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The Role of Information in Consumer Debt and Bankruptcy

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Abstract

Consumer debt and bankruptcy are central issues today because of their explosive trends over the last 20 years in the U.S. economy. However, there is no convincing explanation for these facts. A drop in information costs, a potential cause, has not been evaluated mainly because there is no quantitative theory of consumer debt and bankruptcy where the cost of information plays an important role. This paper provides such a theory and quantifies how much of the rise in debt and bankruptcy can be attributed to the drop in information costs. In the model, lenders offer contracts specifying both interest rates and borrowing limits. In equilibrium, the contracts with low interest rates have tight borrowing limits, while those with high interest rates have loose borrowing limits. Despite being borrowing constrained, low-risk individuals prefer to borrow at the low interest rate. Conversely, high-risk individuals prefer to borrow more at higher interest rates. As the costs of information drop, it may be possible to explicitly condition loans on an individual's risk. This allows previously borrowing constrained individuals to borrow more. As a result, there is also more bankruptcy because the benefits of filing bankruptcy are increasing in the debt size. The quantitative importance of this mechanism is then investigated by calibrating the model's parameters to match moments for the years 1983 and 2004. The model can successfully match key data moments for both years varying only the cost of information and the income distribution. To quantify the effect of the drop in information costs over the last 20 years, two counterfactual economies are computed. The main finding is that the drop in information costs alone generates around 40% of the total rise in consumer bankruptcy.

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1 Introduction

Consumer debt and bankruptcy are central issues today because of their explosive rise over the last 20 years in the U.S. economy. Although many explanations have been proposed, there is still no convincing understanding of these trends. A candidate story is the drop in information costs. This driving force may be important because during the same period there was impressive technological progress in the information sector—often called IT revolution—and the financial sector uses information intensively to evaluate credit risk.[1] This story has not been evaluated mainly because there is no *quantitative theory* of consumer debt and bankruptcy where the cost of information production plays an important role. The purpose of this paper is to provide such a theory and to quantify how much of the rise in debt and bankruptcy can be attributed to the drop in information costs.

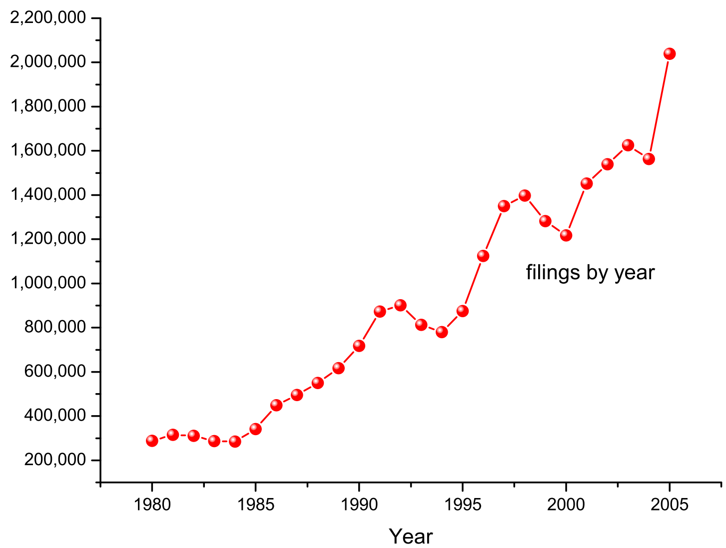
The number of annual *bankruptcy filings* increased by 1.3 million—from 286,444 to 1,563,145, almost 5.5 times—between 1983 and 2004, as depicted in Figure 1. Before the early 1980s, the rise in bankruptcy was moderate. According to Moss and Johnson (1999), "from 1920 to 1985, the growth of consumer filings closely tracked the growth of real consumer credit. Since then, however, the rate of increase of consumer bankruptcies has far outpaced that of real consumer credit." Therefore, a study about the rise in bankruptcy should also consider the trend in consumer debt. According to White (2007), credit card debt rose from 3.2% of median family income to 12% from 1980 to 2004. Other statistic, the ratio of bankruptcy filings to the number of households in debt, is particularly useful because it increases only if the number of filings grow faster than the number of households in debt. This statistic, referred to as the *bankruptcy rate* hereafter, increased from 0.92% to 3% between 1983 and 2004.

This paper builds a quantitative theory of consumer debt and bankruptcy with *asymmetric information* and costly screening. The *type* of an individual, i.e. the income group the individual belongs to, is persistent and unobservable. Lenders would like to know the individual's type because persistence implies that her type is useful to predict the probability of bankruptcy. In particular, individuals with lower income have higher risk of bankruptcy because they are more likely to have low income in the next period. The availability of costly screening divides the lenders into two groups, those that use a screening technology, *informed lenders*; and those that instead design debt contracts to induce borrowers to reveal their type, *uninformed lenders*. Individuals decide, given the cost of information, which kind of lender they prefer to borrow from.[2]

[1] For a careful description of the use of information technologies in the financial sector see the work of Berger (2003). For an analysis of the effect of progress in monitoring technologies on the allocation of capital, firms' financing and capital deepening see the study of Greenwood, Sanchez, and Wang (2007).

[2] Notice that zero profits implies that borrowers pay the cost of information production, directly or through

Figure 1: Consumer debt and bankruptcy



Source: American Bankruptcy Institute

When screening costs go to zero, the model collapses to the one of Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), where there is perfect information so all the individuals borrow from "informed lenders". Instead, if the cost of information is higher, some individuals will borrow from uninformed lenders. Since low-income individuals are more likely to file for bankruptcy, they accept a higher interest rate than high-income individuals to borrow more. As a consequence, uninformed lenders can achieve self-revelation of types: the contracts for high-income individuals have lower interest rates and tighter borrowing constraints. Thus, under these contracts—when information costs are high enough—some individuals are borrowing constrained. This fact is crucial for understanding the effect of information costs on debt and bankruptcy. As information costs drop, individuals borrow more, and the number of bankruptcy filings rises. More debt generates more bankruptcy because the benefit from bankruptcy—discharge of debts—is increasing in the amount owed, while the costs—temporary exclusion from financial markets and income lost—are independent of the individual's debt size. Therefore, a drop in information costs leads to more debt and more bankruptcy, two comparative statics results qualitatively consistent with the facts presented above.

The model is calibrated to account for relevant features of the U.S. data for the year 2004. Specifically, it reproduces the bankruptcy rate, the debt-to-income ratio,

prices, whenever they decide to borrow from informed lenders.

the capital-to-output ratio, and some moments of the joint distribution of debt and income. The model in then used to answer two quantitative questions. The first question is: If the income process is recalibrated for the year 1983, *can* a change in the cost of information reproduce the bankruptcy rate and the debt-to-income ratio for that year? Recalibrating only the income process and the cost of information, the model can actually reproduce both the bankruptcy rate and the debt-to-income observed in 1983. The second question is: How much of the rise in the bankruptcy rate can be explained by the drop in information costs? Two counterfactual exercises are performed to answer this question. First, the model economy is computed with the technology in the information sector for the year 1983 and all the other parameters for the year 2004. The result is that the drop in information costs alone explains 37 percent of the rise in the bankruptcy rate between the years 1983 and 2004. In the second counterfactual exercise, the model economy is computed with the technology in the information sector for the year 2004 and all the other parameters for the year 1983. In that case, the drop in information costs alone accounts for 45 percent of the total rise in the bankruptcy rate between 1983 and 2004.

In addition to the rise in debt and bankruptcy, a drop in information costs has three distinguishing implications. First, it generates changes in the distribution of debt across income groups. Individuals in the second and third decile of the income distribution are borrowing constrained in the economy with costly information and borrow more in the economy with perfect information. Second, a drop in the cost of information production increases the dispersion of interest rates. This is because under asymmetric information, lenders can only use the contracts satisfying self-revelation. In fact, in the economy calibrated for the year 1983 most of the individuals borrow at the risk free interest rate. Third, when lenders use more information, interest rates depend on income. More precisely, given the amount of debt, interest rates are decreasing in income only when the lenders use information.

All the distinguishing implications described above are consistent with the data from the SCF. Table 5 shows that households in the second and third decile of the income distribution hold a bigger proportion of total debt in 2004 than in 1983. The second (third) income decile had 7.8% (10.4%) of the total debt in 1983 and 15% (19%) in 2004. The dispersion of interest rates also rose from 1983 to 2004. Figure 10 depicts the distribution of interest rates for both years. The dispersion of interest rates measured by the standard deviation rose from 4.1 in 1983 to 6.3 in 2004. Finally, Table 4 shows the projection of income on interest rates. In 1983 the effect of income on interest rates is not significantly different than zero. In contrast, in 2004 a rise in income of 1% decreases the interest rate in -0.8%.[3] This result not only supports the

[3] Edelberg (2006) studies risk-based pricing of interest rates for consumer loans and finds similar results.

hypothesis that lenders use more information but also indicates that it is appropriate to focus on information about individuals' income.

This paper builds on previous literature on consumer debt and bankruptcy. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) study a quantitative theory of unsecured consumer credit with risk of bankruptcy. Although they successfully explain some facts about debt and bankruptcy in the U.S., they do not consider asymmetric information, and they do not explain the rise in bankruptcy during the last 20 years.[4] There are three recent papers interested in the rise in bankruptcy: Livshits, MacGee, and Tertilt (2007a), Narajabad (2007), and Drozd and Nosal (2008). The work of Livshits, MacGee, and Tertilt (2007a) is less related to the current paper because they do not consider a change in information technologies. Nevertheless, their work is relevant because they evaluate the role played by several driving forces that have been proposed to explain the rise in bankruptcy. Their result is that the main factor explaining the rise in bankruptcy is a reduction in the preference parameter representing the *cost of bankruptcy*—called stigma. More related to this work, Narajabad (2007) evaluates the role of more informative credit rating technologies in an environment with heterogeneity in the *cost of bankruptcy*. There are two crucial differences with this paper. First, individuals do not know their type—their own cost of bankruptcy—when they sign a debt contract. This assumption is crucial because it makes "direct-revelation contracts" impossible and implies that the key mechanism at work in this model is ruled out by assumption. Second, restrictive assumptions make his model not very suitable for quantitative purposes. For instance, by assuming that individuals cannot save, he makes any comparison between the model and data on the distribution of assets—key in a model of debt—impossible. It is worth noticing that the cost of bankruptcy plays a crucial role in both papers: Livshits, MacGee, and Tertilt (2007a) vary this parameter across time, while Narajabad (2007) uses a distribution of this parameter across individuals. Since this parameter does not have a clear counterpart in the data, both strategies are hard to justify. The current paper explains a sizeable proportion of the rise in the bankruptcy rate with only one value for this parameter; no changes across time or individuals are required. Finally, Drozd and Nosal (2008) present a search model of the market for unsecured credit. They study the effect of a drop in the cost of screening and soliciting credit customers on debt and

[4] Athreya (2002) and Livshits, MacGee, and Tertilt (2007b) use slightly different quantitative models than Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) to understand similar facts as theirs. Chatterjee, Corbae, and Rios-Rull (2007a) and Chatterjee, Corbae, and Rios-Rull (2007b) incorporate asymmetric information in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) model of consumer debt and bankruptcy. However, in their model asymmetric information takes a substantially different form and they do not study quantitative implications. Likewise, Dubey, Geanakoplos, and Shubik (2005) study a general equilibrium theory of secured debt with private information and bankruptcy; they are not interested in the rise in bankruptcy and they do not study a quantitative version of their model.

bankruptcy. The first component of this cost, the cost soliciting credit, plays a similar role to the transaction cost analyzed by Livshits, MacGee, and Tertilt (2007a): it rises debt and therefore it increases the number of bankruptcy filings. The second component, the cost of screening, is potentially closer to the current paper. However, Drozd and Nosal (2008) do not model asymmetric information, lenders have no alternative to paying the cost of screening, and therefore their mechanism differs substantially from the current paper.

The paper is organized as follows. Section 2 starts with an illustrative two-period and two-type model. It then describes the main qualitative implications of a drop in the cost of information. Section 3 develops the quantitative general equilibrium model. In Section 4, the model is calibrated to the U.S. for the year 2004 assuming the cost of information is zero. Additionally, this section examines the effect of shifts in the cost of information production on debt and bankruptcy. Section 5 concludes.

2 Debt, bankruptcy, and information: 2×2 model

This section previews the main driving forces at work in the full model using a simple two-period and two-type model.[5] An important simplification is that the analysis is in partial equilibrium; i.e., the risk-free interest rate, i, and wages, w, are given.

The economy is populated by infinitely many individuals and lenders. Individuals live for 2 periods, $t = 1, 2$, and they are endowed with a quantity of labor measured in efficiency units, l_n, that can take 2 values, $l_n \in \{l_L, l_H\}$. The transition probability between state L and H is $\pi_{L,H}$. Persistence is also assumed: $\pi_{H,H} > \pi_{H,L}$ and $\pi_{L,L} > \pi_{L,H}$. Importantly, it implies $\pi_{H,H} > \pi_{L,H}$.

Lenders compete offering debt contracts. In particular, there are two kinds of lenders: *informed lenders* use a screening technology to learn an individual's type, and charge the discount price function $\widetilde{q}$, while *uninformed lenders* design contracts to induce individuals to reveal their type and charge the discount price function $\widehat{q}$. A discount price $\mathbf{q}$ indicates that a borrower must pay one unit of consumption next period for a loan of size $\mathbf{q}$ today. The cost of screening an individual's type (also referred to as information costs), $\mathcal{C}$, is proportional to the amount borrowed to simplify the graphical analysis.[6]

In period 1, agents decide how much to borrow/save for next period and from which kind of lender. In period 2, after the realization of the labor endowment shock is observed by the agents, they decide whether to file for bankruptcy or pay back the

[5] This 2×2 model is studied in detail in Sanchez (2007), a companion paper that can be found at `troi.cc.rochester.edu/~sncz/research`.

[6] Later, in the quantitative general equilibrium model, this cost is independent of the amount borrowed.

debt. After bankruptcy, they lose a proportion of their income, τ. Thus, the lifetime utility of an individual born with assets $a_1 \in \mathbf{A}$, income $y_n = wl_n$, and facing a discount price function q is

$$U(a_1, y_{1,n}; q) = \max_{a_2 \in \mathbf{A}} u(y_{1,n} + a_1 - q(a_2, n; a_1)a_2) + \beta\pi_{n,H} \max\{u(y_{2,H} + a_2), u(y_{2,H}(1-\tau))\} \\ + \beta\pi_{n,L} \max\{u(y_{2,L} + a_2), u(y_{2,L}(1-\tau))\}.$$

Here the discount price function q is used to represent $\widetilde{q}$ or $\widehat{q}$. Then, the choice of lender implies that lifetime utility is $U(a_1, y_{1,n}) = \max\{U(a_1, y_{1,n}; \widetilde{q}), U(a_1, y_{1,n}; \widehat{q})\}$.

Consider an individual that has to make a decision about bankruptcy. It is clear that this decision is characterized by

$$\begin{array}{ll} y_{2,n} + a_2 \geq y_{2,n}(1-\tau), & \text{pay back,} \\ y_{2,n} + a_2 < y_{2,n}(1-\tau), & \text{declare bankruptcy.} \end{array}$$

This implies simple threshold levels of assets for each level of income at which individuals are indifferent between filing bankruptcy and paying back the debt,

$$\begin{aligned} \underline{a}_{2,L} &= -\tau y_{2,L}, \\ \underline{a}_{2,H} &= -\tau y_{2,H}, \end{aligned}$$

where $\underline{a}_{2,H} < \underline{a}_{2,L}$ because $y_{2,L} < y_{2,H}$. The interpretation of these values is intuitive. If an individual borrows less than the limit for the low level of income, $a_2 > \underline{a}_{2,L}$, she will pay back the debt next period if her income is low. Since $\underline{a}_{2,H} < \underline{a}_{2,L}$, she will also pay back if the level of income next period is high. Notice also that for $\underline{a}_{2,H} < a_2 < \underline{a}_{2,L}$ the individual will file for bankruptcy next period only after a transition toward the low labor endowment (what happens with probability $\pi_{n,L}$).

These threshold values are also important because they are useful to characterize zero-expected-profit discount prices. Lenders expected profits from each contract are

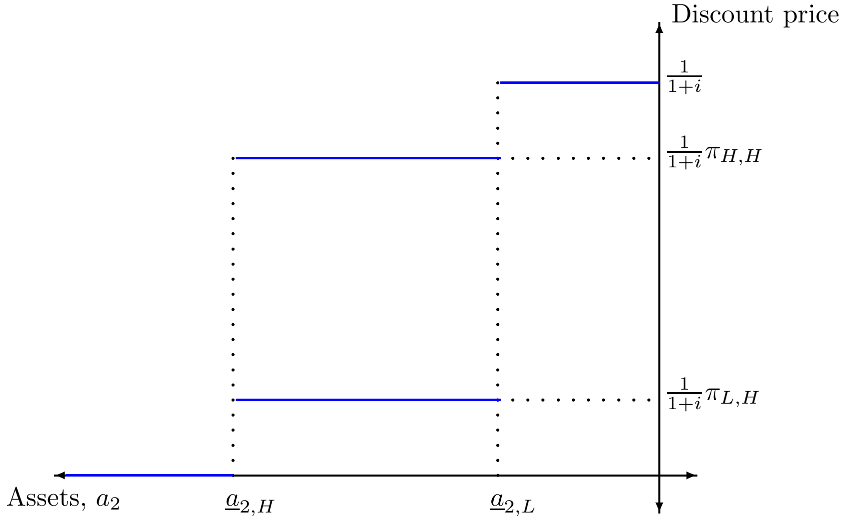
$$\underbrace{q(a_2, n)a_2}_{\text{amount borrowers receive}} - \underbrace{\Pr(\text{repayment} \mid a_2, n)a_2(1+i)^{-1}}_{\text{discounted amount lenders expect to recover}},$$

where $\Pr(\text{repayment} \mid a_2, n)$ is the lender's expectation of repayment given the amount borrowed $(-a_2)$ and the individual's type (n). Then, the zero-expected-profit condition implies discount prices for each type $n = \{L, H\}$

$$q(a_2, n) = \begin{cases} (1+i)^{-1} & \text{if } \underline{a}_{2,L} \leq a_2, \\ \pi_{n,H}(1+i)^{-1} & \text{if } \underline{a}_{2,H} \leq a_2 < \underline{a}_{2,L}, \\ 0 & \text{if } a_2 < \underline{a}_{2,H}. \end{cases}$$

Thus, zero-expected-profit discount prices vary as in Figure 2. Notice that price functions are flat for some ranges of assets and have two jumps. This is because labor endowments can take just 2 values.

Figure 2: Zero-expected-profit discount prices, q



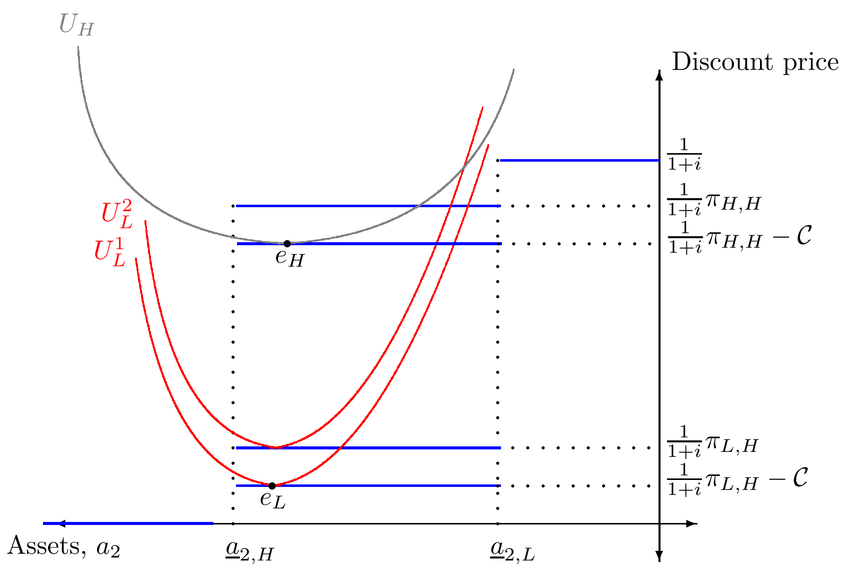
The indifference curves between the discount price, $\mathbf{q}$, and the amount of assets for period 2, a_2, can be described. Specifically, it is important to characterize preferences over a_2 and $\mathbf{q}$ in the range $\underline{a}_{2,H} < a_2 < \underline{a}_{2,L}$, where borrowing implies risk of bankruptcy. The slope of the indifference curves as a function of a_2 is

$$-MRS_{\mathbf{q},a_2}(\mathbf{q},a_2)\begin{cases} <0, & \text{for } \underline{a}_{2,H} < a_2 < a_2^*(\mathbf{q}), \\ =0, & \text{for } a_2 = a_2^*(\mathbf{q}), \\ >0, & \text{for } a_2^*(\mathbf{q}) < a_2 < \underline{a}_{2,L}, \end{cases}$$

where $a_2^*(\mathbf{q})$ is the level of asset accumulation solving the first order condition of the individual's problem given a discount price function constant at $\mathbf{q}$. Moreover, notice that the slope of the indifference curves are different across individuals' types. Take any $\mathbf{q}$ and consider any $a_2^*(\mathbf{q}) < a_2 < \underline{a}_{2,L}$. It is clear that the slope is steeper for the individual with low income in period 1. Intuitively, this follows because the individual with low income in period t is willing to borrow more than an individual with high income.

Equilibrium contracts with *informed lenders*. Informed lenders use a screening technology to learn individuals' types and charge type-specific discount prices. If this is the only kind of lenders in the economy, there are three conditions that contracts must satisfy to be equilibrium contracts: *(i) lenders obtain zero expected profits from each contract, (ii) lenders expectations about repayment probability are realized in equilibrium, and (iii) there is no contract that, if offered, will imply positive expected profits.* Condition *(i)* and *(ii)* imply that equilibrium discount prices are the

Figure 3: Equilibrium discount prices with informed lender, $\widetilde{q}$



zero-expected-profit discount prices minus the cost of information, independently of who pays the information costs. Figure 3 describes an equilibrium allocation considering an economy with only *informed lenders*. The individual with low income will borrow at e_L, where she is maximizing her utility given prices, while the individual with high income will borrow at e_H, also maximizing her utility given prices. Since lenders make zero expected profits with each type of loan, and borrowers' indifference curves at equilibrium are above the zero-expected-profit discount prices, then there is no profitable contract that *informed lenders* could offer and individuals would like to accept. This implies that $\widetilde{q}$ in Figure 3 is the unique equilibrium discount price function with informed lenders.

Importantly, it is clear from Figure 3 why *uninformed lenders* should be allowed. The point e_L cannot be an equilibrium if lenders can offer contracts without paying for information. For instance, lenders could charge the type-L-zero-expected-profit discount prices, $\pi_{L,H}/(1+i)$, to those that decide not to pay for information. Then, type-L individuals should take that offer—they could obtain lifetime utility $U_L^2 > U_L^1$.

Equilibrium contracts with *uninformed lenders*. Uninformed lenders design contracts under the constraint that they must induce individuals to reveal their type; i.e., direct-revelation contracts.[7] Using discount prices and borrowing constraints as instruments it is possible to induce individuals to reveal their type. It is indeed possible to separate individuals using these two variables because in order to obtain looser borrowing limits, low-income individuals are willing to accept a bigger increase in interest rates than high-income individuals; i.e., indifference curves are as depicted in Figure 3. Discount prices and debt limits, $\{q; \underline{a}_2\}$, can be built-in in a discount price function $q(a_2, n; a_1)$ by setting the price at zero for asset levels lower than the constraint, $a_2 < \underline{a}_2(n, a_1)$. Then, q is a direct-revelation contract if and only if

$$\begin{aligned} U(a_1, y_{1,L}; q(\cdot, L; a_1)) &\geq U(a_1, y_{1,L}; q(\cdot, H; a_1)), \ \forall a_1 \in \mathbf{A}, \\ U(a_1, y_{1,H}; q(\cdot, H; a_1)) &\geq U(a_1, y_{1,H}; q(\cdot, L; a_1)), \ \forall a_1 \in \mathbf{A}. \end{aligned}$$

In words, q satisfies direct revelation if and only if individuals are better off borrowing at the discount price designed for their type than pretending to be a different type.

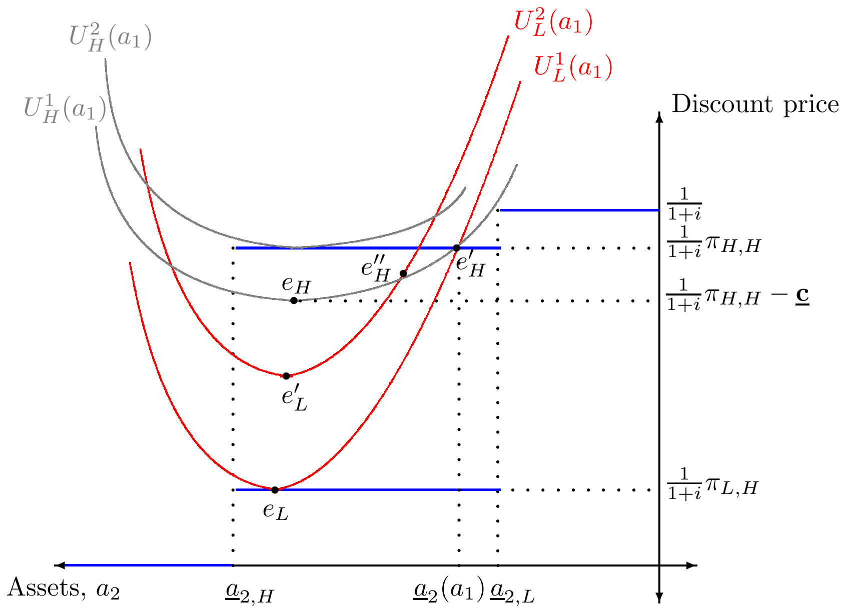
There are three conditions that direct-revelation contracts must satisfy to be equilibrium contracts: *(i) lenders obtain zero expected profits from each contract; (ii) lenders expectations about repayment probability are realized in equilibrium; (iii) there is no direct-revelation contract that, if offered, will imply positive expected profits.* Condition *(i)* and *(ii)* imply that discount prices are the zero-expected-profit discount prices. Condition *(iii)* holds if debt limits are as loose as possible.

The equilibrium debt limits for individuals with assets a_1 can be found with the help of Figure 4 following the next steps. First, maximize the utility of the individual with the lowest income given zero expected profits for this type; this is the point e_L in Figure 4. After that, maximize utility of the other type given zero expected profits and incentive compatibility. The equilibrium allocation for the individual with high income is at the point e'_H, where the incentive compatibility constraint is binding, meaning that high-income individuals are borrowing constrained. Thus, if there is an equilibrium with direct-revelation contracts, the debt limit for assets a_1 is $\underline{a}_2(a_1)$ in Figure 4—to save in notation $\underline{a}_2(a_1)$ is used instead $\underline{a}_2(H, a_1)$. Nevertheless, as it is studied in the next subsection, existence of equilibrium is difficult to guarantee in this environment.

There is another important implication of this allocation. We can find the threshold cost of information, $\underline{\mathbf{c}}$ in Figure 4. This is the cost of information at which the individual is indifferent between signing a contract with informed or uninformed lenders; i.e., the point e_H, at which the high-income individual is borrowing from informed lenders if the cost is $\underline{\mathbf{c}}$, is on the same indifference curve, U_H^1, as the point e'_H, at which the high-income individual is using the contract offered by an uninformed lender. Thus,

[7] This constraint does not prevent uninformed lenders from offering the same discount price for different types. In fact, it can be shown that for levels of debt small enough, the discount price uninformed lenders charge is the same for all the types.

Figure 4: Equilibrium discount price with uninformed lenders, $\widehat{q}$



if the cost $\mathcal{C}$ is lower than $\underline{\mathbf{c}}$, then high-income individuals prefer to take the contract informed lenders offer. On the other hand, low-income individuals will never want to pay to reveal their type.

The problem of existence. It is well known since Jaffee and Russell (1976) that the existence of equilibrium in credit markets with asymmetric information, bankruptcy and strategic lenders is troublesome. Using the equilibrium concept introduced by Rothschild and Stiglitz (1976), a pooling equilibrium cannot exist because at that allocation indifference curves will cross, implying that lenders have incentive to deviate offering a contract that just high-income individuals will accept.[8] As it is explained below, a separating equilibrium, such as the one studied here, cannot be guaranteed either. Two alternative strategies to overcome this problem are introduced next.

First, a question is in order, is the allocation (e_L, e'_H) in Figure 4 an equilibrium contract with *uninformed lenders*? This question can be rephrased as: given the allocation (e_L, e'_H), is there a direct-revelation contract that, if offered, will provide lenders with positive expected profits? A negative answer to this question means that lenders are maximizing—there are no other contracts they could design to make profits. Offers like (e'_L, e''_H) in Figure 4 threaten equilibrium existence. High-income individuals

[8] Dubey, Geanakoplos, and Shubik (2005) assume lenders take the pool as given and find equilibrium with pooling of different types.

should take e''_H because it is slightly above the curve U^1_H. Low-income individuals should also accept this offer because e'_L is at U^2_L, strictly above U^1_L. The remaining question is: is this offer profitable? It may be. It implies losses with low-income individuals—e'_L is above $\pi_{L,H}/(1+i)$—and gains with high-income individuals—e''_H is below $\pi_{H,H}/(1+i)$. Total expected profits depend on the measure of (how many) individuals that take each contract. That is exactly why equilibrium existence cannot be guaranteed.

The ignorance on the measure of each type that takes a contract like (e'_L, e''_H) is the key for the problem of existence. Therefore, both strategies to overcome this problem are focused on this measure. First, for each candidate equilibrium allocation, it is possible to use the stationary distribution for that parametrization to compute if contracts like (e'_L, e''_H) are profitable. Thus, the first strategy to overcome the problem of existence is a *numerical ex-post verification*. If the deviation is profitable, then (e_L, e'_H) is not an equilibrium and there is no equilibrium for that parametrization. Otherwise, if there is no profitable deviation, (e_L, e'_H) is an equilibrium.[9]

The second strategy to overcome the problem of existence is theoretical. Now, off the equilibrium beliefs about the measure of individuals of each type that an offer is not necessarily rational. These beliefs are used in the spirit of Maskin and Tirole (1992): a contract is now an equilibrium if conditions (i) and (ii) above hold, and there is no other direct-revelation contract implying positive expected profits for all *reasonable beliefs*.[10] This simplifies the search for profitable deviations because an equilibrium-breaking contract—like (e'_L, e''_H)—can be discarded if there is a reasonable belief implying negative expected profits. Then, the key is the definition of reasonable beliefs that is adopted: a belief is reasonable if it is consistent with individuals optimal behavior. Thus, this definition rules out clearly false beliefs; in particular, lenders must believe that no high-income individual will accept an offer if that offer makes those individuals worse off. On the other side, if an offer makes both types better off, beliefs about the proportion of individuals of each type taking each contract are not restricted by the concept of reasonable beliefs. As it was explained above, the offer (e'_L, e''_H) should be accepted by both types of individuals. Since lenders make losses with type-L individuals, if they expect that sufficiently many type-L individuals will take the offer, then their expected profits are negative. Thus, since allocations like (e'_L, e''_H) do not imply positive expected profits for all reasonable beliefs, the allocation

[9] The complete procedure is described in the computational appendix to this paper available at `troi.cc.rochester.edu/~sncz/research`.

[10] Closer to Maskin and Tirole (1992), one could state that a contract is an equilibrium if conditions (i) and (ii) hold and there is no other direct-revelation contract implying positive expected profits for all beliefs. In that case there is multiple equilibria. Departing from this alternative definition, the equilibrium with reasonable beliefs is an equilibrium refinement.

(e_L, e'_H) is an equilibrium. In fact, it can be shown that it is the unique equilibrium.

The effect of information costs on debt and bankruptcy. Lower information costs allow high-income individuals to borrow more, making them more likely to file for bankruptcy. This effect is explained with the help of Figure 5. First, assume the cost of information is high enough, $\mathcal{C}_0 > \underline{\mathbf{c}}$. This implies the equilibrium allocation is $(\mathbf{e}, \mathbf{e})$ in Figure 5, meaning that both individuals borrow the same amount at the same discount price. In this allocation both types are better off borrowing less at the risk-free discount price than taking the uninformed lenders contract; i.e. $U_H^2 > U_H^1$ and $U_L^2 > U_L^1$. Here, high-income individuals are clearly borrowing constrained: they would prefer to borrow more at their zero-expected-profit discount price, but those prices cannot be offered because low-income individuals will pretend to be them. Since it was assumed $\mathcal{C}_0 > \underline{\mathbf{c}}$, high-income individuals are also better at $\mathbf{e}$ than paying the cost of information. Notice that at this initial equilibrium allocation, $(\mathbf{e}, \mathbf{e})$, bankruptcy in this economy is actually zero. Both individuals borrow so little that bankruptcy is not optimal at any possible income level next period.

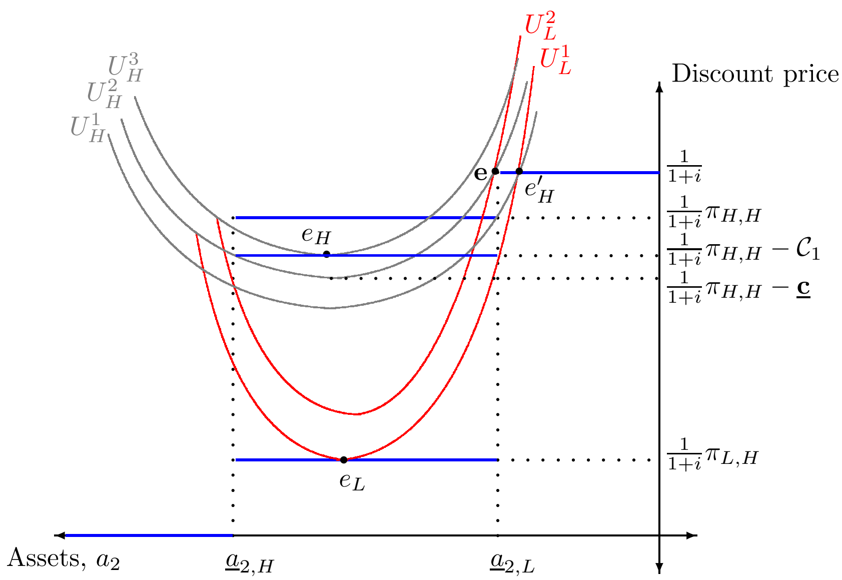
Now, assume technological progress occurred in the information sector implying that $\mathcal{C}_1 < \underline{\mathbf{c}}$, as it is shown in Figure 5. This implies that high-income individuals prefer to pay the cost of information and borrow at e_H; i.e., $U_H^3 > U_H^2$. At the new equilibrium allocation, $(\mathbf{e}, e_H)$, there is more bankruptcy because now high-income individuals will file for bankruptcy with probability $(1 - \pi_{H,H})$. Additionally, notice that there is more debt, since high-income individuals borrow more at the new allocation.

The remaining question is which of the results of this example can be generalized. This is important because preferences can be different than in Figure 5. For instance, if individuals with the lowest income are borrowing while individuals with higher income are only saving, then self-revelation is costless, and changes in information costs have no effect on the equilibrium allocation. The general result is that borrowing-constrained individuals at the initial cost of information ($\mathcal{C}_0 > \underline{\mathbf{c}}$) respond to a drop in information costs (toward $\mathcal{C}_1 < \underline{\mathbf{c}}$) by

1. Increasing the amount borrowed. This is because borrowing constrained individuals at $\mathcal{C}_0 > \underline{\mathbf{c}}$ will prefer to pay for information at $\mathcal{C}_1 < \underline{\mathbf{c}}$ and borrow more.
2. (Weakly) Increasing the probability of bankruptcy. More debt implies more bankruptcy only if at this higher level of debt, there are more levels of income next period at which the individual will prefer to file for bankruptcy, i.e., if the higher level of debt is borrowed at a lower discount price;

These two results are important because they are qualitatively consistent with the facts presented above for the U.S. economy. Moreover, three additional results arise

Figure 5: The effect of information costs on debt and bankruptcy



as the cost of information production drops. First, the proportion of debt is held by individuals with relatively high income rises. This is because the debt of individuals with the lowest income does not change, while it increases for individuals with higher income. Second, the dispersion of interest rates increases. This results arises because when information is cheaper self-revelation is not required, and more contracts are offered. Third, the relationship between interest rates and income becomes stronger. Given the amount borrowed, lenders using information offer lower interest rates to individuals with higher income. Without information, only one interest rate can be offered for each amount of debt.

As it was discussed in the introduction, these distinguishing implications are consistent with the data. Hereafter a general equilibrium model is developed in an attempt to account, quantitatively, for both the rise in consumer debt and bankruptcy.

3 Quantitative General Equilibrium Model

3.1 The Model

Environment. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ At any time there is a unit mass of individuals. They discount future at the rate β. Preferences of individuals are given by the expected value of the discounted sum of momentary utility

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right],$$

where c_t is consumption at period t. The utility function u is strictly increasing, strictly concave, and twice differentiable. Let $n \in \mathbf{N} = \{1, 2, \ \ldots, N\}$ denote the *type* of an individual. Types are persistent, with transition probability $\Pi(n_t, n_{t+1})$. Each individual is endowed with one unit of time. Productivity is exogenously determined by labor endowments that come from different type-specific intervals; for each n, $l \in L(n) = [\underline{l}^n, \bar{l}^n]$. Thus, labor endowments and types at time t are correlated. The transition function is $\phi(l_{t+1} \mid n_{t+1})\Pi(n_t, n_{t+1})$, where $\phi(l_{t+1} \mid n_{t+1})$ is a conditional density function.

Information structure. There is asymmetric information between lenders and borrowers about the latter's types, n. On one side, individuals know their type n. On the other side, if borrowers are not screened, then the type is private information. Nevertheless, each lender has access to a technology that can be used to learn an individual's type at a fixed cost. The stock of assets, a_t, is publicly observable, as well as the credit flag indicating the bankruptcy record (defined later).

Information firm's problem. The information firm uses labor to produce information with the production function

$$z_t^i(\mathbf{m}_t)^{1/\gamma},$$

where z_t^i is the productivity in information production and $\mathbf{m}_t$ is labor demanded in the information industry. This sector is simplified assuming it produces $\{0, 1\}$, where 0 means no information is produced and 1 means "a report with information about the borrower's type is produced". Then, zero expected profits in this sector implies that the cost of learning a borrower's type (or screening cost) is $\mathcal{C}(z_t^i, w_t) = w_t(z_t^i)^{-\gamma}$.

Production firm's problem. It rents capital at the rate r_t and hires labor at the wage w_t. With these factors the firm produces consumption goods in line with a standard Cobb-Douglas production function. Thus, the firm's problem is

$$\max_{\{\mathbf{l}_t, \mathbf{k}_t\}}\left\{z_t^p(\mathbf{k}_t)^{1-\theta}(\mathbf{l}_t)^{\theta} - w_t\mathbf{l}_t - r_t\mathbf{k}_t\right\},$$

where z_t^p is the technology in the production sector, and $\{\mathbf{l}_t, \mathbf{k}_t\}$ are labor and capital in this sector, respectively.

Credit industry. There are two kinds of intermediaries, referred to as *informed lenders* and *uninformed lenders*. There are many lenders of each kind competing among themselves offering debt contracts. They own the stock of capital, which they rent to the firms in the production sector.

Informed lenders' problem. Borrowers have to pay the screening cost to be able to sign a contract with an informed lender. Think of borrowers buying a report at the information industry that proves their type and submitting it to informed lenders. The discount price charged is $\tilde{q}(a_{t+1}, n_t)$; i.e., a different discount price for each level of assets next period, $a_{t+1} \in \mathbf{A}$, and type, $n_t \in \mathbf{N}$. The discount price depends on a_{t+1} because it determines the debt the individual will have to pay back next period, which in turn affects her willingness to pay back the debt. It depends on n_t because this determines the transition probability to different income levels, and thereby the probability of bankruptcy. Let $\widetilde{d}_{a_{t+1},n_t}$ denote the number (measure) of contracts for individuals with $\{a_{t+1}, n_t\}$ that informed lenders sell, $\widetilde{K}_{t+1}$ the stock of capital they accumulate for period $t+1$, and $\Pr(repayment \mid a_{t+1}, n_t)$ the repayment probability of this contract. Then, period-t cash flow is given by

$$\begin{aligned}\widetilde{P}_t = \; & - \; \textstyle\sum_{n_{t-1}} \int_{a_t} \widetilde{d}_{a_t,n_{t-1}} \Pr(repayment \mid a_t, n_{t-1}) a_t da_t \\ & + \; \textstyle\sum_{n_t} \int_{a_{t+1}} \widetilde{d}_{a_{t+1},n_t} \tilde{q}(a_{t+1}, n_t) a_{t+1} da_{t+1} \\ & + \; (1 - \delta + r)\widetilde{K}_t - \widetilde{K}_{t+1}.\end{aligned}$$

Lenders design the contracts and choose $\widetilde{d}_{a_{t+1},n_t}$ and $\widetilde{K}_{t+1}$ to maximize the present discounted value of current and future cash flows,

$$\sum_{t=0}^{\infty} (1 + i_t)^{-t} \widetilde{P}_t,$$

given the risk-free interest rate at period t, i_t, the initial stock of capital, $\widetilde{K}_0$, and the number of each different contract initially sold, $\widetilde{d}_{a_0,n_{-1}}$.

The sequence of cash flows implies a sequence of risk-free bond holdings, $\{\widetilde{B}_{t+1}\}_{t=0}^{\infty}$, which can be obtained by the recursion

$$\widetilde{B}_{t+1} = (1 + i_t)\widetilde{B}_t + \widetilde{P}_t,$$

where $\widetilde{B}_0 = 0$. These bonds, which are issued by the lenders, are incorporated to allow cash flows accumulation. They are not that important hereafter since they will be zero in the stationary equilibrium defined later; i.e., $\widetilde{B}_t = \widetilde{B} = 0$.

Uninformed lenders' problem. These lenders compete offering direct-revelation contracts. The condition a contract has to satisfy to be "direct-revelation" is formally stated later, after the individual's problem is introduced. That condition basically states that, given the contract design, borrowers are better off revealing their type. Since the current stock of assets affects an individual willingness to borrow, discount prices satisfying the revelation constraint depend also on this variable.

Some notation in now introduced. Let $\widehat{d}_{a_{t+1},n_t;a_t}$ denote the number (measure) of contracts uninformed lenders sell for individuals with $\{a_{t+1},n_t,a_t\}$, $\widehat{K}_{t+1}$ the stock of capital they accumulate for period $t+1$, and $\Pr(repayment \mid a_{t+1},n)$ the repayment probability. Then, period-t cash flow is given by

$$\begin{aligned}\widehat{P}_t = \quad & - \quad \textstyle\sum_{n_{t-1}}\int_{a_{t-1}}\int_{a_t}\widehat{d}_{a_t,n_{t-1};a_{t-1}}\Pr(repayment \mid a_t,n_{t-1})a_t da_t da_{t-1}\\ & + \quad \textstyle\sum_{n_t}\int_{a_t}\int_{a_{t+1}}\widehat{d}_{a_{t+1},n_t;a_t}\widehat{q}(a_{t+1},n_t;a_t)a_{t+1}da_{t+1}da_t\\ & + \quad (1-\delta+r)\widehat{K}_t-\widehat{K}_{t+1}.\end{aligned}$$

Lenders design the contract and choose $\widehat{d}_{a_{t+1},n_t;a_t}$ and $\widehat{K}_{t+1}$ to maximize

$$\sum_{t=0}^{\infty}(1+i_t)^{-t}\widehat{P}_t,$$

given i_t, $\widehat{K}_0$, and $\widehat{d}_{a_0,n_{-1},a_{-1}}$. Again, a sequence of cash flows implies a sequence of risk-free bond holdings, $\{\widehat{B}_{t+1}\}_{t=0}^{\infty}$.

Individual's problem. Hereafter, period-t variables will be expressed without any subscripts or superscripts, and period-$t+1$ variables will be represented with superscripts '$\prime$'. Individuals decide on consumption, c, and asset accumulation, a'. In addition, they decide which kind of debt contract they would like to sign, and either to file for bankruptcy or to pay back the debt. These decisions are made taking prices, $\mathrm{S}=(q,w,i,r,\widetilde{q}(\cdot),\widehat{q}(\cdot),\mathcal{C}(\cdot))$, as given.

Several assumptions determine the advantages and disadvantages of bankruptcy. The key advantage is the discharge of debts—assets in the period after bankruptcy are set at zero. Thus, an individual with too much debt may find it profitable to file for bankruptcy. There are many disadvantages of doing so, however.[11] In the period of bankruptcy, consumption equals income, neither saving or borrowing are allowed. Additionally, in the period right after bankruptcy, the defaulter will have a *bad credit flag.* Having a bad credit flag implies that the individual cannot borrow and a proportion of income, τ, is lost.[12] That flag remains in an individual record for a

[11] Here, disadvantages of filing bankruptcy are exogenous. Chatterjee, Corbae, and Rios-Rull (2007b) show how higher interest rates following default arises from the lender's optimal response to limited information about the individual's type and earnings realizations.

[12] Chatterjee, Corbae, and Rios-Rull (2007a) build a model where no punishment is required after filing bankruptcy. There, asymmetric information is crucial to create incentives for debt repayment, because individuals signal their type by paying back their debt.

stochastic number of periods, meaning that the probability of a transition from bad to good credit flag is $\lambda \in (0,1)$—the fresh start probability. The use of λ is a simple way of modeling a bankruptcy flag that remains on an individual's credit history for only a finite number of years.

Lifetime utility for individuals in each possible state is defined as follows.

- *Bad credit flag*: Lifetime utility of an individual excluded from credit markets is

$$B(n,l,a;\mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta \textstyle\sum_{n'} \Pi(n,n')\{\lambda \int_{l'} G(n',l',a';\mathrm{S})\phi(l' \mid n')dl' + (1-\lambda)\int_{l'} B(n',l',a';\mathrm{S})\phi(l' \mid n')dl'\}\}, \tag{1}$$

subject to

$$c + \widehat{q}(a',n)a' = a + lw(1-\tau),$$
$$a' \geq 0, \text{ and } c \geq 0,$$

where G is the lifetime utility for individuals with good credit history (defined below), which is a function of types, n, labor endowments, l, assets, a, and relevant prices, S. Importantly, assets for next period are restricted to be positive. Notice that the individual obtains utility next period just if she survives, what happens with probability ρ. The utility from future periods depends on the probability of a fresh start, λ, while the utility from the current period depends on the proportion of income lost because of bad credit status, τ. Denote the policy functions for asset accumulation and consumption obtained from the solution to this problem as A'_b and C_b.

- *Good credit flag*: Lifetime utility is

$$G(n,l,a;\mathrm{S}) = \max\{\underbrace{V(n,l,a;\mathrm{S})}_{\text{pay back}}, \underbrace{D(n,l;\mathrm{S})}_{\text{bankruptcy}}\}, \tag{2}$$

where V and D (defined below) are lifetime utilities for individuals paying back the debt and filing bankruptcy, respectively. This means that an individual with a good credit flag has the choice of filing bankruptcy. The policy functions for asset accumulation and consumption are A', C, respectively. Additionally, the policy function R indicates whether the individual pays back the debt or not,

$$R(n,l,a;\mathrm{S}) = \begin{cases} 1 \text{ if } V(n,l,a;\mathrm{S}) \geq D(n,l;\mathrm{S}), \\ 0 \text{ otherwise.} \end{cases}$$

  - *Good credit flag and bankruptcy*: Suppose the individual chooses to file for bankruptcy. Then, lifetime utility is

$$D(n,l;\mathrm{S}) = u(lw(1-\tau)) + \rho\beta \textstyle\sum_{n'} \Pi(n,n') \int_{l'} B(n',l',0;\mathrm{S})\phi(l' \mid n')dl'. \tag{3}$$

Neither saving or borrowing is allowed in this period. Therefore the individual's consumption equals net income (labor income minus the proportion lost due to bankruptcy). In the period after bankruptcy, the individual will have a bad credit flag for sure and zero debt.

• *Good credit flag and pay back the debt*: Suppose the individual decides to pay back the debt. Then, she must decide which kind of contract to sign. Thus, the value function is

$$V(n,l,a;\mathrm{S}) = \max\{\underbrace{\widetilde{V}(n,l,a;\mathrm{S})}_{\text{use information}};\underbrace{\widehat{V}(n,l,a;\mathrm{S})}_{\text{no information}}\}, \tag{4}$$

where $\widetilde{V}(n,l,a;\mathrm{S})$ and $\widehat{V}(n,l,a;\mathrm{S})$ (defined below) are lifetime associated with borrowing from informed and uninformed lenders, respectively. The policy function U indicates whether the individual borrow from uninformed lenders or not,

$$U(n,l,a;\mathrm{S}) = \begin{cases} 1 \text{ if } \widehat{V}(n,l,a;\mathrm{S}) \geq \widetilde{V}(n,l,a;\mathrm{S}), \\ 0 \text{ otherwise.} \end{cases}$$

• *Pay back the debt and informed debt contract*: If the individual decides to sign a contract with an informed lender, then she faces the debt price $\tilde{q}(a',n)$, and her lifetime utility is

$\widetilde{V}(n,l,a;\mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta\sum_{n'}\Pi(n,n')\int_{l'}G(n',l',a';\mathrm{S})\phi(l' \mid n')dl'\}$,
subject to

$$\begin{aligned} &c + \tilde{q}(a',n)a' = a - \mathcal{C}(z^i,w) + lw, \\ &a' \geq \underline{a}_N, \text{ and } c \geq 0, \end{aligned} \tag{5}$$

where $\underline{a}_N$ is the natural borrowing limit and $\mathcal{C}(z^i,w)$ the cost of information. Notice that this cost is independent of the amount borrowed, which is consistent with the interpretation that the individual buys a report about her type and then presents it to the lender.

• *Pay back the debt and uninformed debt contract*: Now suppose the individual prefers to sign a contract with an uninformed lender. Then, the relevant debt price is $\widehat{q}(a',n)$, the debt limit is $\underline{a}(a,n)$, and there is no fixed cost to pay. Thus, her lifetime utility is

$\widehat{V}(n,l,a;\mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta\sum_{n'}\Pi(n,n')\int_{l'}G(n',l',a';\mathrm{S})\phi(l' \mid n')dl'\}$,
subject to

$$\begin{aligned} &c + \widehat{q}(a',n)a' = a + lw, \\ &a' \geq \underline{a}(a,n), \text{ and } c \geq 0. \end{aligned} \tag{6}$$

3.2 The Equilibrium

***Informed lenders'* contracts.** These lenders offer a contract for each type and prevent misrepresentation by paying the cost of information.

Definition 1 *A discount price function, $\widetilde{q}$, is an equilibrium with informed lenders if the following conditions hold:*

1. *Zero expected profits are collected by each lender offering the contract; i.e.,*

$$\tilde{q}(a', n) = (1+i)^{-1} \Pr(repayment \mid a', n),$$

for each (a', n).

2. *The expectations about repayment are realized in equilibrium; i.e.,*

$$\Pr(repayment \mid a', n) = \rho \sum_{n'} \Pi(n, n') \int_{l'} R(n', l', a'; \mathrm{S}) \phi(l' \mid n') dl'.$$

3. *There is no other contract that, if offered, will imply positive expected profits; i.e., $\nexists \widetilde{q}' : \widetilde{p}(\widetilde{q}') > 0$, where*

$$\widetilde{p}(\widetilde{q}') = \sum_{n} \int_{a'} P(a', n; \widetilde{q}') \mathrm{E}(\widetilde{d}_{a',n}) da',$$

$$P(a', n; \widetilde{q}') = \left\{ -\frac{\Pr(repayment \mid a', n)}{1+i} a' + \widetilde{q}(a', n) a' \right\},$$

and expectations $\mathrm{E}(\widetilde{d}_{a',n})$ are rational.

Conditions (1) and (2) have a clear implication for equilibrium prices; i.e.,

$$\tilde{q}(a', n) = \frac{1}{1+i} \rho \sum_{n'} \Pi(n, n') \int_{l'} R(n', l', a'; \mathrm{S}) \phi(l' \mid n') dl'.$$

Here it is very clear why the discount price, $\tilde{q}$, depends on (a', n) and is independent of a. It depends on a' because it affects the bankruptcy decision, R, in each possible state. It depends on n because it determines the transition probability to each n' and therefore the next period labor endowment, l'. Finally, it is independent of a because it does not affect either the transition probabilities nor the bankruptcy decision in the next period.

***Uninformed lenders'* contracts.** Now consider lenders offering direct-revelation contracts. Here a contract contains a duple $\{\widehat{q}(a', n), \underline{a}(a, n)\}$ for each $a' \in \mathbf{A}$, $n \in \mathbf{N}$, and $a \in \mathbf{A}$. The duple $\{\widehat{q}(a', n), \underline{a}(a, n)\}$ can be also written as a discount price function $\widehat{q}(a', n; a)$ by setting it at zero for asset levels lower than the debt limit, $a' < \underline{a}(n, a)$. Thus, to save in notation, hereafter the whole contract is referred to as $\widehat{q}$.

Let $\mathbf{M}$ denote the set of direct-revelation contracts. Then, $\widehat{q} \in \mathbf{M}$ if and only if it satisfies

$$\widehat{V}(n, l, a; \widehat{q}(\cdot, n; a)) \geq \widehat{V}(n, \bar{l}, a; \widehat{q}(\cdot, \bar{n}; a)) \forall a \in \mathbf{A},\ \forall n, \bar{n} \in \mathbf{N},\ \forall l \in L(n) \text{ and } \forall \bar{l} \in L(\bar{n}).$$

That is, a contract is a direct-revelation contract if and only if individuals prefer to borrow at the discount price designed for their type instead of pretending to be of a different type. Notice that this constraint must be satisfied for all l. This could imply a debt limit depending also on l. In fact, a limit depending on l can be defined from the constraint above, $\underline{a}(a, n, l)$. Nevertheless, since l is unobservable and zero-expected-profit discount prices are independent of l, debt limits must be independent on l in equilibrium. Thus, the limit satisfying the direct-revelation constraint for all l is the tightest limit, $\underline{a}(a, n) = \max_{l \in L(n)} \underline{a}(a, n, l)$.

Definition 2 *A discount price function satisfying direct revelation, $\widehat{q} \in \mathbf{M}$, is an equilibrium with uninformed lenders if the following conditions hold:*

1. *Zero expected profits are collected from each contract; i.e.,*

$$\widehat{q}(a', n; a) = \begin{cases} (1+i)^{-1} \Pr(\textit{repayment} \mid a', n) \textit{ if } a' \geq \underline{a}(a, n), \\ 0 \textit{ otherwise.} \end{cases}$$

2. *Expectations about repayment are realized in equilibrium; i.e.,*

$$\Pr(\textit{repayment} \mid a', n) = \rho \sum_{n'} \Pi(n, n') \int_{l'} R(n', l', a'; \mathrm{S}) \phi(l' \mid n') dl'.$$

3. *There is no other direct-revelation contract that, if offered, will make positive expected profits; i.e., $\nexists \widehat{q}' \in \mathbf{M} : \widehat{p}(\widehat{q}') > 0$, where*

$$\widehat{p}(\widehat{q}') = \sum_n \int_a \int_{a'} P(a', n, a; \widehat{q}') \mathrm{E}(\widehat{d}_{a', n; a}) da' da,$$

$$P(a', n, a; \widehat{q}') = \left\{ -\frac{\Pr(\textit{repayment} \mid a', n)}{1+i} a' + \widehat{q}(a', n; a) a' \right\},$$

and expectations $\mathrm{E}(\widehat{d}_{a', n; a})$ are rational.

This equilibrium definition is in the spirit of Rothschild and Stiglitz (1976). As it was pointed out in Section 2, it is well known that an equilibrium may not exist in this context. Should an equilibrium exist, the allocation is very interesting. Therefore, this paper focuses on this allocation and introduces two alternative solutions for the problem of existence. The main ideas behind those solutions were exposed in Section 2. Appendix 6.1 explains how Definition 2 has to be modified to incorporate reasonable beliefs.

Stationary equilibrium. Assume technologies in the information sector, z^i, and in the production sector, z^p, are constant. Then, stationary equilibrium requires optimization together with aggregate conditions that guarantee markets clearing and stationarity.

Definition 3 *A stationary equilibrium with costly information is a set of policy functions A_b', C_b, A', C, R, U, $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{k}$, cumulative density functions $\Psi_n(a,l)$, $\Psi_n^g(a,l)$, $\Psi_n^b(a,l)$, and prices w, i, r, $\widetilde{q}$, $\widehat{q}$, and $\mathcal{C}$, such that the following conditions hold:*

1. *The functions A_b', C_b, A', C, R, and U solve the individual's problems, or satisfy problems 1 to 6.*
2. *The function $\widetilde{q}$ and $\widehat{q}$ are equilibrium discount prices, or satisfy Definition 1 and 2, respectively.*
3. *The firm in the production sector maximizes profits given $\{w, r\}$, or*

$$(1-\theta)z^p(\mathbf{k})^{-\theta}(\mathbf{l})^{\theta} = r,$$

$$\theta z^p(\mathbf{k})^{1-\theta}(\mathbf{l})^{\theta-1} = w.$$

4. *The function $\Psi_n(a,l)$ is the stationary c.d.f. over (n,a,l), and $\Psi_n^g(a,l)$ and $\Psi_n^b(a,l)$ are the stationary c.d.f. over (n,a,l) conditional on having good and*

bad credit flag, respectively; or

$$\begin{aligned}
\Psi_n(a,l) =& \Psi_n^g(a,l) + \Psi_n^b(a,l), \\
d\Psi_{n'}^g(a',l') =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n')R(n,a,l)d\Psi_n^g(a,l) \\
& + \lambda \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n')d\Psi_n^b(a,l), \\
d\Psi_n^b(0,l') =& \sum_n \int_a \int_l \Pi(n,n')\phi(l' \mid n')(1-R(n,a,l))d\Psi_n^g(a,l) \\
& + (1-\lambda) \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=0\}} \Pi(n,n')\phi(l' \mid n')d\Psi_n^b(a,l), \\
d\Psi_n^b(a',l') =& (1-\lambda) \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n')d\Psi_n^b(a,l), \;\; a' \neq 0.
\end{aligned}$$

5. *The labor market clears, or*

$$\mathbf{m} + \mathbf{l} = \sum_n \int_a \int_l l d\Psi_n(a,l).$$

6. *The credit market clears, or*

$$\begin{aligned}
\widehat{d}_{a',n;a} =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} U(n,a,l)d\Psi_n^g(a,l) \\
& + \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} U(n,a,l)d\Psi_n^b(a,l), \\
\widetilde{d}_{a',n} =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} (1-U(n,a,l))d\Psi_n^g(a,l).
\end{aligned}$$

7. *The goods market clears, or*

$$z^p(\mathbf{k})^{1-\theta}(\mathbf{l})^\theta = \sum_n \int_a \int_l C(n,a,l)d\Psi_n^g(a,l) + \sum_n \int_a \int_l C_b(n,a,l)d\Psi_n^b(a,l) + \delta\mathbf{k}.$$

Notice that the borrowing limits contained in contracts with uninformed lenders, $\underline{a}(a,n)$, have to be looser than the bankruptcy-free limit $\underline{\mathbf{a}}$,[13] where

$$\underline{\mathbf{a}} = \min_a \{a : V(n,l,a) \geq D(n,l), \forall n \in \mathbf{N}, \forall l \in L(n)\}.$$

This result is implied by the zero-expected-profit condition. Since the best possible discount price is the bankruptcy-free discount price, it is clear that no contract can

[13]This borrowing limit was first introduced by Zhang (1997) and Abraham and Carceles-Poveda (2006).

be sustained in equilibrium offering a borrowing limit tighter than the bankruptcy-free limit. If it was true, then there will be a better contract: one offering the highest possible discount price, $\frac{\rho}{1+i}$, and a looser borrowing limit, $\underline{\mathbf{a}}$. Therefore, equilibrium debt contracts for informed and uninformed lenders imply that discount prices charged by uninformed lenders will be

$$\widehat{q}(a', n; a) = \begin{cases} \frac{\rho}{1+i} \text{ if } a' \geq \underline{\mathbf{a}}, \\ \widetilde{q}(a', n) \text{ if } \underline{\mathbf{a}} > a' \geq \underline{a}(a, n), \\ 0 \text{ if } \underline{a}(a, n) > a'. \end{cases}$$

Thus, there are two key differences between the offers that lenders make. First, uninformed lenders' discount prices are the same that those offered by informed lenders but equal to zero for debt bigger than certain limits. Second, to access informed lenders' discount prices, individuals must pay the cost of information.

4 Quantitative analysis

For the quantitative analysis, the model stationary equilibrium—Definition 3—is computed several times. In that equilibrium, the solution to the problem of uninformed lenders—Definition 2—may or may not exist, as it was explained before, because Condition (iii) may not hold. If it does exist, then the allocation is the same that solves the problem of uninformed lenders considering reasonable beliefs—Definition 5, which always exist. This implies that the solution computed in this section is the equilibrium using Definition 5 for the problem of uninformed lenders and also the equilibrium using instead Definition 2 if this equilibrium exist.[14]

4.1 Calibration

The benchmark calibration is designed such that the model free of informational frictions represents the U.S. economy in the year 2004. This exercise is referred to hereafter as *"2004" calibration.* The cost of information assigned to that year is zero because the model without informational frictions can be computed substantially faster.[15] Notice that this assumption simplifies the computation substantially because uninformed lenders can be ignored. Additionally, if the cost of information production is actually close to zero in the year 2004 and the model is continuous on this cost, then the calibration's targets will be matched also for small information costs.[16]

[14] Checking if the equilibrium with Definition 2 exist, i.e. if condition (iii) holds, is feasible ex-post, after the stationary distribution is found. That ex-post verification is presented in the computational appendix to this paper available at `troi.cc.rochester.edu/~sncz/research`.

[15] Later, in the assessment of the effect of the IT revolution, it is explained how the costs of information will be determined for the year 1983.

[16] Numerically, the model is continuous in the cost of information around zero.

The calibration consists in assigning values to 24 parameters. Some of them can be determined using a priori information. The others are determined jointly using Nelder and Mead (1965) algorithm to minimize the distance between key moments in the data and model. Parameters and targets are explained in detail in the next subsections.

Parameters determined using a priori information (5). The survival probability, ρ, is determined to match a period of financially active life of 40 years. The utility function is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ was chosen to match a coefficient of risk aversion of 2. The labor share of income, 0.64, determines the value of the parameter in the production function, θ. The depreciation rate, δ, is set at 7% annually. The probability of a fresh start, λ, is set to match an average time of exclusion after bankruptcy of 10 years.

Parameters determined jointly (19). There are ten different income groups or types, $\mathbf{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, where 1 and 10 are associated with the lowest and highest labor endowments, respectively. This choice is important because it determines the maximum that lenders can learn about individuals' income when they decide to screen borrowers. Nevertheless, how much information is contained in these ten types depends on their persistence, which is endogenously determined considering several moments from the data. Additionally, the proportion of total income that each group account for is obtained directly from the Survey of Consumer Finance (SCF). The parameters to calibrate are described in detail below.

- *Transition matrix (6 parameters):* $\mathbf{\Pi}$. Several assumptions restrict the number of parameters in this group. In particular, it is assumed that: transitions further than 2 types away than the current type are zero probability events, persistence is the same for all the groups except $\{1, 2, 9, 10\}$, the transition to one and two types higher and lower are proportionally the same for all the types, and the highest type is a low probability state with very high labor endowments, as in Castaneda, Diaz-Gimenez, and Rios-Rull (2003). After these assumptions there are 6 parameters to calibrate.[17] These parameters determine the size of each income group which in turns affects the joint distribution of debt and income.

- *Discount factor (1 parameter):* β. This parameter is crucial determining the economy capital-to-income and debt-to-income ratios and the bankruptcy rate.

[17]More details on the assumptions made on the transition matrix are provided in Appendix 6.2.

- *Income lost during bankruptcy (1 parameter):* τ. Since this parameter represents the fraction of earnings lost when individuals have a bad credit flag, it plays a very important role determining the bankruptcy rate and the debt-to-income ratio.

- *Labor endowments distribution exponent (1 parameter):* φ . Given a type n, the cumulative distribution for labor endowments, $l \in [\underline{l}^n, \bar{l}^n]$, is

$$\int_{\underline{l}^n}^{x} \phi(l \mid n) dl = \left(\frac{x - \underline{l}^n}{\bar{l}^n - \underline{l}^n} \right)^{\varphi},$$

  where the exponent is one more parameter to calibrate. This parameter is particularly important in determining income inequality given the size of income groups.

- *Labor endowment intervals' limits (10 parameters):* $\bar{\mathbf{l}}$. The first one is normalized, $\underline{l}^1 = 1$. The ending limit of a range is equal to the starting limit of the next range; i.e. $\bar{l}^1 = \underline{l}^2, \bar{l}^2 = \underline{l}^3, \ldots, \bar{l}^9 = \underline{l}^{10}$. Thus, there are 10 parameters to be calibrated: $\bar{\mathbf{l}} = [\bar{l}^1, \bar{l}^2, \bar{l}^3, \bar{l}^4, \bar{l}^5, \bar{l}^6, \bar{l}^7, \bar{l}^8, \bar{l}^9, \bar{l}^{10}]$. These limits determine the proportion of total income in each group.

There are 8 parameters that are chosen minimizing the distance between moments from the model and data. Specifically, there are 10 statistics used as targets. Most of them are very important in any model of debt and bankruptcy. Others are relevant given the informational frictions in this paper. Many statistics from the SCF 1983, SCF 2004, and other sources are presented in Table 5. Those chosen as targets of the calibration are described in details below.

- *Capital-to-output ratio (1 target):* This target is fixed assets and consumer durable goods over GDP, both obtained from the Bureau of Economic Analysis (BEA).

- *Bankruptcy rate (1 target):* The number of bankruptcy filings in a year, obtained from the American Bankruptcy Institute, is prorated using 0.53 because income shocks cause 53% of the cases of bankruptcy. Then, to construct the bankruptcy rate, the prorated number of bankruptcy filings is divided by the number of households with a credit card balance, obtained from the SCF 2004.[18]

- *Earnings and wealth inequality (4 targets):* The statistics in this group are all obtained from the SCF 2004. In particular, the targets used are the Gini coefficient of income, the mean-to-median income ratio, the Gini coefficient of wealth, and the mean-to-median wealth ratio.

[18] Similar results are obtained if the number of filings is not prorated and the denominator in this ratio is replaced by the population over 20 years old with a credit card balance.

- *Debt-to-income ratio (1 target):* Debt and income are obtained from SCF 2004. Debt is equal to minus net worth when it is negative and zero otherwise.[19]

- *Debt across income groups (3 targets):* Let D_i denote the percentage of debt hold by individuals with income percentile lower than i. The moments D_{10}, D_{20}, and D_{30} are used in the calibration procedure. They are obtained from the SCF 2004 using the definition of debt introduced above.

Among the targets, the percentage of debt held by individuals with incomes lower than the 10, 20, and 30 percentile were not considered in previous literature. They are important in the calibration for the year 2004 because how much debt is held by individuals in an income group depend on their expected income, which is determined by the transition matrix parameters. More important, given all the other parameters, the cost of information is crucial determining the proportion of debt held by income poorest individuals, as explained at the end of Section 2. Thus, these targets will be useful in the calibration of the cost of information for the year 1983.

The next steps were followed to calibrate the model parameters minimizing the distance between the moments from the data and model.

Step 1: Guess a value for 8 parameters $\{\mathbf{\Pi}, \beta, \tau\}$.

Step 2: Given the value of $\mathbf{\Pi}$, compute the measure of individuals in each type n in the stationary distribution.

Step 3: Obtain 10 parameters, $\bar{\mathbf{l}} = [\bar{l}^1, \bar{l}^2, \bar{l}^3, \bar{l}^4, \bar{l}^5, \bar{l}^6, \bar{l}^7, \bar{l}^8, \bar{l}^9, \bar{l}^{10}]$, using the SCF 2004 to match exactly the measure in each labor endowments' interval.

Step 4: Given the limits $\bar{\mathbf{l}}$, search for the value of φ that minimizes the distance between the Gini coefficient of income from the model and data using the Nelder and Mead (1965) algorithm.

Step 5: At this point there is a value for each of the 19 parameters. Use these parameters to compute the model and calculate the distance between the moments from the data and model. If the distance to the targets is small enough, end. Otherwise, choose a new value for $\{\mathbf{\Pi}, \beta, \tau\}$ according to the Nelder and Mead (1965) algorithm and return to step 2.

The parameters values resulting from the calibration strategy are presented on the top panel of Table 1. The discount factor and the cost of bankruptcy are similar in other quantitative model of consumer bankruptcy and hard to compare with direct evidence. The transition matrix parameters can be compared with data obtained from

[19] This target is constructed exactly as in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007). There and here, debt-to-income is prorated using 0.53 because the model does not have other shocks that may cause borrowing.

Figure 6: Income distribution

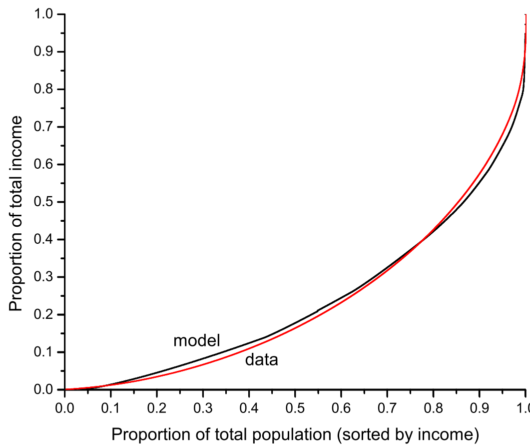
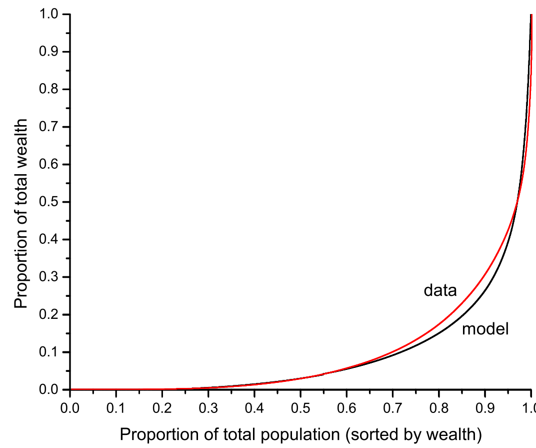


Figure 7: Wealth distribution



the matched March supplement of the Current Population Survey (CPS). The top panel of Table 6 presents the transition matrix from the model and data for the first 6 types, which accumulate 92% of the total population.[20] Although there are several reasons that can explain why the transition matrix from CPS may be different,[21] the transition probabilities obtained in the calibration look relatively similar to those in the data. In particular, the parameter in the main diagonal are quite similar.

How well does the model fit the data? The bottom panel of Table 1 presents the model's goodness of fit. It can replicate key moments for the year 2004. In particular, the first two statistics, the bankruptcy rate and the debt-to-income ratio are closely replicated. Despite it overestimates the percentage of total debt held by the poorest 10, 20, and 30 percentiles of the population, the power of the model to explain debt across income groups is acceptable. Given that income is directly obtained from the data, income distribution statistics are perfectly replicated. The Lorenz curve, depicted in Figure 6, is very similar in the model and in the data. Additionally, the wealth distributions from the model and data are very alike, as shown in Figure 7. Obviously, it implies that the wealth inequality coefficients considered as target of calibration are also closely reproduced. The success of the model reproducing wealth inequality is because the income of rich individuals is well calibrated using the SCF. This result is well known since the work of Castaneda, Diaz-Gimenez, and Rios-Rull (2003).

[20]The comparison for the richest individuals is avoided because of the different treatment of individuals on the top of the income distribution (top-coding) in the SCF and CPS.

[21]For instance, the model is calibrated using data on earnings from SCF which contains information about households earnings, while the information from CPS is about individuals earnings.

Table 1: Parameters and goodness of fit for the year 2004

Parameters		Values
Discount factor, β		0.92
Cost of bankruptcy, τ		0.18
Persistence of type $n = 1$		0.20
Persistence of type $n = 2$		0.76
Persistence of type $n = \{3, ..., 7\}$		0.41
Transition probability from type $n = \{1, ..., 9\}$ to $n = 10$		0.0002
Transition probability from type $n = 2$ to $n = 1$		0.035
Transition probability from type $n = \{3, ..., 8\}$ to $n - 1$		0.23
Statistics	Targets 2004	"2004" calibration
Bankruptcy rate	1.59%	1.45%
Debt-to-income ratio	0.76%	0.73%
Capital-to-output ratio	3.27	3.31
Proportion of debt held by income poorer 10%	10%	14%
Proportion of debt held by income poorer 20%	25%	33%
Proportion of debt held by income poorer 30%	44%	52%
Gini coefficient of income	0.53	0.53
Mean-to-median income ratio	1.64	1.64
Gini coefficient of wealth	0.81	0.82
Mean-to-median wealth ratio	4.81	5.30

4.2 Assessment of the IT revolution

The quantitative model is now used to assess the role of information costs in the rise of the bankruptcy rate over the last 20 years. This cost was set equal to zero for the calibration of the year 2004. To quantify the change in information costs and its effect on other variables, a new value for the technology in the information sector, z^i, is required. This parameter is calibrated jointly with the parameters from the transition matrix, $\mathbf{\Pi}$, the exponent in the income distribution, φ, and the limits for each income type, $\bar{\mathbf{I}}$. Recalibrating z^i for 1983 together with the income process' parameters is required because of two reasons. The first one is that the role of information costs in an economy depends on the income process. For instance, if income is i.i.d across time, then information does not matter at all. Therefore, a correct "estimation" of z^i for 1983 can be obtained only if that year's income distribution is considered. The second reason is that income inequality changed substantially between 1983 and 2004.

The algorithm is similar to the one described above for the year 2004. Given $\mathbf{\Pi}$, the parameters in $\bar{\mathbf{I}}$ are directly obtained from the SCF 1983, and φ is obtained minimizing the distance to the Gini coefficient of income in 1983. This guarantees that the income distribution of the year 1983 will be closely matched. The parameters z^i_{1983} and $\mathbf{\Pi}$ are then chosen by minimizing the distance between the model and data moments for the year 1983. Again, the Nelder and Mead (1965)'s algorithm is used in the minimization. The target variables used from the year 1983 are the same that were used above for 2004.

The goodness of fit of the recalibrated model, referred to hereafter as *"1983" calibration*, is presented in Table 2. The model can replicate the data fairly well without changing the discount factor and the cost of bankruptcy. Specifically, it reproduces very well the bankruptcy rate, which is 0.51% in the model and 0.49% in the data. Importantly, the debt-to-income ratio is also well matched; it is 0.31% in the data and 0.33% in the model. The model's capital-to-output ratio, 3.39, is similar to the value from the data, 3.44. As it happened in the "2004" calibration, the power of the model explaining debt across income groups is acceptable but it overestimates the targets.

Understanding what causes the differences between Table 1 and 2 is crucial to understand the rise in bankruptcy. To that end, two counterfactual exercises are performed. First, the economy with the income process for 2004 is computed with the cost of information for 1983. Second, the economy with the income process for 1983 is computed with the cost of information for 2004. These two exercises are explained in details below.

In the first counterfactual exercise, the technology in the information sector obtained in the "1983" calibration is used to quantify how much of the rise in consumer

Table 2: Parameters and goodness of fit for the year 1983

Parameters	Values
Discount factor, β	0.92
Cost of bankruptcy, τ	0.18
Persistence of type $n = 1$	0.44
Persistence of type $n = 2$	0.77
Persistence of type $n = \{3, ..., 7\}$	0.42
Transition probability from type $n = \{1, ..., 9\}$ to $n = 10$	0.0002
Transition probability from type $n = 2$ to $n = 1$	0.036
Transition probability from type $n = \{2, ..., 8\}$ to $n + 1$	0.12
Technology in the information sector	1.26

Statistics	Targets 1983	Costly-information calibration
Bankruptcy rate	0.49%	0.507%
Debt-to-income ratio	0.31%	0.328%
Capital-to-output ratio	3.44	3.39
Proportion of debt held by income poorer 10%	25%	31%
Proportion of debt held by income poorer 20%	33%	44%
Proportion of debt held by income poorer 30%	43%	57%
Gini coefficient of income	0.45	0.45
Mean-to-median income ratio	1.35	1.35
Gini coefficient of wealth	0.75	0.82
Mean-to-median wealth ratio	2.95	5.2
Information costs to mean income ratio	—	3.9%

Table 3: **The effect of information costs on the bankruptcy rate**

	Bankruptcy rate
Comparison in the economy with the income distribution for 2004	
Difference in the model between high and zero information costs	0.41 pp.
Difference between the data for the years 1983 and 2004	1.10 pp.
Ratio: Model/Data	37.3%
Log difference between the model with high and zero information costs	34.2%
Log difference between the data for the years 1983 and 2004	118.5%
Ratio: Model/Data	28.9%
Comparison in the economy with the income distribution for 1983	
Difference in the model between high and zero information costs	0.50 pp.
Difference in the data between years 1983 and 2004	1.10 pp.
Ratio: Model/Data	45.5%
Log difference in the model between high and zero information costs	68.3%
Log difference in the data between years 1983 and 2004	118.5%
Ratio: Model/Data	58.0%

bankruptcy can be attributed directly to the drop in information costs.[22] Specifically, the model is computed with all the parameters for the year 2004 but setting the technology in the information sector at z^i_{1983}. This exercise is referred to hereafter as *counterfactual calibration 1.* Thus, this new economy represents year 2004 with informational frictions for 1983. The top panel in Table 3 presents the effect of information costs on the bankruptcy rate. It indicates that *information costs alone explain 37.3 percent of the total change in the bankruptcy rate in the U.S. over the last 20 years.*

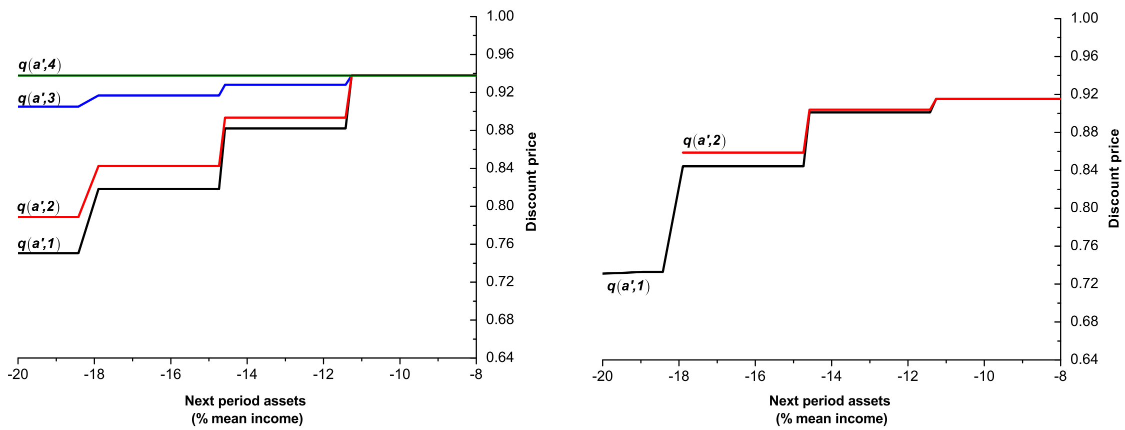
The role of information costs in consumer debt and bankruptcy can be better understood examining changes in other variables when information costs rise. In particular, the cost of information restricts the number of bankruptcy filings through changes in discount prices. Figure 8 contains discount prices in the "2004" calibration and the counterfactual calibration 1. The main difference is that, in the economy with high in-

[22]Notice that a cost of information equal to 3.9% of mean income is relatively high because in general individuals in debt have income lower than the mean.

Figure 8: Equilibrium discount prices

(a) Benchmark calibration ($z \to \infty$) (b) Counterfactual calibration 1 ($z = 1.26$)

(individuals with debt equal to 20% mean income)



formation costs, discount prices for types higher than 2 are not available for debt sizes implying risk of bankruptcy. Additionally, how much can type-2 individuals borrow depends on their current asset holdings. For instance, for an individual with current debt equal to 20% of mean income, type-2 discount prices are available only until debt equal to 18% of mean income. Therefore, the change in discount prices reduces the number of bankruptcy filings because individuals borrowing with current type higher than 1 can borrow just in the range with very low risk (or no risk) of bankruptcy. This effect is clear looking at the next statistics. In the "2004" calibration economy, only 34% of the risky debt was borrowed by type-1 individuals, while 62% the risky debt was borrowed by type-1 individuals in the economy with high information costs—counterfactual calibration 1.

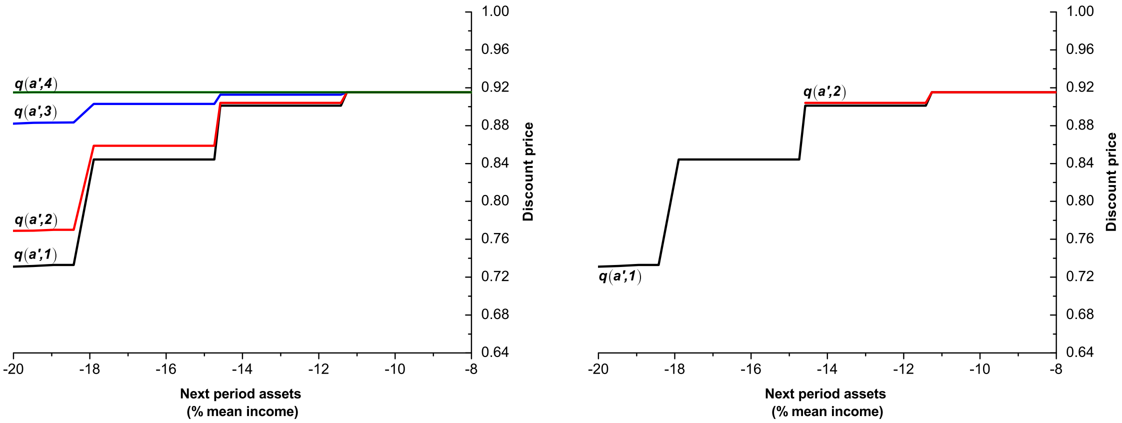
In addition, notice that in the economy with high information costs, discount prices are restricted only if the individual does not pay the cost of information. Otherwise, discount prices look as those presented in Figure 9 (a). Thus, an agent with current debt equal to 10% of mean income must decide between borrowing from informed lenders (after paying the information cost) at discount prices as in Figure 9 (a) and borrowing from uninformed lenders at discount prices as in Figure 9 (b). Moreover, the effect of current debt on debt limits set by uninformed lenders is not that strong. On one hand, individuals with high debt—Figure 8 (b)—can borrow using type-2 discount prices only until next period debt equal to 18% of mean income. On the other hand,

Figure 9: Discount prices and debt limits (counterfactual calibration 1)

(a) Informed lenders

(b) Uninformed lenders

(individuals with debt equal to 10% mean income)



individuals with low debt—Figure 9 (b)—can borrow using type-2 discount prices only until next period debt equal to 15% of mean income. This variation in the limit is because type-1 individuals, who determine the limits, desire to borrow more when their current debt is higher.[23]

The comparison of Figures 9 (a) and (b) is also useful to study the projection of income on interest rates. Informed lenders offer different interest rates for different income groups for each amount of debt (see Figure 9 (a)). In contrast, uninformed lenders offer only one interest rates for each amount of debt (see Figure 9 (b)). This generates that the relationship between interest rates and income is stronger when the cost of information drops.

Other statistics from the model with zero information costs ("2004" calibration) and the counterfactual calibration 1, presented in the last two columns in Table 7, shed light on what happens if informational frictions vanish. The debt-to-income ratio and the proportion in debt rise. This is because individuals that were previously borrowing constrained can now access to their corresponding zero-profit discount prices for any debt size. Therefore, these results are basically explained by the change in discount prices described above. Likewise, as informational frictions vanish, the proportion of debt held by income poorest 10% drops from 19% to 14%. The reason is that previously borrowing constrained individuals are those with types higher than 1. Moreover,

[23]In the two-by-two example, the curves U_H^1 and U_L^1 in Figure 3 move both to the left when a_1 decreases.

as information costs drop, the capital-to-output ratio rises and the risk-free interest rate drops. That happens because the lack of information favors individuals with low income, making the credit market better to insure income shocks. Specifically, discount prices for types 1 and 2 are higher in the economy with high information costs.[24] Now, why do discount prices change in that direction? Again, because low-type individuals prefer the credit market with high information costs and therefore, they are less likely to file for bankruptcy in the economy with high information costs. In summary, better discount prices at low income level generate better insurance possibilities and less precautionary savings that in turn imply a lower capital stock and a higher risk-free interest rate. Precautionary savings are also important determining the difference between income and wealth inequality. Because of precautionary savings, in the economy with perfect information individuals accumulate more assets. Then, greater asset accumulation leads to more wealth inequality given income inequality. Finally, the coefficient of variation of the interest rates paid by borrowers is also presented in Table 7. The result is that as information costs drop, the dispersion in interest rates rises. Basically, this is because the lack of information restrict some interest rates from being offered.

The second exercise quantifies the importance of the change in the income process. This exercise, referred to hereafter as *counterfactual calibration 2*, consists of computing an economy with the cost of information for 2004 and the income process calibrated for 1983. Before analyzing the results, it is important to study the income processes obtained in the "1983" and "2004" calibrations. First, it is clear that the "1983" calibration implies less income inequality than the "2004" calibration and matches exactly the Gini coefficient of income for 1983. This is because the limits for the labor endowments intervals are obtained from the SCF 1983. More surprisingly, the transition parameters are also similar to those obtained from the data. The bottom panel of Table 6 presents the data and model transition parameters for 1983. The persistence of each group is similar in the model and data. Additionally, the transition parameters in the two calibrations are alike in most of the income groups. The main difference is the probability of staying in the first income group: it is 0.44 in 1983 and 0.20 in 2004. Importantly, the same pattern can be observed in the transition matrix in the data: the probability of staying in the first income group is 0.46 in 1983 and 0.36 in 2004. This change generates more debt in that income group because the expected lifetime income of those individuals increases.

One important question can be answered using this quantitative exercise: how much of the rise in the bankruptcy rate is explained by the change in the income

[24] This can be observed comparing Figure 8 (a) and (b) for debt levels between 11% and 18% of mean income.

process? The bankruptcy rate in the counterfactual calibration 2, also presented in Table 7, is very similar to the one in the counterfactual calibration 1. That means that the change in the income process alone also explains around 40 percent of the total rise in the bankruptcy rate in the U.S. over the last 20 years. Additionally, it means that around 20 percent of the difference in the bankruptcy rate between the calibrations for the years 1983 and 2004 is explained by the interaction of the changes in the income process and information costs. The interaction is positive and significant because information is more important for the income process in "1983" calibration. Why? Because types are more persistent in that calibration. This implies that a type is more informative and therefore, learning individuals' types is more valuable.

This exercise is also important because it can be compared with "1983" calibration to obtain another "estimation" of the effect of information costs. The only difference between "1983" calibration and counterfactual calibration 2 is the drop of information costs to zero. Therefore, the change in a variable between these two economies can be attributed to the change in information costs. The salient result is that the effect on the bankruptcy rate is again very significant: it accounts by 45% of the total rise between 1983 and 2004—see bottom panel in Table 3. Additionally, this exercise is useful because it shows that changing information costs has a significant effect on the bankruptcy rate with both income processes. This means that the importance of information costs is robust to changes in the income distribution as the one obtained between "1983" and "2004" calibrations.

5 Conclusions

How do information costs affect consumer debt and bankruptcy? Asymmetric information and costly screening are incorporated into a model of consumer debt and bankruptcy to study this question. When screening is too expensive, uninformed lenders overcome the lack of information by designing contracts to induce individuals to reveal their income. The contracts' design implies that low-risk individuals are borrowing constrained. This is because contracts with low interest rates are linked to tight debt limits to avoid high-risk individuals taking these contracts (they will prefer contracts with higher interest rates and looser borrowing constraints). With technological progress in the IT sector, information costs drop and previously borrowing constrained individuals can now be screened and obtain more debt. Then, the rise in debt generates an increase in bankruptcy filings because the benefits of filing bankruptcy increase with the amount owed.

Can this model account for both the rise in consumer debt and bankruptcy? Several quantitative exercises are performed to answer this question. The parameters are first calibrated to the year 2004 assuming that information costs are zero. The model is successful replicating key moments for this year. Then, the cost of information and the parameters determining the income distribution are recalibrated to the year 1983. Without changing any preference parameters, the model can replicate many important features of the data. In particular, it can reproduce the change in the bankruptcy rate and the debt-to-output ratio. Then, the effect of information costs is isolated by computing an economy with the income distribution for 2004 and the cost of information for 1983. The results show that 37 percent of the total change in the bankruptcy rate over the last 20 years can be explained by the drop in information costs occurred during the same period. Likewise, the effect of information costs can be identified by computing an economy with the income distribution for 1983 and the cost of information for 2004. In that case, the change in the bankruptcy rate account for 46% of the total rise between 1983 and 2004.

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6 Appendices

6.1 Equilibrium with *reasonable beliefs*

Lenders, when offering a contract, need to consider expectations not just about repayment for each type of individual, but also about the measure of (how many) individuals of each type will take the contract. Until now *rational expectations* were considered: what lenders believe will happen if they offer a contract is exactly what will happen if they do offer the contract. As it was mentioned above, with rational expectations, equilibrium may not exist.

Some notation is now introduced. Let $\mathbf{V}$ denote lifetime utility of an individual with $\{n, l, a\}$ borrowing a' at the discount price $\mathbf{q}$; i.e.,

$$\mathbf{V}(\widehat{n}, l, a; a', \mathbf{q}) = u(c) + \rho\beta \sum_{n'} \Pi(\widehat{n}, n') \int_{l'} G(n', l', a')\phi(l' \mid n')dl',$$

subject to

$$c + \mathbf{q}a' = a + lw.$$

Let $\mathcal{B}_m(a, a', \mathbf{q} \mid q') : \mathbf{N} \times \mathbf{A}^2 \times \mathbf{Q} \to [0,1]$ denote the belief about the measure of individuals of type m and assets a that demand assets a' at the discount price $\mathbf{q}$ if the current best offer is the price function q'.[25] Assume these beliefs are reasonable. This concept is defined below.

Definition 4 *Beliefs $\mathcal{B}_m(a, a', \mathbf{q} \mid q')$ are reasonable, i.e. $\mathcal{B}_m(a, a', \mathbf{q} \mid q') \in \mathbf{B_r}$, if they are consistent with individuals optimal behavior. That is, if a discount price q' is the current best price and a lender consider offering $\mathbf{q}$, with*

$$\widehat{V}(m, l, a; q') > \mathbf{V}(m, l, a; a', \mathbf{q}),$$

then $\mathcal{B}_m(a, a', \mathbf{q} \mid q')$ is a ***reasonable belief*** *if and only if $\mathcal{B}_m(a, a', \mathbf{q} \mid q') = 0$.*

Now, *Definition 2* is modified by incorporating reasonable beliefs in condition 3.

Definition 5 *A discount price function satisfying direct revelation, $\widehat{q} \in \boldsymbol{M}$, is a reasonable beliefs equilibrium contract with uninformed lenders if the following conditions hold:*

1. *Zero expected profits are collected from each contract; i.e.,*

$$\widehat{q}(a', n; a) = \begin{cases} (1+i)^{-1} \Pr(\textit{repayment} \mid a', n, \widehat{q}) \textit{ if } a' \geq \underline{a}(a, n), \\ 0 \textit{ otherwise.} \end{cases}$$

[25] This function can be built as the sup of the offers for each (a', a, n).

2. *Expectations about repayment are realized in equilibrium; i.e.,*

$$\Pr(repayment \mid a', n) = \rho \sum_{n'} \Pi(n, n') \int_{l'} R(n', l', a'; \widehat{q}) \phi(l' \mid n') dl'.$$

3. *There is no other direct-revelation contract implying positive expected profits for all reasonable beliefs; i.e.,* $\nexists \widehat{q}' \in \mathbf{M} : \widehat{p}(\widehat{q}') > 0 \; \forall \mathcal{B} \in \mathbf{B_r}$, *where*

$$\widehat{p}(\widehat{q}) = \sum_n \sum_m \int_a \int_{a'} P(a', n, a, m; \widehat{q}) \mathcal{B}_m(a, a', \widehat{q}'(a', n; a) \mid \widehat{q}) da' da,$$

$$P(a', n, a, m; \widehat{q}) = \left\{ -\frac{\Pr(repayment \mid a', m)}{1+i} a' + \widehat{q}(a'; a, n) a' \right\}.$$

6.2 More on the calibration

The transition matrix is described in more detail in this subsection. This matrix can be written as

$$\begin{pmatrix}
\varrho_1 & \chi_1\omega & \chi_2\omega & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \\
\alpha_{-1} & \varrho_2 & \chi_1 & \chi_2 & 0 & 0 & 0 & 0 & 0 & \varepsilon \\
\chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & 0 & 0 & \varepsilon \\
0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & 0 & \varepsilon \\
0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & \varepsilon \\
0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & \varepsilon \\
0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & \varepsilon \\
0 & 0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_4 & \chi_1 & \varepsilon \\
0 & 0 & 0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_5 & \varepsilon \\
1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 0
\end{pmatrix},$$

which is equivalent to

$$\begin{pmatrix}
\varrho_1 & R_1 f_1 & R_1(1-f_1) & 0 & 0 & ... & 0 & 0 & \varepsilon \\
\alpha_{-1} & \varrho_2 & R_2 f_1 & R_2(1-f_1) & 0 & ... & 0 & 0 & \varepsilon \\
R_3(1-f_1) & R_3 f_1 & \varrho_3 & R_2 f_1 & R_2(1-f_1) & ... & 0 & 0 & \varepsilon \\
0 & R_3(1-f_1) & R_3 f_1 & \varrho_3 & R_2 f_1 & ... & 0 & 0 & \varepsilon \\
0 & 0 & R_3(1-f_1) & R_3 f_1 & \varrho_3 & ... & 0 & 0 & \varepsilon \\
0 & 0 & 0 & R_3(1-f_1) & R_3 f_1 & ... & R_2(1-f_1) & 0 & \varepsilon \\
0 & 0 & 0 & 0 & R_3(1-f_1) & ... & R_2 f_1 & R_2(1-f_1) & \varepsilon \\
0 & 0 & 0 & 0 & 0 & ... & \varrho_4 & R_2 f_1 & \varepsilon \\
0 & 0 & 0 & 0 & 0 & ... & R_3 f_1 & \varrho_5 & \varepsilon \\
1/9 & 1/9 & 1/9 & 1/9 & 1/9 & ... & 1/9 & 1/9 & 0
\end{pmatrix},$$

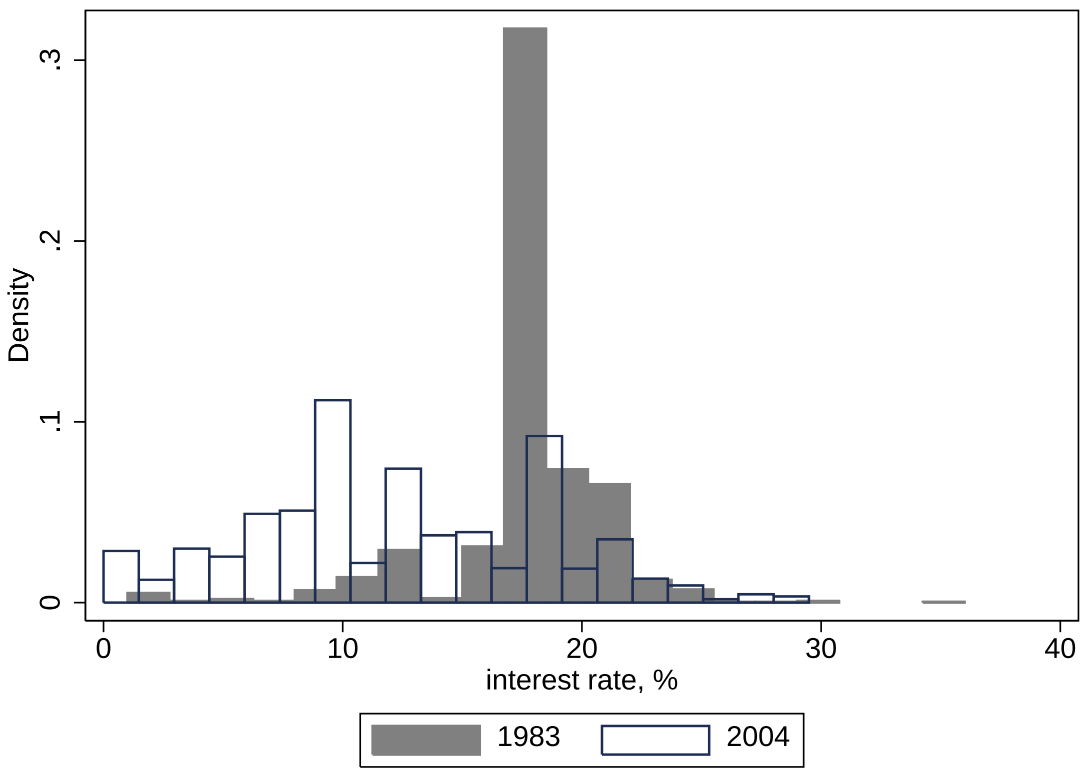
where R_i is obtained taking into account each row must sum up 1. Therefore, R_1, R_2, R_3, ϱ_4 and ϱ_5 are known, since they can be determined by

$$\begin{aligned}
R_1 &= (1-\varrho_1-\varepsilon)\\
R_2 &= (1-\alpha_{-1}-\varrho_2-\varepsilon)\\
R_3 &= (1-\varrho_3-R_2-\varepsilon)\\
\varrho_4 &= (1-R_3-R_2f_1-\varepsilon)\\
\varrho_5 &= (1-R_3-\varepsilon).
\end{aligned}$$

Then, only six parameter need to be calibrated : ρ_1, f_1, ε, α_1, ρ_2, and ρ_3.

6.3 Additional Figures and Tables

Figure 10: Distribution of interest rates on credit cards



Source: SCF restricted to households with credit cards debt.

Table 4: The effect of income on interest rates

Dependent variable: interest rate	Year	
	1983	2004
ln(income)	0.155	-0.768
	(0.219)	(0.103)
ln(credit card debt)	0.061	0.209
	(0.102)	(0.052)
Age of the head of household	0.007	0.012
	(0.012)	(0.008)
Age of the head of household squared	-0.001	-0.001
	(0.001)	(0.000)
Male head of household	0.720	-0.315
	(0.497)	(0.242)
Married head of household	-0.370	0.204
	(0.439)	(0.215)
Constant	15.400	18.720
	(2.114)	(1.076)
Observations	1115	6380
R squared	0.010	0.012

Data from SCF restricted to households with credit card debt.
The difference between the coefficients for ln(income) is 0.923 and the standard deviation of this difference is 0.379.
The difference between the coefficients for ln(credit card debt) is 0.149 and the standard deviation of this difference is 0.178.

Table 5: Descriptive Statistics

	Year 1983	Year 2004
Mean of income	27,037	72,019
Gini of income	0.45	0.53
Income mean-to-median ratio	1.35	1.63
Standard deviation of log income	0.84	0.93
Mean of debt*	171	1,014
Mean-debt-to-mean income ratio	0.63%	1.41%
Proportion of households with negative net worth	5.04%	6.93%
Proportion of households with credit card	65.4%	74.9%
Proportion of households carrying a balance in credit card	37.0%	46.2%
Gini of wealth	0.746	0.806
Wealth mean-to-median ratio	2.95	4.698
Standard deviation of log wealth	1.91	2.091
Proportion of total debt held by income poorest 5%	21.0%	4.7%
Proportion of total debt held by income poorest 10%	24.9%	10.0%
Proportion of total debt held by income poorest 20%	32.7%	25.0%
Proportion of total debt held by income poorest 25%	38.6%	38.2%
Proportion of total debt held by income poorest 30%	43.1%	44.0%
Households (U.S. Bureau of the Census)	83,918,000	112,000,000
Population over 20 years old (U.S. Bureau of the Census)	164,669,767	212,064,902
Population (U.S. Bureau of the Census)	233,792,000	293,655,000
Bankruptcy filings	286,444	1,563,145
Bankruptcy filings over households	0.34%	1.39%
Bankruptcy filings over population over 20 years	0.17%	0.74%
Bankruptcy filings over population	0.12%	0.53%
Bankruptcy filings over households carrying credit card balance	0.92%	3.01%
Bankruptcy filings over population (over 20) carrying credit card balance**	0.47%	1.60%
Bankruptcy filings over population carrying a balance in credit card**	0.33%	1.15%
Fixed assets and consumer durable goods to GDP ratio (BEA)	3.44	3.27

(*) Debt here is defined as minus net worth if net worth is negative and 0 otherwise.

(**) Obtained dividing the bankruptcy filings by the population (over 20) carrying a balance in credit card. The population (over 20) with a credit card balance is obtained multiplying the number of households carrying a balance in credit card by the average number of individuals over 20 years old per household.

Source: SCF 1983 and 2004 or specified between parenthesis otherwise.

Table 6: Transition matrix types 1 to 6 (data and model)

	Data year 2004						Model's "2004" calibration					
	Income groups between percentiles						Income groups between percentiles					
	$[0,6]$	$[6,43]$	$[43,63]$	$[63,77]$	$[77,86]$	$[86,92]$	$[0,6]$	$[6,43]$	$[43,63]$	$[63,77]$	$[77,86]$	$[86,92]$
$[0,6]$	0.36	0.50	0.06	0.03	0.02	0.01	0.20	0.48	0.32	0.00	0.00	0.00
$[6,43]$	0.06	0.68	0.15	0.05	0.02	0.01	0.04	0.76	0.12	0.08	0.00	0.00
$[43,63]$	0.02	0.27	0.45	0.17	0.04	0.03	0.16	0.23	0.41	0.12	0.08	0.00
$[63,77]$	0.02	0.14	0.19	0.46	0.13	0.03	0.00	0.16	0.23	0.41	0.12	0.08
$[77,86]$	0.01	0.10	0.12	0.21	0.34	0.13	0.00	0.00	0.16	0.23	0.41	0.12
$[86,92]$	0.02	0.09	0.09	0.10	0.20	0.36	0.00	0.00	0.00	0.16	0.23	0.41

	Data year 1983						Model's "1983" calibration					
	Income groups between percentiles						Income groups between percentiles					
	$[0,7]$	$[7,45]$	$[45,64]$	$[64,78]$	$[78,86]$	$[86,92]$	$[0,7]$	$[7,45]$	$[45,64]$	$[64,78]$	$[78,86]$	$[86,92]$
$[0,7]$	0.46	0.41	0.06	0.03	0.02	0.01	0.44	0.35	0.21	0.00	0.00	0.00
$[7,45]$	0.07	0.71	0.13	0.04	0.02	0.01	0.04	0.77	0.12	0.07	0.00	0.00
$[45,64]$	0.02	0.28	0.47	0.15	0.05	0.01	0.12	0.19	0.42	0.17	0.10	0.00
$[64,78]$	0.02	0.14	0.22	0.41	0.17	0.03	0.00	0.12	0.19	0.42	0.17	0.10
$[78,86]$	0.01	0.09	0.09	0.21	0.41	0.12	0.00	0.00	0.12	0.19	0.42	0.17
$[86,92]$	0.02	0.09	0.08	0.09	0.26	0.31	0.00	0.00	0.00	0.12	0.19	0.42

Source: Matched population from the Annual Demographic and Income Supplement from the Current Population Survey. The sample was restricted to men in working age.

Table 7: All relevant statistics in the artificial economies

Statistics	"1983" calibration $\bar{\mathrm{I}}_{83}, \Pi_{83}, z_{83}$	Counterfactual calibration 2 $\bar{\mathrm{I}}_{83}, \Pi_{83}, z_{04}$	Counterfactual calibration 1 $\bar{\mathrm{I}}_{04}, \Pi_{04}, z_{83}$	"2004" calibration $\bar{\mathrm{I}}_{04}, \Pi_{04}, z_{04}$
Bankruptcy rate	0.51%	1.01%	1.03%	1.45%
Bankruptcy filings	0.025%	0.077%	0.055%	0.14%
Population in debt	4.93%	7.65%	5.30%	9.48%
Debt-to-income ratio	0.33%	0.61%	0.27%	0.73%
Proportion of total debt held income poorest 10%	31%	22%	19%	14%
Proportion of total debt held income poorest 20%	44%	38%	33%	33%
Proportion of total debt held income poorest 30%	57%	57%	51%	52%
Gini coefficient of income	0.45	0.45	0.53	0.53
Mean-to-median income ratio	1.35	1.35	1.64	1.64
Gini coefficient of wealth	0.82	0.84	0.69	0.82
Mean-to-median wealth ratio	5.2	5.4	2.72	5.28
Capital-to-output ratio	3.39	3.66	2.68	3.31
Proportion of total debt borrowed at $q(\cdot, 1)$	54.5%	38.2%	39.6%	27.0%
Proportion of total debt borrowed at $q(\cdot, 2)$	45.4%	61.5%	60.2%	71.8%
Risky debt as proportion of total debt	11.2%	17.3%	18.9%	35.1%
Proportion of risky debt borrowed at $q(\cdot, 1)$	98.6%	56.0%	62.2%	34.0%
Proportion of risky debt borrowed at $q(\cdot, 2)$	1.4%	43.9%	37.8%	65.7%
Proportion of risky debt borrowed from informed lenders	0.9%	100%	0.6%	100%
Risk free interest rate	4.0%	3.1%	6.5%	4.0%
Mean (paid) discount price	0.938	0.946	0.915	0.937
Std. dev. (paid) discount price	0.010	0.014	0.011	0.014
Std. dev. (paid) log discount price	0.014	0.019	0.014	0.017
Information costs to mean income	3.9%	0%	3.4%	0%