LOCKHEED ELECTRONICS COMPANY

HOUSTON AEROSPACE SYSTEMS

A Division of Lockheed Aircraft Corporation

NASA CR 9?

THE RESPONSE OF HARD-LIMITING
BAND PASS LIMITERS TO PM SIGNALS

HASD 822228

LOCKHEED ELECTRONICS COMPANY
Telecommunications Department
Systems Analysis Section

October 22, 1968

Prepared Under
Contract NAS 9-5191
Task Order 21
Job Order 222

Prepared By: J. C. Chang

Reviewed By: J. A. Webb

Approved By: P. L. Harton

PREFACE

The Systems Analysis Branch of the Information Systems Division performs the task of communication systems engineering and analysis at the Manned Spaceflight Center. This NASA organization provides support to the Apollo program in the areas of performance specifications, testing, tradeoff studies, and predictions.

The Systems Analysis Section, a part of the Houston Aerospace Systems Division of the Lockheed Electronics Company, aids in the accomplishment of these tasks by maintaining and operating a developmental, electronic and mechanical production and testing capability. The capability includes provisions for testing prototype models of telecommunication subsystems and components. In addition, a subsystem and component analysis capability is provided. The Systems Analysis Section performs this function as a part of the broader responsibilities of the Telecommunications Systems Department.

The analysis described in this report was accomplished to provide additional information concerning the behavior of specific elements of the Apollo Communication System. It forms the basis for evaluating results that are to be obtained from laboratory measurements. The plan for making these measurements is described in HASD 822227.

The analysis contained herein concerns the performance of bandpass limiters, when PM signals are received. A continuation of this analysis will be reported in HASD 822229. In that document the analysis is extended to a cascade connected bandpass limiter and a phase detector.

This report has been prepared for the Information Systems Division of NASA's Manned Spacecraft Center under Contract NAS 9-5191. It is distributed for information purposes only. The conclusions and recommendations contained herein should neither be regarded as representing a firm position of NASA's Manned Spacecraft Center, nor should they obligate or commit the Center in any way.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF APPENDICES

SUMMARY

The output signal-to-noise ratio of a hard-limiting bandpass limiter is calculated for a phase-modulated signal. A direct approach using first order statistics of the random phase variable is employed for all input SNR levels. The computer program and the calculated results are given in this report. The resulting limiter SNR transfer characteristics are in close agreement with Blachman's result.

THE RESPONSE OF HARD-LIMITING BANDPASS LIMITERS TO PM SIGNALS

I. Introduction

The bandpass limiter is a non-linear device which is insensitive to the noisy irregularity of the amplitude of an input waveform. The effect of hard limiting of an AM or CW signal plus noise has been analyzed by Davenport (1). Other people (2) and (3) have done similar work concerning the performance of the limiter for AM signals.

Bandpass limiters are also used extensively in PM and FM receivers to remove amplitude fluctuations from the signal. Recently, Kuhar and Schilling (4) obtained a closed form approximation for the bandpass limiter signal-to-noise ratio characteristic, using Rice's approach (5). The work of Kuhar and Schilling agrees reasonably well with Davenport's result in the region of high signal-to-noise input. However, their result differs from that of Davenport in the threshold and very low input signal-to-noise regions. Sevy (6) has analyzed the effect of hard-limiting an angle-modulated signal and his conclusion is that the output signal-to-noise ratio of a limiter and zonal bandpass filter combination is the same for an angle-modulated signal and a CW signal. Lately, Blachman (7) gave a plot of the bandpass limiter output signal-to-noise ratio curve which is originally derived for a power law device (8).

The purpose of this report is to evaluate the concepts of Blachman and Sevy by performing the actual calculation. The calculation of the output signal-to-noise ratio of hard-limiting, bandpass limiters is accomplished in this report by the use of first order statistics. This direct approach will also be used advantageously in the calculation of the output signal-to-noise ratio of a limiter-phase detector combination in another report.

II. PROBABILITY DENSITY FUNCTION FOR THE RANDOM PHASE VARIABLE

As shown in Figure 1, we shall consider the effect of passing a sinusoidal signal and narrow band random noise through a limiter and zonal filter combination. The signal s(t) and noise n(t) can be expressed respectively as:

$$s(t) = a(t) \operatorname{Sin} \omega_c t + b(t) \operatorname{Cos} \omega_c t \quad (1)$$

$$n(t) = x(t) \operatorname{Sin} \omega_c t + y(t) \operatorname{Cos} \omega_c t \quad (2)$$

Where

$$a = \sqrt{2P_s} \operatorname{Cos}\theta = E_s \operatorname{Cos}\theta$$

$$b = \sqrt{2P_s} \operatorname{Sin}\theta = E_s \operatorname{Sin}\theta$$

$$E_s(t) = \text{rms value of signal} = \sqrt{a^2(t) + b^2(t)}$$

$$P_s = \text{signal power}$$

$$\theta(t) = \text{signal phase} = \tan^{-1}\left(\frac{b(t)}{a(t)}\right)$$

$$x(t) = \sigma(t) \operatorname{Cos}\eta(t)$$

$$y(t) = \sigma(t) \operatorname{Sin}\eta(t)$$

$$\sigma(t) = \text{rms value of noise} = \sqrt{x^2(t) + y^2(t)}$$

$$\eta = \text{noise phase angle} = \tan^{-1}\left(\frac{y(t)}{x(t)}\right)$$

Equations (1) and (2) can also be expressed as:

$$s(t) = E_s\,(\text{Cos}\theta\ \text{Sin}\ \omega_c t + \text{Sin}\theta\ \text{Cos}\ \omega_c t)$$

$$= E_s\ \text{Sin}\ (\omega_c t + \theta) \qquad (3)$$

$$n(t) = \sigma(t)\ (\text{Cos}\eta\ \text{Sin}\ \omega_c t + \text{Sin}\eta\ \text{Cos}\ \omega_c t)$$

$$= \sigma(t)\ \ \text{Sin}(\omega_c t + \eta) \qquad (4)$$

The input signal plus noise of the limiter is:

$$e_{in}(t) = s(t) + n(t)$$

$$= [a(t) + x(t)]\ \text{Sin}\ \omega_c t + [b(t) + y(t)]\ \text{Cos}\ \omega_c t$$

$$= R(t)\ \text{Sin}\ (\omega_c t + \phi) \qquad (5)$$

Where

$$R(t) = \sqrt{[a(t) + x(t)]^2 + [b(t) + y(t)]^2}$$

$$\phi(t) = \tan^{-1}\left(\frac{a(t) + x(t)}{b(t) + y(t)}\right)$$

In equation (5), we have the expression of the input signal plus noise, $e_{in}(t)$, in the terms of two orthogonal components, i.e.,

$$e_{in}(t) = X(t)\ \text{Cos}\ \omega_c t + Y(t)\ \text{Sin}\ \omega_c t \qquad (6)$$

Where

$$X(t) = x(t) + a(t)$$

$$Y(t) = y(t) + b(t)$$

The probability density functions of X(t) and Y(t) are:

$$P(X) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(X - a)^2}{2\sigma^2}} \qquad (7)$$

$$P(Y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(Y - b)^2}{2\sigma^2}} \qquad (8)$$

Where

σ^2 = variance of random variables X and Y

= noise power

The joint probability density function of the two independent random variables, X and Y, is:

$$P(X, Y) = P(X)\,P(Y)$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{(X - a)^2 + (Y - b)^2}{2\sigma^2}} \qquad (9)$$

We want to find the joint probability density function, $P(R,\phi)$, of the envelope R and the random phase ϕ, put

$$e_{in}(t) = X(t) + Y(t)$$

$$= R(t)\,\mathrm{Cos}[\phi(t)] + R(t)\,\mathrm{Sin}[\phi(t)] \qquad (10)$$

Where

$$R(t) = \sqrt{X^2(t) + Y^2(t)}$$

$$\phi(t) = \tan^{-1}\left[\frac{Y(t)}{X(t)}\right]$$

Since

$$P(X, Y)dX\,dY = P(X, Y)\,R\,dR\,d\phi$$

$$= P(R, \phi)\,dR\,d\phi \qquad (11)$$

From equation (11), we have

$$P(R, \phi) = R(t)\,P(X, Y)$$

$$= \frac{R}{2\pi\sigma^2}\,e^{-\frac{(X-a)^2 + (Y-b)^2}{2\sigma^2}} \qquad (12)$$

Where

$$X(t) = R(t) \cos[\phi(t)] \quad (13)$$

$$Y(t) = R(t) \sin[\phi(t)] \quad (14)$$

Substituting equations (13) and (14) into equation (12), we get,

$$P(R,\phi) = \frac{R}{2\pi\sigma^2} e^{-\frac{(R \cos\phi - a)^2 + (R \sin\phi - b)^2}{2\sigma^2}}$$

$$= \frac{R}{2\pi\sigma^2} e^{-\frac{[R^2 + a^2 + b^2 - 2aR\cos\phi - 2bR\sin\phi]}{2\sigma^2}} \quad (15)$$

Notice that

$$R^2 + a^2 + b^2 - 2aR \cos\phi - 2bR \sin\phi$$

$$= R^2 + E_s^2 - 2RE_s (\cos\phi \cos\theta - \sin\phi \sin\theta)$$

$$= R^2 + E_s^2 - 2RE_s \cos(\phi - \theta) \quad (16)$$

where

$$E_s^2 = a^2(t) + b^2(t)$$

$$a(t) = E_s \cos\theta$$

$$b(t) = E_s \sin\theta$$

Thus equation (15) becomes

$$P(R,\phi) = \frac{R}{2\pi\sigma^2} e^{-\frac{[R^2 + E_s^2 - 2RE_s \cos(\phi - \theta)]}{2\sigma^2}} \qquad (17)$$

The probability density function $P(\phi)$ of the random phase variable ϕ is:

$$P(\phi) = \int_0^\infty P(R,\phi)dR$$

$$= \int_0^\infty \frac{R}{2\pi\sigma^2} e^{-\frac{[R^2 + E_s^2 - 2RE_s \cos(\phi - \theta)]}{2\sigma^2}} dR \qquad (18)$$

Since

$$R^2 + E_s^2 - 2RE_s \cos(\phi - \theta) = [R - E_s \cos(\phi - \theta)]^2$$

$$+ E_s^2[1 - \cos^2(\phi - \theta)] ,$$

equation (18) can be expressed as:

$$P(\phi) = \frac{e^{-\frac{E_s^2[1-\cos^2(\phi-\theta)]}{2\sigma^2}}}{2\pi\sigma^2} \int_0^\infty Re^{-\frac{[R - E_s \cos(\phi-\theta)]^2}{2\sigma^2}} dR$$

$$= \frac{1}{2\pi\sigma^2} e^{-\frac{E_s^2[1 - \cos^2(\phi-\theta)]}{2\sigma^2}}$$

$$\left\{ \int_{0}^{\infty} 2\sigma^2 \left[\frac{R-E_s \operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}\right] e^{-\frac{[R - E_s \operatorname{Cos}(\phi-\theta)]^2}{2\sigma^2}} \, d\left[\frac{R - E_s \operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}\right] \right.$$

$$\left. + E_s \operatorname{Cos}(\phi-\theta) \int_{0}^{\infty} e^{-[R - E_s \operatorname{Cos}(\phi-\theta)]^2/2\sigma^2} \, dR \right\}$$

$$= \frac{1}{2\pi\sigma^2} \, e^{-\frac{E_s{}^2[1 - \operatorname{Cos}^2(\phi-\theta)]}{2\sigma^2}} \left\{ \int_{-\frac{E_s{}^2 \operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}}^{\infty} \sigma^2 \, e^{-\alpha^2} \, d\alpha^2 \right.$$

$$\left. + \sqrt{2}\,\sigma \, E_s \operatorname{Cos}(\phi-\theta) \int_{-\frac{E_s \operatorname{Cos}(\phi-\theta)}{\sqrt{2}\,\sigma}}^{\infty} e^{-\alpha^2} d\alpha \right\} \qquad (19)$$

The error function erf(x) , is defined as:

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-\alpha^2} d\alpha \qquad (20)$$

and,

$$\operatorname{erf}(-x) = -\operatorname{erf}(x) \qquad (21)$$

Using the relation of equations (20) and (21), equation (19) becomes,

$$P(\phi) = \frac{1}{2\pi\sigma^2} e^{-\frac{E_s^2[1 - \cos^2(\phi-\theta)]}{2\sigma^2}} \left\{ \sigma^2 e^{-\frac{E_s^2\cos^2(\phi-\theta)}{2\sigma^2}} \right.$$

$$\left. + \sqrt{\frac{\pi}{2}}\, \sigma\, E_s \cos(\phi-\theta) \left\{ 1 + \operatorname{erf}\left[\frac{E_s\cos(\phi-\theta)}{2\,\sigma}\right] \right\} \right\}$$

$$= \frac{e^{-\frac{E_s^2}{2\sigma^2}}}{2\pi} + \frac{E_s\cos(\phi-\theta)}{2\sqrt{2\pi}\,\sigma} e^{-\frac{E_s^2[1-\cos^2(\phi-\theta)]}{2\sigma^2}}$$

$$\left[1 + \operatorname{erf}\left(\frac{E_s\cos(\phi-\theta)}{\sqrt{2}\,\sigma}\right)\right] \qquad (22)$$

Now, put

$$Z = \frac{\frac{E_s^2}{2}}{\sigma^2}$$

$$= \text{Input Signal-to-Noise Ratio}$$

and we have

$$P(\phi) = \frac{e^{-Z}}{2\pi} + \frac{\cos(\phi-\theta)}{2} \sqrt{\frac{Z}{\pi}}\, e^{-Z(1 - \cos^2(\phi-\theta))}$$

$$[1 + \operatorname{erf}(\sqrt{Z}\cos(\phi-\theta))] \qquad (23)$$

Equation (23) is the probability density function of the random phase variable ϕ with input signal-to-noise ratio Z and signal phase angle θ.

III. Limiter Output Signal-to-Noise Ratio

Suppose we pass the input signal plus noise e_{in} through a limiter, the output e_L of the limiter is:

$$e_L = \begin{cases} +V_L & \text{Sign of } e_{in} \text{ is positive} \\ -V_L & \text{Sign of } e_{in} \text{ is negative} \end{cases} \qquad (24)$$

In general, e_L is a series of square waves with period T and random phase ϕ as shown in Figure 2. The Fourier series expansion of e_L is:

$$e_L(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \text{ Cos } n\omega t + b_n \text{ Sin } n\omega t) \qquad (25)$$

Where

$$a_n = \frac{2}{T} \int_{-t_\phi}^{T-t_\phi} e_L(t) \text{ Cos}(n\omega t)dt \qquad (26)$$

$$b_n = \frac{2}{T} \int_{-t_\phi}^{T-t_\phi} e_L(t) \text{ Sin}(n\omega t)dt \qquad (27)$$

From equation (26), we have

$$a_1 = \frac{2}{T} \int_{-t_\phi}^{\frac{T}{2} - t_\phi} V_L \text{ Cos}\omega t dt - \frac{2}{T} \int_{\frac{T}{2} - t_\phi}^{T-t_\phi} V_L \text{ Cos } \omega t \text{ dt} \qquad (28)$$

Since $T = \frac{2\pi}{\omega}$, equation (28) becomes

$$a_1 = \frac{V_L}{\pi} \int_{-\phi}^{\pi-\phi} \text{Cos}\phi \, d\phi - \frac{V_L}{\pi} \int_{\pi-\phi}^{2\pi-\phi} \text{Cos}\phi \, d\phi$$

$$= \frac{4V_L \text{ Sin}\phi}{\pi} \qquad (29)$$

Likewise, from equation (27) we get

$$b_1 = \frac{4V_L \text{ Cos}\phi}{\pi} \qquad (30)$$

The statistics of the random phase variable ϕ can be assumed stationary (in wide sense) and ergodic. Therefore, the first zonal filter output voltage can be expressed as

$$e_{out} = \frac{4V_L}{\pi} [\text{Sin}\phi \text{ Cos } \omega_c t + \text{Cos}\phi \text{ Sin } \omega_c t] \qquad (31)$$

where the random phase variable ϕ is described by its probability density function $p(\phi)$.

The in phase part of $e_{out}(t)$ is $\frac{4V_L \text{ Cos } \phi}{\pi}$

and the quadrature part is $\frac{4V_L \text{ Sin } \phi}{\pi}$.

The output signal, S_o may be represented as the ensemble average of $e_{out}(t)$, i.e.

$$V_o = E\left\{e_{out}(t)\right\}$$

$$= \frac{4V_L}{\pi}\left[E\left\{\text{Cos}\phi\right\}\ \text{Sin}\ \omega_c t + E\left\{\text{Sin}\phi\right\}\ \text{Cos}\ \omega_c t\right] \quad (32)$$

Where

$$E\left\{\text{Cos}\phi\right\} = \int_0^{2\pi} \text{Cos}\phi\ P(\phi)\ d\phi$$

$$E\left\{\text{Sin}\phi\right\} = \int_0^{2\pi} \text{Sin}\phi\ P(\phi)\ d\phi$$

$$P(\phi) = \frac{e^{-Z}}{2\pi} + \sqrt{\frac{Z}{\pi}}\ \frac{\text{Cos}(\phi-\theta)}{2}\ e^{-Z(1-\text{Cos}^2(\phi-\theta))}$$

$$\left[1 + \text{erf}(\sqrt{Z}\ \text{Cos}(\phi-\theta))\right]$$

Since $P(\phi)$ is an even function of ϕ,

$E\{\text{Sin}\phi\}= 0$. This fact has been proven in the actual calculation of $E\{\text{Sin}\phi\}$.

Thus equation (32) becomes

$$V_o = \frac{4V_L}{\pi}\ E\{\text{Cos}\phi\}\text{Sin}\ \omega_c t \quad (33)$$

Mean square signal, S_o is then

$$S_o = \frac{1}{2}\left(\frac{4V_L}{\pi}\right)^2 \quad (E\{\cos\phi\})^2 \tag{34}$$

The noise at the output of the first zonal filter consists of whatever is left after the signal portion is subtracted out, i.e.,

$$e_{no} = \frac{4V_L}{\pi} \left[(\cos\phi - E\{\cos\phi\}) \sin \omega_c t + (\sin\phi - E\{\sin\phi\}) \cos \omega_c t\right]$$

$$= e_{ni} + e_{nq} \tag{35}$$

From equation (35), we can get the mean square noise power, i.e.,

$$E\{e_{no}^2\} = \frac{8V_L^2}{\pi^2} E\left\{(\cos\phi - E\{\cos\phi\})^2\right\} + \frac{8V_L^2}{\pi^2} E\left\{(\sin\phi - E\{\sin\phi\})^2\right\}$$

$$= N_{oi} + N_{oq} \tag{36}$$

Where

N_{oi} = in phase noise power

$$= \frac{8V_L^2}{\pi^2} \quad E\left\{(\text{Cos}\phi - E\{\text{Cos}\phi\})^2\right\} \tag{37}$$

N_{oq} = noise power in quadrature

$$= \frac{8V_L^2}{\pi^2} \quad E\left\{(\text{Sin}\phi - E\{\text{Sin}\phi\})^2\right\}$$

And

$$E\left\{(\text{Cos}\phi - E\{\text{Cos}\phi\})^2\right\} = \int_0^{2\pi} (\text{Cos}\phi - E\{\text{Cos}\phi\})^2 P(\phi)\, d\phi \tag{38}$$

$$E\left\{(\text{Sin}\phi - E\{\text{Sin}\phi\})^2\right\} = \int_0^{2\pi} (\text{Sin}\phi - E\{\text{Sin}\phi\})^2 P(\phi)\, d\phi$$

$$= \int_0^{2\pi} \text{Sin}^2\phi\, P(\phi)\, d\phi \tag{39}$$

The noise in quadrature affects the phase parameter ϕ much more than the in-phase noise at high input signal to noise ratio. N_{oi} and N_{oq} are equally important in the very low input signal to noise ratio region.

The effective output noise is:

$$N_o = N_{oq}\left[1 + \left(\frac{N_{oi} \quad N_{oq}}{[S_o + N_{oi}]^2}\right)^{\frac{1}{2}}\right] \qquad (40)$$

Substituting equations (34), (37) and (38) into equation (40), we have

$$N_o = \frac{8V_L^{\,2}}{\pi^2} \quad E\left\{(\mathrm{Sin}\phi - E\{\mathrm{Sin}\phi\})^2\right\}$$

$$\left[1 + \frac{[E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\} \; E\,\{\mathrm{Sin}^2\phi\}]^{\frac{1}{2}}}{[(E\{\mathrm{Cos}\phi\})^2 + E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}]}\right] \qquad (41)$$

From equations (34) and (41), we obtain the output signal to noise ratio of the limiter, i.e.,

$$\frac{S_o}{N_o} = \frac{(E\{\mathrm{Cos}\phi\})^2}{N_{oq}\left[1 + \frac{[E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\} \; E\{\mathrm{Sin}^2\phi\}]^{\frac{1}{2}}}{[(E\{\mathrm{Cos}\phi\})^2 + E\{(\mathrm{Cos}\phi - E\{\mathrm{Cos}\phi\})^2\}]}\right]}$$

(42)

Where

$$E\{f(\phi)\} = \int_0^{2\pi} f(\phi)\; P(\phi)\; d\phi$$

= ensemble average of $f(\phi)$.

$$N_{oq} = E\{(\mathrm{Sin}\;\phi - E\{\mathrm{Sin}\;\phi\})^2\}$$

IV. A COMPARISON OF COMPUTATIONS

A computer program has been compiled to calculate the output signal-to-noise ratio of the limiter (equation (42)). The computer program is listed in Appendix A. The results of the calculation are shown in Appendix B. For each input signal-to-noise ratio Z, the Appendix tabulation gives the calculated results of the output signal-to-noise ratio $\frac{S_o}{N_o}$ and the ratio of $\frac{S_o/N_o}{Z}$ given in dB.

The calculation also shows us that $E\{\sin \phi\} = 0$.

A plot of $\frac{S_o/N_o}{Z}$ versus Z (input signal-to-noise ratio) is shown in Figure 3. Figure 3 includes the results of Davenport, Springett, and Blachman. Our calculated result matches pretty well with Blachman's result.

V. REFERENCES

1. W. B. Davenport, "Signal-to-Noise Ratios in Bandpass Limiters", J. Applied Physics, Vol. 24, pp. 720-727, June 1953.

2. J. Galejs, "Signal-to-Noise Ratios in Smooth Limiters", IRE Trans. Information Theory, Vol. II-5 pp. 79-85, June 1959.

3. J. D. Springett, "A Note on Signal-to-Noise and Signal-to-Noise Spectral Density Ratios at the Output of a Filter-Limiter Combination", JPL Space Programs Summary, No. 37-36, Vol. IV.

4. J. Kuharadi, D. L. Schilling, "Signal-to-Noise Ratio for Band-pass Limiters", IEEE Trans., Vol. AES-4 No. 1, pp. 125-128, January, 1968.

5. S. O. Rice," Noise in FM Receivers", Time Series Analysis, M. Rosenblatt, Ed. New York; Wiley 1963.

6. J. L. Sevy, "The Effect of Hard Limiting on Angle-Modulated Signal-Plus-Noise", IEEE Trans. Vol. AES-4, No. 1, pp. 24-30, January, 1968.

7. N. M. Blachman, "The Output Signal-to-Noise Ratio of a Bandpass Limiter", IEE Trans. Vol. AES-4, No. 1, pp. 635, July 1968.

8. N. M. Blachman, "The Output Signal-to-Noise Ratio of a Power Law Device", J. of Applied Physics, Vol. 24, No. 6, June 1953, pp. 783-785.

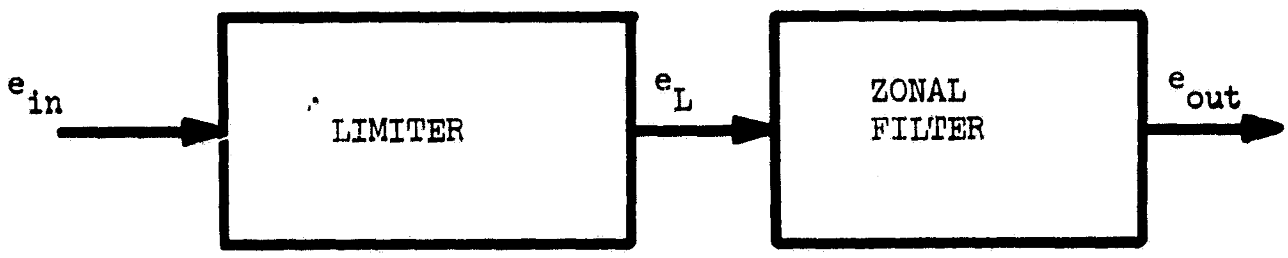

FIGURE 1 LIMITER-ZONAL FILTER COMBINATION

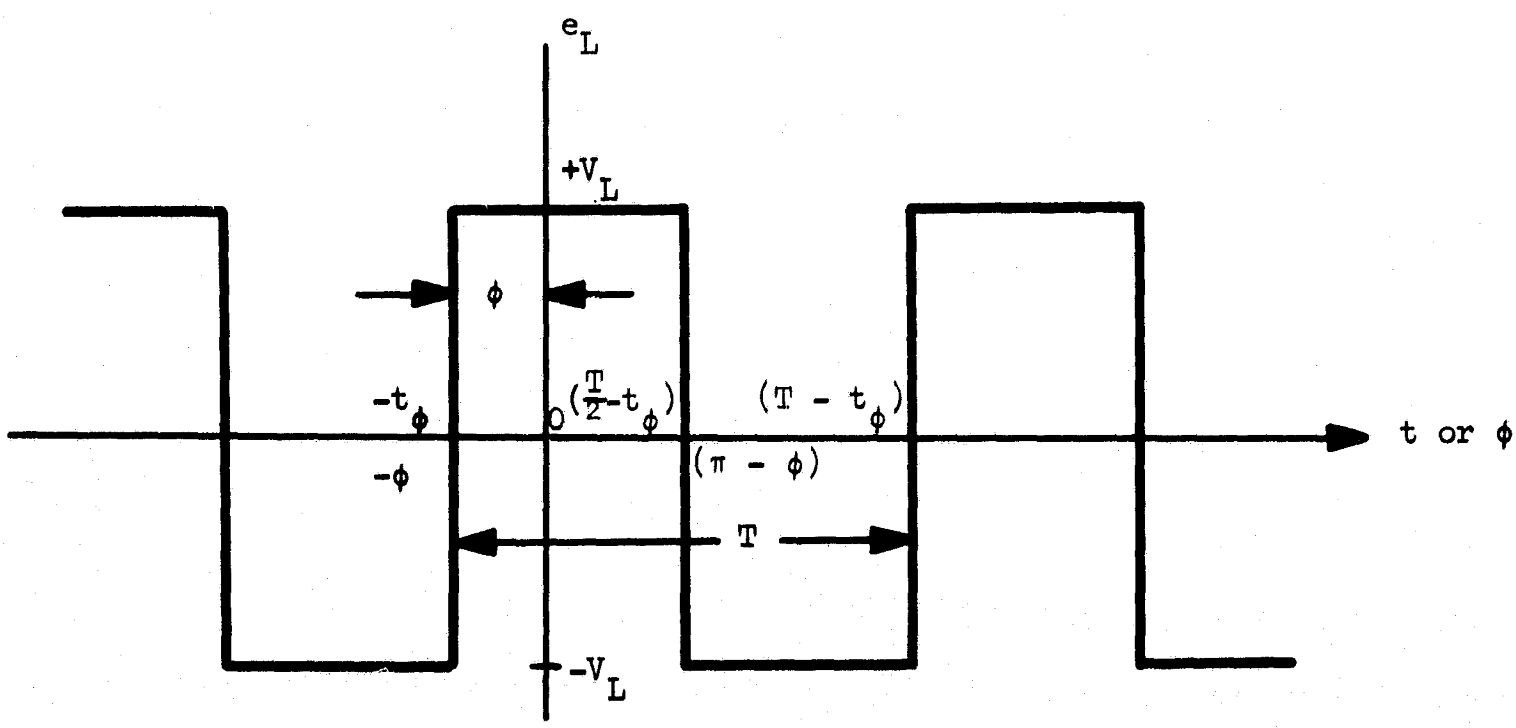

FIGURE 2 LIMITER OUTPUT WAVEFORM

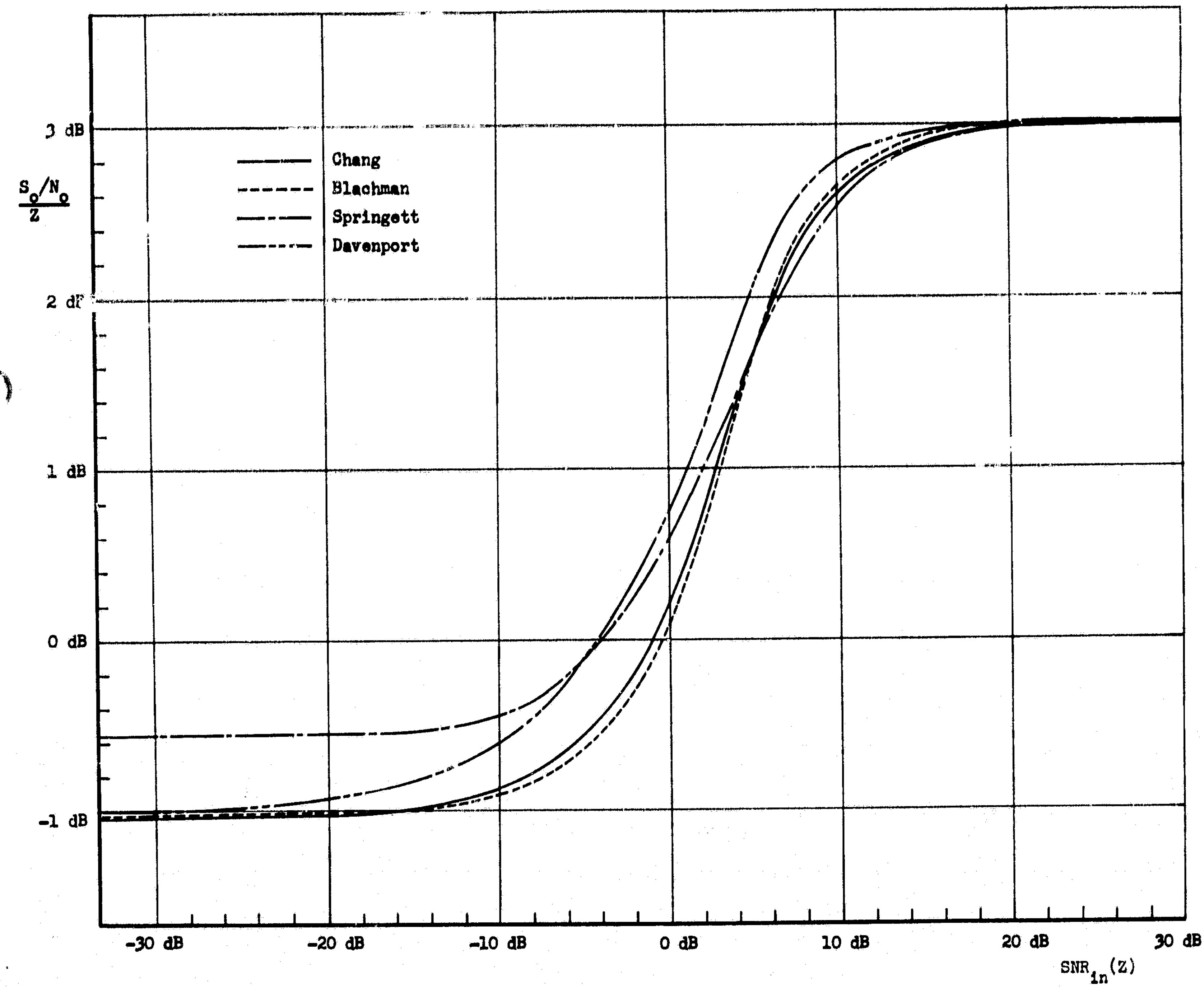

FIGURE 3: LIMITER SNR TRANSFER CHARACTERISTICS

APPENDIX A

COMPUTER PROGRAM

FORTRAN IV   MODEL 44 PS   VERSION 2, LEVEL 1   DATE 68254

```
      DOUBLE PRECISION THETA,XCOS(6000),DEL,YSNRI,XCOS2,X7,X8,SN,SNR2,
     1 Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,DELT,XCOS3,X1,X2,X3,X4,X5,X6,X13,X14,
     2  BB,SNR3,Y9,X9,DEL1,DEL2,DELT1,DELT2,AA1,AA2,AA3,XSIN(6000),
     3 DELT3,SNR4,AA4,A4,Y10,Y11,YSNR,Y12,X10,X11,XSQR,X12,A42,A44
      DO 100 I1=1,2880
      THETA=I1*3.14159265358979D2/1440.D0
       XCOS(I1)=DCOS(THETA)
       XSIN(I1)=DSIN(THETA)
  100  CONTINUE
      DO 800 I8=1,10
      J8=I8-6
       BB=10.D0**J8
       DO 300 I3=1,10
       DEL=0.D0
        DEL1=0.D0
       DEL2=0.D0
      YSNRI=I3*1.D0*BB
      DO 400 I4=1,2880,2
       IF(YSNRI.GE.60.0)GO TO 2
       Y10=DEXP(YSNRI)
      Y11=1.D0/Y10
      GO TO 4
    2  Y11=0.D0
    4  YSNR=DSQRT(YSNRI)
       I7=I4+1
       XCOS2=XSIN(I4)*XSIN(I4)
       Y6=YSNRI*XCOS2
       IF(Y6.GE.60.0)GO TO 10
       Y1=YSNR*XCOS(I4)
       Y2=DERF(Y1)
      Y3=3.14159265358979D2
      Y4=DSQRT(Y3)
       Y5=Y4*(1.D0+Y2)*Y1
       Y7=DEXP(Y6)
       Y12=Y5/Y7
       GO TO 20
   10  Y12=0.D0
   20 Y8=(Y11+Y12)*XSIN(I7)
      Y9=(Y11+Y12)*XCOS(I7)
       DEL2=DEL2+Y8/1440.D0
      DEL1=DEL1+Y9/1440.D0
  400  CONTINUE
       DELT=0.D0
       DELT1=0.D0
       DELT2=0.D0
      DO 500 I5=1,2880,2
       IF(YSNRI.GE.60.0)GO TO 12
       X10=DEXP(YSNRI)
      X11=1.D0/X10
```

```
              GO TO 14
         12   X11=0.D0
         14   XSNR=DSQRT(YSNRI)
              I9=I5+1
              XCOS3=XSIN(I5)*XSIN(I5)
              X6=YSNRI*XCOS3
              IF(X6.GE.60.D0) GO TO 40
              X1=XSNR*XCOS(I5)
              X2=DERF(X1)
             X3=3.14159265[illegible]7932
             X4=DSQRT(X3)
              X5=X4*(1.D0+X2)*X1
              X7=DEXP(X6)
              X12=X5/X7
              GO TO 50
         40   X12=0.D0
         50  X13=XSIN(I9)-DEL2
             X14=XCOS(I9)-DEL1
             X8=(X11+X12)*X13*X13
             X9=(X11+X12)*X14*X14
              DELT1=DELT1+X8/1440.D0
             DELT2=DELT2+X9/1440.D0
        500   CONTINUE
              DEL=DEL1+DEL2
              DELT=DELT1+DELT2
             SNR=DEL*DEL/DELT
             SNR2=DEL*DEL/DELT1
             SNR3=DEL*DEL/DELT2
             AA1=SNR/YSNRI
             AA2=SNR2/YSNRI
              AA3=SNR3/YSNRI
              A42=DELT2/(DEL1*DEL1+DELT2)
              A44=DSQRT(A42)
              DELT3=DELT1*(1.D0+A44)
               SNR4=DEL1*DEL1/DELT3
             AA4=SNR4/YSNRI
              A4=10.*DLOG10(AA4)
             WRITE(6,3000)YSNRI,DEL1,DELT,DELT1,DELT2,DEL2,DEL,SNR,SNR2,
            1 SNR3,AA1,AA2,AA3,DELT3,SNR4,AA4,A4
       3000   FORMAT(1X,5D23.16,/,1X,5D23.16,/,1X,3D23.16,/,1X,4D23.16,//)
        400   CONTINUE
        800   CONTINUE
              END
```

APPENDIX B

CALCULATED DATA

APPENDIX B

CALCULATED DATA

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
1×10^{-5}	0.785401×10^{-5}	-1.049084
2×10^{-5}	0.157081×10^{-4}	-1.049067
3×10^{-5}	0.235622×10^{-4}	-1.049050
4×10^{-5}	0.314164×10^{-4}	-1.049033
5×10^{-5}	0.392707×10^{-4}	-1.049016
6×10^{-5}	0.471250×10^{-4}	-1.048999
7×10^{-5}	0.549794×10^{-4}	-1.048982
8×10^{-5}	0.628338×10^{-4}	-1.048965
9×10^{-5}	0.706883×10^{-4}	-1.048948
1×10^{-4}	0.785429×10^{-4}	-1.048931
2×10^{-4}	0.157092×10^{-3}	-1.048760
3×10^{-4}	0.235647×10^{-3}	-1.048589
4×10^{-4}	0.314209×10^{-3}	-1.048419
5×10^{-4}	0.392776×10^{-3}	-1.048248
6×10^{-4}	0.471350×10^{-3}	-1.048078
7×10^{-5}	0.549930×10^{-3}	-1.047908
8×10^{-5}	0.628516×10^{-3}	-1.047737
9×10^{-5}	0.707108×10^{-3}	-1.047567
1×10^{-3}	0.785707×10^{-3}	-1.047396
2×10^{-3}	0.157203×10^{-2}	-1.045693
3×10^{-3}	0.235897×10^{-2}	-1.043990
4×10^{-3}	0.314652×10^{-2}	-1.042289
5×10^{-3}	0.393469×10^{-2}	-1.040589
6×10^{-3}	0.472348×10^{-2}	-1.038890
7×10^{-3}	0.551288×10^{-2}	-1.037193
8×10^{-3}	0.630290×10^{-2}	-1.035496
9×10^{-3}	0.709353×10^{-2}	-1.033801

CALCULATED DATA (Continued)

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
1×10^{-2}	0.788477×10^{-2}	-1.032107
2×10^{-2}	0.158309×10^{-1}	-1.015234
3×10^{-2}	0.238382×10^{-1}	-0.984784
4×10^{-2}	0.319063×10^{-1}	-0.981840
5×10^{-2}	0.400348×10^{-1}	-0.965317
6×10^{-2}	0.482237×10^{-1}	-0.948907
7×10^{-2}	0.564725×10^{-1}	-0.932610
8×10^{-2}	0.647810×10^{-1}	-0.916423
9×10^{-2}	0.731489×10^{-1}	-0.900345
1×10^{-1}	0.815760×10^{-1}	-0.884374
2×10^{-1}	0.169045	-0.730272
3×10^{-1}	0.262162	-0.585509
4×10^{-1}	0.360708	-0.449045
5×10^{-1}	0.464480	-0.320027
6×10^{-1}	0.573292	-0.197748
7×10^{-1}	0.686968	-0.081614
8×10^{-1}	0.805337	-0.028879
9×10^{-1}	0.928237	0.134162
1	0.105551×10	0.234609
2	0.253507×10	1.029609
3	0.429088×10	1.554252
4	0.620156×10	1.904412
5	0.818429×10	2.140110
6	0.101935×10^{2}	2.301706
7	0.122087×10^{2}	2.415710
8	0.142227×10^{2}	2.498932
9	0.162339×10^{2}	2.561815

CALCULATED DATA (Concluded)

Z	$\frac{S_o}{N_o}$	$\frac{S_o/N_o}{Z}$ in dB
10	0.182425×10^2	2.610843
2×10	0.382725×10^2	2.818563
3×10	0.582800×10^2	2.883980
4×10	0.782834×10^2	2.916099
5×10	0.982934×10^2	2.935192
6×10	0.118287×10^3	2.947848
7×10	0.138288×10^3	2.986854
8×10	0.158288×10^3	2.963589
9×10	0.178289×10^3	2.968817
1×10^2	0.198289×10^3	2.972992
2×10^2	0.398291×10^3	2.991760
3×10^2	0.598292×10^3	2.997917
4×10^2	0.798292×10^3	3.001018
5×10^2	0.998292×10^3	3.002876
6×10^2	0.119829×10^4	3.004115
7×10^2	0.139829×10^4	3.004999
8×10^2	0.159829×10^4	3.005663
9×10^2	0.179829×10^4	3.006178
1×10^3	0.199829×10^4	3.006591
2×10^3	0.399829×10^4	3.008446
3×10^3	0.599829×10^4	3.009064
4×10^3	0.799829×10^4	3.009373
5×10^3	0.999829×10^4	3.009558
6×10^3	0.119983×10^5	3.009682
7×10^3	0.139983×10^5	3.009770
8×10^3	0.159983×10^5	3.009837
9×10^3	0.179983×10^5	3.009888
10×10^4	0.199983×10^5	3.009929
	0.399983×10^5	3.010115