anxiety group to be slower than both of these ($M = 2{,}288$ ms). The low–medium group reversal seemed due to a slight speed–accuracy tradeoff, with 3% errors for low-math-anxiety versus 5% for the medium group (the high-anxiety group's overall error rate was 6%). The RT data also showed a 186-ms speedup of performance on six-letter trials on the part of high-math-anxiety participants, versus a nonsignificant 30 ms difference for low and medium groups, despite the fact that letters did not have to be stored in and retrieved from memory in this task: Anxiety × Memory Load interaction, $F(2, 42) = 6.76$, $MSE = 84{,}401$, $p < .01$.

Accuracy Results

The analysis of error rates revealed straightforward evidence of a math anxiety effect on addition performance in the form of an Anxiety × Carry interaction, $F(2, 42) = 6.04$, $MSE = 128.17$, $p < .01$. No-carry problems showed essentially no variation in error rates across anxiety group (2%, 1%, and 0.4%, respectively), contrasted with the marked increase on carry problems (4%, 8%, and 11%, respectively). This pattern replicates earlier reports (e.g., Faust et al., 1996) of the particular difficulties experienced by

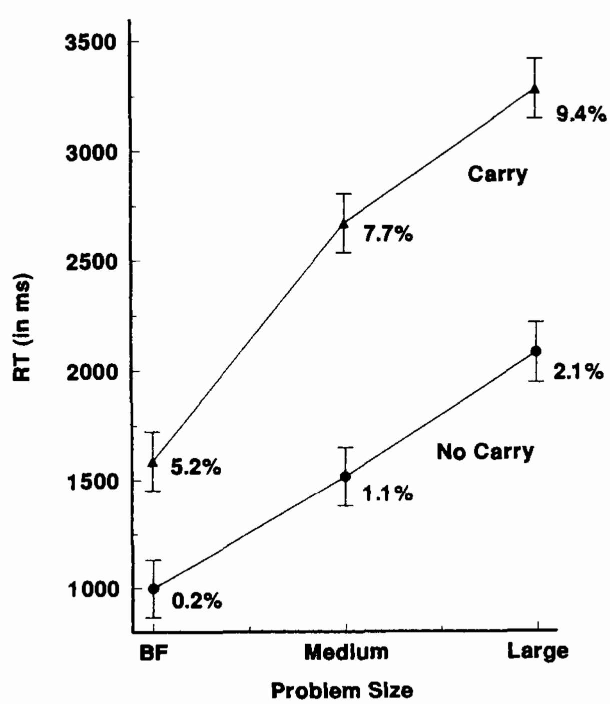

Figure 1. Mean reaction time (RT) and percentage error rates to basic fact (BF; $n + m$), medium ($nn + m$), and large ($nn + mm$) addition problems, separately for no carry and carry problems: Experiment 2. Error bars display 95% confidence interval based on *MSw*.

high-math-anxiety participants on problems involving the carry operation.

The analysis also revealed processing difficulties due to problem variables. Beyond the significant main effects of problem size, $F(2, 84) = 4.13$, $MSE = 101.9$, and carry status of the problems, $F(1, 42) = 42.00$, $MSE = 128.2$, $p < .001$, there was a significant interaction of problem size, memory load, and carry, $F(2, 84) = 8.19$, $MSE = 88.40$, $p < .001$. Error rates climbed to 14% for large carry problems on six-letter trials, suggesting that articulating the six-letter sets generated some within-trial interference with difficult arithmetic processing, even without the memory load requirement.

Dual-Task Analyses

Latencies and error rates were analyzed as a function of control versus dual task, to determine how much performance deteriorated when the two demanding tasks were performed concurrently. We begin with an explanation of the analysis decisions dictated by the dual-task procedure, then present the results of the analyses.

In the dual-task condition, it seemed clear that errors in either task could be taken as evidence of interference in working memory. That is, particularly difficult addition might disrupt working memory sufficiently to depress letter recall, or the working memory load for letter recall might disrupt concurrent addition performance. As such, error rates in the dual task were the percentages of trials in the 12 stimulus conditions that contained an error of either type. Likewise, for the control observations, we summed error percentages across letter-recall-only and math-only tasks, separately for the 12 stimulus conditions. For all scoring, accurate recall was defined as recalling both letters in the two-letter sets and at least five of the six in the six-letter sets. Our rationale here was that accurate recall of five letters still required a substantial involvement of working memory, whereas insistence on perfect six-letter recall depleted the number of usable RT trials in the dual task to a greater degree than was acceptable. The RT analyses considered only accurate trials, which in the dual task meant only those trials scored as correct for both the math and the letter recall.

Latency Results

The analysis of RTs revealed several processing effects due to problem difficulty and memory loads. For example, problem size and the carry factor were significant as main and interaction effects, as found in the math-only analysis (Figure 1). Furthermore, this interaction depended on the control versus dual-task factor; for the three-way interaction, $F(2, 84) = 5.49$, $MSE = 110{,}756$, $p < .01$. The diverging pattern shown in Figure 1 was even more pronounced in the dual-task condition; for example, the slowest RT in Figure 1—3,280 ms for large carry problems—increased to 3,554 ms in the dual task, whereas RTs to the smaller problems and those without a carry remained essentially unchanged. The same (nonsignificant) pattern of means across anxiety groups was obtained again, and there was once again a significant speedup to six-letter trials for only the high-anxiety participants, $F(2, 42) = 5.95$, $MSE = 97{,}316$, $p < .01$.

Accuracy Results

Far more informative were the results from the error analysis. Here, all main effects (math anxiety at a computed p of .052), several double interactions, and three triple interactions were significant. We first focus on the three-way interaction pertaining to the task environment and then the two triple interactions that included math anxiety.

The interaction of task, carry, and memory load, $F(1, 42) = 6.28$, $MSE = 195.20$, was quite straightforward. The pattern in the control condition was of two main effects, showing a modest (6%) increase in error rates for carry problems and a modest (4%) increase with the heavy memory load. The source of the interaction was the diverging pattern found in the dual task. Here, the increase in errors for carry problems was 8% in the two-letter condition but 14% for carry problems done concurrently with the six-letter memory load. In other words, it was the combination of carrying performed under the heavy memory load that surpassed the capabilities of working memory. This is precisely the pattern reported for a sample of participants not selected on the basis of math anxiety (Ashcraft et al., 1999). Having to maintain a heavy load in working memory seems to selectively interfere with performance to difficult additions involving the carry operation.

This diverging Carry × Memory Load pattern also varied across the three math-anxiety groups, as shown in the Anxiety × Carry × Memory Load interaction, $F(2, 42) = 3.94$, $MSE = 116.92$. The four-way interaction of this effect with control versus dual task narrowly missed conventional significance, $F(2, 42) = 2.99$, $MSE = 195.20$, computed $p = .061$. We nonetheless present the four-way interaction in Figure 2 because it both shows the pattern of three-way effect as well as the exaggeration of this effect in the dual task. In the control task, the increases in errors due to carrying were moderate, increased slightly under the heavy memory load, and showed modest increases at higher levels of anxiety. A similar pattern was obtained in the dual task when only two letters had to be held in working memory. The heavy memory load, however, disrupted dual-task performance much more markedly. This disruption was particularly strong in the high-math-anxiety group, which showed a 39% error rate.

Another way of conceptualizing these effects is to define the *cost of carrying* as the difference in error rates between carry and no carry problems. In these terms, although all four sections of the figure show costs in accuracy due to carrying, these costs range from minimal to moderate in the control condition (from 0.8% for low anxiety, two-letter to 9% for high anxiety, six-letter). Costs remained in the 8% range in the dual-task condition but only with the light memory load. With the heavy memory load, however, the cost of carrying increased sharply with math anxiety; for low-, medium-, and high-math anxiety, the costs of carrying here were 10%, 17%, and 27%, respectively. No-carry problems therefore appear to rely only moderately on working memory, showing only moderate increases when memory load increases. When carrying was required, however, the costs due to the memory load were greater and increased substantially with the math-anxiety group. Carrying thus seems especially reliant on working memory processes and is especially prone to disruption among high-math-anxiety individuals.

Finally, the Math Anxiety × Problem Size × Task interaction was also significant, $F(4, 84) = 3.33$, $MSE = 546.23$, $p < .01$. In the control observations, errors rose somewhat across problem size for the medium- and high-anxiety groups, although not for the low-anxiety group (confirmed by a breakdown analysis limited to the control condition). Under the dual-task condition, the increases

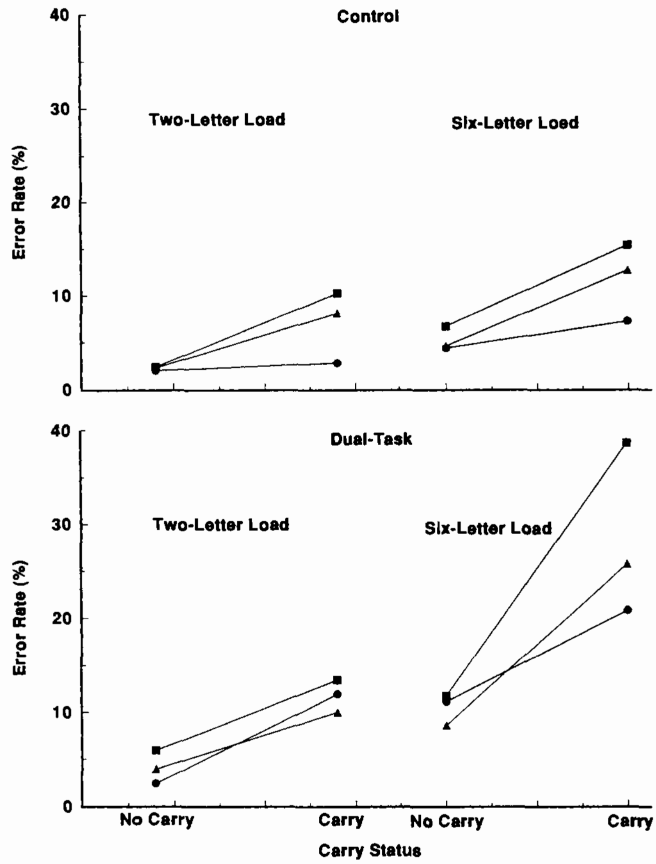

Figure 2. Mean percentage error rate for the control and dual-task conditions, separately by two- versus six-letter memory loads, no carry versus carry problems, and math anxiety (low anxiety [circles], medium anxiety [triangles], high anxiety [squares]): Experiment 2.

in error rates were clear for all three groups but were most pronounced for the high-anxiety group. The four-way interaction of this effect with memory load would have confirmed that this high error rate was particularly due to the heavy memory load; this interaction only approached significance, however (computed $p = .085$).

These results correspond well with the patterns predicted earlier, when we considered the lower working-memory capacity of high-math-anxiety participants and the reliance on working memory during difficult addition processing. All anxiety groups showed increases in error rates under the increasing pressures of problem size and dual-task requirements, indicating reliance on the working memory system for difficult arithmetic processing. However, the striking increase in errors for the high-math-anxiety group is consistent with the interpretation that the high-anxiety group had the least capacity to devote to concurrent processing, which itself taxed working-memory capacity. In passing, we note that despite their overall skill and achievement, low-math-anxiety participants still demonstrated reliance on working memory for carry problems, that is, problems involving procedural processes.

Discussion

Experiment 2 confirmed earlier indications that high-math-anxiety individuals have particular difficulties with addition problems involving a carry operation (Faust et al., 1996). Earlier interpretation had suggested that this was due to the load on working memory engendered by the carry operation. The results of Experiment 2 demonstrated this quite clearly. When carrying was performed concurrently with a task that placed heavy demands on working memory, performance deteriorated sharply for the high-math-anxiety group. This deterioration was not as pronounced at lesser degrees of problem difficulty or on problems not requiring the carry operation, and it was considerably weaker for participants at the low-math-anxiety level. Importantly, there was no evidence of anxiety-related deterioration in the letter-only task, in which processing within working memory involved nonnumerical stimuli. In short, these results conform exactly to the predicted consequences of lower working memory span for high-math anxiety individuals: degradation of their working memory performance in an on-line arithmetic task.

Experiment 3

We wish to extend this relationship between compromised working memory span and math anxiety by examining performance in a task that is working-memory intensive while not explicitly involving learned math. This demonstration is motivated generally by Eysenck and Calvo's (1992) prediction that anxiety will have its primary debilitating effects in tasks that place heavy processing loads on working memory. If the math anxiety reaction compromises participants' available working memory capacity when doing math, as was the case in Experiment 2, then it is of interest to explore the limits of this effect. In brief, must arithmetic and math stimuli be used to trigger the math anxiety reaction, or will tasks that merely rely on number-related processes also demonstrate the effect?

To obtain evidence on this question we tested a transformation task used by Eysenck (1985; originally reported in Hamilton, Hockey, & Rejman, 1977); letter transformation here was basically an "alphabet arithmetic" task (e.g., A + 3 = D; Logan & Klapp, 1991) with a recall requirement. We also repeated our assessment of working memory span from Experiment 1 in hopes of obtaining more definitive results on the relationship between math anxiety and language-based span.

Method

Participants

Participants were recruited from several lower level undergraduate psychology classes and given extra credit for their participation. After completing informed consent procedures, they completed the standard demographic sheet and the sMARS. Participants whose scores categorized them as having low, medium, or high math anxiety were then recruited for individual testing in the two working-memory tasks. A total of 51 participants were tested in the individual sessions, 15 each from the three math-anxiety levels described earlier. Four participants were dropped from the sample because they were unable to complete the tasks with adequate accuracy, and 2 more were replaced as described below. After the 90-min testing session was completed, participants were debriefed about the purpose of the experiment and then excused.

Working Memory Tasks

Participants were given the L-span and C-span working memory tasks described in Experiment 1. Additionally, they were given a letter transformation task (Eysenck, 1985) to test working memory capacity and processing. In this task, participants were given series of letter transformation trials and series of number transformation trials. In both cases, the stimulus set had to be transformed mentally, then reported to the experimenter for final recall.

Letter transformation. Participants saw either two or four letters of the alphabet, one at a time. They were required to transform each letter mentally by moving or counting forward either two or four steps through the alphabet, then holding that outcome in working memory while transforming the next letter in the set. In the simplest type of trial (two letters, transformation size two), a participant would see *2*, indicating that the letters had to be transformed two steps down the alphabet. The participant then pressed a key on the keyboard and saw the first letter, for example, *E*. The participant would mentally transform *E* by moving two letters through the alphabet (*F, G*), press the key again to see the next letter, for example, *S*, then transform *S* to *U* while simultaneously holding the earlier transformation outcome (*G*) in working memory. After pressing the key to indicate the second transformation was completed, the word *Recall* would appear on the screen, and the outcome of the transformations (*G, U*) had to be reported in order.

After 10 practice trials, a total of 40 letter transformation trials were presented, in randomized order. There were 10 trials in each of the cells of a 2 × 2 design, with factors of transformation size (two or four), and number of letters (two or four). Letters were selected randomly from the set *B* through *V* and were ordered randomly within trials subject to two restrictions. First, we eliminated any letter sequences that formed words or meaningful acronyms. Second, we excluded letters that would duplicate part of the transformation sequence required for another letter in that trial (e.g., the transformation sequence for *B*—which is *C, D, E, F*—precluded using the letters *C* through *F* for that trial).

Number transformation. We substituted randomly selected single or double digit numbers in the range 5–25 for the letters and had participants transform those numbers by adding either the value 7 or 13. Thus, for a "transform 7" sequence, the numbers "15 3 25 19" might be presented, again one at a time, and at the end of transformation, the participant would report "22 10 32 26" at the *Recall* prompt. As in the letter task, participants saw either two or four numbers to be transformed, to which they applied either a "plus 7" or a "plus 13" transformation. Again, we presented 10 practice and 40 experimental trials, the latter in randomized order.

Dependent Measures

Accuracy and latency data were collected for both transformation tasks. Participants' recall scores were the percentage of letters and numbers correctly recalled on each trial. The computer recorded the latencies between key presses on the keyboard, yielding a position-by-position record of the time required to encode, transform, and rehearse the letters or numbers to be recalled on each trial. Accuracy and span length on the L- and C-span tasks were scored as described in Experiment 1.

Apparatus

The transformation experiment was conducted using the same apparatus as in Experiment 2. The software presented all instructions and stimuli to the participants and recorded latencies in milliseconds. The span tasks were conducted as in Experiment 1.

Procedure

Participants were selected on the basis of math-anxiety scores obtained in the preliminary screening session. They were given either the two span tasks and then the two transformation tasks, or they were given these tasks in reverse order. A few participants were retested on the sMARS at the end of the experimental session when their performance seemed inordinately out of line with their sMARS score. Two participants were dropped from the sample and replaced because the retest scores would have reclassified

Table 3
Means (and Standard Deviations), Correlation Coefficients, and Math-Anxiety Group Means for Experiment 3

Variable	M	r with sMARS	Anxiety group: Low	Med.	High	$F(2, 47)$	$p<$
sMARS	37.6 (23.4)	1.0	10.6	36.7	66.0	345.11	.001
Demographics							
No. of high school courses	3.62 (1.15)	−.45	4.20	3.67	3.00	4.77	.05
High school math grades	2.73 (0.89)	−.67	3.4	2.80	2.00	15.54	.01
College math grades	2.53 (1.12)	−.35	3.1	2.6	1.9	4.50	.05
Rated math anxiety	3.31 (1.44)	.81	1.87	3.40	4.67	37.91	.01
Rated enjoyment of math	2.78 (1.36)	−.74	4.2	2.4	1.7	31.15	.01
Performance measures							
L-span	2.80 (1.12)	−.20	3.13	2.73	2.53	1.12	ns
C-span	3.04 (1.31)	−.40	3.60	3.27	2.27	4.94	.05
Letters transformation	80.8 (12.0)	−.44	85.0	84.0	73.5	4.25	.05
Numbers transformation	81.5 (11.3)	−.44	85.6	85.8	73.0	6.26	.05

Note. $r_{crit}(43) = .29$ at $p = .05$; $r_{crit}(43) = .39$ at $p = .01$. Letters transformation category reports percentage correct on difficult letter transformations; numbers transformation category reports percentage correct on difficult number transformations. sMARS = short Mathematics Anxiety Rating Scale; Med. = medium; L-span = listening-span task; C-span = computation span task; crit = critical.

them into a different math-anxiety group. The remaining retest scores were not seriously discrepant so these participants were retained in the sample.

Results

Demographic Data

The far right column of Table 1 summarizes the sample of participants in Experiment 3 in terms of demographic variables. The sample here was somewhat older than in the previous experiments, had taken more college math courses, and included a higher percentage of female participants but was otherwise fairly similar to those in the earlier experiments.

Mathematics Anxiety

Table 3 presents means and standard deviations of variables that correlated significantly with sMARS, the correlation coefficients, and group means as a function of math-anxiety group when those means were significantly different in simple ANOVAs.

The patterns of correlations and significant group differences were very similar to those obtained in Experiment 1. High math anxiety was associated with fewer high school math courses, lower high school and college math grades, higher self-rated math anxiety, and lower enjoyment of math than low math anxiety.

Working Memory: C- and L-Span Results

Table 3 also displays the significant correlation of the C-span measure and sMARS, the group means on the C-span measure as a function of anxiety group, and the nonsignificant correlation and group means on the L-span measure; as in Experiment 1, these two span-to-sMARS correlations did not differ significantly ($z = -1.34$). Although the means were somewhat lower than those in Experiment 1 and there was no significant difference between C-span and L-span tasks, the overall effect of anxiety was the same, a significant decline in span length from low to medium to high math anxiety, $F(2, 42) = 3.67$, $MSE = 1.94$. A breakdown ANOVA confirmed that the C-span scores declined significantly with increasing math anxiety but that the L-span scores did not. We conclude that there is a clear decline in working-memory span that is specifically related to an individual's level of math anxiety when the span task uses numerical information. The anxiety effect on working memory span is rather moderate when language stimuli are used, however.

Transformation Task

Latency results. We first examined participants' latencies in the transformation tasks, that is, the time spent in encoding, transforming, and then maintaining the transformation result for recall.[5] Participants occasionally spent an extraordinarily long or short time on one or another item, usually because they "lost" the earlier transformation results as they worked on a subsequent item (according to volunteered reports). To avoid skewing the latency results owing to such on-line lapses, we screened outliers from the

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[5] Eysenck (1985) asked his participants to perform the transformation aloud. He then separated the total latency per position into three subprocesses; long-term-memory access to retrieve the correct starting point in the alphabet, transformation of the letter, and rehearsal and storage of the transformation result. These were defined, respectively, as time from stimulus presentation to onset of vocalization, the duration of the vocalizations while transforming the letter, and the time from the end of vocalization until the keypress to reveal the next letter. According to this analysis, increasing latency across list position was entirely due to the rehearsal and storage component, which increased more sharply for high-anxiety participants. We did not intend to conduct this kind of breakdown analysis because it seems doubtful that the three subprocesses are completely independent and nonoverlapping. Given the serial recall requirement, however, we do not dispute the idea that increasing latencies are probably due to rehearsal and storage. It is unclear, however, whether this subprocess is more heavily dependent on the central executive, as Eysenck concluded, or the articulatory loop, as might be expected for serial rehearsal.

data, removing any latency larger or smaller than 2 standard deviations away from the Participant × Condition × Position mean.

The latency analysis revealed that letter transformation was slower than number transformation, large transformations took more time than small ones, and transformation latency increased across positions. These three main effects, and their double interactions, were significant in both the two- and four-item analyses. There was also a three-way interaction in the four-item analysis, with factors of letter versus number, transformation size, and position, $F(3, 126) = 7.66$, $MSE = 919{,}153$, $p < .001$. Part of the explanation of the interaction involved the larger separation in latencies for the +4 transformations of letters than the parallel +13 transformation for numbers. Beyond that, in all but the large number transformation condition, there was a tendency for latencies to drop off at Position 4, essentially the result reported in Eysenck (1985). Participants seem to have relied on a classic short-term memory strategy in such self-timed study, knowing that the fourth position was the last item, so spending relatively little time rehearsing at that position.

Figure 3 reveals how the transformation latency patterns depended on level of math anxiety. Math anxiety was a significant main effect in the two-item latency analysis, as was the interaction of math anxiety and position, $F(2, 42) = 7.08$, $MSE = 1{,}579{,}721$, $p < .01$. The main effects of anxiety, $F(2, 42) = 6.44$, $MSE = 54{,}311{,}073$, $p < .01$, and position, $F(3, 126) = 47.27$, $MSE = 5{,}612{,}390$, $p < .001$, were significant on the four-item trials, although their interaction was not (computed $p = .18$).

Accuracy results. High-math-anxiety participants spent significantly more time transforming the letters and numbers in the tasks than did the low- and medium-anxiety groups, just as Eysenck (1985) reported for high trait anxious individuals. One might suspect that this greater effort on the part of high-anxiety participants would lead to equivalent accuracy when the letters or numbers had to be recalled. This was not the case, however. In all conditions, the high-math-anxiety participants had poorer recall scores in the transformation tasks, as depicted in Figure 4. Math anxiety was significant as a main effect in the accuracy analysis, $F(2, 42) = 10.17$, $MSE = 232.39$, $p < .001$, and interacted with transformation value (small vs. large), $F(2, 42) = 4.05$, $MSE = 54.14$. The three-way interaction among anxiety, transformation value, and number of items being transformed (two vs. four), depicted in the figure, was significant, $F(2, 42) = 3.73$, $MSE = 36.09$, but seemed to depend on the somewhat unusual pattern obtained for the medium-anxiety group; $F < 1.0$ for the interaction when the medium-anxiety group was excluded. Nonetheless, the accuracy scores for high-anxiety individuals were uniformly lower than the comparable scores for the other two groups. Despite their longer transformation latencies, high-math-anxiety participants recalled fewer transformed items correctly, whether for small transformations on short lists or large transformations on long lists.

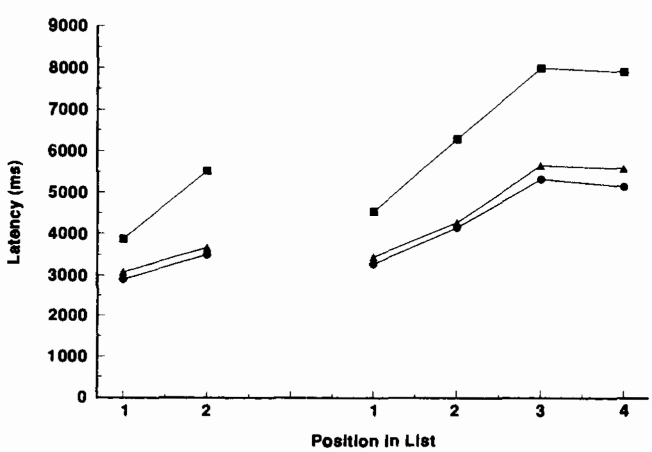

Figure 3. Mean transformation latencies across positions in list, separately for math-anxiety groups (low anxiety [circles], medium anxiety [triangles], high anxiety [squares]): Experiment 3.

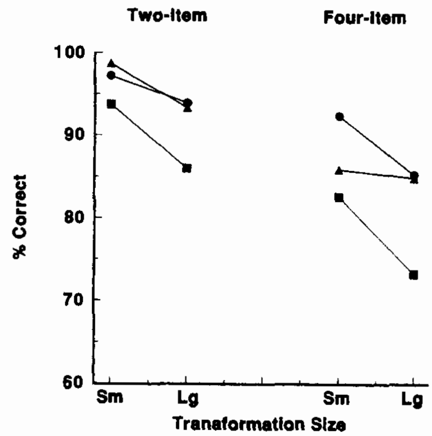

Figure 4. Mean percentage correct recall in the transformation task, as a function of math anxiety (low anxiety [circles], medium anxiety [triangles], high anxiety [squares]), two- versus four-item lists, and small (Sm) versus large (Lg) transformations: Experiment 3.

Thus, there is clear evidence of a math-anxiety-related difference in the processing of information through working memory. The task itself is working memory intensive, in that previous transformation results have to be held and rehearsed while simultaneously performing further transformations. High-math-anxiety participants were significantly slower at this, suggesting more laborious processing. Even with this additional effort, however, they were still less able to recall the transformations accurately. The results directly confirm and extend Eysenck and Calvo's (1992) prediction about processing efficiency and anxiety, that is, that under conditions in which anxiety is aroused, highly anxious individuals must expend additional working memory resources to achieve comparable levels of performance. It remains to be seen whether even greater effort during transformations would have achieved comparable accuracy in recall for the high-math-anxiety group.

We note two further effects in passing. First, transforming letters versus numbers made a consistent difference in the latency analyses but virtually no difference on the accuracy analysis. The closest this factor came to significance in accuracy scores was as a main effect, $F(1, 42) = 3.00$, $p < .10$; the means, however, were very close ($M = 88\%$ for letters, 90% for numbers). Second, there was no differential effect of anxiety on recall accuracy as a function of letter versus number transformation (computed

$F = 0.00$ to two decimals). All three anxiety groups were consistently 2% poorer in letter recall than number recall. Thus, the drop in accuracy depicted in Figure 4 was characteristic of high-math-anxiety participants regardless of whether letters or numbers were being transformed (F for the overall interaction = 0.13).

Discussion

It is not necessary to use conventional arithmetic and math problems to trigger the math-anxiety reaction. Instead, it is apparently enough that the task requires a counting-like process, as in Logan and Klapp's (1991) alphabet arithmetic task. With such a task, we found clear evidence of increased effort on the part of high-math-anxiety participants during input, effort which nevertheless did not equate them with the low-math-anxiety group on output accuracy. Because the task—counting-based transformations with an added recall requirement—is especially demanding of working memory, it highlights the performance decrements predicted for the high-math-anxiety group. Individuals already devoting some of their working memory's capacity to the on-line anxiety reaction show especially compromised performance when the experimental task is itself demanding of working memory.

General Discussion

The math-anxiety literature across the past 20 years attests to pervasive, long-term, damaging consequences of math anxiety (Hembree, 1990). What the literature has lacked until recently, however, is evidence that math anxiety is related to the actual doing of math, to the mental processes involved in working with numbers. The present work (also Ashcraft & Faust, 1994; Faust et al., 1996) has demonstrated that there are indeed such on-line effects, detected when procedures like carrying in multicolumn addition are required.

There are at least two possible reasons for finding such effects. One possibility is that because of their long-term avoidance of math, and their lesser mastery of the math that couldn't be avoided, high-math-anxiety individuals are simply less competent at doing math. This lesser degree of competence could presumably apply even to elementary carrying, introduced almost universally in the U.S. in the second grade math curriculum. It would surely apply at higher levels of math difficulty. This competence explanation is in fact the gist of Fennema's (1989) autonomous learning behavior model: Affect, including both attitudes and math anxiety, influences the behaviors one engages in as learning takes place. These behaviors, in turn, affect performance. We have diagrammed the gist of Fennema's model, as applied to our research, in the top half of Figure 5, in which attitudes and math anxiety influence competence, which then determines performance as measured by standardized tests such as the Wide Range Achievement Test (WRAT).

The present results along with related data argue that this simple competence explanation is insufficient to account for math-anxiety effects. First, we find speed and accuracy effects of math anxiety on multicolumn addition problems when standard on-line laboratory tasks are used (Ashcraft & Faust, 1994; Faust et al., 1996; the present Experiment 2). When the same stimuli were tested in a paper-and-pencil format designed to minimize the anxiety effect, there was no difference in performance across levels of math

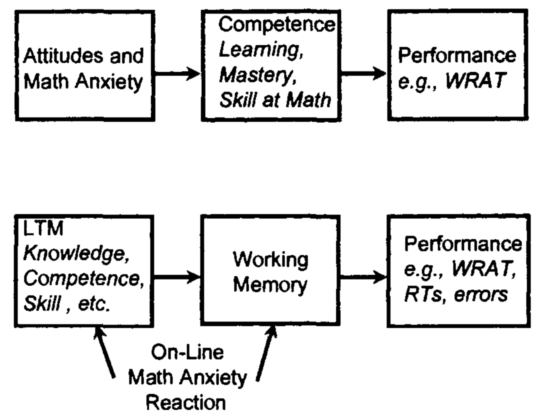

Figure 5. Top: A representation inspired by Fennema's (1989) autonomous learning behavior model of math anxiety. Bottom: A representation of the on-line math anxiety proposal. WRAT = Wide Range Achievement Test; LTM = long-term memory. RT = reaction time.

anxiety (Ashcraft & Kirk, 1998). Further, in that same report, we also found no relationship between competence and math anxiety on the simpler, whole number math problems of a standardized math achievement test.

To be sure, a significant competence differential appears when more complex arithmetic and math are tested (mixed fractions, algebraic equations, etc.; see, e.g., the complex subtraction results in Ashcraft & Kirk, 1998). Thus, as noted elsewhere (Ashcraft, 1995), it may be very difficult to separate math anxiety and competence effects when more advanced math is tested. However, for the simpler math tested here, the lack of a basic competence difference among the anxiety groups, along with the prominence of anxiety effects on timed laboratory tasks, begs for a more acceptable interpretation than is provided by a simple competence model.

We propose a second reason for math-anxiety effects, the on-line math anxiety influence shown in the bottom half of Figure 5. We propose that there is an on-line reduction in the available working-memory capacity of high-math-anxiety individuals when their anxiety is aroused. This reduction should depress levels of performance in any math or math-related task that relies substantially on working memory, including not only addition with carrying but presumably any counting-based task (e.g., alphabet arithmetic; Logan & Klapp, 1991). It specifically includes math in which procedural knowledge is essential, for example, situations requiring carrying, borrowing, or sequencing and keeping track in a multistep problem. Our evidence here is consistent with this working-memory-based explanation; the obtained patterns are exactly what would be expected if the available capacity of working memory were reduced by an anxiety reaction.

As a first approximation to the specific working memory mechanism affected by math anxiety, we suggest adapting Eysenck and Calvo's (1992) explanation of the effect of generalized anxiety on cognition. According to this position, the anxiety reaction is one of attention to or even preoccupation with intrusive thoughts and worry. Because such thoughts and worry are attended, they therefore consume a portion of the limited resources of working memory. This reduces the available pool of resources to be deployed for task-relevant processing. If this model holds here, then math anxiety, when aroused, functions exactly like a dual-task procedure (e.g., Baddeley, 1986); that is, performance to the primary task is degraded because the secondary task, the anxiety reaction, compromises the capacity of working memory. The draining of resources implies continued, inappropriate (and self-defeating) attention to the cognitive components of the math-anxiety reaction and to intrusive thoughts, worry, preoccupation with performance evaluation, and the like. Although not definitive, Faust's (1992) high-math-anxiety participants reported significantly more intrusive or off-task thoughts during math than verbal anagram processing, and they also reported more of them than those reported by a low-math-anxiety group. As such, it may be appropriate to examine explanations involving a failure to inhibit attention to anxiety-induced distractions (e.g., Eysenck, 1992; Hopko, Ashcraft, Gute, Ruggiero, & Lewis, 1998).

We hypothesize that the locus of this effect is in the central executive component of working memory. This is the component most likely to implement the procedures of doing arithmetic and math and presumably is also the component in which intrusive thoughts and worry are registered and attended. A pressing need, therefore, is research on working memory's specific role in mental arithmetic and, in particular, research on the implementation of procedural knowledge.

We offer no theory here of the onset and development of mathematics anxiety. Little if any empirical work addresses the issues of precursors or contributors to math anxiety, and there have apparently been no studies examining math anxiety earlier than the sixth grade (Hembree, 1990). Because the evidence relating math anxiety, attitudes, and competence is correlational, issues concerning directionality or causality cannot be addressed.

The present results on working memory capacity, however, allow us to make some theoretical progress in this situation. It seems implausible to suggest that lower working memory capacity is a permanent consequence of either math anxiety or low math competence. Equally implausible in our view is the notion that lower working memory capacity is a precursor to math anxiety. If it were, then math anxiety should also affect cognitive performance in other domains, a prediction generally disconfirmed by the available evidence (e.g., the correlation between math anxiety and verbal achievement or aptitude is $-.06$, and $-.17$ with IQ). Instead, we suggest that the working memory disruption is temporary—an on-line effect in the context of math testing—whether conducted in the laboratory or the classroom.

We thus propose that two distinct mechanisms affect the performance of high-math-anxiety individuals. The first is adequately captured by Fennema's (1989) model, as diagrammed in the top half of Figure 5: Attitudes, including math anxiety, affect one's opportunities to gain math competence, and an individual's overall competence is one of two major influences on performance. In cases of favorable attitudes and low math anxiety, individuals would enjoy, seek out, and succeed at opportunities to achieve mastery and competence, with positive effects on their performance. In the reverse situation, with poor attitudes and high math anxiety, individuals exhibit global avoidance and attain lower competence, again with obvious effects on performance.

There is a separate on-line influence of math anxiety, however, one with a direct consequence for performance, as shown in the bottom half of Figure 5. Higher levels of math anxiety are related *to lower available* working memory capacity, not as a stable characteristic but as a temporary, functional reduction in processing capacity. The effect may be the result of an inability to inhibit attention to intrusive thoughts or distracting information or, perhaps equivalently, a failure to focus attention and effort on the task at hand. In either case, the available processing capacity of working memory is compromised, with transitory but important effects on cognitive performance.

A second, somewhat speculative pathway is also diagrammed in Figure 5, from the on-line math-anxiety reaction to a long-term memory component. We suggest that on-line math anxiety has an impact during original learning of difficult arithmetic and mathematics, probably beginning in the early years of middle school. Just as math anxiety compromises the functioning of working memory in on-line tasks, it probably exerts the same influence on students in the math classroom, reducing the working memory capacity needed for learning and mastery. This is a second possible explanation for the overall negative effect math anxiety has on higher math competence, an on-line disruption during learning due to transient working memory disruption.

We conclude with two more general remarks. First, math anxiety is not an epiphenomenon—it is not a cognitive appraisal about oneself that is unrelated to the nature of mental processing. For example, statistics students who do poorly on an exam claim that they become confused, are unable to focus on the task at hand, or keep thinking about how poor they are at math. Regardless of the subjectivity of these claims, they are entirely consistent with our main result: Math anxiety disrupts the on-going, task-relevant activities of working memory, slowing down performance and degrading its accuracy.

Second, we note the positive effects of following the advice with which this article began: that affect is an aspect of problem solving that deserves empirical attention (Mandler, 1989). Cognitive investigations that include individual difference characteristics like math anxiety are rare, yet may prove useful in gaining an understanding of domain-specific cognitions. Furthermore, it now appears that customary assessments of working memory span, especially those using arithmetic stimuli, are sensitive to at least two classes of influences: the central capacity and processing characteristics of the individual, to be sure, but also the transitory effects of anxiety in the testing situation. Given current and important efforts that relate working memory mechanisms to processes such as reading comprehension, memory retrieval, and the like, it would be sensible to consider the possibility that anxiety or other individual difference factors may be influencing both the assessments of individuals' working memory span and their on-line performance.

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