# The Role of Dust Charge Fluctuations on Nonlinear Dust Ion-Acoustic Waves

A. A. Mamun and P. K. Shukla

Abstract—The nonlinear propagation of the dust ion-acoustic waves has been investigated accounting for the charge fluctuation dynamics of stationary dust grains in an unmagnetized dusty plasma. The Korteweg-de Vries equation, as well as the Korteweg-de Vries-Burgers equation, are derived by employing the reductive perturbation method. It has been shown that dust charge fluctuations produce a dissipation which is responsible for shock waves. Conditions for the formation of dust ion-acoustic solitary and shock waves as well as their properties are clearly explained. The implications of our investigations to both space and laboratory dusty plasmas are discussed.

Index Terms-Dust ion-acoustic waves, dusty plasmas, shock waves, solitary waves.

#### I. INTRODUCTION

THE WAVE propagation in dusty plasmas has received a great deal attention because of its vital role in understanding different collective processes in space environments [1]–[3] as well as in laboratory devices [4]–[7]. The consideration of charged dust grains in a plasma does not only modify the existing plasma wave spectra [8]–[10], but also introduces a number of novel eigenmodes, such as the dust ion-acoustic (DIA) waves [11], the dust-acoustic (DA) waves [12], the dust lattice waves [13], etc.

Shukla and Silin [11] have theoretically shown that, due to the conservation of equilibrium charge density  $n_{e0}e + n_{d0}Z_{d0}e$  –  $n_{i0}e = 0$  and the strong inequality  $n_{e0} \ll n_{i0}$  [where  $n_{s0}$  is the particle number density of the species s with s = e(i)d for electrons (ions) dust,  $Z_{d0}$  is the number of electrons residing onto the dust grain surface and e is the magnitude of the electronic charge], a dusty plasma (with negatively charged static dust grains) supports low-frequency (in comparison with the ion plasma frequency) DIA waves with a phase speed much smaller (larger) than electron (ion) thermal speed. The DIA wave spectrum is similar to the usual ion-acoustic wave spectrum for a plasma with  $n_{i0} = n_{e0}$  and  $T_i \ll T_e$ , where  $T_i(T_e)$  is the ion (electron) temperature. However, in dusty plasmas we usually have  $n_{i0} \gg n_{e0}$  and  $T_i \simeq T_e$ . So, obviously, a dusty plasma

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cannot support the usual ion-acoustic waves, but the DIA waves of Shukla and Silin [11] can. The DIA waves have also been observed in laboratory experiments [5], [6]. The linear properties of the DIA waves in dusty plasmas are now well understood from both theoretical and experimental points of view [3], [5], [6], [11], [14].

Recently, nonlinear waves associated with the DIA waves, particularly the DIA solitary [15] and shock waves [16] have also received a great deal of interest in understanding the basic properties of localized electrostatic perturbations in space and laboratory dusty plasmas [1]-[7], [15]-[18]. The DIA solitary and shock waves have been investigated theoretically by several authors [15], [16], [19]–[22]. However, all these investigations [15], [16], [19]–[22] are limited to constant dust grain charge which may not be a realistic situation in space and laboratory devices, since in space, as well in laboratory devices, the charge on a dust grain is not constant but varies with space and time [3]. Therfore, in the present paper, we consider an unmagnetized dusty plasma model consisting of the ion fluid, Boltzmann electrons, and stationary charge fluctuating dust grains and analyze nonlinear propagation of the DIA waves. We show here how the effects of dust grain charge fluctuations significantly modify the nonlinear propagation characteristics of the DIA waves as well as produce monotonic shock waves. We note that Popel et al. [23] have also studied shock waves by considering dust grain charge fluctuations, but they have not deduced evolution equations for the DIA shock waves.

This paper is organized as follows. The basic equations describing our dusty plasma model are given in Section II. The DIA solitary waves are studied by deriving the Korteweg-de Vries (K-dV) equation in Section III. The DIA shock waves are investigated by obtaining the Korteweg-de Vries-Burgers (K-dV-B) equation in Section IV. A brief discussion of our results is contained in Section V.

#### II. GOVERNING EQUATIONS

We consider the nonlinear propagation of the DIA waves in an unmagnetized dusty plasma whose constituents are the ion fluid, Boltzmann electrons, and charge fluctuating immobile dust particles. The nonlinear dynamics of DIA waves, whose phase speed is much smaller (larger) than the electron (ion) thermal speed, is governed by

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x} (n_i u_i) = 0$$

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x}$$

$$\frac{\partial^2 \phi}{\partial x^2} = \mu \exp(\phi) - n_i + (1 - \mu) Z_d$$
(3)

$$\frac{\partial u_i}{\partial t} + u_i \frac{\partial u_i}{\partial x} = -\frac{\partial \phi}{\partial x} \tag{2}$$

$$\frac{\partial^2 \phi}{\partial r^2} = \mu \exp(\phi) - n_i + (1 - \mu)Z_d \qquad (3)$$

where  $n_i$  is the ion number density normalized by its equilibrium value  $n_{i0}$ ,  $u_i$  is the ion fluid velocity normalized by  $C_i = (k_B T_e/m_i)^{1/2}$  ( $k_B$  is the Boltzmann constant and  $m_i$  is the ion mass),  $\phi$  is the electrostatic wave potential normalized by  $k_B T_e/e$ , and  $Z_d$  is the number of electrons residing onto the dust grain surface normalized by its equilibrium value  $Z_{d0}$ . The time and space variables are in units of the ion plasma period  $\omega_{pi}^{-1}=(m_i/4\pi n_{i0}e^2)^{1/2}$  and the Debye radius  $\lambda_{Dm}=(k_BT_e/4\pi n_{i0}e^2)^{1/2}$ , respectively. We have denoted  $\mu = n_{e0}/n_{i0}$ . We note that  $Z_d$  is not constant but varies with space and time. Thus, (3) is completed by the normalized dust grain charging equation [3]

$$\eta \frac{\partial Z_d}{\partial t} = \mu \beta \exp\left(\phi - \alpha Z_d\right) - \beta_i n_i u_i \left(1 + \frac{2\alpha Z_d}{u_i^2}\right) \tag{4}$$

where  $\eta = \sqrt{\alpha m_e (1 - \mu)/2m_i}$ ,  $\beta = (r_d/a)^{3/2}$ ,  $\beta_i = \beta \sqrt{\pi m_e/8m_i}$ ,  $\alpha = Z_{d0}e^2/k_BT_er_d$  and  $a = n_{d0}^{-1/3}$ . We note that at equilibrium  $\mu\beta \exp(-\alpha) = \beta_i u_0 (1 + 2\alpha/u_0^2)$ , where  $u_0$  is the ion streaming speed normalized by  $C_i$ .

#### III. DIA SOLITARY WAVES

To study small but finite amplitude DIA solitary waves, we first introduce the stretched coordinates [24]  $\xi = \epsilon^{1/2}(x - v_0 t)$ and  $\tau = \epsilon^{3/2}t$ , where  $\epsilon$  is the expansion parameter, measuring the amplitude of the wave or the weakness of the wave dispersion. We then expand  $n_i$ ,  $u_i$ ,  $\phi$  and  $Z_d$  in a power series of  $\epsilon$ 

$$n_i = 1 + \epsilon n_i^{(1)} + \epsilon^2 n_i^{(2)} + \cdots$$
 (5a)

$$u_i = u_0 + \epsilon u_i^{(1)} + \epsilon^2 u_i^{(2)} + \cdots$$
 (5b)

$$\phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots \tag{5c}$$

$$Z_d = 1 + \epsilon Z_d^{(1)} + \epsilon^2 Z_d^{(2)} + \cdots$$
 (5d)

We develop equations in various powers of  $\epsilon$ . To lowest order in  $\epsilon$ , (1)–(4) give

$$w_0 n_i^{(1)} = u_i^{(1)} (6a)$$

$$w_0 u_i^{(1)} = \phi^{(1)} \tag{6b}$$

$$0 = \mu \phi^{(1)} - n_i^{(1)} + (1 - \mu) Z_d^{(1)}$$
 (6c)

$$0 = \beta_e \phi^{(1)} - \alpha u_\beta Z_d^{(1)}$$

 $-\beta_i u_1 u_i^{(1)} - u_0 \beta_i u_2 n_i^{(1)}$ (6d)

where  $w_0 = v_0 - u_0$ ,  $u_1 = 1 - 2\alpha/u_0$ ,  $u_2 = 1 + 2\alpha/u_0^2$ ,  $u_{\beta} = \beta_e + 2\beta_i/u_0$  and  $\beta_e = \mu\beta \exp(-\alpha)$ . Now, substituting  $n_i^{(1)}, u_i^{(1)}$ , and  $Z_d^{(1)}$  [obtained from (6a), (6b) and (6d)] into (6c) we obtain the dispersion relation

$$aw_0^2 - bw_0 - c = 0 (7)$$

where

$$a = \mu + \frac{(1 - \mu)\beta_e}{\alpha u_\beta} \tag{8a}$$

$$b = \frac{(1-\mu)u_1\beta_i}{\alpha u_\beta} \tag{8b}$$

and 
$$c = 1 + \frac{(1-\mu)u_2u_0\beta_i}{\alpha u_\beta}.$$
 (8c)

Considering the next higher order in  $\epsilon$ , from (1)–(4) we obtain the following set of equations:

$$\frac{1}{w_0^2} \frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{2}{w_0^3} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = 0$$
(9a)

$$\frac{1}{w_0} \frac{\partial \phi^{(1)}}{\partial \tau} - w_0 \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{1}{w_0^2} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} = \frac{\partial \phi^{(2)}}{\partial \xi} \tag{9b}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \mu \phi^{(2)} + \frac{1}{2} \mu \left[ \phi^{(1)} \right]^2 - n_i^{(2)} + (1 - \mu) Z_d^{(2)} \ \ (9c)$$

$$0 = \beta_{e} \phi^{(2)} - \alpha u_{\beta} Z_{d}^{(2)} - \beta_{i} u_{1} u_{i}^{(2)} - u_{0} \beta_{i} u_{2} n_{i}^{(2)} + \beta_{1} \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \epsilon}$$
(9d)

$$\beta_1 = \beta_e \left[ 1 + (\alpha \beta_0 - 2) \, \alpha \beta_0 \right] - \frac{2\beta_i}{w_0^3} \left[ 1 + \frac{w_\alpha}{u_0^3} \right] \quad (10a)$$

$$\beta_0 = \frac{1}{\alpha u_\beta} \left[ \beta_e - \frac{\beta_i w_2}{w_0} + \frac{2\alpha \beta_i}{w_0 u_0} \left( 1 - \frac{1}{w_0} \right) \right]$$
 (10b)

 $w_1 = 1 - u_0/w_0$ ,  $w_2 = 1 + u_0/w_0$  and  $w_\alpha = 2\alpha w_0 w_1(1 - w_0)$  $\beta_0 w_0 u_0$ ). Combining (9a)–(9d), we deduce a K-dV equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0 \tag{11}$$

where

$$A = \frac{3c + bw_0 - \beta_2 w_0^4}{w_0 (2c + bw_0)}$$
 (12a)

$$B = \frac{w_0^3}{2c + bw_0} \tag{12b}$$

$$\beta_2 = \mu + \frac{\beta_1(1-\mu)}{\alpha u_\beta}.$$
 (12c)

The stationary solution of the K-dV equation (11) is obtained by transforming the independent variables  $\xi$  to  $\zeta = \xi - U_0 \tau$ , where  $U_0$  is a constant speed normalized by  $C_i$  and imposing the appropriate boundary conditions for localized perturbations, viz.  $\phi^{(1)} \to 0$ ,  $d\phi^{(1)}/d\zeta \to 0$ ,  $d^2\phi^{(1)}/d\zeta^2 \to 0$  at  $\zeta \to \pm \infty$ . Accordingly, the stationary solitary wave solution of (11) is

$$\phi^{(1)} = \psi \operatorname{sech}^2 \left[ \frac{\xi - U_0 \tau}{\Delta} \right] \tag{13}$$

where  $\psi = 3U_0/A$  and  $\Delta = \sqrt{4B/U_0}$  represent the amplitude and the width of the solitary waves, respectively. It is obvious from (13) that there exists compressive (rarefactive) solitary waves if A > 0 (A < 0). We have numerically analyzed A for the parameters corresponding to space dusty plasma situations (e.g., Saturn's E-ring [2], [3]:  $n_{d0} = 10^{-7} \text{ cm}^{-3}$ ,  $r_d = 1 \mu\text{m}$ ,  $k_BT_e=50~{\rm eV}$  and  $Z_d=10^3$  which correspond to  $\alpha=0.0288$  and  $\beta=3\times 10^{-10}$ ) as well as for the parameters corresponding to laboratory dusty plasma devices [5], [6]  $(n_{d0} = 10^5 \text{ cm}^{-3})$ ,  $r_d = 5 \ \mu \text{m}$ ,  $k_B T_e = 0.2 \text{ eV}$  and  $Z_d = 10^3 \text{ which correspond}$ to  $\alpha = 1.44$  and  $\beta = 3 \times 10^{-4}$ ) and found that A is always positive (which is obvious from the upper plot of Fig. 1, since  $U_0 > 0$ ). This means that our present dusty plasma model can support only compressive solitary waves (solitary waves with  $\phi > 0$ ). The results of our numerical analyses (see Fig. 1) show how the amplitude and the width of these compressive solitary waves vary with  $\mu$  for both the space [2], [3]) ( $\alpha = 0.0288$ 

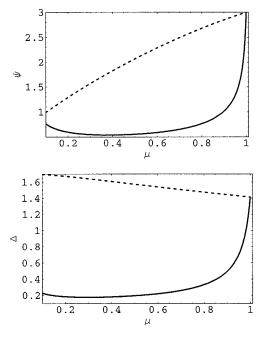


Fig. 1. The variation of the amplitude  $\psi$  (upper plot) and the width  $\Delta$  of the solitary waves with  $\mu$  for  $U_0=1$ . The solid curves correspond to space dusty plasma parameters:  $\alpha=0.0288$  and  $\beta=3\times10^{-10}$ , whereas the dotted curves correspond to laboratory dusty plasma parameters:  $\alpha=1.44$  and  $\beta=3\times10^{-4}$ .

and  $\beta = 3 \times 10^{-10}$ ) and laboratory [5], [6] ( $\alpha = 1.44$  and  $\beta = 3 \times 10^{-4}$ ) dusty plasma situations.

## IV. DIA SHOCK WAVES

In the preceding section, we have seen that the parameter  $\eta$  does not play any role in our analysis of the DIA solitary waves. This is because of the scaling that we have used. We now consider a situation in which we can scale the parameter  $\eta$  as  $\eta = \epsilon^{1/2}\eta_0$ . This additional scaling, remaining stretching/scaling of other variables as before, introduces a new term, namely  $-u_0\eta_0\partial Z_d^{(1)}/\partial \xi$ , in the right-hand side of (9d). Therefore, with  $\eta = \epsilon^{1/2}\eta_0$ , (9d) can be rewritten as

$$-u_0 \eta_0 \frac{\partial Z_d^{(1)}}{\partial \xi} = \beta_e \phi^{(2)} - \alpha u_\beta Z_d^{(2)} + \beta_1 \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} -\beta_i u_1 u_i^{(2)} - u_0 \beta_i u_2 n_i^{(2)}.$$
(14)

Replacing (9d) by (14) and performing all mathematical steps as we did in order for deriving the K-dV equation (8), we obtain a K-dV-Burgers equation

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = C \frac{\partial^2 \phi^{(1)}}{\partial \xi^2}$$
 (15)

where

$$C = \frac{Bv_0\eta_0\beta_0(1-\mu)}{\alpha\left(\beta_e + \frac{2\beta_i}{u_0}\right)}.$$
 (16)

An exact analytical solution of (15) is not possible. However, we can deduce some approximate analytical solutions of (15). Let us try to find an analytical solution of (15) which is valid for both space and laboratory dusty plasmas. We first transform the independent variables  $\xi$  to  $\zeta = \xi - U_0 \tau$ . Under the steady-state

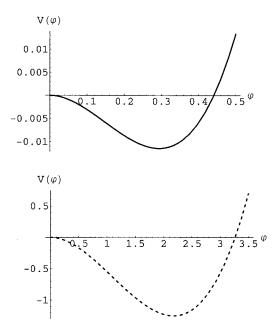


Fig. 2. The behavior of the potential  $V(\varphi)$  for  $\mu=0.5$ . The upper plot corresponds to space dusty plasma parameters:  $\alpha=0.0288$  and  $\beta=3\times 10^{-10}$ , whereas the lower plot corresponds to laboratory dusty plasma parameters:  $\alpha=1.44$  and  $\beta=3\times 10^{-4}$ .

condition, we then find from (15) a third order ordinary differential equation for  $\phi^{(1)}(\zeta) = \varphi$ . The latter can be integrated once, obtaining

$$B\frac{d^2\varphi}{\partial\zeta^2} - C\frac{d\varphi}{d\zeta} + \frac{A}{2}\varphi^2 - U_0\varphi = 0 \tag{17}$$

where we have imposed the appropriate boundary conditions, viz.  $\varphi \to 0$ ,  $d\varphi/d\zeta \to 0$ ,  $d^2\varphi/d\zeta^2 \to 0$  at  $\zeta \to \infty$ . To analyze (17), we can use a simple mechanical analogy [25], [26] based on a fact that it has a form of an equation of motion for a pseudoparticle of mass B, of pseudotime  $-\zeta$  and pseudoposition  $\varphi$  in a force field with potential

$$V(\varphi) = \frac{A}{6}\varphi^3 - \frac{U_0}{2}\varphi^2 \tag{18}$$

and a frictional force with the coefficient C. The nature of the pseudopotential  $V(\varphi)$  for typical space (upper plot) and laboratory (lower plot) dusty plasma parameters is depicted in Fig. 2. If the frictional force were absent, the quasi-particle entering from the left will go the right-hand side of the well ( $\varphi < 0$ ), reflect and return to  $\varphi = 0$ , making a single transit. This corresponds to the DIA solitary waves defined by (13). However, since in (17) a frictional force is practically present, i.e., the particle suffers a loss of energy, it will never return  $\varphi = 0$ , but will oscillate about some negative value of  $\varphi$  corresponding to the minimum of  $V(\varphi)$ . We assume that at pseudotime  $-\zeta = -\infty$ , i.e., at  $\zeta = \infty$ , the quasi-particle is located at the coordinate origin, i.e.,  $\varphi(\infty) = 0$  and at pseudotime  $-\zeta = \infty$ , i.e., at  $\zeta = -\infty$ , the quasi-particle is located at a point corresponding to the minimum of  $V(\varphi)$ , i.e.,  $\varphi(-\infty) = 2U_0/A$ . Thus, the solution of (17) describes a shock wave whose speed  $U_0$  is related to the extreme values  $\varphi(\infty) = 0$  and  $\varphi(-\infty) = 2U_0/A$  by  $\varphi(-\infty) - \varphi(\infty) = 2U_0/A$ .

The nature of these shock structures depends on the relative values between the dispersive and dissipative coefficients B and

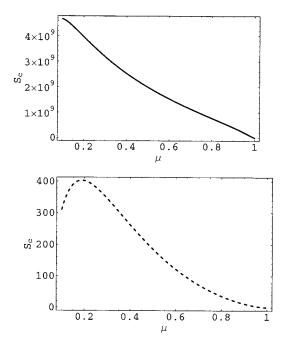


Fig. 3. The variation of  $S_c$  with  $\mu$  for  $U_0=1$  and  $\epsilon=10^{-2}$ . The upper plot corresponds to space dusty plasma parameters:  $\alpha=0.0288$  and  $\beta=3\times10^{-10}$ , whereas the lower plot corresponds to laboratory dusty plasma parameters:  $\alpha=1.44$  and  $\beta=3\times10^{-4}$ .

C. If the value of C is very small, the energy of the particle decreases very slowly and the first few oscillations at the wave front will be close to solitary waves defined by (13). However, if the value of C is larger than a certain critical value, the motion of the particle will be aperiodic and we obtain a shock wave with a monotonic structure. We now determine the condition for monotonic or oscillating shock profiles by investigating the asymptotic behavior of the solutions of (17) for  $\zeta \to -\infty$ . We first substitute  $\varphi(\zeta) = \varphi_0 + \Phi(\zeta)$ , where  $\Phi \ll \varphi_0$ , into (17) and then linearize it with respect to  $\Phi$  in order to obtain

$$B\frac{d^2\Phi}{\partial\zeta^2} - C\frac{\partial\Phi}{\partial\zeta} + U_0\Phi = 0. \tag{19}$$

The solutions of (19) are proportional to  $\exp(p\zeta)$ , where p is given by

$$p = \frac{C \pm \sqrt{C^2 - 4BU_0}}{2B}. (20)$$

It turns out that the shock wave has a monotonic profile for  $S_c=C/2\sqrt{BU_0}>1$  and an oscillating profile for  $S_c<1$ . We have numerically analyzed  $S_c$  for space ( $\alpha=0.0288$  and  $\beta=3\times 10^{-10}$ ) and laboratory ( $\alpha=1.44$  and  $\beta=3\times 10^{-4}$ ) dusty plasma parameters and have found that  $S_c\gg 1$  is always valid for  $0<\mu<1$  (see Fig. 3).

This means that for typical space and laboratory dusty plasma parameters (17) [or (15) under steady state condition] exhibits a monotonic shock wave solution which is given by

$$\phi^{(1)} \simeq \psi_{sh} - \psi_{sh} \tanh \left[ \frac{\xi - U_0 \tau}{\Delta_{sh}} \right]$$
 (21)

where  $\psi_{sh}=U_0/A$  and  $\Delta_{sh}=2C/U_0$  represent the amplitude and the width of the shock wave, respectively. We note that since  $S_c\gg 1$ , we have neglected the dispersive term just to avoid the complexity of the mathematics which does not affect the

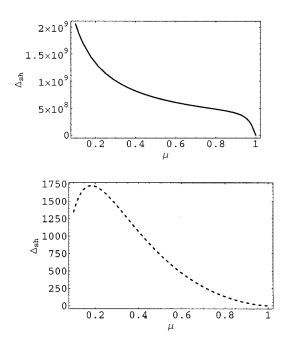


Fig. 4. The variation of the shock width  $\Delta$  with  $\mu$  for  $U_0=1$  and  $\epsilon=10^{-2}$ . The upper plot corresponds to space dusty plasma parameters:  $\alpha=0.0288$  and  $\beta=3\times 10^{-10}$ , whereas the lower plot corresponds to laboratory dusty plasma parameters:  $\alpha=1.44$  and  $\beta=3\times 10^{-4}$ .

properties of the shock structure that may exist in our space and laboratory dusty plasmas. Since  $\psi_{sh} = \psi/3$ , the variation of the shock amplitude  $\psi_{sh}$  with  $\mu$  for both space and laboratory dusty plasmas can be represented by the upper plot of Fig. 1. We have also graphically shown how the width  $\Delta_{sh}$  of the shock wave varies with  $\mu$  for both space ( $\alpha=0.0288$  and  $\beta=3\times 10^{-10}$ ) and laboratory ( $\alpha=1.44$  and  $\beta=3\times 10^{-4}$ ) dusty plasmas. This is depicted in Fig. 4. We note that the shock structures predicted by Popel *et al.* [23] do not follow our shock solution (21) and look like the ion density steepening as observed by Luo *et al.* [18]. However, the shock structures found in our present investigation are very close to those observed by Nakamura *et al.* [17].

### V. DISCUSSION

We have studied the properties of DIA solitary and shock waves in an unmagnetized dusty plasma composed of inertial ions, Boltzmann electrons, and immobile dust grains, including the dust charge fluctuation dynamics. Our results show that the effects of dust grain charge fluctuations modify the properties of the DIA solitary waves (see Fig. 1). It has been found that the effects of dust grain charge fluctuations reduce the speed of compressive DIA solitary waves. The characteristics of these DIA solitary waves in the space dusty plasma condition (see solid curves of Fig. 1) are found to be different from those in laboratory dusty plasma condition (see dotted curves of Fig. 1). It is seen that for the parameters corresponding to space dusty plasma situations (e.g., Saturn's E-ring [2], [3]:  $n_{d0} = 10^{-7} \text{ cm}^{-3}, r_d = 1 \ \mu\text{m}, k_B T_e = 50 \text{ eV}, \text{ and } Z_d = 10^3$ which correspond to  $\alpha = 0.0288$  and  $\beta = 3 \times 10^{-10}$ ) as we increase  $\mu$ , both the amplitude and the width of the DIA solitary waves remains constant for  $\mu$  < 0.5, but increases very rapidly for  $\mu > 0.5$  (see solid curves of Fig. 1). On the other hand, for laboratory dusty plasma parameters [5], [6] ( $n_{d0} = 10^5 \text{ cm}^{-3}$ ,  $r_d = 5 \, \mu\text{m}$ ,  $k_B T_e = 0.2 \text{ eV}$  and  $Z_d = 10^3 \text{ which correspond to}$   $\alpha = 1.44 \text{ and } \beta = 3 \times 10^{-4}$ ) as we increase  $\mu$ , the amplitude increases, but the width decreases (see dotted curves of Fig. 1).

We have shown that dust grain charge fluctuations are responsible for the formation of DIA shock waves in both space and laboratory dusty plasmas (see Fig. 3). The shock wave amplitude is one-third of the solitary wave amplitude, but their dependence on dusty plasma parameters are exactly the same. Furthermore, the shock width decreases with  $\mu$  (when  $1>\mu>0.2$ ) for both space and laboratory dusty plasma parameters. However, for  $0<\mu<0.2$  as we increase  $\mu$ , the shock width increases for laboratory dusty plasma conditions [2], [3], but decreases for space dusty plasma conditions [5], [6].

To conclude, we stress that the results of the present investigation should be useful in understanding the properties of localized DIA waves in space and laboratory dusty plasmas.

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