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The resistive effect of the solar wind on the orbits of solid bodies in the solar system

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Summary. The equations of motion of a body orbiting the Sun and passing through a radially moving stream of gas under the assumption that the body does not accrete any significant quantities of matter are obtained. An exact solution was found on the assumption the velocity of the gas exceeds the orbital velocity of the body. This was applied to the solid bodies found in the solar system for the present solar wind and the enhanced wind present during the Sun's T-Tauri phase. It was found that at present the Sun's radiation will be the dominant source of resistance for all solid objects but that in the past objects of average asteroidal mass or less would have undergone substantial rounding. Interplanetary grains would however have been ejected from the system altogether. This tends to indicate that any such objects observed today are unlikely to have been formed at the same time as the planets.

1 Introduction

In a previous paper (Donnison & Williams 1977, hereafter Paper I) the effect on the orbit of a gaseous protoplanet of a solar-type wind, was investigated under the assumption that accretion of the wind was substantial.

In this investigation we consider the effect of a wind on objects where significant accretion does not occur, the molecules of the wind instead being reflected from the surface of the object. This will be the case for the solid bodies found in the solar system such as zodiacal-light particles, meteorites, asteroids and the terrestrial planets.

2 The equations of motion and their solution

We consider a spherical object of constant mass M and radius R in orbit about a central star from which a radially flowing wind is being emitted. The model for the wind is taken to be identical to that in Paper I, that is where the flow is radial at a speed, V_{SW}, in excess of the escape velocity, the flow rate being $\dot{M}_{SW}$. Consequently, the gravitational field of the body is ignorable for any radius less than 5×10^{10} cm and the resistive force per unit mass taken from

Baines, Williams & Asebimo (1965) is

$$\mathbf{R}_{\text{res}} = \frac{\dot{M}_{\text{SW}} R^2}{4MV_{\text{SW}}} |\mathbf{V}| \mathbf{V} \tag{1}$$

V being the relative velocity. Conveniently, this expression is independent of the mode of reflection of the molecules. Under the given assumptions, the equations of motion of the body, in polar coordinates centred on the Sun, become

$$\ddot{r} - r\dot{\theta}^2 = -\frac{GM_\odot}{r^2} + \frac{\dot{M}_{\text{SW}} V_{\text{SW}}}{4M} \left(\frac{R}{r}\right)^2 \tag{2}$$

and

$$\frac{d(r^2\dot{\theta})}{dt} = -\frac{\dot{M}_{\text{SW}}}{4M} R^2\dot{\theta}. \tag{3}$$

Denoting the specific angular momentum by h, equation (3) can be written

$$\frac{dh}{d\theta} = -\frac{\dot{M}_{\text{SW}} R^2}{4M} = -\gamma h_0 \tag{4}$$

where γ is the constant $\dot{M}_{\text{SW}} R^2/4Mh_0$ which is a non-dimensional quantity, h_0 being the initial value of h. Equation (4) can be integrated immediately and using the boundary condition that $h = h_0$ at $\theta = 0$ gives

$$h = h_0(1 - \gamma\theta) \tag{5}$$

h therefore decreases linearly with increasing θ and is reduced to zero when $\gamma\theta$ reaches 1 and thereafter it remains zero.

When this state is reached equation (2) reduces to

$$\ddot{r} = -\frac{GM_\odot}{r^2}\left(1 - \frac{\gamma h_0 V_{\text{SW}}}{GM_\odot}\right) \tag{6}$$

which clearly yields two types of solution depending on whether the gravitational or the wind force dominates. If gravity dominates then $(\gamma h_0 V_{\text{SW}})/(GM_\odot) < 1$, and the time taken to fall into the Sun, τ_{infall}, is

$$\tau_{\text{infall}} = \tau_{\text{ff}} \Big/ \sqrt{\left(1 - \frac{\gamma h_0 V_{\text{SW}}}{GM_\odot}\right)} \tag{7}$$

where τ_{ff} is the free fall time. For $(\gamma h_0 V_{\text{SW}})/(GM_\odot) > 1$, the time taken for a body to be blown outwards to large distances by the wind is similarly given by

$$\tau_{\text{out}} \sim (r/r_0)\left[\tau_{\text{ff}} \Big/ \sqrt{\left[\left(\frac{\gamma h_0 V_{\text{SW}}}{GM_\odot}\right) - 1\right]}\right] \tag{8}$$

where r_0 is the initial distance of the body from the Sun. Since $\tau_{\text{ff}} \sim 1$ yr for distances of the order of an astronomical unit, it is clear that once the angular momentum of a body has been reduced to zero, it rapidly either falls into the Sun or is blown out of the system, depending on whether gravity or the wind dominates.

We now turn to the more interesting situation before h has been reduced to zero where $\gamma\theta < 1$. Let $u = 1/r$ and substitute for h from equation (5), then equation (2) takes the form

$$\frac{d^2u}{d\theta^2} + u = \frac{GM_\odot}{h_0^2}\,\frac{(1 - \gamma h_0 V_{SW}/GM_\odot)}{(1-\gamma\theta)^2}. \tag{9}$$

This equation can be solved fairly simply by using standard methods, and has the general solution

$$u = A\sin\theta + B\cos\theta + \frac{GM_\odot}{h_0^2}\left(1 - \frac{\gamma h_0 V_{SW}}{GM_\odot}\right)\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n-1)!\,\gamma^{2(n-1)}}{(1-\gamma\theta)^{2n}} \tag{10}$$

where A and B are constants determined by the boundary conditions. With $u = 1/q_0$ (q_0 being the initial perihelion distance) and $du/d\theta = 0$ at $\theta = 0$, the complete solution is

$$u = -\frac{1}{q_0} - \left(1 - \frac{\gamma h_0 V_{SW}}{GM_\odot}\right)\left[\sum_{n=1}^{\infty}(-1)^{n+1}(2n-1)!\,\gamma^{2(n-1)}\left(2n\gamma\sin\theta + \cos\theta - \frac{1}{(1-\gamma\theta)^{2n}}\right)\right] + \frac{\cos\theta}{q_0}. \tag{11}$$

Equations (6) and (11) are the exact solution to the equations of motion (4) and (9) Equation (11) is obviously difficult to handle, however the quantity γ will in general be small (for the majority of solid objects within the solar system it has a value in the range 10^{-3}–10^{-10}). In this case equation (11) reduces to

$$u = \frac{1}{q_0}\left[\left(\frac{e_0 + \alpha}{1 + e_0}\right)\cos\theta + \frac{1-\alpha}{(1+e_0)(1-\gamma\theta)^2}\right] \tag{12}$$

where $\alpha = (\gamma h_0 V_{SW})/(GM_\odot)$, and also is a non-dimensional quantity. On rearranging this equation we obtain

$$\left[\frac{q_0(1+e_0)(1-\gamma\theta)^2}{1-\alpha}\right]u = \left[1 + \left(\frac{e_0+\alpha}{1-\alpha}\right)(1-\gamma\theta)^2\cos\theta\right] \tag{13}$$

which is the equation of a conic with *semi-latus rectum* $l = [q_0(1+e_0)(1-\gamma\theta)^2]/(1-\alpha)$ and eccentricity $e = [(e_0+\alpha)/(1-\alpha)](1-\gamma\theta)^2$. The type of orbit actually pursued is determined by the value of α and γ. Clearly if $\alpha > (1-e_0)/2$ (remembering that γ is small) then the orbit will be hyperbolic and the body will escape to infinity. This case is not however applicable to most solid bodies in the solar system.

For most physical situations $\alpha \ll 1$ and equation (13) reduces to that of an ellipse given by

$$l_0(1-\gamma\theta)^2 u = 1 + e_0(1-\gamma\theta)^2\cos\theta \tag{14}$$

where l_0 is the initial *semi-latus rectum* of the orbit. Advancement of the perihelion is clearly not therefore a feature of this type of motion. From equation (13) we can draw the various types of orbit provided we select values for the parameters e_0, γ, α.

This was done for three particular cases, the value of e_0 being set at 0.7 and q_0 being used as the scale of length in each case. Fig. 1 shows the case with $\gamma = 10^{-8}$ and $\alpha \ll 1$ and clearly illustrates that the orbit is undergoing rounding and is moving inwards towards the Sun. It also indicates that there is virtually no precession of the semi-major axis. Fig. 2, for which

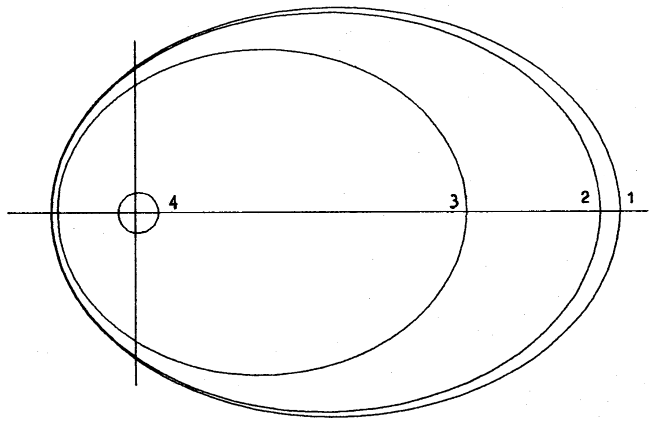

Figure 1. Indicates how the orbit changes with increasing values of θ for $\gamma = 10^{-8}$, $e_0 = 0.7$ and $\alpha \ll 1$. Curve (1) is the first orbit, (2) is after 10^5 orbits, (3) is after 10^6 orbits and (4) is after 10^7 orbits. It is clear from this diagram that both rounding and an inward drift of the orbit is occurring with no perihelion advance.

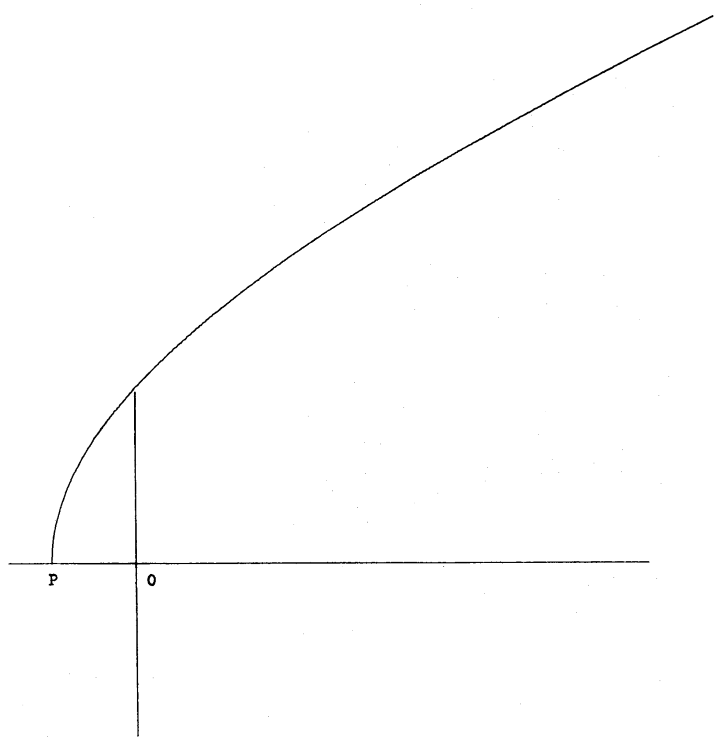

Figure 2. Illustrates the escape of a body on a hyperbolic orbit when the wind becomes important with $\gamma = 10^{-3}$, $e_0 = 0.7$ and $\alpha = 0.2$. *P* is the initial position of the body and the Sun is at 0.

$\gamma = 10^{-3}$ and $\alpha = 0.2$, illustrates the escape of the body on a hyperbolic orbit. Fig. 3 shows the variation of *r* with θ for $\gamma = 2 \times 10^{-3}$, $\alpha = 0.0505$. This is a case when the angular momentum of the body approaches zero when the motion becomes entirely radial. The orbit obviously contracts rapidly and the body will eventually fall into the Sun.

It is clear that we cannot, using the diagrams, specify the change in eccentricity and then obtain the number of orbits necessary for this to occur. It is only possible to use the reverse procedure of specifying a given number of orbits and then obtaining the change in eccentricity which occurs.

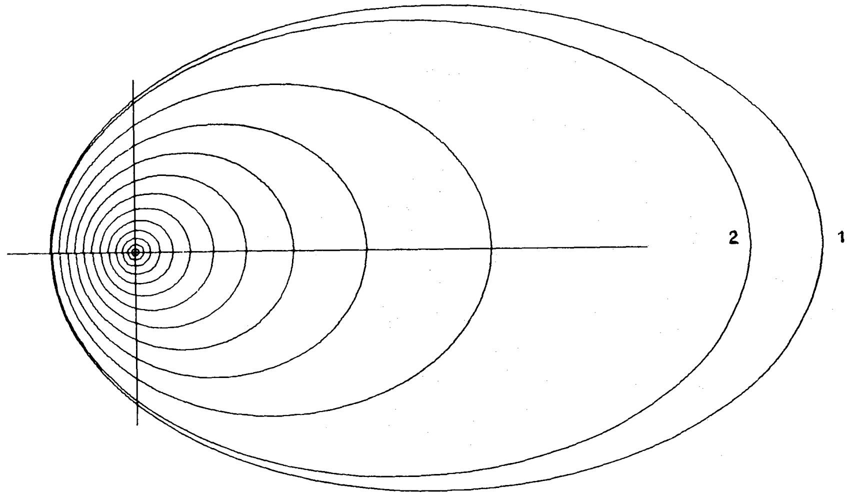

Figure 3. Shows the variation of r with θ for $\gamma = 2 \times 10^{-3}$, $e_0 = 0.7$, $\alpha = 0.0505$ as the angular momentum decreases very rapidly and the body spirals inwards. Curve (1) is the first orbit, (2) is after the second orbit, and every sixth subsequent orbit is shown until 79 is reached after which the body falls into the Sun.

We can, however, obtain the number of orbits necessary for a specified amount of rounding. From equation (14) the change in the *semi-latus rectum* of the orbit is

$$(l - l_0)/l_0 = -2\gamma\theta \tag{15}$$

while the change in the eccentricity is

$$(e - e_0)/e_0 = -2\gamma\theta. \tag{16}$$

The eccentricity and *semi-latus rectum* therefore decrease with θ (and hence the number of orbits) at the same rate so that total rounding can only occur when l itself goes to zero, so that complete rounding of an orbit cannot be achieved. If the initial semi-major axis is a_0 then since $l = a(1 - e^2)$ we obtain the change in semi-major axis using (15) and (16) as

$$(a - a_0)/a_0 = -2\gamma\theta\,[(1 + e_0^2)/(1 - e_0^2)] \tag{17}$$

which again obviously decreases with increasing θ. Combining equations (15), (16) and (17) we have

$$(l - l_0)/l_0 = (e - e_0)/e_0 = [(a - a_0)\,a_0]\,[(1 - e_0^2)/(1 + e_0^2)] = -2\gamma\theta. \tag{18}$$

This expression can be used to estimate the number of orbits required to obtain a specified degree of rounding or a specified change in the size of the orbit. We use this equation to obtain the characteristic rounding times for solid objects found in the solar system.

3 Application to physical situations

Equations (4) and (5) governing the motion of a body under the influence of a radial wind are similar to those due to the Poynting–Robertson effect (Robertson 1937), where the radiation emitted from the Sun exerts a force sufficient to cause a body to lose angular momentum and spiral inwards. As both the wind and the radiation will be acting at the same time it is of interest to determine under what conditions each particular process will be important. This basically reduces to a comparison of the wind momentum flow rate $\dot{M}_{SW}V_{SW}$ to the radiation momentum flow rate L/c, where L is the luminosity of the Sun and c the speed of light. For the present-day Sun $\dot{M}_{SW} = 10^{12}\,g\,s^{-1}$ and $L = 3.826 \times 10^{33}\,erg\,s^{-1}$ and $V_{SW} = 4.5 \times 10^7\,cm\,s^{-1}$ at the Earth's present distance (Allen 1973). Clearly $L/c \gg \dot{M}_{SW}V_{SW}$ and the effect of solar radiation on a body exceeds that due to the solar wind. The radiation pressure of the Sun is therefore more effective than the solar wind at the present time in causing all types of solid bodies in the solar system to move inwards and fall into the Sun or have their orbits rounded. Consequently we will not discuss the present-day solar wind any further.

There is, however, strong evidence from the observations of T-Tauri stars (Herbig 1962, 1977; Kuhi 1964; Rydgren, Ström & Ström 1976) that the mass loss rate and consequently the density of the wind may have been considerably larger during the pre-main sequence phase of the Sun's evolution. Typical mass loss rates for T-Tauri stars are of the order of $10^{-8}\,M_\odot\,yr^{-1}$, while levels as high as $10^{-6}\,M_\odot\,yr^{-1}$ have been estimated by Kuhi (1964).

The typical luminosity of such T-Tauri stars does not exceed $25\,L_\odot$ (Rydgren *et al.* 1976) so that the solar wind momentum flow rate now becomes dominant for both the typical and maximum mass flows. The resistive effect of the wind therefore dominates over that of the radiation during this phase and it is physically meaningful to determine the characteristic timescale necessary for various types of solid objects to have their orbits substantially altered by this large mass outflow. From equation (16) the time taken, τ, for an object to have its eccentricity changed from an initial value e_0 to a value e, on resubstituting for γ, is

$$\tau = 1.3\,\frac{\rho^{2/3}M^{1/3}}{\dot{M}_{SW}}\,l_0^2(1-e_0^2)^{3/2}\left(\frac{e_0-e}{e_0}\right) \tag{19}$$

For the typical T-Tauri mass loss rate this becomes

$$\tau_{typ} = 0.21 \times 10^{-17}\rho^{2/3}M^{1/3}l_0^2(1-e_0^2)^{3/2}\left(\frac{e_0-e}{e_0}\right) \tag{20}$$

while for the maximum value

$$\tau_{max} = 0.21 \times 10^{-19}\rho^{2/3}M^{1/3}l_0^2(1-e_0^2)^{3/2}\left(\frac{e_0-e}{e_0}\right) \tag{21}$$

For substantial rounding, e will be close to zero, we will therefore set $e = 0$ throughout. It is now of interest to determine the timescale for rounding of specific objects within the solar system.

3.1 THE TERRESTRIAL PLANETS

We take $M = 5 \times 10^{27}\,g$ and $\rho \sim 5\,g\,cm^{-3}$ as typical values for the terrestrial planets, so that equations (20) and (21) become

$$\tau = \left.\begin{matrix} 1.03 \times 10^{-8} \\ 1.03 \times 10^{-10} \end{matrix}\right\}\,l_0^2(1-e_0^2)^{3/2} \tag{22}$$

(the upper value is that for the typical outflow rate and the lower for the maximum, this convention will be used throughout). As a typical initial value we shall take $e_0 = 0.7$. It is immediately obvious that for the typical mass loss value rounding can only occur on a timescale of $4.5 \times 10^9\,yr$. This exceeds the duration of this type of wind which cannot have lasted longer than the pre-main-sequence phase of the Sun (of the order of $10^7\,yr$). The rounding time for the maximum outflow value is a factor 10^2 less than this but this again exceeds the duration of the wind which cannot reasonably have lasted for more than 5×10^5 yr ($M_\odot/2$ is the largest observed quantity lost during the outflow phase). Thus in neither case does the wind last sufficiently long enough for significant rounding of terrestrial planets to occur.

3.2 ASTEROIDS

Observations of asteroids indicate that they have a wide mass and size distribution, and we consider the effects of the wind on two typical types: the large asteroids which are usually several hundred kilometres in extent with masses $\sim 10^{22}$ g and the smaller kilometre-size bodies with masses $\sim 10^{16}$ g. In both cases the probable density is around 3.5 g cm^{-1} (Allen 1973). For large asteroids the rounding times for the two wind rates are

$$\tau = \left.\begin{matrix} 1.04 \times 10^{-10} \\ 1.04 \times 10^{-12} \end{matrix}\right\} l_0^2(1-e_0^2)^{3/2}. \tag{23}$$

It is clear that if we assume asteroidal distances for l_0 then the typical wind rate yields a time of $\sim 10^9$ yr which far exceeds the time for the T-Tauri phase. Similarly for the maximum outflow rate which will take 10^7 yr exceeds 5×10^5 yr, so that in neither case will the large asteroids undergo significant rounding. For the kilometre-size asteroids the timescales of rounding are

$$\tau = \left.\begin{matrix} 1.04 \times 10^{-12} \\ 1.04 \times 10^{-14} \end{matrix}\right\} l_0^2(1-e_0^2)^{3/2}. \tag{24}$$

For the typical mass loss rate the rounding time for the asteroids is comparable to the duration of the wind, so that we should expect fairly significant rounding of their orbits. The corresponding timescale for the maximum rate indicates rounding in $\sim 10^5$ yr, so that again substantial rounding would be expected. This fits in fairly well with the observations of the asteroids which indicate a mean eccentricity as low as 0.14 (Allen 1973).

3.3 METEORITES

In the case of meteorites we take as a typical mass $M = 10^7$ g and density ~ 0.25 g cm^{-3} (Allen 1973) so that equation (21) becomes

$$\tau = \left.\begin{matrix} 0.18 \times 10^{-15} \\ 0.18 \times 10^{-17} \end{matrix}\right\} l_0^2(1-e_0^2)^{3/2}. \tag{25}$$

With l_0 as 1 AU this gives a rounding time of 500 yr for typical T-Tauri rate. The maximum wind decreases this value by a factor of 10^2 so that the meteorites will be swept into the Sun very rapidly. This suggests that most, if not all, the material left over from the planetary formation process was removed during this phase and that the meteoritic material now observed is not primordial but has been introduced by the disintegration of comets on their close approach to the Sun or by some other means subsequent to the T-Tauri phase. Conversely, if the meteorites are proved to be primordial in origin, then the Sun cannot have passed through the T-Tauri wind phase.

3.4 INTERPLANETARY GRAINS

In all cases discussed so far $\gamma \ll 1$ and the equations (15), (16) and (17) are valid. For typical grains where $R \sim 10^{-5}$ cm and $M \sim 4 \times 10^{-15}$ g (density ~ 1 g cm^{-3}) the value of γ for a T-Tauri flow is of the order of 10^4 which means that total loss of angular momentum is very rapid. The relevant timescale is therefore given by equation (8) which indicates that such grains will be totally removed from the system in a matter of a year or so.

4 Summary and conclusions

We have derived the equation of motion for a body orbiting the Sun and moving through a radially moving stream of gas under the assumption that the body does not accrete any significant amount of matter. An exact analytical solution, given by equations (6) and (9), was found on the assumption that the velocity of the wind greatly exceeds the orbital velocity of the body. Figs 1, 2 and 3 indicate how the dimensions and eccentricity of such orbits vary with θ, the angle swept out, for a number of values of the parameters α and γ. It was shown that at the present time the resistive effect due to radiation from the Sun will dominate that of the solar wind for all solid objects in the solar system. In the past, however, provided the solar wind was far larger than at present it was shown that it would have been the dominant resistive agent. Its effect on the terrestrial planets would have been small however and even the large asteroids would have undergone little rounding of their orbits during this phase. The smaller kilometre-size asteroids in contrast would have undergone substantial rounding, with the meteoritic material falling into the Sun within a relatively short time. The meteoritic material observed today cannot therefore be primordial in sense that it is a remnant of the original formation process and hence cannot have passed through the same evolutionary sequence as the planets. This lends support to the theory that most meteoritic material observed is derived from comets which have been tidally disrupted during their close approaches to the Sun. Finally, it is evident that any material that is present in the form of grains during the T-Tauri phase will be rapidly expelled from the vicinity of the Sun. It is therefore clear that any material left over during the formation process of the planets that is of standard meteoritic material size or smaller would have been removed on a fairly short timescale either by falling into the Sun or being blown out of the solar system.

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