cA

# Riccion,Instanton and Primordial Inflation

**S.K.Srivastava**
1.Department of Mathematics,
North Eastern Hill University,
Permanent Campus,
Shillong - 793022
(INDIA)
2.IUCAA , Post Bag 4 ,
Ganeshkhind , Pune - 41100
(INDIA)



It is shown here that $R^2$ and $R^3$ terms dominate over Einstein - Hilbert term in the gravitational action till energy mass scale $M \geq 1.19 \times 10^{10} GeV$ . In the presence of these higher - derivative terms , action for Riccions is obtained with quartic self interaction potential. It is interesting to see that instanton solution for Riccions gives rise to primordial inflation without any phase transition and symmetry breaking . PACS nos. 04.62.+v,98.80.-k,98.80Cq,98.80.Bp

Submitted to Gen.Rel.Grav.Journal for publication.

## 1.Introduction

Einstein's theory of gravity needs modification as it is non - renormalizable and exhibits solutions having point - like singularities,where physical laws collapse [1]. In this context, efforts have been made to study higher - derivative gravity adding terms$R^2$, $R_{\mu\nu}R^{\mu\nu}$,$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $\Box R$ and$R^3$ etc. to the Einstein - Hilbert lagrangian$\frac{R}{16\pi G}$ (G is the gravitational constant ).Here $R_{\mu\nu\rho\sigma}$ are components of Riemann Curvature tensor , $R_{\mu\nu}$ are components of Ricci tensor and R is trace of Ricci tensor , which is called Ricci scalar . $\Box$ is defined as $\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}(\sqrt{-g}g^{\mu\nu}\frac{\partial}{\partial x^\nu})$. These curvature terms depend on components of the metric tensor $g_{\mu\nu}$ given by the distance function

$$dS^2 = g_{\mu\nu}dx^\mu dx^\nu. \tag{1.1}$$

Higher - derivative gravity obeys the principle of covariance and the principle of equivalence , which are the basic principles of the general relativity . These theories have problem at perturbation level where ghost terms appear in the Feynman propagator of graviton [2].

Recently, an entirely different physical aspect of $R^2$-gravity has been discussed in refs.[3 - 6]. In these references , it is found that at high energy level , Ricci scalar $R$ manifests itself in two different ways (1) as a spinless matter field and (2) as a geometrical field (which is its usual nature at low energy),whereas at low energy it behaves as a geometrical field only .

Before going into further details,it is better to give brief discussion on dual role of Ricci scalar at high energy level.In refs.[3 - 6], the action for $R^2$- gravity is taken as

$$\bar{S}_g = \int d^4x \sqrt{-g}\Big[\frac{R}{16\pi G} + \tilde{\alpha} R_{\mu\nu}R^{\mu\nu} + \tilde{\beta} R^2\Big], \qquad (1.2)$$

where $G$ is the Newtonian gravitational constant which is equal to $M_P^{-2}$ ($M_P$ is the Planck mass) in natural units ($\hbar = c = 1$,where $\hbar$ andc have their usual meaning) adopted throughout the paper.Here $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless coupling constants. Imposing invariance of $\bar{S}_g$ under transformations $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$,one obtains field equations

$$(1/16\pi G)(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) + \tilde{\alpha}(R^{\rho}_{\mu;\nu\rho} - \Box R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}\Box R$$
$$+2R^{\rho}_{\mu}R_{\rho\nu} - \frac{1}{2}g_{\mu\nu}R^{\gamma\delta}R_{\gamma\delta}) + \tilde{\beta}(2R_{;\mu\nu} - 2g_{\mu\nu}\Box R$$
$$-\frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}) = 0. \qquad (1.3)$$

Trace of these field equations is given as

$$\Box R + m^2 R = 0, \qquad (1.4)$$

where $m^2 \equiv [8\pi G(5\tilde{\alpha} + 12\tilde{\beta})]^{-1/2}$. Here $\tilde{\alpha}$ and $\tilde{\beta}$ are chosen such that $(5\tilde{\alpha} + 12\tilde{\beta}) \geq 0$ to avoid the ghost problem.

It is known that $R$ and $R_{\mu\nu}$ are combinations of second order partial derivatives and square of first order partial derivatives of $g_{\mu\nu}$ w.r.t. space - time coordinates. Moreover, $g_{\mu\nu}$ are dimensionless. So, mass dimension of $R$ and $R_{\mu\nu}$ are 2. Thus,in terms of mass scale,$\frac{R}{16\pi G}$ corresponds to $\frac{M^2 M_P^2}{16\pi}$ and $(\tilde{\alpha} R_{\mu\nu}R^{\mu\nu} + \tilde{\beta} R^2)$ corresponds to $(\tilde{\alpha} + \tilde{\beta})M^4$. So, it is found that when $M > [16\pi(\tilde{\alpha} + \tilde{\beta})]^{-1/2}M_P$, $R^2$-terms will dominate over $\frac{R}{16\pi G}$ and contrary to it, $\frac{R}{16\pi G}$ will dominate over $R^2$-terms when $M < [16\pi(\tilde{\alpha} + \tilde{\beta})]^{-1/2} M_P$ [6].

It is clear from the above discussion that eq.(1.4) can be obtained only when $R^2$-terms are not insignificant compared to linear term $\frac{R}{16\pi G}$ in $\bar{S}_g$ which is possible at the energy mass scale

$$M \geq [16\pi(\tilde{\alpha} + \tilde{\beta})]^{-1/2} M_P \qquad (1.5).$$

Eq.(1.5) shows that $M \geq 100 Gev$ only when $(\tilde{\alpha} + \tilde{\beta}) \simeq 10^{33}$, which means that coupling constants associated with $R^2$-terms should be extremely large.Now one can think that if coupling constants of $R^2$-terms are so strong, these terms should manifest themselves at low energy also. But we do not observe manifestation of higher - derivative terms in gravity at low energy level. It indicates that these coupling constants should be small enough so that $R^2$-terms be relevant at high energy only. On the basis of above discussion,$\tilde{\alpha}$ and $\tilde{\beta}$ are chosen to satisfy the following equations

$$5\tilde{\alpha} + 12\tilde{\beta} = 2 \qquad (1.6a)$$

and

$$\tilde{\alpha} + \tilde{\beta} = 1. \qquad (1.6b)$$

for the sake of convenience. Under these conditions ( given by eq.(1.6)), $m$ is given by

$$m = 1.41 \times 10^{18} Gev$$

and the inequality (1.5) looks like

$$M \geq 1.41 \times 10^{18} Gev.$$

Now the question arises whether eq.(1.4) can be treated as Klein - Gordon equation of $R$ like Klein - Gordon equation for other scalar fields $\phi$ given as

$$\Box\phi + m_\phi^2\phi = 0. \tag{1.7}$$

On comparing eqs.(1.4) and (1.7), one finds two problems. Firstly, mass dimension of $\phi$ is 1 , whereas mass dimension of $R$ is 2. Second problem is more serious in the sense that operator $\Box$ and $R$ both depend on $g_{\mu\nu}$,while $\phi$ does not depend on $g_{\mu\nu}$.

To evade the first problem, eq.(1.4) can be multiplied by $\eta$ ( where $\mid \eta \mid= 1$ and dimension of $\eta$ is $(mass)^{-1}$) and it is rewritten as

$$\Box\tilde{R} + m^2\tilde{R} = 0, \tag{1.8}$$

where $\tilde{R} = \eta R$. Mass dimension of $\tilde{R}$ is 1.But even eq.(1.8) is not free from the second problem.

High energy modes excite the physical system at small length scales. Thus one finds that high frequency modes probe the geometry in the small vicinity of a space - time point with co-ordinates $\{\acute{x}^\mu; \mu = 0,1,2,3\}$. $g_{\mu\nu}$ can be expanded around this point as

$$g_{\mu\nu}(x) = g_{\mu\nu}(\acute{x}) + \frac{1}{3}R_{\mu\alpha\nu\beta}(\acute{x})y^\alpha y^\beta - \frac{1}{6}\partial_\gamma R_{\mu\alpha\nu\beta}(\acute{x})y^\alpha y^\beta y^\gamma +$$

$$\left[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta} + \frac{2}{45}R_{\alpha\mu\beta\lambda}R^\lambda_{\gamma\nu\delta}\right](\acute{x})y^\alpha y^\beta y^\gamma y^\delta + ........, \tag{1.9}$$

where $y^\alpha = x^\alpha - \acute{x}^\alpha (\alpha = 0,1,2,3)$ and $g_{\mu\nu}(\acute{x}) = \eta_{\mu\nu}$. Using these expansions ( given by eq.(1.9)), one obtains operator $\Box$ as

$$\Box = g^{\mu\nu}\frac{\partial^2}{\partial x^\mu \partial x^\nu} + B^\nu \frac{\partial}{\partial x^\nu} \tag{1.10a}$$

where

$$g^{\mu\nu}(x) = g^{\mu\nu}(\acute{x}) - \frac{1}{3}R^{\mu\nu}_{\alpha\beta}(\acute{x})y^{\alpha}y^{\beta} - \frac{1}{6}\partial_{\gamma}R^{\mu\nu}_{\alpha\beta}(\acute{x})y^{\alpha}y^{\beta}y^{\gamma} -$$

$$\Big[\frac{1}{20}R_{\mu\alpha\nu\beta;\gamma\delta} + \frac{2}{45}R_{\alpha\mu\beta\lambda}R^{\lambda}_{\gamma\nu\delta}\Big](\acute{x})y^{\alpha}y^{\beta}y^{\gamma}y^{\delta} + \ldots\ldots.., \qquad (1.10b)$$

and

$$B^{\nu} = \Big[\frac{1}{6}\partial_{\gamma}R^{\gamma\nu}_{\alpha\beta} - \frac{1}{12}\partial^{\nu}R_{\alpha\beta}\Big](\acute{x})y^{\alpha}y^{\beta} - \Big[\frac{1}{20}R^{\nu}_{\beta;\gamma\delta} +$$

$$\frac{2}{45}R_{\beta\lambda}R^{\lambda\nu}_{\gamma\delta}\Big](\acute{x})y^{\beta}y^{\gamma}y^{\delta} + \Big[\frac{1}{20}R^{\nu}_{\alpha;\gamma\delta} + \frac{2}{45}R_{\beta\lambda}R^{\lambda\nu}_{\gamma\delta}\Big](\acute{x})\times$$

$$y^{\alpha}y^{\gamma}y^{\delta} - \Big[\frac{1}{20}R^{\mu\nu}_{\alpha;\mu\delta} + \frac{2}{45}R^{\mu}_{\alpha\beta\gamma}R^{\gamma\nu}_{\mu\delta}\Big](\acute{x})y^{\alpha}y^{\beta}y^{\delta} -$$

$$\Big[\frac{1}{20}R^{\mu\nu}_{\alpha\beta;\gamma\mu} + \frac{2}{45}R^{\mu}_{\alpha\beta\lambda}R^{\lambda\nu}_{\gamma\mu}\Big](\acute{x})y^{\alpha}y^{\beta}y^{\gamma} - \frac{1}{6}R_{\gamma\delta}(\acute{x})\Big[\frac{1}{6}\partial_{\gamma}R^{\gamma\nu}_{\alpha\beta}$$

$$-\frac{1}{12}\partial^{\nu}R_{\alpha\beta}\Big](\acute{x})y^{\alpha}y^{\beta}y^{\gamma}y^{\delta} + \ldots\ldots \qquad (1.10c)$$

Thus, at high energy level, one can work in the small neighbourhood of a point $\{\acute{x}\}$.where $\Box$ depends on curvature terms evaluated at this particular point and $\tilde{R}(x)$ is defined at an arbitrary point in the neighbourhood. So, at high energy , it is possible to have $\tilde{R}$ independet of $\Box$ and it can be treated as $\phi$ , given by eq.(1.7).Physically, it means that, at high energy scales, $\tilde{R}$ behaves as a spinless matter field, which is not possible at low energy scales due to insignificance of $R^2$ - terms compared to $\frac{R}{16\pi G}$in the gravitational action. Moreover, expansion of $g_{\mu\nu}$ in the small neighbourhood of a point is not physically viable at low energy scales, because extremely small distances can not be observed by low frequency modes.

From the above discussion , it is clear that Ricci scalar has dual role at high energy (1) as a spinless matter field and (2) as a geometrical field. The matter aspect of Ricci scalar is exhibited by $\tilde{R}$. The geometrical aspect is exhibited by $g_{\mu\nu}$.

Taking the matter aspect of $R$, the lagrangian corresponding to eq.(1.8) can be written as

$$L = \frac{1}{2}\Big[g^{\mu\nu}\partial_\mu \tilde{R}\partial_\nu \tilde{R} - m^2 \tilde{R}^2\Big] \tag{1.11}$$

with action $\int d^4x L$ , where $\tilde{R}$ is taken as a basic physical field and invariance of this action under transformation $\tilde{R} \rightarrow \tilde{R} + \delta\tilde{R}$ yields eq.(1.8). When $R$ is treated as a geometrical field, $g_{\mu\nu}$ are considered as basic fields. In quantum field theory, fields are treated as physical concepts describing elementary particles. So particles , described by $\tilde{R}$, are hereafter called riccions (which are new particles different from gravitons in the scenario of pure gravitational theories).Riccions are massive spinless particles, whereas gravitons are spin - 2 massless particles. Propagator for riccions is expected to be ghost - free contrary to gravitons from higher - derivative gravity.

In the present paper, $R^3$ - terms are also added to the gravitational action . As a result, riccions are obtained with a quartic self potential. It is interesting to see that instanton solution for riccions gives rise to primordial inflation without any phase transition and spontaneous symmetry breaking.

The main text of the paper starts from section 2 , where an extended form of gravitational is considered and instanton solution is obtained for riccions. Section 3 contains discussion on primordial inflation obtained using the instanton solution. Section 4 is the concluding section, where it is shown that entropy problem can be solved in Einstein phase using results obtained in previous sections.

## 2.Instanton solution for riccions

Here the gravitational action is taken as

$$S_g = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi G} - \tilde{\alpha}R_{\mu\nu}R^{\mu\nu} - \tilde{\beta}R^2 + \lambda\eta^2R^3\Big] \qquad (2.1)$$

where $\tilde{\alpha}$ , $\tilde{\beta}$ and $\lambda$ are dimensionless coupling constants satisfying eqs.(1.6). Here G is the Newtonian gravitational constant as given in the previous section.

Invariance of $S_g$ under transformations $g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$ yields the gravitational field equations

$$\begin{aligned}(1/16\pi G)(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - \tilde{\alpha}(R^{\alpha}_{\mu;\nu\alpha} - \Box R_{\mu\nu}\\ -\frac{1}{2}g_{\mu\nu}\Box R + 2R^{\alpha}_{\mu}R_{\alpha\nu} - \frac{1}{2}g_{\mu\nu}R^{\gamma\delta}R_{\gamma\delta}) - \tilde{\beta}(2R_{;\mu\nu}\\ -2g_{\mu\nu}\Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}) + \lambda\eta^2(6R^2_{;\mu\nu}\\ -6g_{\mu\nu}\Box R^2 - \frac{1}{2}g_{\mu\nu}R^3 + 3R^2R_{\mu\nu}) = 0\end{aligned} \qquad (2.2)$$

Trace of these field equations yields

$$-\frac{R}{16\pi G} + \Box R + \lambda\eta^2R^3 - 18\lambda\eta^2\Box R^2 = 0 \qquad (2.3)$$

Using Gauss's divergence theorem

$$\int_{\Omega} d^4x\sqrt{-g}\Box R^2 = \int_{\partial\Omega} d^3x\sqrt{-g}n^{\alpha}R^2_{,\alpha},$$

which yields

$$M \geq 1.19 \times 10^9 \lambda^{-1/4} GeV \tag{2.7}$$

as $\tilde{\alpha}$ and $\tilde{\beta}$ satisfy the equation $5\tilde{\alpha} + 12\tilde{\beta} = 2$.

It means that when $M < 1.19 \times 10^9 \lambda^{-1/4} GeV$, $\frac{R}{16\pi G}$ will dominate over higher - derivative terms and Einstein's theory of gravity will be effective.

Energy - momentum tensor components for Riccions are obtained as

$$T_{\mu\nu} = \partial_\mu \tilde{R} \partial_\nu \tilde{R} - g_{\mu\nu}\Big[\frac{1}{2} g^{\rho\sigma} \partial_\rho \tilde{R} \partial_\sigma \tilde{R} - \frac{\lambda}{4}\Big(\tilde{R}^2 - \frac{M_P^2}{16\pi\lambda}\Big)^2\Big]$$

$$-2\eta R_{\mu\nu} \Box \tilde{R} - 2\lambda\eta\Big(\tilde{R}^2 - \frac{M_P^2}{16\pi\lambda}\Big)\tilde{R} R_{\mu\nu} - 8\lambda\eta\Big[\Big(\tilde{R}^2$$

$$-\frac{M_P^2}{16\pi\lambda}\Big)\tilde{R}\Big]_{;\mu\nu} + 8\lambda\eta g_{\mu\nu} \Box \Big(\tilde{R} - \frac{M_P^2}{16\pi\lambda}\Big)\tilde{R} \tag{2.8}$$

After getting the lagrangian for $\tilde{R}$, given by eq.(2.10) , one can find the condition for the finite Euclidean action corresponding to this lagrangian , which gives the classical path for imaginary time called instanton.

If early universe is spatially flat , homogeneous and isotropic, its geometry can be described by the Robertson - Walker line - element

$$dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \tag{2.9}$$

In the background geomatry of the model , given by eq.(2.9) , the Riccion energy can be written as

$$E = \frac{1}{2} g^{00} \Big(\frac{d\tilde{R}}{dt}\Big)^2 + V(\tilde{R}). \tag{2.10}$$

Now using $t = -i\tau$,one obtains that

$$\tilde{R} \to -\tilde{R} \tag{2.11}$$

and

$$E = -\frac{1}{2}\Big(\frac{d\tilde{R}}{d\tau}\Big)^2 + V(\tilde{R}) \tag{2.12}$$

Thus , in imaginary time , $\tilde{R}$ can go from $\tilde{R} = -\frac{M_P}{\sqrt{16\pi\lambda}}$ state to $\tilde{R} = +\frac{M_P}{\sqrt{16\pi\lambda}}$ state with $E = 0$. Setting $E = 0$ in eq.(2.12), one obtains the classical trajectory in $\tilde{R} - V(\tilde{R})$ plane with imaginary time as

$$\tilde{R} = \frac{M_P}{\sqrt{16\pi\lambda}} tanh\Big(\frac{M_P\tau}{4\sqrt{2\pi}}\Big), \tag{2.13a}$$

which is an instanton solution . The action for this trajectory in imaginary time system can be calculated as

$$S_E = l^3 \int_{-\infty}^{\infty} d\tau a^3(\tau)\Big[\frac{1}{2}\Big(\frac{d\tilde{R}}{d\tau}\Big)^2 + \frac{\lambda}{4}\Big(\tilde{R}^2 - \frac{M_P^2}{16\pi\lambda}\Big)^2\Big]$$

$$= \frac{l^3\lambda}{2} \int_{-\infty}^{\infty} d\tau a^3(\tau)\Big(\tilde{R}^2 - \frac{M_P^2}{16\pi\lambda}\Big)^2, \tag{2.13b}$$

using eq.(2.12)with $E = 0$. Here $l^3$ is the spatial volume. After getting $a(\tau)$ , it will be shown below that $S_E$ is finite, which is must for the trajectory , given by eq.(2.13a), to be an instanton.

## 3.Primordial inflation

The basic equation showing physical aspect of $\tilde{R}$ is eq.(2.5). So $\tilde{R}$ , given by eq.(2.13a),should satisfy eq.(2.5) for the sake of consistency. In the background geometry of the early universe, given by eq.(2.9) , eq.(2.5) is written as

$$\ddot{\tilde{R}} + \frac{3\dot{a}}{a}\dot{\tilde{R}} = -\lambda\tilde{R}(\tilde{R}^2 - \frac{M_P^2}{16\pi\lambda}). \tag{3.1}$$

Connecting eqs.(2.13) and (3.1), one finds that eq.(3.1) is satisfied by $\tilde{R}$ , given by eq.(2.13), only when

$$\tau \geq \frac{48.8\sqrt{2\pi}}{M_P} = 1.22 \times 10^{-17}GeV^{-1} \tag{3.2}$$

as $tanh 12.2 = 1$ and $Sech 12.2 = 0$ . For $\tau < 1.22\times10^{-17}GeV^{-1}$, eq.(3.1) yields $\frac{\ddot{a}}{a} = 0$, which satisfies eq.(2.13) only when $\tau = 0$. Moreover , components of energy - momentum tensor is complex for $\tau < 1.22 \times 10^{-17}GeV^{-1}$. So , to be on safe side , one can take $\tau \geq 1.22 \times 10^{-17}GeV^{-1}$ which yields

$$\tilde{R} = \frac{M_P}{\sqrt{16\pi\lambda}}. \tag{3.3}$$

With $t = -i\tau$,$\tilde{R}$ is obtained from the line - element , given by eq.(2.9), as

$$\tilde{R} = -6\eta\Big[\frac{a''}{a} + \Big(\frac{a'}{a}\Big)^2\Big], \tag{3.4}$$

where prime denotes derivation with respect to $\tau$. Thus for $\tau \geq 1.22 \times 10^{-17}GeV^{-1}$ , one obtains from eqs.(3.3) and (3.4)

$$\frac{a''}{a} + \Big(\frac{a'}{a}\Big)^2 = -\frac{M_P}{24\eta\sqrt{\lambda\pi}}. \tag{3.5}$$

The first integral of the differential equation (3.5) is

$$a^2 \acute{a}^2 = A - \frac{M_P a^4}{96\eta\sqrt{\lambda}\pi}, \tag{3.6}$$

where $A$ is an integration constant which can be absorbed through rescaling $a \rightarrow A^{1/4} a$. As a result , eq.(3.6) can be re - written as

$$a^2 \acute{a}^2 = 1 - \frac{M_P a^4}{96\eta\sqrt{\lambda}\pi} \tag{3.7}$$

which yields the solution

$$a^2 = a_0^2 cos[2(\frac{M_P}{96\eta\sqrt{\lambda}\pi})^{1/2}(\tau - \tau_0)], \tag{3.8a}$$

with

$$a_0^2 = (\frac{96\eta\sqrt{\lambda}\pi}{M_P})^{1/2}. \tag{3.8b}$$

Using eqs.(3.8) in eq.(2.13b)

$$S_E \leq \frac{\lambda l^3}{2} \int_{-\infty}^{\infty} d\tau (\frac{96\eta\sqrt{\lambda}\pi}{M_P})^{3/4} \frac{M_P^4}{(16\pi\lambda)^2} Sech^4(\frac{M_P \tau}{4\sqrt{2\pi}})$$

$$= 1.23 \times \lambda^{-5/8} {M_P}^{9/4} l^3.$$

Now using $\tau = it$ , one obtains from eqs.(3.8)

$$a = a_0 cosh^{1/2}\Big[2\Big(\frac{M_P}{96\eta\sqrt{\lambda}\pi}\Big)^{1/2}(t - t_0)\Big]. \tag{3.9}$$

As discussed above , solutions (given by eqs.(3.8) and (3.9) are valid for $t = \tau \geq 1.22 \times 10^{-17} GeV^{-1}$ . So one can take

$$t_0 = 1.22 \times 10^{-17} GeV^{-1}.$$

When $t \geq t_0 + 1.03 \times 10^{-9} \lambda^{1/4} GeV^{-1}$,

$$a \cong a_0 exp\Big[\Big(\frac{M_P}{96\eta\sqrt{\lambda}\pi}\Big)^{1/2}(t - t_0)\Big]. \qquad (3.10)$$

Thus it is obtained that inflationary scenario starts at $1.22 \times 10^{-17} GeV^{-1} \simeq 8.03 \times 10^{-42} Sec.$ ie. well before Grand Unified phase transition which is expected to occur at $10^{-35} Sec.$[7]. This phenomenon is called primordial inflation [ 8,9].

It is interesting to see that primordial inflation is obtained here without any kind of phase transition and spontaneous aymmetry breaking . So the model is free from problems arising out of phase transitions . It is free from fine - tuning problem also . Moreover, it can explain existence of extremely large entropy in the early universe. This model can provide sufficient inflation when

$$t = t_0 + 2.68 \times 10^{-7} \lambda^{1/4} GeV^{-1}, \qquad (3.11)$$

as at this particular time $a = 1.69 \times 10^{28} a_0 = 1.09 \times 10^{24} \lambda^{1/8}$. The entropy in the universe can be calculated as

$$S \simeq (4/3) a^3 T^3. \qquad (3.12)$$

As discussed above , higher - derivative terms will dominate over Einstein - Hilbert action till $1.19 \times 10^9 \lambda^{-1/4} GeV$. So inflationary scenario (obtained here) can continue till energy falls to this level as below this level Einstein phase will start and Ricci scalar will behave as a geometrical field only .

## 4.Concluding remarks

In the Einstein phase , the amount of entropy required to solve the fine - tuning problem is $\geq 10^{87}$ [7,10]. The entropy in the above model can be calculated using eq.(3.12) after sufficient inflation as

$$S \geq 2.9 \times 10^{99} \lambda^{-3/8}, \tag{4.1}$$

which shows that the required amount of entropy can be generated provided that

$$\lambda = 1. \tag{4.2}$$

Determination of dimensionless coupling constant $\lambda$ removes arbitrariness in energy mass scales and time for various stages obtained above . Using this value of $\lambda$ , one obtains that higher - derivative terms will dominate over Einstein - Hilbert term in $S_g$ till energy mass scale comes down to

$$M \geq 11.9 \times 10^{9} GeV. \tag{4.3}$$

Exponential expansion of the model will start when

$$t \geq t_0 + 1.03 \times 10^{-9} GeV^{-1}$$

and sufficient inflation will be obtained when

$$t = t_0 + 2.68 \times 10^{-7} GeV^{-1}.$$

At this particular time ( time of sufficient inflation )

$$a = 1.04 \times 10^{24}.$$

Author is thankful to Prof. K.P.Sinha for encouragement and helpful suggestions during preparation of this paper .

## References

1.S.W.Hawking and R.Penrose , Proc.R.Soc.(London)A **314** (1970)229.

2.I.L.Buchbinder , S.D.Odintsov and I.L.Shapiro , Effective action in Quantum Gravity ( IOP Publishing Ltd., 1952 ).

3.S.K.Srivastava and K.P.Sinha , Phys.Lett.B **307**(1993) 40.

4.K.P.Sinha and S.K.Srivastava , Jour.Ind.Math.Soc.,**61**(1994) 80.

5.K.P.Sinha and S,K,Srivastava , Pramana , **44** (1995) 383.

6.S.K.Srivastava and K.P.Sinha,Int.J.Theo.Phys.**35**(1996)135.

7.A.H.Guth,"Phase transitions in the very early universe" from 'The Very Early Universe" ed. G.W.Gibbons,S.W.Hawking and S.Siklos, Cambridge University Press (1983).

8.J.Ellis,D.V.Nanopoulos,K.A.Olive and K.Tamvakis,Nucl.Phys. B**221** (1983),524.

9.S.K.Srivastava, Gen.Rel.Grav.J., **19** (1987) , 537.

10.R.D.Tenreiro and M.Quiros," An Introducytion to Cosmology and Particle Physics", World Scientific (1988).