The Return On Investment for Taxi Companies Transitioning to Electric Vehicles

A Case Study in San Francisco

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Abstract

We study whether taxi companies can simultaneously save petroleum and money by transitioning to electric vehicles. We propose a process to compute the return on investment (ROI) of transitioning a taxi corporation's fleet to electric vehicles. We use Bayesian data analysis to infer the revenue changes associated with the transition. We do not make any assumptions about the vehicles' mobility patterns; instead, we use a time-series of GPS coordinates of the company's existing petroleum-based vehicles to derive our conclusions.

As a case study, we apply our process to a major taxi corporation, Yellow Cab San Francisco (YCSF). Using current prices, we find that transitioning their fleet to battery electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEVs) is profitable for the company. Furthermore, given that gasoline prices in San Francisco are only 5.4% higher than the rest of the United States, but electricity prices are 75% higher; taxi companies with similar practices and mobility patterns in other cities are likely to profit more than YCSF by transitioning to EVs.

Keywords: Electric Vehicles, Bayesian Networks, Public Transportation

1 Introduction

Replacing standard petroleum taxis with EVs can save significant amounts of petroleum. For triple-shift taxis (those that are driven 24 hours a day) we estimate savings of approximately 15,000 liters each year per taxi, and 10,000 liters for double-shift (16 hours a day) taxis. However, taxi operators will only invest in EVs if it is economically viable. Therefore, we design a process to determine a taxi company's ROI in transitioning to electric vehicles. We first build a Bayesian model of taxi fleet mobility. We then show how to use the model to determine the ROI.

Existing research on transitioning petroleum vehicles to EVs has focused on bus transit and personally owned vehicles. Taxis do not travel on fixed routes like transit buses and are not parked a majority of the time like personally owned vehicles. As a consequence of these different mobility patterns, we were not able to apply any existing model to study the cost of transitioning a taxi fleet to EVs.

Our main contributions are:

1. *We propose a data-oriented process to compute a given taxi company's ROI in transitioning to electric vehicles.* The process uses time-series of GPS and passenger data collected from instrumented petroleum taxis. We use Bayesian data analysis to compute the costs of operating a taxi fleet consisting of petroleum vehicles, plug-in hybrid vehicles (PHEVs) or battery electric vehicles (BEVs).
2. *As a case study, we analyze adoption of PHEVs or BEVs by a taxi company with over 530 vehicles, Yellow Cab San Francisco.* We study different infrastructure scenarios, including battery switching and roadside charging. For each scenario, we quantify the revenue losses or gains, quantify the investment payback period, and extrapolate the analysis to a wide array of electricity and petroleum prices.
3. *We formulate the problem of locating battery switching stations that serve the taxi fleet as an optimization problem, and present a framework to compute this placement for a given city.* Using our algorithm, we find only three battery switching stations are needed for Yellow Cab San Francisco for BEVs to be profitable.

Our case study shows that both PHEVs and BEVs have a positive ROI given current vehicle and fuel prices in San Francisco.

The rest of this paper is organized as follows. Background and related work is given in Section 2. We present our process in Section 3. The results from our case study can be found in Section 4. Future work and limitations are discussed in Section 5, and finally we present our conclusions in Section 6. A brief background on Bayesian networks is given in Appendix A and the details of our optimization algorithm are presented in Appendix B.

2 Background and Related Work

This section presents an overview of electric vehicles, existing taxi corporation practices, and related work.

2.1 Electric Vehicle Types

We study two types of electric vehicles:

1. *Plug-in Hybrid Electric Vehicles (PHEVs).* PHEVs have a grid-chargeable battery and an internal combustion engine (ICE). PHEVs are powered completely from the battery for first portion of a trip, without using the ICE. The standard notation to describe a PHEV is "PHEVxxm", where xx refers to the distance in kilometers (km) the PHEV is expected to drive using only the battery. Once the battery has been nearly depleted, the ICE is used to propel the vehicle for the remainder of the trip and the battery may continue to power electronics onboard the vehicle.
2. *Battery Electric Vehicles (BEV).* BEVs are fully powered by batteries and do not have an ICE; these vehicles are not reliant on petroleum for transportation.

2.2 Terminology

We now define a few terms used throughout the paper.

- *SV.* SV is an abbreviation for fully ICE-powered petroleum vehicles.
- *Kilowatt-hour.* A kilowatt-hour (kWh) is a unit of energy. One kWh is the amount of energy used by a device consuming power at a rate of 1 kilowatt for 1 hour, equal to $3.6\text{x}10^6$ joules of energy.
- *Capacity.* The capacity of an EV is the amount of energy (usually stated in kWh) that can be stored in its battery.
- *Discharge Rate.* The discharge rate of an EV is the rate at which the EV consumes energy. It is analogous to the km per liter of an SV.
- *Battery Charging and Battery Switching.* There are two ways to refuel an EV. *Battery charging* is when the car is plugged into a charging unit with a connection to the electrical grid. These units transfer power from the grid into the battery at a rate depending on the type of charging unit and the availability of power in that location. This process takes several hours depending on the type of charging unit. The other option is *battery switching*, where a battery switching station physically changes the batteries in an EV. Here, a user comes to the station with a nearly-depleted battery and the battery is replaced with a fully charged battery. The depleted batteries are then charged at the switching station. This type of refueling is similar to petroleum vehicles, where the vehicle is refueled in minutes instead of hours. The first major manufacturer of switching station infrastructure states that the entire switching process takes 80 seconds [22]. We note that currently, battery switching is only available for BEVs.
- *Charge Rate.* The charge rate of an EV is the rate at which the vehicle draws power from the grid into the battery at a charging station. This rate is not limited by the battery, but by the amount of power that can be supplied by a charging unit [31] (which depends on the type of unit and the electrical connection the charger has to the grid). There are three STANDARD levels of charging. Level 1 charging is the slowest form and uses a 110V connection found in any standard electrical outlet in North America. Level 2 charging uses a 220V connection, which most homes and businesses also have for large appliances. Level 3 is known as "quick charging," and uses a 480V connection [11,12]. However, quick charging has not yet been widely deployed.
- *Range.* The range of a vehicle is the expected distance it can travel given normal driving conditions when fully fueled (using petroleum for SVs, electricity for BEVs, and both for PHEVs). PHEV manufacturers additionally state the range that can be driven using electric power before the ICE is used.

2.3 Taxi Operation Overview

We briefly describe the operation of a taxi company as it relates to our work and how its operating practices may change if it transitions its fleet to EVs. We model a specific type of taxi company: one where employee-drivers operate company-owned vehicles in *shifts* that typically last eight hours. At the end of the shift, drivers return the vehicle to the company premises where the vehicles are re-fueled and handed over to another driver for another shift. Drivers may only refuel the vehicles while they are not carrying passengers.

Fares may be pre-arranged by calling the company to schedule a pickup, or they can be arranged on-the-fly by signaling taxis as they drive past. We use the term *fare* to refer to a contract between a driver and a passenger to transport the passenger to a desired destination for some price. Each taxi company has its own pricing model, that is, how it charges for fares. It is usually a function of time, distance, and other incidental charges. A driver's goal is to complete as many fares as possible during their shift, as this is the sole source of revenue for the company. We assume if the vehicle does not have sufficient fuel to carry a potential customer to their destination, the customer is refused, leading to a loss in revenue.

To maximize the likelihood of fares, taxi drivers may continuously drive around looking for passengers or may wait at busy locations such as airports and city centers. This behavior is not fuel efficient, but the revenue from additional fares currently compensates for the cost of wasted fuel.

We now note how this existing operation may change if the taxi company were to convert their fleet to BEVs or PHEVs. Such a change can impact the frequency of refueling, the potential introduction of refueling delays between shifts, and driver behavior between fares.

- *BEVs* The range of a BEV is about a third of an SV. Thus, BEVs must be refueled about three times more often than SVs. Consequently, either drivers must refuel more often between fares or turn down more fares. Note that installing battery switching stations allows BEVs to be refueled as quickly as SVs. Therefore, there is no additional delay at the company premises between shifts.

  Many equations in the following sections are dependent on a variable τ, which represents the battery charge threshold below which taxi drivers switch their battery if they are at a location with a switching station. We use the notation $\cdot(\tau)$ to represent a variables value assuming the switching threshold is τ. We assume in our analysis that a BEV driver switches their battery whenever:

  1. Their battery's charge level is less than τ,
  2. The driver is at a location with a switching station—we assume drivers never modify their trajectories to switch their batteries.

  We discuss computing the optimal value of τ in Section 3.4.8.
- *PHEVs* PHEVs do not have to be refueled more often than SVs. However, the primary gain from switching to PHEVs is to reduce fuel costs by driving the taxis primarily using the battery. This reduction is possible only if taxis rarely switch to their ICE mode, which requires their batteries to be fully charged after the end of a shift. PHEV batteries cannot be switched in today's models. This introduces a large delay between shifts while the vehicles are charged. To avoid this delay, the taxi company could purchase additional PHEVs to ensure vehicle availability for the next shift. This issue is discussed in detail in Section 3.4.7.
- In both cases, a driver's practice of opportunistically attracting fares by driving around would be affected. Drivers need to trade off the benefit from additional fares for the cost of battery depletion.

2.4 Related Literature

Prior work on EVs can be divided into four main categories.

- *Public Transit.* Several papers have addressed the feasibility of hybrid and electric bus transit [1, 17, 55]. Gao and Kitirattragarn study the likely hybrid EV (HEV) penetration level in New York City and the potential environmental benefits of transitioning [2]. We are unaware of any economic study of a taxi company transitioning to EVs. We note that Better Place, a manufacturer of EV infrastructure, has recently completed a feasibility study of EVs in Tokyo, Japan, but the results from this study have yet to be published. They have also stated they will be conducting experiments in San Francisco [10], the location of our case study.
- *Optimal charging strategies.* Several papers address the issue of optimal EV charging times [9,12,14,21,24, 29]. Many of these studies conclude off-peak overnight charging of EVs is the best charging strategy for personally owned vehicles, because nearly all such vehicles are parked at this time and overnight charging puts the least burden on the electricity grid. However, we find taxi companies can only transition to EVs if the company can charge their fleet at any time.
- *Vehicle-to-grid services.* Several authors have considered scenarios where EVs can provide power to the grid during times of peak demand [29–31, 40]. Their work is not applicable here because public transit vehicles are not parked long enough to serve as vehicle to grid stores. Specifically, the authors of [21] show at any given point in a day, even during rush hour, more than 90% of all residential vehicles are parked. In comparison, however, the typical taxi in our study parked on average only 12% of each day.
- *Effect on electrical grid.* Electrifying transportation that is currently petroleum based adds a large load to the existing electrical grid. Many locations may not be able to accommodate this new load with current infrastructure. Studies considering the impact of EV penetration on the grid include [4, 14, 24, 25, 49]. Transitioning taxi fleets to EVs would affect the grid; however, studying these impacts are beyond the scope of their work. Similarly, the effect on electricity prices due to large scale adoption of EVs is beyond the scope of this work.

3 Data Oriented Process to Estimating ROI

Our goal is to calculate the company's ROI in transitioning their taxi fleet to EVs. To do this, we first build a model that allows us to study the company's existing SVs as if they have the constraints and specifications of EVs described in Section 2.2; this is explained in detail in Section 3.3.1. Then we use this model to compute the costs associated with operating a taxi fleet of BEVs or PHEVs as opposed to SVs. This process is described in Section 3.4

3.1 Inputs

Our process for determining the changes in revenue for the company as a result of switching to EVs requires the following inputs:

1. *Mobility Data.* A critical input to our taxi model is mobility data from the existing SV fleet. We require the periodic collection, from each taxi, of its geographical location and fare status, for a period of several weeks. This could be obtained by collecting a log file for each SV, where each record of the log file has a

time stamp, the GPS location of the SV, and whether there is a paying passenger currently in the vehicle.

The input dataset must be a set of *shift files*, where a shift file represents data for one drivers working shift as defined in Section 2.3. We require a set of shift files for each driver and each taxi.

2. *Reduced Coordinate Space.* A second input to our model is a reduced coordinate space that minimizes model dimensionality without overly affecting its correctness. We overlay the taxi company's geographical operating region with the set of points specified in the reduced space and we map GPS data to its closest grid coordinate using Euclidean distances.
3. *Fare Pricing Model.* Every taxi company has their own pricing function they use to charge for fares, and this needs to be given as input. Let r_{fare} be the cost of one fare. Most taxi companies use a function of the following form:

$$r_{\text{FARE}}(C_{\text{I}}, d, C_{\text{D}}, p, C_{\text{T}}, M) = C_{\text{I}} + dC_{\text{D}} + pC_{\text{T}} + M \quad (1)$$

where C_{I} is an initial cost, d is the distance traveled during the fare, C_{D} is a cost per kilometer, p is the time parked at traffic lights, C_{T} is the cost per minute of waiting at lights, and M is miscellaneous fees.
4. *Operating Costs.* Gasoline, electricity, and vehicle prices vary between different cities around the world.
5. *Vehicle Specifications* We require specification of EV parameters such as battery size, range, and charging rates.

3.2 Outputs

The process produces the following outputs:

1. The company's ROI based on the fraction of the fleet transitioned BEVs or PHEVs.
2. Assuming a PHEV transition, the number of additional vehicles that must be purchased so that each driver can begin their shift with a fully-charged vehicle.
3. Assuming a BEV transition, the total number of extra batteries the company must purchase. The number and location of battery switching stations needed is also determined.

3.3 Estimating Charge Levels

A necessary intermediate step in executing our process is to estimate the charge level of an EV battery based on its mobility pattern and initial state of charge. We first describe how we obtain this estimate in Section 3.3.1, then use this model to determine the ROI in Section 3.4.

3.3.1 Estimating EV Battery Charge Level

We develop a Bayesian model to infer an EV's charge level at any time t given the time-series of GPS coordinates from the cooresponding SV. We found Bayesian networks are a natural fit for inferring the hidden charge-level variable. The problem of estimating battery charge levels can be modeled as a *causal* graphical model, as explained in the following sections, and Bayesian networks are designed to infer variables' values in casual models. We refer the reader to Appendix A for further information on the type of Bayesian network we are using, and to Koller et al. [34] for detailed background information. Here, we present our specific model.

A *dynamic Bayesian network* tracks variables that change over time by observing them at discrete *timeslices*. A *timeslice* is an instantaneous point in time that we observe the network, and the kth timeslice is denoted t_k. These timeslices are spaced by a *timestep*, a period of time between two timeslices, which can be constant or variable. The choice of the timestep duration is difficult in many applications. In our case however, we have a natural solution to this problem—we associate one timestep for every GPS measurement. This can be thought of as observing constantly changing variables at "random" points in time; random because two GPS measurements can have an arbitrary length of time between them, so any two timeslices can have an arbitrary timestep between them.

It is important not to confuse the relationship between continuous variables and discrete timeslices. Continuous variables in dynamic networks are real-valued but are observed at discrete timeslices. For example *time* is a continuous variable in our network even though we observe this variable at discrete timeslices.

We define $P(X(t_k)|Pa(X(t_0, ..., t_{k-1}, t_k)))$ as the conditional distribution of X at time t_k, given the values of its parents at times $t_0, ..., t_{k-1}, t_k$. For some models, computing this distribution may be extremely complex, as the set $Pa(X(t_0, ..., t_{k-1}, t_k))$ may be large. We assume that our model follows the *Markov assumption*, i.e., $P(X(t_k)|Pa(X(t_0, ..., t_{k-1}, t_k))) = P(X(t_k)|Pa(X(t_k, t_{k-1}))$ for discrete timestep Bayesian networks. This assumption greatly reduces the computational cost of querying the network. Charge level is the variable we ultimately estimate from the network, and the charge level at timestep t_{k+1} is independent of $t_0, ..., t_{k-1}$; it is only dependent upon its state at t_k and the energy used or gained between t_k and t_{k+1}. Furthermore, because fares may be requested by random passengers to and from anywhere, it not unreasonable to assume the location of a taxi is independent of its prior locations. (In reality, the location is actually dependent; for example taxis that travel from the downtown portion of a city out to the airport may be more likely to return downtown than to serve fares near the airport.)

There are many varieties of Bayesian networks. We use a *dynamic conditional linear Gaussian network (DCLGN)*. DCLGNs allow inference (querying) of both continuous and discrete variables, and are designed for querying *expectations of variables* instead of probabilities. We are interested in the query "what is the expected current charge level of the taxi". Standard Bayesian networks can only answer probability queries, e.g., "what is

Variables			
Name	Symbol in Eq.	Meaning	Type
Location	$L(t_k)$	Taxi's Current location	O, D
Fare	$F(t_k)$ (0/1)	Whether a passenger is in the vehicle at t_k	O, D
Time		Value of time	O, C
Distance Traveled	$d(t_{k-1}, t_k)$	Km traveled between last two timesteps	H, C
Time Difference		Time elapsed between last two timesteps	H, C
Time Parked	$p(t_{k-1}, t)$	Seconds parked between last two timesteps	H, C
Energy Used	$u(t_{k-1}, t_k)$	kWh used between last two timesteps	H, C
Energy Gained	$g_x(t_{k-1}, k)$	Energy gained between last two timesteps	H, C
Charge Level	$CL(t_k)$	Current charge level	Hd, C

Table 1: Table of variables in the Bayesian network. O = Observeable, H = Helper, Hd = Hidden, D = Discrete, C = Continuous

the probability the charge level is currently X?". In the latter case, our only option is to compute $P(X|Pa(X))$ for all values of $Pa(X)$ and then find $E(X)$. With DCLGNs, we already have $E(X)$ and can easily query its value. Therefore, even though DCLGNs are more theoretically complex, they are far more efficient for querying means as opposed to probabilities; further details are presented in Appendix A.

3.3.2 Inferring Charge Level

We now discuss querying the network for the battery charge level. We first note two simplifying assumptions:

1. For all formulas and calculations, we assume batteries charge at a constant rate. This is not true in reality, as batteries charge faster when they have a lower state of charge and slower otherwise [38]. This assumption is further discussed in Section 5.
2. Charge level is a hidden variable and is never observed. We maintain the mean μ of charge level which is assumed to be a Gaussian distribution. We do not maintain the variance in our model, but we explain this limitation in Appendix A.

Our goal is to find the mean of the charge level Gaussian at every timestep (every GPS data point). Figure **??** shows the graphical model that is used to estimate the battery charge levels over time. The dotted arrows represent variables that have an effect on the next timeslice called *persistence edges*. The solid lines represent *intertime edges* that do not affect variables at the next timeslice.

In Bayesian networks, the complexity of querying a variable X grows exponentially with $|Pa(X)|$. Therefore we introduce *helper variables* that reduce the number of parents of variables we are interested in querying. Table 1 shows the variables in our network, and whether they are observed, helper variables, hidden, discrete or continuous.

We now explain the three most important variables in the network.

- *Energy Used.* This variable represents the energy the taxi consumes between two timesteps. Let D be the discharge rate of the EV (kWh/km), and e represent the most significant energy usage of an EV other than propelling the vehicle: air conditioning (AC). Then

$$u(t_{k-1}, t_k) = d(t_{k-1}, t_k)D + e \tag{2}$$

  The US National Renewable Energy Laboratory states "Air conditioning loads can reduce EV range and HEV fuel economy by nearly 40% depending on the size of air conditioner and driving cycle". We discuss our AC assumptions in Section 4, and note that AC usage by taxi companies is different depending on climate in their regions.

- *Energy Gained.* This variable is only used when studying the effect of roadside charging on battery charge level. If a taxi is parked between two timesteps, we assume the taxi could have been charged during this time. Let $g_x(t_{k-1}, t_k)$ be the energy gained between two timesteps assuming level x charging (kWh), and B_{gx} is the amount of energy gained per second (kWh/second) assuming level x charging (this depends on the BEV or PHEV model). Because we assume drivers never charge with passengers in the vehicle, we model Level 1 and charging and use the formulas:

$$g_1(t_{k-1}, t_k) = F(t_k)p(t_{k-1}, t_k)B_{g1} \tag{3}$$
$$g_2(t_{k-1}, t_k) = F(t_k)p(t_{k-1}, t_k)B_{g2} \tag{4}$$

- *Charge Level.* Charge level is the variable our network is designed to query. This variable has 4 parents: the charge level from the previous state, the current location, the energy used, and the energy gained. The edge between the two variables *charge level* and *location* is because the *charge level* is dependent upon *location* because of battery switching. Let $CL(t_k)$ be the charge level at timestep t_k, then

$$CL(t_k) = \begin{cases} \mathit{full}^{*} \\ \quad \text{if } L(t_k) \text{ has a switching station} \\ \quad \text{and } CL(t_k) < \tau \\ \quad \text{and } F(t_k) = 0 \\ CL(t_{k-1}) + u(t_{k-1}, t_k) + g_x(t_{k-1}, k) \\ \quad \text{otherwise} \end{cases} \tag{5}$$

  Note that one of $u(t_{k-1}, t_k), g_x(t_{k-1}, t_k)$ will always be zero—either the taxi parked and acquired energy or the taxi traveled and used energy (we assume the taxi does not charge with the AC on).

3.4 Using The Model To Infer Costs

We now describe the process to use this Bayesian model to determine company's ROI in transitioning their fleet to EVs.

*This equation only applies when studying BEVs with switching stations. The different scenarios we study are given in Section 4.4

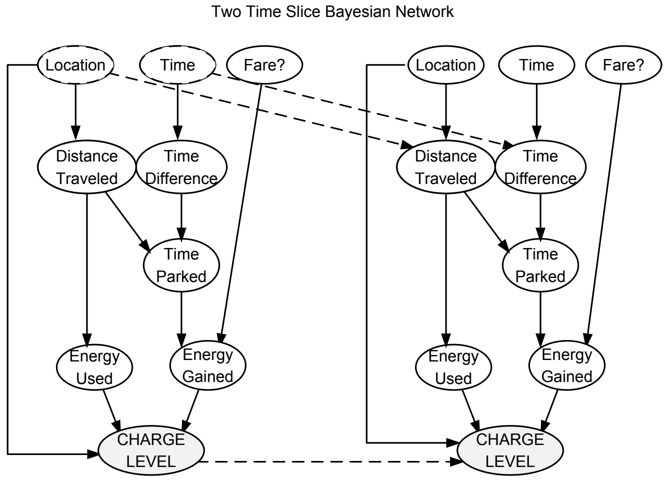

Figure 1: Our Bayesian network

3.4.1 Notation

In the following equations, we use i to index a specific taxi, k to index a specific shift, and j to index a specific battery switching station.

3.4.2 Methodology

For BEVs and PHEVs respectively, our process is to compute

$$r_{\mathrm{V-BEV}}(\tau) = r_{\mathrm{E}} - r_{\mathrm{L}}(\tau) + s_{\mathrm{BEV}}(\tau) - C_{BEV} - c_{\mathrm{EB}}(\tau) \quad (6)$$

$$r_{\mathrm{V-PHEV}} = r_{\mathrm{E}} + s_{\mathrm{PHEV}} - C_{PHEV} - c_{\mathrm{EP}} \quad (7)$$

$$\Delta_{\mathrm{BEV}}(\tau) = r_{\mathrm{V-BEV}}(\tau)x - c_{\mathrm{BSS}}(\tau) \quad (8)$$

$$\Delta_{\mathrm{PHEV}} = r_{\mathrm{V-PHEV}}x \quad (9)$$

where the terms in this equation are defined in Table 2.

Our model ignores overheads that remain fixed whether the company uses EVs or SVs, such as driver salaries and dispatch expenses. We now explain how we compute each of these costs.

3.4.3 Determining Existing Taxi Revenue

We compute the company's existing revenue r_{E} using the fare data and the companies pricing model. Let F_i be the set of all fares completed by taxi i. Using the input r_{FARE} from Section 3.1, the pricing function the company uses for a fare,

$$r_{\mathrm{E_i}} = \sum_{k \in F_i} r_{\mathrm{FARE}}(C_{\mathrm{I}}, d_k, C_{\mathrm{D}}, p_k, C_{\mathrm{T}}, M) \quad (10)$$

$$r_{\mathrm{E}} = \frac{\sum_i r_{\mathrm{E_i}}}{i} \quad (11)$$

where d_k, p_k come from fare k and $C_{\mathrm{I}}, C_{\mathrm{D}}, C_{\mathrm{T}}, M$ come from the input pricing function for a fare.

3.4.4 Revenue Loss from Lost Fares

We now show how to compute the revenue loss due to transitioning to BEVs, $r_{\mathrm{L}}(\tau)$ (PHEVs do not have revenue losses as they use the ICE after battery depletion). We assume that if a taxi depletes its battery during a shift, all revenue the taxi would have generated during the remainder of that shift is lost. This upper bounds

Name	*Description*
$\Delta_{\mathrm{BEV}}(\tau)$	total ROI in transitioning x SVs to BEVs
Δ_{PHEV}	total ROI in transitioning x SVs to PHEVs
$r_{\mathrm{V-BEV}}(\tau)$	average ROI per BEV over its lifetime
$r_{\mathrm{V-PHEV}}$	average ROI per PHEV over its lifetime
x	no. of vehicles the company transitions to EVs
r_{E}	total revenue generated by the company's existing SV taxis
$r_{\mathrm{E_i}}$	revenue of the company's SV taxi i
$r_{\mathrm{L}}(\tau)$	average revenue lost per taxi from missed fares
$r_{\mathrm{L_i}}(\tau)$	revenue lost by taxi i from missed fares
$r_{\mathrm{T_i}}(k,\tau)$	revenue generated during shift k of SV taxi i
$r_{\mathrm{B_i}}(k,\tau)$	revenue generated during shift k of taxi i *before battery depletion* assuming it was a BEV
$s_{\mathrm{BEV}}(\tau)$	average fuel savings per taxi from using BEVs instead of SVs
$s_{\mathrm{BEV_i}}(\tau)$	fuel savings from taxi i generated by using a BEV instead of an SV
s_{PHEV}	average fuel savings per taxi from using PHEVs instead of SVs
$s_{\mathrm{PHEV_i}}$	fuel savings from taxi i generated by using a PHEV instead of an SV
$c_{\mathrm{EB}}(\tau)$	average cost of BEV batteries needed per taxi
c_{EP}	average cost of extra PHEVs,per taxi, needed so drivers start their shift with full batteries
$c_{\mathrm{BSS}}(\tau)$	cost of all battery switching stations needed
C_{BEV}	cost(BEV -SV); incremental BEV cost
C_{PHEV}	cost(PHEV - SV); incremental PHEV cost

Table 2: Variables used in determining ROI

revenue losses because we assume drivers can only switch batteries if they are in a location with a switching station and they do not modify their paths to drive to a switching station. As a result of this worst-case restriction, the drivers may deplete their battery on their shift under our model. Let δ_i be the set of all shifts completed by taxi i. We determine the revenue loss by:

$$r_{\mathrm{L_i}}(\tau) = \sum_{k \in \delta_i} r_{\mathrm{T_i}}(k,\tau) - r_{\mathrm{B_i}}(k,\tau) \quad (12)$$

$$r_{\mathrm{L}}(\tau) = \frac{\sum_i r_{\mathrm{L_i}}(\tau)}{i} \quad (13)$$

3.4.5 Fuel Cost Reduction

Transitioning to EVs has the one primary financial benefit: electricity is a cheaper form of fuel than petroleum so it costs less to fuel EVs than SVs. Note that PHEVs usually have a higher gasoline efficiency after battery depletion compared to SVs due to regenerative breaking. Thich is taken into account in our model.

The fuel savings are computed as follows,

$$s_{\mathrm{BEV_i}}(\tau) = G\left(\frac{d_{\mathrm{S_i}}}{V_{\mathrm{E}}}\right) - E\left(\frac{d_{\mathrm{B_i}}}{B_{\mathrm{E}}}\right) \quad (14)$$

$$s_{\mathrm{PHEV_i}} = G\left(\frac{S_{\mathrm{D}}}{V_{\mathrm{E}}}\right) - \left(E\left(\frac{d_{\mathrm{E_i}}}{P_{\mathrm{E}}}\right) + G\left(\frac{d_{\mathrm{G_i}}}{P_G}\right)\right) \quad (15)$$

$$s_{\mathrm{BEV}}(\tau) = \frac{\sum_i s_{\mathrm{BEV_i}}(\tau)}{i} \quad (16)$$

$$s_{\mathrm{PHEV}} = \frac{\sum_i s_{\mathrm{PHEV_i}}}{i} \quad (17)$$

where the variables are defined in Table 3. Variables $d_{\mathrm{B_i}}(\tau), d_{\mathrm{E_i}}$, and $d_{\mathrm{G_i}}$ come from the Bayesian network. We start the analysis with the first datapoint of the first

Name	*Description*
G	price of gas per liter
E	price of electricity per kWh
V_E	efficiency of an SV (mpg)
B_E	efficiency of the BEV (mpkWh)
P_E	efficiency of the PHEV (mpkWh)
P_G	efficiency of the PHEV after battery depletion (mpg)
d_{S_i}	the total distance driven by the company's SV taxi i
$d_{B_i}(\tau)$	total distance driven by taxi i assuming it is a BEV: sum of distance driven before depletion over all shifts
d_{E_i}	total distance driven by taxi i assuming it is a PHEV on electricity
d_{G_i}	total distance driven by taxi i assuming it is a PHEV on petroleum

Table 3: Variables used in computing fuel savings

shift for each taxi and assume the charge level of the vehicle is full. At each datapoint (GPS reading), we update the total distance driven by the taxi so far, and query the Bayesian network for the charge level of the vehicle. Assuming we are analyzing PHEVs, if the charge level ever reaches zero, then d_{E_i} is the distance driven to that point and d_{G_i} is the distance driven throughout the remainder of the shift. If we are analyzing BEVs, if the charge level reaches zero, $d_{B_i}(\tau)$ is the distance driven to that point (then Equations (12) and (13) must be used to compute the revenue losses).

3.4.6 Switching Station Infrastructure

Due to BEV range limitations, battery switching is necessary for BEVs to be feasible for use by taxi companies. Battery switching allows drivers to have a fully charged battery within minutes. This mitigates the range limitations of BEVs, assuming there are enough switching stations to service the taxi fleet. Switching stations have a large upfront cost—the infrastructure cost is estimated to be \$500,000 by Better Place, a manufacturer of EV switching infrastructure [22, 56]. This does not take into account the cost of real estate in a given area.

To provide an adequate coverage area, the fleet may need to be served by several switching stations spread across a city. Given the expense of switching stations, we want to find the minimal number and optimal location of stations to supply the fleet *without wasting money on buying unnecessary stations.* This problem can be stated as an optimization problem: given a set of taxis and the mobility data, find the optimal location(s) for switching stations such that the taxi company's profits are maximized. We formally define this problem and present an algorithm to find $c_{BSS}(\tau)$ in Appendix B.

The switching station location problem is a generalization of the well-studied facility location problem (see, e.g., [18,35]). The facility location problem is NP-hard, so theoreticians believe it to be a computationally difficult problem, and by extension, the switching station location problem is also computationally difficult. Details of the NP-hardness reduction are given in Appendix B.

3.4.7 Battery and Extra Vehicle Costs

This section presents the computation of the cost of batteries ($c_{EB}(\tau)$) and additional PHEVs (c_{EP}). Taxis should start each shift with a fully charged battery. This requires purchasing extra batteries to be kept at each switching station (BEVs) or storing extra PHEVs at the headquarters (PHEVs).

Name	*Description*
n	the number of switching stations needed
$q_j(\tau)$	average no. of batteries needed at i per taxi
$\lambda_j(\tau)$	average no. of battery switches at k per taxi per day
$r_j(\tau)$	rate (batteries per day) at which batteries become charged at k
$\mu_j(\tau)$	average remaining charge level (kWh) of batteries switched at k
λ_H	rate at which PHEVs return to headquarters (PHEVs/day)
B_{Fx}	time it takes to charge a fully depleted battery (days/battery) assuming level x charging
C_B	the capacity of each battery (kWh)
R_P	PHEV charging rate (days/PHEV) assuming level x charging
B_C	the cost of one battery (dollars)
P_C	full cost of a PHEV (dollars)

Table 4: Variables used in calculating battery and additional vehicle costs

Using Little's law [33], $c_{EB}(\tau)$ and c_{EP} can be computed on a per taxi basis as follows:

$$c_{EB}(\tau) = \sum_{i=1}^{n} q_j(\tau) * B_C \tag{18}$$

$$q_k(\tau) = \lambda_j(\tau) r_j(\tau) = \lambda_j(\tau) B_{Fx} \left(\frac{C_B - \mu_j(\tau)}{C_B} \right) \tag{19}$$

$$c_{EP} = \lambda_H R_{px} * P_C \tag{20}$$

where the terms are defined in Table 4. Equation (18) multiplies the number of batteries needed at switching station i (per taxi) by the cost of each battery, and sums over all needed switching stations.

Note that $\mu_j(\tau)$ and $r_j(\tau)$ are proportional to τ. If τ increases, batteries are switched with higher remaining capacity and take less time to charge. As τ decreases, batteries are switched with lower remaining capacity and take more time to charge.

We assume additional PHEVs are kept only at the headquarters and drivers only switch PHEVs at the end of their shifts. We are not considering storing and charging PHEVs at the BEV battery switching stations. This is because it is less expensive to store batteries than vehicles—batteries can be stacked and stored in the same building but vehicles require expensive real estate for parking.

3.4.8 Optimal Switching Threshold

The optimal value of τ is unknown. We cannot find the optimal value of τ by optimizing $c_{EB}(\tau)$, $s_{BEV}(\tau)$, or $r_L(\tau)$ alone because this will not globally maximize Δ_{BEV}. Therefore we numerically evaluate $\Delta_{BEV}(\tau)$ for each value of τ in the set {10%, 20%, ... ,100%} and choose the value of τ that maximizes $\Delta_{BEV}(\tau)$.

4 San Francisco Case Study

We applied our process to a data set collected by Yellow Cab San Francisco (YCSF) as part of the Cabspotting project [16, 50]. In this section, we describe the dataset and perform the revenue analysis outlined in Section 3.4.2.

4.1 Dataset and Preprocessing

The dataset includes the following information for 536 YCSF taxis during May 17, 2008 – June 10, 2008. Each measurement includes:

- Latitude and longitude to 5 decimal places
- Whether a paying passenger is inside the vehicle
- The current time of the data point

The average timestep between each data point is 60-90 seconds. As discussed in Section 3.1, we split the data from each taxi into shifts.

GPS devices sometimes report erroneous data, so we preprocessed the dataset to remove inconsistencies For example, in some cases a taxi's position would be incorrectly reported between two correct readings. We noticed data from a taxi was either >99.5% correct or very erroneous due to a faulty GPS device in that taxi. For the taxis that only had few erroneous points we simply removed those points, whereas the taxis with many problems were simply discarded and excluded from all results. We discarded all data from seven out of 536 taxis.

4.2 Clustering Locations

We clustered the GPS coordinates in our data using the reduced coordinate space displayed in Figure 2. The clustering locations are spaced 4km apart with the exception of downtown San Francisco. For the downtown area, we used a denser grid (1km x 1km) because of the higher density of data within this region. After collapsing each GPS datapoint into its closest grid point, the GPS data was discarded.

4.3 Assumptions For BEV/PHEV Revenue Analysis

Here we state the assumptions we made while performing the revenue analysis.

- EVs may use up to 40% of their battery for AC assuming the AC was always on [20]. We assume that ACs doubled in efficiency since 2000 and taxi drivers in San Francisco use AC 50% of the time—hence we assume 10% of the battery is used for AC.
- We earlier quoted the price of a Better Place battery switching station to be $500,000 [56]. We also use their current estimates for battery prices. The company executives state "EV batteries are approaching $500 a kilowatt hour" and "[Better Place] is now purchasing batteries for cars at $400 per kilowatt hour for delivery in early 2012" [27]. We assume Nissan Leaf batteries (to be kept at switching stations) cost $450/kWh for a total of $11,000 based on these estimates. We also note several sources indicate battery prices are likely to continue decreasing [11, 26, 28].

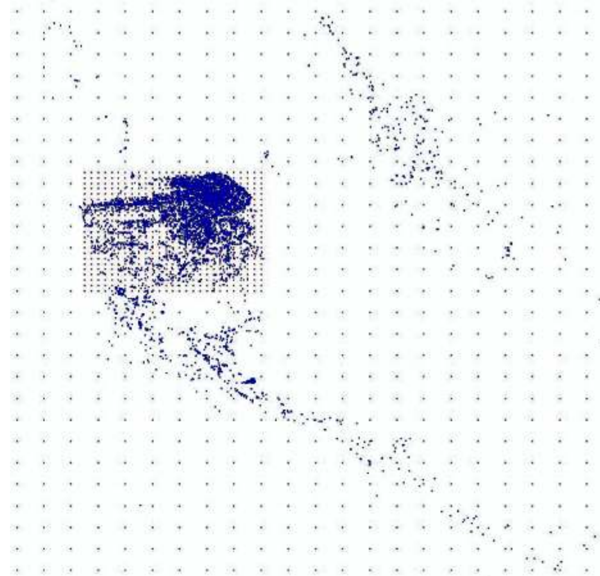

Figure 2: Points represent taxi mobility data. The grid shows the reduced coordinate space. We used a denser grid in downtown San Francisco due to the large number of data points in this region.

- We assume the companies existing taxis have an efficiency of 25mpg [45].
- Our data set indicates that each taxi is driven 112,000–177,000 mms per year. Based on this figure, we assume the company replaces each SV, BEV, PHEV, and battery after four years of use. (This does not include replacing parts over the four years). Companies do not disclose their vehicle replacement rates, which makes estimating this figure difficult. However, the The Taxi and Limousine Commission of New York City states "Cars brought into service as taxicabs must be brand new vehicles and generally must be replaced five years after being placed into service. [44].
- We assume the company currently spends $15,000 for a new SV when replacing an old SV. We use this figure to compute the incremental cost to purchase an EV instead of an SV. YCSF does not provide this figure thus we estimate it based off the vehicles listed on their website. We do not know how much the company actually pays for their vehicles due to bulk discounts.
- On May 12th 2011, the average gas price in San Francisco was $1.08/liter, and electricity was $.22/kWh [5, 6]. Although we extend our revenue analysis to a wide array of fuel prices, our discussion is based on these prices.
- We assume maintenance costs for a fleet of EVs is the same as a fleet of SVs. This limitation of our study is discussed in Section 5.

4.4 Case Study EV Scenarios

We studied ten different scenarios which are as follows:

- (1-2) BEVs with Level 1 and 2 roadside charging only
- (3-4) BEVs with Llevel 1 and 2 roadside charging and battery switching

- (5) BEVs with only battery switching
- (6-7) PHEVs with Level 1 and 2 roadside charging only
- (8-9) PHEVs with Level 1 and 2 roadside charging and PHEV switching at YCSF headquarters
- (10) PHEVs with PHEV switching at YCSF headquarters only

However, we found that only scenarios five and ten were interesting. Taxis in our case study were parked only 12% of the time (constantly driving the other 88%). Even when we assume level 2 roadside charging is available *everywhere* in San Francisco (an extremely unrealistic assumption), the results changed by less than 15% for both PHEVs and BEVs. Thus, for BEVs we only show results for scenario 5, and for PHEVs we show results for only scenario 10.

4.5 Existing Taxi Revenue

As discussed in Section 3.1, each company has their own fare pricing model. For YCSF, $r_{\mathrm{FARE}}(f)$ is given on their website [57]:

$$r_{\mathrm{FARE}}(f) = 3.10 + .45(p + (d - .2)) + 2\delta \qquad (21)$$

where d is the distance of the trip in miles, p is the time the taxi was parked (at traffic lights) during the fare, and δ is one if the passengers' destination was the airport and zero otherwise (the company charges an airport surcharge fee).

4.6 Revenue Analysis for BEVs

We now compute $\Delta_{\mathrm{BEV}}(\tau)$ given current prices and vehicle specifications using Equations 8 and 9. First, we show how we compute the revenue losses, r_{E}, in Section 4.5. The cost of the BEV we study, C_{BEV}, is derived in Section 4.6.1. In Section 4.6.2 we derive the cost of the battery switching stations, $c_{\mathrm{BSS}}(\tau)$, and show its relationship to $r_{\mathrm{L}}(\tau)$. In section 4.6.3 we measure the relationship between the threshold τ, $r_{\mathrm{L}}(\tau)$ and the cost of extra batteries needed, $c_{\mathrm{EB}}(\tau)$. Roadside charging is briefly discussed in Section 4.4. We incorporate $r_{\mathrm{L}}(\tau)$, the fuel savings $s_{\mathrm{BEV}}(\tau)$, and $c_{\mathrm{EB}}(\tau)$ into Section 4.6.4 which shows the overall return on investment we are after, $\Delta_{\mathrm{BEV}}(\tau)$.

4.6.1 Nissan Leaf Specifications

For our BEV experiments, we study the Nissan Leaf, a consumer available BEV. The price of the Nissan Leaf in California is $33,720 [37]. However, current federal tax rebates for the purchase of electric vehicles provide a tax credit of $7,500 for the Leaf in California [15]. Therefore, each Leaf can be purchased for $26,220, which is consistent with the post-tax credit price listed at [37]. Using our assumption that the company replaces their SVs for $15,000, C_{BEV} = $11,220.

Even though manufactures list the full capacity of a battery, the full capacity is not actually used—the battery is not fully charged or discharged to preserve the life of the battery [11]. However, manufactures list the expected range based on the *usable* portion of the battery—the figure we are interested in. Table 5 gives the values of the constants needed for our revenue analysis for the Nissan Leaf. These figures were derived from the specifications given on their website [38].

Constant	*Value for Nissan Leaf*
D_r in Eq. 2 (kWh/km)	0.384
B_{g1} in Equation 3 (kWh/s)	.00033
B_{g2} in Equation 3 (kWh/s)	.001
B_{f1} in Eq. 19 (days/battery)	.83
B_{f2} in Eq. 19 (days/battery)	.29
C_{B} in Eq. 19 (kWh)	24
C_{BEV} in Table 2	$11,220

Table 5: Nissan Leaf Specifications [38]

4.6.2 Switching Station Location and Distribution Over Locations

We now calculate the cost of battery switching stations, $c_{\mathrm{BSS}}(\tau)$, and show its relationship to the revenue losses incurred, $r_{\mathrm{L}}(\tau)$. We find the locations of switching stations by applying the algorithm presented in Appendix B. We were able to find a global optimal solution using brute force because there are fewer than 500 locations in our data set.

With the optimal value of τ (discussed in the next section), we find that three switching stations are optimal, so $c_{\mathrm{BSS}} = \$1,500,000$.

The relationship between $r_{\mathrm{L}}(\tau)$ and $c_{\mathrm{BSS}}(\tau)$ is shown in Table 6. Without battery switching, even if we assume charging infrastructure is available everywhere (i.e., whenever a taxi is stopped, its battery charges while it is parked), a third of all fares are lost. However, with additional switching stations at the San Francisco airport and Yellow Cab headquarters, only 3% of fares are lost. Adding additional stations to these three has negligible impact on $r_{\mathrm{L}}(\tau)$ but greatly drives up $c_{\mathrm{BSS}}(\tau)$; three stations represents the optimal value for YCSF.

We find the distribution over all locations where fares began and ended to gain intuition as to why three stations are adequate. Figure 3 shows this distribution and the corresponding heat map superimposed over a map of San Francisco. We find approximately 90% of all pickups and drop-offs occur in only 20% of the locations. This explains why a small number of switching station locations suffice; switching stations near these locations will be heavily used.

4.6.3 Switching Threshold Analysis

We find the ROI $\Delta_{\mathrm{BEV}}(\tau)$as a function of τ as discussed in Section 3.4.8. This threshold is an optimization between $r_{\mathrm{L}}(\tau)$, $s_{\mathrm{BEV}}(\tau)$, and $c_{\mathrm{EB}}(\tau)$ as follows. Consider

No Charging or Switching	41.5%
L2 Roadside charging only	37 %
Union Square BSS (no charging)	15%
YC, Union Square, Airport BSS	3%

Table 6: Percentage of fares lost in different BEV scenarios. BSS denotes battery switching station(s).

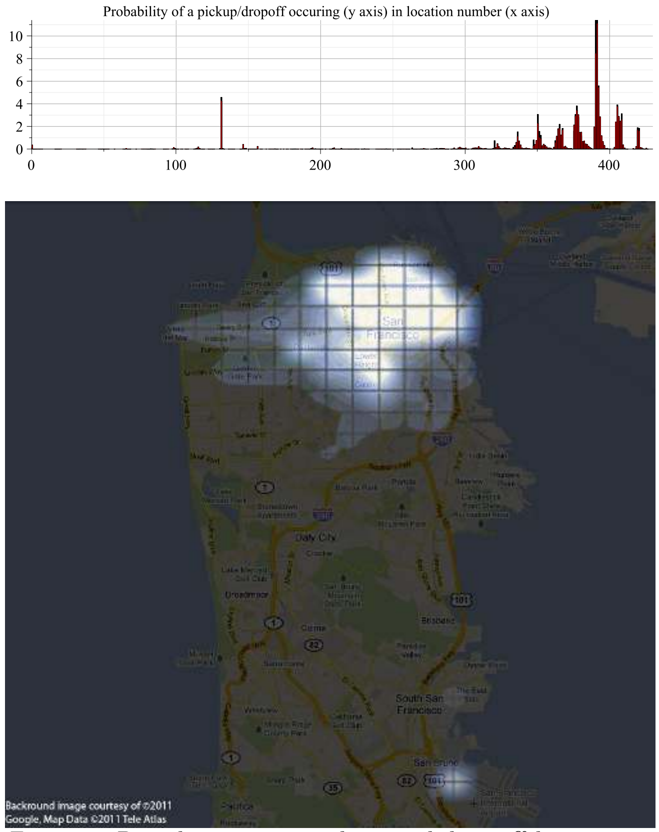

Figure 3: Distribution over pick-up and drop-off locations as distribution and corresponding heat map ("whiter" areas represent more activity).

Figures 4 and 5 which show $r_{\mathrm{V-BEV}}(\tau)$ vs. τ. Note the single peaked distribution. Increasing the threshold increases the taxis' average charge levels, which increases $s_{\mathrm{BEV}}(\tau)$ and decreases $r_{\mathrm{L}}(\tau)$, but at some point the threshold is too high and c_{EB} is large enough to offset these benefits. Therefore an optimal value of τ exists and we find its value by computing the ROI for all values of τ.

We emphasize that we do not allow taxis to deviate from the routes in the data set. They only switch batteries if they are at a location with a switching station and their threshold is less than τ. In practice, taxi drivers would be likely to actively monitor their battery charge level and travel to switching stations when needed to avoid depletion. Thus, our analysis is conservative.

4.6.4 Overall BEV Transition Cost

Figure 6 shows the cost to transition each SV to a BEV ($r_{\mathrm{V-BEV}}$) for a wide array of gas and electricity prices. We see that at current prices, BEVs are more profitable than SVs in San Francisco. We find $r_{\mathrm{V-BEV}} \approx \$4100$, which is $\approx 0.68\%$ of the company's existing revenue r_{E}. Because current prices of gasoline and electricity may vary, we determine the gasoline price for which they become profitable. For a fixed electricity price of \$.22kWh, the price point where BEVs are exactly at parity with petroleum vehicles, without considering the cost of the switching stations ($c_{\mathrm{BSS}}(\tau)$) is \$1.02/liter. With gasoline prices above this point, the company can pay back $c_{\mathrm{BSS}}(\tau)$.

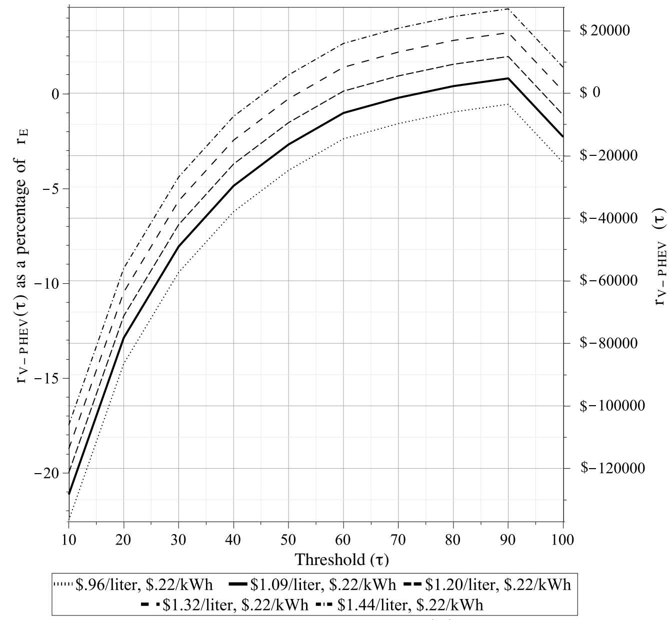

Figure 4: Switching threshold τ vs. $r_{\mathrm{V-BEV}}(\tau)$ as a percentage of r_{E}, $r_{\mathrm{V-BEV}}(\tau)$ for varying gasoline prices, fixed electricity price of \$.22/kWh.

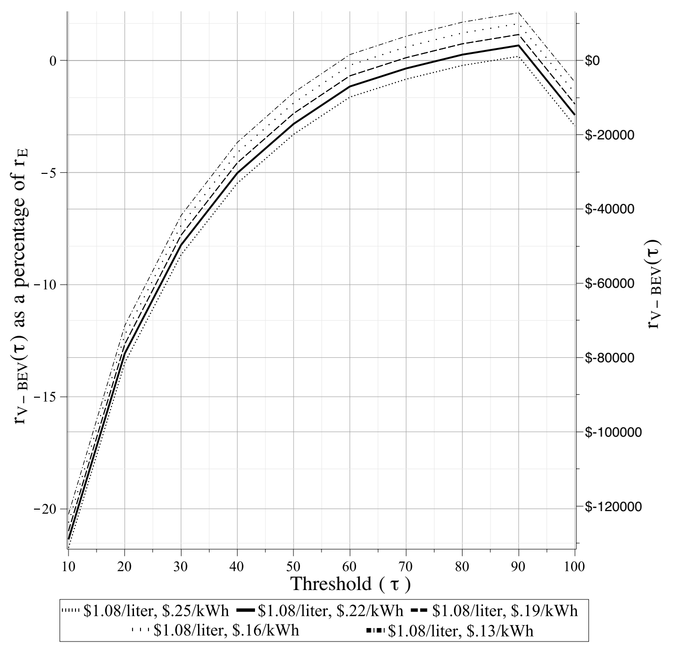

Figure 5: Switching threshold τ vs. $r_{\mathrm{V-BEV}}$ as a percentage of r_{E}, $r_{\mathrm{V-BEV}}$ for varying electricity prices, fixed gas price of \$1.08/liter.

We now compute $\Delta_{\mathrm{BEV}}(\tau) = r_{\mathrm{V-BEV}}x - c_{\mathrm{BSS}}(\tau)$. We are specifically interested in the point where $\Delta_{\mathrm{BEV}}(\tau) = 0$; this represents the "break even" point where x BEVs can be operated for the exact cost that x SVs can. We assume that switching infrastructure lasts 15 years and amortize the \$1.5M cost of three switching stations accordingly, yielding a cost of \$100k/year. Figure 7 shows the when $\Delta_{\mathrm{BEV}}(\tau) = 0$ for a fixed electricity price of \$.22/kWh. For a given point on the line, if gas prices rise, the company accrues profits. We note that gas prices in San Francisco are currently above all points on this line (\$1.08/liter).

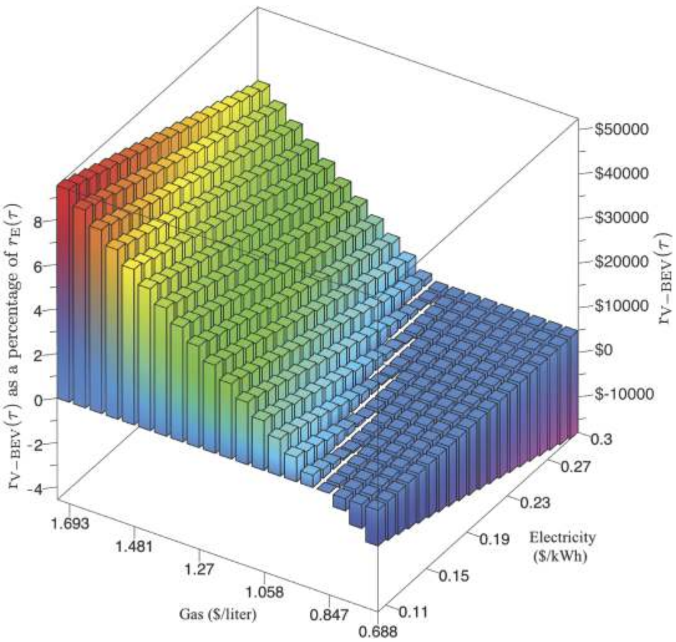

Figure 6: Gas and electricity prices vs.$r_{\mathrm{V-BEV}}$ as a function of r_{E}, $r_{\mathrm{V-BEV}}$. Three switching stations.

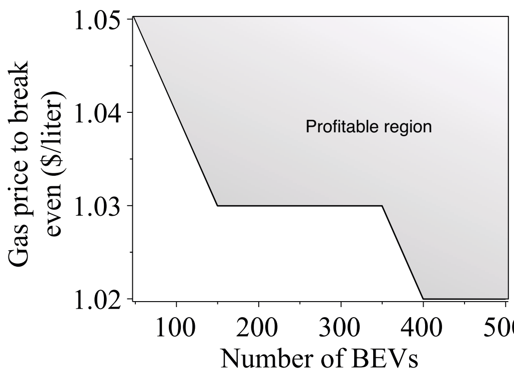

Figure 7: Profitable region for a given BEV penetration, gas price. Electricity fixed at $.22/kWh.

4.7 Revenue Analysis for PHEVs

We now compute the costs of switching to PHEVs instead of BEVs and compare the two scenarios. We therefore derive C_{PHEV} and charging rates next, and we find Δ_{PHEV} in the following section.

4.7.1 Vehicle Cost and Specifications

For our PHEV analysis, we study the 4 cylinder Chevrolet Volt. The Chevrolet Volt has a retail cost of $41,000 [41]. After the $7,500 tax credit, the price is $33,500. Using the $15,000 taxi replacement figure, $C_{PHEV} = \$18,500$. Table 7 gives the values of the constants needed for our revenue analysis for the Chevrolet Volt. These figures were derived from the specifications given on their website [13].

4.7.2 Overall PHEV Transition Cost

Figure 8 shows the ROI in PHEVs, $r_{\mathrm{V-PHEV}}$, without any roadside charging. As with BEVs, at current prices PHEVs are less expensive to operate than SVs. We find $r_{\mathrm{V-PHEV}}$ corresponding to current prices of $1.08/liter and $.22kWh is $\approx$ $3400, which is $\approx 0.57\%$ of r_{E}.

For PHEVs there is no fixed cost investment, so

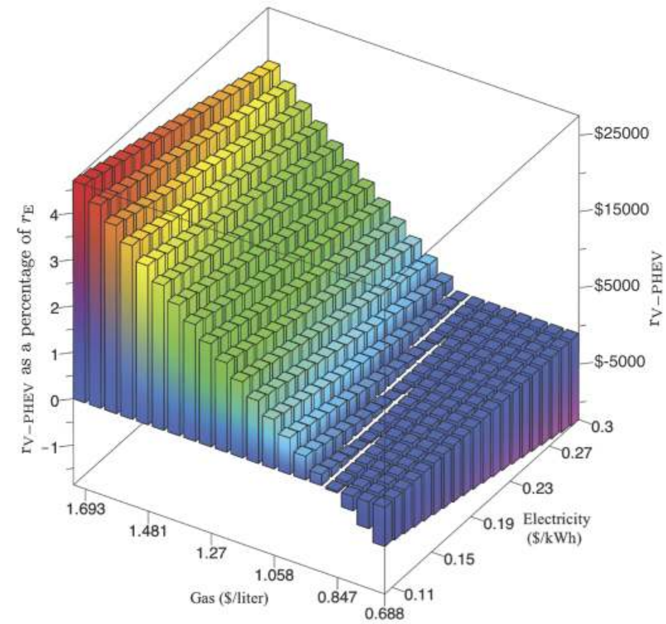

Figure 8: Gas and electricity prices vs.$r_{\mathrm{V-PHEV}}$ as a function of r_{E}, $r_{\mathrm{V-PHEV}}$. No roadside charging.

Constant	*Value for Chevrolet Volt*
D in Eq. 2 (kWh/km)	0.4
B_{g1} in Equation 3 (kWh/s)	.0004
B_{g2} in Equation 3 (kWh/s)	.0011
R_{p1} in Eq. 19 (days/PHEV)	.45
R_{p2} in Eq. 19 (days/PHEV)	.16
C_{B} in Eq. 19 (kWh)	16
C_{PHEV} in Table 2	$18,500

Table 7: Chevrolet Volt Specifications [13]

Δ_{PHEV} can be derived by multiplying $r_{\mathrm{V-PHEV}}$ by any given x to get the cost of transitioning x SVs to PHEVs. Therefore, the company can switch to PHEVs on a per vehicle basis. However, in the next section, we discuss why BEVs have more advantages than PHEVs.

4.8 PHEV vs. BEV Comparison

Even though PHEVs and BEVs are both currently profitable, BEVs are a likely better investment. Gas prices are volatile, and in the past three years we have seen the two highest prices ever for a liter of gas in the United States. Figure 9 shows gasoline prices per gallon (one gallon = 3.78 liters) since 1971 adjusted for inflation [3]. In contrast, electricity prices have not been volatile; Figure 10 shows the average electricity price *in California* (adjusted for inflation) since 1980 [51]. Note that Figure 10 does not show a price of $.22/kWh electricity (the figure used throughout this paper) because electricity prices in San Francisco are nearly double than in the rest of California and the U.S. average, as shown in Table 8 [54]. We could not find a long history of electricity prices in San Francisco alone. Because most PHEVs are still about 60% petroleum based (for example, the Volt uses electricity for 40% of its useable range [13]), if both of these trends continue as they have for the past 30+ years, BEVs are a better investment. Furthermore, bat-

May 2011: San Francisco Electricity Prices vs. U.S. Average			
Item	U.S. Average	San Francisco	Percent Difference
Electricity ($ per kWh)	0.129	0.226	75.2
Gasoline ($ per liter)	1.06	1.12	5.4

Table 8: San Francisco vs. U.S. fuel prices [54]

tery prices are decreasing and are expected to continue decreasing [11, 26–28, 36]. Because PHEVs are more expensive due to their ICEs, and the majority of the price of a BEV is its battery, BEVs are expected to decrease in price faster than PHEVs. Finally, if our ultimate goal is *complete petroleum independence*, PHEVs can only be used as an interim solution and would need to be replaced.

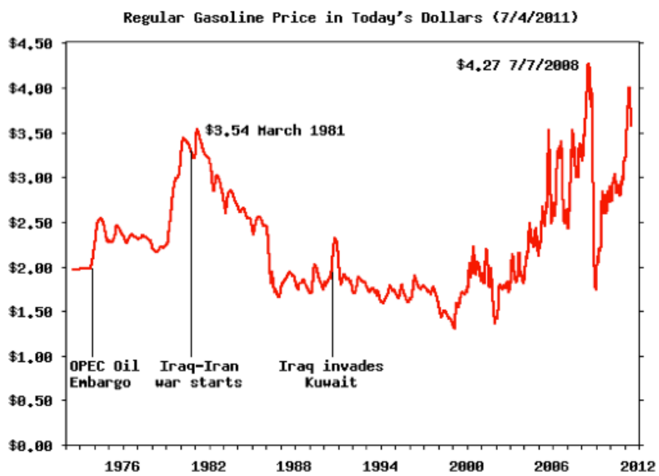

Figure 9: Average U.S. price per gallon of gas since 1971 [3]

4.9 Sensitivity Analysis

We now analyze whether our case study results would generalize to taxi companies in different cities. Although we cannot draw definite conclusions without re-running our study for a taxi company in a different city, we attempt to answer this question here.

The following factors are important to determine the ROI of EVs in a region.

1. *Average Trip Length and City Density.* Switching infrastructure is expensive, so it will initially be sparsely deployed, in contrast with current petroleum infrastructure. Consequently, the geography of a city affects the feasibility of BEVs. Large cities with widespread points of interest are less suitable than dense cities with concentrated points of interest. One way we can measure this for a given city is to determine the distribution over fare trip lengths. We can use the distribution of how far people commonly travel as a heuristic to estimate how many switching stations will be needed. Figure 11 shows the distribution of trip lengths for all fares the YCSF taxis completed during the study period. From the cumulative density function of this distribution, we find 85% of all fares are less than 10 km, which is why few stations are needed in San Francisco. This figure shows a two-peaked distribution. From the probability mass function, we find 8% of the fares are between 20 and 30 km (roughly 7% of all trips are to the San Francisco International Airport, which is 24 km by highway from Union Square in downtown San Francisco).

   The average trip length can also help us determine whether PHEVs or BEVs are better suited for a region. For a PHEVxxm, it does not matter how many trips are completed before battery depletion, because the financial benefit comes from the transportation savings on the first xx km. BEVs are range limited, however, and completing a large number of short trips (before battery depletion) is more profitable than a short number of long trips, due to the initial charge to each passenger that requests a fare. Therefore, PHEVs are likely better suited for cities with many long trips, whereas BEVs will be more profitable in cities with a large number of short trips, like San Francisco.
2. *Distribution over locations.* Closely related to the distribution over trip lengths is the distribution over locations discussed in Section 4.6.2. We found roughly 90% of YCSF fares start or end in fewer than 20% of the grid locations. If this distribution was less concentrated, the average trip length shown in Figure 11 may have increased. In our case study, trips in the downtown area within 3 km of Union Square accounted for more than half of all trips. This is highly conducive to centralized switching station placement. Taxis in larger cities may find they have to travel a greater distance out of their way to refuel.
3. *Gas and electricity prices.* Although current gas prices in San Francisco ($1.08/liter) are higher than the rest of the United States, they are lower compared to the rest of the world. For example, the average price in London, England is $1.39/liter [8], and the average price in Toronto, Canada is $1.29/liter [7]. If the mobility patterns of taxis in these regions are similar to those in San Francisco, transitioning to EVs would be even more profitable. We also note that electricity prices in San Francisco are twice the United States national average, while gas prices are not [52].
4. *Temperature and weather.* Reference [46] shows that at cold temperatures ($< 32°F$), over 10% of the energy in a battery is lost compared to at $68°F$. It rarely snows or drops below freezing in San Francisco, even during

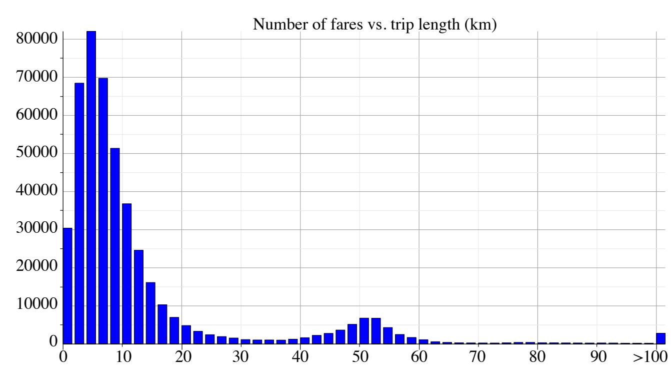

Figure 11: Distribution of fare trip lengths (the bar for 50 represents all trips over 50km)

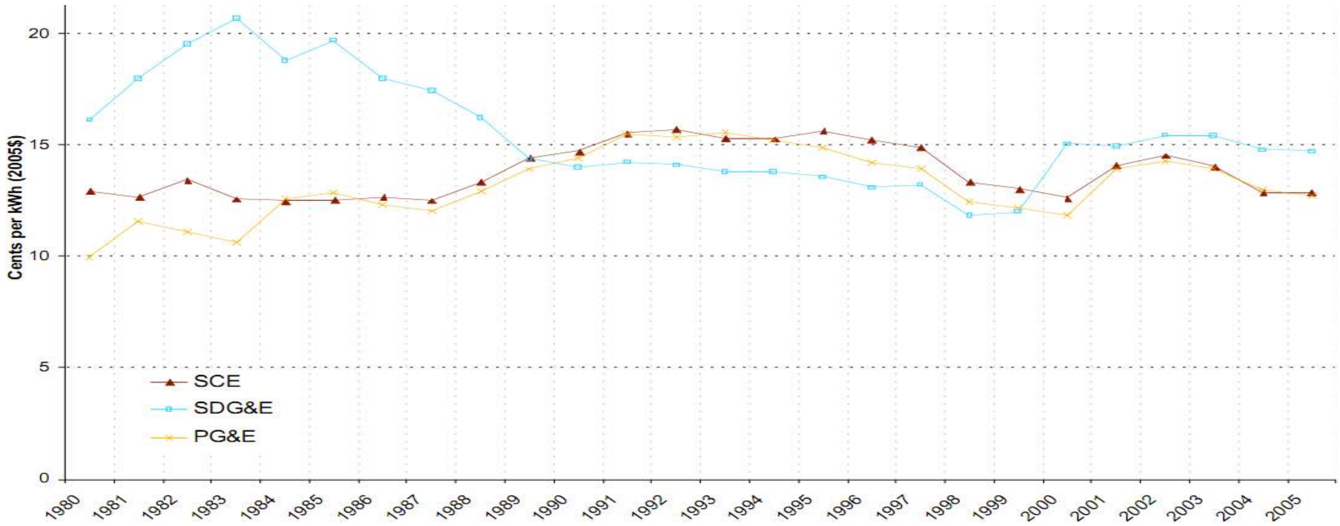

Figure 10: Average electricity price given by three major utilities in California since 1980 [51]

the winter months, but taxi companies in cities with colder climates should expect worse performance. Furthermore, passengers in cities with extreme weather temperatures require more heating and cooling, which further drains the battery.

5 Limitations and Future Work

To obtain realistic results for our case study, we used only commercially available vehicles and their manufacturer specifications. However, predicting the outcome of a major transition prior to it occurring is an error-prone process. We now discuss some avenues for future work.

1. Our process can only be used with taxi companies whose vehicles are brought back to a common location after each driver's shift. Future work could generalize the process to different types of taxi companies.
2. We have not included any analysis of vehicle maintenance costs. Maintenance costs for a fleet of EVs is thought to be lower than SVs [11], but we are not aware of any quantitative analysis comparing the two. A maintenance cost analysis for a large fleet of PHEVs/BEVs would greatly improve our cost model.
3. Our switching station optimization assumes that the locations can charge any number of batteries and can be placed anywhere in the city. In reality, distribution network limitations may place some restrictions on switching station placement and battery charging; areas with a fully utilized distribution network may not be able to accommodate the new load.
4. We have not considered real estate prices for switching stations, other than the cost of the stations themselves. We should account for the cost of acquiring space to build the switching station.
5. Obtaining a second data set from a different city would provide a better foundation for the sensitivity analysis section.
6. Batteries do not charge at a constant rate as we have assumed. A better assumption would be to use a two phase linear approximation; have a higher charge rate while the state of charge (SOC) is less than 80%, and a lower rate when the SOC is above 80% [38].
7. If battery switching for PHEVs is implemented in the future, this would drastically change the PHEV revenue analysis. Currently, PHEVs are not manufactured in this fashion.

We note that EVs are still manufactured using petroleum; the study of the overall petroleum use of a vehicle, including manufacturing is known as *life cycle* analysis [19, 43].

6 Conclusions

In this paper, we proposed a process to determine the ROI for a taxi corporation transitioning to electric vehicles. We first built a model of taxi fleet transportation, and then used the model to compute the economic costs of the transition. The model can be configured with a wide array of input parameters, including the type of vehicle to be tested, electricity and gasoline prices, and roadside charging/battery switching infrastructure assumptions. We then used our process to analyze a fleet of over 500 taxis in San Francisco. We found that PHEVs and BEVs are both currently profitable.

Electric vehicles are expected to play a large role in reducing petroleum consumption and global carbon emissions. The transition to EVs is a necessary but will likely be a difficult transition, but we can mitigate the negative effects of major transitions, such as a complete overhaul of the transportation industry, with careful planning. Careful planning requires analysis of several aspects of the transition, including financial feasibility and social factors. Our work presents a step towards providing information to taxi companies as to the extent, either positively or negatively, to which they might be affected.

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Appendix A: Brief CLGN Background

In this appendix we provide a brief background on CLGNs. We assume knowledge of standard Bayesian networks; an excellent reference text is Koller et. al [34].

We first present some necessary definitions.

- The *graphical model* of a problem is a directed acyclic graph $G(V, E)$, where each vertex is a variable and each edge represents a causal effect. The variables may be known (we can directly observe or compute their values) or hidden (we estimate their value because we cannot observe their values directly).
- The set of parents $Pa(X)$ of a node X in a graphical model is defined as all nodes Y such that $(Y, X) \in E$ and $Y \neq X$.
- Variables can be either *discrete* or *continuous*; discrete variables can only take values from a countable set of values, such as the integers, whereas continuous variables can be any real number.
- A *Bayesian network* is a directed acyclic graph that defines the relationship $P(X|Pa(X))$ between every variable and its parents. The probability of any variable X is independent of all other variables in the network given its parents.

Hybrid Models

Standard Bayesian networks contain only discrete variables. A *hybrid* model contains a mix of both continuous and discrete variables. Several different hybrid models exist; we chose to use *conditional linear Gaussian networks (CLGNs)*. In linear Gaussian models, each variable X is modeled as a linear combination of its parents. CLGNs are extensions of linear Gaussian models that allow for both discrete and continuous variables.

In CLGNs, three types of relationships are defined:

- A discrete child with only discrete parents
- A continuous child with only continuous parents
- A continuous child with a mixture of continuous and discrete parents.

Note that CLGNs do not allow for discrete variables with continuous parents. Other models address this issue, but we do not need these extensions for our application.

Querying Conditional Linear Gaussian Networks

To query a variable is to return its Gaussian distribution. To query each of the three types of variables, we use the following formulas based [34].

We consider the simplest case first; a discrete variable with only discrete parents. To express this conditional relationship, we use a discrete conditional probability table (CPT) as in standard Bayesian networks.

Next we consider a continuous variable with only continuous parents. A continuous variable X can take on any real number in the domain of X. Therefore, we cannot have a finite CPT because it would be infinitely large. Instead, we maintain a function of its parents' values that is used to generate a Gaussian over X. Let X have k parents with means $pa_1, pa_2, ...pa_k$. Under the CLGN model, we specify $k + 2$ parameters $\alpha_0, \alpha_1, ..., \alpha_k$, and a variance σ^2 and compute $P(X|pa_1, pa_2, ...pa_k)$ as

$$P(X|pa_1, pa_2, ...pa_k) = \mathcal{N}\left(\alpha_0 + \sum_{i=1}^{k} \alpha_i(t)pa_i(t),\ \sigma^2\right) \tag{22}$$

That is, the set of αs are linear combination constants; we are calculating a new Gaussian that is a linear combination of other Gaussians (its parents).

Before we examine the third case, we note how σ^2 is obtained. There are two widely used versions of CLGN's: those where the variance of each variable depends on the variances of its parents, and those where the variance is assumed to not depend on its parents [34]. We are using the latter simpler model because we do not have data for the variables in Table 1. This model is not as accurate because it ignores covariance between variables and their parents, but is commonly used when the variances of variables in the network are not known, and still captures most of the meaningful relationships [34]. Because we use this model, we do not present background on CLGNs where the variance of each variable X depends on $Pa(X)$; but note these models rely on the theory of *multivariate Gaussian distributions*.

Finally, we consider the most complex case, a continuous variable with both continuous and discrete parents. Let X be a continuous random variable with j discrete parents and k continuous parents. Let $\mathbf{D} = \{D_1, ..., D_j\}$ represent the discrete parents of X. Let $\mathbf{C} = \{C_1, ..., C_k\}$ represent the continuous parents of X; we denote the mean of the i^{th} continuous parent c_i. Together, $\mathbf{D} \bigcup \mathbf{C} = Pa(X)$. For every combination $\mathbf{d}$ chosen from $\mathbf{D}$, we have a (possibly different) vector of $k + 2$ constants $\alpha_{d_0}, \alpha_{d_1}, ..., \alpha_{d_k}, \sigma_d^2$, and a variance σ_d^2 such that

$$P(X|\mathbf{D} = \mathbf{d}, \mathbf{C} = \mathbf{c}) = \mathcal{N}\left(\alpha_{d_0} + \sum_{i=1}^{k} \alpha_{d_i} c_i\ ; \sigma_d^2\right) \tag{23}$$

Again, the set of αs are the linear combination constants.

The problem with this approach is that the set of all combinations of $\mathbf{d}$ may be massive; even if each discrete parent was binary, we would still have 2^d combinations and would need to store $(k + 2)2^d$ constants *for every variable*. A better idea is to store a function for each variable that calculates these $k + 2$ constants based on its parents at any time. This also allows us to set the α values based on $X's$ discrete *and* continuous parents, if needed. Therefore, we introduce a function $\phi_X(\mathbf{d}, \mathbf{c}) : \mathbb{R}^{j+k} \rightarrow \mathbb{R}^{k+1}$. This function ϕ_X takes in *all* of $Pa(X)$ and generates the α values used in the linear combination. Creating the ϕ functions requires knowledge of the problem; we need encode our knowledge of how variables are dependent upon each other into the network via the ϕ functions. If we were instead storing the constants, then we would need to derive the constants for each variable based on our knowledge of the problem.

Having introduced the ϕ_X function, we rewrite Equation(23) as:

$$P(X|\mathbf{D}=\mathbf{d},\mathbf{C}=\mathbf{c}) = \mathcal{N}\left(\alpha_{d_0} + \sum_{i=i}^{k} \alpha_i * c_i \; ; \sigma^2_{\mathbf{d}}\right) \quad (24)$$

$$\{\alpha_0, ..., \alpha_k\} = \phi_X(\mathbf{d},\mathbf{c}) \quad (25)$$

Whenever we query a variable, we use Equations 24 and 25 to calculate the distribution.

Appendix B: General Switching Station Optimization Algorithm

Shukla et. al. [47] provide an optimization framework for EV infrastructure placement based on *flow interception facility location*. A more complex flow based model is given by [53]. These models are designed to reduce the total amount of fuel used while placing restrictions on the number of vehicles that can be serviced by each refueling station. We assume the taxi company's objective is to maximize their overall revenue, but maximizing miles traveled does not necessarily maximize revenue. We therefore introduce a new optimization framework based on the discretized locations of the taxis and their charge levels. It is also a variation of the flow based facility location model.

We now provide the details of our approach to computing locations for switching stations. First, we show that the problem is NP-hard, which implies that it is unlikely to be able to be solved by an algorithm that runs in polynomial-time. Then, we formulate it as an integer program, and propose an algorithm for the problem that works on small instances.

We outline a proof that the switching station location problem is NP-hard by a reduction to the facility location problem. The facility location problem is stated as: given a set of clients has some demand from a facility and a cost to build each facility, find the optimal placement of facilities to minimize the cost of the facilities and the cost of serving the clients. The switching station location problem can be reduced to the facility location problem by treating the taxis as clients whose demand varies over time and the switching stations as the facilities that can meet that demand. Therefore, finding the set of optimal switching station locations is also NP-hard.

We now formally describe the switching station location problem. First, we introduce the necessary notation. Let L be the set of locations where a switching station can be placed. We denote a taxi by x and the set of all taxis by X. We assume knowledge of a cost function $\mathrm{cost}(l)$ for each location $l \in L$ that is the price of placing a station at l. We use $\mathrm{Loc}_t(x)$ to be the location of x at time t and $\mathrm{fare}_t(x)$ is True when x has a passenger and False when it does not. We use a binary variable $y(l)$ to indicate if a location has been selected for a switching station. The charge level of $x \in X$ at timestep t_k is denoted by $\mathrm{CL}(x, t_k)$. Let $\mathrm{o}_t(x, r)$ be the opportunity cost of an EV with charge level r at time t. For a taxi x, $\mathrm{o}_t(x, r)$ should be zero when x's battery is sufficiently charged; however, as its charge level drops, there is some opportunity cost because the driver will not be able to complete trips over some length, and thus may lose revenue because some passengers cannot be transported to their destination. In our analysis, we define $\mathrm{o}_t(x, r)$ to be the sum of taxi x's fares for the remainder of its shift, once it cannot complete a trip because its charge level r is too low. That is, if x cannot complete a trip at time t, then its opportunity cost is the fares for the trips it would have completed from time t until the end of its shift. Finally, we use tau to be the battery level at which a taxi will always swap its battery if is at the same location as a switching station.

The objective of our optimization problem is stated as *given the set of taxis and their temporal mobility patterns, find the optimal location(s) for switching stations such that the taxi company's profits are maximized.* Our mathematical formulation of the switching station location problem is as follows:

$$\min \sum_{l \in L} \mathrm{cost}(l) y(l) + \sum_{x \in T} \sum_{t} \mathrm{o}_t(x, \mathrm{c}_t(x)) \quad (26)$$

subject to:

- $y(l) = \{0, 1\}$ for all $l \in L$
- $CL(x, t_k) =$
$$\begin{cases} Full & \text{if } y(\mathrm{Loc}_t(x)) = 1 \\ & \quad \text{and } CL(x, t_{k-1}) < \text{thresh} \\ & \quad \text{and } \mathrm{fare}_t(x) = \text{False} \\ CL(x, t_{k-1}) - u(t_{k-1}, t_k), & \text{otherwise} \end{cases}$$
where $u(t_{k-1}, t_k) =$ the energy used from t_{k-1}, t_k.

When L does not contain too many locations (for example, as in our case study below), we can solve the switching station location problem optimally using brute force. That is, we find the value of Equation 26 for all possible locations of $1, 2, \ldots, k$ switching stations. The value of k is found by determining the number of switching stations sufficient so that no revenue is lost due to opportunity costs (i.e., we have $\sum_{x \in T} \sum_t \mathrm{o}_t(x, \mathrm{charge}_t(x)) = 0$). At this point, Equation 26 is monotonically increasing when more switching stations are added, so we can safely conclude that Equation 26 is minimized with k or fewer switching stations.

This brute force approach may not be feasible over larger areas with more locations. In this case, it is possible to use heuristic algorithms to find a solution, though these heuristics cannot guarantee the optimality of their solution. Algorithms such as simulated annealing, tabu search, and hill climbing are general optimization methods, and could be used to find approximate solutions to the switching station location problem [23, 32, 42].

Our formulation of the switching station location problem relies on time series locations of the vehicles that will use the switching stations. Ideally, this location data is collected from multiple vehicles over multiple weeks; however, this data may not be obtainable in some situations. In this case, it is still possible to optimize the placement of switching stations using stochastic facility location algorithms (e.g.,. [39, 48]). Such algorithms are designed to optimize facility locations when there is a high amount of uncertainty in the input. These algorithms take a probability distribution of the amount of time vehicles spend at given locations as input. This distribution could be estimated from, e.g., road congestion statistics or logs of passenger pickups and drop-offs.

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