The Rigorous Analysis of Cascaded Step Discontinuities in Microstrip

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Abstract —**A rigorous analysis of boxed microstrip single step discontinuities and cascades of strongly coupled discontinuities is presented. Use is made of a variational formulation involving the expansion of the transverse E field at the step in terms of suitable basis functions. Strongly coupled steps are analyzed using the concept of "localized" and "accessible" modes and making use of a network model. The method is applied to a five-section low-pass filter.**

I. Introduction

It is becoming increasingly important to be able to predict accurately the behavior of microstrip circuits before manufacture. This is especially true in the design of microwave integrated circuits where adjustments after fabrication are very difficult or impossible to carry out.

The currently available methods for use in the computer-aided design of microwave components, e.g. [1], [2] rely heavily on quasi-static approximations which are only correct in the limit of low frequency and which suffer significant error as the frequency increases.

Cascades of step discontinuities constitute a basic configuration for the design of filters and impedance transformers, and it is to these in particular that the work described herein is addressed. Methods by which a more accurate frequency-dependent solution have previously been attempted include the equivalent waveguide model (e.g. [3]), the transmission line matrix method (e.g. [4]), and the finite element method. The method of mode matching has been applied directly to finline [5] and microstrip [19] and also to the parallel-plate waveguide model [6], although it is well known that this method may suffer from the "relative convergence" problem [7].

More recently a rigorous formulation of the single step discontinuity in microstrip, such as that shown in Fig. 1, has been published [8] and a wide variety of results presented. In this method, the portion of microstrip including the step is enclosed by electric walls to form a resonant cavity. By varying the length of the cavity and evaluating the resonant frequencies, the S parameters of the step can be obtained. While this method gives good results for the single step, it does not lend itself readily to the treatment of cascades of strongly coupled discontinuities. This is due to the fact that the amount of computation becomes very large when a complicated metallization pattern is analyzed.

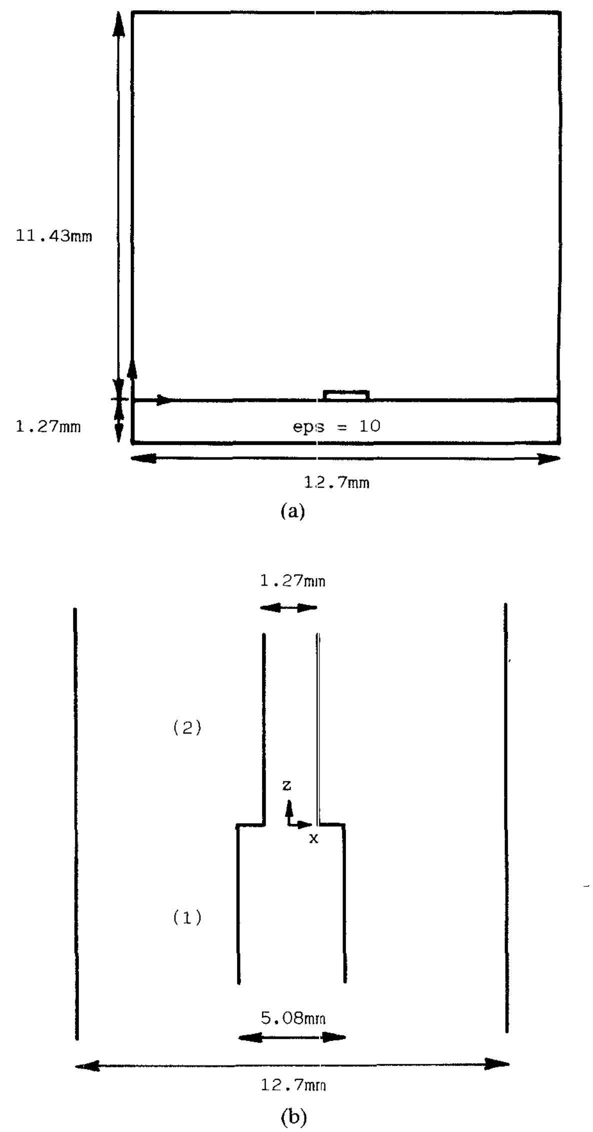

Fig. 1. (a) Microstrip cross section. (b) Plan of step discontinuity.

The formulation presented herein makes use of variational principles for the generalized S parameters of a single step discontinuity. This lends itself to the treatment of strongly interacting discontinuities by means of the concept of accessible and localized modes [9]–[11]. In this approach the higher order modes excited at the discontinuity are treated according to their effect at the neighboring

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discontinuities. If they have a significant effect, then they are deemed to be "accessible"; otherwise they are deemed to be "localized." Since there is no localized mode incident at a discontinuity, these scattered modes are effectively terminated in their characteristic impedances. Each discontinuity is treated as a multiport device, each port corresponding to an accessible mode. Likewise the microstrip section which connects neighboring discontinuities is modeled as a set of transmission lines, each carrying one accessible mode. In this way the coupling between the discontinuities can be accurately accounted for.

The single step discontinuity is analyzed using the Galerkin variational method. The E field at the discontinuity is expanded in the set of microstrip modes at each side of the step and also in a suitable set of vector basis functions appropriate to the step itself.

In order to analyze a microstrip discontinuity in this way, it is necessary to calculate the field patterns of a large number of microstrip modes, typically 100. An efficient method for achieving this has been presented [12], [13].

II. The Formulation of the Single Step Discontinuity

Most formulations of the microstrip step discontinuity make use of the equivalent circuit shown in Fig. 2, where the element values are evaluated by quasi-static approximation. This model, however, suffers from the disadvantages that it is only correct in the limit of low frequency, and that as it stands cannot be used to model strongly coupled steps.

The formulation presented here uses the model shown in Fig. 3. The step is represented by a multiport device with frequency-dependent S parameters. Each port on the model corresponds to an accessible mode, that is, a mode which does not decay to negligible levels by the time it reaches the next discontinuity. Combination of these S matrices, by standard network methods, makes possible the characterization of cascades of strongly coupled discontinuities (see Fig. 4). In principle, the accuracy of the model can be systematically improved by increasing the number of modes which are treated as accessible. In practice, however, as the number of modes deemed to be accessible is increased, the increase in numerical error becomes greater than the improvement from the formally more accurate representation.

Referring to the plan of Fig. 1, we start from the continuity equations for the transverse E and H fields, expanded in terms of the transverse modal fields at either side of the step:

$$\sum_n \left(a_n^{(1)} + b_n^{(1)}\right) \boldsymbol{E}_n^{(1)} = \sum_n \left(a_n^{(2)} + b_n^{(2)}\right) \boldsymbol{E}_n^{(2)} = \boldsymbol{E}(\boldsymbol{r}) \quad (1)$$

$$\sum_n \left(a_n^{(1)} - b_n^{(1)}\right) \boldsymbol{H}_n^{(1)} = \sum \left(a_n^{(2)} - b_n^{(2)}\right) \boldsymbol{H}_n^{(2)} + \hat{z} \times \boldsymbol{J} \quad (2)$$

where

$\boldsymbol{E}(\boldsymbol{r})$ is the transverse electric field at the discontinuity;
$\hat{z}$ is the unit vector in the z direction;

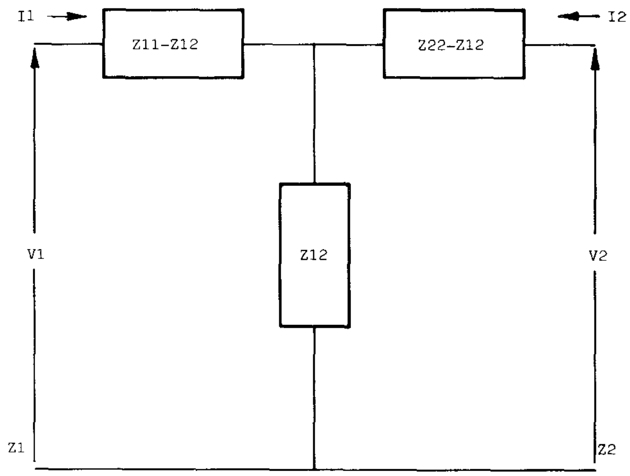


Fig. 2. Quasi-static equivalent circuit of step.

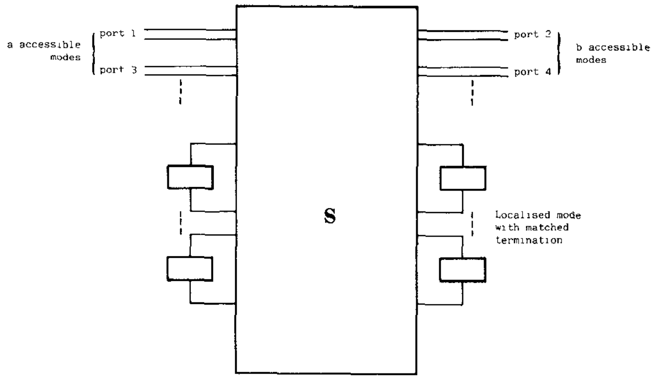


Fig. 3. Network model of single discontinuity.

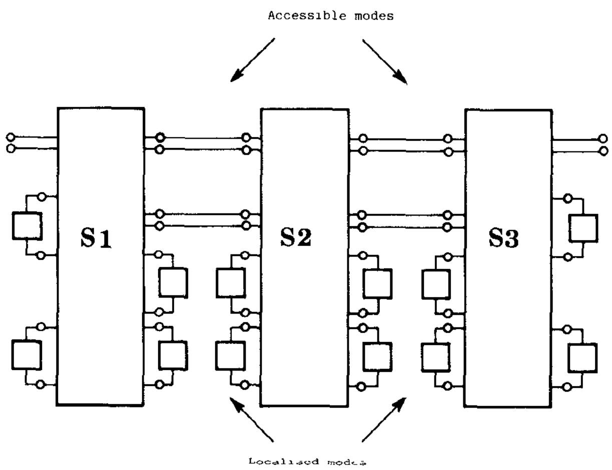


Fig. 4. Network model of cascaded discontinuities.

$\boldsymbol{J}$ is the current at the step;
the coefficients a represent the incident wave amplitudes;
the coefficients b represent the scattered wave amplitudes;
the superscripts (1) and (2) refer to the regions defined in Fig. 1.

Note that due to the hybrid nature of the modes, there is no simple relationship between E and H transverse. Consequently, we specify the E field and the H field separately. This contrasts with the case of simple LSE/LSM modes, where it is possible to define a simple scalar wave impedance linking E and H transverse.

We normalize the modes so that

$$\langle \boldsymbol{E}_n, \boldsymbol{H}_m \rangle = \delta_{mn} \tag{3}$$

where

$$\delta_{mn} = 0 \qquad m \neq n \qquad \delta_{mm} = 1$$

and the inner product is defined as

$$\int \boldsymbol{E}_n \times \boldsymbol{H}_m \cdot \hat{z}\, dS \tag{4}$$

with the integral taken over the box cross sections.

By taking the inner products of each side of (1) with each of the microstrip modes in turn we get

$$a_n^{(1)} + b_n^{(1)} = \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(1)} \rangle}{\langle \boldsymbol{E}_n^{(1)}, \boldsymbol{H}_n^{(1)} \rangle} \tag{5}$$

$$a_n^{(2)} + b_n^{(2)} = \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(2)} \rangle}{\langle \boldsymbol{E}_n^{(2)}, \boldsymbol{H}_n^{(2)} \rangle}. \tag{6}$$

To proceed we choose the inputs to the ports to satisfy the following conditions:

$$\begin{aligned} a_t^{(1)} &= 1 \\ a_p^{(1)} &= 0 \qquad p \neq t \\ a_p^{(2)} &= 0. \end{aligned} \tag{7}$$

Substituting into (5) and (6) gives

$$1 + b_t^{(1)} = \frac{\langle \boldsymbol{E}, \boldsymbol{H}_t^{(1)} \rangle}{\langle \boldsymbol{E}_t^{(1)}, \boldsymbol{H}_t^{(1)} \rangle} = 1 + S_{tt} \tag{8}$$

$$b_p^{(1)} = \frac{\langle \boldsymbol{E}, \boldsymbol{H}_p^{(1)} \rangle}{\langle \boldsymbol{E}_p^{(1)}, \boldsymbol{H}_p^{(1)} \rangle} = S_{pt} \tag{9}$$

$$b_p^{(2)} = \frac{\langle \boldsymbol{E}, \boldsymbol{H}_p^{(2)} \rangle}{\langle \boldsymbol{E}_p^{(2)}, \boldsymbol{H}_p^{(2)} \rangle} = S_{qt}, \qquad q = a + p \tag{10}$$

where a is the number of accessible modes in region (1).

We now substitute these expressions into (2):

$$\left(1 - b_t^{(1)}\right)\boldsymbol{H}_t^{(1)} - \sum_{n \neq t} \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(1)} \rangle}{\langle \boldsymbol{E}_n^{(1)}, \boldsymbol{H}_n^{(1)} \rangle} \boldsymbol{H}_n^{(1)} = \sum_n \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(2)} \rangle}{\langle \boldsymbol{E}_n^{(2)}, \boldsymbol{H}_n^{(2)} \rangle} \boldsymbol{H}_n^{(2)}. \tag{11}$$

Therefore

$$2\boldsymbol{H}_t^{(1)} = \sum_n \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(1)} \rangle}{\langle \boldsymbol{E}_n^{(1)}, \boldsymbol{H}_n^{(1)} \rangle} \boldsymbol{H}_n^{(1)} + \sum_n \frac{\langle \boldsymbol{E}, \boldsymbol{H}_n^{(2)} \rangle}{\langle \boldsymbol{E}_n^{(2)}, \boldsymbol{H}_n^{(2)} \rangle} \boldsymbol{H}_n^{(2)}. \tag{12}$$

We now take inner products of both sides of this equation with E, yielding

$$\langle \boldsymbol{E}, \boldsymbol{H}_t^{(1)} \rangle = \langle \boldsymbol{E}, \boldsymbol{G}, \boldsymbol{E} \rangle \tag{13}$$

where the kernel g of the integral operator G is given by

$$2g(r, r') = \sum_{n=1}^{\infty} \left[\frac{\boldsymbol{H}_n^{(1)}(r) \boldsymbol{H}_n^{(1)}(r')}{\langle \boldsymbol{E}_n^{(1)}, \boldsymbol{H}_n^{(1)} \rangle} + \frac{\boldsymbol{H}_n^{(2)}(r) \boldsymbol{H}_n^{(2)}(r')}{\langle \boldsymbol{E}_n^{(2)}, \boldsymbol{H}_n^{(2)} \rangle} \right]. \tag{14}$$

Making use of (8)–(10) and the normalization given by (3), we get the following expressions for the elements of the S matrix:

$$\frac{\langle \boldsymbol{E}, \boldsymbol{G}, \boldsymbol{E} \rangle}{\langle \boldsymbol{E}, \boldsymbol{H}_p^{(1)} \rangle \langle \boldsymbol{E}, \boldsymbol{H}_t^{(1)} \rangle} = R_{pt} \tag{15}$$

where

$$\begin{aligned} R_{pt} &= \frac{1}{S_{pt} - \delta_{pt}}, \qquad p \leqslant a \\ R_{pt} &= \frac{1}{S_{pt}}, \qquad p > a. \end{aligned} \tag{16}$$

We expand the unknown function for the electric field at the discontinuity in terms of a complete set of two-dimensional vector basis functions which satisfy the boundary conditions:

$$\boldsymbol{E} = \sum_{q=1}^{\infty} c_q \phi_q(x, y). \tag{17}$$

Substituting these expressions into (15) and taking partial derivatives of each side of the equation with respect to c_u $(1 \leqslant u < \infty)$, we obtain the following:

$$\langle \boldsymbol{E}, \boldsymbol{H}_p^{(1)} \rangle \langle \boldsymbol{E}, \boldsymbol{H}_t^{(1)} \rangle \frac{\partial R_{pt}}{\partial c_u} + \sum_q c_q \langle \phi_u, \boldsymbol{H}_t^{(1)} \rangle \langle \phi_q, \boldsymbol{H}_p^{(1)} \rangle R_{pt} = \sum_q c_q \langle \phi_q, \boldsymbol{G}, \phi_u \rangle. \tag{18}$$

Substituting for R_{pt} we get

$$\left\{ \langle \phi_u, \boldsymbol{H}_t^{(1)} \rangle - \sum_q c_q \langle \phi_q, \boldsymbol{G}, \phi_u \rangle \right\} + \langle \boldsymbol{E}, \boldsymbol{H}_p^{(1)} \rangle \langle \boldsymbol{E}, \boldsymbol{H}_t^{(1)} \rangle \frac{\partial R_{pt}}{\partial c_q} = 0. \tag{19}$$

From (12) we get

$$\langle \phi_u, \boldsymbol{H}_t^{(1)} \rangle = \langle \phi_u, \boldsymbol{G}, \boldsymbol{E} \rangle \tag{20}$$

which, if substituted into (19), yields

$$\frac{\partial R_{pt}}{\partial c_u} = 0 \tag{21}$$

$$\langle \phi_u, \boldsymbol{H}_t^{(1)} \rangle = \sum_q c_q \langle \phi_q, \boldsymbol{G}, \phi_u \rangle. \tag{22}$$

The first result shows that the expressions for the elements of the S matrix are stationary with respect to small changes in the trial field function and hence we have a variational principle. The second result has the form of an infinite set of simultaneous equations from which the coefficients c_q may be calculated. Hence the field may be found from (17), and the left half of the S matrix can be found from

(8)–(10). The other half of the S matrix is found by means of a similar analysis with inputs to the ports satisfying the conditions

$$a_p^{(1)} = 0$$
$$a_t^{(2)} = 1 \qquad p \neq t$$
$$a_p^{(2)} = 0$$

instead of those specified in (7).

We note that (22) is the same as would have been obtained if Galerkin's method had been applied to (12). This is a consequence of the fact that the operator G is self-adjoint, which in turn is a consequence of the law of conservation of energy.

In practice, of course, we approximate the field with a small number of basis functions, chosen to well approximate the actual field at the discontinuity. This leads to an efficient and accurate formulation. The form of the chosen basis functions is discussed later.

III. Choice of Basis Functions for the Step

In (17) we made use of a set of basis functions in which to expand the transverse E field at the step. It is crucial that a good choice be made here; otherwise the result will be inaccurate. It is this aspect of Galerkin's method, and other methods of a similar nature, which has attracted criticism [14]. Where it is possible, from physical considerations, to know *a priori* the important characteristics of the unknown function, then basis functions can be chosen which ensure fast convergence. Such a procedure has been used to good effect for the solution of the modes of uniform microstrip [12] and finline [15] where the singularity of the fields at the edge of the infinitely thin strip or fin are known exactly.

Unfortunately, for the case of the step discontinuity, it is not obvious what the form of the fields will be. There is no simply applicable condition, corresponding to the edge condition at a wedge, which can be used. Possible ways of deriving a suitable set of basis functions may involve numerical methods to solve the static problem [16] or making the order of the singularity at the corner of the step a variable with respect to which we can extremize the variational expression [17].

Various sets of basis functions which satisfy the boundary conditions, but which incorporate no beliefs concerning the form of the field at the step, have been tried. In most cases, however, the result has been a very ill-conditioned set of equations (22) from which no satisfactory answer could be obtained.

A simple set of basis functions which can be used is the wave patterns of E_x and E_y of the modes of the microstrip containing the wider of the two strips. These functions meet the boundary conditions, but do not have the correct singularity at the corner. From physical considerations, however, it is likely that the field at the step will be similar to the field in the wider continuous microstrip. The ratio of E_x to E_y is left as a parameter to be found during the solution of the variational expression. If this were not done, then the higher order modes of the wider microstrip excited by the discontinuity would be orthogonal to the basis functions and therefore would not contribute to the sum in (12). Fig. 5 shows the results of calculating the phase of $S12$ for a step discontinuity using different numbers of basis functions at frequencies up to 12 GHz. It can be seen that convergence is achieved at various frequencies when nine basis functions are used and the ratio of the strip widths is 4:1. While not ideal, this result means that only a moderately small matrix need be handled. This contrasts with the large matrices which result from employing mode matching methods such as [19].

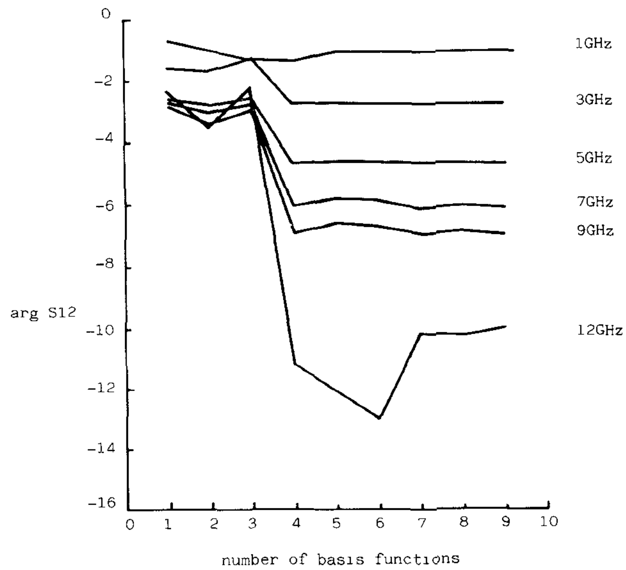


Fig. 5. Convergence of $S12$ as basis functions increase.

In addition, numerical experiments have been carried out using the modes of the wider strip multiplied by an expression of the form

$$\left\{\left(\frac{a-w}{2}\right)^2 - x^2\right\}^{\mu}$$

where a is the box width, w is the wider strip width, and μ is a parameter which is chosen to achieve best convergence. The multiplication was carried out by taking the convolution of the Fourier transform of the above expression, expressed in terms of Bessel functions, with the previously calculated Fourier components of the modal fields. By this means it was hoped to improve on the results obtained by using the unchanged microstrip modes as basis functions by bringing the edge behavior close to what it really was. Results for various values of μ were obtained but the convergence showed no improvement over that achieved using the unmodified modes.

IV. Convergence of the Green's Function

The Green's function, (14), is built up as an infinite sum of the eigenmodes of the continuous microstrip. In practice, of course, it is necessary to truncate this sum after a finite number of terms. The effect of such a truncation on the calculated value of the equivalent circuit impedance

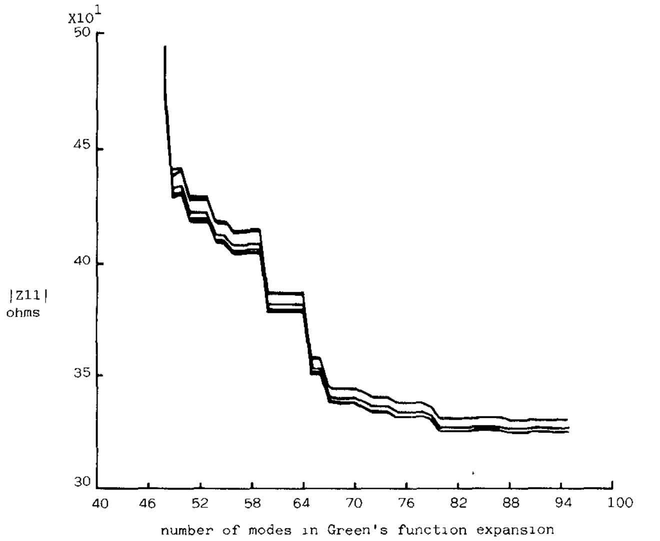


Fig. 6. Convergence of $|Z11|$ with number of modes for different numbers of basis functions.

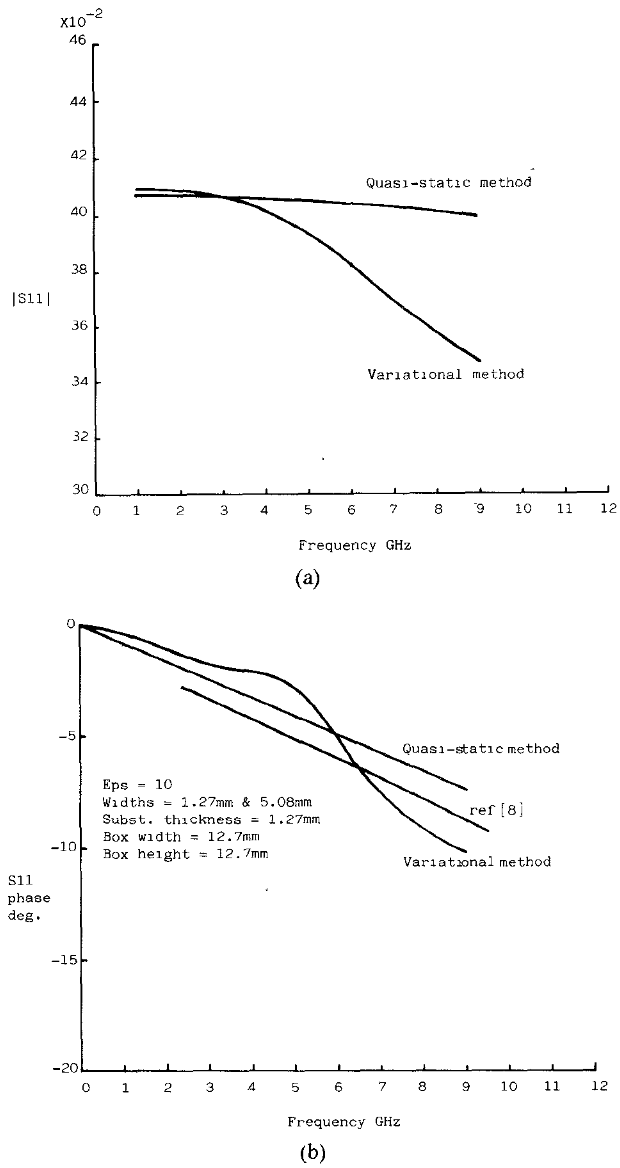


Fig. 7. (a) $S11$ modulus versus frequency. (b) $S11$ phase versus frequency.

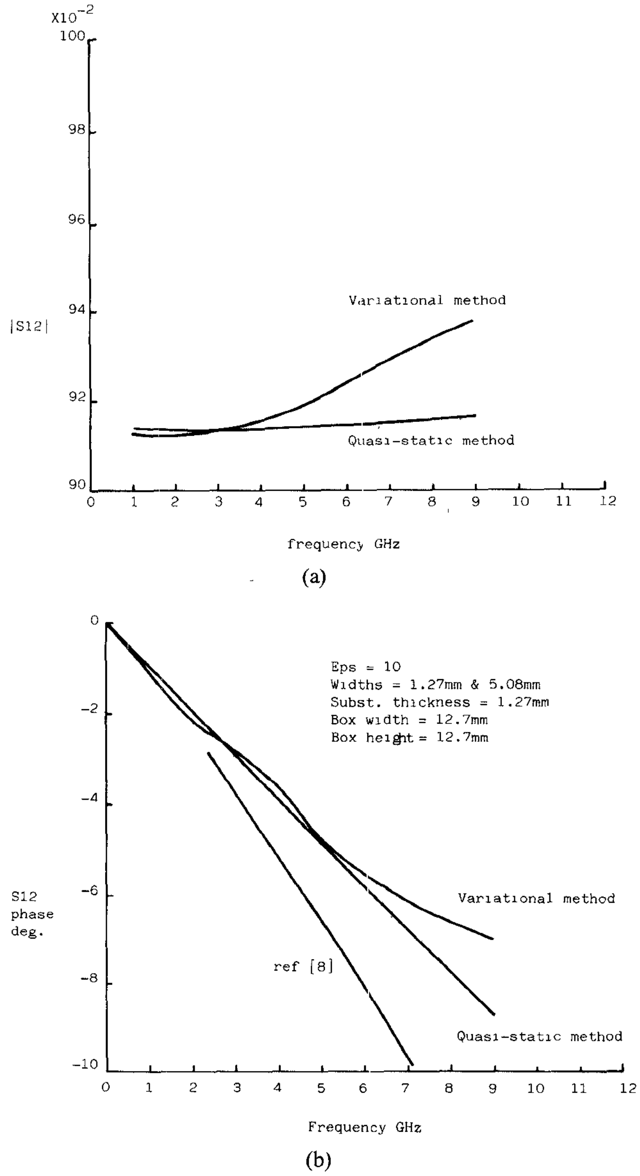


Fig. 8. (a) $S12$ modulus versus frequency. (b) $S12$ phase versus frequency.

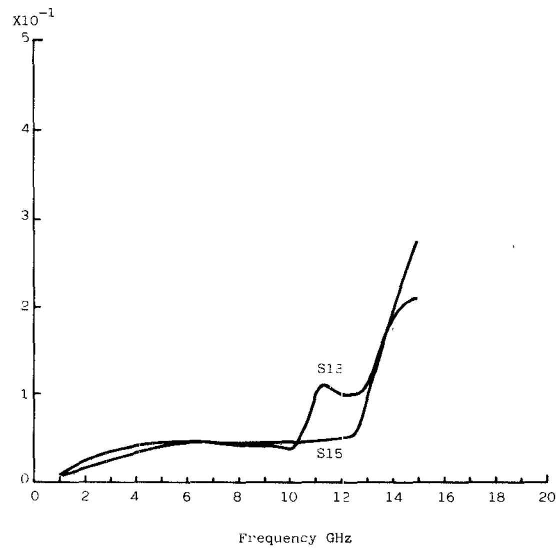


Fig. 9. Coupling to first and second higher order modes.

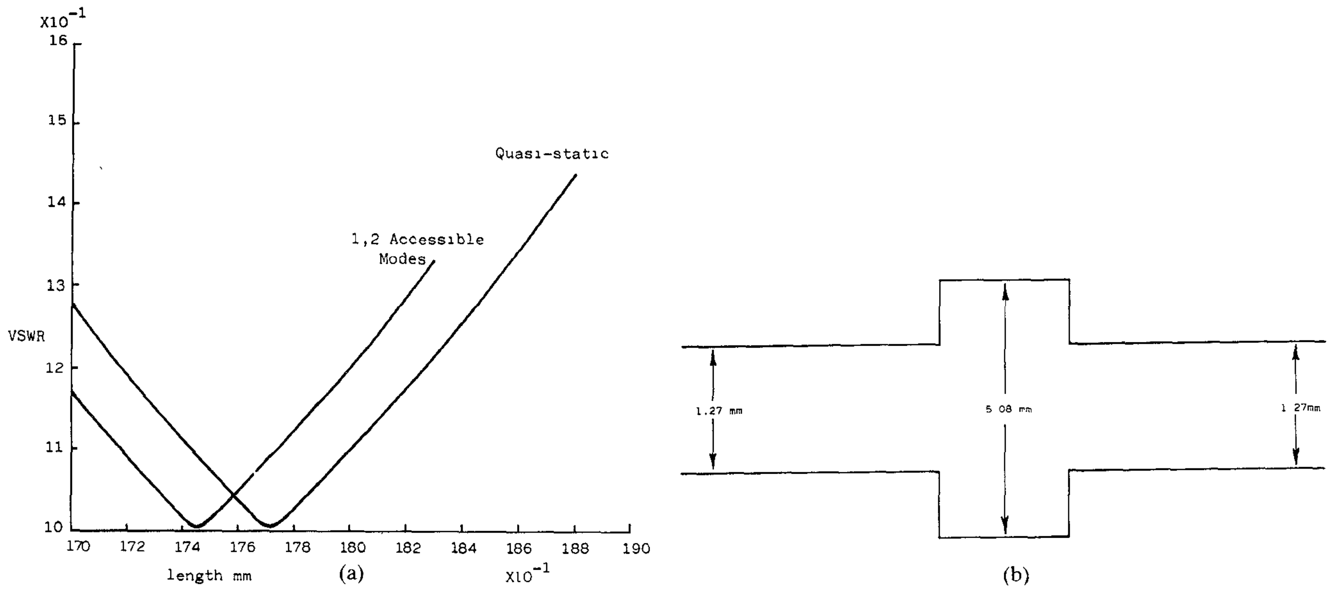


Fig. 10. (a) VSWR and (b) geometry of a double step discontinuity 3 GHz. $a = 12.7$ mm, $d = 1.27$ mm, $h = 11.43$ mm, $eps = 10$, $w1 = w3 = 1.27$ mm, $w2 = 5.08$ mm.

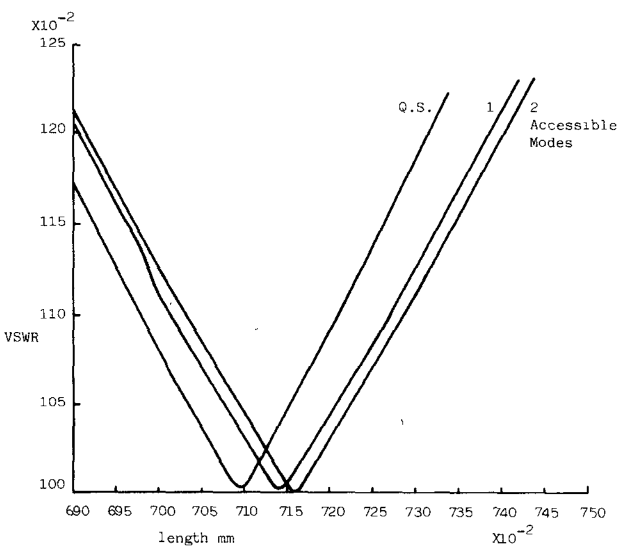


Fig. 11. VSWR of a double step discontinuity 7 GHz. $a = 12.7$ mm, $d = 1.27$ mm, $h = 11.43$ mm, $eps = 10$, $w1 = w3 = 1.27$ mm, $w1 = 5.08$ mm.

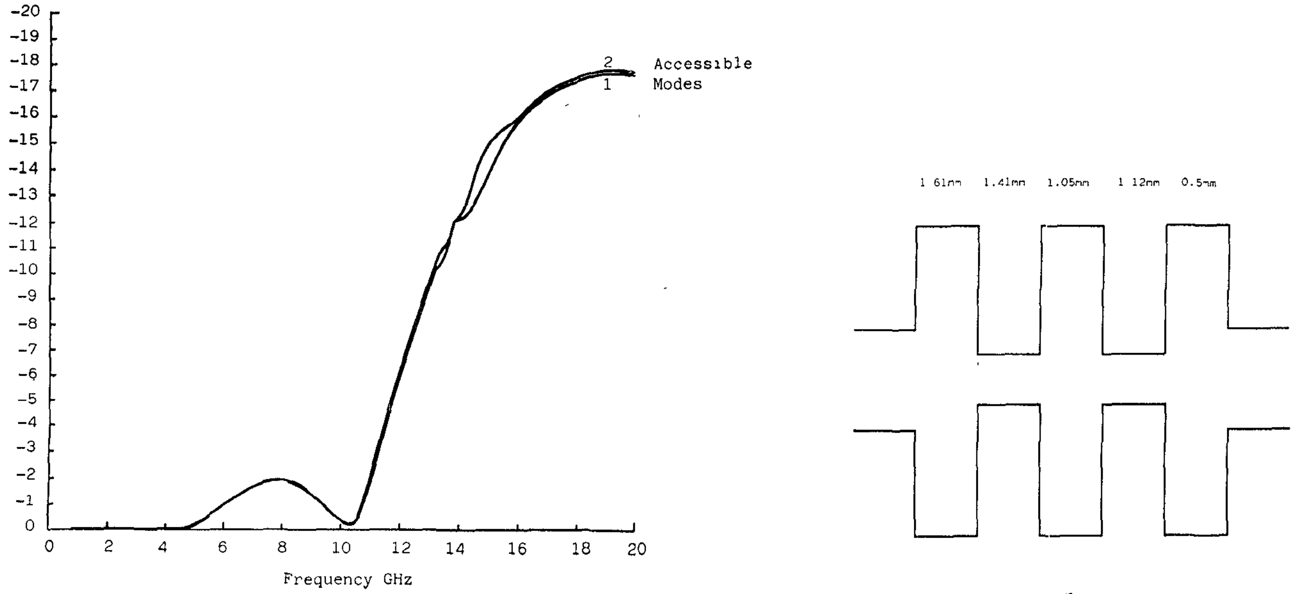
z_{11}, defined in Section VI, is shown in Fig. 6. It can be seen here that for accurate results it is necessary to take into account about 100 eigenmodes each side of the step. Examination of the geometry shows that this should be expected. We are essentially dealing with three complete sets of functions. Any transverse electric field pattern which satisfies the boundary conditions may be expressed as a linear combination of any of these sets. These are the microstrip modes for the continuous microstrip each side of the step and the basis functions chosen to express the field at the step itself. Each of these sets contains singular functions where the singularities may occur at different places and have different strengths in each set. Clearly if we are to express a singular function as a linear combination of a set of singular functions, when the singularities do not coincide, we need many terms in order to obtain an accurate representation. Thus in expressing the Green's function in terms of a summation of eigenmodes, many eigenmodes must be included.

It is interesting to compare the situation existing here to that of the analysis of uniform microstrip [12]. In the latter case we also have a Green's function expressed as a sum of eigenmodes; in this case they are the eigenmodes of a slab-loaded waveguide. Unlike the present case these functions are not singular, but the microstrip modes, which are to be expressed as a linear combination of them, do contain a singularity. In that form, it would also be necessary to take a large number of terms in order to achieve convergence. It was possible, however, in that case to find an asymptotic form of the expression to be summed, with a consequent decrease in computer time. In the present case, however, no such asymptotic form has so far been found.

V. Results for the Single Step Discontinuity

The S parameters for a step discontinuity calculated using the formulation described above are shown in Figs. 7 and 8. These show the modulus and the phase, respectively. Also shown, in Figs. 7(b) and 8(b), are the rigorous results read from the graphs presented in [8] and results using published quasi-static approximations [1]. It can be seen that at low frequencies, the agreement between rigorous methods and the quasi-static approximation is good, especially for the transmission coefficient. However as the frequency rises and we approach the cutoff frequency of the second mode, there is considerable deviation.

In Fig. 9 we see the coupling between the dominant mode and the first two higher order modes at the step. It can be seen that the coupling increases almost linearly with frequency so long as we are well below the cutoff of the higher order modes.



(a) (b)

Fig. 12. (a) Frequency response and (b) plan of a low-pass filter. $a = 12.7$ mm; $d = 0.625$ mm; $h = 5.715$ mm; $eps = 10$. Strip widths are 0.59 mm, 1.92 mm, and 0.125 mm.

VI. Network Formulation of Multiple Discontinuities

In the conventional equivalent circuit model for a step discontinuity, the parasitic effects are represented by two series inductors and a shunt capacitor (see Fig. 2). This model has the following limitations.

First, the validity of the equivalent circuit presupposes that a characteristic impedance can be defined for microstrip. Because of the hybrid nature of the microstrip modes, such a definition is unambiguous only at zero frequency.

Second, the values of the components are frequency dependent. This fact limits the usefulness of a simple equivalent circuit.

Third, no account is taken of the existence of higher order modes excited by the discontinuity other than as a means of energy storage. If we have closely spaced discontinuities, then the effect of these modes will be significant.

We can circumvent the first difficulty by modeling the longitudinal behavior of the transverse modal fields, i.e., the scalar amplitudes of $\boldsymbol{E}_t$ and $\boldsymbol{H}_t$, of each mode by means of the amplitudes of the voltage V and current I of an ideal transmission line having scalar characteristic impedance, say unity, and with propagation constant identical to that of the microstrip mode. The effect of an isolated step discontinuity can then be modeled by means of an equivalent circuit, as in Fig. 2, between transmission lines representing the fundamental mode of microstrip at either side of the step. The second problem is a difficult one and is as yet unsolved.

In order to overcome the third difficulty, it is possible to model the discontinuity as a multiport device with in-built storage elements. Such a model has previously been used for cascades of interacting irises and steps in rectangular waveguide [9]–[11].

The basic model is shown in Fig. 3. We split the mode spectrum of the microstrip into "accessible" and "localized" modes. The former are considered to have a significant amplitude at the next discontinuity. These include all the propagating modes and the first few evanescent modes. The localized modes are considered to have decayed to negligible amplitude at the next discontinuity. The distinction is obviously dependent on the geometry, the frequency of the operation, and the accuracy required.

For each accessible mode there exists an input/output port. The microstrip which connects successive discontinuities is then modeled as a set of transmission lines, one for each transported mode, each with its own propagation coefficient. The localized modes which are excited propagate outwards from the discontinuity and do not see any reflection; therefore they can be treated as being terminated with a matched termination.

The complete cascade can therefore be treated as a cascade of multiport networks connected as shown in Fig. 4. The first and last of these networks have all but the dominant modes terminated in their characteristic impedances. Once the S matrices for each discontinuity are known and the propagation coefficients of the intervening microstrip for each accessible mode is known, then the overall S matrix can be calculated using standard methods (e.g. [1]).

VII. Results for the Double Step Discontinuity

The above method has been applied to the double step discontinuity, the plan of which is shown in Fig. 10. For given frequencies of 3 GHz and 7 GHz the input VSWR was calculated as a function of the length of the step. The results are shown in Figs. 10 and 11. Here we have the results of taking just one accessible mode, i.e., assuming the steps have negligible coupling, and the results of taking two accessible modes. In addition the results using quasi-static formulas are shown. It can be seen that at 7 GHz the calculated resonant length is noticeably changed when the second accessible mode is included, thus indicating a significant amount of coupling. At 3 GHz the results are almost indistinguishable, implying that there is no significant coupling. The quasi-static results are significantly different in both cases.

VIII. Application to a Low-Pass Filter

A five-section low-pass filter made up of a cascade of microstrip step discontinuities has been analyzed using the rigorous method, in order to see the effect of including more than one accessible mode in the model. The geometry of the filter is shown in Fig. 12. It has been designed using 50 Ω input and output lines, 25 Ω capacitive lines, and 90 Ω inductive lines. The cutoff frequency is 10 GHz. In Fig. 12 we see the frequency response calculated by taking one and two accessible modes into account. It can be seen that, at high frequencies, the effect of the second accessible mode becomes noticeable.

In order to produce Fig. 12, the steps were characterized at 1 GHz frequency intervals and the parameters at intervening frequencies were calculated using interpolation. This produces accurate results except in the region of the cutoff of the higher order modes, where the parameters and their derivatives vary rapidly.

It is noted that there are only two different step discontinuities contained in the filter, a step from 50 Ω to 25 Ω and a step from 25 Ω to 90 Ω. Once these steps have been characterized, optimization of the filter consists of varying only the lengths of the lines between each step. Thus for each iteration of the optimization procedure, the only calculations involved are those of the S parameters of the lines and the resulting network problem. The computationally more expensive rigorous analysis of the step need not be repeated.

IX. Conclusions

A variational method for the analysis of single step discontinuities in microstrip has been presented. By treating such a step as a multiport network, we can take account of the scattered higher order modes when they are of significant amplitude at a neighboring discontinuity. Thus we are able to analyze cascades of closely spaced step discontinuities. The results for the single step are in agreement with quasi-static formulas at low frequency. Results for the VSWR of a double step discontinuity and the frequency response of a low-pass filter have been presented which show the effect of including the scattered higher order modes.

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