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Department of Statistics

ROBUSTNESS OF RELIABILITY PREDICTIONS FOR A SERIES SYSTEM OF IDENTICAL COMPONENTS*

by

Clifford D. Leitao**
and
Harry O. Posten

Research Report No. 29

May 1967

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**Presently with the National Aeronautics and Space Administration, Wallops Island, Virginia.

1. INTRODUCTION

In the field of reliability analysis one is often faced with the problem of choosing a failure distribution to represent the pattern of failure for a particular component or subsystem. If the assumption of a uniform hazard rate can be justified, the appropriate failure density function is the familiar "exponential" failure density and reliability predictions may be made upon the determination or specification of a single parameter. Alternatively, since the "exponential" distribution does not allow for degradation effects over the period of use, one may prefer to use a failure distribution which exhibits a hazard rate that increases with age. An important class of failure densities with this property frequently used in reliability analysis is the Weibull family.[1] This family includes the exponential distribution as a special case but, in general, requires two parameters to identify the distribution instead of simply the "mean-time-to-failure" as in the "exponential" case. Therefore, to use a member of the Weibull family of failure densities one requires more knowledge than in the exponential case since, in a sense, one must also specify the type of degradation involved.

If sufficient failure data is available, the problem presents no difficulty, for the data may be used to specify whether the exponential

[1]The Weibull family allows for degradation effects only when its shape parameter, α, is greater than unity. When $\alpha = 1$ the hazard rate is constant and when $\alpha < 1$ the hazard rate is decreasing and the corresponding component would have the unusual property of improving reliability with increasing age. In practice $\alpha \geq 1$ is the usual case; however, the present research covers the entire range $\alpha > 0$.

or another member of the Weibull family is valid. However, when failure data is insufficient or entirely absent (for example, in the design stage of a system) one may wish to use the assumption of an exponential distribution in lieu of attempting to identify and specify a degradation effect. Or, one may feel that an exponential assumption is quite reasonable but would like some sort of assurance that if this assumption is modestly in error, the use of it will produce only small prediction errors. In either of these cases, therefore, it would be useful to know how much of a deviation from the exponential assumption may be allowed without producing a significant prediction error. Obviously, the exponential assumption would be more useful if the magnitude of this deviation were large than if it were small. Specifically, if the magnitude is large we may say that the prediction procedure is "robust" with respect to deviations from the exponential assumption.[2]

The purpose of this paper is to investigate the robustness of prediction procedures with respect to deviations from an exponential assumption, where the deviations are within the family of Weibull densities. Since the concept of robustness of a particular procedure is often quite vague due to the arbitrariness of what one means by "acceptable" error or a "modest" deviation, the author chose to attempt to quantify the idea of robustness by determining regions in the Weibull family such that if the true failure distribution is in the region, the use of the exponential assumption will produce a prediction error within a prespecified bound. These regions are called "regions of robustness" and the bound identifies the level of robustness. The region itself is identified in terms of the

[2]Statistical procedures which are "healthy" with respect to deviations from a particular assumption (i.e., modest deviations produce only modest errors in the probability level) are called "robust procedures" with respect to deviations from this assumption.

shape parameter of the Weibull distribution. The foregoing concepts are defined specifically in the next section.

Because of the complexity of the problem, the present research is limited to a series system with N independent and identical components. Results are obtained from N = 1, 2, 3, 4, 5, 8, and 10.

2. TERMINOLOGY AND NOTATION

2.1 Region of Robustness

If a reliability function $R_o(t)$ is assumed when the true reliability function R(t) is a member of a broader family of reliability functions, then a region of robustness of size δ is defined as any region of the parameters of the family R(t) which satisfies the condition.

$$\max_t |R_o(t) - R(t)| \leq \delta \quad . \tag{1}$$

If this region is the largest such region, it may be called a maximal region of robustness of size δ. The quantity δ may be called the level of robustness since it specifies the maximum absolute prediction error which can be made in this region. This level is valid over the entire range of $t > 0$ unless it is specified that a restricted range of t is being considered.

2.2 Exponential Distribution

The exponential density function has the functional form

$$f(t) = (1/\theta)\, e^{-t/\theta} \quad t > 0,\ \theta > 0 \quad ,$$

where $E(t) = \theta$ is known as the mean-time-to-failure (MTTF). The reliability function for this failure distribution is

$$R_e(t) = \int_t^\infty f(t;\theta)dt = e^{-t/\theta} \quad ,$$

where the subscript e identifies it as exponential.

2.3 Weibull Distribution

The Weibull density has the form

$$g(t;\alpha,\beta) = (\alpha t^{\alpha-1}/\beta) e^{-t^\alpha/\beta} \quad t > 0, \alpha > 0, \beta > 0 \quad ,$$

and with $\alpha = 1$ is the exponential density. The reliability function, identified by a subscript w, is

$$R_w(t) = \int_t^\infty g(t;\alpha,\beta)dt = e^{-t^\alpha/\beta}$$

and the mean (MTTF) is $E(t) = \beta^{1/\alpha} \Gamma(1 + 1/\alpha)$.

3. REGIONS OF ROBUSTNESS

3.1 Standardization of the Problem

To investigate the effects of using an exponential assumption when Weibull is correct, a reasonable approach is to require that the mean of the Weibull distribution be equal to that of the exponential. This results in the forms

$$R_e(t) = e^{-t/m} \quad \text{and} \quad R_w(t) = e^{-t^\alpha \Gamma^\alpha(1+1/\alpha)/m^\alpha} \quad ,$$

where m is the mean in each case.

A change of scale from t to t/m leads to the standardized forms

$$R_e(t) = e^{-t} \qquad \text{and} \qquad R_w(t) = e^{-t^\alpha \Gamma^\alpha(1+1/\alpha)} \tag{2}$$

which will be used in the present paper. The standardization equates the means to unity, thus making the analysis simpler without loss of generality.

3.2 Single Component System

In the case of a system which consists of a single component, the maximal region of robustness, using (1) and (2) is defined as the region in α such that

$$\max_t |\Delta(t,\alpha)| < \delta \quad ,$$

where

$$\Delta(t,\alpha) = e^{-t} - e^{-t^\alpha \Gamma^\alpha(1+1/\alpha)} \quad . \tag{3}$$

The level of robustness, previously defined, is δ. An analysis of the function (3) indicates that $\Delta(t,\alpha)$ possesses two local extremum points with respect to t for fixed α (see Appendix A). Also, it can be shown that $R_e(t)$ and $R_w(t)$ intersect at a single point $t_* = [\Gamma(1 + 1/\alpha)]^{\alpha/(1-\alpha)}$

and that t_* is greater than unity for $\alpha \neq 1$. Therefore, from an inspection of the curves for $R_w(t)$ for a range of α (see Figure 1) it is evident that the two local extrema exist one on each side of t_* and that for limiting values of the parameter α (i.e., $\alpha \to 0$ and $\alpha \to \infty$) the absolute extremum appears to be to the left of t_*. Because of the complexity of the function $\Delta(t;\alpha)$, the local extrema were not determined analytically, but rather a numerical technique[3] was employed to calculate both extremum points. The extremum point yielding the largest value of $|\Delta(t;\alpha)|$ was identified as the absolute extremum point t_α. The numerical results showed that t_α was always the smaller of the two extremum points and was always less than unity.

The results of the calculations are shown in Figure 2, where $|\Delta(t_\alpha;\alpha)|$ is plotted as a function of α. The graph indicates that the level of robustness is monotonic decreasing with respect to α as α increases from zero to unity and monotonically increasing $\alpha > 1$. Certain additional numerical results further strengthen these indications; however, to date a rigorous proof of this monotonicity property has not been found. This, however, presents no difficulties since the definition of a region of robustness of size δ requires only that the error level δ be satisfied. It does not require that the region be the largest (maximal) such region. In the present paper, however, the regions obtained are the maximal regions of robustness for the specified δ. The latter statement is justified by the fact that $|\Delta(1;\alpha)|$ is monotonic increasing as α deviates from unity in either direction (see Appendix C) and by the fact that $|\Delta(t_\alpha;\alpha)| \geq |\Delta(1;\alpha)|$. A bound on the maximal region of robustness

[3]The value of t_α was found using a Newton-Raphson approximation for the root of $\Delta'(t;\alpha)$. The computations were conducted on a GE-625 computer with error tolerance 10^{-5} in α.

for a given level δ may therefore be obtained by solving the equation $|\Delta(1;\alpha)| = \delta$ for the two values α_1 and α_2 ($\alpha_1 < 1 < \alpha_2$). Then to obtain the maximal region of robustness of size δ, one need evaluate $|\Delta(t_\alpha;\alpha)|$ only over values of α in $[\alpha_1, \alpha_2]$. This was the procedure for the present research and hence all regions of robustness are maximal for the specified level. These regions are given in Table 1 for a system of $n = 1$ components.

One obtains a region of robustness "about $\alpha = 1$", at a specified level δ, by reading the end points of the region in Table 1 from the entries for a single component system. If a required level δ is not given in Table 1, an approximation may be obtained by reading the two abcissae corresponding to the desired robustness level from Figure 2 which provides a graphical representation of $|\Delta(t_\alpha;\alpha)|$ as a function of α.

3.3 Identical Components in Series

In the case of a system of n identical independent components in series, the reliability functions, using the assumptions (2) of an exponential and Weibull reliability function for the individual components, are respectively

$$R_e^n(t) = e^{-nt}$$

and

$$R_w^n(t) = e^{-nt^\alpha \Gamma^\alpha(1+1/\alpha)} .$$

As in Section 3.2, regions of robustness may be obtained by maximizing

$$|\Delta(t;\alpha,n)| = |e^{-nt} - e^{-nt^\alpha \Gamma^\alpha(1+1/\alpha)}| \qquad (4)$$

with respect to t for a given α and n. Two local extrema of $\Delta(t;\alpha,n)$ exist (see Appendix A) and again they straddle $t_* = [\Gamma(1+1/\alpha)]^{\alpha/(1-\alpha)}$. The same process of evaluating $\Delta(t;\alpha,n)$ at the two local extrema and selecting $t_{\alpha,n}$ = the extremum producing the maximum value of (4) was used. Numerical results again showed this maximum to always be at the lower extremum point which was less than unity. Also, the same monotonicity arguments used in Section 3.2 may be used, with $\Delta(1;\alpha,n)$ replacing $\Delta(1;\alpha)$, for the n component system (see Appendix A). Repeating this approach, the numerical evaluations were extended over the appropriate region to produce the maximal regions of robustness for a given level δ. The endpoints of the regions of robustness for various δ and n = 1, 2, 3, 4, 5, 8, 10 are given in Table 1[4] and $|\Delta(t_{\alpha,n};\alpha,n)|$ is plotted versus α for the same range of n in Figure 3. Table 1 and Figure 3 may be used in the same manner for the n component system as Table 1 and Figure 2 were for a single component. It is of interest to note that Figure 3 shows a decrease in the region of robustness for fixed δ as the number of components increases.

3.4 Series System of Identical Components with High Reliability

Thus far no restriction has been placed on the range of t for which $\max_t|\Delta(t;\alpha,n)|$ was evaluated to obtain a region of robustness. That is, in the indicated region the error bound was valid for any prediction time $t > 0$. It is quite possible, however, that a particular system might be used only over a period of time where it is known that the reliability of each component is extremely high (e.g., components used in the Manned Space Flight Program) and, therefore, it might be possible that the region of robustness for a specified level might be larger than when the use is considered over the entire range $t > 0$. This section therefore considers a system of n identical components with the individual components to

[4]The same numerical techniques used for n = 1 were used for all values of n, with the same error tolerance (10^{-5}) in α.

be used only over the period where its reliability is at least ρ. Obviously, the regions previously obtained for unrestricted t are valid regions of robustness for the present case, but they may no longer be maximal such regions for the restricted range of t.

In this section, it is assumed that the identical components under investigation are to be used over the time interval $(0, t_\rho)$ where t is the length of time which provides a reliability of exactly ρ (t_ρ actually depends upon α but the additional argument α will be understood and not indicated). The problem therefore is to determine the range of α such that

$$\max_{t \leq t_\rho} |\Delta(t;\alpha,n)| < \delta \tag{5}$$

for fixed n.

By appeal to Figure 1 and the development in Sections 3.2 and 3.3 it is clear that the range of α satisfying (5) are the same as in these previous sections when t_ρ is greater than or equal to the smaller of the two local extremum point of $\Delta(t;\alpha,n)$ (α and n fixed) and, when t_ρ is smaller than the same extrema, is the range of α satisfying

$$|\Delta(t_\rho;\alpha,n)| < \delta \quad . \tag{6}$$

In the latter case the region is enlarged because of the restricted period of use of the system.

In summary, the calculations for this section were performed by solving

$$e^{-t^\alpha \Gamma^\alpha (1+1/\alpha)} = \rho$$

to obtain

$$t_\rho = -\Gamma^{-1}(1 + 1/\alpha)(\ln \rho)^{1/\alpha} \quad . \tag{7}$$

The value of (7) was obtained for fixed α and if t_ρ proved to be smaller than $t_{\alpha,n}$, (i.e., the smaller local extremum of $\Delta(t;\alpha,n)$), $|\Delta(t_\rho;\alpha,n)|$ was evaluated. This procedure was repeated until the range of α satisfying (6) was determined.

Regions of robustness were determined, in the restricted case, for $\rho = .95$ and .99 and for n = 1, 2, 3, 4, 5, 8, 10. The end points of these regions are given in Table 2 and Table 3 and, in general, indicate a larger region than in the unrestricted case for the same n.

4. EXAMPLES

Example 1: A Weibull density function with parameters $\alpha = 1.1$ and $\beta = 2.0$ has effectively represented the failure distribution of oiled bearings. What maximum error can be expected in a reliability prediction if an exponential reliability function is used?

Since $\alpha = 1.1$ falls in the interval $.8748 < \alpha < 1.1524$ we may use the exponential reliability function and make an error no greater than .05, regardless of the value of t for which we predict.

Example 2: In a study on the fatigue life of deep groove ball bearings [1], it was shown that for Record Number 2-35 the parameter α of the Weibull distribution was estimated to be 1.05 from a sample of 14 ball bearings. Using one ball bearing as a component, the exponential reliability function can be used to calculate the reliability of the ball bearing if a maximum error of .05 is accepted. The region of robustness $.8748 < \alpha < 1.1524$ yields the desired level of robustness of .05.

Example 3: An auxiliary system composed of four identical independent components in series is used in a particular radar system. Suppose that it is necessary for each component of the auxiliary system to have a reliability of at least .99 over an intended period of use. If the reliability prediction for the system is based upon an exponential reliability assumption, what range of deviation from the exponential reliability assumption (in terms of a bound on α) can be allowed which will result in an error of less than .005? Table 3 indicates that $.9733 < \alpha < 1.0249$ will satisfy the requirement.

5. SUMMARY

The present research investigates the robustness of reliability prediction procedures based on the assumption of an exponential failure distribution. A system of n independent identical components in series is considered and a maximal region of robustness, in terms of the shape parameter of the Weibull failure distribution (having mean identical to that of the assumed exponential distribution), is determined. This region of robustness has the property that within this region the use of the exponential assumption will produce a prediction error no larger than the specified level of robustness for the region. Regions are obtained for an unrestricted time period and a restricted time period in which the individual components have a minimum reliability ρ (ρ = .95, .99).

In general, the results indicate a decrease in robustness as the number of components increases. That is, for a fixed level of robustness, the size of the region of robustness decreases as n increases. Alternatively, one could state that for a fixed region the level of robustness worsens (increases) as n increases.

APPENDIX A

STATEMENT: The function $\Delta(t;\alpha,n) = e^{-nt} - e^{-nt^{\alpha}\Gamma^{\alpha}(1+1/\alpha)}$, where $t > 0$ and $\alpha > 0$, has exactly two relative extrema.

PROOF: From the definition of $\Delta(t;\alpha,n)$,

$$\Delta'(t;\alpha,n) = -ne^{-nt} + n\alpha t^{\alpha-1}\Gamma^{\alpha}(1+1/\alpha)e^{-nt^{\alpha}\Gamma^{\alpha}(1+1/\alpha)} \quad . \qquad \text{(A1)}$$

Therefore, equating (A1) to zero, dividing by ne^{-nt}, and taking logaithms, yields

$$-\ln[\alpha\Gamma^{\alpha}(1+1/\alpha)] - (\alpha-1)\ln t + nt^{\alpha}\Gamma^{\alpha}(1+1/\alpha) - nt = 0 \quad . \qquad \text{(A2)}$$

The solutions to (A2) are the desired extrema of $\Delta(t;\alpha,n)$. Now set

$$g(t) = -\ln[\alpha\Gamma^{\alpha}(1+1/\alpha)] - (\alpha-1)\ln t + nt^{\alpha}\Gamma^{\alpha}(1+1/\alpha) - nt \quad .$$

Then

$$g'(t) = (1-\alpha)/t + \alpha n\Gamma^{\alpha}(1+1/\alpha)t^{\alpha-1} - n \quad .$$

Let $h(t) = tg'(t)$. Then $h(t) = x(t) - y(t)$, where $x(t) = \alpha n\Gamma^{\alpha}(1+1/\alpha)t^{\alpha}$ and $y(t) = nt+\alpha-1$. It can be seen that for $\alpha > 1$, $x(t)$ is convex and for $\alpha < 1$ is concave. Therefore, $x(0) = 0$ and $y(0) = \alpha-1$ imply that $h(t)$ has only one root, say t_o.

Since the only difference between $g'(t)$ and $h(t)$ is the factor $1/t$, the two functions have the same sign. This implies that when $\alpha < 1$, $g(t)$ increases in $(0,t_o)$, is maximum at t_o, and decreases for $t > t_o$; and when $\alpha > 1$, $g(t)$ decreases in $(0,t_o)$, is minimum at t_o, and increases for $t > t_o$. Therefore, both $g(t)$ and $\Delta'(t;\alpha,n)$ have at most two roots. Hence, if $g(t)$ can be shown to be positive for some value of t when $\alpha < 1$ and negative for some value of t when $\alpha > 1$, the proof will be complete.

Now, it can be shown that for $\alpha < 1$, $\alpha\Gamma^{\alpha}(1+1/\alpha) < 1$ and $\Gamma^{\alpha}(1+1/\alpha) > 1$, and also for $\alpha > 1$, $\alpha\Gamma^{\alpha}(1+1/\alpha) > 1$ and $\Gamma^{\alpha}(1+1/\alpha) < 1$. Therefore, from

$$g(1) = -\ln[\alpha\Gamma^{\alpha}(1+1/\alpha)] + n[\Gamma^{\alpha}(1+1/\alpha) - 1] \quad ,$$

it is clear that g(1) is positive for $\alpha < 1$ and negative for $\alpha > 1$.

APPENDIX B

STATEMENT: $\Gamma^{\alpha}(1+1/\alpha)$ is monotonically decreasing in α.

PROOF: Using the Liapounoff Inequality [2], if $a \geq b \geq c$, then

$$(E|x|^{b})^{a-c} \leq (E|x|^{c})^{a-b} (E|x|^{a})^{b-c}$$

which, with $c = 0$, simplifies to

$$(E|x|^{b})^{a} \leq (E|x|^{a})^{b} \quad . \tag{B1}$$

Letting the random variable x have the exponential density with unit mean, and applying (B1) where $a \geq b \geq 0$,

$$\left(\int_0^{\infty} |x|^{b}\, e^{-x}\, dx\right)^{1/b} \leq \left(\int_0^{\infty} |x|^{a}\, e^{-x}\, dx\right)^{1/a} \quad .$$

Since the range on x is non-negative, the absolute value signs can be dropped and we have

$$[\Gamma(b+1)]^{1/b} \leq [\Gamma(a+1)]^{1/a} \quad .$$

Equating $\beta = 1/b$ and $\alpha = 1/a$, the inequality becomes

$$\Gamma^{\beta}(1+1/\beta) \leq \Gamma^{\alpha}(1+1/\alpha) \tag{B2}$$

where $\alpha \leq \beta$.

APPENDIX C

STATEMENT: The function $|\Delta(1;\alpha,n)|$ is monotonic decreasing in α as α increases from zero to unity, and is monotonic increasing in α for $\alpha > 1$.

PROOF: $\Delta(1;\alpha,n) = e^{-n} - e^{-n\Gamma^{\alpha}(1+1/\alpha)}$. Since $\Gamma^{\alpha}(1+1/\alpha)$ is monotonic decreasing for $\alpha > 0$ (Appendix B), $\Delta(1;\alpha,n)$ is also monotonic decreasing in this range. Therefore, since $\Delta(1;1,n) = 0$, $\Delta(1;\alpha,n)$ is positive and decreasing to zero at $\alpha = 1$ and thereafter becomes negative and continues decreasing. This proves the statement by considering the absolute value of $\Delta(1;\alpha,n)$ over the range of α.

TABLE 1. Regions of Robustness (unrestricted case)

Number of Components	Level of Robustness									
	.10		.05		.01		.005		.001	
1	.7705	1.3410	.8748	1.1524	.9730	1.0280	.9864	1.0139	.9972	1.0028
2	.8379	1.2073	.9143	1.0969	.9820	1.0184	.9910	1.0092	.9982	1.0018
3	.8643	1.1647	.9291	1.0781	.9853	1.0150	.9926	1.0075	.9985	1.0015
4	.8791	1.1429	.9372	1.0683	.9870	1.0132	.9935	1.0066	.9987	1.0013
5	.8888	1.1293	.9424	1.0621	.9881	1.0120	.9941	1.0060	.9988	1.0012
8	.9052	1.1072	.9512	1.0518	.9900	1.0101	.9950	1.0050	.9990	1.0010
10	.9115	1.0991	.9546	1.0480	.9907	1.0094	.9953	1.0048	.9991	1.0009

TABLE 2. Regions of Robustness (restricted case $\rho = .95$)

Number of Components	Level of Robustness							
	.05		.01		.005		.001	
1	.0600	1.2769	.9373	1.0588	.9693	1.0298	.9939	1.0061
2	.8227	1.1520	.9677	1.0314	.9840	1.0158	.9968	1.0032
3	.8820	1.1088	.9775	1.0222	.9888	1.0111	.9978	1.0022
4	.9090	1.0869	.9823	1.0176	.9912	1.0088	.9982	1.0018
5	.9243	1.0737	.9851	1.0148	.9926	1.0074	.9985	1.0015
8	.9460	1.0546	.9892	1.0108	.9946	1.0054	.9989	1.0011
10	.9525	1.0487	.9904	1.0096	.9952	1.0048	.9990	1.0010

TABLE 3. Regions of Robustness (restricted case $\rho = .99$)

Number of Components	Level of Robustness							
	.01		.005		.001		.0005	
1	.2804	1.1632	.8793	1.0889	.9794	1.0195	.9898	1.0097
2	.8781	1.0899	.9453	1.0474	.9875	1.0099	.9949	1.0050
3	.9239	1.0627	.9642	1.0326	.9931	1.0067	.9966	1.0034
4	.9442	1.0484	.9733	1.0249	.9948	1.0051	.9974	1.0026
5	.9557	1.0396	.9786	1.0203	.9958	1.0041	.9979	1.0020
8	.9722	1.0260	.9863	1.0132	.9973	1.0027	.9987	1.0013
10	.9775	1.0214	.9889	1.0108	.9978	1.0022	.9989	1.0011

APPENDIX E

Figures

Figure 1: Plot of the function $f(t) = e^{-t^{\alpha}\Gamma^{\alpha}(1+1/\alpha)}$ for several values of α

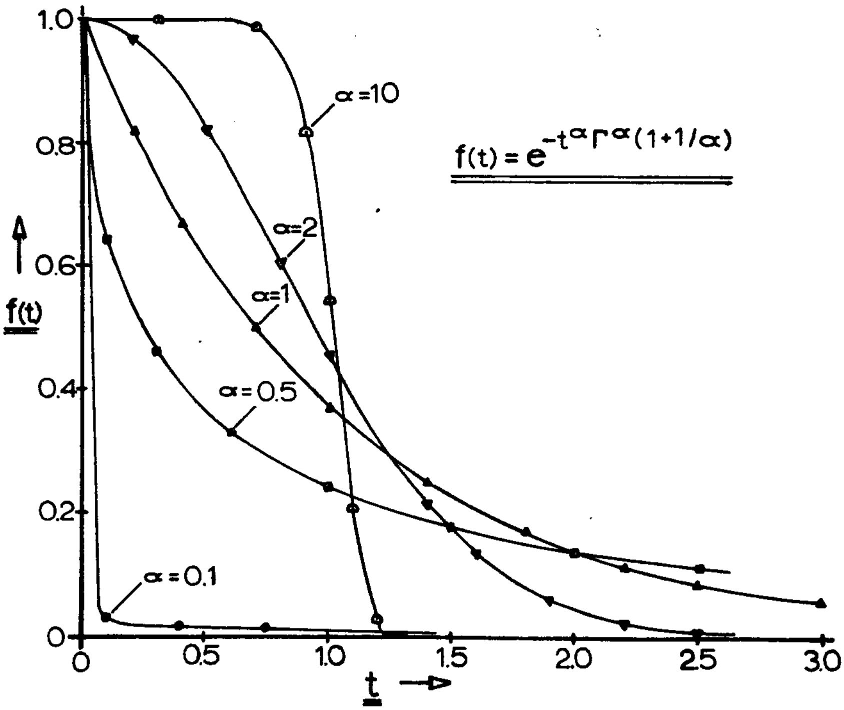


Figure 2: Plot of the function $|e^{-t_\alpha} - e^{-t_\alpha^\alpha \Gamma^\alpha(1+1/\alpha)}|$

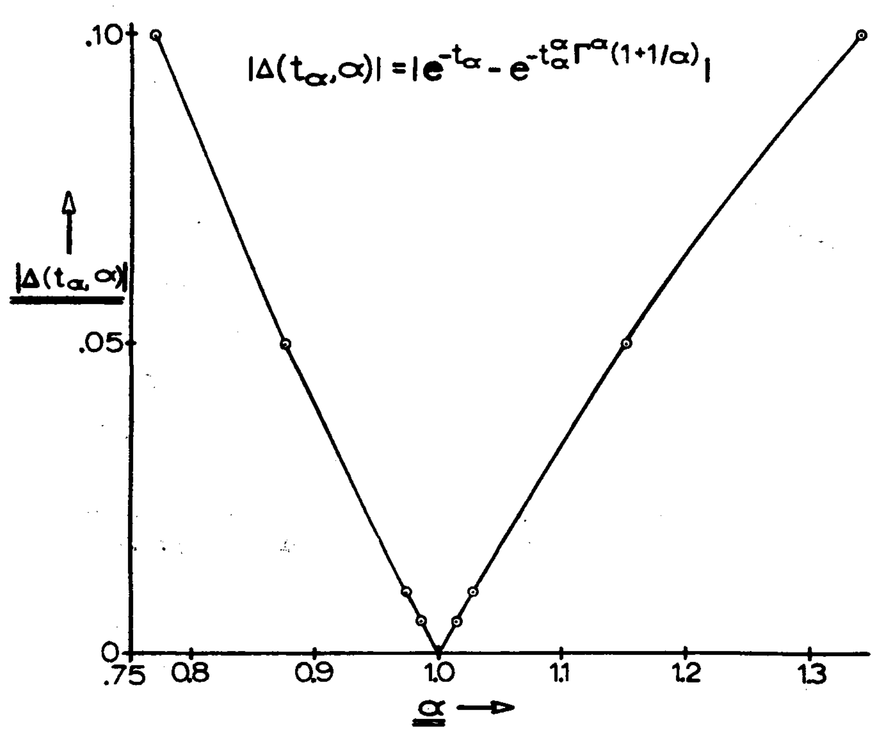
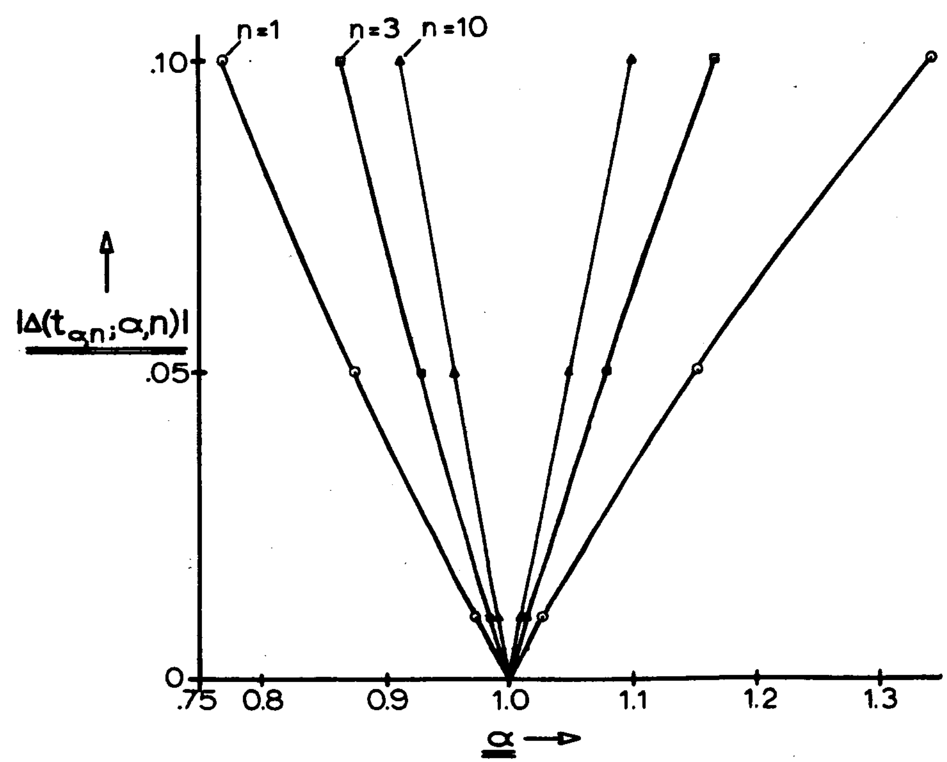


Figure 3: Plot showing the reduction of the Region of Robustness for increasing values of **n**



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13. ABSTRACT

The present research investigates the robustness of reliability prediction procedures based on the assumption of an exponential failure distribution. A system of n independent identical components in series is considered and a maximal region of robustness, in terms of the shape parameter of the Weibull failure distribution (having mean identical to that of the assumed exponential distribution), is determined. This region of robustness has the property that within this region the use of the exponential assumption will produce a prediction error no larger than the specified level of robustness for the region. Regions are obtained for an unrestricted time period and a restricted time period in which the individual components have a minimum reliability ρ (ρ = .95, .99).

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Weibull reliability function						
Robustness						
Exponential distribution						
Exponential failure distribution						

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