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The role of studs in the sound transmission of double walls

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Abstract

Steel studs are used in double walls to provide structural stability. This creates a vibration transmission path between leaves that can often be more critical than the airborne path through the cavity. Some of the existing models for sound transmission consider the studs as elastic springs. The spring stiffness may be taken as the cross-section elastic stiffness of the stud, but this leads to an underestimation of the vibration transmission. A procedure to obtain more accurate parameters to be used in vibration and sound insulation models is presented. The results show that they must be obtained from dynamic models and/or experiments.

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1 Introduction

Double walls are a common solution in lightweight structures. They are typically constructed by means of two thin leaves (plasterboards, wood plates or similar) and

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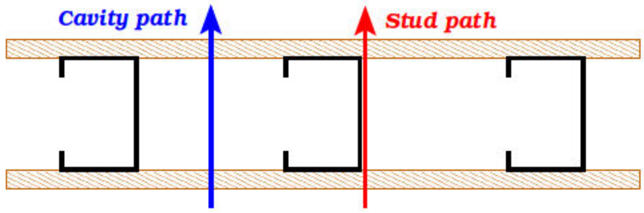


Figure 1: Sound transmission paths in a double wall with studs.

some kind of absorbing material placed inside the air cavity to improve the acoustic insulation capacity of the system. In order to satisfy construction requirements and to give a certain stiffness to the wall, wood beams or steel studs are used. Studs act as sound bridges between the two leaves.

These connections between leaves cause the actual acoustic response of the wall to be worse than that of an ideal double wall. A new vibration transmission path (besides the airborne or cavity path) is created, see Fig. 1. The decrease in the sound reduction index of the double walls highly depends on the mechanical properties (mainly stiffness) of these connecting elements. In this paper, the attention is focused on the study and characterisation of lightweight cold-formed steel studs. Their effect is greatly influenced by the cross-section shape.

Two direct applications can be mentioned. On the one hand, stud manufacturers are interested in knowing which stud is better from an acoustic (vibration transmission) point of view. On the other hand, wave approach or statistical energy analysis (SEA) models cannot reproduce the exact geometry of the stud and require some parameters describing its mechanical response. Some of these parameters can be provided by a numerical model because it can deal with accurate geometry descriptions. Numerical models can also deal with the whole problem. However, working with two different levels of detail (rooms - double wall and stud shape) increases the meshing tasks and the computational cost. Thereby, it is also interesting for a numerical approach to simplify the modelling of the stud.

An outline of the paper follows. A literature review of sound transmission models of double walls with connection between leaves and experimental characterisation of studs is done in Section 2. The method to characterise studs is presented in Section 3.1. The studs and wall typologies used in the analysis are described in Section 3.2. Numerical results are shown in Section 4. The influence of the stud mass and shape in the transmission of vibrations is shown in Sections 4.1 and 4.2 respectively. Some studs are characterised by means of an averaged response corrected stiffness in Section 4.3. The values obtained have been used as input data for SEA models of vibration transmission in Section 4.4 and numerical models of sound transmission in Section 4.5. The concluding remarks of Section 5 close the paper.

2 Literature review

2.1 Models

Several models dealing with double walls with connections can be found in the literature. In the simplest cases the studs are considered as infinitely rigid connections between the leaves [1]. Such models can be quite correct for rigid studs (i.e. wood studs) but underestimate the isolating capacity of lightweight double walls by neglecting the benefits of the use of more flexible steel studs. In [2] and [3] the two leaves of the double walls are supposed to be connected by means of springs. Both translational and rotational springs are considered. The value of the stiffness is considered to be constant in the whole frequency range and it is typically taken from an elastic measurement (i.e. elastic stiffness of the flange of the stud). In [4] three different models of sound transmission through double walls are proposed: *i)* two infinite leaves linked by means of point connections; *ii)* one infinite leave and one infinite leave with rib-stiffeners linked by means of point connections; *iii)* one infinite leave and one infinite leave with rib-stiffeners linked through the lines defined by the stiffeners. It has been found that the stiffness of the connector has important effects on the radiated acoustic power. A complete model of sound transmission through double walls is developed in [5]. Both the cavity and the stud paths between infinite homogeneous plates are accounted for. Finite cavities are considered and the pressure field inside solved by means of cosine-series. The stud is considered to be an Euler beam and the cross-sectional stiffness is modelled by means of a spring.

In the model proposed in [6], each transmission path is described by a different expression. The sound insulation of double walls can be calculated by means of very simple expressions, which is attractive for practical computations. Note, however, that these expressions (piecewise-defined in the frequency domain) are based on ad-hoc considerations, not in any governing equation. This contrasts with [2, 3, 4, 5], which solve the vibroacoustic equations of the double wall (i.e. clear physical foundations) by means of a wave approach. In [7] a model for lightweight double walls with studs is presented. The leaves are modelled as infinite plates and solved with a wave approach. The steel studs connecting them are considered by means of line connections or point connections depending on the frequency range (mid and high respectively). The second option uses statistical energy analysis expressions. Finally, in [8] the effect of studs is modelled by assuming valid a simple model of vibration isolation devices. The parameters of the model (damping and critical frequency) are adjusted. The influence in the low-frequency range of resilient connections between leaves is discussed. The mass-air-mass resonance frequency of lightweight double walls is shifted due to the studs.

The characterisation of the studs can be used in order to simplify numerical-based models of sound transmission through double walls. To consider the full vibroacoustic problem often implies a very expensive calculation. A common approach is to describe the rooms by means of modal analysis and concentrate the use of the numerical method (i.e. finite elements) in the analysed wall [9, 10, 11, 12]. This simplification is even more relevant when the analysed structure is a double wall. In [13] the rooms are described by means of modal analysis and the double wall using finite elements. The connection between leaves is done by means of plane homogeneous plates. The rooms are supposed to be infinite in the direction orthogonal to the wall in [14] where the sound transmission through single and double walls is also predicted.

All the models cited above predict the acoustic isolation of the double wall with studs, but the characterisation of the connecting element is not their main goal. A review of the more referenced simplified models of sound transmission in double walls is done in [15]. It shows that only five of the seventeen models considered take into account the possible existence of studs. Moreover only two of them allow these studs to be flexible.

2.2 Experimental measurements

Some laboratory measurements of the effect of the studs in the sound reduction index can be found in [16] and [17]. In [18] lightweight steel studs are considered. Measurements of the dynamic stiffness of isolated studs (using the methodology proposed in [19]) and its effect on the sound reduction index of double walls are reported. A set of measurements illustrating the importance of studs in the sound insulation of exterior walls can be found in [20]. The discussion is focused on the effect of stud size and the spacing between studs. This is relevant in the low-frequency range and affects the primary structural resonance. In [21], studs with non-conventional cross-section shape are tested in order to check the improvement in sound isolation. However, studies dealing with a deterministic approach to the problem (exact descriptions of stud geometry and solution of the problem by means of analytical or numerical methods) can be rarely found. No practical rule on how to choose the correct value of the stud stiffness has been found. It is a necessary parameter in most of the models mentioned above. In [22], the laboratory measurements that motivated the stud characterisation methodology proposed below are presented.

The dynamic properties of resilient studs have also been measured in the laboratory. Most of the techniques were originally developed in order to determine the properties of rubber-like materials that are used as vibration isolators. An extensive literature review can be found in [23]. The consequence of this background is a set of ISO regulations establishing the basis of this kind of measurements [24]. Two laboratory tests are proposed: direct [25] and indirect [26].

The dynamic stiffness of resilient steel elements is measured by means of the indirect method in [27]. Some guidelines are proposed in order to increase the frequency range where the method is valid. In [28] an extensive review of measurement techniques on resilient elements can be found. Moreover an iterative procedure that corrects some of the simplifications done in the indirect technique is proposed. Common steel studs used in façades and inner walls are characterised.

In all the measurement techniques described in these references the resilient element is loaded by means of an static load and the behaviour is supposed to be one-dimensional. As reported in [27], the measurements can differ depending on this load value. Moreover, the specimen must be placed between plates that force the behaviour to be one-dimensional. Thus, these load and boundary conditions largely differ from the ones of a stud placed between leaves in a floor or a wall.

The results obtained in this work are in line with those obtained previously with the various approaches mentioned in this section.

3 The characterisation method

The aim of our research is to study in detail how to characterise the steel studs (especially their dynamic cross-section behaviour). Studs are characterised taking into account their final role inside the structure (double wall or floor), instead of the laboratory set-ups described in [24]. The analyses are done using two-dimensional models describing the cross-section of a wall. This is enough in order to illustrate the proposed methodology and study the differences in the vibration behaviour due to the stud shape. The connection between the stud and the leave is considered to be a line-connection (i.e. a point in the two-dimensional cross-section). The study of the connection type has been omitted but this is an important issue that should be analysed by means of three-dimensional models [7].

The literature review of Section 2 reveals that resilient devices tested in the laboratory are usually characterised by means of a four-pole parameter analysis [23] (also called two-port parameter analysis in [28]), but double wall models usually consider the stud to be a translational or rotational elastic spring.

In the four-pole parameter analysis a relationship between the input (applied point force and velocity) in one side of the resilient device and the output (point force and velocity) on the other is established. The parameters are frequency-dependent and take into account mass and stiffness effects. In the low-frequency range the device behaves like a single spring and the four parameters are equal (their value is the low-frequency stiffness). For higher frequencies their value, depending on symmetry considerations, can be different. In practise, it requires to perform several measurements in order to determine each frequency-dependent parameter (i.e. blocking some of the displacements of the stud).

The option chosen here is to consider the studs as translational or rotational springs. It is a more simplified approach appropriate to provide data for existing double wall models. The mass of the stud is divided in two concentrated masses. These springs are then characterised by means of an averaged response corrected stiffness. Thus, for each stud, the cross-section dynamic behaviour is described by two parameters, the averaged response corrected translational and rotational stiffness. The idea is to have a single device with a frequency-dependent parameter that provides the same vibration level difference as the stud.

As shown in [22], if the stud is isolated it is difficult to distinguish between translational and rotational effects and the boundary conditions highly modifies the vibration response. For these reasons, we will consider from now on the entire package leave-studs-leave, see Fig. 2. This situation is closer to the actual use of studs in the double wall.

In this section the characterisation procedure is described. The procedure to determine the averaged response corrected stiffness values is described in Section 3.1 and the analysed studs and material data used are defined in Section 3.2.

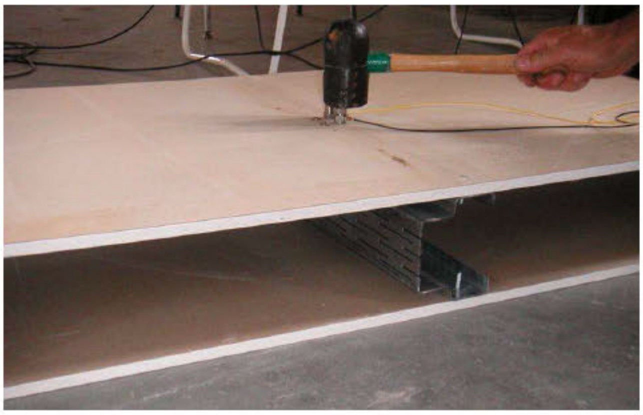


Figure 2: Laboratory tests of the entire package (studs and leaves). The lower leave is blocked.

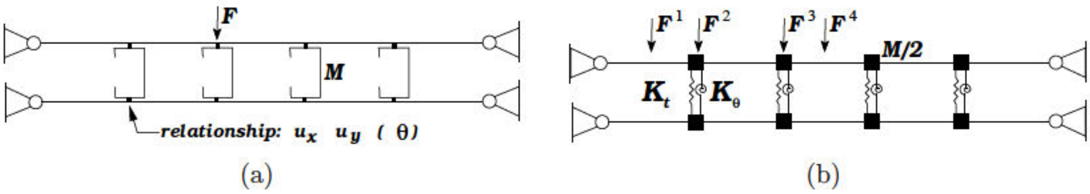


Figure 3: Models of the leave-stud-leave package: (a) detailed model with the actual geometry of the studs; (b) simplified model with studs modelled as a translational stiffness K_t, a rotational stiffness K_θ and two concentrated masses $M/2$. F^i indicates the four load positions considered.

3.1 Cross-section structural vibration models

In order to study the vibration transmission path, the two structures of Fig. 3 are considered. It is assumed that the transmission of vibrations between leaves of a double wall can be studied at cross-section level. The required parameter for the studs in sound transmission models is the averaged response corrected cross-section stiffness per unit length. In addition, the main difference between double walls using different stud types is found at cross-section level. This is represented by the models of Fig. 3.

The output of interest is the vibration level difference between the upper and the lower leaves

$$D_{\nu,ij} = -10\log_{10}(d_{ij}) \quad \text{with} \quad d_{ij} = \frac{< v^2_{rms,j} >}{< v^2_{rms,i} >} \tag{1}$$

where $< \bullet >$ is the spatial average of $\bullet$, $v_{rms,i}$ is the root mean square velocity in leave i (upper), where the force is applied, j is the receiving leave (lower) and d_{ij} is the vibration reduction factor. $D_{\nu,ij}$ can be calculated from the data of the numerical model as

$$D_{\nu,ij} = 10\log_{10}\left(\frac{< |u_{upper}|^2 >}{< |u_{lower}|^2 >}\right) \tag{2}$$

where u_{upper} and u_{lower} are the phasors of displacements for the upper and lower leaves. The spatial average is done along the leave. A larger value of $D_{\nu,ij}$ means a better vibration isolation.

The vibration level difference $D_{\nu,ij}$ is a useful parameter in order to compare the performance of different studs used in the same double wall. However, it is an environment-dependent parameter: the values of $D_{\nu,ij}$ do not only depend on the stud type. They also depend on other variables, such as leave properties and boundary conditions. Thus, $D_{\nu,ij}$ is not the the most suitable characterising parameter. As shown in the following sections, an averaged response corrected stiffness of the stud is a better parameter, less dependent on each particular situation and more related with every stud type.

In the detailed model of Fig. 3(a), the actual geometry of the stud is discretised. In the connection between the stud flanges and the leaves, continuity of displacements and rotations is imposed. In the simplified model of Fig. 3(b), the stud is replaced by a translational spring, a rotational spring and two concentrated masses. The equivalence between both models has been established by comparison of the vibration level difference $D_{\nu,ij}$.

A set of admissible values of averaged response corrected rotational and translational stiffness can be obtained by comparing the two deterministic models presented in Fig. 3. The key is to find pairs of values that provide, for a given frequency, the same vibration level difference. This requires to generate surfaces of vibration level difference in the plane K_t - K_θ for every 1/3 octave frequency band.

Once the surfaces have been generated, the admissible values of rotational and translational stiffness can be obtained by imposing the same value of vibration level difference for both models of Fig. 3:

$$D_{\nu,ij}^{(\text{simplified})}(K_t, K_\theta) = D_{\nu,ij}^{(\text{detailed})} \tag{3}$$

Both the surface $D_{\nu,ij}^{(\text{simplified})}$ (obtained with the simplified model) and the value $D_{\nu,ij}^{(\text{detailed})}$ (obtained with the detailed model) are known. The equality provides a set of admissible values (K_t, K_θ), which yield the same vibration isolation in the simplified model and the detailed model.

For all the results presented here, four load configurations have been considered, see Fig. 3(b). The positions of point loads have been chosen in order to be representative: load applied over the stud or between studs, and at the centre of the double wall or on the side. The study is done in terms of averaged responses: average of the load position, average in time (over a period), average in frequency (the results are given in 1/3 octave frequency bands) and average in space. One analysis per Hz is carried out in order to describe the response spectrum.

The structural spectral finite element method (SFEM, see [29] and [30]) has been chosen. In two-dimensional situations, the exact solution is reached using only the necessary elements in order to describe the geometry (i.e. 5 elements are required for the case of the TC cross-section, 9 for the AWS and 4 for the O, see Fig. 4). This is a very important advantage since typical mesh requirements of the finite element method can be ignored. Results presented in Sections 4.2 and 4.3 are obtained using this model.

3.2 Studs and leaves analysed

The analysis has been extended to several stud sections. The results presented here are by default obtained using double walls with an air space between leaves (stud height) of 70 mm. Two other stud heights (125 mm and 175 mm) have been considered in the analysis. For every height, several cross-section shapes have been studied. Five of them will be employed in order to illustrate the most interesting aspects of the study. They are plotted in Fig. 4, and the dimensions for the 70 mm series can be found in Table 1. Both conventional studs (TC, S, O) and acoustic studs (AWS, LR) are analysed. Acoustic studs have a similar bending stiffness but are more flexible at cross-section level than conventional studs. Experimental studies of the influence of cross-sectional shape on acoustic performance can be found in [18] and [21].

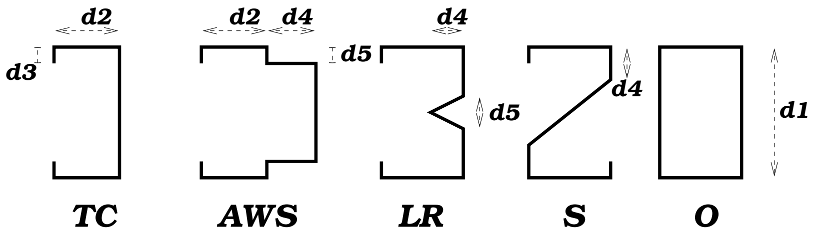


Figure 4: Sketch of the stud cross sections.

Section	**d1**	**d2**	**d3**	**d4**	**d5**	**e**
TC	70	40	10	–	–	0.47
LR	70	40	10	0.7·d2	0.2·d1	0.47
S	70	40	10	0.2·d1	–	0.47
O	70	40	–	–	–	0.47
AWS	70	40	10	24	13	0.47

Table 1: Dimensions (in mm) of the cross sections shown in Fig. 4. e is the thickness of the section.

Case	**Upper leave**	**Lower leave**	**Connection**	ℓ **[m]**	**no. studs**
Wall 1	GN	GN	u_x, u_y and θ	3	4
Wall 2	GN	GEK	u_x, u_y and θ	3	4
Wall 3	GEK	GN	u_x, u_y and θ	3	4

Table 2: Description of the three wall typologies.

Meaning	**Symbol**	**Value, GN**	**Value, GEK**
Thickness	t	13 mm	13 mm
Young's modulus	E	$2.5 \cdot 10^9$ N/m^2	$4.5 \cdot 10^9$ N/m^2
Density	ρ_{solid}	692.3 kg/m^3	900 kg/m^3
Damping	η	3 %	3 %

Table 3: Geometrical and mechanical properties of the leaves [31]

Besides the changes at cross-section level, three different double walls have been considered. Each of them has different leaves, see Tables 2 and 3. For all the situations, the length of the double wall is 3 m and the separation between studs is 0.6 m (four studs employed in every double wall). The plasterboards (GN and GEK) are supported at the beginning and ending points.

4 Results, analysis and discussion

All the results shown in this Section are obtained by means of two-dimensional models and no experimental result is shown. The two-dimensional models can reproduce the more relevant aspects of vibration transmission at cross-section level and deal with the different shape of the studied studs. Sections 4.1, 4.2 and 4.3 present results obtained with the SFEM models of Fig. 3. In Section 4.4 these results are used as input data in SEA models and in Section 4.5 the whole vibroacoustic problem is solved by means of the finite element method or a combination with modal analysis [11, 12, 13]. Note that to characterise the stud, both the simplified and detailed models have to be solved. However, these are purely structural problems and the results obtained can be used as input for vibroacoustic and SEA models dealing, for example, with a sound transmission problem.

4.1 Effect of concentrated masses

The effect of concentrated masses in the vibration level difference for the case of the simplified model can be seen in Fig. 5. Differences are not larger than 4 dB. The stud mass provides, in general terms, an increase in the vibration isolation. This value would be larger for heavier studs. In the low-frequency range the mass effect can be neglected, however it is relevant for high frequencies. The increase of isolation caused by the concentrated masses is more important for stiffer springs connecting the upper and lower leaves.

4.2 Influence of stud shape in the vibration level difference

The vibration level difference between the leaves of a wall can be different depending on the stud cross-section shape. This is shown in Fig. 6 where the results obtained with several studs are compared. For all of them, the isolation of vibrations in the low-frequency range is really poor, see Fig. 6(a). It cannot be improved by changing the stud shape. Note that standard sections like TC, O or S provide an almost constant level of vibration isolation. On the contrary, acoustic sections like LR or AWS improve the isolation of vibrations in the mid and high-frequency range, see Fig. 6(b) (but they can be worse than the others for some frequencies in the low-frequency range).

The important variations of $D_{\nu,ij}$ with frequency found in Fig. 6 can be understood by means of a modal analysis of the structure. The vibration level difference of the double walls with TC and AWS studs shown in Fig. 6 is compared in Fig. 7 with the vibration level difference of the eigenmodes of the structure. The vertical red lines indicate the value of $D_{\nu,ij}$ of each mode. Not all the eigenmodes in the range

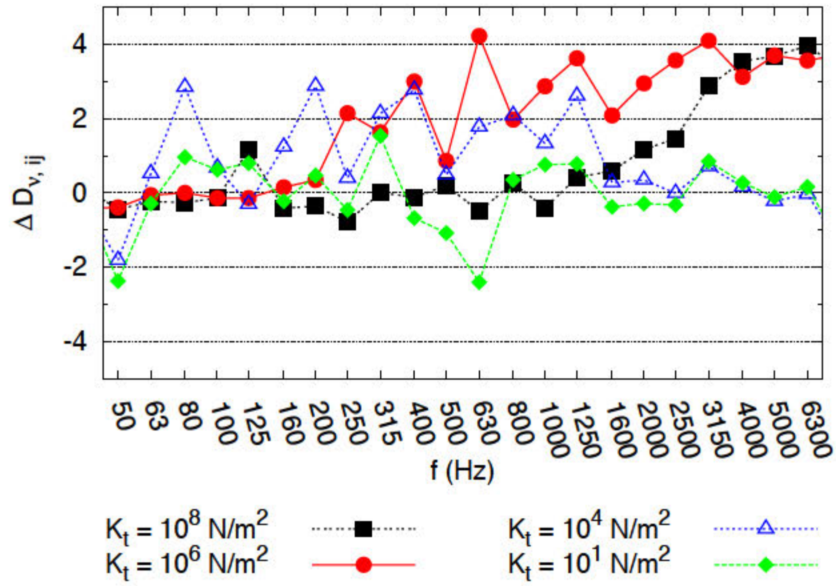
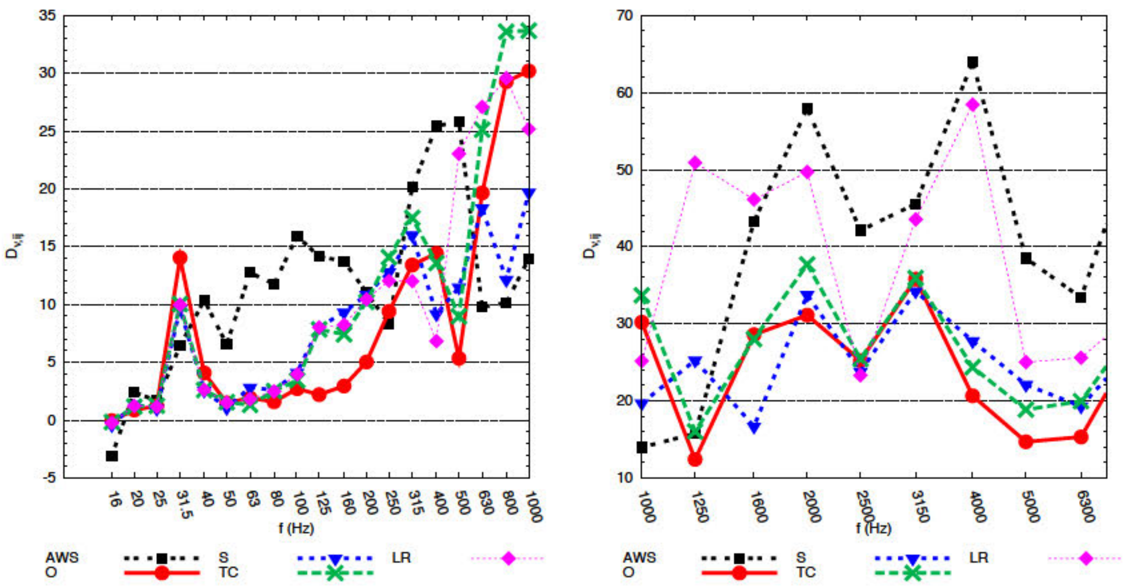


Figure 5: Vibration level difference in the simplified model (wall 2) considering or not concentrated masses ($\Delta D_{\nu,ij} = D_{\nu,ij}^{(\text{with masses})} - D_{\nu,ij}^{(\text{no masses})}$).



(a) (b)

Figure 6: Comparison of the vibration level difference for several studs with different cross-section: (a) low-frequency range; (b) mid-frequency range. Note the different vertical scales.

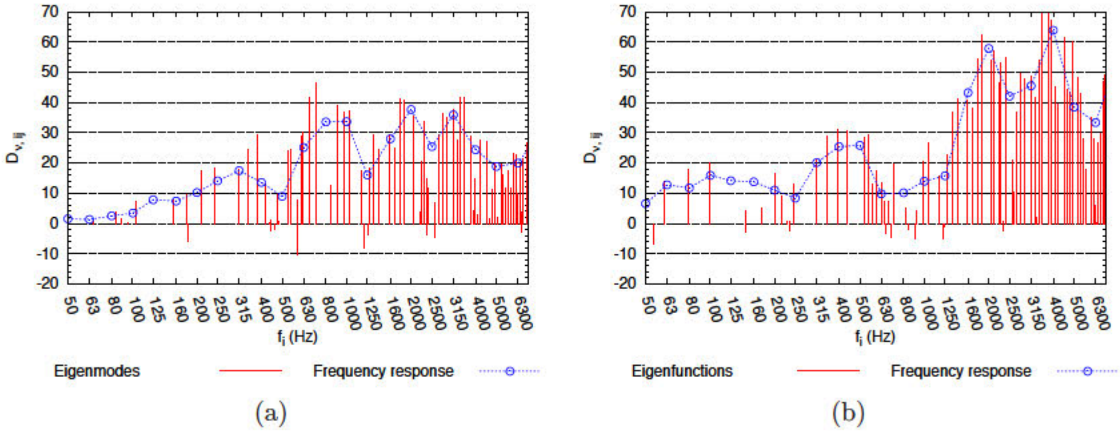


Figure 7: Comparison of the vibration level difference between leaves of a double wall (analysis of the frequency response) with the vibration level difference of the more contributive eigenfunctions: (a) double wall with TC studs; (b) double wall with AWS studs.

50 – 6300 Hz are shown in Fig. 7. Only the eigenmodes that have more contribution to the frequency response have been plotted. The criterion used to identify such eigenmodes is to verify if the displacement at the application point of the force is larger than the mean displacement of the structure. This is related with the modal expression of a point force applied at x_F, for mode m: $\Psi_m(x_F)F$ (Ψ_m is the eigenmode and F the point force applied at x_F). With this selection criterion, most of the modes with negative vibration level difference (the vibration in the lower leave is larger than in the upper leave) are discarded and do not appear in the plot. (Note, however, that a few modes with negative $D_{\nu,ij}$ do pass this filter). In the discarded modes, $\Psi_m(x_F)F$ is small when compared with the plotted modes.

Fig. 7 shows that the evolution of the vibration level difference of the structure has a correct agreement with the vibration level difference of the structural eigenmodes. This has been found for all the analysed studs. Therefore, it can be concluded that the variations in the vibration level difference observed in Fig. 6 are related to the modal behaviour of the studied system.

4.3 Stud equivalent stiffness

The procedure to obtain averaged response corrected values of stiffness is shown here. The vibration level difference surfaces for several values of the translational and rotational springs in the simplified model of Fig. 3(b) are shown in Fig. 8. The four surfaces are for 200, 500, 1000 and 3150 Hz and for wall 2 (see Table 2). They have been generated analysing 36 different situations combining values of $K_t = 10^4, 10^5, 10^6, 10^7, 10^8, 10^9$ N/m^2 and $K_\theta = 10^1, 10^2, 10^3, 10^4, 10^5, 10^6$ N $\cdot$ m/ (rad $\cdot$ m). The vibration level difference increases with frequency but only for the lower values of rotational and translational stiffness.

When these surfaces are intersected by the vibration level differences obtained from the detailed model of Fig. 3(a), a set of admissible average response corrected

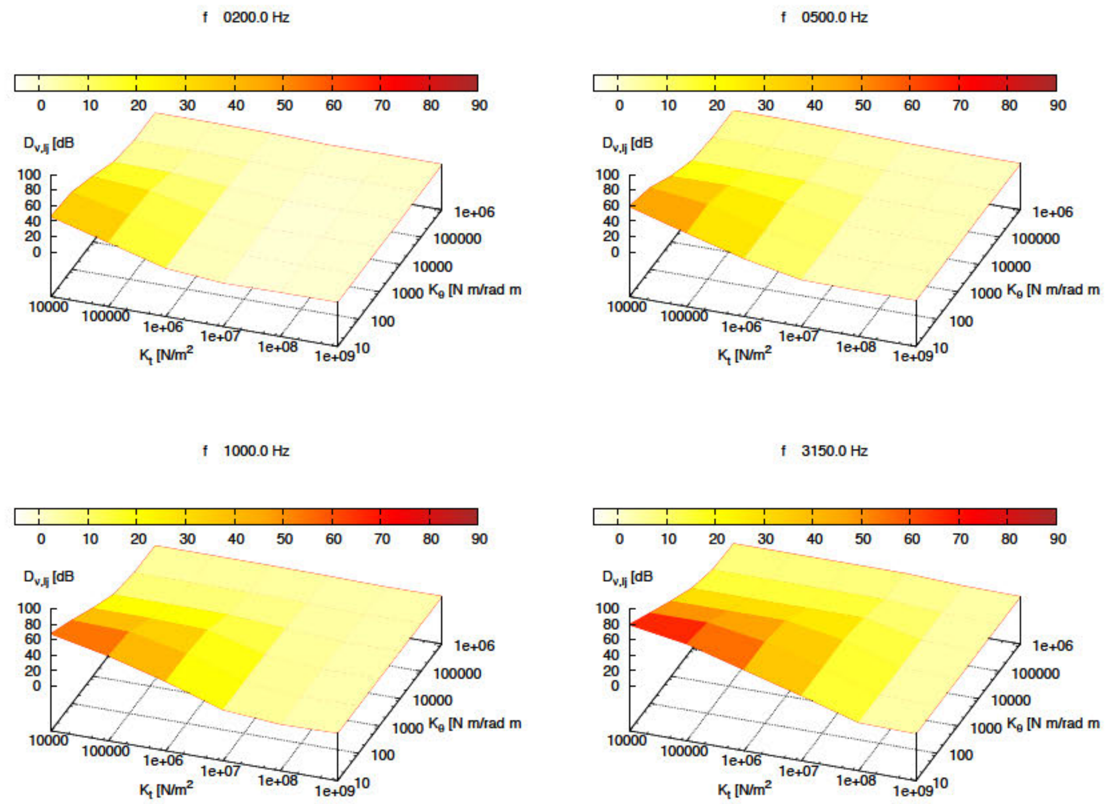


Figure 8: Vibration level difference $D_{\nu,ij}$ in wall 2 as a function of translational stiffness K_t and rotational stiffness K_θ, for various frequencies.

values of stiffness is obtained. The values for the TC section and wall 2 (see Table 2) can be seen in Fig. 9.

Several important aspects have to be highlighted. On the one hand, a frequency dependence of the parameters is observed. On the other hand, an equivalent effect in terms of $D_{\nu,ij}$ can be obtained using: *i)* only a translational spring; *ii)* only a rotational spring; *iii)* and adequate combination of both. The option chosen here is to use $K_\theta = 0$ and a frequency-dependent K_t. Most of the models available in the literature tends to consider only translational springs. This is also a more intuitive idea than rotational springs. For low frequencies the values of stiffness are smaller (around the typical values of elastic measurements), and they generally increase with frequency.

For a given cross-section, similar results are obtained for the three different walls. In Fig. 10, the results of walls 1, 2 and 3 (see Table 2) for the AWS section are compared. The influence of the wall type in the $K_t - K_\theta$ curves can be measured by considering the dimensionless parameter σ/μ along the curve. σ is the standard deviation and μ the mean value. It has to be measured in the normal direction. For the three different curves at $f = 200$ Hz, σ/μ is evaluated at five discrete positions: *i)*$K_t = 10^4$ N/m and K_θ variable; *ii)*$K_t = 10^5$ N/m and K_θ variable; *iii)*K_t variable

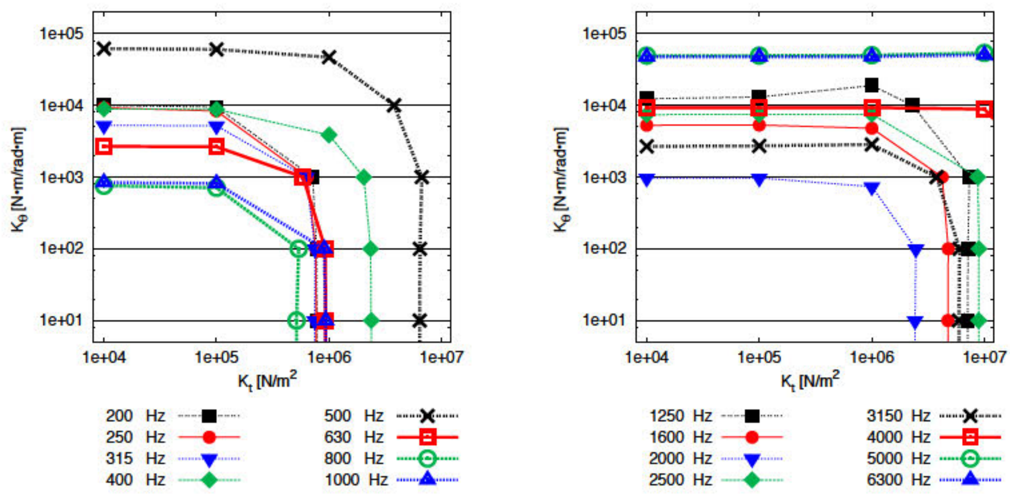


Figure 9: Admissible values of rotational and translational stiffness for the TC section obtained by comparison of deterministic models. Wall 2.

and $K_\theta = 10^3$ N·m/rad·m; *iv)* K_t variable and $K_\theta = 10^2$ N·m/rad·m; *v)* K_t variable and $K_\theta = 10^1$ N·m/rad·m. Three values (of K_t or K_θ) corresponding to each wall type are used to calculate the ratio σ/μ. The five σ/μ values are averaged in order to obtain an approximation to the curves variation. The final outputs obtained are for the curves in Fig. 10 are: $f = 200Hz$, $\sigma/\mu = 11.0\%$; $f = 1250Hz$, $\sigma/\mu = 69.4\%$; $f = 4000Hz$, $\sigma/\mu = 78.1\%$. These values are not large enough in order to modify the global response of the vibroacoustic system.

The difference between walls 1, 2 and 3 is more important for low frequencies, where the modal behaviour governs the response and the number of modes in the 1/3 octave band is smaller. In the low-frequency range (frequencies under 200 Hz) it is difficult to obtain the stiffness variation with frequency. The values of $D_{\nu,ij}^{(\text{simplified})}$ oscillate between 0 dB and 5 dB depending on the value of the stiffnesses and the mechanical and geometrical characteristics of the leaves. Thus, the intersection of the $D_{\nu,ij}^{(\text{simplified})}$ surface with a constant value $D_{\nu,ij}^{(\text{detailed})}$ does not provide smooth curves like in Fig. 9.

In Fig. 11 a typical final output for this kind of analysis can be seen. Frequency-dependent translational stiffness laws are presented for every section.

4.4 Using the stiffness values in a SEA model

The values of stiffness characterising the studs can be used as input data for other modelling techniques. A clear example is its use as input data for Statistical Energy Analysis (SEA). The transmission of vibrations between two leaves (i and j) connected by means of a spring has been studied by [32]. This SEA model is two-dimensional and the two leaves are considered to be infinite or semi-infinite Euler beams. A mechanical load is applied to the upper leave ($i = 1$). The SEA vibration level difference can be

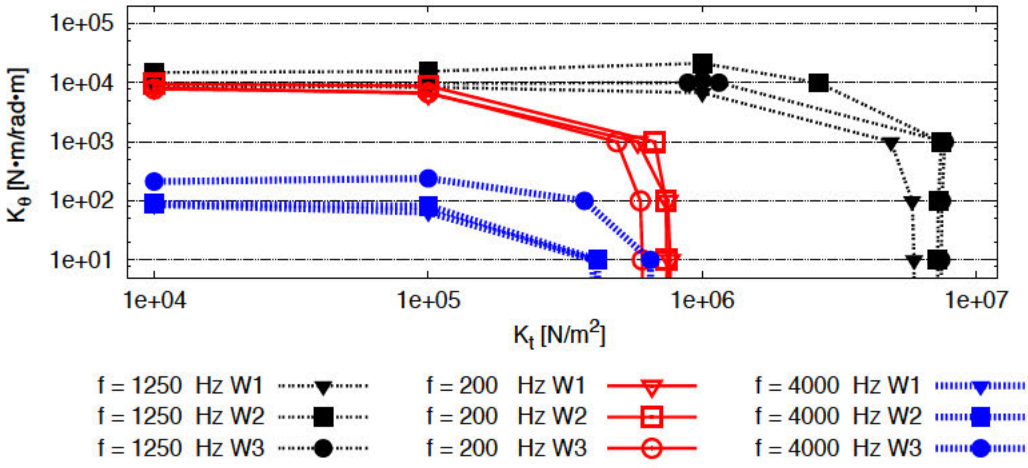


Figure 10: Comparison of the admissible values of rotational and translational stiffness of section AWS obtained by means of different walls.

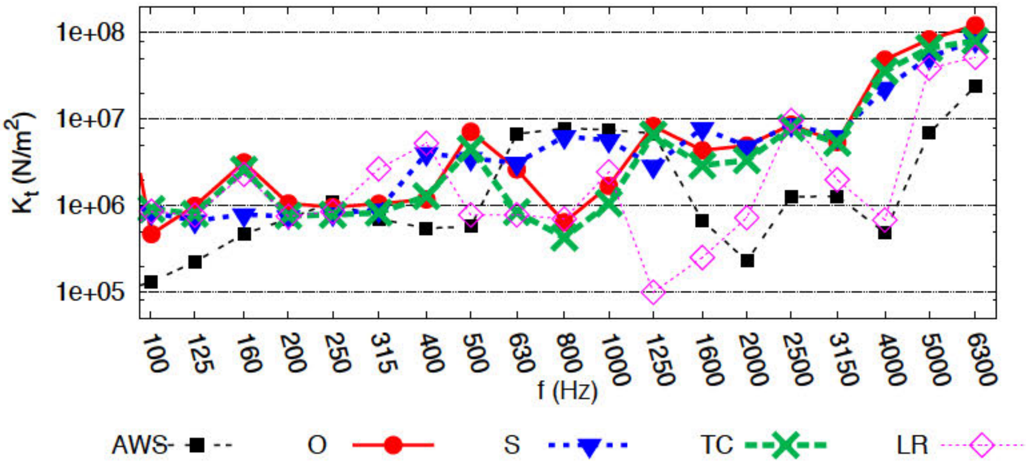


Figure 11: Laws of translational stiffness for $K_\theta = 0$. Averaged values between walls 1, 2 and 3.

written as

$$D_{\nu,ij} = D_{12} = 10 \log_{10} \left(m_2 \eta_2 \omega \frac{|Y_1 + Y_2 + Y_t|^2}{n\mathrm{Re}\left\{Y_2\right\}} \right) \tag{4}$$

where m_2 is the mass per unit length, η_2 is the loss factor, n is the number of connections (springs) per unit length, Y_1 and Y_2 are the mobility of the two leaves and Y_t is the mobility of a point connecting tie ($Y = v/F$, v being the velocity and F the force). The last parameter can be related to the dynamic stiffness of this tie, $Y_t = \mathrm{i}\omega/K_t$, where K_t is the stiffness (in this SEA model, only translational stiffness is considered). For the mobility of the two leaves, the formulas given in [33]

$$Y_{\infty\mathrm{leave}} = \frac{1}{4} Y_{\text{semi-}\infty\mathrm{leave}} = \frac{(1-\mathrm{i})}{4 m_\ell c_B} \quad \text{with} \quad c_B = \frac{\omega}{\sqrt[4]{\omega^2 m_\ell / E\,I}} \tag{5}$$

are used, where m_ℓ is the density per unit length of the leave, E the Young's modulus and I the moment of inertia. Note that two reference cases have been considered: infinite and semi-infinite leaves. The former provides better approximations for high frequencies where the influence of boundaries is not important and the vibrations are localised due to damping. The latter is a better approximation for low frequencies or situations where the mechanical load is close to the boundary.

We have considered first the case of two leaves connected by means of springs, see Fig. 3(b). In this case the value of the stiffness is known and a 'pure' comparison between the SEA model and the numerical method (SFEM) can be established without additional errors caused by the uncertainties due to the characterisation of the stud shape. Note that the SEA model does not consider concentrated masses. In order to make the comparison, concentrated masses have also been removed from the numerical model.

The results are presented in Fig. 12. The transmission of vibrations in wall 2 has been calculated with the numerical model and with the SEA model with two variants: infinite and semi-infinite leaves. The agreement is correct. The numerical results are closer to the infinite leave curve. Differences are smaller for mid and high frequencies.

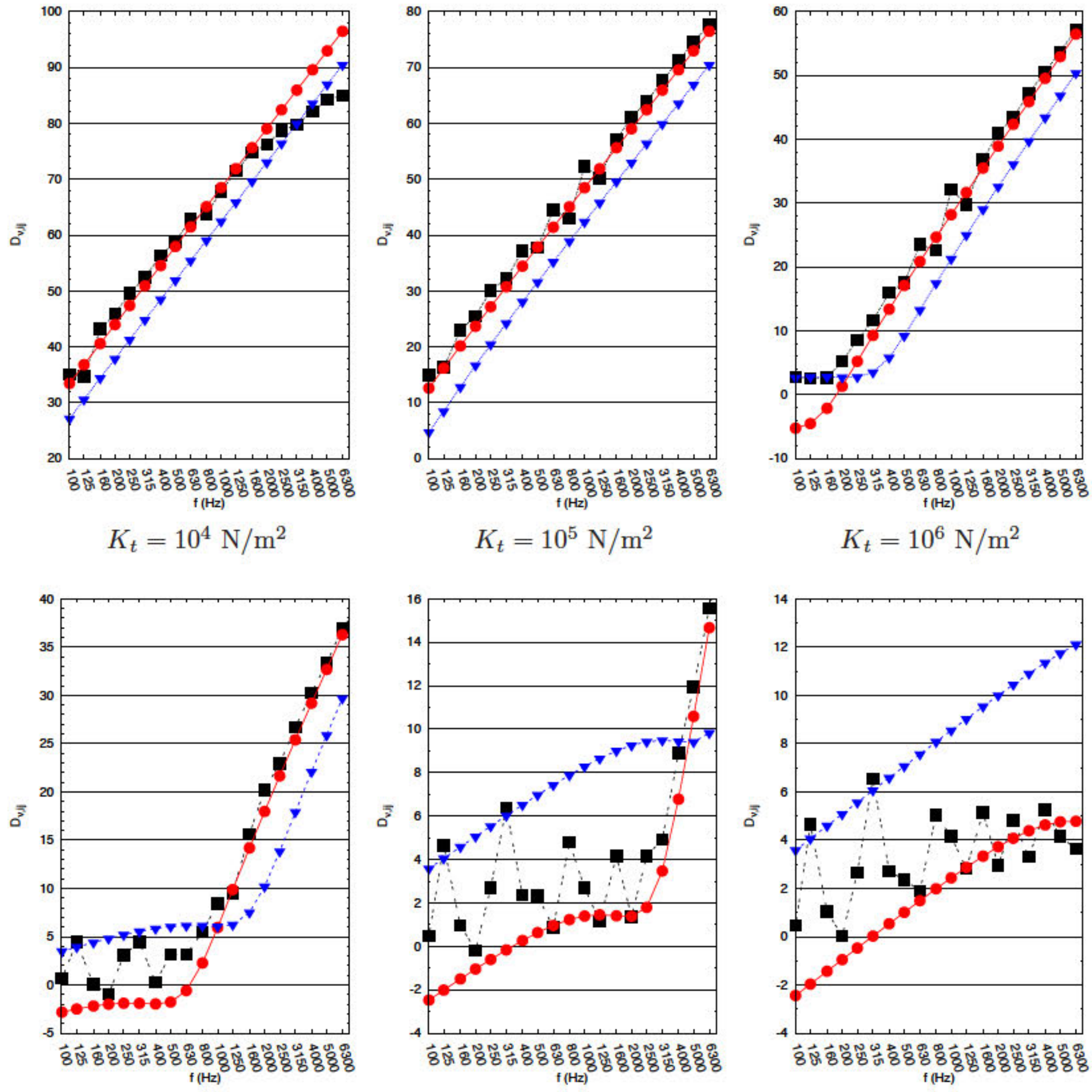
The modal behaviour at low frequencies is affected by the value of the stiffness. For the smaller values of K_t, the two leaves are weakly connected and vibrations develop all along the span length (3 m). Nevertheless, for larger values of K_t the link between leaves becomes stronger, and each leave cannot be considered as a 3 m long structure. Due to the connections it behaves like a group of short cells (space between springs). Oscillations in the response curve are then important for frequencies below 400 Hz ($K_t = 10^7$ N/m^2), 2000 Hz ($K_t = 10^8$ N/m^2) and 4500 Hz ($K_t = 10^9$ N/m^2).

Results presented here are also important in order to understand the type of laws obtained for the translational and rotational stiffnesses. The values of $D_{\nu,ij}$ for some of the studied sections have a small variation range. See for example the variation of O, TC and S studs in Fig. 6(b), where the values of $D_{\nu,ij}$ are between 15 and 35 dB. On the contrary, the variation range of $D_{\nu,ij}$ for some cases of constant spring stiffness is very large (see Fig. 12, $D_{\nu,ij} \in [25, 90]$ for $K_t = 10^4$ N/m^2, $D_{\nu,ij} \in [10, 80]$ for $K_t = 10^5$ N/m^2, $D_{\nu,ij} \in [0, 60]$ for $K_t = 10^6$ N/m$^2 \ldots$). This means that the studied sections behave like a spring of variable, rather than constant, stiffness. If the required stiffness was constant, the variation of $D_{\nu,ij}$ obtained with the model considering the geometry of the stud would be larger. This is not the case.

The same method has been used for the detailed model, see Fig. 3(a). In this case, two different errors are possible. On the one hand the agreement between a SEA model and a numerical model (shown with the previous example). On the other hand, the correct characterisation of steel studs ($K_t - f$ frequency dependent behaviour).

The results for AWS and TC studs are presented in Fig. 13. Again the vibration level differences calculated by means of a numerical model and by means of the SEA model are compared. Now, for the case of the SEA model, the value of stiffness is variable with frequency. The laws obtained in Section 4.3 have been used as input data.

This example shows how the double wall behaviour predicted by a model that considers the geometrical detail of the studs can be reproduced by means of a SEA



$K_t = 10^4$ N/m^2 $K_t = 10^5$ N/m^2 $K_t = 10^6$ N/m^2

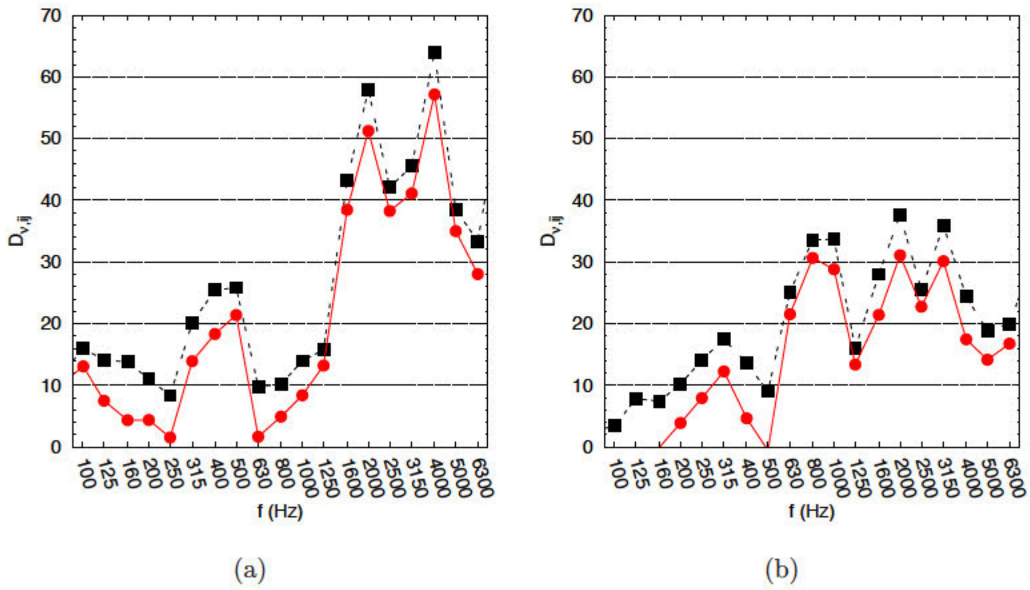
$K_t = 10^7$ N/m^2 $K_t = 10^8$ N/m^2 $K_t = 10^9$ N/m^2

In all figures: —■— is SFEM; —●— is SEA infinite; —▼— is SEA semi-infinite.

Figure 12: Two leaves connected with springs. Comparison of the vibration level difference obtained by means of a numerical model (SFEM) and statistical energy analysis (SEA). Note the different scale for $D_{\nu,ij}$.

model, where the geometrical complexity has been reduced to the use of a frequency-dependent stiffness law.

As done in Section 4.2, the frequency response can be explained by means of a modal analysis of the structure, see Fig. 14. The general tendency is an increase of the vibration level difference of the eigenfunctions with frequency. Note also that the oscillations observed for the detailed model (i.e. studs connecting elements, cf. Fig. 7) do not occur for the simplified model (i.e. springs as connecting elements).



In both figures: - - ■ - - is SFEM; —●— is SEA infinite.

Figure 13: Comparison of vibration level difference obtained by *i)* a detailed numerical model where the actual geometry of the studs is discretised (SFEM) and *ii)* a SEA model, which uses a frequency-dependent stiffness provided by the numerical model: (a) acoustic stud AWS; (b) standard stud TC.

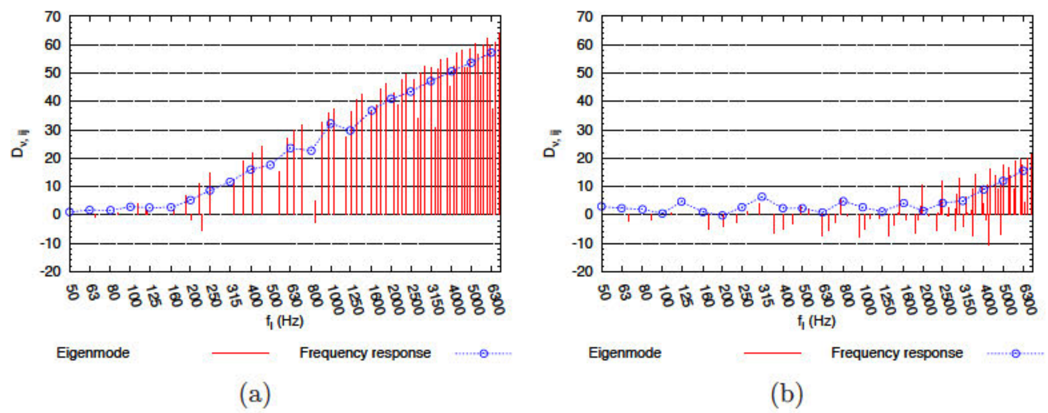


Figure 14: Comparison of the vibration level difference between leaves of a double wall with spring connecting the two leaves (analysis of the frequency response) with the vibration level difference of the more contributive eigenfunctions: (a) $K_t = 10^6$ N/m^2; (b) $K_t = 10^8$ N/m^2.

4.5 Global response of double walls

In previous sections the effort has been focused on the characterisation of flexible steel studs and the study of the vibration transmission path (vibration level difference). It is only one of the parts of the problem of sound transmission. The performance of the studs and the validity of results obtained in Section 4.3 is now verified in a two-

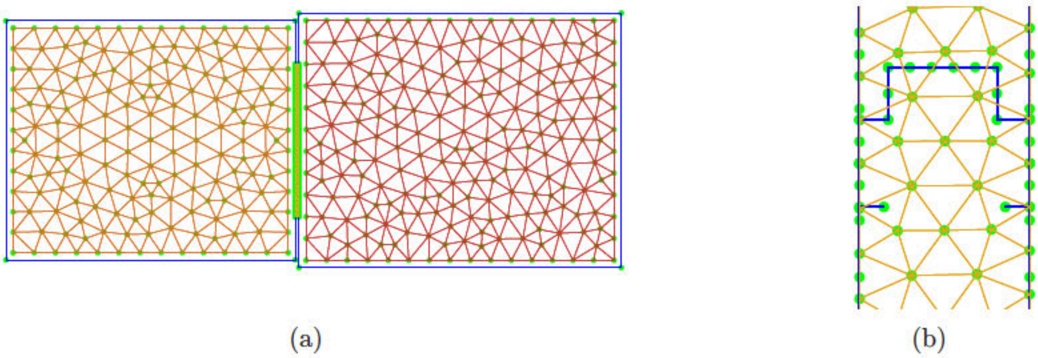


Figure 15: Finite element mesh used to solve the vibroacoustic problem: (a) general view of the two rooms and the double wall; (b) detail in the double wall zone.

dimensional vibroacoustic problem. This means that both the stud path and the cavity path are considered at the same time, see Fig. 1.

The relevance of the acoustic design of studs depends on the type of double wall. If the cavity path is very insulating, the stud path will be the critical path and then the type of stud used is very important. On the contrary, if the isolation of the cavity path is poor, the type of stud used will not be relevant because most of the sound is not transmitted through the stud.

The effect of mechanical connections between leaves has been studied in [12]. The main conclusion is that there are limit values of stiffness below and above which the effect of the stud is not important. These limit values depend on the type of double wall (air space between leaves, use or not of absorbing material, type of leaves). If the modifications in the stud (shape, thickness, materials, damping) can change its stiffness in this frequency range the optimisation is possible, otherwise the type of stud used is not an important variable of the problem.

The model presented in [12] has been used. The studs have been discretised, see the finite element mesh of Fig. 15. Two rectangular acoustic domains, the two leaves connected by means of a steel stud and the cavity between leaves can also be seen. The dimensions of the rooms are 5.7 m $\times$ 4.7 m and 6.35 m $\times$ 5 m. The double wall is 3 m long. For some cases absorbing material (resistivity $\sigma = 8000\,\mathrm{Pa{\cdot}s/m^2}$) is placed inside the air cavity. The absorbing material has been modelled by means of the equivalent fluid model proposed in [34]. The cavity has been considered continuous through the studs. The opposite situation where small cavities between the studs are modelled instead of a large single cavity can also be reproduced with the numerical model. However, the thermal slots in the studs web establish a continuity in the air between leaves that justifies the use of a single air cavity. In addition, [5] concludes that 'the error of letting the cavity field passing through the beams probably is minor'. A sound source has been placed in the lower left corner of the sending room (separated 0.5 m from each wall). The value of impedance in the absorbing contours of the acoustic cavity is $Z/\left(\rho_0 c\right) = 19.03$ (absorption $\alpha = 20\%$).

In Fig. 16 the sound reduction index for the 70 mm thick double wall with and without absorbing material in the cavity can be seen. Several values of translational

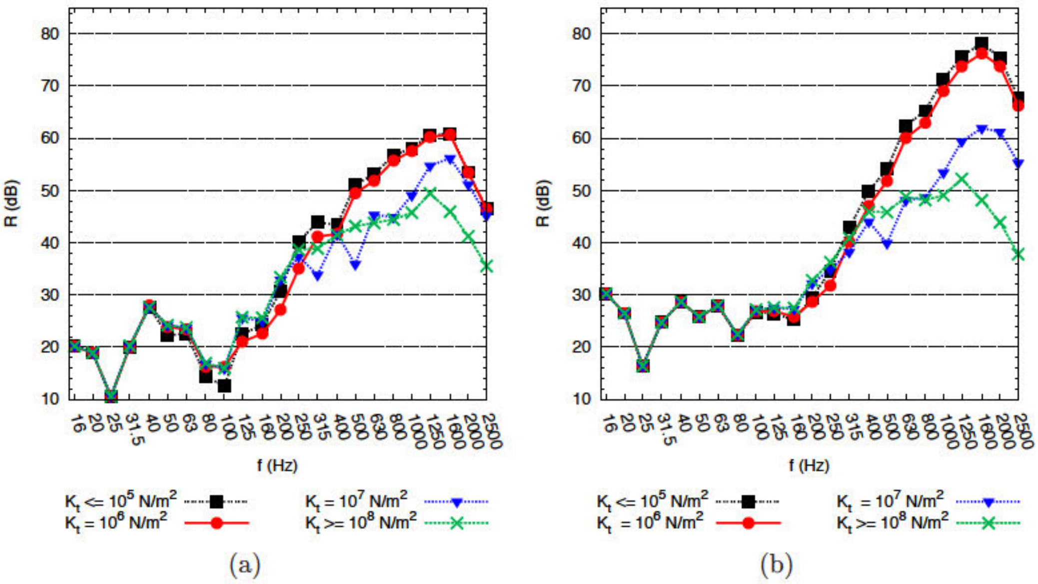


Figure 16: Influence of the value of translational stiffness K_t ($K_\theta = 0$). Sound reduction index R: (a) double wall with air cavity; (b) double wall with absorbing material. Thickness of double wall: $d = 70$ mm.

stiffness have been considered. The limit values for this double wall are 10^5 N/m^2 and 10^8 N/m^2. It can be seen that the improvement by the use of flexible studs is larger if there is absorbing material in the cavity (high cavity path isolation) than in the case of air cavity (poor cavity path isolation). This improvement is found in the high-frequency range.

Fig. 17 shows the results obtained from the vibroacoustic model of sound transmission where: *i)* the leaves are connected with springs of constant value (like in the simplified model); *ii)* the leaves are connected by means of studs (like in the detailed model).

A set of TC studs with increasing value of thickness (0.47 mm, 1 mm and 3 mm) have been considered. The translational stiffness laws obtained by means of the methodology proposed in Section 3.1 are shown in Fig. 17(a). In Fig. 17(b) the sound reduction index of double walls with leaves connected by means of springs of constant stiffness or by means of the TC studs is presented.

The results for the studs and the springs are in good agreement. As an example, consider the case of the 1 mm thick TC stud. The calculated values of translational stiffness go from 10^7 N/m^2 to 10^8 N/m^2, see Fig. 17(a). The 1 mm thick stud curve in Fig. 17(b) shows how the stud behaves as a spring of frequency-dependent stiffness as predicted in Fig. 17(a).

5 Concluding remarks

The main conclusions of the paper can be summarised as follows:

- Studs can be successfully modelled as a translational spring with a frequency-dependent averaged response corrected stiffness. It is not necessary to use both

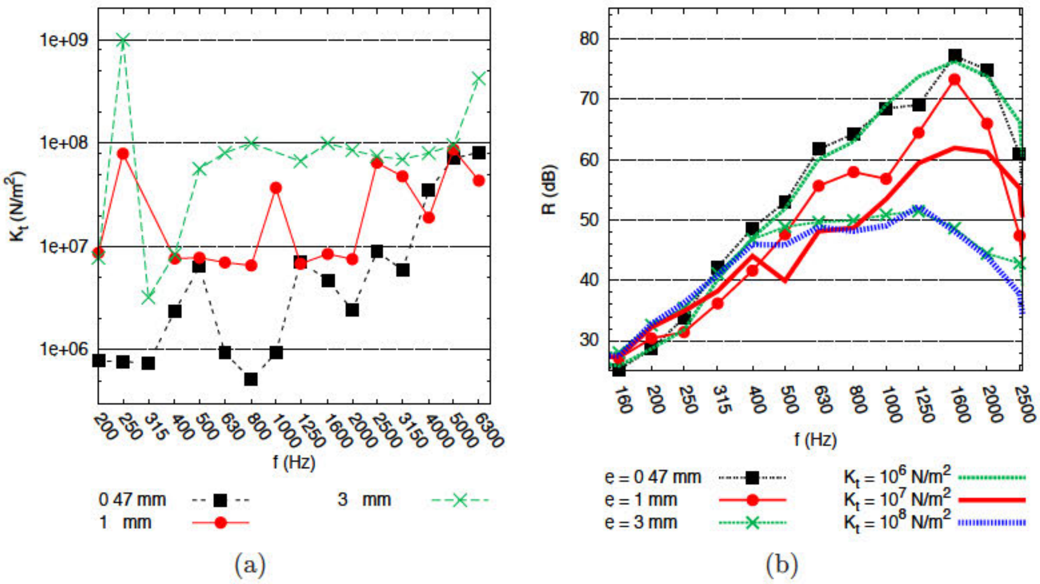


Figure 17: Influence of the thickness e of the studs: (a) $K_t - f$ laws for a set of TC studs whose thickness is: e $= 0.47$ mm, e $= 1$ mm, e $= 3$ mm; (b) sound reduction index of a double wall with TC-studs (e $= 0.47$ mm, e $= 1$ mm, e $= 3$ mm) and with springs of constant stiffness ($K_t = 10^6 \mathrm{N/m}^2$, $K_t = 10^7 \mathrm{N/m}^2$ and $K_t = 10^8 \mathrm{N/m}^2$).

a translational and a rotational spring in order to match the vibration level difference between leaves provided by the stud.

- The steps to follow in order to determine the frequency-dependent averaged response corrected stiffness law for each stud are: *i)* solve the detailed model; *ii)* solve the simplified model with translational springs; *iii)* identify, for each frequency, the translational stiffness K_t that provides the same vibration level difference for the two models.

- For each new stud geometry, it is necessary to solve the two models (simplified and detailed). Note, however, that these are purely structural problems (i.e. no acoustics) and that the resulting frequency-dependent stiffness law can be used as input data for SEA or numerical simulations of the full vibroacoustic problem.

- According to our numerical simulations, the stud shape is not relevant at low frequencies but has a larger influence at higher frequencies. Two reasons explain the improvement in the isolation of vibrations of acoustic studs. On the one hand, they are more flexible due to their cross-section shape. However, this intuitive increase of flexibility is only true around the eigenfrequencies where the central part of the stud acts as a spring, see Fig. 18(a). On the other hand, acoustic studs have a larger perimeter. This represents an increase of the effective damping in the vibration transmission path from the upper to the lower parts of the stud. This is mainly relevant for high frequencies where the length of vibration waves is smaller than a characteristic length of the stud. The displacements are small but fast and dissipation is more relevant. This is

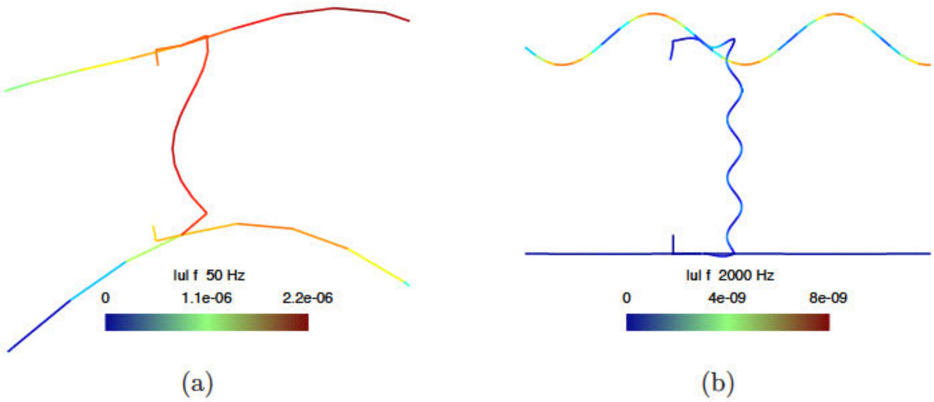


Figure 18: Different vibration shapes of a TC stud placed between leaves (zoom of the entire package): (a) $f = 50$ Hz, the stud vibrates by separating its upper and lower parts like an spring; (b) $f = 2000$ Hz, the vibration wave length is shorter than the stud. The colour indicates the amount of vibration.

the case of Fig. 18(b). The increase of the vibration isolation in this frequency range could also be achieved by means of some damping mechanism in the stud (i.e. use of materials with high damping).

- There is an important variability of the vibration level difference between leaves with frequency. This can be explained by means of a modal analysis.

- There is a range of stiffness where the stud shape affects the overall response. By considering realistic designs of double walls (air space between leaves, material parameters,...), the stiffness of the considered steel studs fall into this range. This means that in usual double wall typologies, improving the stud design will lead to and increase of the sound insulation.

- The cavity path (i.e. air space between double wall leaves, type of absorbing material) and the stud path must be controlled simultaneously.

Acknowledgements

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