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Abstract

Consumer bankruptcies rose sharply over the last 20 years in the U.S. economy. During the same period, there was impressive technological progress in the information sector. This paper provides a theory to understand and quantify the role of improvements in information technologies in consumer credit markets. Informational frictions restrict the amount of debt that can be borrowed. In fact, in the equilibrium in which investing in information is too expensive, many households borrow such small amounts that the default risk is very low. When information costs drop and informational frictions vanish, those households borrow more and default is likely after a bad shock. Quantitative exercises show that information costs have a significant effect on the bankruptcy rate. Additionally, a drop in information costs generates changes in other variables (e.g. interest rate dispersion) similar to what has occurred over the last 20 years.

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1 Introduction

Consumer bankruptcy is a central issue today because of its explosive rise over the last 20 years in the U.S. economy. Although many explanations have been proposed, there is still no conclusive understanding of these trends. A possible reason is the drop in information costs. This driving force may be important because during the same period, there was impressive technological progress in the information sector—often called the IT revolution—and the financial sector uses information intensively to evaluate credit risk.[1] The purpose of this paper is to provide a theory to understand the effect of information on bankruptcy and to quantify its importance.

The number of annual *bankruptcy filings* increased by 1.3 million—from 286,444 to 1,563,145, almost 5.5 times—between 1983 and 2004, as depicted in *Figure 1*. Before the early 1980s, the rise in bankruptcy was moderate. According to Moss and Johnson (1999), "from 1920 to 1985, the growth of consumer filings closely tracked the growth of real consumer credit. Since then, however, the rate of increase of consumer bankruptcies has far outpaced that of real consumer credit." This conclusion is drawn looking at the ratio of bankruptcies filings to real consumer credit.[2] A similar measure, the ratio of bankruptcy filings to the number of households in debt, will be used here. This statistic, referred to as the *bankruptcy rate* hereafter, increased from 0.92% to 3% between 1983 and 2004.

To study the role of the IT revolution on consumer bankruptcy, this paper extends Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) to incorporate *informational frictions*. The key ingredient is that the persistent component of income, referred to as the *productivity group* hereafter, is unobservable. Lenders would like to know this component because persistence implies that the productivity group will be useful to estimate the risk of bankruptcy. To capture the existence of debt contracts with different intensity in the use of information technologies, two alternative contracts will be considered. *Screening contracts* require the use of an screening technology to identify a household's permanent component of productivity. Think of a household answering some questions to a credit card company's employee (job characteristics, ZIP code, total household income, monthly rent/mortgage payment, etc.) who enters the information into a computer and after a few minutes, tells the applicant about the offer's characteristics (mainly interest rate and amount that can be borrowed).[3] Two things

[1] For a careful description of the use of information technologies in the financial sector see the work of Berger (2003). For an analysis of the effect of progress in monitoring technologies on the allocation of capital, firms' financing, and capital deepening, see the study of Greenwood, Sanchez, and Wang (2007).

[2] See Figure 2 in Moss and Johnson (1999).

[3] Similarly, think of a household filing a credit card application on Internet, or credit card companies

are important to notice about this contract: (i) the offer's characteristics depend on information about the applicant, and (ii) information is processed using information technologies and therefore the state of information technologies will affect the cost of this contract. The alternative contracts are called *revelation contracts*. Think of credit markets populated by lenders who compete with one another in the design of contracts intended to separate the different type of borrowers.[4] These contracts do not require information about a particular household nor do they require the use of information technologies. Lenders design ex ante a menu with combinations of interest rates and amounts of debt and it is up to the households to choose which pair of debt amount and interest rate they prefer.[5] The contract's design induces the households to choose the contract that was designed for them. The possibility of self-revelation using interest rates and amounts borrowed in the Eaton and Gersovitz (1981) model of default has not been noted before.[6] This self-revelation is possible because the trade-off between interest rates and borrowing amounts is different for households in different productivity groups. In particular, since low-productivity-group households are more likely to file for bankruptcy, they accept a higher interest rate to be able to borrow more. As a consequence, with revelation contracts, households above the lowest productivity group obtain lower interest rates at the cost of borrowing smaller amounts.

Given the state of information technologies, which determines the cost of screening contracts, households (lenders) decide which debt contract they prefer to use (offer). The advantage of screening contracts is that households can borrow as much as they want at their zero-profit interest rates. The main disadvantage of these contracts is that they have an additional cost as they require the use of information technologies. As a consequence, when screening costs are high enough, revelation contracts are preferred. That is the key to understand how a drop in information costs generates more debt and bankruptcy: Under revelation contracts some households cannot borrow as much as they would like at their zero-profit interest rates, i.e., they are borrowing-constrained. Then, as information costs drop, those households switch to screening contracts to borrow more, and the number of bankruptcy filings rises. More debt generates more bankruptcy because the benefit from bankruptcy—discharge of debts—is increasing in the amount owed, while the costs—temporary exclusion from financial markets and

gathering information on a household and sending the offer by mail.

[4]See chapter 11 in Hirshleifer and Riley (1992) for an excellent survey of theoretical results in this environment.

[5]Similarly, they could mail all the alternative offers to the households or offer all of them in a bank branch.

[6]The applicability of this result goes beyond credit card markets. Notice that the Eaton and Gersovitz (1981) model of default has been widely used in international finance (Aguiar and Gopinath, 2006; Arellano, 2008; Cuadra and Sapriza, 2006, 2008; Mendoza and Yue, 2008).

income lost—are independent of the household's debt size.

This paper also evaluates the effect of information costs on bankruptcy quantitatively. First, the model is calibrated to account for relevant features of the U.S. data for the year 1983. Specifically, it reproduces the bankruptcy rate, the debt-to-income ratio, the capital-to-output ratio, and some moments of the joint distribution of debt and income. The model also matches well non-targeted moments. Then, the model is used to answer a quantitative question: Can the rise in the bankruptcy rate be explained by the drop in information costs? Unfortunately, there is not a direct measure of the cost of obtaining and processing household information about the risk of default. Therefore, to answer this question, the technology in the information sector is recalibrated. A simple strategy is followed: This technological parameter is calibrated to match the bankruptcy rate in 2004. Then, the model economy is computed with all the parameters for 1983 but with the technology in the information sector for 2004. The model is then challenged by analyzing *implications* of a drop in information costs for: (i) the distribution of debt across income groups, (ii) the dispersion of interest rates, and (iii) the projection of interest rates on income, are analyzed. The model does reasonably well, even though these facts were not targeted.

The first quantitative paper studying the rise in consumer bankruptcies is Livshits, MacGee, and Tertilt (2007a). They argue that a drop in stigma (utility cost of bankruptcy) together with a drop in transaction costs can explain the rise in bankruptcy. Narajabad (2007) proposes an alternative explanation to understand the same fact: the information technology revolution. Together with the current paper, Athreya, Tam, and Young (2008); Drozd and Nosal (2008); and Livshits, MacGee, and Tertilt (2009) present different models to evaluate this driving force.[7]

Narajabad (2007) evaluates the role of more informative credit rating technologies in an environment with heterogeneity in the *cost of bankruptcy*. There are two important differences between Narajabad (2007) and the current paper. First, households do not know their type—their own cost of bankruptcy—when they sign a debt contract. This assumption is crucial because it makes revelation contracts impossible and implies that the key mechanism at work in this model is ruled out by assumption. Second, restrictive assumptions make his model not very suitable for quantitative purposes. For instance, by assuming that households cannot save, he makes any comparison between the model and data on the distribution of assets—key in a model of debt—impossible.

[7]Other recent papers on consumer debt and bankruptcy with informational frictions are Chatterjee, Corbae, and Rios-Rull (2008, 2007). They incorporate asymmetric information in the Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) model of consumer debt and bankruptcy. They find that informational frictions can generate exclusion after bankruptcy as an equilibrium outcome.

Athreya, Tam, and Young (2008) present a quantitative model of unsecured debt with informational frictions. The equilibrium in the environment called "partial information" could be compared with the equilibrium in the current paper when the cost of information is high enough that all lenders use revelation contracts. However, there are two crucial differences. First, they consider signalling equilibria. Lenders' beliefs are crucial. Basically, beliefs must be used to select which equilibrium will be analyzed.[8] Given the choice of beliefs, they find an equilibrium in which there is practically no borrowing in the environment with partial information. This differs substantially from the current paper in which, as explained above, lenders (partially) offset the lack of information by designing debt contracts accordingly. This difference is very important if we bear in mind that what was puzzling about the last 20 years is that the rate of increase of consumer bankruptcies has far outpaced that of real consumer credit. Second, only extreme cases in terms of information availability can be compared. That is, only special cases of the model in the current paper can be studied.

Drozd and Nosal (2008) present a search model of the market for unsecured credit. They study the effect of a drop in the cost of screening and soliciting credit customers on debt and bankruptcy. The cost of screening is potentially close to the current paper. However, Drozd and Nosal (2008) do not model asymmetric information, lenders have no alternative to paying the cost of screening, and therefore their cost is more related to a transaction cost as the one analyzed by Livshits, MacGee, and Tertilt (2007a). Livshits, MacGee, and Tertilt (2009) present a very stylized model with informational frictions in which lenders must pay a cost to design a contract. The model is used to explore, qualitatively, the implications of technological progress in consumer lending. The most remarkable prediction of the model that is supported by the data is that the empirical density of credit card interest rates has become more dispersed since 1983. The quantitative model in the current paper also has this prediction, and the results are directly compared to the data.

2 The mechanism in a 2-period model

This section previews the main driving forces at work in the full model using a simple 2-period model. An additional simplification is that the analysis is in partial equilibrium; i.e., the risk-free interest rate, i, and wages, w, are given. All the proof of this section's lemmas are in the appendix.

[8] That is why Hirshleifer and Riley (1992) conclude that the Nash equilibrium concept has little predictive power in that environment.

Physical environment. The economy is populated by households and lenders. Households live for 2 periods, $t = 1, 2$, and they are endowed with a quantity of labor measured in efficiency units, l_n, that can take 2 values, $l_n \in \{l_L, l_H\}$, meaning low or high productivity. The transition probability between state L and H is $\pi_{L,H}$. Persistence is also assumed: $\pi_{H,H} > \pi_{H,L}$ and $\pi_{L,L} > \pi_{L,H}$. Importantly, it implies $\pi_{H,H} > \pi_{L,H}$. Here, current productivity and "the persistent component of productivity," or productivity group, coincide. Importantly, this does not need to be the case for the results in this section to hold. For instance, all the results in this section go through if all the households have the same income in the first period but some of them have a higher probability of a transition to high productivity than others.

Credit markets. Lenders compete offering debt contracts. In particular, there are two types of contracts. On the one hand, lenders sell *revelation contracts*. These contracts are designed to induce the households to reveal their productivity. The price function in this case is $\widehat{q}$, and depends only on the amount borrowed. On the other hand, they sell *screening contracts*. These contracts require the use of information technologies to learn a household's productivity. The price function in this case is $\widetilde{q}$ and depends on the household's productivity and amount borrowed. The cost of screening a household's productivity (also referred to as information costs), $\mathcal{C}$, is proportional to the amount borrowed to simplify the analysis.[9] It is important to bear in mind that this cost is paid only if screening contracts are used.

Household's problem. In period 1, households decide which type of contract they prefer as well as how much to borrow.[10] In period 2, after the realization of the productivity shock, they decide whether to file for bankruptcy or to pay back the debt. If they decide to file for bankruptcy, they lose a proportion of their income, τ. Thus, the lifetime utility of a household born with assets $a_1 \in \mathbf{A}$, income $y_n = wl_n$, and facing a price function q is

$$\begin{aligned} U(a_1, y_{1,n}; q) = \max_{a_2 \in \mathbf{A}} & \, u(y_{1,n} + a_1 - q(a_2, n; a_1)a_2) \\ & + \beta \pi_{n,H} \max\{u(y_{2,H} + a_2), u(y_{2,H}(1-\tau))\} \\ & + \beta \pi_{n,L} \max\{u(y_{2,L} + a_2), u(y_{2,L}(1-\tau))\}. \end{aligned} \tag{1}$$

[9] Later, in the quantitative general equilibrium model, this cost is independent of the amount borrowed.

[10] The fact that households choose the contract (screening vs. revelation) is for exposition and without loss of generality.

Here the price function q is used to represent $\widetilde{q}$ or $\widehat{q}$. Then, the choice of lender implies that lifetime utility is $\mathbf{U}(a_1, y_{1,n}) = \max\{U(a_1 - \mathcal{C}, y_{1,n}; \widetilde{q}), U(a_1, y_{1,n}; \widehat{q})\}$. Similarly, it is useful to define

$$\begin{aligned}\mathcal{U}(a_1, y_{1,n}; a_2, \mathbf{q}) \equiv & u(y_{1,n} + a_1 - \mathbf{q}a_2) + \beta\pi_{n,H} \max\{u(y_{2,H} + a_2), u(y_{2,H}(1-\tau))\} \\ & + \beta\pi_{n,L} \max\{u(y_{2,L} + a_2), u(y_{2,L}(1-\tau))\},\end{aligned}$$

as the lifetime utility of a household born with assets a_1, income y_n, borrowing $-a_2$ at the price $\mathbf{q}$. This function will be used later to define self-revelation.

Zero-expected-profit prices. Lenders' expected profits from a contract $q(a_2, n)$ are

$$\underbrace{q(a_2, n)a_2}_{\text{amount borrowers receive}} \quad - \quad \underbrace{\Pr(\text{repayment} \mid a_2, n)a_2(1+i)^{-1}}_{\text{discounted amount lenders expect to recover}},$$

where $\Pr(\text{repayment} \mid a_2, n)$ is the lender's expectation of repayment given the amount borrowed ($-a_2$) and the household's type (n). Before the equilibrium prices for each type of contracts (screening and revelation) are derived, it is useful to characterize the prices implying zero-expected profits.

Lemma 1 *Zero-expected-profit prices for each $n = \{L, H\}$ are*

$$\overline{q}(a_2, n) = \begin{cases} (1+i)^{-1} & \text{if } \underline{a}_{2,L} \leq a_2, \\ \pi_{n,H}(1+i)^{-1} & \text{if } \underline{a}_{2,H} < a_2 < \underline{a}_{2,L}, \\ 0 & \text{if } a_2 \leq \underline{a}_{2,H}, \end{cases}$$

where $\underline{a}_{2,L} = -\tau y_{2,L}$ and $\underline{a}_{2,H} = -\tau y_{2,H}$.

Lemma 1 implies that zero-expected-profit prices vary as in *Figure 2*. The intuition is simple. If debt is small enough ($\underline{a}_{2,L} \leq a_2$), households will prefer to pay back their debt for both levels of next period income. This is because the benefit of bankruptcy—discharge of debt—depends directly on the amount borrowed, while the cost—income lost—is independent of it. For bigger debt ($\underline{a}_{2,H} \leq a_2 < \underline{a}_{2,L}$), households will find beneficial to file for bankruptcy only if next period income is low. This is because the cost of bankruptcy is directly related to the current income. Notice that in this range, prices depend on current productivity. This is due to the fact that productivity is persistent, implying that households with higher productivity have also higher expected productivity. For amounts of debt big enough, households will file bankruptcy for both level of income. Clearly, this implies that prices must be zero to obtain zero-expected profits.

Preferences over debt and prices. Before the equilibrium can be characterized, it is also useful to study the preferences of households over prices, $\mathbf{q}$, and amounts of assets for period 2, a_2. Specifically, it is important to characterize preferences over a_2 and $\mathbf{q}$ in the range $\underline{a}_{2,H} < a_2 < \underline{a}_{2,L}$, where borrowing implies risk of bankruptcy. In this simple model, it is only there where lenders would like to charge different prices to households with different current productivity.

Lemma 2 *The slope of the indifference curves at* $\mathbf{q}$ *as a function of* $a_2 \in [\underline{a}_{2,H}, \underline{a}_{2,L}]$ *is*

$$-MRS_{\mathbf{q},a_2}(\mathbf{q}, a_2) \begin{cases} < 0, & \text{for } \underline{a}_{2,H} < a_2 < a_2^*(\mathbf{q}), \\ = 0, & \text{for } a_2 = a_2^*(\mathbf{q}), \\ > 0, & \text{for } a_2^*(\mathbf{q}) < a_2 < \underline{a}_{2,L}, \end{cases}$$

where a_2^* *solves* $-u'(y_{1,n} + a_1 - \mathbf{q}a_2^*)\mathbf{q} + \beta\pi_{n,H}u'(y_{2,H} + a_2^*) = 0$.

The level of assets $a_2^*(\mathbf{q})$ is the level of asset accumulation solving the first order condition of the household's problem given a price function constant at $\mathbf{q}$. By construction, the slope is zero at that level of debt. Starting from there, it is simple to understand the shape of the indifference curve. Any deviation from $a_2^*(\mathbf{q})$ reduces the household's utility, implying that any deviation must be compensated with a higher $\mathbf{q}$ to keep the household with the same utility.

Importantly, the range of a in which the slope is negative is not relevant to characterize equilibrium outcomes. This is because for any point in that range lenders and households would be better off reducing the amount borrowed.[11] Therefore, hereafter (and in the figures) we focus on the increasing part of the indifference curves. More importantly, indifference curves of households with different current productive have different slopes at a given amount of debt. This result is crucial for the existence of revelation contracts. Intuitively, this follows because (i) households with low productivity in period t are willing to borrow more than an households with high productivity, and (ii) they are also more likely to file for bankruptcy in the second period, so they are less affected by the interest rate.

Lemma 3 *Take any value* $\mathbf{q}$ *and consider any* $a_2 \in (a_2^*(\mathbf{q}), \underline{a}_{2,L})$. *Then, the slope is bigger (steeper) for households with low productivity than for those with high productivity.*

Equilibrium contracts. First, notice that the choice of contracts is irrelevant for $a_2 > \underline{a}_{2,L}$. This is because for those amounts of debt the probability of bankruptcy is

[11] The household is better off by construction (decreasing indifference curve). The lender is better off because the repayment probability is increasing (but not strictly) in a_2.

zero for both levels of current productivity. In that case, equilibrium prices are equal to the zero-expected-profit prices described above. The rest of this subsection describes equilibrium prices for screening and revelation contract for $a_2 \in [\underline{a}_{2,H}, \underline{a}_{2,L}]$.

In equilibrium, competition between lenders implies that prices must imply zero profits. First, focus on *screening contracts*. Zero-expected profits implies that prices must take into account the cost of information.

Lemma 4 *Equilibrium prices of screening contracts with* $a_2 \in [\underline{a}_{2,H}, \underline{a}_{2,L}]$ *are* $\tilde{q}(a_2, n) = \bar{q}(a_2, n) - \mathcal{C}$.

Figure 3 shows an allocation in an economy with only screening contracts. It is clear there that high-risk (low-productivity) households would be better off if they could avoid paying $\mathcal{C}$—they would be in a higher indifference curve. It is also clear that lenders will be willing to offer $\mathbf{q}$ above the price of screening contracts (in that range) and below zero-profit prices for low-productivity households, and do not pay for information. They would obtain profits because the worst that can happen is that only low-productivity households take it. This explains why contracts that do not require the use of information technologies must be offered in equilibrium.

Now, focus on *revelation contracts*. Think that lenders design contracts under the constraint that they must induce households to reveal their productivity. Using prices and amounts of debt as instruments, it is possible to induce households to reveal their type. It is indeed possible to separate households because in order to obtain more debt, low-productivity households are willing to accept a bigger increase in interest rates than high-productivity households; i.e., indifference curves are as described by Lemma 3.

Suppose the price of a revelation contract, q, depend on the amount borrowed, $-a_2$, the household's report on productivity, m, and the current stock of assets, a_1. Notice that the price depends also on a_1 because households' willingness to borrow depends on it. Then, we can define self-revelation.

Definition 1 *A function* q *satisfies self-revelation if and only if for each given current assets* a_1, $\max_{a_2} \mathcal{U}(a_1, y_{1,m}; a_2, q(a_2, m; a_1)) \geq \max_{\hat{a}} \mathcal{U}(a_1, y_{1,m}; \hat{a}, q(\hat{a}, n; a_1)),\ n \neq m$.

In words, q satisfies the self-revelation constraint if and only if households are better off borrowing at the price designed for their productivity than misrepresenting their productivity. Notice that for a given a_1, revelation implies that there must be at most one value of $\mathbf{q}$ for each a_2—it cannot depend on the report. Otherwise, borrowers will

make the report that implies the highest $\mathbf{q}$. Thus, q can be written only as a function of (a_2, a_1).

But, what are the equilibrium prices of revelation contracts? In general, equilibrium requires that lenders make zero profits and there is no other profitable contract that borrowers would prefer.[12] However, it is well known that using that definition an equilibrium may not exist in this environment. Then, to guarantee existence, we consider a similar concept, introduced by Riley (1979): prices of revelation contracts are equilibrium prices if they imply zero profits and any other contract that is profitable and the borrowers would prefer imply subsidies across borrowers with different risk. Under this definition, lenders do not deviate from the equilibrium allocation because those deviations will become unprofitable after a reaction that skims the cream and produces losses for the defector.[13] The following lemma characterizes equilibrium prices of revelation contracts.

Lemma 5 *The equilibrium prices of revelation contracts are*

$$\hat{q}(a_2; a_1) = \begin{cases} \bar{q}(a_2, H) & \text{if } a_2 \geq \underline{a}_2(a_1), \\ \bar{q}(a_2, L) & \text{if } \underline{a}_2(a_1) \geq a_2, \end{cases}$$

where $\underline{a}_2(a_1)$ *is such that* $\mathcal{U}(a_1, y_{1,L}; \underline{a}_2(a_1), \bar{q}(\underline{a}_2(a_1), H)) = max_{\hat{a}} \mathcal{U}(a_1, y_{1,L}; \hat{a}, \bar{q}(\hat{a}, L))$.

Screening or revelation contract? Now, the choice of contracts can be described. Given the cost of information, households compare the utility from both types of contracts and choose the one associated with higher utility.

Lemma 6 *The choice of contracts is characterized by*

1. *Households with low productivity never prefer screening contracts.*
2. *There exists a cost of information* $\underline{\mathbf{c}}$ *such that households with high productivity are indifferent between screening and revelation contracts. Households with high productivity prefer screening contracts if and only if* $\mathcal{C} < \underline{\mathbf{c}}$.

The effect of information costs on debt and bankruptcy. Lower information costs allow high-productivity households to borrow more, making them more likely to file for bankruptcy.

[12] This would be a Nash equilibrium.

[13] The existence problem of Nash equilibrium and the existence of a unique Reactive equilibrium—the concept used here—are explained in details by Hirshleifer and Riley (1992), Chapter 11.

Lemma 7 *If information costs fall from $\mathcal{C}_0$ to $\mathcal{C}_1$ and a household does not pay for information at $\mathcal{C}_0$ but she pays for information at $\mathcal{C}_1$, then:*

1. *This household's debt increases.*
2. *This household's probability of bankruptcy increases or stays constant.*

Assume initially the cost of information is high enough, $\mathcal{C}_0 > \underline{\mathbf{c}}$. If the debt constraint associated with revelation contracts is tight enough, high-productivity households may prefer to borrow at risk-free price. Assume this is the case. In this allocation, high-productivity households are borrowing-constrained: they would prefer to borrow more at their zero-expected-profit price, but those prices cannot be offered because low-productivity households will pretend to have higher productivity. Now, assume technological progress occurred in the information sector implying that $\mathcal{C}_1 < \underline{\mathbf{c}}$. This implies that high-productivity households prefer to pay the cost of information. If they do so, it is because they borrow more in the new allocation. Actually, they would only pay for information to borrow with some risk of bankruptcy. Thus, in this new allocation there is also more bankruptcy because this household now files for bankruptcy with probability $(1 - \pi_{H,H})$.

These two results are important because they are qualitatively consistent with the facts presented above for the U.S. economy. Hereafter a general equilibrium model is developed in an attempt to quantify the importance of these results.

3 Quantitative General Equilibrium Model

3.1 The Model

Environment. Time is discrete and denoted by $t = 0, 1, 2, \ldots$ At any time there is a unit mass of households. They discount the future at the rate β. Preferences of households are given by the expected value of the discounted sum of momentary utility

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

where c_t is consumption at period t. The utility function u is strictly increasing, strictly concave, and twice differentiable. Let $n \in \mathbf{N} = \{1, 2, \ldots, N\}$ denote the *productivity group* of a household. Productivity groups are persistent, with transition probability $\Pi(n_t, n_{t+1})$. Each household is endowed with one unit of time. Inside each group, productivity is exogenously determined by labor endowments that come from different group-specific intervals; for each n, $l \in L(n) = [\underline{l}^n, \bar{l}^n]$. Thus, labor

endowments and productivity groups at time t are correlated. The transition function is $\phi(l_{t+1} \mid n_{t+1})\Pi(n_t, n_{t+1})$, where $\phi(l_{t+1} \mid n_{t+1})$ is a conditional density function. Notice that the cumulative density function of l_{t+1} can be written as a function of n_t directly,

$$F(l_{t+1} \mid n_t) = \sum_{n_{t+1}} \int_{\underline{l}^0}^{\bar{l}^n_{t+1}} \phi(l_{t+1} \mid n_{t+1})\Pi(n_t, n_{t+1}).$$

With this notation it is simpler to state the assumption on the transition function.

Assumption 1 *n_t is an index of first order stochastic dominance; i.e., if $n_t > \hat{n}_t$, then $F(l_{t+1} \mid n_t) \leq F(l_{t+1} \mid \hat{n}_t)$ for all l_{t+1}.*

Assumption 1 is very important because it implies that individuals with higher n_t will have lower default risk. Thus, this infinite-horizon model resembles the 2-period model presented above.

Information structure. There is asymmetric information between lenders and borrowers about the latter's productivity group, n. On one side, households know their n. On the other side, if borrowers are not screened, then the productivity group is private information. Nevertheless, each lender has access to a technology that can be used to learn a household's productivity group at a fixed cost. The stock of assets, a_t, is publicly observable, as well as the credit flag indicating the bankruptcy record (defined later).

Information firm's problem. The information firm uses labor to produce information with the production function

$$z_t^i(\mathbf{m}_t)^{1/\gamma},$$

where z_t^i is the productivity in information production and $\mathbf{m}_t$ is labor demanded in the information industry. This sector is simplified assuming it produces $\{0, 1\}$, where 0 means no information is produced and 1 means "a report with information about the borrower's productivity group is produced." Then, zero-expected profits in this sector implies that the cost of learning a borrower's productivity group (or screening cost) is $\mathcal{C}(z_t^i, w_t) = w_t(z_t^i)^{-\gamma}$.

Production firm's problem. It rents capital at the rate r_t and hires labor at the wage w_t. With these factors the firm produces consumption goods in line with a standard Cobb-Douglas production function. Thus, the firm's problem is

$$\max_{\{\mathbf{l}_t, \mathbf{k}_t\}} \left\{ z_t^p(\mathbf{k}_t)^{1-\theta}(\mathbf{l}_t)^{\theta} - w_t\mathbf{l}_t - r_t\mathbf{k}_t \right\},$$

where z_t^p is the technology in the production sector, and $\{\mathbf{l}_t, \mathbf{k}_t\}$ are labor and capital in this sector, respectively.

Credit industry. There are two types of debt contract: *screening* and *revelation* contracts. There are many lenders competing among themselves offering debt contracts. They own the stock of capital, which they rent to the firms in the production sector.

Lenders offering screening contracts. Borrowers have to pay the screening cost to be able to use a screening contract. Think of borrowers buying a report at the information industry that proves their productivity group and submitting it to lenders. The price charged is $\tilde{q}(a_{t+1}, n_t)$; i.e., a different price for each level of assets next period, $a_{t+1} \in \mathbf{A}$, and productivity group, $n_t \in \mathbf{N}$. The price depends on a_{t+1} because it determines the debt the household will have to pay back next period, which in turn affects her willingness to pay back the debt. It depends on n_t because this determines the transition probability to different productivity levels, and thereby the probability of bankruptcy. Let $\widetilde{d}_{a_{t+1},n_t}$ denote the number (measure) of contracts for households with $\{a_{t+1}, n_t\}$ that lenders sell, $\widetilde{K}_{t+1}$ the stock of capital they accumulate for period $t+1$, and $\Pr(repayment \mid a_{t+1}, n_t)$ the repayment probability of this contract. Then, period-t cash flow is given by

$$\begin{aligned} \widetilde{P}_t = \; & - \; \textstyle\sum_{n_{t-1}} \int_{a_t} \widetilde{d}_{a_t,n_{t-1}} \Pr(repayment \mid a_t, n_{t-1}) a_t da_t \\ & + \; \textstyle\sum_{n_t} \int_{a_{t+1}} \widetilde{d}_{a_{t+1},n_t} \tilde{q}(a_{t+1}, n_t) a_{t+1} da_{t+1} \\ & + \; (1-\delta+r)\widetilde{K}_t - \widetilde{K}_{t+1}. \end{aligned}$$

Lenders design the contracts and choose $\widetilde{d}_{a_{t+1},n_t}$ and $\widetilde{K}_{t+1}$ to maximize the present discounted value of current and future cash flows,

$$\sum_{t=0}^{\infty} (1+i_t)^{-t} \widetilde{P}_t,$$

given the risk-free interest rate at period t, i_t, the initial stock of capital, $\widetilde{K}_0$, and the number of each different contract initially sold, $\widetilde{d}_{a_0,n_{-1}}$.

The sequence of cash flows implies a sequence of risk-free bond holdings, $\{\widetilde{B}_{t+1}\}_{t=0}^{\infty}$, which can be obtained by the recursion

$$\widetilde{B}_{t+1} = (1+i_t)\widetilde{B}_t + \widetilde{P}_t,$$

where $\widetilde{B}_0 = 0$. These bonds, which are issued by the lenders, are incorporated to allow for the accumulation of cash flows. They are not that important hereafter since they

will be zero in the stationary equilibrium defined later; i.e., $\widetilde{B}_t = \widetilde{B} = 0$.

Lenders offering revelation contracts. These lenders compete offering self-revelation contracts. The condition a contract has to satisfy to be "self-revelation" is formally stated later, after the household's problem is introduced. That condition basically states that, given the contract design, borrowers are better off revealing their productivity group. Since lenders offering revelation contracts do not observe n, prices depend on the households' reports on n. Additionally, since the current stock of assets affects a household willingness to borrow, prices satisfying the revelation constraint depend also on this variable.

Some notation in now introduced. Let $\widehat{d}_{a_{t+1},n_t;a_t}$ denote the number (measure) of contracts uninformed lenders sell for households with $\{a_{t+1}, n_t, a_t\}$, $\widehat{K}_{t+1}$ the stock of capital they accumulate for period $t+1$, and $\Pr(repayment \mid a_{t+1}, n)$ the repayment probability. Then, period-t cash flow is given by

$$\begin{aligned}\widehat{P}_t = \; & - \; \textstyle\sum_{n_{t-1}} \int_{a_{t-1}} \int_{a_t} \widehat{d}_{a_t,n_{t-1};a_{t-1}} \Pr(repayment \mid a_t, n_{t-1}) a_t da_t da_{t-1} \\ & + \; \textstyle\sum_{n_t} \int_{a_t} \int_{a_{t+1}} \widehat{d}_{a_{t+1},n_t;a_t} \widehat{q}(a_{t+1}, n_t; a_t) a_{t+1} da_{t+1} da_t \\ & + \; (1-\delta+r)\widehat{K}_t - \widehat{K}_{t+1}.\end{aligned}$$

Lenders design the contract and choose $\widehat{d}_{a_{t+1},n_t;a_t}$ and $\widehat{K}_{t+1}$ to maximize

$$\sum_{t=0}^{\infty} (1+i_t)^{-t} \widehat{P}_t,$$

given i_t, $\widehat{K}_0$, and $\widehat{d}_{a_0,n_{-1},a_{-1}}$. Again, a sequence of cash flows implies a sequence of risk-free bond holdings, $\{\widehat{B}_{t+1}\}_{t=0}^{\infty}$.

Household's problem. Hereafter, period-t variables will be expressed without any subscripts or superscripts, and period-$t+1$ variables will be represented with superscripts '$'$'. Households decide on consumption, c, and asset accumulation, a'. In addition, they decide which kind of debt contract they would like to sign, and either to file for bankruptcy or to pay back the debt. These decisions are made taking prices, $\mathrm{S} = (q, w, i, r, \widetilde{q}(\cdot), \widehat{q}(\cdot), \mathcal{C}(\cdot))$, as given.

Several assumptions determine the advantages and disadvantages of bankruptcy. The key advantage is the discharge of debts—assets in the period after bankruptcy are set at zero. Thus, a household with too much debt may find it profitable to file for bankruptcy. There are many disadvantages of doing so, however.[14] In the

[14]Here, disadvantages of filing bankruptcy are exogenous. Chatterjee, Corbae, and Rios-Rull (2008) show how higher interest rates following default arise from the lender's optimal response to limited information about the household's type and earnings realizations.

period of bankruptcy, consumption equals income, and neither saving nor borrowing are allowed. Additionally, in the period right after bankruptcy, the defaulter will have a *bad credit flag*. Having a bad credit flag implies that the household cannot borrow and a proportion of income, τ, is lost.[15] That flag remains in a household record for a stochastic number of periods, meaning that the probability of a transition from bad to good credit flag is $\lambda \in (0,1)$—the fresh start probability. The use of λ is a simple way of modeling a bankruptcy flag that remains on a household's credit history for only a finite number of years.

Lifetime utility for households in each possible state is defined as follows.

- *Bad credit flag*: Lifetime utility of a household excluded from credit markets is

$$\begin{aligned} B(n,l,a;\mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta \textstyle\sum_{n'} \Pi(n,n')\{\lambda \int_{l'} G(n',l',a';\mathrm{S})\phi(l' \mid n')dl' \\ +(1-\lambda)\int_{l'} B(n',l',a';\mathrm{S})\phi(l' \mid n')dl'\}\}, \end{aligned} \tag{2}$$

subject to

$$\begin{aligned} &c + \bar{q}(a',n)a' = a + lw(1-\tau), \\ &a' \geq 0, \text{ and } c \geq 0, \end{aligned}$$

where G is the lifetime utility for households with good credit history (defined below), which is a function of productivity, n, labor endowments, l, assets, a, and relevant prices, S. Importantly, assets for the next period are restricted to be positive. Notice that the household obtains utility in the next period just if she survives, and that happens only with probability ρ. The utility from future periods depends on the probability of a fresh start, λ, while the utility from the current period depends on the proportion of income lost because of bad credit status, τ. Denote the policy functions for asset accumulation and consumption obtained from the solution to this problem as A'_b and C_b.

- *Good credit flag*: Lifetime utility is

$$G(n,l,a;\mathrm{S}) = \max\{\underbrace{V(n,l,a;\mathrm{S})}_{\text{pay back}}, \underbrace{D(n,l;\mathrm{S})}_{\text{bankruptcy}}\}, \tag{3}$$

where V and D (defined below) are lifetime utilities for households paying back the debt and filing bankruptcy, respectively. This means that a household with a good credit flag has the choice of filing bankruptcy. The policy functions for asset accumulation and

[15] Chatterjee, Corbae, and Rios-Rull (2008) build a model where no punishment is required after filing bankruptcy. There, asymmetric information is crucial to create incentives for debt repayment, because households signal their type by paying back their debt.

consumption are A' and C, respectively. Additionally, the policy function R indicates whether the household pays back the debt or not,

$$R(n,l,a;\mathrm{S}) = \begin{cases} 1 \text{ if } V(n,l,a;\mathrm{S}) \geq D(n,l;\mathrm{S}), \\ 0 \text{ otherwise.} \end{cases}$$

• *Good credit flag and bankruptcy*: Suppose the household chooses to file for bankruptcy. Then, lifetime utility is

$$D(n,l;\mathrm{S}) = u(lw(1-\tau)) + \rho\beta \textstyle\sum_{n'} \Pi(n,n') \int_{l'} B(n',l',0;\mathrm{S})\phi(l' \mid n')dl'. \tag{4}$$

Neither saving nor borrowing is allowed in this period. Therefore the household's consumption equals net income (labor income minus the proportion lost due to bankruptcy). In the period after bankruptcy, the household will have a bad credit flag for sure and zero debt.

• *Good credit flag and pay back the debt*: Suppose the household decides to pay back the debt. Then, she must decide which kind of contract to sign. Thus, the value function is

$$V(n,l,a;\mathrm{S}) = \max\{\underbrace{\widetilde{V}(n,l,a;\mathrm{S})}_{\text{use information}}; \underbrace{\widehat{V}(n,l,a;\mathrm{S})}_{\text{no information}}\}, \tag{5}$$

where $\widetilde{V}(n,l,a;\mathrm{S})$ and $\widehat{V}(n,l,a;\mathrm{S})$ (defined below) are lifetime associated with borrowing using screening and revelation contracts, respectively. The policy function U indicates whether the household borrows using revelation contracts or not,

$$U(n,l,a;\mathrm{S}) = \begin{cases} 1 \text{ if } \widehat{V}(n,l,a;\mathrm{S}) \geq \widetilde{V}(n,l,a;\mathrm{S}), \\ 0 \text{ otherwise.} \end{cases}$$

• *Pay back the debt and screening debt contract*: If the household decides to sign a screening contract with a lender, then she faces the debt price $\tilde{q}(a',n)$, and her lifetime utility is

$$\begin{aligned} &\widetilde{V}(n,l,a;\mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta \textstyle\sum_{n'} \Pi(n,n') \int_{l'} G(n',l',a';\mathrm{S})\phi(l' \mid n')dl'\}, \\ &\text{subject to} \\ &\qquad c + \tilde{q}(a',n)a' = a - \mathcal{C}(z^i,w) + lw, \\ &\qquad \text{and } c \geq 0, \end{aligned} \tag{6}$$

where $\mathcal{C}(z^i,w)$ is the cost of information. Notice that this cost is independent of the amount borrowed, which is consistent with the

interpretation that the household buys a report about her type and then presents it to the lender.

• *Pay back the debt and revelation debt contract*: Now suppose the household prefers to use a revelation contract. Then, the relevant debt price is $\widehat{q}(a', a)$ and there is no fixed cost to pay. Thus, her lifetime utility is

$$\widehat{V}(n, l, a; \mathrm{S}) = \max_{a',c}\{u(c) + \rho\beta \textstyle\sum_{n'} \Pi(n, n') \int_{l'} G(n', l', a'; \mathrm{S})\phi(l' \mid n')dl'\},$$

subject to

$$c + \widehat{q}(a', a)a' = a + lw, \quad \text{and } c \geq 0. \tag{7}$$

3.2 The Equilibrium

Equilibrium prices for *screening contracts* must imply zero-expected profits. Therefore, they can be written as

$$\tilde{q}(a', n) = \frac{1}{1+i}\rho \sum_{n'} \Pi(n, n') \int_{l'} R(n', l', a'; \mathrm{S})\phi(l' \mid n')dl'. \tag{8}$$

Here it is very clear why the price, $\tilde{q}$, depends on (a', n) and is independent of a. It depends on a' because it affects the bankruptcy decision, R, in each possible state. It depends on n because it determines the transition probability to each n' and therefore the next period labor endowment, l'. Finally, it is independent of a because it does not affect either the transition probabilities or the bankruptcy decision in the next period. The difference with screening contracts in the 2-period model (Lemma 4) is that the (constant) cost of information that must be paid to use these contracts is now afforded directly by the borrower.

To characterize *revelation contracts*, two differences with the 2-period example are important. First, there are more than two types, so the limit for the zero-expected-profit prices corresponding to a productivity group n will be the tighter of those set by households with productivity group $j < n$. The second difference is that households with any labor endowment $l \in L(j)$ could pretend to be of productivity group j. Again, the solution is to use the tighter limit among those set by $l \in L(j)$. To write this formally, additional notation must be introduced. First, consider the bankruptcy-free limit,[16]

$$\underline{\mathbf{a}} = \min_a\{a : V(n, l, a) \geq D(n, l), \forall n \in \mathbf{N}, \forall l \in L(n)\}.$$

[16]This borrowing limit was first introduced by Zhang (1997) and Abraham and Carceles-Poveda (2006).

Then, let the function

$$\begin{aligned}&\mathbf{V}(n,l,a;a',j)=\max_{a',c}\{u(c)+\rho\beta\textstyle\sum_{n'}\Pi(n,n')\int_{l'}G(n',l',a';\mathrm{S})\phi(l'\mid n')dl'\},\\&\text{subject to}\\&\qquad c+\bar{q}(a',j)a'=a+lw,\\&\qquad\text{and } c\geq 0,\end{aligned}\tag{9}$$

represent the lifetime utility if the amount of assets chosen for the next period is a' and the price used is the one satisfying the zero-expected-profit condition for households in the productivity group j. The auxiliary limit for the price of households with productivity group n set by households in the group j and with labor endowment l is $\underline{a}(a,n;j,l)$ such that

$$\max_{a'}\mathbf{V}(j,l,a;a',j)=\mathbf{V}(j,l,a;\underline{a}(a,n;j,l),n).\tag{10}$$

Since l is unobservable, the limit for the price of households with productivity group n set by households in the group j is

$$\underline{a}(a,n;j)=\max_{l\in L(j)}\underline{a}(a,n;j,l).\tag{11}$$

Similarly, since we want to exclude all $j<n$ from the price of households with productivity group n, the minimum a' that can be offered at that price is

$$\underline{a}(a,n)=\min\{\underline{\mathbf{a}};\max_{j<n}\underline{a}(a,n;j)\}.\tag{12}$$

Notice that the limit cannot be higher than the risk-free limit because information about productivity is irrelevant for $a>\underline{\mathbf{a}}$. Now, equilibrium prices of screening contracts can be written as

$$\widehat{q}(a',a)=\begin{cases}\frac{\rho}{1+i}\text{ if } a'\geq\underline{\mathbf{a}},\\ \tilde{q}(a',N)\text{ if }\underline{\mathbf{a}}>a'\geq\underline{a}(a,N),\\ \tilde{q}(a',N-1)\text{ if }\underline{a}(a,N)>a'\geq\underline{a}(a,N-1),\\ \ldots\\ \tilde{q}(a',2)\text{ if }\underline{a}(a,3)>a'\geq\underline{a}(a,2),\\ \tilde{q}(a',1)\text{ if }\underline{a}(a,2)>a'.\end{cases}\tag{13}$$

These prices resemble those in the 2-period model. The only difference is that there are model debt limits.[17]

Stationary equilibrium. Assume technologies in the information sector, z^i, and in the production sector, z^p, are constant. Then, stationary equilibrium requires optimization together with aggregate conditions that guarantee markets clearing and stationarity.

[17]It is actually simple to see that those limits satisfy $\underline{a}(a,n)>\underline{a}(a,n-1)$. Two things are important for this result. First, zero-expected-profit prices are increasing in n. This is clearly implied by *Assumption 1*. Second, higher prices make borrowers better off, so tighter limits are required to keep them indifferent.

Definition 2 *A stationary equilibrium with costly information is a set of policy functions A'_b, C_b, A', C, R, U, $\mathbf{l}$, $\mathbf{m}$, and $\mathbf{k}$, cumulative density functions $\Psi_n(a,l)$, $\Psi^g_n(a,l)$, $\Psi^b_n(a,l)$, and prices w, i, r, $\widetilde{q}$, $\widehat{q}$, and $\mathcal{C}$, such that the following conditions hold:*

1. *The functions A'_b, C_b, A', C, R, and U solve the household's problems, or satisfy problems 2 to 7.*
2. *The function $\widetilde{q}$ and $\widehat{q}$ are equilibrium prices, or satisfy 8 and 13, respectively.*
3. *The firm in the production sector maximizes profits given $\{w, r\}$, or*

$$(1-\theta)z^p(\mathbf{k})^{-\theta}(\mathbf{l})^{\theta} = r,$$
$$\theta z^p(\mathbf{k})^{1-\theta}(\mathbf{l})^{\theta-1} = w.$$

4. *The function $\Psi_n(a,l)$ is the stationary c.d.f. over (n,a,l), and $\Psi^g_n(a,l)$ and $\Psi^b_n(a,l)$ are the stationary c.d.f. over (n,a,l) conditional on having good and bad credit flags, respectively; or*

$$\begin{aligned}
\Psi_n(a,l) =& \Psi^g_n(a,l) + \Psi^b_n(a,l),\\
d\Psi^g_{n'}(a',l') =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n') R(n,a,l) d\Psi^g_n(a,l)\\
&+ \lambda \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n') d\Psi^b_n(a,l),\\
d\Psi^b_n(0,l') =& \sum_n \int_a \int_l \Pi(n,n')\phi(l' \mid n')(1-R(n,a,l)) d\Psi^g_n(a,l)\\
&+ (1-\lambda) \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=0\}} \Pi(n,n')\phi(l' \mid n') d\Psi^b_n(a,l),\\
d\Psi^b_n(a',l') =& (1-\lambda) \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} \Pi(n,n')\phi(l' \mid n') d\Psi^b_n(a,l), \;\; a' \neq 0.
\end{aligned}$$

5. *The credit market clears, or*

$$\begin{aligned}
\widehat{d}_{a',n;a} =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} U(n,a,l) d\Psi^g_n(a,l)\\
&+ \sum_n \int_a \int_l \mathbf{1}_{\{A_b(n,a,l)=a'\}} U(n,a,l) d\Psi^b_n(a,l),\\
\widetilde{d}_{a',n} =& \sum_n \int_a \int_l \mathbf{1}_{\{A(n,a,l)=a'\}} (1-U(n,a,l)) d\Psi^g_n(a,l).
\end{aligned}$$

6. *The labor market clears, or*

$$\mathbf{m} + \mathbf{l} = \sum_n \int_a \int_l l d\Psi_n(a,l).$$

7. *The goods market clears, or*

$$z^p(\mathbf{k})^{1-\theta}(\mathbf{l})^{\theta} = \sum_n \int_a \int_l C(n,a,l) d\Psi^g_n(a,l) + \sum_n \int_a \int_l C_b(n,a,l) d\Psi^b_n(a,l) + \delta\mathbf{k}.$$

4 Calibration

The strategy here is to calibrate the model for 1983 and then evaluate the impact of information costs on bankruptcy, debt, and other variables that can be used to test the model with the data.

4.1 Calibration Strategy

The benchmark calibration, also referred as *"1983 calibration,"* is designed such that the model represents the U.S. economy in the year 1983. The choice of this year is very important. As mentioned by Moss and Johnson (1999), "from 1920 to 1985, the growth of consumer filings closely tracked the growth of real consumer credit. Since then, however, the rate of increase of consumer bankruptcies has far outpaced that of real consumer credit." The year 1983 will then represent a steady state before the transformation of the market that started in 1985. The calibration consists in assigning values to 25 parameters. Some of them can be determined using a priori information. The others are determined jointly using the Nelder and Mead (1965) algorithm to minimize the distance between key moments in the data and model. Parameters and targets are explained in detail in the next subsections.

Parameters determined using a priori information (5). The survival probability, ρ, is determined to match a period of a financially active life of 40 years. The utility function is

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

where σ was chosen to match a coefficient of risk aversion of 2. The labor share of income, 0.64, determines the value of the parameter in the production function, θ. The depreciation rate, δ, is set at 7% annually. The probability of a fresh start, λ, is set to match the average time of exclusion after bankruptcy (10 years).[18]

Parameters determined jointly (20). There are ten different productivity groups or types, $\mathbf{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, where 1 and 10 are associated with the lowest and highest labor endowments, respectively. The number of groups is an important choice because it determines the maximum that lenders can learn about households' incomes when they decide to screen borrowers. Nevertheless, how much information is contained in these ten types depends on their persistence, which is endogenously determined considering several moments from the data. Additionally, the

[18] The same value is used by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007).

proportion of total income that each group accounts for is obtained directly from the Survey of Consumer Finance (SCF). The parameters to calibrate are described below.

- *Transition matrix (6 parameters):* $\mathbf{\Pi}$. Several assumptions restrict the number of parameters in this group. In particular, it is assumed that: (i) transitions further than 2 types away than the current type are zero probability events, (ii) persistence is the same for all the groups except $\{1, 2, 9, 10\}$, (iii) the transition to one and two types higher and lower are proportionally the same for all the types, and (iv) the highest type is a low probability state with very high labor endowments, as in Castaneda, Diaz-Gimenez, and Rios-Rull (2003). After these assumptions there are 6 parameters to calibrate.[19] These parameters determine the size of each productivity group which in turn affects the joint distribution of debt and income.

- *Technology in the information sector (1 parameter):* z^i. This parameter is important in determining the distribution of debt across income groups and the bankruptcy rate.

- *Discount factor (1 parameter):* β. This parameter is crucial in determining the economy capital-to-income and debt-to-income ratios and the bankruptcy rate.

- *Income lost during bankruptcy (1 parameter):* τ. Since this parameter represents the fraction of earnings lost when households have a bad credit flag, it plays a very important role determining the bankruptcy rate and the debt-to-income ratio.

- *Labor endowments distribution exponent (1 parameter):* φ . Given a type n, the cumulative distribution for labor endowments, $l \in [\underline{l}^n, \bar{l}^n]$, is

$$\int_{\underline{l}^n}^{x} \phi(l \mid n) dl = \left(\frac{x - \underline{l}^n}{\bar{l}^n - \underline{l}^n} \right)^{\varphi},$$

  where the exponent is one more parameter to calibrate. This parameter is particularly important in determining income inequality given the size of income groups.

- *Labor endowment intervals' limits (10 parameters):* $\bar{\mathbf{l}}$. The first one is normalized, $\underline{l}^1 = 1$. The ending limit of each group is equal to the starting limit of the next range; i.e., $\bar{l}^1 = \underline{l}^2$, $\bar{l}^2 = \underline{l}^3$, ... , $\bar{l}^9 = \underline{l}^{10}$. Thus, there are 10 parameters to be calibrated: $\bar{\mathbf{l}} = [\bar{l}^1, \bar{l}^2, \ \ldots \ , \bar{l}^{10}]$. These limits determine the proportion of total income in each group.

[19] More details on the assumptions made on the transition matrix are provided in Appendix 7.2.

From these parameters, the last 10 will be taken directly from the data on income.[20] The labor endowments distribution exponent will be calibrated in a separate step to match the Gini coefficient of income. The remaining 9 parameters are chosen minimizing the distance between moments from the model and data. Specifically, there are 12 statistics used as targets. Most of them are very important in any model of debt and bankruptcy. Others are relevant given the informational frictions in this paper. The moments chosen as targets of the calibration are described in detail below. Their values are presented in *Table 1*.

- *Capital-to-output ratio (1 target):* This target is fixed assets and consumer durable goods over GDP, both obtained from the Bureau of Economic Analysis (BEA).
- *Bankruptcy rate (1 target):* The number of bankruptcy filings in a year, obtained from the American Bankruptcy Institute, is prorated using 0.53 because income shocks cause 53% of the cases of bankruptcy.[21] Then, to construct the bankruptcy rate, the prorated number of bankruptcy filings is divided by the number of households with a credit card balance, obtained from the SCF.
- *Earnings and wealth inequality (4 targets):* The statistics in this group are all obtained from the SCF 1983. In particular, the targets used are the Gini coefficient of income, the mean-to-median income ratio, the Gini coefficient of wealth, and the mean-to-median wealth ratio.
- *Debt-to-income ratio (1 target):* Two alternative approaches have been followed to compare the debt measure of the model and the data. The first one, followed by Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), is to define debt as minus net worth when it is negative and zero otherwise. The alternative approach, followed by Livshits, MacGee, and Tertilt (2007b), is to use credit card debt. The "net worth approach" is followed here. The main advantage is that it is consistent with a model with only one asset.[22] The data is obtained from the SCF.
- *Debt across income groups (5 targets):* Let D_i denote the percentage of total debt held by households with an income percentile lower than i. The moments D_{10}, D_{15}, D_{20}, D_{30}, and D_{40} are used in the calibration procedure. They are obtained from the SCF using the definition of debt introduced above.

[20] More details are discussed below.

[21] This adjustment is necessary because the model only has income shocks and in reality other shocks are also important. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) applied the same procedure when they calibrated their model with only income shocks. Then, they show that the model can match the total number of bankruptcies if other shocks are included.

[22] As in Chatterjee, Corbae, Nakajima, and Rios-Rull (2007), debt-to-income is also prorated using 0.53.

Among the targets, the percentage of debt held by households with incomes lower than the 10, 15, 20, 30, and 40 percentiles were not considered in previous literature. They are important in the calibration because how much debt is held by households in a productivity group depends on their expected income, which is determined by the transition matrix parameters. More important, given all the other parameters, the cost of information is crucial in determining the proportion of debt held by different income groups, as explained at the end of Section 2. Thus, these targets will be useful for the calibration of z^i, too.

The next steps were followed to calibrate the model parameters minimizing the distance between the moments from the data and model.

Step 1: Guess a value for 9 parameters $\{\mathbf{\Pi}, z^i, \beta, \tau\}$.

Step 2: Given the value of $\mathbf{\Pi}$, compute the measure of households in each type n in the stationary distribution.

Step 3: Obtain 10 parameters, $\bar{\mathbf{l}} = [\bar{l}^1, \bar{l}^2, \bar{l}^3, \bar{l}^4, \bar{l}^5, \bar{l}^6, \bar{l}^7, \bar{l}^8, \bar{l}^9, \bar{l}^{10}]$, using the SCF 1983 to match exactly the measure in each labor endowment's interval.

Step 4: Given the measure in each interval and the limits $\bar{\mathbf{l}}$, search for the value of φ that minimizes the distance between the Gini coefficient of income from the model and data using the Nelder and Mead (1965) algorithm.

Step 5: At this point a value was assigned to each of the 20 parameters. Use these parameters to compute the model and calculate the distance between the moments from the data and model. If the distance to the targets is small enough, end. Otherwise, choose a new value for $\{\mathbf{\Pi}, z, \beta, \tau\}$ according to the Nelder and Mead (1965) algorithm and return to step 2.

Step 3 is different from previous literature. For instance, Chatterjee, Corbae, Nakajima, and Rios-Rull (2007) also search on $\bar{l}^i$ to match some targets. This procedure can be better understood using *Figure 7*. First, households' incomes from the SCF is normalized such that the minimum non-zero income is 1. This will be the value of $\underline{l}^1$ in the calibration. Let Δ_1 represent the measure of households in the first productivity group given $\mathbf{\Pi}$. Then, pick the upper limit of the first interval from the (normalized) SCF data such that between 1 and $\bar{l}^1$, the measure of households is Δ_1. This is $\bar{l}^1$, in *Figure 7*. The same procedure can be applied to determine the next $\bar{l}^i$ taking $\bar{l}^{i-1} = \underline{l}^i$ and Δ_i as given. Thus, all $\bar{l}^i$ are obtained directly from the SCF.

4.2 Calibration Results

The results from the calibration strategy are presented on *Table 1*. How well does the model fit the data? It can replicate key moments for the year 1983. In particular, the first two statistics, the bankruptcy rate and the debt-to-income ratio, are closely replicated. Despite the fact it overestimates the percentage of total debt held by the poorest 10, 15, 20, 30, and 40 percentiles of the population, the power of the model to explain debt across income groups is acceptable.[23] Given that income is directly obtained from the data, income distribution statistics are replicated very well. The wealth inequality statistics considered as a target of calibration are also closely reproduced. The success of the model reproducing wealth inequality is because the income of rich households is well calibrated using the SCF and because of the structure of the transition matrix.

Now the question is: What parameters are necessary to match the selected moments? The parameters obtained from the calibration are presented in *Table 2*. The discount factor and the punishment after bankruptcy are similar in other quantitative models of default and hard to compare with direct evidence. The parameter representing the technology in the information sector implies a cost of information around 5% of mean income. This is very high. Remember that if $\mathcal{C}$ is high, households will avoid this cost by using revelation contracts. This in fact happens; more than 99% of the borrowers use contracts that do not require the use of information. The parameters obtained for transition matrix parameters can be compared with data obtained from the matched March supplement of the Current Population Survey (CPS). *Table 3* presents the transition matrix from the data and the model for the first 6 types, which accumulate 93% of the total population.[24] Although there are several reasons that can explain why the transition matrix from the CPS may be different,[25] the transition probabilities obtained in the calibration look relatively similar to those in the data. In particular, the parameter in the main diagonal are quite similar.

While it is important to obtain a good fit of the targets with reasonable parameters, it is perhaps more important to check that non-targeted moments are also reasonably matched. *Table 4* compares the data and model predictions for four non-targeted moments. The charge-off rate, which is the value of loans removed from the books

[23] If those target were not included, the model would do a terrible job. This is because, in general, only the poorest individuals have debt in this type of model.

[24] The comparison for the richest households is avoided because of the different treatment of households on the top of the income distribution (top-coding) in the SCF and the CPS.

[25] For instance, the model is calibrated using data on earnings from the SCF, which contains information about households earnings, while the information from the CPS is about individuals' earnings.

and charged against loss reserves as a percentage of average loans, and the number of bankruptcy filings over the total population are bigger in the data than in the model. However, given that they were not targeted, they are relatively similar. The proportion of households with negative net worth is quite similar in the model and the data. More importantly, the coefficient of variation of (paid) interest rates is very similar. This is very important because the dispersion of interest rates is a measure of how much information lenders use about borrowers' characteristics.

5 Assessment of the IT Revolution

The quantitative model is now used to assess the role of information costs in the rise of the bankruptcy rate over the last 20 years. Three questions are answered in this section: (i) Can a drop in the cost of information replicate the rise in consumer bankruptcies between 1983 and 2004?, (ii) What are the implications of that drop in information costs for other variables?, and (iii) Does the mechanism in the infinite horizon model resemble the one in the 2-period model? Finally, this section discusses distinguishing implications of a drop in information costs.

5.1 Recalibrating z

To evaluate the role of the IT revolution, a new value for the technology in the information sector, z, is required. A simple methodology is followed here to obtain that value: pick the new value of z such that the bankruptcy rate in the model matches that in 2004.[26]

The value of z obtained in this exercise is 2.85, which implies a cost of information (relative to mean income) smaller than 1%. Thus, the answer to the first question is *yes*, the model can reproduce the rise in consumer bankruptcies between 1983 and 2004 if the cost of information drops from 5% to 1% of mean income.

5.2 Effect of z on other variables

The role of information costs in consumer debt and bankruptcy can be better understood by examining changes in other variables when information costs rise. The results are presented in *Table 5*. In the comparison with the data for 2004, notice

[26] Although this methodology has been widely used in macroeconomics, it has the disadvantage that the value of z for 2004 is not identified. To do so, an exercise similar to the one done for 1983 should be performed for 2004. However, in that exercise, all changes between 1983 and 2004 will have to be explained by the parameters recalibrated. If other changes occurred during that period, the parameters obtained will also be biased.

that all the statistics but the the bankruptcy rate *were not targeted*. By construction, the bankruptcy rate is very similar in the data (1.60%) and the model for $z = 2.85$ (1.52%). Importantly, all the variables move in the same direction in the data (1983 vs. 2004) and the model ($z = 1.26$ vs. $z = 2.85$). More importantly, the changes in three of the most relevant statistics—the charge-off rate, the ratio of bankruptcy filings to the population, and the coefficient of variation of interest rates—are very similar in the data and model. Changes in the distribution of debt across income percentiles must be interpreted carefully. Since the statistic used before was the cumulative share of debt, we must take differences to see which group is borrowing a bigger share in 2004 than in 1983. Notice that in the model, households between percentiles 10 and 40 are borrowing more. The same happens in the statistics computed from the SCF,[27] although the magnitudes are much larger. Thus, changes in information costs alone are not enough to explain changes in the distribution of debt across income groups. However, notice that income inequality does not change by construction, while changes in the data are very significant. These changes may be important in explaining changes in the distribution of debt across income groups. Additionally, changes in information costs do not modify wealth inequality. Finally, in the model, the rise in the debt-to-income ratio is significantly smaller (17%) than the rise in the bankruptcy filings over the population (115%). Although this exaggerates what happens in the data (debt rose 81% and bankruptcy 149%), this result is important because, as mentioned by Moss and Johnson (1999), what is special about this period is that "the rate of increase of consumer bankruptcies has far outpaced that of real consumer credit." Other driving forces, such as a drop in transaction costs, have problems generating this asymmetric response of debt and bankruptcy.[28]

5.3 The mechanism in the infinite horizon model

When information costs drop, households in income percentiles 10 to 40 increase their share of total debt. Thus, as in the 2-period model, after technological progress in the information sector, some households can borrow more and the default rate increases. To analyze this effect in greater details, the distribution of debt (among those in debt) is depicted in *Figure 8*. On the left, an economy with very high information costs ($z = 0.8$) is presented. To understand the mechanism, the key is that there is almost

[27] A similar result is reported in Figure 6 of Livshits, MacGee, and Tertilt (2009) following the alternative approach to measure debt. Using their metric, the 40% poorest individuals had around 10% of the debt in 1983 and 20% in 2000. Also in their case, this change is not explained by an increase in the bottom 10%—they had less than 3% of the debt in both years.

[28] See Livshits, MacGee, and Tertilt (2007a).

no debt with risk of bankruptcy. In that economy, the default rate is really low (0.08%) and there is almost no dispersion in the interest rate paid (the coefficient of variation is 1.9%, compared to 28% and 43% in 1983 and 2000 data, respectively), since almost all households pay the risk free rate. On the right panel of *Figure 8*, an economy with very low information costs ($z = 10$) is presented. Now, there are much more households borrowing with risk of default. The bankruptcy rate in that economy is 2.2% and the coefficient of variation of interest rates is 61.2%, both much higher than for the economy with high information costs. Importantly, notice that while the bankruptcy rate and the dispersion of interest rates increased dramatically, the debt-to-income ratio rose from 0.35% to 0.50%. *The key is not the rise in debt but the shift from non-risky to risky debt that is generated by a drop in information costs.*

5.4 Distinguishing implications

Finally, the model is challenged analyzing *distinguishing implications* that a drop in information costs generates. This driving force has two distinguishing implications.

5.4.1 Changes in the dispersion of interest rates

Livshits, MacGee, and Tertilt (2009) point out that during the last 20 years the dispersion of interest rates increased significantly. Since the model was not calibrated to match those moments, it is really challenging to compare the dispersion in the model and the data. In the model, a drop in the cost of information production increases the dispersion of interest rates. This is because when screening is too costly, lenders use fewer contracts (only those satisfying self-revelation). Actually, this implies that most of the households borrow at the risk free rate. When information costs drop, more contracts are used and the dispersion of interest rates increase.

The dispersion of interest rates for the model with $z = 1.26$ and $z = 2.85$ and the data for 1983 and 2004 was presented in *Tables 4 and 5*, respectively. The values are surprisingly similar. In 1983, the coefficient of variation of interest rates was 28.3% and the 1983 calibration's number is 27.4%. In the model with low information costs ($z = 2.85$) this coefficient is 43.7%, while in the data for 2004 it is 43.2%. Thus, this driving force is very successful replicating the dispersion of interest rates.

5.4.2 Changes in the projection of income on interest rates

A very specific implication of this model is that the relation between income and interest rates (after controlling by other variables) becomes stronger as the cost of information drops. More precisely, given the amount of debt borrowed, interest rates

are decreasing in income only "when lenders use screening contracts." Thus, another test for information costs as the driving force behind the rise in consumer bankruptcies is to compare the coefficient for income in an interest rates regression.[29]

The first two columns of *Table 6* shows the projection of income on interest rates using data from the SCF. In 1983 the effect of income on interest rates is not significantly different than zero. In contrast, in 2004, borrowers with higher income pay lower interest rates.[30] The last two columns present the results obtained using the model. The value and the change in the coefficient between the different cases are quite similar. This result not only supports the hypothesis that lenders use more information but also indicates that it is appropriate to focus on information about households' productivity.

6 Conclusions

How do information costs affect consumer bankruptcy? Asymmetric information and costly screening are incorporated into a model of consumer debt and bankruptcy to study this question. When screening is too expensive, uninformed lenders overcome the lack of information by designing contracts to induce households to reveal their income. The design of these contracts implies that low-risk households are borrowing-constrained. This is because contracts with low interest rates are linked to tight debt limits to avoid high-risk households taking these contracts (they will prefer contracts with higher interest rates and looser borrowing limits). With technological progress in the IT sector, information costs drop and previously borrowing-constrained households can now be screened and obtain more debt. Then, the rise in debt generates an increase in bankruptcy filings because the benefits of filing bankruptcy increase with the amount owed.

Can this model account for the changes in consumer credit markets over the last 20 years? Quantitative exercises are performed to answer this question. The parameters are first calibrated to the year 1983. The model is successful replicating key (targeted and non-targeted) moments for this year. Then, the cost of information is recalibrated to the year 2004 to match the bankruptcy rate in that year. Without changing any other parameter, the model can replicate many important features of the data. Importantly, changes in the variables describing unsecured credit markets move in the same direction as in the data. More importantly, in most of the cases changes are also quantitatively similar. The fit of non-targeted moments—like the dispersion in interest rates or the

[29] Thanks to Mark Bils for suggesting this exercise.

[30] Edelberg (2006) studies risk-based pricing of interest rates for consumer loans and finds similar results.

charge-off rates—supports a drop in information costs as the driving force behind the rise in bankruptcies over the last 20 years.

References

Abraham, A., and E. Carceles-Poveda (2006): "Endogenous Trading Constraints in Incomplete Asset Markets," Manuscript.

Aguiar, M., and G. Gopinath (2006): "Defaultable debt, interest rates and the current account," *Journal of International Economics*, 69, 64–83.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98(3), 690–712.

Athreya, K., X. S. Tam, and E. R. Young (2008): "A Quantitative Theory of Information and Unsecured Credit," Federal Reserve bank of Richmond Working Paper 08-6.

Berger, A. N. (2003): "The Economic Effects of Technological Progress: Evidence from the Banking Industry," *Journal of Money, Credit, and Banking*, 35, 141–176.

Castaneda, A., J. Diaz-Gimenez, and J. V. Rios-Rull (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.

Chatterjee, S., D. Corbae, M. Nakajima, and J. V. Rios-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525–1591.

Chatterjee, S., D. Corbae, and J. V. Rios-Rull (2007): "A Theory of Credit Scoring and Competitive Pricing of Default Risk," Manuscript.

——— (2008): "A Finite-Life Private-Information Theory of Unsecured Consumer Debt," *Journal of Economic Theory*, 142, 149–177.

Cuadra, G., and H. Sapriza (2006): "Sovereign default, terms of trade, and interest rates in emerging markets," Working Paper 2006-01, Banco de Mexico.

——— (2008): "Sovereign default, interest rates and political uncertainty in emerging markets," *Journal of International Economics*, forthcoming.

Drozd, L., and J. B. Nosal (2008): "Competing for Customers: A Search Model of the Market for Unsecured Credit," Manuscript.

Eaton, J., and M. Gersovitz (1981): "Debt with potential repudiation: theoretical and empirical analysis," *Review of Economic Studies*, 48, 289–309.

Edelberg, W. (2006): "Risk-based Pricing of Interest Rates for Consumer Loans," *Journal of Monetary Economics*, 53(8), 2283–2298.

Greenwood, J., J. M. Sanchez, and C. Wang (2007): "Financing Development: The Role of Information Costs," Economie davant garde, Research Report No. 14.

Hirshleifer, J., and J. G. Riley (1992): *The Analytics of Uncertainty and Information*. Cambridge University Press.

Livshits, I., J. MacGee, and M. Tertilt (2007a): "Accounting for the Rise in Consumer Bankruptcies," NBER Working Paper 13363.

——— (2007b): "Consumer Bankruptcy: A Fresh Start," *The American Economic Review*, 97(1), 402–418.

——— (2009): "Costly Contracts and Consumer Credit," Manuscript.

Mendoza, E., and V. Yue (2008): "A Solution to the Default Risk-Business Cycle Disconnect," Manuscript, New York University.

Moss, D. A., and G. A. Johnson (1999): "The Rise of Consumer Bankruptcy: Evolution, Revolution, or Both," *American Bankruptcy Law Journal*, 73, 311–351.

Narajabad, B. N. (2007): "Information Technology and the Rise of Household Bankruptcy," Manuscript.

Nelder, J. A., and R. Mead (1965): "A Simplex Method for Function Minimization," *The Computer Journal*, 7(4), 308–313.

Riley, J. G. (1979): "Informational Equilibrium," *Econometrica*, 47(2), 331–59.

Zhang, H. H. (1997): "Endogenous Borrowing Constraints with Incomplete Markets," *Journal of Finance*, 52(5), 2187–2209.

7 Appendix

7.1 Proofs

7.1.1 Proof of Lemma 1

Proof: The model is solved backward. First, consider a household who has to make a decision about bankruptcy. It is clear that this decision is characterized by

$$\begin{array}{ll} y_{2,n} + a_2 \geq y_{2,n}(1-\tau), & \textit{pay back}, \\ y_{2,n} + a_2 < y_{2,n}(1-\tau), & \textit{declare bankruptcy}. \end{array}$$

This implies simple threshold levels of assets for each level of income at which households are indifferent between filing bankruptcy and paying back the debt,

$$\begin{array}{l} \underline{a}_{2,L} = -\tau y_{2,L}, \\ \underline{a}_{2,H} = -\tau y_{2,H}, \end{array}$$

where $\underline{a}_{2,H} < \underline{a}_{2,L}$ because $y_{2,L} < y_{2,H}$. Notice that if a household borrows less than the limit for the low level of income, $a_2 \geq \underline{a}_{2,L}$, she will pay back the debt next period if her income is low. Since $\underline{a}_{2,H} < \underline{a}_{2,L}$, she will also pay back if the level of income next period is high. This implies that $\overline{q}(a_2, n) = (1+i)^{-1}$ if $\underline{a}_{2,L} \leq a_2$. Notice also that for $\underline{a}_{2,H} < a_2 < \underline{a}_{2,L}$ the household will file for bankruptcy next period only after a transition toward the low productivity. Since this happens with probability $\pi_{n,L}$, we have that $\overline{q}(a_2, n) = \pi_{n,L}(1+i)^{-1}$ if $\underline{a}_{2,H} < a_2 < \underline{a}_{2,L}$. Finally, notice that for $a_2 \leq \underline{a}_{2,H}$ the households will file for bankruptcy for sure. Thus, in this case, the price is equal to 0. ■

7.1.2 Proof of Lemma 2

Proof: The slope of the indifference curve between $\mathbf{q}$ and a_2 is

$$-MRS_{\mathbf{q},a_2}(\mathbf{q}, a_2) = \frac{-u'(y_{1,n} + a - \mathbf{q}a_2)\mathbf{q} + \beta\pi_{n,H}u'(y_{2,H} + a_2)}{u'(y_{1,n} + a_1 - \mathbf{q}a_2)a_2}.$$

Notice that the denominator is always positive. The numerator is decreasing in a_2. Define $a_2^*(\mathbf{q})$ as the value of a_2 that set the numerator equal to zero. Then, we have that the slope will be

$$-MRS_{\mathbf{q},a_2}(\mathbf{q}, a_2) \begin{cases} < 0, & \text{for } \underline{a}_2^H < a_2 < a_2^*(\mathbf{q}), \\ = 0, & \text{for } a_2 = a_2^*(\mathbf{q}), \\ > 0, & \text{for } a_2^*(\mathbf{q}) < a_2 < \underline{a}_2^L. \end{cases}$$

■

7.1.3 Proof of Lemma 3

Proof: We need to show that the slope is bigger for households with low productivity at period 1,

$$\frac{-u'(y_{1,L}+a_1-\mathbf{q}a_2)\mathbf{q}+\beta\pi_{L,H}u'(y_{2,H}+a_2)}{u'(y_{1,L}+a_1-\mathbf{q}a_2)a_2} > \frac{-u'(y_{1,H}+a_1-\mathbf{q}a_2)\mathbf{q}+\beta\pi_{H,H}u'(y_{2,H}+a_2)}{u'(y_{2,H}+a_1-\mathbf{q}a_2)a_2}.$$

To see this, rewrite

$$\frac{-u'(y_{1,n}+a_1-\mathbf{q}a_2)\mathbf{q}+\beta\pi_{n,H}u'(y_{2,H}+a_2)}{u'(y_{1,n}+a_1-\mathbf{q}a_2)a_2} = \frac{1}{a_2}\left[-\mathbf{q}+\frac{\beta\pi_{n,H}u'(y_{2,H}+a_2)}{u'(y_{1,n}+a_1-\mathbf{q}a_2)}\right].$$

Then, we only need to show that

$$\frac{\beta\pi_{H,H}u'(y_{2,H}+a_2)}{u'(y_{1,H}+a_1-\mathbf{q}a_2)} > \frac{\beta\pi_{L,H}u'(y_{2,H}+a_2)}{u'(y_{1,L}+a_1-\mathbf{q}a_2)}.$$

This holds because $\pi_{H,H} > \pi_{L,H}$ and $u'(y_{1,L}+a_1-\mathbf{q}a_2) > u'(y_{1,H}+a_1-\mathbf{q}a_2)$. ■

7.1.4 Proof of Lemma 4

Proof: It is clear that $\tilde{q}$ implies zero profits given $\bar{q}$. Also, it is clear that there is no other contract that is profitable—and the borrowers would prefer—to $\tilde{q}$. For this, just notice that a borrower would take $\mathbf{q}$ only if it is above $\tilde{q}$ (for a given a) and those contracts imply negative profits if the cost of information is paid. ■

7.1.5 Proof of Lemma 5

Proof: To show that these prices are the Reactive equilibrium of this economy, we need to prove that: (i) they imply zero-expected profits and (ii) any profitable deviation implies cross-subsidization between borrowers with different risk of default. *Figure 4* will be used to show these two points.

First, it is easy to see that these prices will imply zero-expected profits if borrowers with low productivity borrow more than $\underline{a}_2(a_1)$ and borrowers with high productivity borrow less than $\underline{a}_2(a_1)$. This is because for those ranges of a_2, prices $\hat{q}$ are actually equal to $\bar{q}$. Then, to see that borrowers separate themselves according to $\underline{a}_2(a_1)$, look at $\underline{a}_2(a_1)$ in 4. By construction, households with low productivity are indifferent borrowing more than $\underline{a}_2(a_1)$ at the prices for low productivity households than borrowing $\underline{a}_2(a_1)$ at the prices for high productivity. This implies that high productivity households prefer to borrow less. Therefore, prices $\hat{q}$ achieve separation.

Second, notice that any contract from which a low-productivity household would deviate should have a $\mathbf{q}$ above $\bar{q}$ for that level of a_2—remember that low productivity borrowers are choosing a_2 freely from $\bar{q}(\cdot, L)$ in the equilibrium. But then, to be

profitable, this deviation should have a cross subsidy from high productivity borrowers. Thus, any profitable deviation must have cross-subsidization. ■

7.1.6 Proof of Lemma 6

Proof: First, notice that type L households never prefer screening contracts. The point e_L in *Figure 3* cannot be an equilibrium if lenders can offer contracts without paying for information. In that allocation, the riskiest households are paying to reveal their risk. For instance, some lenders could set the price at $\pi_{L,H}/(1+i)-\epsilon$ and do not pay for information. Then, low productivity households would prefer that offer and lenders will make profits.

To find the threshold cost of information, $\underline{\mathbf{c}}$, we use *Figure 4*. There, notice that the point e'_H, at which the high-income household is borrowing using screening contracts if the cost is $\underline{\mathbf{c}}$, is on the same indifference curve, U_H, as the point e_H, at which the high-income household is using revelation contracts. Thus, if the cost $\mathcal{C}$ is lower than $\underline{\mathbf{c}}$, then high-income households prefer to take the screening contracts , and vice versa. ■

7.1.7 Proof of Lemma 7

Proof: There are two possibilities in which borrowers switch from not paying the cost of information to paying it. Both cases are analyzed below:

- In *Figure 5*, the initial cost of information is high enough, $\mathcal{C}_0 > \underline{\mathbf{c}}$, and the type-$H$ household is borrowing-constrained at e_H. When the cost of information drops to $\mathcal{C}_1 > \underline{\mathbf{c}}$, these households prefer to pay for information, $U_H^2 > U_H^1$, and debt increases. Since both levels of debt are in the range between $\underline{a}_2^L$ and $\underline{a}_2^H$, the default probability does not change. Notice that default does not change because initially these households were already borrowing in the risky debt range.

- In *Figure 6*, the initial equilibrium allocation is at (e_L, e_H). Notice that there, high-type households are clearly borrowing-constrained. At this initial equilibrium allocation, default of H-types is actually zero. They borrow so little that default is not optimal at any possible income level next period. When information costs fall to $\mathcal{C}_0$, the new equilibrium allocation is (e_L, el_H). There, type-H households pay the cost of information, borrow more, and file for bankruptcy with probability $\pi_{H,L}$. Thus, in this case, H-type households' debt rises and their probability of bankruptcy increases from 0 to $\pi_{H,L}$.

■

7.2 Transition matrix assumptions

The transition matrix is described in more detail in this subsection. This matrix can be written as

$$\begin{pmatrix} \varrho_1 & \chi_1\omega & \chi_2\omega & 0 & 0 & 0 & 0 & 0 & 0 & \varepsilon \\ \alpha_{-1} & \varrho_2 & \chi_1 & \chi_2 & 0 & 0 & 0 & 0 & 0 & \varepsilon \\ \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & 0 & 0 & \varepsilon \\ 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & 0 & \varepsilon \\ 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & 0 & \varepsilon \\ 0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_3 & \chi_1 & \chi_2 & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_4 & \chi_1 & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & 0 & \chi_{-2} & \chi_{-1} & \varrho_5 & \varepsilon \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & 0 \end{pmatrix},$$

which is equivalent to

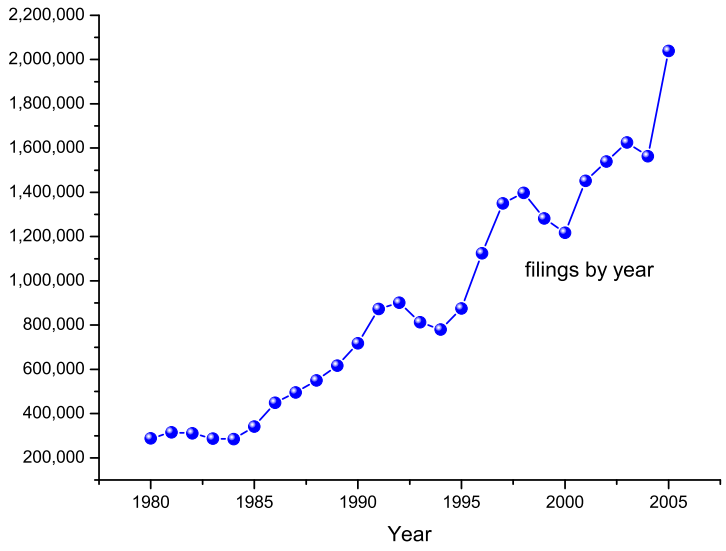
$$\begin{pmatrix} \varrho_1 & R_1 f_1 & R_1(1-f_1) & 0 & 0 & ... & 0 & 0 & \varepsilon \\ \alpha_{-1} & \varrho_2 & R_2 f_1 & R_2(1-f_1) & 0 & ... & 0 & 0 & \varepsilon \\ R_3(1-f_1) & R_3 f_1 & \varrho_3 & R_2 f_1 & R_2(1-f_1) & ... & 0 & 0 & \varepsilon \\ 0 & R_3(1-f_1) & R_3 f_1 & \varrho_3 & R_2 f_1 & ... & 0 & 0 & \varepsilon \\ 0 & 0 & R_3(1-f_1) & R_3 f_1 & \varrho_3 & ... & 0 & 0 & \varepsilon \\ 0 & 0 & 0 & R_3(1-f_1) & R_3 f_1 & ... & R_2(1-f_1) & 0 & \varepsilon \\ 0 & 0 & 0 & 0 & R_3(1-f_1) & ... & R_2 f_1 & R_2(1-f_1) & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & ... & \varrho_4 & R_2 f_1 & \varepsilon \\ 0 & 0 & 0 & 0 & 0 & ... & R_3 f_1 & \varrho_5 & \varepsilon \\ 1/9 & 1/9 & 1/9 & 1/9 & 1/9 & ... & 1/9 & 1/9 & 0 \end{pmatrix},$$

where R_i is obtained taking into account each row must sum up 1. Therefore, R_1, R_2, R_3, ϱ_4 and ϱ_5 are known, since they can be determined by

$$\begin{aligned} R_1 &= (1 - \varrho_1 - \varepsilon) \\ R_2 &= (1 - \alpha_{-1} - \varrho_2 - \varepsilon) \\ R_3 &= (1 - \varrho_3 - R_2 - \varepsilon) \\ \varrho_4 &= (1 - R_3 - R_2 f_1 - \varepsilon) \\ \varrho_5 &= (1 - R_3 - \varepsilon). \end{aligned}$$

Then, only six parameter need to be calibrated : ρ_1, f_1, ε, α_1, ρ_2, and ρ_3.

Figure 1: The rise in consumer bankruptcies



Source: American Bankruptcy Institute.

Figure 2: Zero-expected-profit prices, $\overline{q}$

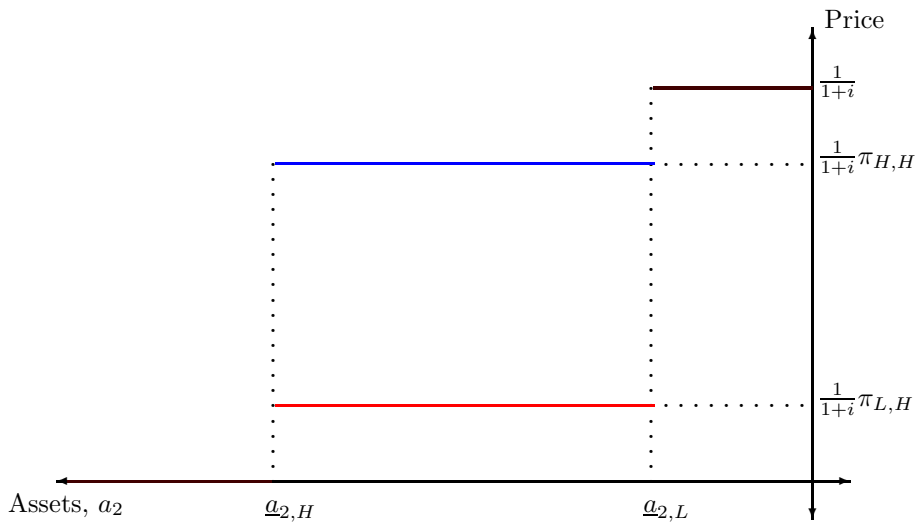


Figure 3: Equilibrium prices of screening contracts, $\widetilde{q}$

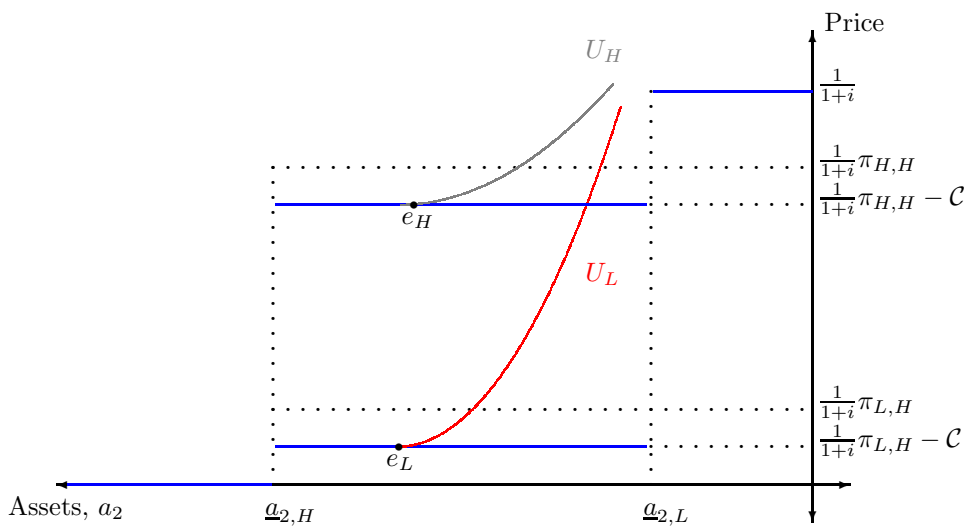


Figure 4: Equilibrium prices of revelation contracts, $\hat{q}$

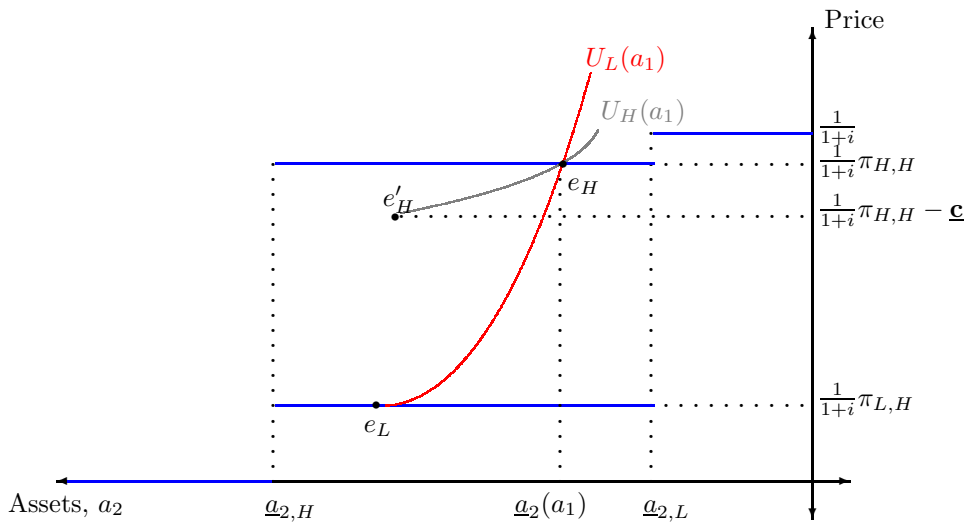


Figure 5: Cheaper information ⇒ more debt

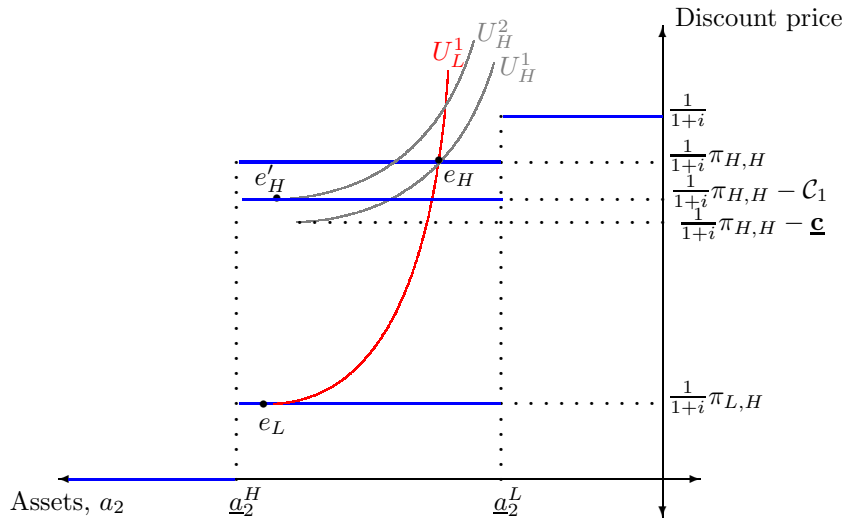


Figure 6: Cheaper information ⇒ more debt and default

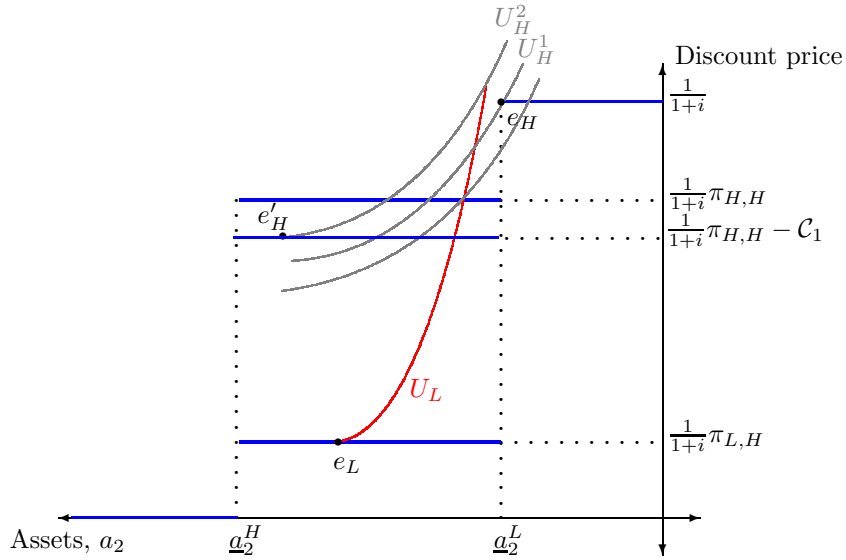


Figure 7: Calibration of limits to income groups

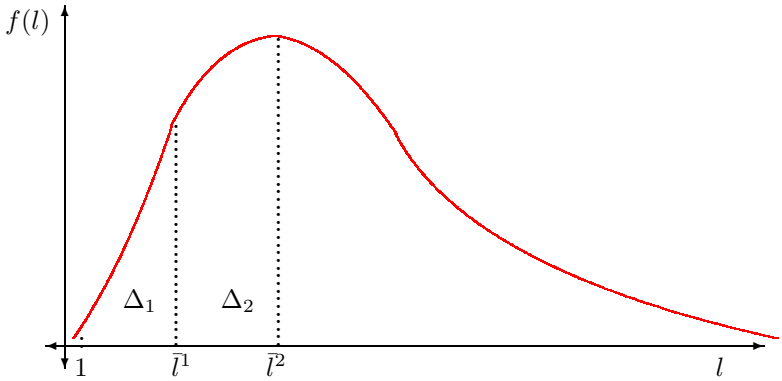
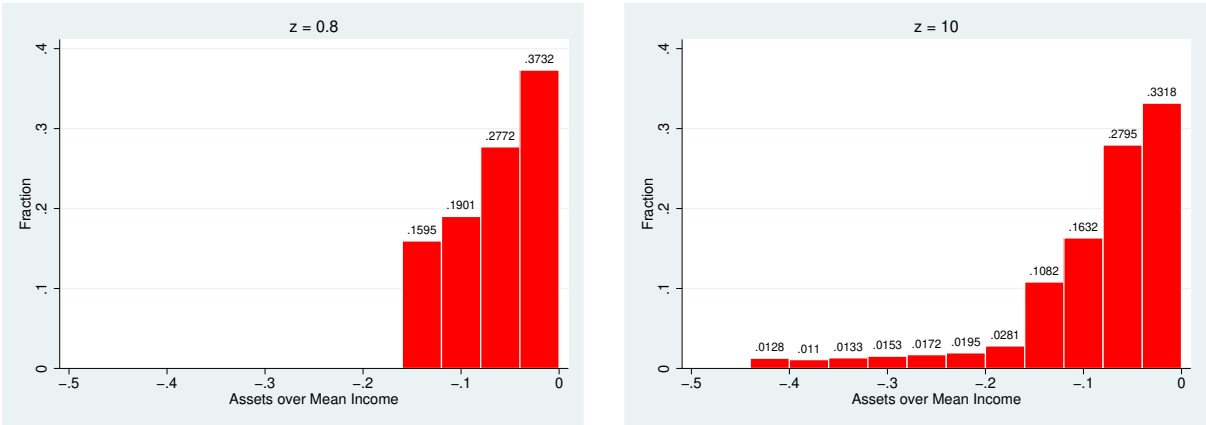


Figure 8: The effect of information costs on the debt distribution



(a) High cost of information

(b) Low cost of information

Table 1: Goodness of fit for the year 1983

Statistics	Targets 1983	"1983" calibration
Bankruptcy rate	0.49%	0.49%
Debt-to-income ratio	0.33%	0.36%
Capital-to-output ratio	3.44	3.35
Proportion of debt held by income poorest 10%	27.3%	28.7%
Proportion of debt held by income poorest 15%	30.9%	36.4%
Proportion of debt held by income poorest 20%	35.3%	44.0%
Proportion of debt held by income poorest 30%	45.3%	58.9%
Proportion of debt held by income poorest 40%	59.4%	73.9%
Gini coefficient of income	0.46	0.45
Mean-to-median income ratio	1.35	1.50
Gini coefficient of wealth	0.75	0.83
Mean-to-median wealth ratio	3.01	6.25

Table 2: Parameters for the "1983" calibration

Parameters	Values
Discount factor, β	0.921
Cost of bankruptcy, τ	0.189
Technology in the information sector, z^i	1.261
Persistence of type $n = 1$	0.442
Persistence of type $n = 2$	0.766
Persistence of type $n = \{3, ..., 7\}$	0.416
Transition probability from type $n = \{1, ..., 9\}$ to $n = 10$	0.0001
Transition probability from type $n = 2$ to $n = 1$	0.032
Transition probability from type $n = \{2, ..., 8\}$ to $n + 1$	0.144

Table 3: Transition matrix types 1 to 6 (data and model)

Data year 1983

	Income groups between percentiles						
	$[0,6]$	$[6,46]$	$[46,65]$	$[65,79]$	$[79,87]$	$[87,93]$	...
$[0,6]$	0.46	0.41	0.06	0.03	0.02	0.01	
$[6,46]$	0.07	0.71	0.13	0.04	0.02	0.01	
$[46,65]$	0.02	0.28	0.47	0.15	0.05	0.01	
$[65,79]$	0.02	0.14	0.22	0.41	0.17	0.03	
$[79,87]$	0.01	0.09	0.09	0.21	0.41	0.12	
$[87,93]$	0.02	0.09	0.08	0.09	0.26	0.31	
...							

Model's "1983" calibration

	$[0,6]$	$[6,46]$	$[46,65]$	$[65,79]$	$[79,87]$	$[87,93]$	...
$[0,6]$	0.44	0.35	0.21	0.00	0.00	0.00	
$[6,46]$	0.03	0.77	0.14	0.06	0.00	0.00	
$[46,65]$	0.11	0.27	0.42	0.14	0.06	0.00	
$[65,79]$	0.00	0.11	0.27	0.42	0.14	0.06	
$[79,87]$	0.00	0.00	0.11	0.27	0.42	0.14	
$[87,93]$	0.00	0.00	0.00	0.11	0.27	0.42	
...							

Source: Matched population from the Annual Demographic Income. Supplement from the Current Population Survey.
The sample was restricted to men in working age.

Table 4: Goodness of fit to non-targeted moments

Moments	Data 1983	"1983" Calibration
Charge-Off Rates	2.02%	0.59%
Bankruptcy filings over population*	0.06%	0.03%
Proportion of households with negative net worth	5.04%	5.49%
Coefficient of variation of interest rates	28.3%	27.4%

Source: SCF and Federal Reserve Statistical Release.
(*) Prorated using 0.53 as targeted moments for debt and bankruptcy.

Table 5: The effect of information on several variables

Statistics	Data 2004	(Diff ln data)* '04 and '83	Model with $z = 2.85$	(Diff ln model)** $z = 2.85$ and 1.26
Bankruptcy rate	1.60%	118.0%	1.52%	113.2%
Debt-to-income ratio	0.75%	80.9%	0.42%	17.0%
Capital-to-output ratio	3.44	-5.0%	3.35	-0.1%
Proportion of debt held by income poorest 10%	10.0%	-100.8%	27.2%	-5.4%
Proportion of debt held by percentiles 10 to 15%	5.0%	33.1%	7.8%	1.6%
Proportion of debt held by percentiles 15 to 20%	10.0%	81.7%	7.7%	0.8%
Proportion of debt held by percentiles 20 to 30%	19.0%	63.9%	15.2%	2.0%
Proportion of debt held by percentiles 30 to 40%	16.7%	17.7%	15.2%	1.8%
Gini coefficient of income	0.53	15.2%	0.45	0.0%
Mean-to-median income ratio	1.63	18.8%	1.50	0.0%
Gini coefficient of wealth	0.81	7.3%	0.83	0.0%
Mean-to-median wealth ratio	4.70	44.6%	6.25	0.0%
Charge-Off Rates	3.67%	98.0%	1.64%	102.9%
Bankruptcy filings over population	0.28%	148.5%	0.09%	114.7%
Proportion of households with negative net worth	5.38%	31.8%	3.67%	1.5%
Coefficient of variation of interest rates	43.2%	42.4%	43.7%	57.5%

* The data for each year was first logged and then the difference was computed.
** The data for each value of z was first logged and then the difference was computed.

Table 6: The effect of income on interest rates

Dependent variable: interest rate	Year		Model	
	1983	2004	"1983"	"2004"
ln(income)	0.155	-0.768	0.063	-0.587
	(0.219)	(0.103)	(0.237)	(0.175)
Controls				
ln(credit card debt)	0.061	0.209	0.468	1.117
	(0.102)	(0.052)	(0.124)	(0.091)
Age of the head of household	0.007	0.012		
	(0.012)	(0.008)		
Age of the head of household squared	-0.001	-0.001		
	(0.001)	(0.000)		
Male head of household	0.720	-0.315		
	(0.497)	(0.242)		
Married head of household	-0.370	0.204		
	(0.439)	(0.215)		
Constant	15.400	18.720		
	(2.114)	(1.076)		
Observations	1115	6380	1078	6357
R squared	0.010	0.012	0.013	0.027

Note: the data from the SCF restricted is to households with credit card debt. The difference between the coefficients for ln(income) is 0.923 and the standard deviation of this difference is 0.379. The difference between the coefficients for ln(credit card debt) is0.149 and the standard deviation of this difference is 0.178. The regressions with the data generated with the model have a similar number of observations as those from the SCF because a random sample of similar size was generated with the model.