The Ricci Flow: An Introduction

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AMS *Mathematical Surveys and Monographs*, Vol. 110

List of corrigenda

April 2011.

1. On page 2 of Chapter 1, the word 'separating' should not appear in the definition of an irreducible 3-manifold.

2. The "Rosenau solution" (Section 3.3) should be called the King–Rosenau solution. See:

   King, J. R. Exact polynomial solutions to some nonlinear diffusion equations. Phys. D 64 (1993), no. 1-3, 35–65.

3. On page 32, the coordinates for the Rosenau solution should be $x \in \mathbb{R}$ and $\theta \in \mathcal{S}^1_1 = \mathbb{R}/4\pi\mathbb{Z}$.

4. On page 35, definition (2.31) is off by a factor of 2 and should be $X(t) \doteqdot \frac{1}{2}\frac{r}{tf(r)} \cdot \frac{\partial}{\partial r}$. The covector field metrically dual to X then becomes $\xi = \frac{1}{2} r f(r)\, dr$. With this change, the subsequent display changes to

$$\begin{aligned}
(\mathcal{L}_X g)_{11} &= 2\nabla_1 \xi_1 = \frac{\partial}{\partial r}\left(r f(r)\right) - \Gamma^1_{11}\xi_1 = f(r) \\
(\mathcal{L}_X g)_{12} &= 2\nabla_1 \xi_2 = 0 \\
(\mathcal{L}_X g)_{21} &= 2\nabla_2 \xi_1 = 0 \\
(\mathcal{L}_X g)_{22} &= 2\nabla_2 \xi_2 = -\Gamma^1_{22}\xi_1 = \frac{r^2}{f(r)},
\end{aligned}$$

   and the remainder of the section goes through without modification.

   We are indebted to Danny Calegari for noticing this.

5. On page 49, the formula for U should read

$$U = (n-1)\left\{(K_1 - K_0)\, ds^2 - K_1 \psi^2 \hat{g}\right\}.$$

   We are indebted to Sven Strohmer for noticing this.

6. A square root was inadvertently left out of several related estimates in Chapter 2. The corrected statements below reflect what is actually proved there.

   (a) On page 40, equation (2.45) should read:

$$\frac{\psi(x,t)}{\sqrt{T-t}} \le C \frac{s-\bar{s}}{\sqrt{-(T-t)\log(T-t)}} \sqrt{\log \frac{s-\bar{s}}{\sqrt{-(T-t)\log(T-t)}}} \tag{2.45}$$

(b) On page 53, the 'intermediate layer' estimate in statement (3) of Proposition 2.36 should read:

$$\frac{\psi}{r_{\min}} \leq C \frac{s-\bar{s}}{r_{\min}\sqrt{-\log r_{\min}}}\sqrt{\log \frac{s-\bar{s}}{r_{\min}\sqrt{-\log r_{\min}}}}$$

(c) Similarly, equation (2.52) on page 55 should be:

$$\frac{\psi(x,t)}{r_{\min}(t)} \leq C \frac{\sigma}{r_{\min}(t)\sqrt{-\log r_{\min}(t)}}\sqrt{\log \frac{\sigma}{r_{\min}(t)\sqrt{-\log r_{\min}(t)}}} \tag{2.52}$$

(d) Finally, the last display in the proof of Lemma 2.40 should be:

$$\frac{\psi}{r_{\min}} \leq C \frac{\sigma}{r_{\min}\sqrt{-\log r_{\min}}}\sqrt{\log \frac{\sigma}{r_{\min}\sqrt{-\log r_{\min}}}}$$

7. The ninth display on page 50 should read

$$\psi(x_1,t) - \psi(x_2,t) \geq \frac{\left(\frac{5}{8}-\frac{1}{2}\right)D^2}{\left(\sqrt{\frac{3}{4}}+\sqrt{\frac{3}{8}}\right)D} > \frac{D}{12}.$$

Then the tenth display should read

$$-\psi_s > \frac{\psi(x_1,t)-\psi(x_2,t)}{|s(x_1,t)-s(x,t)|} > \frac{D/12}{|s(x_1,t_1)-s(1,t_1)|} \doteqdot \delta.$$

We are indebted to Mohammad Javaheri for this observation.

8. Remark 5.2 on page 106 was correct when written. However, there now exists a complete proof of the Uniformization Theorem via Ricci flow. For the final step, see:

   Chen, XiuXiong; Lu, Peng; Tian, Gang. A note on uniformization of Riemann surfaces by Ricci flow. Proc. Amer. Math. Soc. 134 (2006), no. 11, 3391–3393 (electronic).

9. In the titles of Sections 7 and 15 of Chapter 5, $\chi\left(\mathcal{M}^2 > 0\right)$ should read $\chi(\mathcal{M}^2) > 0$.

10. In the first display on page 178, the term $\nabla_j \nabla_p R_{qik}^{\ell}$ should be preceded by a minus sign. We are indebted to Hee Kwon Lee for noticing this error.

11. On page 181, the statement of Claim 6.21 that was intended and is proved should read as follows:

    Claim 6.21. Define a metric h on V by $h \doteqdot \iota_0^*(g_0)$. If we evolve the isometry $\iota(t)$ by

$$\frac{\partial}{\partial t}\iota = \operatorname{Rc}\circ\iota,$$
$$\iota(0) = \iota_0,$$

    then the bundle maps

$$\iota(t) : (V,h) \to (T\mathcal{M}^n, g(t))$$

    remain isometries.

12. On page 182, the third line from the bottom is missing a minus sign and should read

$$B_{abcd} \doteqdot -h^{eg} h^{fh} R_{aebf} R_{cgdh} .$$

13. On page 186, ι^* should appear on the right-hand side of formula (6.27), which should read

$$\frac{\partial}{\partial t} (\iota^* \mathrm{Rm}) = \Delta_D (\iota^* \mathrm{Rm}) + (\iota^* \mathrm{Rm})^{2} + (\iota^* \mathrm{Rm})^{\#}.$$

14. In Theorem 6.30, the displayed estimate should read

$$\frac{\lambda - \nu}{\nu + \mu + \lambda} \leq \frac{C}{(\nu + \mu + \lambda)^{\delta}}.$$

(This is equivalent to the original statement, but is better adapted to the modification in #10, below.

15. On page 191, in the First proof of Theorem 6.30, replace the text starting with 'Hence' with the following:

Hence

$$\begin{aligned}
&\frac{d}{dt} \log \left[\frac{\lambda - \nu}{(\nu + \mu + \lambda)^{1-\delta}} \right] \\
&= \delta (\nu + \lambda - \mu) - (1 - \delta) \frac{(\nu + \mu) \mu + (\mu - \nu) \lambda + \mu^2}{\nu + \mu + \lambda} \\
&\leq \delta (\nu + \lambda - \mu) - (1 - \delta) \frac{\mu^2}{\nu + \mu + \lambda},
\end{aligned}$$

where we used $\nu \leq \mu$ to obtain the last inequality. By Lemma 6.28, one has

$$\nu + \lambda - \mu \leq \lambda \leq 2C\mu$$

and

$$\frac{\mu}{\nu + \mu + \lambda} \geq \frac{\nu + \mu}{6\lambda} \geq \frac{1}{6C}.$$

Thus, choosing $\delta > 0$ small enough so that

$$\frac{\delta}{1 - \delta} \leq \frac{1}{12C^2},$$

we obtain

$$\frac{d}{dt} \log \left(\frac{\lambda - \nu}{(\nu + \mu + \lambda)^{1-\delta}} \right) \leq 0.$$

Now define the convex set

$$\mathcal{K} = \left\{ \mathbb{P} : [\lambda (\mathbb{P}) - \nu (\mathbb{P})] - C [\nu (\mathbb{P}) + \mu (\mathbb{P}) + \lambda (\mathbb{P})]^{1-\delta} \leq 0 \right\}$$

and continue as before. In particular, if g_0 has positive Ricci curvature, then there exist constants $C' < \infty$ and $\delta > 0$ such that

$$\frac{\lambda (\mathrm{Rm}) - \nu (\mathrm{Rm})}{R^{1-\delta}} \leq C'.$$

16. On page 200, the third display from the bottom needs an extra constant and should read

$$V - \beta R^{2-\gamma} \leq C_2 + C_1 t.$$

The rest of the proof goes through as written, with the last display replaced by

$$\frac{|\nabla R|^2}{R} \leq V \leq \beta R^{2-\gamma} + C_3.$$

Indeed, this implies that

$$\frac{|\nabla R|^2}{R^3} \leq 2\beta R^{-\gamma} + C_4 R^{-3},$$

which is equivalent to the statement of Theorem 6.35.

17. On page 258, the definitions of $\mathcal{K}$ and K are each missing a / symbol. They should be

$$\mathcal{K} = \left\{ \mathbb{P} \;\middle|\; \begin{array}{c} \operatorname{tr}\mathbb{P} \geq -3/\left(1+t\right), \\ \\ \text{and if } \nu\left(\mathbb{P}\right) \leq -1/\left(1+t\right), \text{ then} \\ \\ \operatorname{tr}\mathbb{P} \geq \left|\nu\left(\mathbb{P}\right)\right| \left(\log\left|\nu\left(\mathbb{P}\right)\right| + \log\left(1+t\right) - 3\right) \end{array} \right\}$$

and

$$K = \left\{ (u,v) \;\middle|\; \begin{array}{c} v \geq -3/\left(1+t\right), \\ \\ v \geq -3u, \\ \\ \text{and if } u \geq 1/\left(1+t\right), \text{ then} \\ \\ v \geq u\left(\log u + \log\left(1+t\right) - 3\right) \end{array} \right\},$$

respectively. (The correct definitions are used in the proof.)

We are indebted to Peter Landweber for supplying the following list of corrections:

1. page 4, definition of a model geometry: The subgroup G_* needs to be a compact topological group, i.e. the isotropy groups need to be compact. This is essential for the proof of Lemma 1.9 where one averages a metric over an isotropy group, and is clearly intended.

2. page 5, Proposition 1.10: The manifold should be connected.

3. page 6, line after proof of Lemma 1.12: add "simply-connected" to read "complete simply-connected homogeneous space".

4. page 9, line 1: sentence ends with two periods.

5. page 10, two line display above Lemma 1.16: those primes appearing 4 times should really be asterisks, indicating adjoints of $(\operatorname{ad} X)$ and of $(\operatorname{ad} Y)$.

6. page 10, proof of Lemma 1.16, next to last line: "1/2" should be dropped. By comparing with Milnor's account in [98], it appears that the computation can be made to yield a seemingly stronger result, that $R(F_3, F_1)F_2 = 0$; compare the proof of Theorem 4.3 in [98].
7. page 11, first sentence: It seems you just need Lemma 1.16 as justification for the simultaneous diagonalization of g and $\mathrm{Rc}(g)$. You could also cite Milnor's Theorem 4.3 in [98]; that paper gives a superb exposition of curvature properties of left invariant metrics, beyond the classification you use in dimension 3.
8. page 12, Prop 1.17, line 2: the forward reference to (1.5) on page 13 is a bit strange.
9. page 13, third display: just want $C - D - B$ in the numerator, remove the exponent 2.
10. page 13, paragraph after Remark 1.18, line 2: delete "an"
11. page 63, Figure 5: dumbbell (lost a "b")
12. page 173, paragraph after Conjecture 6.2, second sentence: the period should be after [126], not before it.
13. page 283, section 3, line 6: change Y_1 to Y_0.
14. page 283, section 3, definition of $d\theta$: In the first sum, Y_i should be applied to the i^{th} term (but this appearance of Y_i was omitted).
15. page 318, [32]: Hamilton.
16. page 321, [91]: Yau, Shing-Tung (needs hyphen).
17. page 323, degenerate neckpinch (first word lost an "e").