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THE RISE OF THE SERVICE ECONOMY

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The Rise of the Service Economy[*]

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Abstract

This paper analyzes the role of specialized high-skilled labor in the growth of the service sector as a share of the total economy. Empirically, we emphasize that the growth has been driven by the consumption of services. Rather than being driven by low-skill jobs, the importance of skill-intensive services has risen, and this has coincided with a period of rising relative wages and quantities of high-skilled labor. We develop a theory where demand shifts toward ever more skill-intensive output as income rises, and because skills are highly specialized this lowers the importance of home production relative to market services. The theory is also consistent with a rising level of skill and skill premium, a rising relative price of services that is linked to this skill premium, and rich product cycles between home and market, all of which are observed in the data.

1 Introduction

Two of the most salient, interesting trends in the post-1950 U.S. economy have been the rising importance of the service sector and the growth in the premium to skill despite a large expansion in the relative supply of high-skilled workers. The growth of the service sector and the relative demand for high-skilled workers have been well-studied independently, but theorists have not formally linked the two phenomena. Lacking theory, political rhetoric and alarmist concerns

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that the rise of the service economy has replaced industrial work with low-skill, low-paying services (e.g., "McJobs") have arisen. This paper addresses these concerns, and complements the two existing strands of literature, by both documenting the relevant facts and providing a theoretical framework for understanding the connection between skill accumulation and the growth of the service sector. Contrary to the conventional view, we argue that the growth in services is driven by the movement of consumption into more skill-intensive output.

Several empirical trends involving services and skills motivate our analysis. The share of the service sector in value-added has grown steadily from 60 percent in 1950 to 80 percent in 1980, and this increase is explained by the growth in the consumption of services. This twenty percentage point increase is also explained entirely by the growth of skill-intensive services, and is contemporaneous with an increasing relative quantity of high-skilled labor and a rising skill premium: The output of high-skill industries increases by more than twenty-five percent, while the share of low-skill industries actually declines. Over the same period, the wages of college-educated workers rose from 125 percent to more than twice the wage of high school-educated workers, and the fraction of workers who were college-educated rose from just fifteen percent to over sixty percent. Finally, the growth in college-educated labor, the skill premium, the relative size of skill-intensive services, and the share of the service sector all accelerate around 1950.

Our key theoretical idea linking these three phenonomena is that skills are specialized, and specialization plays an important role in the decision between home and market provision of services. General skills increase the productivity of workers equally in all activities. In contrast, specialized workers are highly productive on the market, but less productive in other work, including almost every potential home production activity. Thus, service output for which specialized labor is relatively productive will tend to be market purchased by all consumers. Moreover, given the higher opportunity cost of their time, specialized workers will tend to purchase a wider range of services on the market.

We incorporate this idea into a growth model. Agents face three stages of decisions. First, they decide whether or not to become high-skilled. Since the acquisition of any particular skill entails paying a fixed cost, each agent chooses to attain specialized skills in at most one service. The marginal cost of attaining skill is increasing in the fraction of agents who become high-skilled so that there is an upward sloping supply of high-skilled workers. Second, agents have preferences over a continuum of individual services, each of which is satiable. They order their wants, and satiate these desires sequentially, starting with the least complex, and therefore cheapest, first. Each want is produced using intermediate manufactured inputs and labor in a final stage. We refer to this final stage as the production of services. Finally, for each want that they satisfy, agents further decide whether to home produce or market purchase the service. Market production has potential cost advantages due to the use of more productive specialized skills, but home production is more customized and therefore provides more utility.

The driving force of growth in the model is exogenous technical change, but this technical change is both skill- and sector-neutral. Instead, the changing margin between home production and market services arises from the movement toward the consumption of services for which high-skilled labor has a larger relative productivity advantage. As labor productivity grows, income rises, and the consumption set expands to ever more complex (costly) services. For the more necessity services consumed at low incomes, high-skilled workers hold a stable absolute advantage in production. In this initial stage, the skill premium, the fraction of workers becoming high-skilled, and the share of services are therefore stable. Eventually, however, demand begins shifting continually toward services for which high-skilled workers hold an ever larger productivity advantage.

The increased importance of specialized high-skilled labor leads to the rise of the service economy. We identify four related forces leading to the rising importance of services. The first two combine to shift the real consumption bundles of individual agents toward market services. First, for fixed relative prices and wages, higher income shifts consumption towards services in which high-skilled labor has a comparative advantage. These skill-intensive market services have a larger cost advantage over their home production counterparts, and so the share of market services in consumption rises for every agent. Second, an increase in the demand for high-skilled labor tends to increase the relative wage of high-skilled agents, which increases their opportunity cost of home production, reinforcing their shift toward market consumption. A third force that leads to the growth in real services is a compositional effect. As more agents become high-skilled, total consumption can shift toward high-skilled workers who consume a higher share of market services. Finally, a rising relative wage of high-skilled workers increases the relative price of services, which increases their current-value share.

The non-homotheticity toward skill-intensive output is consistent with the observed compositional shift toward skill-intensive services. Our story has additional implications, however, which we examine. First, in the model, the shift in demand manifests itself as an increase in both the relative price and relative quantity of services. In the data, the growth in the relative shares of services is indeed driven by both quantities and prices. Second, in the model, the relative price of services rises because of the rising relative price of a scarce resource, high-skilled labor. We show that there is a strong, positive time-series relationship between the skill premium and the relative price of services.

The model has also rich implications on the nature of product cycles of activities from market to home, and from home to market production. As income rises, individual purchase new market produced services. At the same time, as the cost advantage of market services declines with neutral productivity growth, the lost utility from market services becomes more important and so previously market-produced services become home-produced. Recent examples of market to home product cycles include medical acticivities like checking blood sugar/ pressure or home dialysis. At the same time, the home production of other services move into the market as generations of low-skilled workers are replaced by high-skilled workers with high opportunity costs, and consumption basket weighted toward market services.

Though policy evaluation is not the focus of this paper, we conjecture that the model would have different implications than existing theories in several areas, including the elasticity of labor supply and productivity growth. The home-market decision makes labor supply more elastic than otherwise, but this elasticity may fall as market production becomes more skill-intensive. This would have implications for the welfare costs of distortions to labor supply or the service sector (relative to Rogerson, 2008). The model also illustrates that service sector growth need not rely on slower productivity growth in the service sector. Hence, our theory has no "Baumol's disease" implications of slower long run growth prospects (Baumol, 1967).

The rest of the paper is organized as follows. We conclude this introduction with a review of related literature. Section 2 then establishes the facts that motivate our analysis. The model and theoretical results are presented in Section 3. We evaluate testable implications of the model vis-a-vis the data in Section 4, and Section 5 concludes.

1.1 Related Literature

Our paper is related to a vast existing literature on structural change, for which we provide a (very) incomplete summary in order to delineate our relative contribution.

Earlier discussions of the facts and explanations for the changes in the structure of production include Clark (1941), Stigler (1956), Kuznets (1957), Baumol (1967), Chenery and Syrquin (1975), Fuchs (1968), Kravis et. al. (1984), and Maddison (1987). They observed an early growth of the employment share of the service sector, and posited that a combination of biased productivity rates and non-homothetic preferences and were important in explaining labor shifts across sectors. A recent literature has adapted these ideas to explain long run structural change within models that are consistent with Kaldor facts (e.g., Kongsamut, Rebelo and Xie, 2001, Acemoglu and Guerrieri, 2008, and Ngai and Pissarides, 2007). We complement this literature by studying the role human capital as a driving force of structural change. This approach helps address a series of empirical observations, including the late rise of the share of services in value-added, the skill composition of services, and the joint movement of relative prices and quantities. We also focus on output and consumption rather than the allocation of raw labor.

There is also an existing literature on the role of the home vs. market production. Ngai and Pissarides (2008) and Rogerson (2008) are two recent contributions examining the role of home production in explaining the labor market shift toward services.[1] They model differential rates of productivity growth across market and home production sectors in order to explain labor movements. Greenwood et al. (2005) also emphasize technological change in

[1] A related literature has emphasized the role of home production in less developed economies, including an emphasis on structural change out of agriculture (e.g., Gollin, et al, 2004, Buera and Kaboski, 2008).

the home, but their focus is on the the growth of the female labor force rather than the service sector. Buera and Kaboski (2008) argue that a rising optimal scale of production has caused structural reallocations across home and market, and broad sectors of the economy. Locay (1990) analyzes the role of customization and scale in the home vs. market decision. Finally, the work by de Vries (1994) emphasizes the changes between home and market production over development, including the importance of two-way movements, with market production rising in the early stages of the industrious revolution, and home production gaining importance with the decline in female market labor in the latter phases of the industrial revolution. This paper complements these papers by analyzing the relationship between home-production, human capital acquisition, and the service sector.

Our analysis of human capital is related to several other papers, however. Becker and Murphy (2007) examine the effect of general, rather than specialized, human capital on non-market productivity. Caselli and Coleman (2001) use human capital accumulation to explain discrepancies in labor and output trends in the decline of agriculture. Kaboski (2009) shows that human capital investments are often related to reallocations of labor across industries. Our paper's emphasis on the role of specialization, home vs. market-production, and the feedback on services is unique.

Our particular non-homothetic preferences build on those of Matsuyama (2000, 2002), Murphy, Shleifer, and Vishny (1989), and Zweimueller (2000) in their work on structural change. These preferences have shown to be a tractable way of modeling non-homotheticities over dissagregated components of consumption. Our twist is to introduce a decision between home and market production. Hall and Jones (2007) provide an important contribution in explaining the underlying non-homotheticity for one important area of consumption: healthcare. Our model of disaggregated activities and non-homothetic preferences is also closely related to Foellmi and Zweimueller (2005). Their analysis posits a direct preference explanation in which hierarchical wants are satisfied first as agriculture, then industry, and finally services. Our model has no direct exogenous non-homotheticity toward services, but we emphasize how this can arise endogenously through the home production margin and a non-homothetic shift toward skill-intensive wants. Finally, given our focus on the consumption of heterogeneous services, with more complex, newer ones contributing to the rise of the service economy, our paper relates to the earlier work by Katouzian (1970).

2 Empirics

This section provides empirical motivation for our analysis of the link between skill accumulation and the growth in the relative size of the service sector.[2] We review evidence on the delayed acceleration in services, extend this evidence by

[2] Our data, source documentation and calculations are avaiable at http://kaboski.econ.ohio-state.edu/servicesdataappendix_AERrevision.xls.

showing that growth in services is accounted for by growth in the consumption of services, and establish that this acceleration occurs at a particular income threshold.[3] We then establish that growth in the service share subsists almost exclusively in skill-intensive services, and then discuss the fact that the timing of the growth in services coincides with two well-known trends in the literature: the spread of college-education and the rising return to skill.

Figure 1 shows a strong mid-century break in the current-value shares of services in consumption and value-added in the United States. Consumption data is based on final output purchases by consumers. Data is first available from Lebergott (1996) in 1900, while the data post-1929 are based on national income and product (NIPA) accounts. The value-added data is from Martin (1939) from 1869 to 1920, the only source to give value-added in current values for the full service sector, and from NIPA accounts after 1929. For the value-added data, we show a broad measure of services (including those provided by government, public utilities and transportation), but the same substantial trend exists in more narrow concept of services as well. The 25 percentage point difference in the levels of the series is a result of consumption services excluding government services and distribution services (i.e., retail, wholesale and transportation services) on goods, both of which are included in services in the value-added data. The two series are roughly parallel but consumption actually exhibits the larger absolute increase from 1950 to 2000. Our model is a model of the consumption of services and the home production margin, so we stress that growth in services is accounted for by growth in their consumption.[4]

The break point after which growth in share of services accelerates is common to many countries. In the U.S., this breakpoint coincides with the year 1950, but across countries the break is more strongly tied to income per capita than chronological year. We show this using Buera and Kaboski (2008)'s panel data assembled for 30 countries spanning six continents and constituting two-thirds of the world's population and eighty percent of global output.[5] In 1950, the U.S. had an income per capita of \$9200, in Gheary-Khamis 1990 international dollars. We divide the sample of country-year observations using this \$9200

[3] Kuznets (1957) noted the late acceleration in the value-added share of services for a small sample of countries, but it has nevertheless been overlooked in the literature (e.g., Maddison, 1987), probably because raw labor numbers tend to be more readily available.

[4] Government services are 12 percent in 2000, while transportation, wholesale and retail together account for 16 percent in 2000. Another difference between the two series is the treatment of non-distribution intermediates. The consumption of goods includes value-added from the service sector, and the consumption of services includes value-added from the goods sector. The magnitudes and changes in these intermediates approximately net out, however.

[5] These countries include Argentina, Australia, Austria, Brazil, Canada, Chile, China, Colombia, Denmark, Egypt, France, Germany, India, Indonesia, Israel, Italy, Japan, Korea, Mexico, Netherlands, Norway, Pakistan/Bangladesh, Spain, Sri Lanka, Sweden, Switzerland, United Kingdom, United States, and Thailand. Based on Maddison (2006), our data covers: 68 percent of world population and 81 percent of world GDP in 2000; 71 percent and 75 percent, respectively, in 1950; and 40 percent and 60 percent, respectively in 1900. Although the numbers are lower for 1900, since the longer time series include Western Europe and its offshoots, we cover a much larger share of the population and economic activity undergoing large structural change at the time.

threshold, and then run the following regressions on the low- and high-income samples:

$$\textit{services share of value-added}_{i,t} = \alpha_i + \beta \ln y_{i,t} + e_{i,t} \qquad (1)$$

where $\ln y_{i,t}$ is log per capital income of country i at time t. α_i is a country i fixed effect. (We include to control for level differences in the series, some of which are the result of differences in measurement across countries.[6]) Here $\hat{\beta}$ captures the effect of the within country variation of income on the service share. Beyond the \$9200 threshold income, the services share is strongly related to income. While the estimated coefficient on log income is positive and significant in both samples, the coefficient $\hat{\beta}$ increases more than three-fold from just 0.06 (std. error of 0.01) for the <\$9200 sample to 0.22 (0.02) for >=\$9200 sample. In contrast, splitting the sample by the year 1950 yields similar coefficients of 0.08 (0.01) before 1950 and 0.11 (0.01) from 1950 on.[7]

Thus, an acceleration of the share of services in consumption and value-added at higher incomes appears to a be a common feature of structural transformation. This paper is about growth in the *consumption* of services, but we should note that in contrast to the delayed acceleration of services observed for consumption and value-added, the share of *labor* in services increases much more gradually with income per capita, both over time in the U.S. (Ngai and Pissarides, 2008) and in the cross-section of countries (Kuznets, 1957).[8] These numbers imply large differences in output per worker across sectors in the earlier period for the U.S (Caselli and Coleman, 2001; Buera and Kaboski, 2009). If skill levels differ across sectors, the numbers may reflect large discrepancies between raw labor and effective labor. Raw labor numbers may not be as informative for our purposes, especially given our emphasis on human capital. Still, the reason for the discrepancy between sectoral output and labor allocations pre-1950 is an open question, and not one that our theory will explain.

Our theory associates the increase in the consumption of market services with an income effect toward skill-intensive output, and indeed the growth in services has been toward skill-intensive services. Figure 2 separates the growth in services into the contributions of high- and low-skill industries. We rank industries according to their skill intensity as measured by the fraction of workers college-educated in 1940 (the last available data preceding the acceleration) and divide the value-added of the service sector in half in 1950. High-skill industries are therefore industries with at least 12.5 percent of labor-college educated in

[6] For example, in several countries utilities cannot be separated from mining and so are excluded from services. Countries also differ to the extent that small-scale handicrafts are classified as services or manufacturing. Another interesting example is China whose historical data show an extremely low share of services, probably because services were not viewed as producing value under Marxist ideology. After the Economic Census of 2004, the service share was revised upwards by nine percentage points in the current official data.

[7] That is, the growth in services appears to be a feature of development rather than driven by a common shock to the world economy such as a commonly available new technology or adoption of a common policy.

[8] The difference between the labor and value-added trends in a small sample of developed countries was noticed quite early by Kuznets (1957).

1940.[9] We apply the same threshold to map disaggregated components of service consumption into high- and low-skill. We again see a breakpoint at 1950, and the rise of the service economy has been clearly driven by high-skill industries. The importance of low-skill service industries in value-added has actually declined, though it has remained roughly constant for consumption. Again, distribution costs account for the discrepancy between consumption and the other measures. In any case, growth patterns clearly differ across skill-intensity.

We can look at a more disaggregate level, if we focus on labor compensation data. Labor compensation numbers are nearly identical to value-added numbers in Figures 1 and 2, but are available at a more detailed level.[10] Figure 3 shows that there are many quantitatively important industries in the growth of high-skill services. It plots the absolute change in the share of different service industries in total labor compensation between 1950 and 2000 against the skill-intensity of the industry (measured as the fraction of workers with college-education in 1940). Again, given available data, this positive relationship appears to be particular to only the high income, post-1950 period.[11]

The absolute importance of each industry to the total growth in services is its vertical distance from the zero growth line. The growing high-skill services include education (especially higher education), legal services, banking, real estate and accounting, broadcasting and television, air transportation, and health care. We emphasize that the growth in services is a broad increase in the demand for output that is intensive in specialized skills.

Of course, two important industries are health care and education, whose growth may be driven at least in part by growth in government subsidies or other policies. While important, however, these industries are simply not the full story. For example, health and hospitals together account for an almost 8 percentage point increase, but they constitute less than one-quarter of the total rise in high-skill services, and the five percentage point increase in education constitutes less than one-fifth.

Moreover, the trends we highlight are robust to the exclusion of health care, education, and government from the data. Namely, the remaining service industries do not rise until after 1950, but then rise 16 percentage points thereafter, and growth of the remaining high-skill services (19 percentage points) again exceeds service growth overall. The remaining consumption categories grow substantially too, though without health care and education, the increase is now 8 percentage points, or about half of the growth in value-added.

[9] These rankings are remarkably stable over time. We could have produced identical results if we had used data in 2000 to rank industries, but we would need a cutoff of 50 percent.

[10] Output and consumption share cannot be merged precisely with workforce education data at this detailed level. The detailed industry and education data comes from IPUMS census. After 1950, census labor and compensation numbers closely mirror NIPA numbers, except that census compensation does not include benefits. Using manhours instead of labor compensation yields a very similar picture.

[11] Although only a single decade of data are available, census data show absolutely no relationship between skill-intensity and growth in the share of disaggregated services from 1940 to 1950. At an even more disaggregate level, it is clear that many high-skill services were simply not consumed in earlier periods.

More generally, we emphasize that our theory involves not only *what* is being demanded, but *how* it is being delivered. Indeed, even within the categories of health care and education, there has been a rise in the service economy. Health care is provided as both services (medical services, hospitals) and commodities (medical equipment, pharmaceuticals), but the share of services in health care consumption rose from 77 percent in 1950 to 84 percent in 2000. Similarly, if we combine educational services and books together, we see that the share of services in this broad educational consumption category increases from 73 percent in 1950 to 83 percent in 2000.

The broad increase in the demand for skill-intensive output that we propose should manifest itself in the market for high-skilled labor. We therefore view the growth in services as related to the well-known post-1950 increase in the demand for a broad range of "high-skilled" workers (see Juhn, Pierce, and Murphy, 1993). Using college-educated workers as a proxy for high-skilled workers, Figure 4 shows the growth in the relative price and relative quantity of services. The average wage of college-educated workers rose from 125 percent of the average high school-graduate wage in 1950 to over 200 percent by 2000. At the same time, the ratio of college- to high school-educated labor in the workforce rose from about fifteen percent to sixty percent. The rising disparity between high- and low-skill workers is of great policy interest. Policy makers and journalists often argue paradoxically that the falling relative wage of low skill workers is a result of the growing prevalence of low skill service jobs, but we stress that the growth in services is a result of the growth in *high*-skill services.

Based on available evidence for the U.S., the timing of trends involving the service economy and the market for high-skilled labor correspond. That is, the year 1950, or the $9200 threshold, appears to be a turning point in terms of trends related to the schedule for the excess demand for skill. Wage and education questions were first introduced in the 1940 census, and so representative data are scant before that. There was a sharp decrease in premiums to skill, including the college-premium between 1940 and 1950. Broader returns to education, and other proxies for the skill-premium such as white collar-blue collar occupation differentials, did not increase and most likely declined before 1940 (see Goldin and Katz, 1999).

Levels of education and other measures of skill increased well prior to this, and the growth in skills in the labor force is clearly part of a more continuous process. Still, we model skill dichotomously, and college education appears to be a convenient measure of the level of skills associated with the rise of the service economy.[12] The college boom is overwhelmingly a post-1950 phenomenon, since college educated workers accounted for just 11 percent of the labor force in the 1940 U.S. census. The college boom also coincides with the $9200 threshold

[12] Empirically, an increase in elementary and high-school education precedes the college boom. This took place in a time of falling skill premia, suggesting that it could have been, at least partially driven by an increase in the supply of skills, (see Goldin and Katz, 1999, Kaboski, 2004). From the point of view of a more general model with multiple levels of high-skill, these lower levels of education could be viewed as allowing individuals to be specialized in the production of less complex output, where skill has merely an absolute advantage.

in other countries as well. Using Cohen and Soto (2007) data, the fraction of the adult (25+) population of a country that has some college-education averages just 0.08 (std. dev. of 0.03) at real incomes near \$9200. Split-sample regressions analogous to equation (1) but where the service share is replaced with the fraction of college-educated adults, yield a five-fold increase in the coefficient on log income between the low- and high-income samples, from 0.04 (<\$9200) to 0.23 (>=\$9200).[13] That is, both growth in the service economy and investment in college education coincide with an income per capita of \$9200.

3 Model

In this section, we develop a model of specialized skill accumulation that leads to the rise (acceleration) of the service economy.

Specifically, we model an economy with a continuum of differentiated manufactured goods and a continuum of differentiated services, both indexed by their complexity, $z \in \mathbb{R}^{+}$. Manufactured goods are inputs into the production of services, and services are what individuals ultimately consume. Services can be produced either in the market or at home. In the latter case, households directly purchase manufactured goods to home produce services.

In our theory, we posit a utility advantage in consuming home-produced services, reflecting the household's gains from customizing services to its own need. On the other hand, the production of services in the market can be more cost effective.

Labor is the lone resource in the economy and is either general low-skilled labor or specialized high-skilled labor. Given a fixed time cost of becoming high-skilled in the production of each differentiated service, $z \in \mathbb{R}^{+}$, individuals choose to specialize in at most one service. The home production of almost all services will therefore use low-skilled labor. Exogenous productivity improvement is the only source of growth in the economy. For simplicity, we assume that agents are infinitesimally-lived, and so the model is static except for this technical change.

3.1 Technologies

The technology for producing type-z service output $y_s(z,t)$ requires labor and type-z manufactured goods as intermediates, $y_m(z,t)$. Production is Leontieff in intermediates and labor value-added, where high-skilled labor $h_s(z,t)$ and low-skilled labor $l_s(z,t)$ are perfect substitutes in producing this value-added:

$$y_s(z,t) = \min\left\{A_h(z,t)\, h_s(z,t) + A_l(z,t)\, l_s(z,t),\, q y_m(z,t)\right\}$$

[13] For each country, we use all countries with a year of income between \$8500-\$9500 and choose the year closest to \$9200. In comparison, at this income level, primary education is nearly complete (the fraction of the adult population averages 0.97), while secondary schooling is well underway (0.37).

The levels of productivity of high- and low-skilled workers, $A_h(z,t)$ and $A_l(z,t)$, respectively, are specific to the production of service z. We make several assumptions regarding these functions:

- First, z indexes complexity, so we assume that productivity is decreasing in z, i.e., $\frac{\partial A_l(z,t)}{\partial z} < 0$ and $\frac{\partial A_h(z,t)}{\partial z} < 0$.
- Second, for all t, $A_h(z,t) > A_l(z,t)$, so that high-skilled labor is more productive. Still, since skill is specialized, $A_h(z,t)$ is specific to one particular z (the agent's specialty), so the high-skilled agent has the low-skilled productivity $A_l(z',t)$ for all other output $z' \neq z$, with $A_h(z,t) \geq A_l(z,t)$ for all z and t.
- Finally, high-skilled labor has a (weak) comparative advantage in the production of more complex services, i.e., $\frac{\partial [A_h(z,t)/A_l(z,t)]}{\partial z} > (=)\, 0$.

This technology can be used to produce services by competitive firms in the market, or at home by individual households. In the case of home-production, high-skilled labor can only be used to produce a particular service z if the household is specialized in the production of that service (e.g., only accountants use high-skilled labor to process their own taxes).

For simplicity we abstract from the use of intermediates in the production of manufactured goods, and the output of manufactured goods $y_m(z,t)$ is simply linear in labor:

$$y_m(z,t) = A_h(z,t)\, h_m(z,t) + A_l(z,t)\, l_m(z,t)$$

The production of manufactures can only be done in the market.[14]

3.2 Firms' Problem

Given wages, free entry of service and manufacturing firms implies that firms will price at average cost:

$$p_m(z,t) = \min\left\{\frac{w(t)}{A_h(z,t)}, \frac{1}{A_l(z,t)}\right\} \tag{2}$$

and

$$p_s(z,t) = q p_m(z,t) + \min\left\{\frac{w(t)}{A_h(z,t)}, \frac{1}{A_l(z,t)}\right\} \tag{3}$$

[14] See Buera and Kaboski (2008) for an extension of this model in which manufactures are produced, as is true in the data, using technologies requiring large fixed costs and having a large efficient scale. In this extension, it is not cost-effective to produce manufactured goods at the very small scale of home-production.

3.3 Preferences

Agents hold preferences over the continuum of discrete, satiable wants indexed by the service that satisfies them, z. Thus, final consumption takes the form of services, while manufactured goods are purchase as inputs in the home-production of services.[15] Let the function $C(z) : \mathbb{R}^{+} \to \{0,1\}$ indicate whether a particular want is being satisfied. Wants can either be satisfied by procuring the service directly from the market, or by purchasing the required manufactured goods and producing the service at home. Define the function $H(z) : \mathbb{R}^{+} \to \{0,1\}$ to take the value 1 if want z is satisfied by home production and 0 otherwise. Together the set of indicator functions mapping $\mathbb{R}^{+}$ into $\{0,1\}^2$ defines the consumption set. Preferences over wants and the method of satisfying those wants, i.e., over indicator functions $C(z)$ and $H(z)$, are represented by the following utility function:

$$u(C,H) = \int_0^\infty [H(z) + \gamma(1 - H(z))] C(z) \, dz$$

where $H(z) \le C(z)$. The parameter $\gamma \in (0,1)$ reflects the utility of market consumption, which is less than the utility of home-produced consumption because it is not as customized (e.g., driving precisely when and where you want rather than riding the bus on fixed schedules).

These preferences, though somewhat non-standard, have certain advantages. First, the continuum of satiable wants is simple way of modeling disaggregate non-homotheticities that has been used in the existing literature on structural transformation.[16] This disaggregation is not without content and we show, in Section 4, the extent to which they can be represented in a more standard form, as preferences over total consumption of goods and services. Also, Matsuyama (2003) has shown that they can be easily generalized to allow for asymmetries in the utility provided by satiating different wants, in which case our cost of satisfying a particular want can be interpreted as a cost per unit of utility. In any case, we view the preferences as a simple abstraction capturing shifting demand patterns and the home production decisions associated with them. As such, the preferences may be reduced form for a host of idiosyncratic explanations for changes in disaggregate demand patterns. For example, the growth in demand for health-related goods and services may stem from a desire to invest in health capital (as in Hall and Jones, 2007).[17] Our emphasis will be on the importance of skill-intensity in determining whether these are home or market produced, and these preferences will give parsimonious framework for analyzing a common dimension to a wide set of micro phenomena.

[15] One could easily introduce a second continuum of wants that are directly satisfied by manufactured goods, but it would contribute little to the analysis.

[16] See Murphy et al (1989), Zweimueller (2000), and Matsuyama (2001,2003), for example.

[17] Indeed, increases in the demand for health- , education-, and finance-related could all be driven by exogenous increases in life expectancy.

3.4 Schooling

Individuals make a dichotomous education decision $e \in \{l, h\}$, but becoming specialized high-skilled workers, $e = h$, requires spending a fraction θ of their time endowment acquiring skills.[18] This cost of becoming educated is a continuous increasing function of f, the fraction of workers who decides to acquire education, i.e., $\theta'(f) > 0$. We assume that $\theta(0) = 0$, in order to ensure $f > 0$, and $\theta(1) = \bar{\theta} < 1$. The $\theta(f)$ function is a simple way of generating an upward sloping supply curve for education without introducing underlying heterogeneity. One can motivate this in the typical way, where individual agents draw θ after choosing their skill level, but are completely insured against their draw. Thus, high- and low-skilled agents all receive the same utility in equilibrium. In this interpretation, $\theta(f)$ is the average cost of education as a function of the fraction of individuals that decide to acquire education.

In principle, agents could acquire skills for multiple z in order to increase their productivity in home production, but this would never be optimal. Since the time cost is strictly positive for any strictly positive fraction of agents acquiring skills (i.e., $\theta(f) > 0$ for any $f > 0$), in any allocation with positive high-skilled workers, individuals can acquire specialized skills for at most a finite number of services. Given that individuals consume a continuum of them, this finite number would constitute a measure zero of home-produced services. Therefore, individuals will acquire skills in at most one service, and the home-production of all (but a measure zero of) services will be done with low-skilled productivity.

3.5 Consumer's Problem

The demand for market services and manufactured goods of an individual with skill e, solves:

$$
\begin{aligned}
& V^e = \max_{H(z)\leq C(z)} \int_0^{+\infty} \left[H(z) + \gamma(1 - H(z))\right] C(z)\, dz \\
& \text{s.t.} \\
& \underbrace{\int_0^{\infty} \left[C(z)H(z) q p_m(z,t)\, dz\right.}_{C_m} + \underbrace{\int_0^{\infty} C(z)\left[1 - H(z)\right] p_s(z,t)\, dz}_{C_s} \\
= \quad & w_e \underbrace{\left(1 - \theta(f)\mathcal{I}(e) - \int_0^{\infty} C(z)H(z)\frac{1}{A_l(z,t)}dz\right)}_{\text{labor supply}} \qquad (4)
\end{aligned}
$$

[18] The key assumption in our analysis of the growth of services will be that the comparative advantage of high-skilled labor is increasing after a threshold complexity, i.e., $\partial \ln A_h(z,t)/\partial z > 0$ for $z > 1$. An alternative modeling choice that would have allowed for multiple (a continuum of) skill levels, and assignment of skills to the production of want of different complexity. If skills and complexity are complementary, and equilibrium would exhibit positive sorting. We choose to abstract from multiple levels of skill and postulate a dichotomous set of skills as this assumption greatly simplifies the analysis.

Market expenditures are on the left-hand side of the budget constraint, with the first term in the integral capturing expenditures on goods used in home production (C_m), and the second term capturing expenditures on market services (C_s). The right-hand side of the budget constraint is labor income. Labor income is the product of the wage w_e and labor supplied to the market, both of which depend on the educational decision e. Labor supply is net of the amount of time used for schooling $\theta(f)\mathcal{I}(e)$, where $\mathcal{I}(e)$ is an indicator function that equals one if $e = h$ and zero otherwise. Market labor supply is also net of home production time. Note that home production is performed using the low-skilled productivity, regardless of educational decision e.

Given a marginal utility of consumption μ, for each z, the consumer decides to satisfy it (i.e., $C(z) = 1$) iff:

$$\gamma \geq \mu p_s(z) \text{ and/or } 1 \geq \mu \left[q p_m(z) + \frac{w_e}{A_l(z,t)} \right] \tag{5}$$

where the left inequality relates to the decision to market consume and the right inequality is the decision to home produce. Using our knowledge of equilibrium prices from equations (3) and (2), any z that will be satisfied, will be home produced iff:

$$1 - \gamma \geq \left[\left(\frac{w_e}{A_l(z,t)} - \min\left\{ \frac{1}{A_l(z,t)}, \frac{w}{A_h(z,t)} \right\} \right) \right] \mu \tag{6}$$

The right-hand side of expression (6) above captures the difference in the cost (in utility terms) of producing the service at home or purchasing on the market. This difference is higher for more complex (i.e., high z) services, especially those that are more efficiently produced on the market using high-skilled labor, and for agents with high opportunity cost of home production time w_e. Thus, a want will be home-produced if the gains from customizing our consumption exceed the productivity gains from market production.

Recall that wants enter utility symetrically, and production costs, as well as the additional costs of home production, are increasing in z. Therefore, consumers will satisfy and home produce the least complex wants first, and the consumer's problem can be simplified to the choice over the restricted consumption set defined by step functions of the type:

$$C(z) = \begin{cases} 1 \text{ if } z \leq \bar{z}_e \\ 0 \text{ if } z > \bar{z}_e \end{cases}$$

and

$$H(z) = \begin{cases} 1 \text{ if } z \leq \underline{z}_e \\ 0 \text{ if } z > \underline{z}_e \end{cases}$$

where $\bar{z}_e$ denotes the most complex want that is satisfied, and $\underline{z}_e$ denotes the most complex want that is home-produced.

Preferences over the restricted consumption set can then be represented as a utility function over two thresholds $\underline{z}_e$ and $\bar{z}_e$:

$$u\left(\underline{z}_e, \bar{z}_e\right) = (1-\gamma)\,\underline{z}_e + \gamma \bar{z}_e$$

with $0 \leq \underline{z}_e \leq \bar{z}_e$. On the margin, agents can increase utility in two ways: by satisfying an additional not-yet-satiated want (i.e., increasing $\bar{z}_e$) or by moving the least expensive market-satisfied want into home production (i.e., increasing $\underline{z}_e$).

Since the main interest of the paper is the service economy, we focus on parameters in which market services are purchased in equilibrium. The following assumption guarantees $\underline{z}_h < \bar{z}_h$, i.e., high-skilled workers consume some market services.

$$\frac{\gamma}{1-\gamma} > \frac{(1+q)}{\frac{A_h(0,0)}{A_l(0,0)} - 1}. \tag{7}$$

Intuitively, to ensure market services are used, we need the utility of market services γ and cost benefits of market services, governed by the minimum relative productivity of high-skilled workers, to be sufficiently high. The sufficiency of this condition follows from high-skill workers having a comparative advantage in the production of more complex services, i.e., $\partial\left[A_h(z,t)/A_l(z,t)\right]/\partial z \geq 0$, and productivity growth being neutral.

An agent chooses to be high-skilled if $V^h > V^l$, and vice-versa. Given our assumptions on $\theta(f)$, only an interior $f \in (0,1)$ can be an equilibrium, which will require agent indifference:

$$V^h = (1-\gamma)\,\underline{z}_h + \gamma \bar{z}_h = (1-\gamma)\,\underline{z}_l + \gamma \bar{z}_l = V^l \tag{8}$$

Although high- and low-skilled workers earn the same utility in equilibrium, their consumption bundles differ. A higher w_e in the first-order condition (6) above makes the relative cost of home production higher for high-skilled workers than for low-skilled workers. Substitution leads high-skilled workers to home-produce fewer services and market consume more services. Moreover, since agents are indifferent to being high- or low-skilled, the higher wage captures a pure substitution effect (see condition 8). Therefore, since all consumers prefer home-produced services, high-skilled workers consume fewer home-produced services but remain indifferent to being low-skilled because they consume a larger range of services overall.

Proposition 1 *High-skill workers consume some market services that low-skill workers do not consume, $\bar{z}_h > \bar{z}_l$, home produce a smaller range of services $\underline{z}_h < \underline{z}_l$.*

Defining C^h (C^l), C^h_s (C^l_s), and c^h_s $\left(c^l_s\right)$ as the market consumption, market consumption of services, and the services share of market consumption of high-skilled (low-skilled) agents[19], we state the following corollary:

Corollary 2 *High-skilled workers spend a larger fraction of their income in services, $c^h_s = C^h_s/C^h > C^l_s/C^l = c^l_s$.*

[19] Specifically, $C^e_s = \int_{\underline{z}_e}^{\bar{z}_e} p_s(z)dz$, $C^e_m = \int_0^{\underline{z}_e} p_m(z)dz$, $C^e = C^e_s + C^e_m$, and $c^e_s = C^e_s/C^e$.

3.6 Competitive equilibrium

A competitive equilibrium is given by price functions $p_m(z,t)$, $p_s(z,t)$, $w(t)$, a fraction of people who attain schooling f, quantities of manufactured goods and market services determined by $\underline{z}_l(t)$, $\bar{z}_l(t)$, $\underline{z}_h(\theta,t)$, $\bar{z}_h(\theta,t)$, and labor allocations such that consumers' demands and schooling decisions solve (4); prices solve zero profits conditions (2) and (3); and labor markets clear.

4 Growth of Services

The dynamics of the model are fully driven by the functions $A_h(z,t)$ and $A_l(z,t)$. In this section, we analyze specifications of these functions that lead to an initial balanced phase followed by a phase with growth in the share of services, the fraction of high-skilled workers, and the skill premium. First, we characterize restrictions on these functions that are consistent with balanced growth as the result of neutral technical change. We then study a variation in which neutral technical change shifts demand toward wants in which high-skilled workers have higher relative productivity. This variation leads to growth in services and has additional implications about the skill composition, the rising relative price of services, and product cycles, which we later evaluate relative to the data.

4.1 Balanced Growth

We begin by showing that given our assumptions on $A_h(z,t)$ and $A_l(z,t)$, a power parameterization of these functions is necessary and sufficient to generate balanced growth with a constant share of services in consumption. While the focus of the paper is the *rise* of the service economy, we restrict our analysis to variations on this power parameterization of productivity, since we are interested in models that can also reconcile the relatively constant share of services in output and consumption in the pre-1950 data.

Proposition 3 *Consider the consumer's problem for workers parameterized by constants q and γ and a strictly positive and strictly decreasing function $\tilde{A}(z)$, so that $A_h(z,t) = \tilde{A}(z)\bar{A}_l e^{gt}$ and $A_h(z,t) = \tilde{A}(z)\bar{A}_h e^{gt}$, and $w = \bar{A}_h/\bar{A}_l > 1$. Then, the share of service in consumption $c_s(t) \equiv C_s(t)/[C_s(t) + C_m(t)]$ is a constant for all q and γ if and only if $\tilde{A}(z) = z^{-\lambda}$.*

Proof. See Appendix ■

Proposition 3 states that the relative wage is set at the relative productivity of high- and low-skilled workers, which is common across z. Hence, all firms are indifferent between using high- and low-skilled workers. (See the cost minimization in equations (3) and (2).) In this case, the cost of home producing services for low-skilled workers is the same as the cost of market purchasing services, but since $\gamma < 1$, consumers get more utility from home production. Therefore, low-skilled workers consume no market services.[20] High-skilled workers face higher costs to home produce, however, since their wage is higher but not their productivity in home production, and assumption (7) ensures that they indeed consume positive market services.

A rough intuition for the above result is that the power function has a memoryless shape, which keeps the first-order conditions for the marginal $\underline{z}$ and $\bar{z}$ constant, even as these margins increase linearly. Indeed, one can rewrite the consumer's problem for the high-skilled agents into a problem with homothetic quasi-preferences over total expenditures on services and manufactured goods, C_s^h and C_m^h respectively:

$$\max_{C_m^h, C_s^h} \gamma_1 \left(C_m^h\right)^{1-\sigma} + \gamma_2 \left(C_s^h + \gamma_3 C_m^h\right)^{1-\sigma} \tag{9}$$
$$s.t.$$
$$p_m^h C_m^h + C_s^h = w e^{gt} \left(1-\theta\right)$$

We call these "quasi"-preferences because the preference parameters γ_1, γ_2, γ_3 and σ depend on underlying preference *and* technology, while the "price" on manufacturing expenditures p_m^h is a function of the (constant equilibrium) wage.[21] It is straightforward to see that the above preferences are homothetic, however, and so the share of services remains constant as income grows. While the preferences are homothetic with respect to income, consumption patterns nonetheless vary with respect to the wage/opportunity cost of time through its effect on p_m^h. That is, a pure wealth effect (i.e., an increase in income as t increases) does not affect the relative shares of service and manufactured goods consumption, but a higher wage will have a substitution effect away from manufactured goods (which are inputs into home production) and toward services.

Product Cycles We conclude this section by highlighting an interesting implication of this balanced model for product cycles between home and market production. As productivity grows, individuals consume new services. In the case of high-skill workers, they start purchasing these services on the market, since production of the marginal services is particularly time-consuming, and the opportunity cost of high-skill workers' time is too high to be used in low-skill home-production. Eventually, labor productivity increases enough making the absolute cost advantage of market-production smaller, and leading individuals to home produce customized versions of these services which yield higher utility. (See the first-order equation (6).) That is, there is a clear product cycle of services being first market produced and later home produced. This follows naturally from the assumption of a preference for home production, a cost benefit

[20] This result is done for simplicity but is not particularly necessary. Low-skilled workers could consume services if some wants had $\gamma > 1$ or if technologies were characterized by efficient scales greater than the home production scale as in Buera and Kaboski (2008).

[21] In particular, $\sigma = 1 - 1/(\lambda+1)$, $\gamma_1 = (1-\gamma)\left[(\lambda+1)\bar{A}_l/q\right]^{1/(\lambda+1)}$, $\gamma_2 = \gamma[(\lambda+1)\bar{A}_l/(1+q)]^{1/(\lambda+1)}$, $\gamma_3 = (1+q)/q$ and $p_m^h = 1 + w/q$.

to market production, and neutral productivity growth. The following remark formalizes this discussion:

Remark: *In an economy with neutral productivity growth where* $A_e(z,t) = \bar{A}_e z^{\lambda} e^{gt}$, $e = l, h$, *the thresholds defining the demand of low- and high-skill individuals grow at the constant rate* $g/(\lambda+1)$, *with* $\underline{z}_h(t) < \underline{z}_l(t) = \bar{z}_l(t) < \bar{z}_h(t)$. *For high-skill workers, for any particular* z, *the model yields a product cycle from not consumed, to purchased on the market, and finally home-produced.*

In the variation of the model we discuss in the following section, a richer set of product cycles arises.[22]

4.2 Comparative Advantage of Skill in More Complex Output

In this section, we study the growth of the service economy, which arises from agents satisfying more complex and skill-intensive wants as incomes grow. High-skilled labor has a comparative advantage in satisfying these wants, hence more of these services are produced on the market, and the demand for high-skilled labor increases. We proceed by establishing four related effects that all contribute to the growth in services. The leading effect, which drives the others, comes directly from high-skilled workers having a greater comparative advantage in more complex output, which we call the *high-skill advantage effect*. Two other effects, the *opportunity cost effect* and *skill-deepening effect*, come through the effect of comparative advantage on the demand for high-skilled labor. We then show how comparative advantage-drive sorting leads to growth in skill-intensive services, and also growth in the relative price of services, *the relative price effect*, which also contributes to the growth in the share of services. Finally, we combine these results, and show that over time these forces lead to growth in the aggregate share of services in the economy.

The underlying assumption that gives rise to a growing share of services is that high-skilled labor has a comparative advantage in the more complex goods and services, which will first be consumed at a later date given their high cost of production. Specifically, over a range of less complex (i.e., $z < 1$) production, productivity falls with z at a rate λ_l for both low- and high-skilled workers alike. For the range of more complex output (i.e., $z > 1$), the productivity falls more slowly for high-skilled workers than for low-skilled workers. Specifically, we assume:

$$\begin{aligned} A_l(z,t) &= \bar{A}_l e^{gt} z^{-\lambda_l} \\ A_h(z,t) &= \bar{A}_h e^{gt} \max\left\{z^{-\lambda_l}, z^{-\lambda_h}\right\} \end{aligned} \tag{10}$$

with $\lambda_h < \lambda_l$.

Given these productivity assumptions, consumption eventually moves into the $z > 1$ region where high-skilled workers hold a stronger productivity advan-

[22] See also Buera and Kaboski (2008) for an analysis of richer, efficient scale-driven product cycles that are important in the early phase of development.

tage. This creates a sorting of high-skilled workers into high z activities. When the wage exceeds 1, there is a strict sorting of all activities into those that are produced with low-skilled labor, $z < \hat{z}$, and those produced with high-skilled labor. The critical $\hat{z}$ is the one z at which firms are indifferent between hiring low- and high-skilled workers. Given (2) and (3), it is trivial to show that:

$$\hat{z}(t) = \left[w(t) \frac{\bar{A}_l}{\bar{A}_h} \right]^{\frac{1}{\lambda_l - \lambda_h}} .$$

As the skill premium increases, more of the less complex services and manufactured goods are going to be produced with low-skilled labor. Costs using high-skilled workers will be equal to low-skilled worker costs at $\hat{z}$ but strictly less for $z > \hat{z}$.

Chronologically, for low t the economy grows in a balanced fashion with high-skilled workers satisfying more wants overall but fewer of them on the market. In this initial phase, the quantity and price of skills remain constant, as do the shares of services and manufacturing in consumption. We summarize this below.

Proposition 4 *Assume (10). Define t_0 as the unique value sastifying $\bar{z}_h(t_0) = 1$. For $t < t_0$:*
(a) $\underline{z}_h(t) \leq \underline{z}_l(t) = \bar{z}_l(t) < \bar{z}_h(t) < 1$;
(b) $c_s^l(t) = 0 < c_s(t) = c_s(0) < c_s^h(t) = c_s^h(0)$;
(c) $w(t) = \bar{A}_h/\bar{A}_l$ and $f(t) = f(0) \in (0, 1)$.

4.2.1 Direct Effect of High-Skilled Comparative Advantage

As productivity rises, individuals' consumption moves into ever more complex wants, eventually those in which high-skilled labor has a comparative advantage. Indeed, after t_0 the most complex want satisfied by high-skilled workers is produced at a lower cost with high-skilled labor, i.e., $\bar{z}_h > 1$. At a later date, the consumption of low-skilled workers also moves in this set of wants, and it becomes profitable for them to purchase market-produced services, i.e., $1 < \underline{z}_l < \bar{z}_l$. The comparative advantage of high-skilled workers in complexity creates a direct force leading to the growth in services, which we call the *high-skill advantage effect*. Namely the cost difference between market and home production increases with complexity (z) and the share of services in the consumption of both low- and high-skilled workers is increasing. We call the force direct because the change in consumption patterns exists, even when the relative wage w and relative prices are held fixed. (Note that constant relative prices imply that current-value consumption shares c_s^l and c_s^h equal real consumption shares.) We formalize this effect below.

Proposition 5 *Assume (10) and consider an economy with a fixed wage. The set of want that are satisfied, and the set of home-produced wants expands, with the former expanding at larger rate, i.e., $\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} \geq \frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)} > 0$, $e = l, h$. Furthermore, the share of services consumed by low-skilled (high-skilled) individuals*

is non-decreasing, $\partial c_s^l(t)/\partial t \geq 0$ *(*$\partial c_s^h(t)/\partial t \geq 0$*). These inequalities are strict provided* $1 < \underline{z}_l < \bar{z}_l$ *(* $\bar{z}_h > 1$*).*
Proof. *See Appendix.* ■

The intuition for this result is fairly straightforward. Neutral productivity growth leads to more wants being satisfied and to the home-production of a larger set of services. As consumption moves into goods and services for which high-skill labor has a strict comparative advantage, it is cheaper to expand the set of wants that are satisfied than to expand the set of home-produced wants, since home-production is done with the productivity of low-skilled labor. Thus, each individual service remains market-produced for longer, implying that the set of market produced services becomes larger, and the share of services increases.

Intuition for this result can also be seen from a restatement of the problem in terms of the quasi-preferences over total expenditures on manufactured goods and services, respectively. These preferences are now non-homothetic. For example, for high-skilled workers with $\underline{z}_h < \hat{z} < \bar{z}_h$, the high-skilled consumer's problem can be written:

$$\begin{aligned} &\max \gamma_1 \left[C_m^h\right]^{1-\sigma_m} + \gamma_2 \left[C_s^h + \gamma_3 C_m^h + \overline{C}_s\right]^{1-\sigma_s} \qquad (11) \\ &s.t. \\ &p_m^h C_m^h + C_s^h = e^{gt} w \left(1-\theta\right) \end{aligned}$$

Again, preference parameters γ_1, γ_2, γ_3, σ_m, σ_s, and $\bar{C}_s$ depend on underlying preferences *and* technology.[23]

Two factors lead to a non-homotheticity toward services. First, the Stone-Geary-like constant $\overline{C}_s$ is positive if and only if $\lambda_h < \lambda_l$. Second, the exponent on the term with services, $1-\sigma_s$, exceeds the exponent on the purely manufactured good term if and only if $\lambda_h < \lambda_l$. Perhaps the most interesting fact is that because these quasi-preferences depend on technology, their shape changes as consumption moves into services for which high-skill labor has a larger productivity advantage. For $\bar{z}_h < 1$, the preferences are homothetic as in (9), while for $\bar{z}_h > 1$ they are non-homothetic as in (11). That is, the transition captured by the disaggregate model *cannot* be aggregated into *stable* preferences over aggregate service and manufactured good expenditures.

4.2.2 Effects that Operate through the Market for High-Skills

With comparative advantage, movement of consumption into more complex output leads to an increase in the demand for high-skilled workers. This causes a rise in the equilibrium price and quantity of skills, and a further rise in the share of services. In this section we highlight these forces.

[23] In particular, $\overline{C}_s = (1+n)(\lambda_l - \lambda_h)\hat{z}^{1+\lambda_l} / \left[\overline{A}_l (1+\lambda_l)(1+\lambda_h)\right] > 0$, $\sigma_m = 1 - 1/(\lambda_l+1) > \sigma_s = 1 - 1/(\lambda_l+1)$, $\gamma_1 = (1-\gamma)\left[(\lambda_l+1)\bar{A}_l/q\right]^{1/(\lambda_l+1)}$, $\gamma_2 = \gamma\left[(\lambda_h+1)\bar{A}_l/(1+q)\right]^{1/(\lambda_h+1)}$, and γ_3, p_m^h as defined in (9).

Define high-skilled labor demand as total (home+market) labor, where the market uses high-skilled labor for output above a $\hat{z}$. Since high z output will be produced by the high-skilled, the growth in $\bar{z}_h$ and $\bar{z}_l$ increases the demand for high-skilled workers.[24] However, the increase in $\underline{z}_l$ can lower the demand by low-skilled workers for high-skilled labor in market services, if their home production time rises.

By Walras' Law, an increase in the total demand for high-skilled labor is equivalent to a decrease in the total demand for low-skilled labor (since budget constraints must be satisfied.) Hence, a simple and intuitive sufficient condition for an increase (decrease) in demand for high-skilled (low-skilled) labor is:

$$\underbrace{(1-f)\int_0^{\underline{z}_l}\frac{z^{\lambda_l}}{\bar{A}_l}dz}_{(I)}\underbrace{(\lambda_l+1)\left\{\frac{\partial \underline{z}_l}{\partial t}\frac{1}{\underline{z}_l}-\frac{g}{\lambda_l+1}\right\}}_{(II)}$$

$$\leq \quad g\left\{\underbrace{(1-f)\left[q\int_0^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right]}_{(III)}+\underbrace{f\left[\begin{array}{c} q\int_0^{\min(\underline{z}_h,\hat{z})}\frac{z^{\lambda_l}}{\bar{A}_l}dz+ \\ (1+q)\int_{\min(\underline{z}_h,\hat{z})}^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz \end{array}\right]}_{(IV)}\right\}. \quad (12)$$

The right-hand side of the expression is the net change in low-skilled labor used in home production, which is the product of the amount of labor used in home production (term I) and its rate of its increase (term II). The left-hand side is the decrease in demand for low-skilled labor on the market coming from the increase in their productivity. Term III is the market labor demanded by low-skilled workers, and Term IV is the market labor demanded by high-skilled workers.

One case that clearly satisfies this condition is as $\lambda_h \to 0$, since preferences become quasilinear and the $\underline{z}_e$ do not change. The condition also holds for any parameters $\lambda_l < \lambda_h$ as $t \to \infty$, since home production time converges to zero (see Proposition 10).

Proposition 6 *Assume (10), (12), and a fixed quantity of high-skill individuals, $f(t) = f$. For any w, the demand for high-skilled labor increases over time.*

An increase in the demand for high-skilled labor, given a fixed supply curve for high-skilled labor, will lead to a rise in the skill premium w. A rise in the skill premium leads to an increase in the price of home-production relative to market services for high-skilled individuals. This gives an additional force towards the rise of services that we label the *opportunity cost effect*. The rise in the opportunity cost causes a decline in the set of home-produced wants and

[24] The demand for high-skilled labor is perfectly elastic at $w = \bar{A}_h/\bar{A}_l$, i.e., it is a correspondence. For this case, the demand for high-skill labor increases with productivity in the sense that the lower bound of the demand correspondence is increasing. The lower bound will be strictly increasing if $\bar{z}_h > 1$.

an increase in the set of wants that are satisfied by high-skilled individuals, $\partial \underline{z}_h/\partial w < 0$ and $\partial \bar{z}_h/\partial w > 0$. To isolate this effect, we keep output prices fixed here, so that the current value shares c_s^h and c_s^l again equal real shares.

Proposition 7 *For given prices of goods and services, $p_m(z,t)$ and $p_s(z,t)$, the higher the skill premium, the more wants that high-skilled agents satisfy, but the fewer wants they satisfy through home production, i.e, $\partial \bar{z}_h/\partial w > 0$ and $\partial \underline{z}_h/\partial w < 0$. Moreover, the share of services in the consumption of high-skilled agents is increasing in the opportunity cost of their time, $\partial c_s^h/\partial w > 0$.*

The result that $\partial \underline{z}_h/\partial w < 0$ (i.e., home production and consumption of manufactured goods fall in absolute terms) is quite strong and deserves further discussion. An increase in the wage has both an income effect and a substitution effect. The income effect alone would lead the individual to increase both $\bar{z}_h$ *and* $\underline{z}_h$, but the above proposition shows that the substitution effect always dominates. The reason is that individual are highly substitutable. Given our assumption that preferences over individual wants are linear with a satiation point, substitutability depends on how quickly the costs change with z. The higher λ_h is the harder it is to substitute between lower z (home-produced) and higher z (market-purchased) output, since costs increases rapidly. One can see the relationship between the elasticity of substitution and λ_h from the quasi-preferences in (9).[25] The elasticity of substitution between total expenditure on services and manufacturing is $\frac{1}{\sigma} = \frac{\lambda+1}{\lambda}$. As $\lambda \to \infty$, the lowest degree of substitutability is Cobb-Douglas. Thus, for any finite λ, the substitution effect will always dominate, and a higher relative wage lowers the amount of home production done by high-skilled workers.

In equilibrium, an outward-shifting demand for skilled labor will also result in an increase in the quantity of high-skilled labor f, provided that the supply of skilled-labor is upward sloping, i.e., $\partial\theta/\partial f < \infty$. Recall that Corollary 2 stated that services are a higher share of consumption for high-skilled workers than low-skilled workers. Thus, this compositional change of the workforce can also increase the share of services. We call this the *skill-deepening effect.*[26]

Another potential effect of skill-deepening, not present in the model, is the direct increase in services coming from educational expenditures. This is not present in the model, since the only cost of schooling is the time cost/foregone labor, but could easily be captured by making schooling Leontieff in foregone

[25] A similar intuition can be gleaned from the non-homothetic case in equation (11), though the mapping of the elastiicty of substitution is less straightforward.

[26] In general, there could also be a shift in the supply curve of high-skill workers. For example, after $\bar{z}_h$ crosses 1, due to the non-homothetic nature of the budget set, high-skill individuals face a more attractive consumption expansion path than low-skill individuals, and more individuals are willing to acquire skills for any given skill-premium. This effect will reinforce the increase in the equilibrium quantity of high-skill workers. At the same time, this effect could temporarily cause a decline in the skill premium, a counteracting force against the rise in the share of services. Furthermore, an increase in f itself does not necessarily lead to a compositional increase in services, since f could increase without the high-skilled increasing their share of market income. This could happen because the wage could decrease, or not increase enough to offset the forgone labor from education.

labor and purchased educational services. Such a modification would be important in a quantitative analysis.

4.2.3 Skill Intensity and the Effect on the Relative Price of Services

A higher skill premium ($w > \bar{A}_h/\bar{A}_l$) will also imply a well-defined pattern of specialization where the most complex good and service are produced by high-skilled workers with $\hat{z} > 1$. We show below that given this higher skill premium and given some consumption of market services produced by low-skilled workers (implied by $\underline{z}_h(t) < \hat{z}$), the services produced by low-skilled labor will decline, while market services produced with high-skilled labor will grow. This result is (qualitatively) consistent with the changing composition of the service industry presented in Figure 2 of Section 2.

The following proposition establish this results:

Proposition 8 *Assume (10), $w(t) = w > \bar{A}_h/\bar{A}_l$, and $\underline{z}_h(t) < \hat{z}$, then $\partial C_{s|l}/\partial t < 0$ and $\partial C_{s|h}/\partial t > 0$, where $C_{s|l}$ ($C_{s|h}$) is the quantity consumed of services produced with low-skilled (high-skilled) labor.*[27]

Proof. See Appendix. ■

The sorting of high-skill individual into the production of high z goods and services predicts that market services are more skill-intensive than manufactured goods on average, since both high and low z manufactured goods will be produced on the market, but low z services will be produced at home. As the relative wage of high-skilled labor increases, the model therefore also predicts an increase in the relative price of services. Thus, while the earlier results have highlighted forces leading to increases in the real share of services, the model also predicts an increase in the aggregate relative price of services. This *increasing relative price* is a fourth channel through which the current value share of services grows. We state this formally below.

We start by defining our price indices $P_s(t,\tau)$ and $P_m(t,\tau)$ as the values of the time τ consumption baskets of services and manufactured goods, respectively, evaluated at time t prices.[28] We then define $\Pi_{s/m}(t)$ as the instantaneous percentage change in the relative price of the service basket relative to the manufactured good basket at time τ is then $\Pi_{s/m}(t) = \frac{\partial[P_s(t,\tau)/P_m(t,\tau)]/\partial t}{P_s(t,\tau)/P_m(t,\tau)}|_{\tau=t}$.[29]

Proposition 9 *If $w(t) > \bar{A}_h/\bar{A}_l$ and $\partial w(t)/\partial t > 0$, then $\Pi_{s/m}(t) > 0$.*

---

[27] Specifically, $C_{s|l} = f\int_{\min\{\underline{z}_h,\hat{z}\}}^{\hat{z}} p_s(z)dz$, and
$C_{s|h} = f\int_{\max\{\underline{z}_h,\hat{z}\}}^{\max\{\hat{z},\bar{z}_h\}} p_s(z)dz + (1-f)\int_{\max\{\hat{z},\underline{z}_l\}}^{\max\{\hat{z},\bar{z}_l\}} p_s(z)dz.$

[28] Specifically, $P_s(t,\tau) = f(\tau)\int_{\underline{z}_h(\tau)}^{\bar{z}_h(\tau)} p_s(t)dz + [1-f(\tau)]\int_{\underline{z}_l(\tau)}^{\bar{z}_l(\tau)} p_s(t)dz$ and
$P_m(t,\tau) = f(\tau)\int_0^{\underline{z}_h(\tau)} p_m(t)dz + [1-f(\tau)]\int_0^{\underline{z}_l(\tau)} p_m(t)dz$

[29] This corresponds to changes in a relative price index constructed from continous time chain-weighted price indexes, where the indices are continously chained.

Proof. See Appendix. ■

4.2.4 Aggregate Growth in the Share of Services

Using a common time scale, Figures 5 and 6 illustrates a simulated example of the above dynamics in the service share of consumption, relative wage, fraction becoming high-skilled, and fraction of time spent on home production. The simulation is not quantitative but purely illustrative.[30,31]

In our characterization, we have identified four related forces leading to the rise of services, all of which are the result of consumption expanding into goods and services for which high-skill labor has a productivity advantage. We therefore decompose the growth in services in the top panel into these four elements.

- The diamonds show the increase in the real share of services in consumption for fixed wages and fixed quantity of skills. That is, this is the *high-skill advantage effect* from Proposition 5.
- The circles give the response of the real share of services once we also adjust the skill-premium in agents' maximization. That is, it includes the *opportunity cost effect* from Proposition 7.
- The solid line show the combined effect on the real share of services as we further allow for the increase in the quantity of skills. That is, it includes the *skill deepening effect*.
- The above three lines have looked at real consumption in that they have been valued at the initial prices, $w(0) = \bar{A}_h/\bar{A}_l$. The dashed line shows the full effect on the current value share after we allow for the *relative price effect* from Proposition 9.

In the limit the relative wage grows unbounded, all agents become high-skilled, and the share of services in consumption converges to one.[32] The following proposition summarizes the assymptotic behavior of the economy.

Proposition 10 *Assume (10). As* $t \to \infty$,
(a) all workers become high-skilled, $\lim_{t\to\infty} f(t) = 1$*;*
(b) the relative wage converges to $1/\left(1-\overline{\theta}\right) > \bar{A}_h/\bar{A}_l$*;*
(c) both the sets of satisfied and home-produced wants grow in the limit, but

[30] The parameter values behind the simulation reported in Figure 5 are: $\bar{A}_l = 1$, $\bar{A}_h = 1.2$, $\lambda_l = 1.5$, $\lambda_h = 1$; $\gamma = 0.98$, $q = 1$, $g = 0.02$, and $\theta(f) = (1/\phi) f^{\phi}$, with $\phi = 0.9$. In this simulation, the rise of services starts after the period $t = 40$, when $\bar{z}_h$ crosses 1; but the rise of services only become significant once $\underline{z}_l < \bar{z}_l$ after the period $t = 49$.

[31] The transitions has several kinks which could easily be smoothed by allowing for heterogeneity in wealth or earnings as in a previous version of the model.

[32] The skill deepening effect makes it difficult to fully characterize the transition path of aggregate services and the skill premium, as discussed in Footnote 26. That said, in all of the numerical examples that we explore, the transitions of the service share, relative wage, and the fraction high-skilled are all monotone.

the set of satisfied wants grows at a higher rate, i.e., $0 < \lim_{t\to\infty} \dot{\underline{z}}_e(t)/\underline{z}_e(t) < \lim_{t\to\infty} \dot{\bar{z}}_e(t)/\bar{z}_e(t) = g/(\lambda_h + 1)$*, and home production time converges to zero,* $\lim_{t\to\infty} \int_0^{\underline{z}_e(t)} z^{\lambda_l}/\bar{A}_l dz = 0$*, for* $e = l, h$*;*
(d) the share of services in consumption converges to one, $\lim_{t\to\infty} c_s(t) = 1$*, and the share of services in value-added to a number strictly less than one,* $\lim_{t\to\infty} y_s(t) = \frac{1}{1+q}$*.*[33]

Product Cycles, Market-to-Home and Home-to-Market Along with the rise of the service economy, the model generates a rich set of product cycles. The earlier product cycle of not consumed → market consumed → home-produced remains for some services. In addition, however, the production of some services transition from being home-produced by low-skilled workers to market produced, as more individuals become high-skill. Furthermore, in an initial phase, the home-production of service by high-skilled individuals can decline with the increase in the skill-premium.

Indeed, if we assume that for high-skilled workers complexity does not affect their productivity, i.e., $\lambda_h = 0$, then the set of services home produced by high-skill individuals shrinks along the transition with the increase in the skill-premium. If $\lambda_h = 0$, the first-order conditions of high-skill individuals' problem imply

$$\frac{1-\gamma}{\gamma} = \frac{\bar{A}_h}{\bar{A}_l} \frac{\underline{z}_h(t)^{\lambda_l} - 1/w(t)}{(1+q)}.$$

In this case, the set of home-produced goods by high-skill individuals is influenced by the skill-premium and not directly by technical change.

5 Additional Implications

This section evaluates the additional implications of the assumption of comparative advantage of skill in more complex output and discuss evidence consistent with this explanation playing a role. In particular, the growth in the share of services relative to goods is driven by both growth in the relative real quantity of services but also the relative price of services. This growth in the relative price of services is also tightly linked with the skill premium. Finally, examples of product cycles fitting our story abound.

Figure 7 plots the growth in the current-price output (i.e., value-added) of services relative to commodities, and decomposes it into the growth in the measured relative price of services, and the growth in the relative *real* quantity of services (after deflating). Both relative quantities and relative prices show a positive trend, and both play a substantial role in the overall growth of the relative share in services.[34]

[33] Specifically, $y_s = Y_s/(Y_s + Y_m)$, where $Y_s = \frac{1}{1+q} C_s$ and $Y_m = C_m + \frac{q}{1+q} C_s$.
[34] Lee and Wolpin (2006) established the same facts but over a shorter period of 1968-2000.

This decomposition is introduced with the caveat that changes in prices are measured imperfectly because of changes in quality over time. Quality improvements exist for both goods and services, but the rates of change and ability to control or adjust for quality may also vary across sectors. Moreover, many real quantities of services are only measured implicitly, and indeed Bosworth and Triplett (2007) and Griliches (1992) argue that growth in the real quantity of services is understated, and price growth is therefore overstated. Indeed, although we find a similar pattern for the relative prices and quantities of the consumption of goods and services, a much larger share of growth is in relative prices. Nonetheless, all available data show an increase in both the relative real quantity and the relative price of services between 1950 and 2000.

An increase in both relative quantities and relative prices is consistent with a demand explanation. In our comparative advantage model, the increase in demand for services stems from an increase in the demand for complex output. Again, the reason relative prices increase in the comparative advantage story is that the sorting of workers causes market services to be more skill-intensive. The rising relative wage therefore leads to a greater increase in the relative price of services.

Figure 8 shows the wage of college-educated workers relative to high-school educated workers together with the relative price of services over time for the United States. We have normalized the two to be equal in year 1940. There appears to be a tight relationship between the two. In particular, the decade-to-decade fluctuations mirror each other, and the percentage movements are even of similar magnitude.[35]

Another novel implication of the model is our prediction of rich product cycles. Recall that the model allows for movement of productive activities out of the home as the opportunity cost of time rises. This marketization of home production and its effect on the service sector has been modeled by Ngai and Pissarides (2008) and Rogerson (2008). Examples of such activities include child care, elderly care, lawn care, and substituting restaurant meals for home cooked meals, all of which are plausibly driven by rising opportunity costs of time among high-skilled workers. The more novel and surprising implication, however, is the prediction that as the costs of producing fall, the preference for the benefits associated with home production will move activities from the market to the home. The model predicts that the higher the productivity advantage of high-skilled labor, the longer the product cycle, which could make many product cycles difficult to discern. Still, there are numerous examples of this product cycle, even among skill-intensive activities such as medicine and education. For example, in health care, patients now do home dialysis, check blood sugar levels

[35] The growth in the skill premium after 1970 coincides with the well-known slowdown in measured productivity growth. Our model may not be consistent with a rapidly rising skill premium during a time of slower productivity growth, though this will certainly depend on the particular form of the $\theta(f)$. In any case, the literature has proposed several explanation (e.g., Caselli, 1999, Greenwood and Yorukoglu, 1997) for these joint phenomena, including mismeasurement of productivity growth, and our model is consistent with a rising share of service and relative price of services during a period of rising skill premium.

and give insulin shots.[36] In education, self-guided foreign language instruction now exists, and home schooling is a small but rapidly growing segment of the education market, particularly primary schooling.[37] Again, for these examples the utility benefit of home production appears to play a role.

6 Conclusions

To explain the rise of the service economy in the U.S. over the last half century, we have focused on the household's decision between home production and market production in explaining the rise of the service economy. Modeling this margin has yielded insight into understanding the high-skill nature of the rising service economy.

As mentioned in the introduction, we conjecture that our model would have particular implications for several policy-relevant issues. First, the model has a rich theory of labor supply and its elasticity. We have avoided reference to female labor supply, which has strongly impacted the U.S. labor market over the period studied, and is of great importance in considering the home production vs. market purchase margin. Indeed, labor supply decisions has been recently linked to the growth in services (Lee and Wolpin, 2006). Second, we have mentioned government subsidies that exist in important growing service industries like education and health care. A detailed quantitative analysis of how such subsidies affect the margin between home and market production along the lines of Rogerson (2008) would be of great interest. Third, our theory can explain both the rising share of services and rising relative price of services without requiring slower productivity in services. Indeed, slower productivity growth in services would tend to lessen the quantitative implications of our theory for structural change. On the other hand, if productivity growth in services is understated, and comparable or higher than that in manufacturing, then our model has greater potential in quantitatively reconciling structural change and the (smaller) increase in the relative price of services. All of these questions are subjects of ongoing research.

[36] See Blagg (1997, 2006) for a discussion of home dialysis.

[37] Princiotta and Bielick (2006) document the recent growth in the prevalence of home schooling.

A Characterization of the Consumer's Budget Set

In this section we characterize the properties of the budget set for the case where high-skilled labor has a (weak) comparative advantage in the production of more complex wants, $\frac{\partial A_l(z)}{\partial z}\frac{1}{A_l(z)} \leq \frac{\partial A_h(z)}{\partial z}\frac{1}{A_h(z)} < 0$, and market production of a service is a viable option, $\frac{w_e}{A_l\left(\underline{z}_e,t\right)} - \min\left\{\frac{1}{A_l\left(\underline{z}_e,t\right)}, \frac{w}{A_h\left(\underline{z}_e,t\right)}\right\} > 0$. The budget set is given by the following inequality:

$$F\left(\underline{z}_e, \bar{z}_e\right) \leq 0 \tag{13}$$

where

$$\begin{aligned} & F\left(\underline{z}_e, \bar{z}_e\right) \\ = \quad & q\int_{-\infty}^{\underline{z}_e} p_m\left(z,t\right)dz + \int_{\underline{z}_e}^{\bar{z}_e} p_s\left(z,t\right)dz - w_e\left(1 - \int_{-\infty}^{\underline{z}_e}\frac{1}{A_l\left(z,t\right)}dz - \theta\mathcal{I}\left(e\right)\right) \end{aligned}$$

The slope of the budget set equals:

$$\begin{aligned} \frac{d\underline{z}_l}{d\bar{z}_l} &= -\frac{F_{\bar{z}}\left(\underline{z}_e, \bar{z}_e\right)}{F_{\underline{z}}\left(\underline{z}_e, \bar{z}_e\right)} \\ &= -\frac{\min\left\{\frac{1}{A_l(\bar{z}_e,t)}, w\frac{1}{A_h(\bar{z}_e,t)}\right\}}{\frac{w_e}{A_l\left(\underline{z}_e,t\right)} - \min\left\{\frac{1}{A_l\left(\underline{z}_e,t\right)}, \frac{w}{A_h\left(\underline{z}_e,t\right)}\right\}} < 0. \end{aligned}$$

Provided that the function $F\left(.,.\right)$ is concave, condition (13) defines a convex set. In the case $\frac{1}{A_l\left(\underline{z}_e,t\right)} > \frac{w}{A_h\left(\underline{z}_e,t\right)}$, the Hessian of $F\left(.,.\right)$ is given by:

$$-\begin{pmatrix} \frac{w_e}{A_l\left(\underline{z}_e\right)^2}\frac{\partial A_l\left(\underline{z}_e\right)}{\partial \underline{z}_e} - \frac{w}{A_h\left(\underline{z}_e\right)^2}\frac{\partial A_h\left(\underline{z}_e\right)}{\partial \underline{z}_e} & 0 \\ 0 & w\frac{1}{A_h(\bar{z}_e)^2}\frac{\partial A_h(\bar{z}_e)}{\partial \bar{z}_e} \end{pmatrix}$$

Clearly, as long as $\frac{1}{A_l\left(\underline{z}_e\right)}\frac{\partial A_l\left(\underline{z}_e\right)}{\partial \underline{z}_e} \leq \frac{1}{A_h\left(\underline{z}_e\right)}\frac{\partial A_h\left(\underline{z}_e\right)}{\partial \underline{z}_e}$, the budget set is guaranteed to be convex. A similar analysis follows for the cases $\frac{1}{A_l\left(\underline{z}_e,t\right)} < \frac{w}{A_h\left(\underline{z}_e,t\right)}$ and $\frac{1}{A_l(\bar{z}_e,t)} < \frac{w}{A_h(\bar{z}_e,t)}$.

B Proofs of Results in the Paper

Proof of Proposition 3. We first show the sufficiency of $\tilde{A}\left(z\right) = z^{-\lambda}$. Given $\tilde{A}\left(z\right) = z^{-\lambda}$, after substituting in for equilibrium prices and integrating, the problem of a high-skilled individual with neutral productivity e^{gt} simplifies to:

$$\max_{0\leq \underline{z}_h(t)\leq \bar{z}_h(t)} (1-\gamma)\,\underline{z}_h(t)+\gamma\bar{z}_h(t)$$
s.t.
$$\frac{\bar{A}_h/\bar{A}_l-1}{\lambda+1}\underline{z}_h(t)^{\lambda+1}+\frac{1+q}{\lambda+1}\bar{z}_h(t)^{\lambda+1}=\bar{A}_l e^{gt}(1-\theta)$$

From the first-order conditions of this problem we obtain:

$$\frac{\bar{A}_h/\bar{A}_l-1}{1-\gamma}\underline{z}_h(t)^{\lambda}=\frac{1+q}{\gamma}\bar{z}_h(t)^{\lambda}. \tag{14}$$

Substituting (14) into the expression for the share of services in consumption, we obtain the desired result of a constant service share for high-skill individuals, for all q and γ,

$$c_s^h(t)=\frac{C_s^h(t)}{C_m^h(t)+C_s^h(t)}=\frac{(1+q)\int_{\underline{z}_h(t)}^{\bar{z}_h(t)} z^{\lambda}dz}{q\int_0^{\underline{z}_h(t)} z^{\lambda}dz+(1+q)\int_{\underline{z}_h(t)}^{\bar{z}_h(t)} z^{\lambda}dz}$$

$$=\frac{(1+q)\left[1-\left(\frac{1-\gamma}{\gamma}\frac{1+q}{\bar{A}_h/\bar{A}_l-1}\right)^{(\lambda+1)/\lambda}\right]}{q\left(\frac{1-\gamma}{\gamma}\frac{1+q}{\bar{A}_h/\bar{A}_l-1}\right)^{(\lambda+1)/\lambda}+(1+q)\left[1-\left(\frac{1-\gamma}{\gamma}\frac{1+q}{\bar{A}_h/\bar{A}_l-1}\right)^{(\lambda+1)/\lambda}\right]}.$$

The overall share of services is:

$$c_s(t)=\frac{fC_s^h(t)+(1-f)C_s^l(t)}{fC^h(t)+(1-f)\,C^l(t)}$$

Since $C_s^l(t)=0$, $w=\bar{A}_h/\bar{A}_l$ and therefore f, and $C^h(t)/C^l(t)$ are constant, we have shown that $c_s(t)$ is constant.
We next show that $\tilde{A}(z)=z^{-\lambda}$ is a necessary condition for a constant share of services. From the individual's decision problem, we obtain the following two restrictions, the budget constraint:

$$\left(\bar{A}_h/\bar{A}_l+q\right)\int_0^{\underline{z}_h(t)}\frac{dz}{\tilde{A}(z)}+(1+q)\int_{\underline{z}_h(t)}^{\bar{z}_h(t)}\frac{dz}{\tilde{A}(z)}=\bar{A}_l e^{gt}(1-\theta), \tag{15}$$

and the combined first-order conditions:

$$\frac{\gamma}{1-\gamma}=\frac{1+q}{\bar{A}_h/\bar{A}_l-1}\frac{\tilde{A}(\bar{z}_h(t))}{\tilde{A}(\underline{z}_h(t))}. \tag{16}$$

An additional equation is given by the condition requiring that the share of services in consumption is constant:

$$c_s^h(t)=\frac{(1+q)\int_{\underline{z}_h(t)}^{\bar{z}_h(t)}\frac{dz}{\tilde{A}(z)}}{q\int_0^{\underline{z}_h(t)}\frac{dz}{\tilde{A}(z)}+(1+q)\int_{\underline{z}_h(t)}^{\bar{z}_h(t)}\frac{dz}{\tilde{A}(z)}} \tag{17}$$

Totally differentiating (17) gives:

$$\frac{d\underline{z}_h(t)}{\tilde{A}(\underline{z}_h)}(1+q-c_s) = \frac{d\bar{z}_h(t)}{\tilde{A}(\bar{z}_h)}(1+q-c_s(1+q)). \tag{18}$$

Using (16) and (18) we obtain:

$$\frac{d\underline{z}_h(t)}{d\bar{z}_h(t)} = \frac{1+q-c_s(1+q)}{(1+q-c_s)} \frac{1+q}{\bar{A}_h/\bar{A}_l - 1} \frac{1-\gamma}{\gamma}$$

Thus,

$$\bar{z}_h(t) = a + b\underline{z}_h(t) \tag{19}$$

for a constant a and $b = \frac{1+q-c_s(1+q)}{(1+q-c_s)} \frac{1+q}{\bar{A}_h/\bar{A}_l-1} \frac{1-\gamma}{\gamma} < 1$. Furthermore, $a = 0$ since otherwise c_s must equal zero for sufficiently large or low income. Together, (16), (19), and $a = 0$ imply:

$$\frac{\tilde{A}'(b\underline{z}_h(t))}{\tilde{A}(b\underline{z}_h(t))} b\underline{z}_h(t) = \frac{\tilde{A}'(\underline{z}_h(t))}{\tilde{A}(\underline{z}_h(t))}\underline{z}_h(t) \tag{20}$$

Since (20) must hold for any $b \leq 1$ (as we vary q and γ) and all $\underline{z}_h(t)$ (as we vary t), for any z the left-hand side must be a constant, call it λ. Simple integration yields the desired result:

$$\begin{aligned} \frac{\tilde{A}'(z)}{\tilde{A}(z)} z &= \lambda \\ \tilde{A}(z) &= z^{\lambda} \end{aligned}$$

■

To prove Proposition 5 we first establish the following lemma characterizing the evolution of individual thresholds, for a fixed wage, as productivity grows.

Lemma 11 *Assume (10) and consider an economy with a fixed wage. The set of wants that are satisfied and the set of home-produced wants expand, with the former expanding at larger rate, i.e.,* $\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} \geq \frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)} > 0$, $e = l, h$.

Proof. In the case of a corner solution, $\underline{z}_e(t) = \bar{z}_e(t)$, we get this result trivially as $\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} = \frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}$ (in the following discussion, since the wage is assume to be constant, individual thresholds are only a function of time through technology). Given Assumption 7, this can only be the case for low-skill individuals.

In the case of an interior solution, this lemma follows from log-differentiation of the first order condition. There are three interior cases to consider: (i) $\underline{z}_e(t) < \bar{z}_e(t) < \hat{z}$ (all wants consumed by type e are produced with low-skill labor), (ii) $\underline{z}_e(t) < \hat{z} < \bar{z}_e(t)$ (market services consumed by type e are produced with both low and high-skill labor), (iii) $\hat{z} < \underline{z}_e(t) < \bar{z}_e(t)$ (all market services consumed by type e are produced with high-skill labor).

Case (i) Given $\underline{z}_e(t) < \bar{z}_e(t) < \hat{z}$, the dynamics of individual follows from the proof of Proposition 3, and therefore $\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} = \frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)} = g/(\lambda_l + 1)$.
Cases (ii) and (iii): Given $\hat{z} < \bar{z}_e(t)$, individual thresholds satisfy the following first-order condition

$$\frac{1-\gamma}{\gamma} = \frac{w_e \underline{z}_e(t)^{\lambda_l}/\bar{A}_l - \min\left\{\underline{z}_e(t)^{\lambda_l}/\bar{A}_l, w\underline{z}_e(t)^{\lambda_h}/\bar{A}_h\right\}}{(1+q)\, w\underline{z}_e(t)^{\lambda_h}/\bar{A}_h}. \tag{21}$$

Case (ii): Given $\underline{z}_h(t) < \hat{z} < \bar{z}_h(t)$ (for low-skill individuals, $e = l$, the only relevant case is $\hat{z} \leq \underline{z}_e(t)$, as low-skilled workers will never strictly prefer to purchase market services produced by low-skilled workers, since $\gamma < 1$), log-differentiatiion of condition (21) yields

$$\frac{\partial \bar{z}_h(t)/\partial t}{\bar{z}_h(t)} = \frac{\lambda_l}{\lambda_h}\frac{\partial \underline{z}_h(t)/\partial t}{\underline{z}_h(t)} > \frac{\partial \underline{z}_h(t)/\partial t}{\underline{z}_h(t)}.$$

Case (iii): Given $\hat{z} < \underline{z}_e(t) < \bar{z}_e(t)$, $e = l, h$

$$\begin{aligned}
\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} &= \frac{w_e(\lambda_l/\lambda_h)\,\underline{z}_e(t)^{\lambda_l}/\bar{A}_l - w\underline{z}_e(t)^{\lambda_h}/\bar{A}_h}{w_e\underline{z}_e^{\lambda_l}/\bar{A}_l - w\underline{z}_e^{\lambda_h}/\bar{A}_e}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)} \\
&> \frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}.
\end{aligned}$$

■

Proof of Proposition 5. We use the results on the dynamics of individual thresholds from the previous lemma to characterize the evolution of the share of services for each type, given a fixed wage, as productivity grows. The share of services in consumption for an individual of skill e equals

$$\begin{aligned}
c_s^e(t) &= \frac{C_s^e(t)}{C^e(t)} \\
&= \frac{(1+q)\int_{\underline{z}_e(t)}^{\bar{z}_e(t)} \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w\frac{z^{\lambda_h}}{\bar{A}_h}\right\}dz}{q\int_0^{\underline{z}_e(t)} \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w\frac{z^{\lambda_h}}{\bar{A}_h}\right\}dz + (1+q)\int_{\underline{z}_e(t)}^{\bar{z}_e(t)} \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w\frac{z^{\lambda_h}}{\bar{A}_h}\right\}dz}
\end{aligned}$$

For the case of a corner solution, $\underline{z}_e(t) = \bar{z}_e(t)$, we trivially get that the share of services is constant. For Case (i) from the above lemma, the analysis of Proposition 3 applies, and we also get a constant share of services for both types.
We are left with Cases (ii) and (iii) from the above lemma.
Case (ii): Given $\underline{z}_h(t) < \hat{z} < \bar{z}_h(t)$

$$\partial c_s^h(t)/\partial t = \frac{1}{(C^h(t))^2}\left\{\begin{array}{c}
(1+q)\left[w\frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h}\frac{\partial \bar{z}_h(t)/\partial t}{\bar{z}_h(t)} - \frac{\underline{z}_h^{\lambda_l+1}}{\bar{A}_l}\frac{\partial \underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\right]C^h(t) \\
-\left[\begin{array}{c}(1+q)\left[w\frac{\bar{z}_h(t)^{\lambda_h}}{\bar{A}_h}\frac{\partial \bar{z}_h(t)/\partial t}{\bar{z}_h(t)} - \frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l}\frac{\partial \underline{z}_h(y)/\partial t}{\underline{z}_h(t)}\right] \\ +q\frac{\underline{z}_h(t)^{\lambda_l}}{\bar{A}_l}\frac{\partial \underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\end{array}\right]C_s^h(t)
\end{array}\right\}$$

replacing $C^h(t) = C^h_m(t) + C^h_s(t)$, rearranging and cancelling terms

$$\partial c^h_s(t)/\partial t = \frac{1}{\left(C^h(t)\right)^2}\left\{\begin{array}{c}(1+q)\left[w\frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h}\frac{\partial\bar{z}_h(t)/\partial t}{\bar{z}_h(t)} - \frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\right]C^h_m(t)\\ -q\frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}C^h_s(t)\end{array}\right\}.$$

Replacing $C^h_m(t) = q\int_0^{\underline{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz = q\frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)}$ and $C^h_s(t) = (1+q)\left[\int_{\underline{z}_h(t)}^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz + w\int_{\hat{z}}^{\bar{z}_h(t)}\frac{z^{\lambda_h}}{\bar{A}_h}dz\right]$
$= (1+q)\left[w\frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} - \frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)} + \frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)}\right]$,

$$\partial c^h_s(t)/\partial t = \frac{1}{\left(C^h(t)\right)^2}\left\{\begin{array}{c}(1+q)\left[w\frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h}\frac{\partial\bar{z}_h(t)/\partial t}{\bar{z}_h(t)} - \frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\right]q\frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)}\\ -q(1+q)\frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\left[w\frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} - \frac{\underline{z}_h(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)} + \frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)}\right]\end{array}\right\}$$

cancelling and rearranging terms

$$\partial c^h_s(t)/\partial t = \frac{1}{\left(C^h(t)\right)^2}\left\{\begin{array}{c}\frac{(1+q)qw}{\bar{A}_l\bar{A}_h}\bar{z}_h(t)^{\lambda_h+1}\,\underline{z}_h(t)^{\lambda_l+1}\left[\frac{1}{\lambda_l+1}\frac{\partial\bar{z}_h(t)/\partial t}{\bar{z}_h(t)} - \frac{1}{\lambda_h+1}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\right]\\ -q(1+q)\underline{z}_h^{\lambda_l+1}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_l\bar{A}_h(\lambda_l+1)(\lambda_h+1)}\end{array}\right\}$$

Using that $\frac{\partial\bar{z}_h(t)/\partial t}{\bar{z}_h(t)} = \frac{\lambda_l}{\lambda_h}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}$

$$\begin{aligned}\partial c^h_s(t)/\partial t &= \frac{1}{\left(C^h(t)\right)^2}\frac{\partial\underline{z}_h(t)/\partial t}{\underline{z}_h(t)}\left\{\begin{array}{c}\frac{(1+q)qw}{\bar{A}_l\bar{A}_h}\bar{z}_h(t)^{\lambda_h+1}\,\underline{z}_h(t)^{\lambda_l+1}\left[\frac{\lambda_l-\lambda_h}{\lambda_h(\lambda_l+1)(\lambda_h+1)}\right]\\ +q(1+q)\,\underline{z}_h(t)^{\lambda_l+1}\,\hat{z}^{\lambda_h+1}d\underline{z}_h\frac{w(\lambda_l-\lambda_h)}{\bar{A}_l\bar{A}_h(\lambda_l+1)(\lambda_h+1)}\end{array}\right\}\\ &> 0\end{aligned}$$

where the last inequality uses that $\lambda_l > \lambda_h$.
Case (iii): Given $\hat{z} < \underline{z}_e(t) < \bar{z}_e(t)$

$$\partial c^e_s(t)/\partial t = \frac{1}{\left(C^e(t)\right)^2}\left\{\begin{array}{c}(1+q)\frac{w}{\bar{A}_h}\left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial\bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial\underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]C^e_m(t)\\ -q\frac{w}{\bar{A}_h}\underline{z}_e(t)^{\lambda_h+1}\frac{\partial\underline{z}_e(t)/\partial t}{\underline{z}_e(t)}C^e_s(t)\end{array}\right\}.$$

Replacing $C^e_m(t) = q\left[\int_0^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz + w\int_{\hat{z}}^{\underline{z}_e(t)}\frac{z^{\lambda_h}}{\bar{A}_h}dz\right] = q\left[\frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + w\frac{\underline{z}_e(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)}\right]$
and $C^h_s(t) = (1+q)\,w\int_{\underline{z}_e(t)}^{\bar{z}_e(t)}\frac{z^{\lambda_h}}{\bar{A}_h}dz = (1+q)\frac{w}{\bar{A}_h(\lambda_h+1)}\left[\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right]$,

$$\partial c^e_s(t)/\partial t = \frac{1}{\left(C^e(t)\right)^2}\left\{\begin{array}{c}(1+q)\frac{w}{\bar{A}_h}\left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial\bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial\underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\\ q\left[\frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + w\frac{\underline{z}_e(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)}\right]\\ -q(1+q)\left(\frac{w}{\bar{A}_h}\right)^2\frac{1}{(\lambda_h+1)}\underline{z}_e(t)^{\lambda_h+1}\frac{\partial\underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\left[\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right]\end{array}\right\}$$

Using that $\bar{z}_e(t)^{\lambda_h+1}\frac{\partial\bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial\underline{z}_e(t)/\partial t}{\underline{z}_e(t)} > 0$, $\lambda_h - \lambda_l < 0$ and $\hat{z} < \underline{z}_h$, imply that

$$\begin{aligned}
& \left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\left[\frac{w(\lambda_h-\lambda_l)\hat{z}^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + w\frac{\underline{z}_e(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)}\right] \\
> & \left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\left[\frac{w(\lambda_h-\lambda_l)\underline{z}_e(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + w\frac{\underline{z}_e(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)}\right] \\
= & \frac{w}{\bar{A}_h(\lambda_h+1)}\left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\underline{z}_h^{\lambda_h+1}\frac{1+\lambda_h}{\lambda_l+1},
\end{aligned}$$

we can obtain the following lower bound for the change in the share of services

$$\begin{aligned}
\partial c_s^e(t)/\partial t &> \frac{1}{(C^e(t))^2}\left\{\begin{array}{c} (1+q)\,q\frac{w}{\bar{A}_h}\frac{w}{\bar{A}_h(\lambda_h+1)}\left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\underline{z}_h^{\lambda_h+1}\frac{1+\lambda_h}{\lambda_l+1} \\ -q(1+q)\left(\frac{w}{\bar{A}_h}\right)^2\frac{1}{(\lambda_h+1)}\underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\left[\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right] \end{array}\right\} \\
&= \frac{(1+q)\,q\left(\frac{w}{\bar{A}_h}\right)^2\frac{\underline{z}_e(t)^{\lambda_h+1}}{(\lambda_h+1)}}{(C^e(t))^2}\left\{\begin{array}{c} \left[\bar{z}_e(t)^{\lambda_h+1}\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} - \underline{z}_e(t)^{\lambda_h+1}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\right]\frac{1+\lambda_h}{\lambda_l+1} \\ -\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\left[\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right] \end{array}\right\}
\end{aligned}$$

Finally, using that $\frac{\partial \bar{z}_e(t)/\partial t}{\bar{z}_e(t)} > \frac{\lambda_l}{\lambda_h}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}$,

$$\begin{aligned}
\partial c_s^e(t)/\partial t &> \frac{(1+q)\,q\left(\frac{w}{\bar{A}_h}\right)^2\frac{\underline{z}_e(t)^{\lambda_h+1}}{(\lambda_h+1)}}{(C^e(t))^2}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\left\{\begin{array}{c} \left[\frac{\lambda_l}{\lambda_h}\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right]\frac{1+\lambda_h}{\lambda_l+1} \\ -\left[\bar{z}_e(t)^{\lambda_h+1} - \underline{z}_e(t)^{\lambda_h+1}\right] \end{array}\right\} \\
&= \frac{(1+q)\,q\left(\frac{w}{\bar{A}_h}\right)^2\frac{\underline{z}_e(t)^{\lambda_h+1}}{(\lambda_h+1)}}{(C^e(t))^2}\frac{\partial \underline{z}_e(t)/\partial t}{\underline{z}_e(t)}\left\{\frac{\lambda_l-\lambda_h}{\lambda_h(\lambda_l+1)}\bar{z}_e(t)^{\lambda_h+1} + \frac{\lambda_l-\lambda_h}{\lambda_l+1}\underline{z}_e(t)^{\lambda_h+1}\right\} \\
&> 0.
\end{aligned}$$

■

Proof of Proposition 6. Taking as given the quantity of high-skill individuals (f), the demand for high-skill labor ($f^d(w,t)$) as a function of the skill premium and the level of technology (we use time as an index of technology, as technology is a monotone function of time, e^{gt}) equals

$$f^d(w,t) = (1-f)\cdot f_l^d(w,t) + f\cdot f_h^d(w,t)$$

where $f_e^d(w,t)$ is the demand of high-skill labor associated with the consumption of individuals with skill e (to save on notation we don't explicitly write the dependence of the individual thresholds on the skill premium). If $w = \bar{A}_h/\bar{A}_l$,

$$f_e^d(w,t) \in e^{-gt}\left[\begin{array}{c} q\int_{\min\{1,\underline{z}_e(t)\}}^{\underline{z}_e(t)}\frac{z^{\lambda_l}}{\bar{A}_h}dz + (1+q)\int_{\max\{1,\underline{z}_e(t)\}}^{\max\{1,\bar{z}_e(t)\}}\frac{z^{\lambda_l}}{\bar{A}_h}dz + \mathcal{I}(e)\int_0^{\underline{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz \\ ,\, q\int_0^{\underline{z}_e(t)}\frac{z^{\lambda_l}}{\bar{A}_h}dz + (1+q)\int_{\underline{z}_e(t)}^{\bar{z}_e(t)}\frac{z^{\lambda_l}}{\bar{A}_h}dz + \mathcal{I}(e)\int_0^{\underline{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz \end{array}\right]$$

where the lower bound is the case where high-skilled labor is used only if it has a strict cost-advantage, and the upper bound is when it is used whenever their is indifference. If $w > \bar{A}_h/\bar{A}_l$**,** $e = l, h$**.**

$$f_e^d(w,t) = e^{-gt}\left[q\int_{\min\{\hat{z},\underline{z}_e(t)\}}^{\underline{z}_e(t)} \frac{z^{\lambda_l}}{\bar{A}_h}dz + (1+q)\int_{\max\{\hat{z},\underline{z}_e(t)\}}^{\max\{\hat{z},\bar{z}_e(t)\}} \frac{z^{\lambda_l}}{\bar{A}_h}dz + \mathcal{I}(e)\int_0^{\underline{z}_h(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right]$$

In the following discussion we concentrate on the case $w > \bar{A}_h/\bar{A}_h$, but a similar argument can be use to prove that the lower and upper bound of the high-skilled demand correspondence at $w = \bar{A}_h/\bar{A}_l$ increase over time.
Using the budget constraint of low-skilled individuals, $f_l^d(w,t)$ can be written as the difference between the labor endowment and low-skilled labor demanded over the wage:

$$f_l^d(w,t) = 1/w - e^{-gt}\left[q\int_0^{\hat{z}} \frac{z^{\lambda_l}}{\bar{A}_l}dz + \int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right]/w.$$

Implying,

$$\begin{aligned}\frac{\partial f_l^d(w,t)}{\partial t} &= g\frac{e^{-gt}}{w}\left[q\int_0^{\hat{z}} \frac{z^{\lambda_l}}{\bar{A}_l}dz + \int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right] \\ &\quad - \frac{e^{-gt}}{w}\frac{\underline{z}(t)^{\lambda_l+1}}{\bar{A}_l}\frac{1}{\underline{z}(t)}\frac{\partial \underline{z}(t)}{\partial t}\end{aligned}$$

Using $\frac{\underline{z}(t)^{\lambda_l+1}}{\bar{A}_l} = (\lambda_l+1)\int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz$

$$\begin{aligned}\frac{\partial f_l^d(w,t)}{\partial t} &= g\frac{e^{-gt}}{w}\left[q\int_0^{\hat{z}} \frac{z^{\lambda_l}}{\bar{A}_l}dz + \int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right] \\ &\quad - \frac{\lambda_l+1}{\underline{z}_l(t)}\frac{\partial \underline{z}_l(t)}{\partial t}\frac{e^{-gt}}{w}\int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l}dz\end{aligned}$$

Similarly for high-skilled,

$$f_h^d(w,t) = 1-\theta-\left[q\int_0^{\min\{\hat{z},\underline{z}_h(t)\}} \frac{z^{\lambda_l}}{\bar{A}_l}dz + (1+q)\int_{\min\{\hat{z},\underline{z}_h(t)\}}^{\hat{z}} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right]/w.$$

Implying,

$$\begin{aligned}\frac{\partial f_h^d(w,t)}{\partial t} &= g\frac{e^{-gt}}{w}\left[q\int_0^{\min\{\hat{z},\underline{z}_h(t)\}} \frac{z^{\lambda_l}}{\bar{A}_l}dz + (1+q)\int_{\min\{\hat{z},\underline{z}_h(t)\}}^{\hat{z}} \frac{z^{\lambda_l}}{\bar{A}_l}dz\right] \\ &\quad + \frac{e^{-gt}}{w}\frac{\underline{z}(t)^{\lambda_l+1}}{\bar{A}_l}\frac{1}{\underline{z}(t)}\frac{\partial \underline{z}(t)}{\partial t}\mathbf{1}(\hat{z} > \underline{z}_h(t)),\end{aligned}$$

where $\mathbf{1}(\hat{z} > \underline{z}_h(t))$ is an indicator function taking the value 1 if $\hat{z} > \underline{z}_h(t)$.
Thus,

$$\frac{\partial f^d(w,t)}{\partial t} = (1-f)\frac{\partial f_l^d(w,t)}{\partial t} + f\frac{\partial f_h^d(w,t)}{\partial t}$$

$$
\begin{aligned}
&= g\frac{e^{-gt}}{w}\left[\begin{array}{c}(1-f)\left(q\int_0^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz+\int_0^{\underline{z}_l(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right)\\ f\left(q\int_0^{\min\left\{\hat{z},\underline{z}_h(t)\right\}}\frac{z^{\lambda_l}}{\bar{A}_l}dz+(1+q)\int_{\max\{\hat{z},\bar{z}_h(t)\}}^{\bar{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right)\end{array}\right]\\
&\quad-\frac{\lambda_l+1}{\underline{z}_l(t)}\frac{\partial\underline{z}_l(t)}{\partial t}\frac{e^{-gt}}{w}\int_0^{\underline{z}_l(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz+\frac{e^{-gt}}{w}\frac{\underline{z}(t)^{\lambda_l+1}}{\bar{A}_l}\frac{1}{\underline{z}(t)}\frac{\partial\underline{z}(t)}{\partial t}\mathbf{1}\left(\hat{z}>\underline{z}_h(t)\right)
\end{aligned}
$$

Using that $\frac{\partial\underline{z}_h(t)}{\partial t}>0$,

$$
\begin{aligned}
&\geq g\frac{e^{-gt}}{w}\left[\begin{array}{c}(1-f)\left(q\int_0^{\hat{z}}\frac{z^{\lambda_l}}{\bar{A}_l}dz+\int_0^{\underline{z}_l(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right)\\ f\left(q\int_0^{\min\left\{\hat{z},\underline{z}_h(t)\right\}}\frac{z^{\lambda_l}}{\bar{A}_l}dz+(1+q)\int_{\max\{\hat{z},\bar{z}_h(t)\}}^{\bar{z}_h(t)}\right)\end{array}\right]\\
&\quad-\frac{\lambda_l+1}{\underline{z}_l(t)}\frac{\partial\underline{z}_l(t)}{\partial t}\frac{e^{-gt}}{w}\int_0^{\underline{z}_l(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\\
&\geq 0
\end{aligned}
$$

where the last inequality follows from our sufficient condition, condition (12). ■

Proof of Proposition 7. In this proposition, the price of goods and market services were assumed fixed in order to concentrate on the effect of changes in the opportunity cost of time for high-skilled workers. This amounts to holding fixed the cost of high-skilled labor when pricing market goods and services, i.e., $p_m(z)=\min\left\{z^{\lambda_l}/\bar{A}_l,\bar{w}z^{\lambda_h}/A_h\right\}$ and $p_s(z)=(1+q)\,p_m(z,t)$, where $\bar{w}$ denotes the cost of high-skilled labor implicit in the pricing of goods and market services. We use w to denote the opportunity cost of time faced a high-skilled individual. The first-order conditions for a high-skilled individual imply

$$
\frac{1-\gamma}{\gamma}=\begin{cases}\frac{w\underline{z}_h^{\lambda_l}/A_l-\underline{z}_h^{\lambda_l}/A_l}{(1+q)\bar{z}_h^{\lambda_l}/A_l}\text{ if }\bar{z}_h<1\\ \frac{w\underline{z}_h^{\lambda_l}/A_l-\underline{z}_h^{\lambda_l}/A_l}{(1+q)\bar{w}\bar{z}_h^{\lambda_h}/A_h}\text{ if }\underline{z}_h\leq\max\{1,\hat{z}\}<\bar{z}_h\\ \frac{w\underline{z}_h^{\lambda_l}/A_l-\bar{w}\underline{z}_h^{\lambda_h}/A_h}{(1+q)\bar{w}\bar{z}_h^{\lambda_h}/A_h}\text{ if }\max\{1,\hat{z}\}<\underline{z}_h<\bar{z}_h\end{cases}
$$

Differentiating with respect to the opportunity cost (w) and rearranging yields

$$
\frac{\partial\bar{z}_h}{\partial w}\frac{1}{\bar{z}_h}=\begin{cases}\frac{\partial\underline{z}_h}{\partial w}\frac{1}{\underline{z}_h}+\frac{1}{\lambda_l}\frac{1}{w-1}\text{ if }\bar{z}_h<1\\ \frac{\lambda_l}{\lambda_h}\frac{\partial\underline{z}_h}{\partial w}\frac{1}{\underline{z}_h}+\frac{1}{\lambda_h}\frac{1}{w-1}\text{ if }\underline{z}_h\leq\max\{1,\hat{z}\}<\bar{z}_h\\ \frac{\frac{\lambda_l}{\lambda_h}w\underline{z}_h^{\lambda_l}/A_l-\bar{w}\underline{z}_h^{\lambda_h}/A_h}{w\underline{z}_h^{\lambda_l}/A_l-\bar{w}\underline{z}_h^{\lambda_h}/A_h}\frac{\partial\underline{z}_h}{\partial w}\frac{1}{\underline{z}_h}+\frac{1}{\lambda_h}\frac{\underline{z}_h^{\lambda_l}/A_l}{w\underline{z}_h^{\lambda_l}/A_l-\bar{w}\underline{z}_h^{\lambda_h}/A_h}\\ \text{if }\max\{1,\hat{z}\}<\underline{z}_h<\bar{z}_h\end{cases}\tag{22}
$$

The budget constraint of high-skilled individuals is

$$
q\int_0^{\underline{z}_h}p_m(z)\,dz+\int_{\underline{z}_h}^{\bar{z}_h}p_s(z)\,dz=w\left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right)
$$

where $p_m(z) = \min\left\{\frac{z^{\lambda_l}}{A_l}, \bar{w}\frac{z^{\lambda_l}}{A_h}, \bar{w}\frac{z^{\lambda_h}}{A_h}\right\}$ and $p_s(z) = (1+q)\,p_m(z)$. Similarly, differentiating with respect to w obtains

$$\left[w\frac{\underline{z}_h^{\lambda_l}}{A_l} - p_m(\underline{z}_h)\right]\frac{\partial \underline{z}_h}{\partial w} + p_s(\bar{z}_h)\frac{\partial \bar{z}_h}{\partial w} = \left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right)$$

We present the analysis for the case $\bar{z}_h < 1$, but the argument that follows can be mirrored for the other two cases in equation (22). Using (22) (under the assumption $\bar{z}_h < 1$) to substitute in for $\frac{\partial \bar{z}_h}{\partial w}$ into the differentiated budget constraint yields

$$\begin{aligned} & \left[w\frac{\underline{z}_h^{\lambda_l}}{A_l} - p_m(\underline{z}_h)\right]\frac{\partial \underline{z}_h}{\partial w} + p_s(\bar{z}_h)\,\bar{z}_h\left[\frac{\partial \underline{z}_h}{\partial w}\frac{1}{\underline{z}_h} + \frac{1}{\lambda_l}\frac{1}{w-1}\right] \\ = & \left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right) \end{aligned}$$

$$\begin{aligned} & \left[w\frac{\underline{z}_h^{\lambda_l}}{A_l} - p_m(\underline{z}_h) + p_s(\bar{z}_h)\right]\frac{\partial \underline{z}_h}{\partial w} \\ = & \left[\left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right) - p_s(\bar{z}_h)\,\bar{z}_h\frac{1}{\lambda_l}\frac{1}{w-1}\right] \end{aligned}$$

Note that $p_s(\bar{z}_h)\,\bar{z}_h\frac{1}{\lambda_l}\frac{1}{w-1} = \frac{\bar{w}}{w-1}\frac{\lambda_l+1}{\lambda_l}(1+q)\int_0^{\bar{z}_h}\frac{z^{\lambda_l}}{A_h}dz$. Substituting this in yields

$$\begin{aligned} & \left[w\frac{\underline{z}_h^{\lambda_l}}{A_l} - p_m(\underline{z}_h) + p_s(\bar{z}_h)\right]\frac{\partial \underline{z}_h}{\partial w} \\ = & \left[\left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right) - \frac{\bar{w}}{w-1}\frac{\lambda_l+1}{\lambda_l}(1+q)\int_0^{\bar{z}_h}\frac{z^{\lambda_l}}{A_h}dz\right] \\ < & \; 0 \end{aligned}$$

where the last inequality follows from $\frac{\bar{w}}{w-1}\frac{\lambda_l+1}{\lambda_l} > 1$, $\bar{w} = w$, and

$$\begin{aligned} w\left(1-\theta-\int_0^{\underline{z}_h}\frac{z^{\lambda_l}}{A_l}dz\right) &= (1+q)\int_{\underline{z}_h}^{\bar{z}_h}\min\left\{\frac{z^{\lambda_l}}{A_l}, \bar{w}\frac{z^{\lambda_l}}{A_h}\right\}dz \\ &\quad + q\int_0^{\underline{z}_h}\min\left\{\frac{z^{\lambda_l}}{A_l}, \bar{w}\frac{z^{\lambda_l}}{A_h}\right\}dz \\ &\leq (1+q)\int_0^{\bar{z}_h}\bar{w}\frac{z^{\lambda_l}}{A_h}dz \end{aligned}$$

The first line is the budget constraint for high-skilled workers, and actual market expenditures must be less than market expenditures when all output is purchased as market services using high-skilled labor. ■

Proof of Proposition 8. Taking as given the quantity and price of high-skilled labor, $f(t) = f$ and $w(t) = w > \bar{A}_h/\bar{A}_l$ (therefore, allocations will only be a function of time through technology), recalling that $\hat{z} < \underline{z}_l(t)$**,** and using $\underline{z}_h(t) < \hat{z}$, the quantities consumer of services produced with low ($C_{s|l}(t)$) and high-skill ($C_{s|h}(t)$) labor equal

$$C_{s|l}(t) = f\int_{\underline{z}_h(t)}^{\hat{z}} p_s(z)dz$$

and

$$C_{s|h}(t) = f\int_{\hat{z}}^{\bar{z}_h(t)} p_s(z)dz + (1-f)\int_{\underline{z}_l(t)}^{\bar{z}_l(t)} p_s(z)dz.$$

Differentiating with respect to time

$$\frac{\partial C_{s|l}(t)}{\partial t} = -fp_s(\underline{z}_h(t))\frac{\partial \underline{z}_h(t)}{\partial t} < 0$$

and

$$\begin{aligned}\frac{\partial C_{s|h}(t)}{\partial t} &= fp_s(\bar{z}_h(t))\frac{\partial \bar{z}_h(t)}{\partial t} \\ &\quad + (1-f)\frac{w}{\bar{A}_h}\left[\bar{z}_l(t)^{\lambda_h+1}\frac{1}{\bar{z}_l(t)}\frac{\partial \bar{z}_l(t)}{\partial t} - \underline{z}_l(t)^{\lambda_h+1}\frac{1}{\underline{z}_l(t)}\frac{\partial \underline{z}_l(t)}{\partial t}\right] \\ &> 0,\end{aligned}$$

where these inequalities use $\frac{1}{\bar{z}_h(t)}\frac{\partial \bar{z}_h(t)}{\partial t} > 0$ and $\frac{1}{\bar{z}_l(t)}\frac{\partial \bar{z}_l(t)}{\partial t} > \frac{1}{\underline{z}_l(t)}\frac{\partial \underline{z}_l(t)}{\partial t} > 0$ from Proposition 5. ■

In order to prove Proposition 9 we first prove a lemma stating that the share of services produced with high-skill labor in total service consumption is greater than the share of manufacturing goods produced with high-skilled labor in total manufacturing consumption, provided $w > \bar{A}_h/\bar{A}_l$ and $\bar{z}_h > \hat{z}$.

Lemma 12 *If $w > \bar{A}_h/\bar{A}_l$, then $C_{s|h}/C_s > C_{m|h}/C_m$.*

Proof. The condition $w > \bar{A}_h/\bar{A}_l$ guarantees that there is a strict sorting of skills into the production of different wants, with low-skill workers producing goods and services of complexity below $\hat{z}$, and high-skill workers producing the most complex goods and services, $z \geq \hat{z}$. This condition also guarantees that $\bar{z}_h > \hat{z}$, since otherwise, we would have that there is no market demand for high-skilled labor, a clear contradiction of $w > \bar{A}_h/\bar{A}_l$.
The share of services produced with high-skilled labor in total services equals

$$\frac{C_{s|h}}{C_s} = \frac{(1-f)\int_{\max\{\hat{z},\underline{z}_l\}}^{\max\{\hat{z},\bar{z}_l\}} p(z)\,dz + f\int_{\max\{\hat{z},\underline{z}_h\}}^{\bar{z}_h} p(z)\,dz}{(1-f)\int_{\underline{z}_l}^{\bar{z}_l} p(z)\,dz + f\int_{\underline{z}_h}^{\bar{z}_h} p(z)\,dz} \tag{23}$$

where $p(z) = \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w\frac{z^{\lambda_h}}{\bar{A}_h}\right\}$. Similarly, the share of manufactured goods produced with high-skilled labor in total manufacturing consumption equals

$$\frac{C_{m|h}}{C_m} = \frac{(1-f)\int_{\hat{z}}^{\max\left\{\hat{z},\underline{z}_l\right\}} p(z)\,dz + f\int_{\hat{z}}^{\underline{z}_h} p(z)\,dz}{(1-f)\int_0^{\bar{z}_l} p(z)\,dz + f\int_0^{\bar{z}_h} p(z)\,dz}.$$

There are three cases to consider: i) $\hat{z} < \underline{z}_h < \underline{z}_l$, ii) $\underline{z}_h < \hat{z} < \underline{z}_l \leq \bar{z}_l < \bar{z}_h$, and iii) $\underline{z}_h < \underline{z}_l = \bar{z}_l < \hat{z} < \bar{z}_h$. In case i) we get $\frac{C_{s|h}}{C_s} = 1 > \frac{C_{m|h}}{C_m}$, while in case iii) we get $\frac{C_{s|h}}{C_s} > 0 = \frac{C_{m|h}}{C_m}$. In case ii), $\underline{z}_h < \hat{z} < \underline{z}_l < \bar{z}_l < \bar{z}_h$ (we focus on the case $\underline{z}_l < \bar{z}_l$, a similar argument holds for the case $\underline{z}_l = \bar{z}_l$), we obtain

$$\begin{aligned}
&\frac{C_{s|h}}{C_s} - \frac{C_{m|h}}{C_m} \\
&= \frac{C_{s|h}\left(C_{m|l} + C_{m|h}\right) - C_{m|h}\left(C_{s|l} + C_{s|h}\right)}{C_s C_m} \\
&= \frac{C_{s|h}C_{m|l} - C_{m|h}C_{s|l}}{C_s C_m}
\end{aligned}$$

$$\begin{aligned}
= \quad & \frac{1}{C_s C_m}\left\{\left[(1-f)\int_{\underline{z}_l}^{\bar{z}_l} p(z)\,dz + f\int_{\hat{z}}^{\bar{z}_h} p(z)\,dz\right]\right. \\
& \left[(1-f)\int_0^{\hat{z}} p(z)\,dz + f\int_0^{\underline{z}_h} p(z)\,dz\right] \\
& \left. -(1-f)\int_{\hat{z}}^{\underline{z}_l} p(z)\,dz f\int_{\underline{z}_h}^{\hat{z}} p(z)\,dz\right\}.
\end{aligned}$$

Using that $\int_{\hat{z}}^{\bar{z}_h} p(z)\,dz = \int_{\hat{z}}^{\underline{z}_l} p(z)\,dz + \int_{\underline{z}_l}^{\bar{z}_l} p(z)\,dz + \int_{\bar{z}_l}^{\bar{z}_h} p(z)\,dz$ and $\int_0^{\hat{z}} p(z)\,dz = \int_0^{\underline{z}_h} p(z)\,dz + \int_{\underline{z}_h}^{\hat{z}} p(z)\,dz$,

$$\begin{aligned}
= \quad & \frac{1}{C_s C_m}\left\{\left[\int_{\underline{z}_l}^{\bar{z}_l} p(z)\,dz + f\int_{\bar{z}_l}^{\bar{z}_h} p(z)\,dz + f\int_{\hat{z}}^{\underline{z}_l} p(z)\,dz\right]\right. \\
& \left[\int_0^{\underline{z}_h} p(z)\,dz + (1-f)\int_{\underline{z}_h}^{\hat{z}} p(z)\,dz\right] \\
& \left. -(1-f)\int_{\hat{z}}^{\underline{z}_l} p(z)\,dz f\int_{\underline{z}_h}^{\hat{z}} p(z)\,dz\right\}.
\end{aligned}$$

Cancelling terms,

$$
\begin{aligned}
&= \frac{1}{C_s C_m}\left\{\left[\int_{\underline{z}_l}^{\bar{z}_l} p(z)\,dz + f\int_{\bar{z}_l}^{\bar{z}_h} p(z)\,dz\right]\right. \\
&\quad \left[\int_0^{\underline{z}_h} p(z)\,dz + (1-f)\int_{\underline{z}_h}^{\hat{z}} p(z)\,dz\right] \\
&\quad \left.+f\int_{\hat{z}}^{\underline{z}_l} p(z)\,dz \int_0^{\underline{z}_h} p(z)\,dz\right\} \\
&> 0.
\end{aligned}
$$

■

Proof of Proposition 9. The values of the time τ consumption baskets of services and manufactured goods, respectively, equal

$$P_s(t,\tau) = (1+q)\left[(1-f)\int_{\underline{z}_l(\tau)}^{\bar{z}_l(\tau)} e^{-gt} p(z, w(t))\,dz + f\int_{\underline{z}_h(\tau)}^{\bar{z}_h(\tau)} e^{-gt} p(z, w(t))\,dz\right]$$

and

$$P_m(t,\tau) = q\left[(1-f)\int_0^{\underline{z}_l(\tau)} e^{-gt} p(z, w(t))\,dz + f\int_0^{\underline{z}_h(\tau)} e^{-gt} p(z, w(t))\,dz\right]$$

where $p(z,w) = \min\left\{z^{\lambda_l}/\bar{A}_l, wz^{\lambda_h}/\bar{A}_h\right\}$. The evolution of a continuous time chain-weighted relative price index of service to manufacturing equals:

$$\frac{\partial}{\partial t}\left(\frac{P_s(t,\tau)}{P_m(t,\tau)}\right)\bigg|_{\tau=t} = \frac{P_s(t,t)}{P_m(t,t)}\left\{\frac{\partial P_s(t,\tau)/\partial t}{P_s(t,\tau)} - \frac{\partial P_m(t,\tau)/\partial t}{P_m(t,\tau)}\right\}\bigg|_{\tau=t}$$

Using that $\partial P_i(t,\tau)/\partial t = \frac{\partial w(t)}{\partial t}\frac{1}{w(\tau)}C_{i|h}(\tau)$ and $P_i(t,t) = C_i(t)$, $i = s, m$,

$$
\begin{aligned}
&= \frac{\partial w(t)/\partial t}{w(t)}\frac{P_s(t,t)}{P_m(t,t)}\left\{\frac{C_{s|h}}{C_s} - \frac{C_{m|h}}{C_m}\right\} \\
&> 0
\end{aligned}
$$

where the last inequality follows from the previous lemma. ■

Proof of Proposition 10

We proceed by establishing a series of lemmas.

Lemma 13 $\lim_{t\to\infty} e^{-gt}\underline{z}_h(t)^{\lambda_l+1} = 0$, *with* $\lim_{t\to\infty} \dot{\underline{z}}_h(t)/\underline{z}_h(t) = \frac{\lambda_h}{\lambda_l}\lim_{t\to\infty} \dot{\bar{z}}_h(t)/\bar{z}_h(t)$.

Proof. Rearranging the first-order conditions of a high-skilled individual (see equations 5 and 6) we obtain

$$(1+q)\frac{1-\gamma}{\gamma} = \frac{\bar{A}_h}{\bar{A}_l}\frac{\underline{z}_h(t)^{\lambda_l}}{\bar{z}_h(t)^{\lambda_h}} - \min\left\{\frac{\bar{A}_h}{\bar{A}_l}\frac{\underline{z}_h(t)^{\lambda_l}}{w(t)\,\bar{z}_h(t)^{\lambda_h}}, \frac{\underline{z}_h(t)^{\lambda_h}}{\bar{z}_h(t)^{\lambda_h}}\right\}.$$

Taking the limit as $t \to \infty$ and using that $\lim_{t\to\infty} \dot{\underline{z}}_h(t)/\underline{z}_h(t) \leq \lim_{t\to\infty} \dot{\bar{z}}_h(t)/\bar{z}_h(t)$

$$\lim_{t\to\infty} \frac{\underline{z}_h(t)^{\lambda_l}}{\bar{z}_h(t)^{\lambda_h}} = (1+q)\frac{1-\gamma}{\gamma}\frac{\bar{A}_l}{\bar{A}_h} + \lim_{t\to\infty} \min\left\{\frac{\underline{z}_h(t)^{\lambda_l}}{w(t)\,\bar{z}_h(t)^{\lambda_h}}, \frac{\bar{A}_l}{\bar{A}_h}\frac{\underline{z}_h(t)^{\lambda_h}}{\bar{z}_h(t)^{\lambda_h}}\right\} < \infty.$$

Hence:

$$\begin{aligned}
\lim_{t\to\infty} \dot{\underline{z}}_h(t)/\underline{z}_h(t) &\leq \frac{\lambda_h}{\lambda_l}\lim_{t\to\infty} \dot{\bar{z}}_h(t)/\bar{z}_h(t) \\
&\leq \frac{\lambda_h}{\lambda_l}\frac{g}{\lambda_h+1} \\
&< \frac{g}{\lambda_l+1},
\end{aligned}$$

where the first (weak) inequality follows from expenditures growth being bounded by productivity growth (g). This implies that

$$\lim_{t\to\infty} e^{-gt}\underline{z}_h(t)^{\lambda_l+1} = 0.$$

■

Lemma 14 $\lim_{t\to\infty} \bar{z}_l(t)/\bar{z}_h(t) = 1$, *with* $\lim_{t\to\infty} \dot{\bar{z}}_e(t)/\bar{z}_e(t) = g/(\lambda_h+1)$, $e = l, h$.

Proof. In terms of the thresholds $\underline{z}_e$ and $\overline{z}_e$, the budget constraint for high-skilled, see equation (4), can be written as

$$\begin{aligned}
&\int_0^{\underline{z}_h(t)} e^{-gt}\min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w(t)\frac{z^{\lambda_h}}{\bar{A}_h}\right\}dz + \int_{\underline{z}_h(t)}^{\bar{z}_h(t)} e^{-gt}\min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w(t)\frac{z^{\lambda_h}}{\bar{A}_h}\right\}dz \\
= \;& w(t)\left[1-\theta(t) - e^{-gt}\int_0^{\underline{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right]
\end{aligned}$$

Now using $\hat{z}$ as the threshold in the min function, after simple integration and algebra, we have:

$$\begin{aligned}
&q\int_0^{\min\left\{\hat{z}(t),\underline{z}_h(t)\right\}} e^{-gt}\frac{z^{\lambda_l}}{\bar{A}_l}dz + q\int_{\min\left\{\hat{z}(t),\underline{z}_h(t)\right\}}^{\underline{z}_h(t)} e^{-gt}w(t)\frac{z^{\lambda_h}}{\bar{A}_h}dz + \\
&(1+q)\int_{\min\left\{\hat{z}(t),\underline{z}_h(t)\right\}}^{\hat{z}(t)} e^{-gt}\frac{z^{\lambda_l}}{\bar{A}_l}dz + (1+q)\int_{\max\left\{\hat{z}(t),\underline{z}_h(t)\right\}}^{\bar{z}_h(t)} e^{-gt}w(t)\frac{z^{\lambda_h}}{\bar{A}_h}dz \\
= \;& w(t)\left[1-\theta(t) - e^{-gt}\int_0^{\underline{z}_h(t)}\frac{z^{\lambda_l}}{\bar{A}_l}dz\right]
\end{aligned}$$

Integrating and simplifying using $w(t) = \bar{A}_h/\bar{A}_l \hat{z}(t)^{\lambda_l - \lambda_h}$ yields

$$\begin{aligned} & [\mathbf{1}(\hat{z} > \underline{z}_h) + q]\, e^{-gt} \frac{(\lambda_h - \lambda_l)\, \hat{z}(t)^{\lambda_l+1}}{\bar{A}_l\,(\lambda_l+1)\,(\lambda_h+1)} \\ & -e^{-gt}\left[[1 - \mathbf{1}(\hat{z} > \underline{z}_h)]\, \frac{w\,(t)\,\underline{z}_h\,(t)^{\lambda_h+1}}{\bar{A}_h\,(\lambda_h+1)} + \mathbf{1}(\hat{z} > \underline{z}_h) \frac{\underline{z}_h\,(t)^{\lambda_l+1}}{\bar{A}_l\,(\lambda_l+1)} \right] \\ & + (1+q)\, e^{-gt} w\,(t)\, \frac{\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h = (\lambda_h+1)} \\ = \;& w\,(t) \left[1 - \theta\,(t) - e^{-gt} \frac{\underline{z}_h\,(t)^{\lambda_l+1}}{\bar{A}_l\,(\lambda_l+1)} \right] \end{aligned}$$

where $\mathbf{1}(\hat{z} > \underline{z}_h)$ is an indicator taking the value 1 if $\hat{z}\,(t) \geq \underline{z}_h\,(t)$ and zero otherwise. Taking the limit as $t \to \infty$ and using $\lim_{t\to\infty} e^{-gt}\underline{z}_h\,(t)^{\lambda_h+1} \leq \lim_{t\to\infty} e^{-gt}\underline{z}_h\,(t)^{\lambda_l+1} = 0$, and $\hat{z}\,(t) < \bar{z}_l\,(t) \leq \bar{z}_h\,(t)$, we obtain

$$\begin{aligned} & \frac{[\mathbf{1}(\hat{z} > \underline{z}_h) + q]\,(\lambda_h - \lambda_l)\,\hat{Z}}{\bar{A}_h\,(\lambda_l+1)\,(\lambda_h+1)} + (1+q) \lim_{t\to\infty} \frac{e^{-gt}\bar{z}_h\,(t)^{\lambda_h+1}}{\bar{A}_h\,(\lambda_h+1)} \\ = \;& \left(1 - \lim_{t\to\infty} \theta\,(t)\right). \end{aligned} \tag{24}$$

where $\hat{Z} \equiv \lim_{t\to\infty} e^{-gt}\hat{z}\,(t)^{\lambda_h+1} < \infty$, since low-skilled labor is bounded. Since θ is bounded between 0 and $\overline{\theta} < 1$, this implies

$$0 < \bar{Z}_h \equiv \lim_{t\to\infty} e^{-gt}\bar{z}_h\,(t)^{\lambda_h+1} < \infty$$

and

$$\lim_{t\to\infty} \dot{\bar{z}}_h\,(t)\,/\bar{z}_h\,(t) = g/\,(\lambda_h+1)\,.$$

Finally, rearranging the indifference condition between low- and high-skilled individuals (see equation 8 in the main text)

$$(1-\gamma)\,\frac{\underline{z}_l\,(t)}{\bar{z}_h\,(t)} + \gamma \frac{\bar{z}_l\,(t)}{\bar{z}_h\,(t)} = (1-\gamma)\,\frac{\underline{z}_h\,(t)}{\bar{z}_h\,(t)} + \gamma,$$

taking the limit as $t \to \infty$, and using that $\lim_{t\to\infty} \dot{\underline{z}}_e\,(t)\,/\underline{z}_e\,(t) \leq g/\,(\lambda_l+1) < g/\,(\lambda_h+1) = \lim_{t\to\infty} \dot{\bar{z}}_h\,(t)\,/\underline{z}_h\,(t)$,

$$\lim_{t\to\infty} \frac{\bar{z}_l\,(t)}{\bar{z}_h\,(t)} = 1,$$

implying that

$$\lim_{t\to\infty} \dot{\bar{z}}_l\,(t)\,/\bar{z}_l\,(t) = g/\,(\lambda_h+1)\,.$$

■

For later use, we define $\bar{Z}_l = \lim_{t\to\infty} e^{-gt}\bar{z}_l\,(t)^{\lambda_h+1}$, satisfying $0 < \bar{Z}_l < \infty$.

Lemma 15 $\lim_{t\to\infty} w(t) \equiv \bar{w} < \infty$.

Proof. In terms of the thresholds $\underline{z}_l$ and $\overline{z}_l$, the budget constraint, see equation (4), can be written as

$$\begin{aligned} & \int_0^{\underline{z}_l(t)} e^{-gt} \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w(t)\frac{z^{\lambda_h}}{\bar{A}_h}\right\} dz + \int_{\underline{z}_l(t)}^{\bar{z}_l(t)} e^{-gt} \min\left\{\frac{z^{\lambda_l}}{\bar{A}_l}, w(t)\frac{z^{\lambda_h}}{\bar{A}_h}\right\} dz \\ = \;& 1 - e^{-gt}\int_0^{\underline{z}_l(t)} \frac{z^{\lambda_l}}{\bar{A}_l} dz \end{aligned}$$

Recalling that $\hat{z}(t) \leq \underline{z}_l(t)$, integrating, and substituting in $\hat{Z}$ yields:

$$\begin{aligned} & q\frac{(\lambda_h - \lambda_l)\hat{Z}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + (1+q)\lim_{t\to\infty} w(t)\frac{e^{-gt}\bar{z}_l(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} \\ = \;& 1 - \lim_{t\to\infty}\frac{e^{-gt}\underline{z}_l(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)} \leq 1 \end{aligned}$$

By Lemma 14, consumption of low-skilled individuals of services produced with high-skilled labor grows without bounds at the rate $g/(\lambda_h+1)$, i.e., $0 < \lim_{t\to\infty}\frac{e^{-gt}\bar{z}_l(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} = \bar{Z}_l < \infty$. Therefore, since labor income (r.h.s) is finite, expenditures (l.h.s.) must also be finite, hence $\lim_{t\to\infty} w(t) \equiv \bar{w} < \infty$. ■

Lemma 16 $\lim_{t\to\infty} e^{-gt}\underline{z}_l(t)^{\lambda_l+1} = 0$, *with* $\lim_{t\to\infty} \dot{\underline{z}}_l(t)/\underline{z}_l(t) = \frac{\lambda_h}{\lambda_l} g/(\lambda_h+1)$.

Proof. We first show that $\lim_{t\to\infty}\frac{\underline{z}_l(t)}{\bar{z}_l(t)} < 1$ by contradiction. The first order conditions of a low-skill individual imply

$$(1+q)\frac{1-\gamma}{\gamma} \geq \lim_{t\to\infty}\left\{\frac{\bar{A}_h}{\bar{A}_l w(t)}\frac{\underline{z}_l(t)^{\lambda_l}}{\bar{z}_l(t)^{\lambda_h}} - \min\left\{\frac{\bar{A}_h}{\bar{A}_l w(t)}\frac{\underline{z}_l(t)^{\lambda_l}}{\bar{z}_l(t)^{\lambda_h}}, \frac{\underline{z}_l(t)^{\lambda_h}}{\bar{z}_l(t)^{\lambda_h}}\right\}\right\}.$$

Assume $\lim_{t\to\infty}\frac{\underline{z}_l(t)}{\bar{z}_l(t)} = 1$, this becomes:

$$(1+q)\frac{1-\gamma}{\gamma} > \lim_{t\to\infty}\left\{\frac{\bar{A}_h}{\bar{A}_l w(t)}\bar{z}_l(t)^{\lambda_l-\lambda_h} - \min\left\{\frac{\bar{A}_h}{\bar{A}_l w(t)}\bar{z}_l(t)^{\lambda_l-\lambda_h}, 1\right\}\right\}.$$

Given the finite limiting wage $\overline{w}$ from the previous lemma, the first term of the r.h.s. is infinite since $\lambda_h < \lambda_l$, while the min function is finite, since the second term is finite. Thus, the infinite r.h.s. exceeds the finite l.h.s., leading to a contradiction. Thus, $\lim_{t\to\infty}\frac{\underline{z}_l(t)}{\bar{z}_l(t)} < 1$. We can then use the first-order condition at an interior solution:

$$(1+q)\frac{1-\gamma}{\gamma} = \frac{\bar{A}_h}{\bar{A}_l w(t)}\frac{\underline{z}_l(t)^{\lambda_l}}{\bar{z}_l(t)^{\lambda_h}} - \min\left\{\frac{\bar{A}_h}{\bar{A}_l w(t)}\frac{\underline{z}_l(t)^{\lambda_l}}{\bar{z}_l(t)^{\lambda_h}}, \frac{\underline{z}_l(t)^{\lambda_h}}{\bar{z}_l(t)^{\lambda_h}}\right\}.$$

Taking the limit as $t \to \infty$ and using that $\lim_{t\to\infty} \dot{\underline{z}}_l(t)/\underline{z}_l(t) < \lim_{t\to\infty} \dot{\bar{z}}_l(t)/\bar{z}_l(t)$ and $\lim_{t\to\infty} w(t) = \bar{w} < \infty$ the second term of the min goes to zero, hence

$$\lim_{t\to\infty} \frac{\underline{z}_l(t)^{\lambda_l}}{\bar{z}_l(t)^{\lambda_h}} = (1+q)\frac{1-\gamma}{\gamma}\frac{\bar{A}_l}{\bar{A}_h}\bar{w}.$$

Thus,

$$\begin{aligned}
\lim_{t\to\infty} \dot{\underline{z}}_l(t)/\underline{z}_l(t) &= \frac{\lambda_h}{\lambda_l}\lim_{t\to\infty} \dot{\bar{z}}_l(t)/\bar{z}_l(t) \\
&= \frac{\lambda_h}{\lambda_l}\frac{g}{\lambda_h+1} \\
&< \frac{g}{\lambda_l+1},
\end{aligned}$$

The second equality follows from Lemma 14, and the inequality follows from $\lambda_h < \lambda_l$. Hence,

$$\lim_{t\to\infty} e^{-gt}\underline{z}_l(t)^{\lambda_l+1} = 0.$$

■

Lemma 17 $\lim_{t\to\infty} f(t) = 1$.

Proof. By Lemma 15, $w(t) \to \overline{w}$. This implies $\hat{z} \to \bar{\hat{z}} < \infty$. Now demand for low-skilled labor is bounded above by:

$$(1+q)\int_0^{\hat{z}} e^{-gt} z^{\lambda_l} dz$$

Integrating this, the resource constraint ensures that demand cannot exceed supply. Hence

$$(1+q)\,e^{-gt}\frac{\hat{z}^{\lambda_l+1}}{\lambda_l+1} \geq (1-f)$$

As $t \to \infty$, the limit of the l.h.s. is zero, hence $f \to 1$, and $\theta \to \bar{\theta}$ ■

Lemma 18 $\overline{w} = 1/\left(1-\bar{\theta}\right)$.

Proof. Consider the limiting budget constraints for high- and low-skilled agents respectively.

$$\begin{aligned}
&\frac{(1+q)(\lambda_h-\lambda_l)\hat{Z}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + (1+q)\lim_{t\to\infty}\frac{e^{-gt}\bar{z}_h(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} \\
=\ &\left(1-\lim_{t\to\infty}\theta(t)\right).
\end{aligned}$$

and

$$\begin{aligned}
&q\frac{(\lambda_h-\lambda_l)\hat{Z}}{\bar{A}_h(\lambda_l+1)(\lambda_h+1)} + (1+q)\lim_{t\to\infty} w(t)\,\frac{e^{-gt}\bar{z}_l(t)^{\lambda_h+1}}{\bar{A}_h(\lambda_h+1)} \\
=\ &1-\lim_{t\to\infty}\frac{e^{-gt}\underline{z}_l(t)^{\lambda_l+1}}{\bar{A}_l(\lambda_l+1)}
\end{aligned}$$

We have already established that $\hat{Z} = 0, \lim_{t\to\infty} w(t) = \overline{w},$ and $\lim_{t\to\infty} e^{-gt} \underline{z}_l(t)^{\lambda_l+1}$, and $\lim_{t\to\infty} \bar{z}_h(t) = \lim_{t\to\infty} \bar{z}_l(t)$. Hence, the two equations imply:

$$\overline{w} = 1/\left(1-\bar{\theta}\right)$$

■

Lemma 19 $\lim_{t\to\infty} c_s(t) = 1$ *and* $\lim_{t\to\infty} y_s(t) = \frac{1}{1+q}$.
Proof. *This results follows from Lemmas 13, 14, and 16, and the fact that* $C_s(t) = (1+q)\,Y_s(t)$. ■

References

[1] Acemoglu, D. and V. Guerrieri. "Capital Deepening and Non-Balanced Economic Growth." *Journal of Political Economy*, 2008, 116 (3), 467-498

[2] Baumol, William J. "Macroeconomics of Unbalanced Growth: The Anatomy of the Urban Crisis," *The American Economic Review* 57 (1967) : 415-426.

[3] Becker, G. S. and Murphy, K. "Education and Consumption: The Effects of Education in the Household Compared to the Marketplace," *Journal of Human Capital*, 1 (Winter 2007): 9-35.

[4] Blagg, C.. "The History of Home Hemodialysis: A View from Seattle," *Home Hemodialysis International* 1: p 1-7, 1997.

[5] Blagg, C. "It's Time to Look at Home Hemodialysis in a New Light," in Hemodialysis Horizons: Patient Safety & Approaches to Reducing Errors, February 2006. http://www.aami.org/publications/hh/index.html.

[6] Bosworth, B. and J. Triplett. "Services Productivity in the United States: Griliches' Services Volume Revisited," in Hard-to-Measure Goods and Services: Essays in Honor of Zvi Griliches, eds. E. R. Berndt and C. M. Hulten, The University of Chicago Press, 2007.

[7] Buera, F. and Kaboski, J. "Scale and the Origins of Structural Change." mimeo, 2008

[8] Caselli, F. "Technological Revolutions", *American Economic Review*, 89 (March 1999):78-102

[9] Caselli, F. and Coleman, W. J., II, "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy*, 109 (June 2001): 584-616.

[10] Clark, C. *The Conditions of Economic Progress.* London: Macmillan, 1941.

[11] Cohen, D. and Soto, M. "Growth and human capital: Good data, good results" *Journal of Economic Growth*, 12(1), 2007, 51-76.

[12] Chenery H. *Structural Change and Development Policy.* Washington, D.C.: Oxford University Press, for the World Bank (1979).

[13] Chenery H. and M. Syrquin. *Patterns of Development.* Oxford University Press (1975).

[14] Clark, C. *The Conditions of Economic Progress.* London: Macmillan (1941).

[15] de Vries, J. "The Industrial Revolution and the Industrious Revolution." The Journal of Economic History 54 (2): 249-270 (1994).

[16] Echevarria, C. "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review* 38 (1997): 431-452.

[17] Foellmi, R. and J. Zweimuller. "Structural Change, Engel's Consumption Cycles and Kaldor's Facts of Economic Growth." *mimeo*, University of Zurich (2005).

[18] Fuchs, V. *The Service Economy.* New York: National Bureau of Economic Research (1968).

[19] Goldin, C. and Katz, L., 1999, "The Returns to Skill in the United States Across the Twentieth Century." NBER working paper WP 7126.

[20] Gollin, D., Parente, S. and Rogerson, R. "Farm Work, Home Work, and International Productivity Differences," *Review of Economic Dynamics*, 7 (October 2004): 827-850

[21] Greenwood, J., A. Seshadri and M. Yorukoglu "Engines of Liberation." *Review of Economic Studies*, Jan 2005, v.72, n.1: 109-134.

[22] Greenwood, J. and Yorukoglu, M., 1997. "1974," *Carnegie-Rochester Conference Series on Public Policy*, 46 (1997): 49-95

[23] Griliches, Zvi, ed. Output Measurement in the Service Sectors. National Bureau of Economic Research. Studies in Income and Wealth. Volume 56. Chicago and London: University of Chicago Press, 1992.

[24] Hall, R. and C. Jones. "The Value of Life and the Rise in Health Spending," *The Quarterly Journal of Economics,* 122 (February 2007): 39-72

[25] Juhn, C., Murphy, K. M., and Pierce, B. "Wage Inequality and the Rise in Returns to Skill." *Journal of Political Economy,* 101 (June 1993): 410-442

[26] Kaboski, J. 2004. "Supply Factors and the Mid-Century Fall in the Skill Premium." mimeo, 2004

[27] Kaboski, J. 2009. "Education, Sectoral Composition and Growth," *Review of Economic Dynamics*, 12 (January 2009): 168-182

[28] Katouzian, M. A. “The Development of the Service Sector: A New Approach”, *Oxford Economic Papers*, 22 (3): 362-82 (1970).

[29] Kongsamut, P., S. Rebelo, and D. Xie. “Beyond Balanced Growth,” *Review of Economic Studies* 68 (2001): 869-882.

[30] Kravis, I., A. Heston and R. Summers. “The Share of Services in Economic Growth,” in Global Econometrics: Essays in Honor of Lawrence R. Klein, eds. F. Gerard Adams and Bert G. Hickman, (1984).

[31] Kuznets, S. “Quantitative Aspects of the Economic Growth of Nations: II. Industrial Distribution of National Product and Labor Force.” *Economic Development and Cultural Change,* Vol. 5, No. 4, Supplement (Jul., 1957), pp. 1-111.

[32] Kuznets, S. “Modern Economic Growth: Findings and Reflections.” *The American Economic Review,* Vol. 63, No. 3 (Jun., 1973), pp. 247-258.

[33] Lebergott, S. *Consumer Expenditures: New Measures and Old Motives.* Princenton: Princeton University Press, 1996.

[34] Lee, D. and K. Wolpin. “Intersectoral Labor Mobility and the Growth of the Service Sector,” *Econometrica*, 47 (January, 2006), pp. 1-46.

[35] Locay, L. “Economic Development and the Division of Production between Households and Markets.” *Journal of Political Economy*, 98 (1990): 965-82.

[36] Maddison, A. “Growth and Slowdown in Advanced Capitalist Economies: Techniques of Quantitative Assessment,” *Journal of Economic Literature* 25 (1987): 649-698.

[37] Maddison, A. *The World Economy.* Paris, France : Development Centre of the Organisation for Economic Co-operation and Development, 2006.

[38] Martin, R. *National Income in the United States, 1799-1938.* National Industrial Conference Board., 1939

[39] Matsuyama, K. “A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade, *Journal of Political Economy* 108 (2000): 1093-1120.

[40] Matsuyama, K. “The Rise of Mass Consumption Societies,” *Journal of Political Economy* 110 (2002): 1035-1070.

[41] Matsuyama, K. “Structural Change”, forthcoming in: New Palgrave Dictionary of Economics, London, Macmillan (2005).

[42] Murphy, K., Shleifer, A., and R. Vishny “Income Distribution, Market Size, and Industrialization,” *Quarterly Journal of Economics* 104 (August 1989): 537-64.

[43] National Resources Planning Board, *Family Expenditures in the United States: Statistical Tables and Appendixes,* Washington: U.S. GPO, 1941, 209 pp.

[44] Ngai, R. and C. Pissarides. "Structural Change in a Multi-Sector Model of Growth," *American Economic Review* 97 (March 2007) 429-443.

[45] Ngai, R. and C. Pissarides. "Trends in Hours and Economic Growth." *Review of Economic Dynamics*, 11 (2). 2008: 239-256.

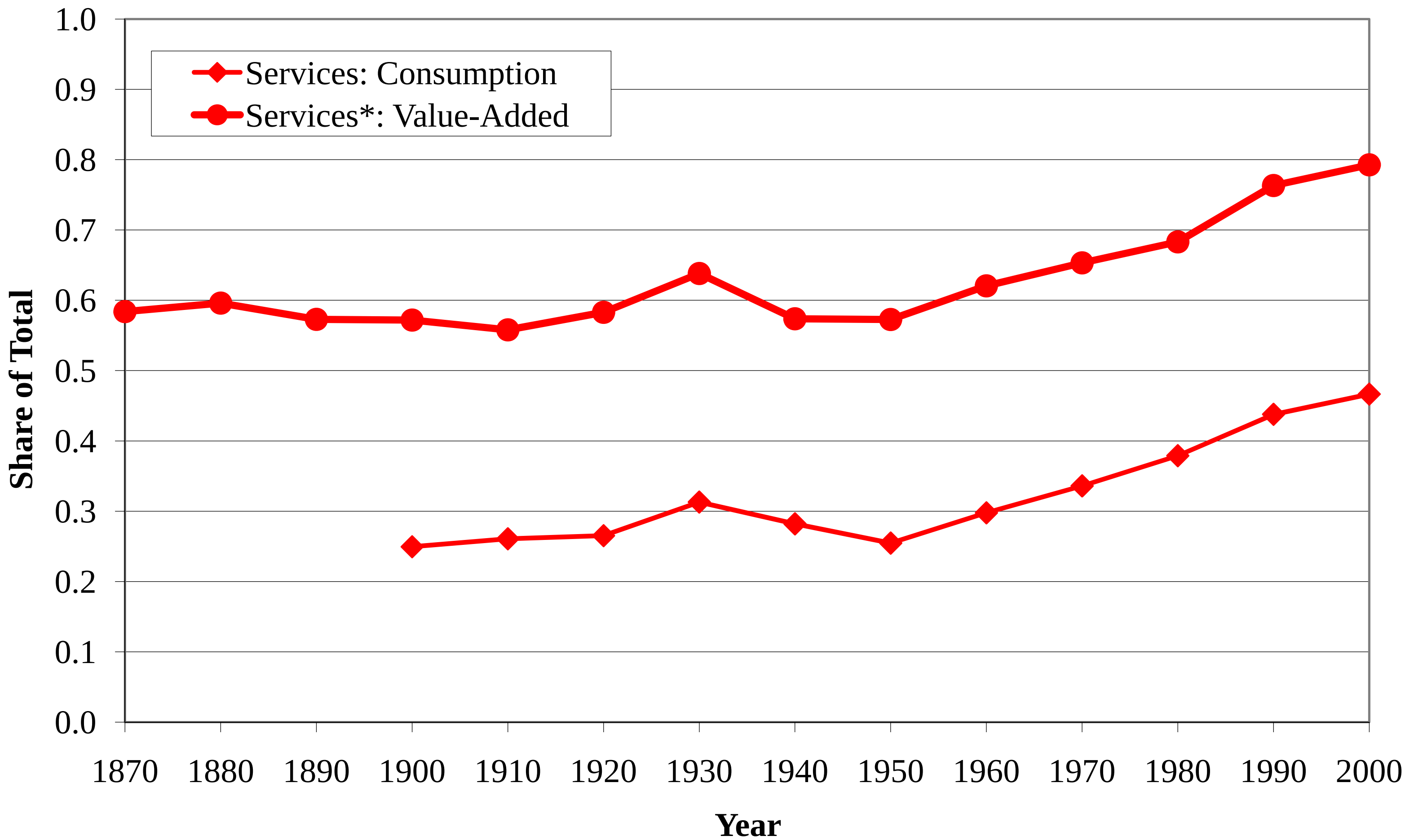
[46] Princiotta, N. and Bielick, S. "Homeschooling in the United States, 2003" National Center for Education Statistics, NCES 2006042, February 2006, 54 pp.

[47] Reid, M. *Economics of household production.* New York, J. Wiley & Sons, Inc., 1934

[48] Rogerson, R. "Structural Transformation and the Deterioration of European Labor Market Outcomes" *Journal of Political Economy*, 116 (2), 2008: 235-259

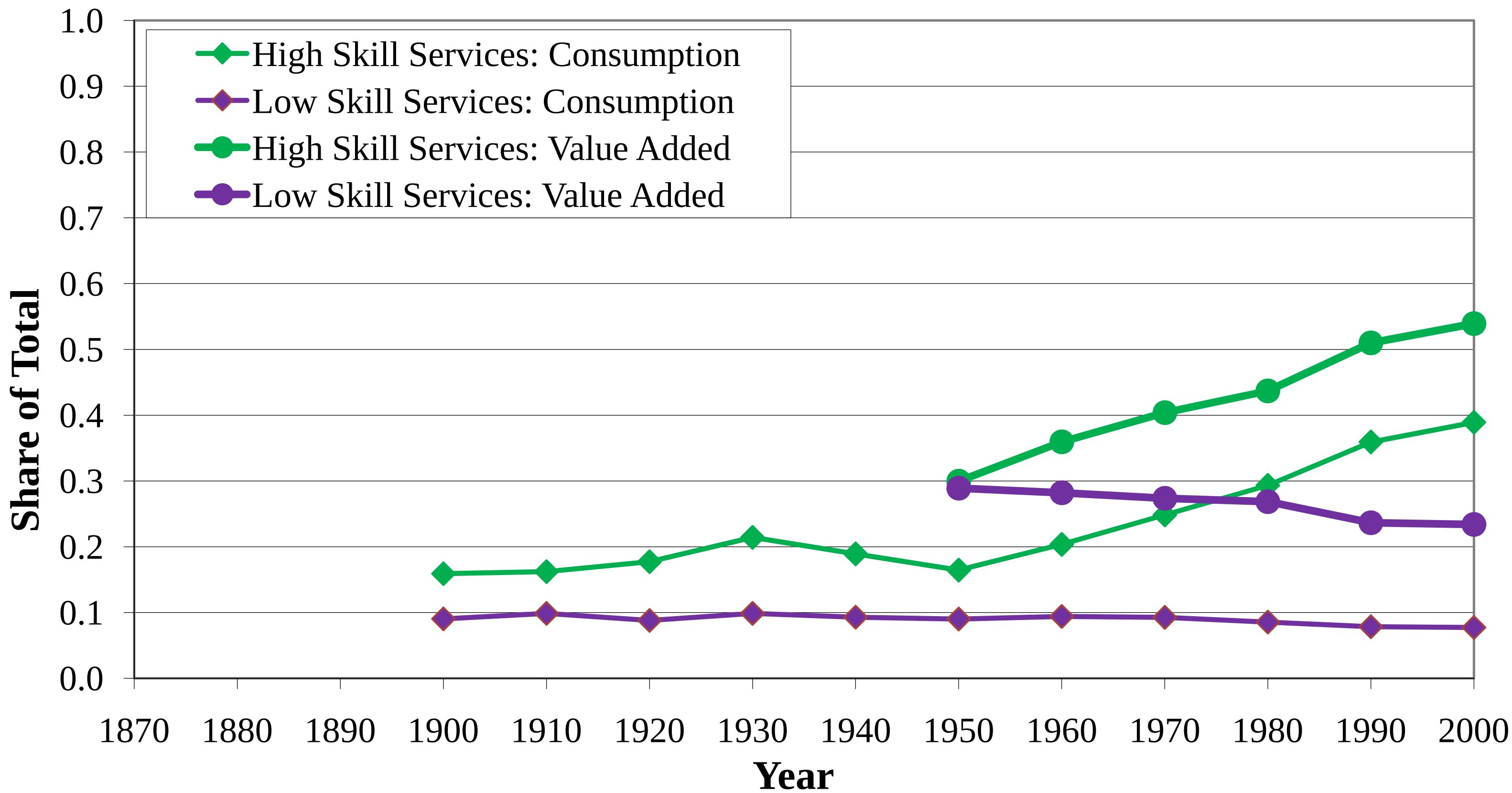
[49] Stigler, G. *Trends in Employment in the Service Industries.* National Bureau of Economic Analysis, 59, General Series.

[50] Zweimuller, J. "Schumpeterian Entrepreneurs Meet Engel's Law: The Impact of Inequality on Innovation-Driven Growth," *Journal of Economic Growth*, 5 (June 2000): 185-206



*Services includes services, retail and wholesale trade, public administration, utilities, and transportation.

Figure 1: Growth of Share of Services in Consumption and Output



*High-skill services include all industries with at least 12.5% of workers college-educated in 1940.

Figure 2: Growth of Low and High Skill Service Shares

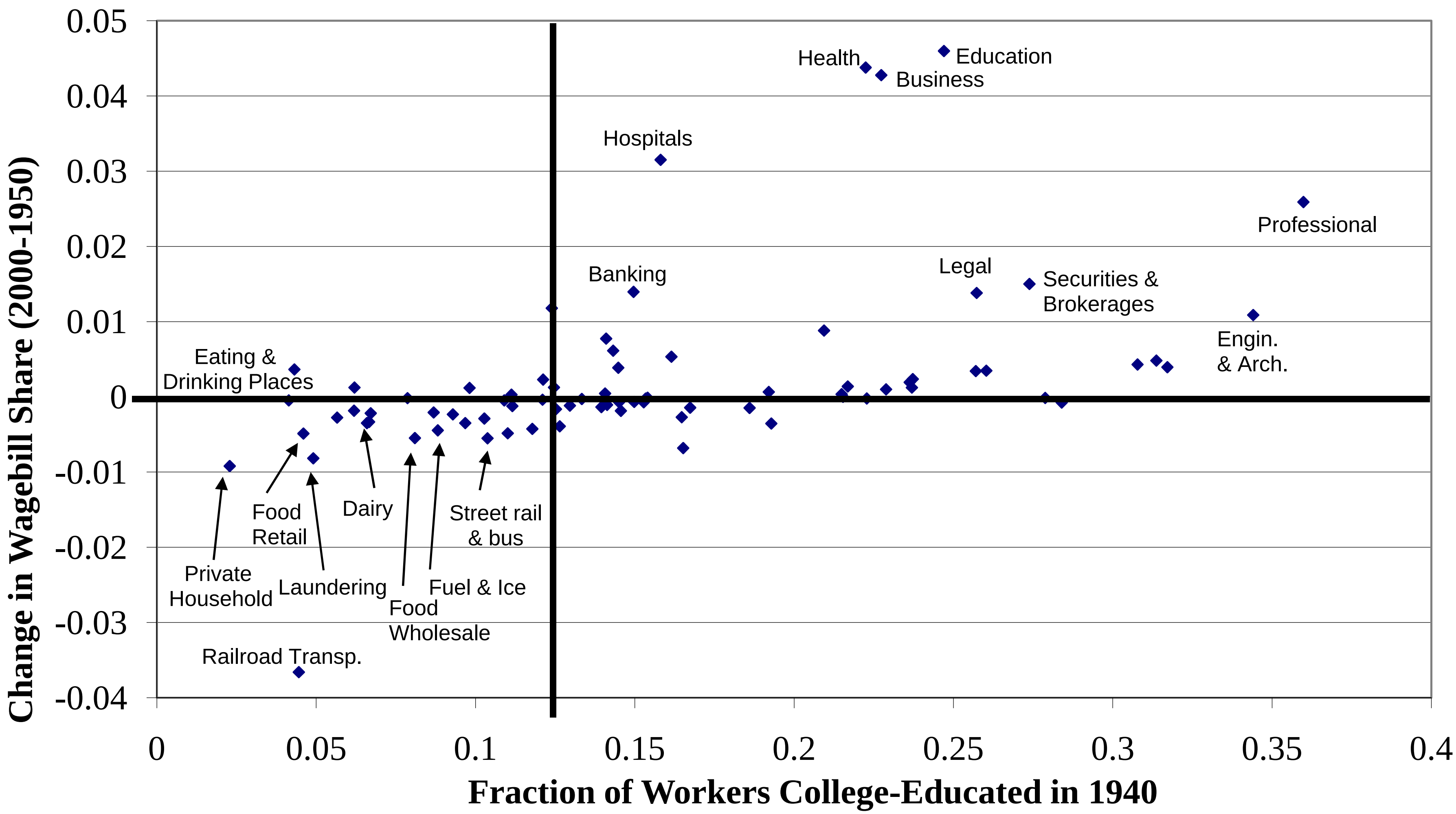


Figure 3: Growth vs. Skill Intensity of Disaggregate Service Industries

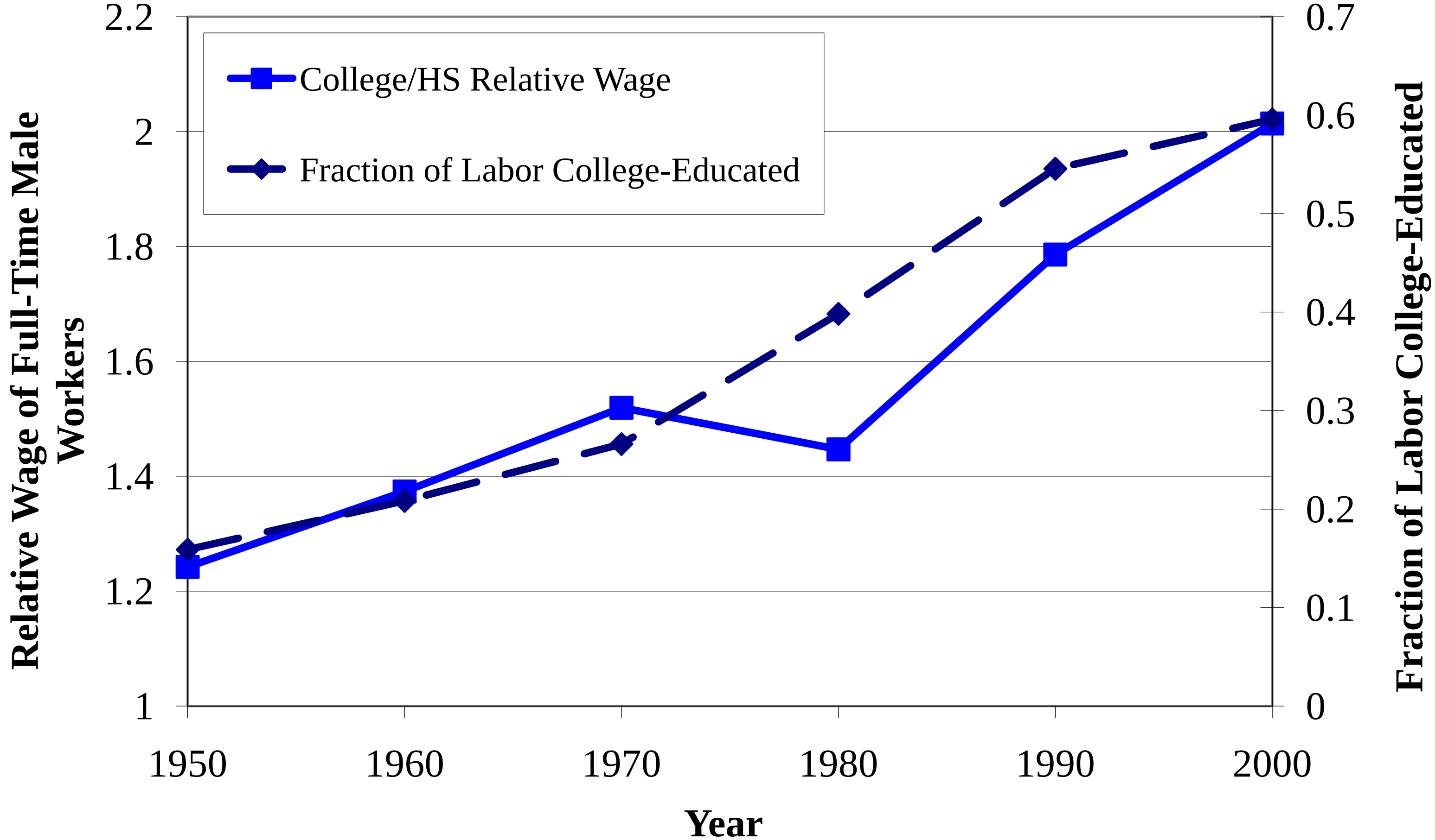


Figure 4: Growth of College Premium and Fraction College-Educated

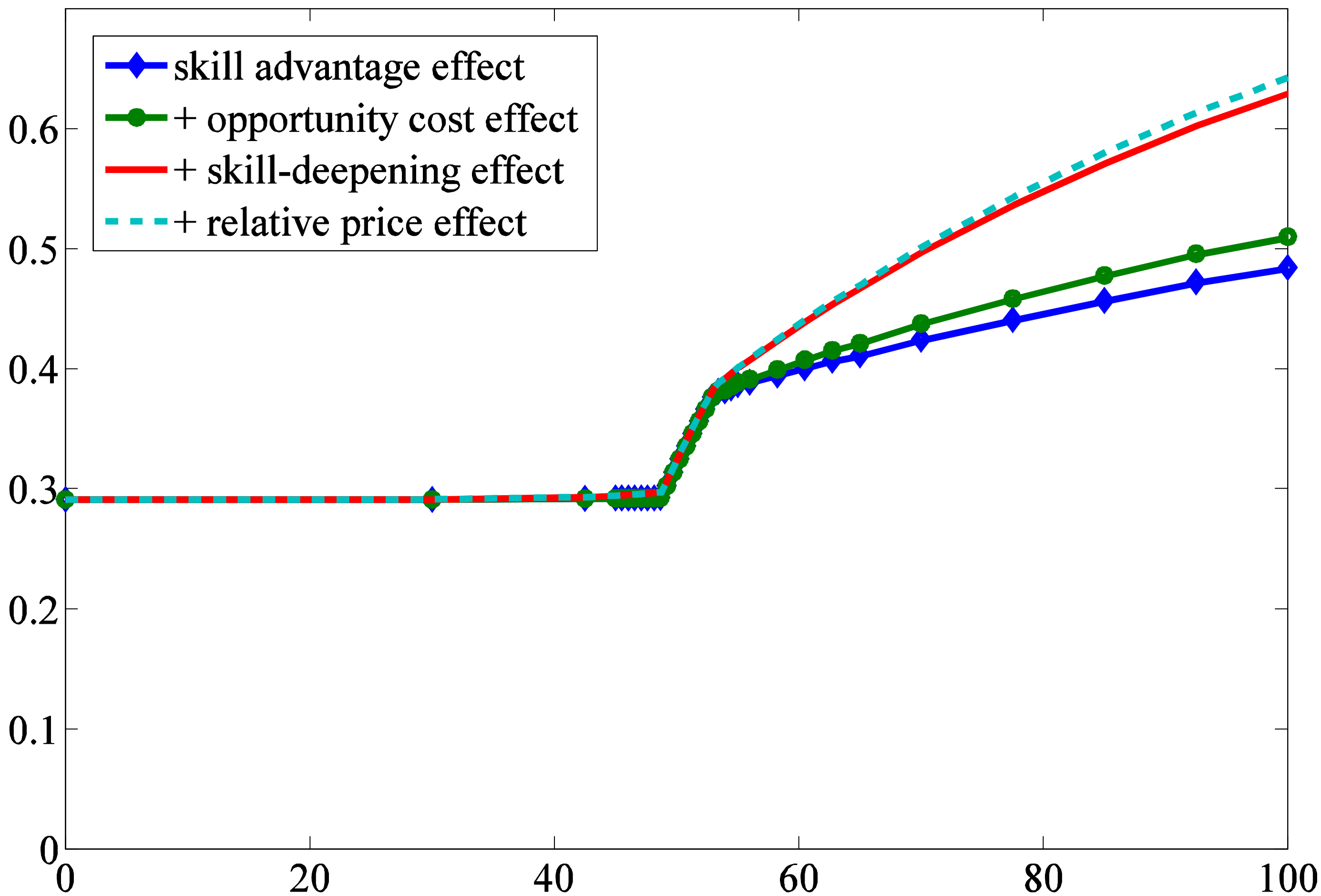


Figure 5: Decomposition of the Growth Service Consumption in the Model. See footnote 30 for a discussion of the parameter values used in the simulation.

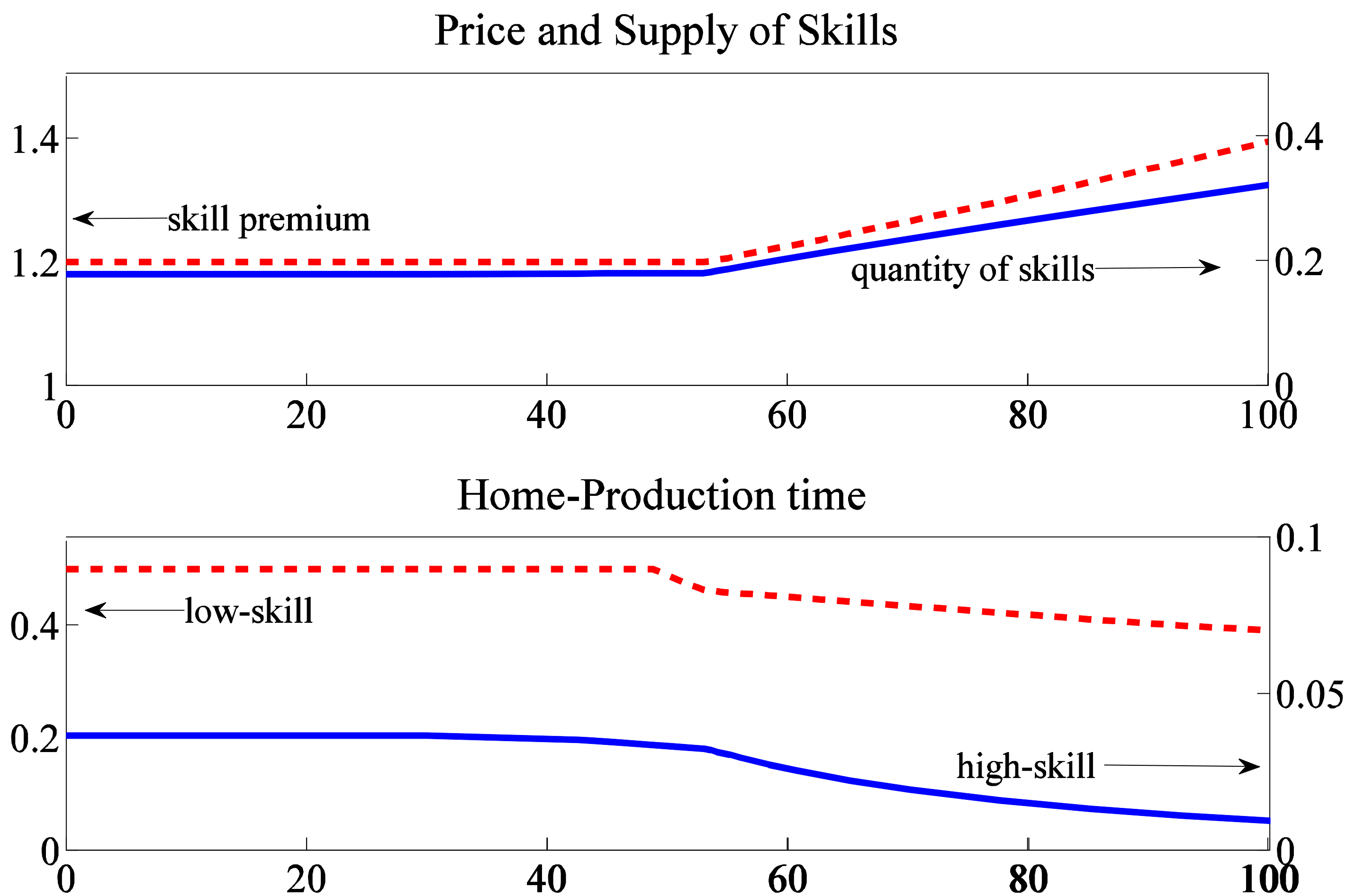


Figure 6: Simulation of the Quantity and Price of High-Skill Labor (upper panel), and Home Production Time (lower panel).

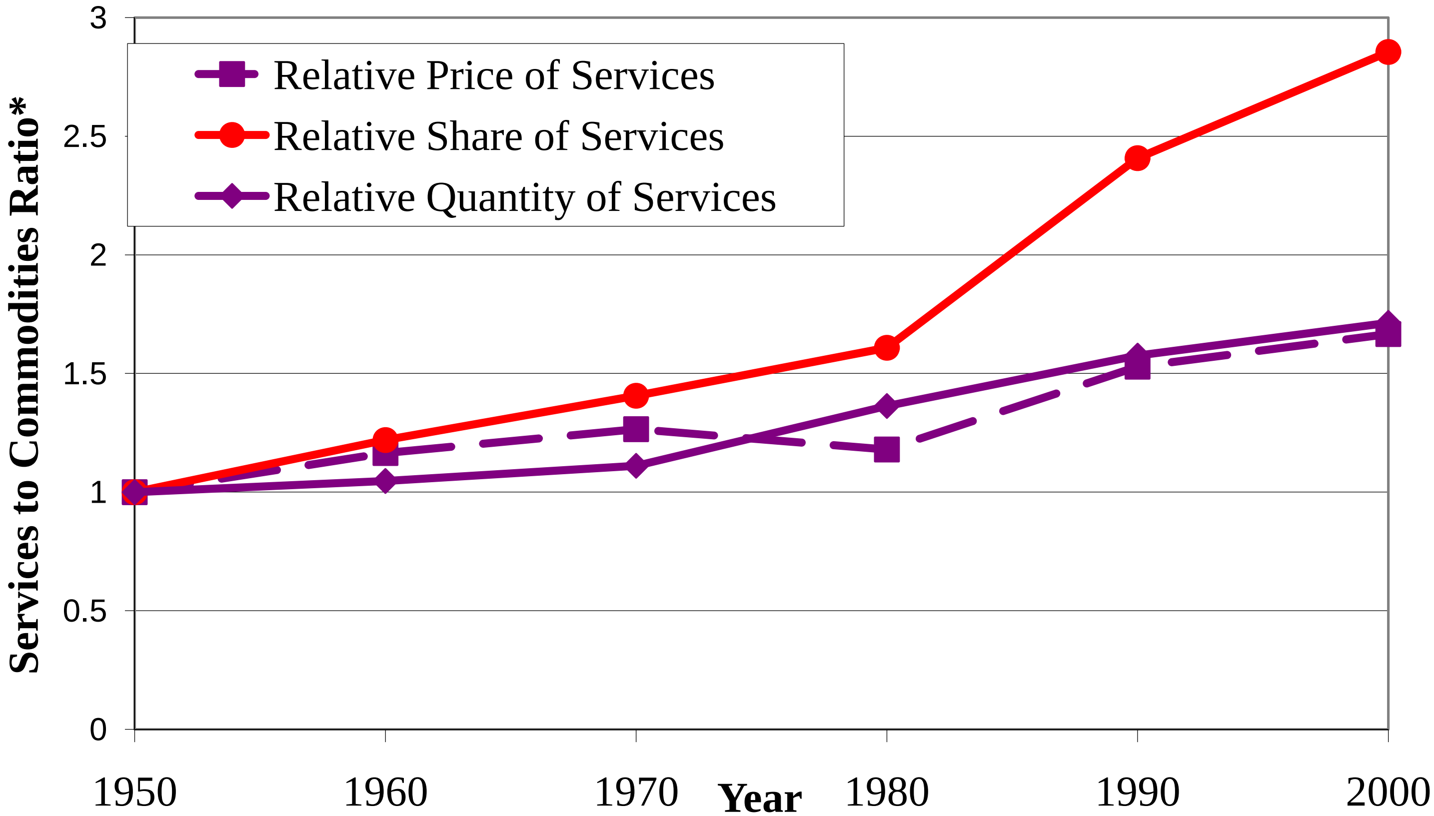


Figure 7: Growth of Relative Price and Relative Quantity of Services

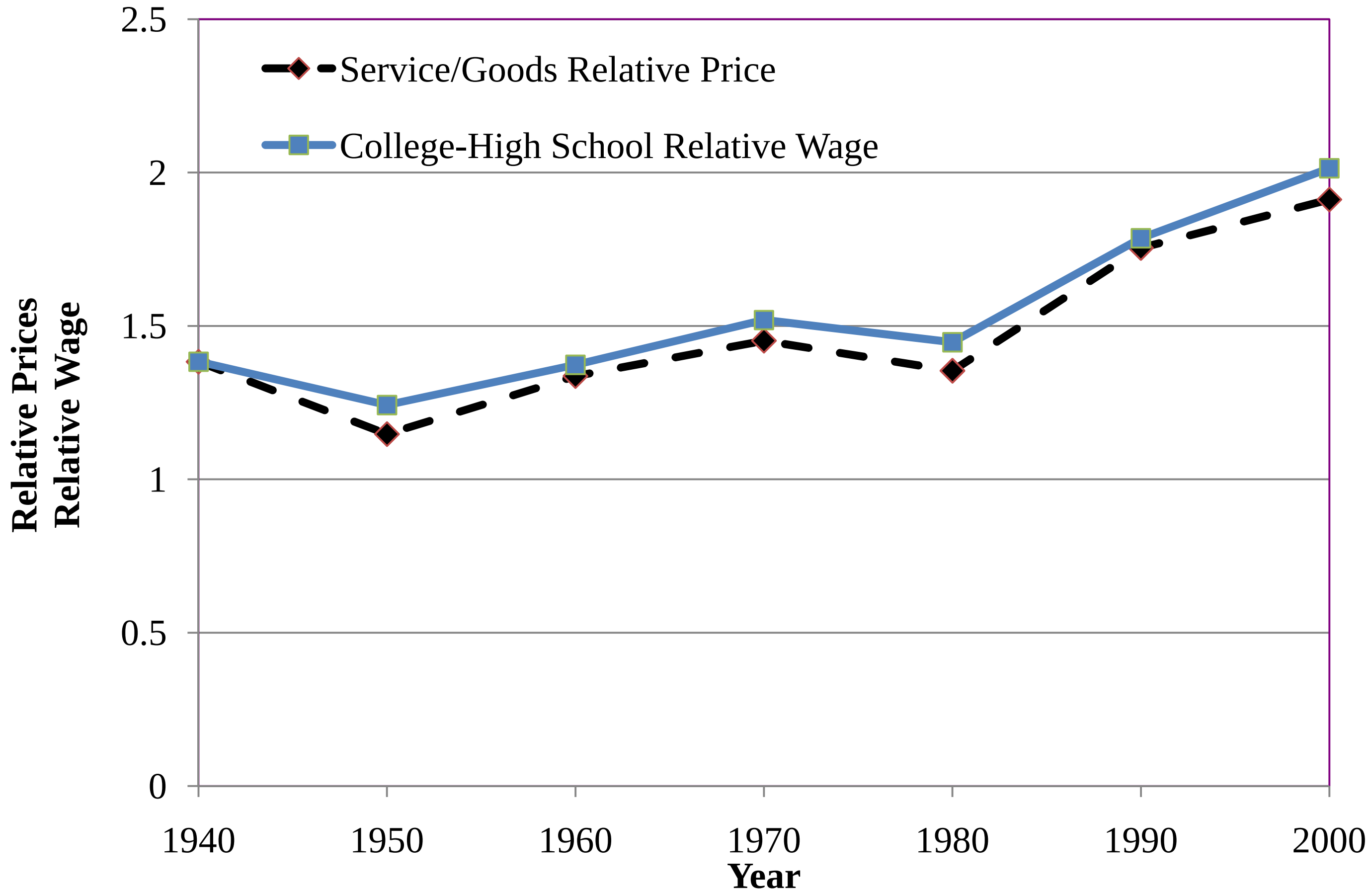


Figure 8: Correlation of Skill Premium and Relative Price of Services