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Abstract.--Reliable radar observations and some of the generally unreliable optical observations of Mercury are shown to be consistent with its rotating in a direct fashion with a period just two-thirds of its orbital period. This possibility may be understood as a consequence of the combined solar torques exerted on tidal deformations and on a permanent asymmetry in Mercury's equatorial plane, as suggested by Colombo. A simple model illustrating this superharmonic resonance phenomenon is developed in some detail; several alternative paths by which Mercury could have reached its present state of motion are discussed briefly.

Author

Introduction

Radar observations of the planets have uncovered some startling facts. In the forefront of these are the discoveries that Venus has a retrograde rotation with a period of 247 ± 5 days (Carpenter, 1964; Goldstein, 1964; and Shapiro, 1964) and that Mercury's rotation, while direct, has a period of 59 ± 5 days (Pettengill and Dyce, 1965). These radar data consist of delay-doppler maps of the planetary surfaces (Pettengill and Shapiro, 1965) and seem to be beyond question. Solar-system theorists are forced to find an explanation.

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Our main purpose here is to present a preliminary, semiquantitative examination of a recent proposal made by Colombo (1965) to explain Mercury's rotational period. He noticed that this period might be exactly two-thirds of the orbital period[3] and conjectured that such a rotational motion might be stable under the influence of solar torques, provided that there is a sufficiently large difference between the two permanent principal moments of inertia lying in Mercury's equatorial plane (i.e., provided that Mercury's inertia ellipsoid deviates sufficiently from rotational symmetry). Previously, in presenting their exclusively tidal explanation of Mercury's present rotation, Peale and Gold (1965) explicitly discounted the possibility that a permanent deviation from axial symmetry could lead to any stable rotational period other than 88 days.

Before setting up a simple model with which to study this superharmonic resonance condition, we shall discuss the optical observations of Mercury's surface. We originally examined these data for two reasons: 1) to determine whether or not they are consistent with the radar result, and 2) to determine whether, if consistent, they allow a more precise value of the rotation period to be deduced. Our final decision to include this discussion was based more on its historical interest.

Following our preliminary analysis of the model of Mercury's interaction with the sun, we outline several evolutionary paths that Mercury may have followed and discuss means of distinguishing between them based on further study of the dynamics of its rotational motion.

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[3] Note that two-thirds of the 87.9693-day sidereal orbital period is approximately 58.65 days.

Analysis of optical data

Because of Mercury's small size, low reflectivity, and close proximity to the Sun, its markings are very difficult to observe telescopically and are even more difficult to photograph. The optical observations have consequently not been noted for their reliability.

At the start of the 19th century, Schröter claimed to have seen mountains rising to 20 km on the planet, and he produced drawings of the surface. Bessel (1813) deduced from these drawings a rotation period of 24 hr 0 min 53 sec, with the axis being inclined 70° to the orbital plane. Although some astronomers remained skeptical, many found it especially appealing aesthetically to think of Mercury, like Mars, as having approximately the same length of day as the Earth. This "fact" was not finally discredited until the 1880's when Schiaparelli's extended series of observations (Schiaparelli, 1889) convinced almost everyone that Mercury was rotating slowly. He himself actually concluded that the rotation was uniform with a period equal to the orbital period of 88 days. From that time until this spring all observations were interpreted as being consistent with the 88-day rotational period. Danjon (1924) concluded unequivocally, but illogically, that Mercury's rotation period was 88 days; and Antoniadi (1934) stated even more positively that this period was beyond question. The details of Antoniadi's proof, however, were omitted; only a table indicating the number of times he had observed each of a large number of surface markings was included. More recently, Dollfus (1953), in comparing Schiaparelli's map with his own, concluded that Mercury's rotational period equalled its orbital period " with a precision greater than one part in ten thousand" since the features of each map coincided to "within 10° of hermocentric longitude," and since the maps were separated by

"53 years or 220 revolutions of Mercury." It is hard to assess the meaning of this statement for several reasons: 1) the placing of features on the map, at least in Schiaparelli's case, presupposed an 88-day rotational period; and 2) whereas Dollfus's map was based on Lyot's visual and photographic observations of 1924, Schiaparelli's was based mainly on observations, over several years, in the early 1880's (although his map was published only in 1889, i.e., 53 years before Lyot's observations).

We have examined much of the original data, starting with those in Schiaparelli's 1889 paper. The 88-day period was there based primarily on observations of a particularly prominent surface feature, labelled q. These observations were made during 6 sets of intervals in 1882 and 1883. Taken literally, Schiaparelli's tabular summary of results is definitely not consistent with a 59-day period. Sets of his observations of q were separated by approximately 1 synodic period (116 days). Hence, were q visible at both ends of this interval, on the basis of an assumed 88-day rotational period, it could not have been seen at the "proper" place were the period 59 days; the difference in the change in angular position on the 2 assumptions is about 125° after a synodic period. We are forced to conclude that on at least some days of observation, Schiaparelli mistakenly thought he had identified the same feature seen at other times. Nonetheless, Schiaparelli must be given full credit for successfully exploding the myth of rapid rotation.

Schiaparelli's 1889 planispheric-map drawing of the only hemisphere of Mercury presumably illuminated by the Sun if libration is ignored, apparently exerted a strong influence on subsequent observers; e.g., Dollfus (1953) did not consider the later maps of Rudaux (1928) and Antoniadi (1934) to be independent of Schiaparelli's. Lowell's map (1897) is almost universally appraised as being more a product of his imagination than of the actual surface of Mercury. Despite these indications of mutual distrust among observers and of the probable unreliability of the data, we studied the dated drawings of the appearance of Mercury's surface that were published by Antoniadi (1934), by Danjon (1924), and by Dollfus (1953). We paid especially careful attention to the sets of drawings made over short time spans so as to infer the rotational period unambiguously (granted the slow-rotation hypothesis) although inaccurately. We also thought, at first, that the reliability of the identification of the same surface features by the observer might be greater for observations made closer together in time. Even taking the drawings quite literally, we found difficulty in convincing ourselves of the proper identification of the same feature on 2 that were separated by more than about 10 days. We found the results of measurements on the relative location of "identical" features that moved through the region of least foreshortening to be somewhat more consistent with the 59-day period than with the 88-day period. A typical result was a period of 70 ± 15 days, which we obtained from the drawings of 6 October and 19 October 1950 (Dollfus, 1953). In principle, a more accurate determination of the period can be made by working with drawings widely separated in time. But obstacles still remain. Not only is there the same problem of reliably associating markings, but also the wider the time separation of two observations the greater is the ambiguity in interpreting the results. We do not know _a priori_ how many complete revolutions intervened between the two observations. Rather than computing the whole set of possible solutions for each pair of drawings, we confined ourselves to those within the range of ambiguity allowed by the radar data. Given two observations and the assumption that Mercury's rotation axis is approximately perpendicular to its orbital plane, the direct rotation period P in days may be expressed as

$$P = \frac{\Delta t}{n + \frac{\Delta \Phi}{360}} \quad , \qquad (1)$$

where Δt in days is the elapsed time between the observations, n is zero or a positive integer, and $\Delta \Phi$ denotes the hermocentric sidereal longitude of the Earth at the time of the later observation minus the corresponding quantity for the earlier observation if the location of the feature being compared has the same relation to the subearth point on Mercury in both cases. If the relative location is otherwise, $\Delta\Phi$ must be adjusted accordingly. For the drawings dated 29 March 1912 and 2 May 1923 (Danjon, 1924), we find possible solutions at $P = 57.0, 57.8, 58.6, 59.5, 60.4$ days, etc., with the uncertainty in each being about $\pm$ 0.15 days. The drawings of 27 September 1927 and 23 August 1929 (Antoniadi, 1934) yield $P = 50.0, 53.9, 58.4, 63.7$ days, etc., with a probable error of $\pm$ 0.4 days. Those of 22 July 1942 and 12 October 1950 (Dollfus, 1953) lead to $P = 56.4, 57.5, 58.7, 59.8$ days, etc., with a probable error of $\pm$ 0.1 days. We compared the 1923 and 1927 drawings to check for consistency in the "epoch" and found $P = 54.9, 56.9, 58.9$ days, etc., with a probable error of $\pm$ 0.3 days. (Even if the same features could be discerned on each, the comparing of Antoniadi's and Dollfus's drawings would not be too useful because the intervals between possible solutions would be very small in the region of interest.) In all cases, the errors quoted are inversely proportional to Δt and are based on the inaccuracy in determining the relative hermocentric sidereal longitude of the common feature selected from a pair of drawings. No attempt was made to include the possible systematic error caused by observer bias.

These results appear to be not significantly inconsistent with Mercury's rotation period being 58.65 days. We might even suppose that, except for Schiaparelli's, almost all optical observations were made at intervals corresponding in essence to even multiples of Mercury's orbital period and hence to integral multiples of 58.65 days. The extended series of observations made by Antoniadi and Lyot seem to rule out this possibility, and the facts that prominent features on the one hemisphere were never noted as being in the "wrong" location and that the other hemisphere of Mercury was never recognized as such provide powerful evidence for considering all drawings of Mercury's surface to be suspect. Prudence probably demands that we look to future rather than to past observations for a reliable reduction in the present uncertainty given by the radar determination of Mercury's rotational period.[4]

A simple model of Sun-Mercury interactions

Mercury's physical constitution and shape are too poorly known to allow us to formulate an accurate model of the solar interactions that affect its rotational motion. Moreover, since our main goal here is merely to demonstrate the stability of a rotational period equal to two-thirds of the orbital period, we shall confine our analysis to a very simple model. First of all, we shall consider Mercury's angular velocity vector to be normal to its orbital plane so that we may restrict ourselves to a two-dimensional problem. We assume that the total torque exerted on Mercury by the Sun is composed of two parts: a tidal torque and a torque caused by Mercury's lack of axial symmetry. How shall we represent these terms analytically? Consider the tidal term first. The theory of a linear, slightly damped oscillator shows the response to lag in phase behind the forcing function by an amount proportional to the damping coefficient. In our case, the analog of the damping coefficient may depend on the strength and frequency of the forcing function as well as on the structural and compositional properties of the planet. For simplicity, we shall take the phase lag to be a constant angle. Since it is a lag, its sign will of course depend on the sign of $\dot{\theta} - \dot{v}$, where $\dot{\theta}$ is the inertial (direct) rotational angular velocity of Mercury about its center of mass, and $\dot{v}$ is the orbital angular velocity. For $\dot{\theta} > \dot{v}$, the lag angle will be positive (i.e., will increase in a counterclockwise direction from the Mercury-Sun line when one looks down on the orbital plane from the north). We must still model the body distortion or tidal bulge on Mercury. We may argue that the strain

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[4] McGovern *et al.* (1965) also reanalyzed the optical observations of Mercury's surface, but reached the stronger conclusion that these data show the rotation period to be 58.4 ± 0.4 days.

induced in the planet is directly proportional to the small differential stress caused by the difference in solar gravity at different distances from the Sun. The motion of this strained configuration depends again on solar gravity, thus giving rise to the oft-quoted result that the tidal torque is inversely proportional to the sixth power of the distance from the primary (in this case, the Sun). The differential gravitational attraction is proportional to the inverse cube of the distance and so both the stress and the torque on the distorted configuration are individually proportional to the inverse cube. We may therefore approximate the tidal torque $\vec{T}_t$ by the expression

$$\vec{T}_t = -\frac{\eta}{r^6} \operatorname{sgn}(\dot{\theta} - \dot{v})\, \hat{k} \quad , \tag{2}$$

where $\hat{k}$ is a unit vector normal to the orbital plane and directed generally north, η is a positive constant, and r is the Mercury-Sun distance.

We may for simplicity treat the permanent axial asymmetry of Mercury as a dipole in the equatorial plane, superposed on an otherwise spherically symmetric planet. If the dipole consists of two points each of mass m and each a distance d from the planet's center of mass, with the line joining them passing through the center of mass, then the resultant solar torque $\vec{T}_{pd}$ is easily shown to be

$$\vec{T}_{pd} = -\frac{3GM_\odot(B - A)\sin 2(\theta - v)}{2r^3}\left[1 + O\left(\frac{d}{r}\right)\right]\vec{k} \quad , \tag{3}$$

where G is the gravitational constant, $M_\odot$ the mass of the Sun, θ the orientation of the dipole line with respect to the apsidal line of the orbit, v the orbital true anomaly, and $B - A = 2md^2$ the difference between the maximum and minimum moments of inertia lying in the planet's equatorial plane. The axis of minimum moment of inertia obviously lies along the dipole line. A more realistic model of a planet with an elliptical equator yields the same result as eq. (3).

There will of course be coupling between Mercury's rotational and orbital motion. However, even if Mercury's rotational period had once been only one day, its total angular momentum of rotation would have been less than 1 part in 10^7 of its present orbital angular momentum. We are therefore safe in neglecting the effects on orbital motion. Mercury also raises a tide on the Sun and the resultant interaction will affect Mercury's orbital motion. A preliminary estimate indicates that this effect too may be neglected. But the orbital motion is affected significantly by the perturbing effects of other planets. As shown by Brouwer and Clemence (1961), Mercury's eccentricity e apparently undergoes a long-period oscillation between the values of 0.11 and 0.24; its apsidal line rotates with a period of about 2.2×10^5 years. For the moment, we neglect these variations as well and write the decoupled equation of motion for rotational motion as

$$I \ddot{\theta} = - \frac{\eta}{r^6} \operatorname{sgn} (\dot{\theta} - \dot{v}) - \frac{3GM_\odot(B - A)}{2r^3} \sin 2 (\theta - v) \quad , \tag{4}$$

where the orbital parameters are presumed constant and where I is the moment of inertia about the $\hat{k}$ axis. Consolidating constants we find

$$\ddot{\theta} = - \frac{\alpha}{r^6} \operatorname{sgn} (\dot{\theta} - \dot{v}) - \frac{\beta}{r^3} \sin 2 (\theta - v) \quad . \tag{5}$$

The order of magnitude of the important ratio α/β will be considered later.

For the rotational motion to be stable, we require that the average total torque vanish, and that if Mercury's orientation and frequency of rotation are perturbed then the resultant torque tends to oppose the perturbations. Since α and β are very small, we may replace the right side of eq. (5) by its average value over an orbital period, keeping $\dot{\theta}$ and the orbital elements constant. A somewhat tedious calculation shows that

$$\frac{1}{\tau}\int_0^\tau dt\left[-\frac{\alpha}{r^6}\,\mathrm{sgn}\,(\dot{\theta}-\dot{v})\right] = -\frac{\alpha}{a^6(1-e^2)^{9/2}}\begin{cases} -\left[1+3e^2+\frac{3}{8}e^4\right] & ;\ \dot{\theta} \le \frac{\mu^{1/2}}{p^{3/2}}[1-e]^2 \\ \frac{1}{2\pi}\left[2(\pi-2v_c)-16e\sin v_c\right] & \\ +6e^2\left[\pi-2v_c-\sin 2v_c\right] & \\ -\frac{16}{3}e^3\left[3\sin v_c-\sin^3 v_c\right] & \\ +\frac{e^4}{8}\left[6(\pi-2v_c)-8\sin 2v_c-\sin 4v_c\right] & ; \\ \qquad \frac{\mu^{1/2}}{p^{3/2}}[1-e]^2 \le \dot{\theta} \le \frac{\mu^{1/2}}{p^{3/2}}[1+e]^2 & \\ \left[1+3e^2+\frac{3}{8}e^4\right] & ;\ \dot{\theta} \ge \frac{\mu^{1/2}}{p^{3/2}}[1+e]^2 \end{cases} \tag{6}$$

where

$$\left.\begin{aligned} \cos v_c &= \frac{1}{e}\left[\left(\frac{\dot{\theta}\,p^{3/2}}{\mu^{1/2}}\right)^{1/2} - 1\right] \\ \sin v_c &= \frac{1}{e}\left[e^2 + 2\left(\frac{\dot{\theta}\,p^{3/2}}{\mu^{1/2}}\right)^{1/2} - 1 - \left(\frac{\dot{\theta}\,p^{3/2}}{\mu^{1/2}}\right)\right]^{1/2} \end{aligned}\right\} , \tag{7}$$

and where μ denotes $GM_{\odot}$, p the semilatus rectum, a the semimajor axis, and τ the period of Mercury's orbit. If $\dot{\theta} \le \frac{\mu^{1/2}}{p^{3/2}} [1 - e]^2$, which corresponds to a direct rotation with period $\ge$ 132 days and to any retrograde rotation, the tidal torque will be positive throughout the orbit; if $\dot{\theta} \ge \frac{\mu^{1/2}}{p^{3/2}} [1 + e]^2$, which corresponds to a direct rotation with period $\le$ 56 days, the tidal torque will be negative throughout the orbit. For intermediate values of $\dot{\theta}$, the torque changes sign when $v = v_c$ and when $v = 2\pi - v_c$.

A similarly tedious calculation leads to

$$\frac{1}{\tau}\int_0^\tau dt \left[-\frac{\beta}{r^3} \sin 2(\theta - v)\right] = -\frac{\beta}{4\pi p^3}\left[\cos\left(2\theta_0 + 4\pi\frac{\dot{\theta}}{n}\right) - \cos 2\theta_0\right]$$

$$\cdot \left[-\frac{1}{\left(\frac{\dot{\theta}}{n} - 1\right)} + \frac{e}{2}\left\{\frac{1}{\left(\frac{\dot{\theta}}{n} - \frac{1}{2}\right)} - \frac{7}{\left(\frac{\dot{\theta}}{n} - \frac{3}{2}\right)}\right\} + \frac{e^2}{2}\left\{\frac{11}{\left(\frac{\dot{\theta}}{n} - 1\right)} - \frac{17}{\left(\frac{\dot{\theta}}{n} - 2\right)}\right\} + O(e^3)\right] , \qquad (8)$$

where $n = \mu^{1/2}/a^{3/2}$ is Mercury's mean motion and θ_0 is the initial orientation of the equatorial axis of minimum moment of inertia with respect to the apsidal line at perihelion. Equation (8) exhibits the expected zero-order resonance at $\dot{\theta} = n$; in first order in e, we find a resonance at $\dot{\theta} = n/2$ and a surprisingly strong resonance at $\dot{\theta} = 3n/2$, corresponding to the 58.65-day rotation period. In second order in e, we find resonances only at $\dot{\theta} = n$ and $\dot{\theta} = 2n$. In the limit as $\dot{\theta}/n$ approaches an integral or half-integral value, it is easy to show that

$$\cos\left(2\theta_0 + 4\pi\frac{\dot{\theta}}{n}\right) - \cos 2\theta_0 \xrightarrow[\frac{\dot{\theta}}{n} \to \frac{m}{2} \pm \delta]{} \mp 4\pi\,\delta \sin 2\theta_0 + O(\delta^2) \quad , \qquad (9)$$

where $m = 0,1,\ldots,$ and $\delta \ll 1$. Hence the average gravitational torque is a maximum when the equatorial axis of minimum moment of inertia is oriented at 45° to the apsidal line at the time Mercury passes through perigee. It is clear that for nonresonant values of $\dot{\theta}/n$, the gravitational torque tends to average out over sufficiently long time intervals. The effective width of a gravitational resonance depends on the length of time over which it is valid to consider $\dot{\theta}$ constant. In Figure 1 we have plotted the right sides of eqs. (6) and (8); the latter is shown for $\theta_0 = \pm 45^\circ$ with the positive values of the average torque corresponding to $\theta_0 = -45^\circ$, except for the resonance at $\frac{\dot{\theta}}{n} = \frac{1}{2}$, for which $\overline{T}_{pd}$ has the opposite sign. The resonance widths have been drawn only schematically. Figures 2 and 3 show the corresponding curves for $e = 0.15$ and $e = 0.25$, respectively. All ordinate scales are arbitrary; in particular the ordinates for $\overline{T}_t$ and for $\overline{T}_{pd}$ are not drawn to the same scale. For $\overline{T}_t$ only the coefficient of $\frac{\alpha}{a^6(1 - e^2)^{9/2}}$ is shown, whereas for $\overline{T}_{pd}$ the coefficient of $\frac{\beta}{4p^3}$ is given.

It is clear that if α/β is sufficiently small, there will be some value of $\theta_0 (0 \le \theta_0 \le 180^\circ)$ for which the average total torque will be zero when $\dot{\theta} = 3n/2$. As an indication of the actual value of α/β, we note that were the fractional difference in Mercury's equatorial moments of inertia identical to that of the Moon (i.e., 2×10^{-4}), then

$$\beta \approx 10^{-2} \quad , \qquad (10)$$

when time is expressed in units of Mercury's orbital period and distances are expressed in units of its semimajor axis. To obtain a crude estimate of α, we set $\overline{T}_t$ equal to $\frac{1}{\dot{\theta}} \overline{\frac{dE}{dt}}$, where $\overline{\frac{dE}{dt}}$ is the approximate expression given by Munk and MacDonald (1960) for the tidal energy dissipated into heat by the Earth, but with parameter values appropriate for the Mercury-Sun case substituted for those suitable for the Earth-Moon system. For a phase lag of 2.5×10^{-3} rad, which corresponds approximately to the Q of the Earth's interior (Munk and MacDonald, 1960), we find

$$\alpha \approx 2 \times 10^{-7} \tag{11}$$

in the same units as were used to express β. Even were the fractional difference in Mercury's equatorial moments of inertia much smaller than the difference for the Moon, and even were the tidal torque far greater than estimated above, we would find that a very small negative value of θ_0 would suffice to make the total average torque vanish at $\frac{\dot{\theta}}{n} = \frac{3}{2}$. That such an orientation and frequency will indeed be stable is illustrated in Figure 4, where we show the results of a numerical integration of eq. (5). The values of θ and $\dot{\theta}$ are presented at successive perigee passages. (In order to show the convergence to a stable point in the phase plane without excessive use of computer time, we increased the value of α to 10^{-5}.) The tendency to converge toward the stable point at $\theta \approx \frac{3}{2}$ nt - 0.01 rev, $\dot{\theta} \approx \frac{3}{2}$ n + 0.001 rev/orb. per. is apparent.

We reserve for another report an analytical demonstration of this stability. There we will also discuss the possibility of achieving stability at, for example, $\dot{\theta} = n$ and at $\dot{\theta} = 2n$ for various values of e. A qualitative indication of the "favored" position of $\dot{\theta} = \frac{3}{2}$ n can be gleaned from the relative strengths of $\overline{T}_{pd}$, especially for e = 0.25.

For comparison, we show in Figures 5 and 6 the orientations of the axis of minimum moment of inertia at different orbital positions for $\theta_0 = 0$, $\dot{\theta} = \frac{3}{2}n$, and $\theta_0 = 0$, $\dot{\theta} = n$, respectively. Note that for $\dot{\theta} = \frac{3}{2}n$, the axis of minimum moment of inertia is aligned very closely with the Mercury-Sun line throughout the region of strong interaction near perihelion, thus giving a geometric indication of the relatively large magnitude of $\overline{T}_{pd}$ at $\dot{\theta} = \frac{3}{2}n$. That is, for a relatively small value of θ_0, the torque $\vec{T}_{pd}$ will maintain the same sign throughout this region.

Problems for future study

We may conclude at least that the observations and our theoretical analysis are consistent with Mercury's present rotation being stable with a period equal to two-thirds of its orbital period. Many questions, of course, remain; we shall consider some of these briefly in the context of two possible evolutions of Mercury's state of motion. Without delving into the problem of planet formation, we may start with the following possible initial configurations: 1) Mercury orbiting about the Sun and rotating in a direct fashion with a period on the order of 10 hr; and 2) Mercury moving in a retrograde orbit about Venus (Shapiro, 1965). For configuration 1) we must show that enough time has been available for solar tidal friction (or other devices) to have reduced the rotational angular velocity to its present value. If we assume that e has always varied between 0.1 and 0.25 and that the effective Q of Mercury is the same as that of Earth, then 5×10^9 yr appears to be a sufficient interval. For the Earth, however, most of the energy loss seems to be through land-water friction which is most likely not true for Mercury. Light reflected from Mercury yields a polarization curve very similar to the Moon's (Antoniadi, 1934) and lends support to the supposition that its surface may be of the same porous, but solid structure. Recent temperature measurements also seem consistent with the surface being solid. It therefore appears more reasonable to take for the effective Q of Mercury a value corresponding more closely to that for the interior of the Earth as was done in evaluating α above. The age of the solar system then only provides barely enough time for tidal dissipation to account for the present rotational speed. For the first configuration we must also explain the origin of the permanent equatorial deformation. Another question is: "If Mercury's rotation is now stable with $\dot{\theta} \approx \frac{3}{2}$ n why were earlier states (e.g., $\dot{\theta} \approx 2n$) not stable?"

Given the second initial configuration, we may envision that tidal interaction eventually resulted in both Mercury and Venus constantly presenting the same face to each other, thus accounting for Venus's retrograde rotation. But one must then find a plausible model of the Sun's capture of Mercury and of the latter's evolution into its present orbit, without violating order of magnitude estimates of the energies and angular momenta involved. At least an axial asymmetry of Mercury's inertia ellipsoid, comparable to that of the Moon, would be more easily explained on the basis of this initial configuration. We must, of course, still show how Mercury's rotation evolved to its present value from an initial post-Sun-capture retrograde motion, i.e., we must show, for example, that its angular velocity would not have remained near the value $\dot{\theta} = n$.

Depending on the outcome, a stability analysis of $\dot{\theta} = n$ and of $\dot{\theta} = 2n$ may enable us to distinguish between these two evolutionary paths; at least, we will be able to place limits on the values that α and β may have possessed at various stages of the evolution. Future radar determinations of Mercury's equatorial ellipticity and its orientation will also help place bounds on α and β. Knowledge of the actual direction of Mercury's rotational angular velocity vector might be useful to establish the present stage of evolution of the rotational motion in comparison with the Moon's (i.e., to establish the proximity to the stable minimum of the precession angle of the angular velocity vector).

Although the conclusions concerning the amplitude of oscillation of e may be in error, the rotation of the apsidal line can hardly be in doubt. We must, therefore, demonstrate that for the stable rotation the axis of minimum moment of inertia maintains its orientation at perihelion as the latter rotates. Any change in eccentricity will alter the balance of $\overline{T}_{pd}$ and $\overline{T}_{p}$ in virtue of their different radial dependencies. The effect on stability of such changes must be investigated; in particular the possibility of the rotational angular velocity evolving from being near one harmonic of n to being near another must be considered. But if α was always on the order of 10^{-7}, then the time scale of evolution of Mercury's rotational motion might be sufficiently long for us to average over the oscillations in e caused by planetary perturbations.

We intend to return to many of these problems in a later study.

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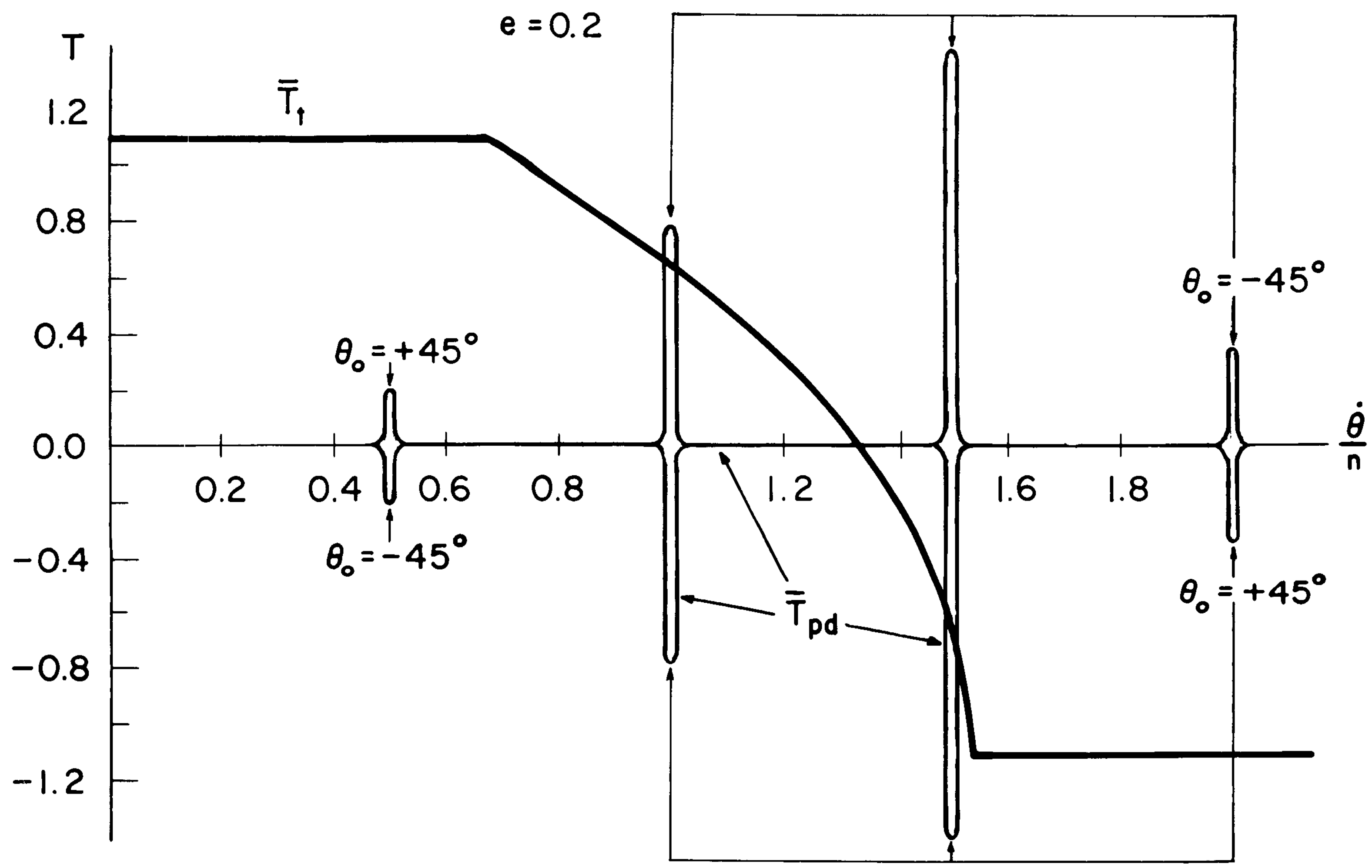

Figure 1.--Curves of $\overline{T}_t$ and $\overline{T}_{pd}$ versus $\frac{\dot{\theta}}{n}$ for e = 0.2. Two curves are shown for $\overline{T}_{pd}$, one with $\theta_0 = 45°$ and the other with $\theta_0 = -45°$. The positive values of $\overline{T}_{pd}$ correspond to $\theta_0 = -45°$, except for $\frac{\dot{\theta}}{n} = \frac{1}{2}$ where the negative values correspond to this value of θ_0. The ordinate scale is arbitrary and, in particular, the ordinates for $\overline{T}_t$ and $\overline{T}_{pd}$ are not drawn to the same scale (see text).

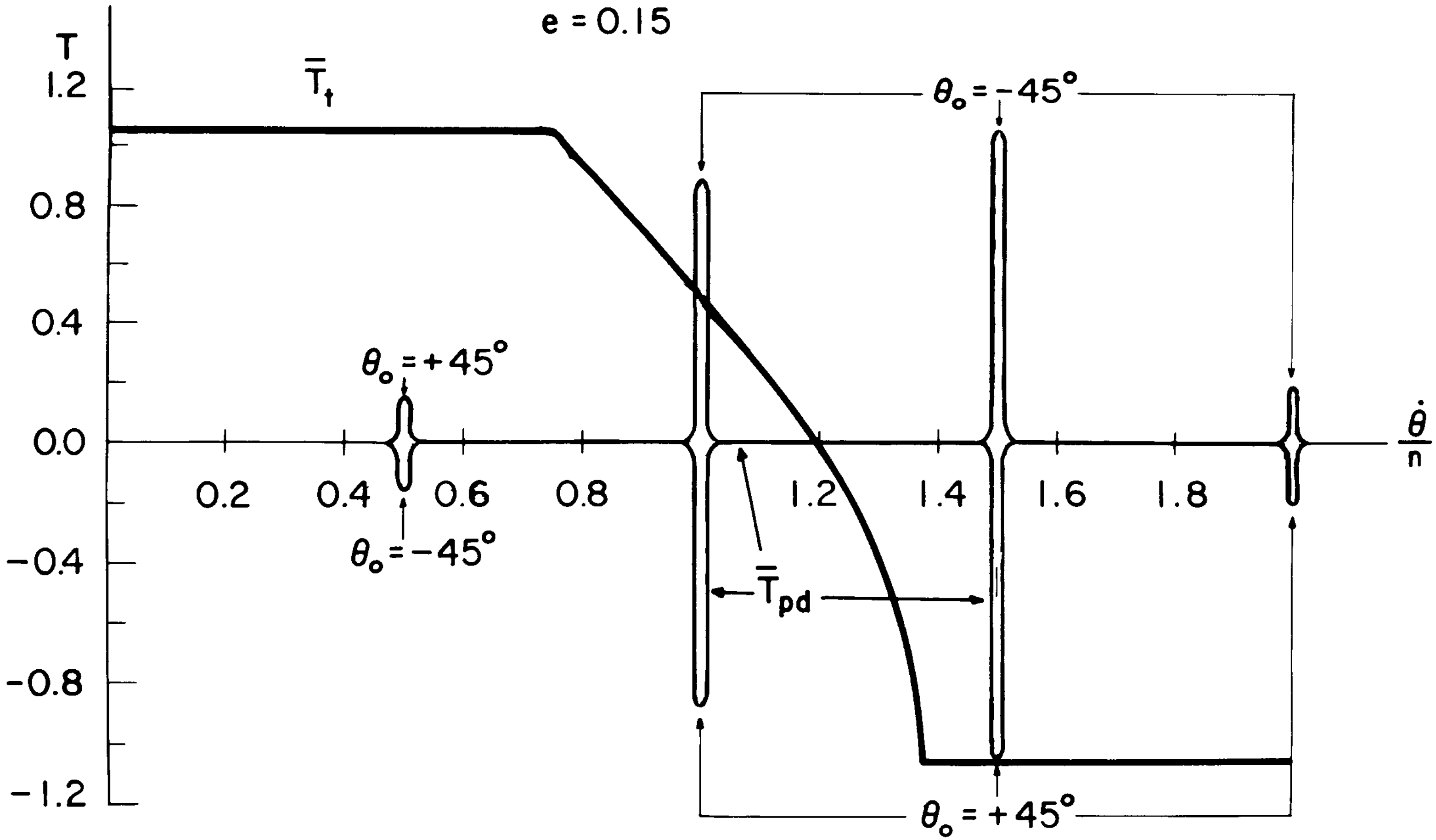

Figure 2.--Curves of $\overline{T}_t$ and $\overline{T}_{pd}$ versus $\frac{\dot{\theta}}{n}$ for e = 0.15. Two curves are shown for $\overline{T}_{pd}$, one with $\theta_0 = 45°$ and the other with $\theta_0 = -45°$. The positive values of $\overline{T}_{pd}$ correspond to $\theta_0 = -45°$, except for $\frac{\dot{\theta}}{n} = \frac{1}{2}$ where the negative values correspond to this value of θ_0. The ordinate scale is arbitrary and, in particular, the ordinates for $\overline{T}_t$ and $\overline{T}_{pd}$ are not drawn to the same scale (see text).

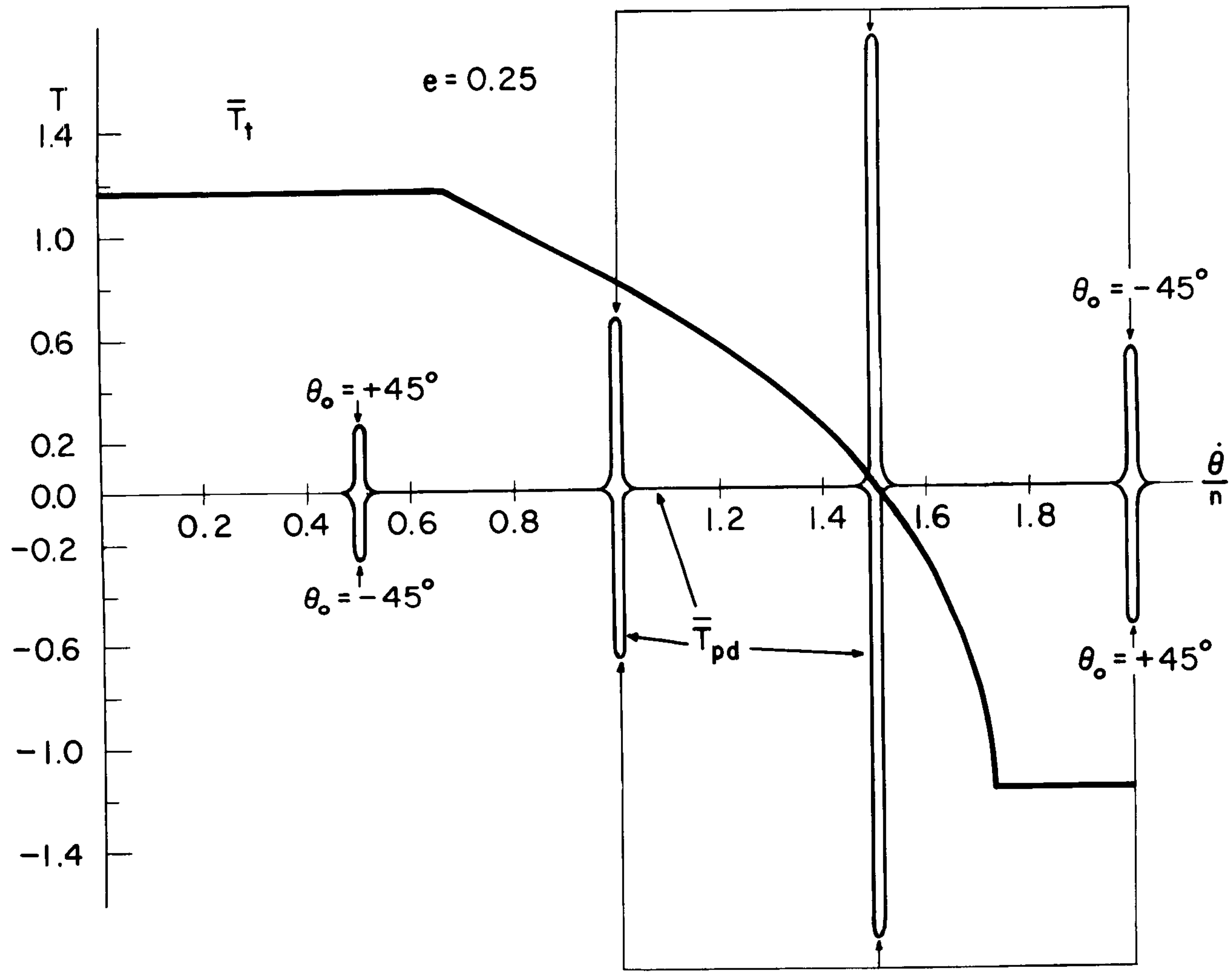

Figure 3.--Curves of $\overline{T}_t$ and $\overline{T}_{pd}$ versus $\frac{\dot{\theta}}{n}$ for e = 0.25. Two curves are shown for $\overline{T}_{pd}$, one with $\theta_0 = 45°$ and the other with $\theta_0 = -45°$. The positive values of $\overline{T}_{pd}$ correspond to $\theta_0 = -45°$, except for $\frac{\dot{\theta}}{n} = \frac{1}{2}$ where the negative values correspond to this value of θ_0. The ordinate scale is arbitrary and, in particular, the ordinates for $\overline{T}_t$ and $\overline{T}_{pd}$ are not drawn to the same scale (see text).

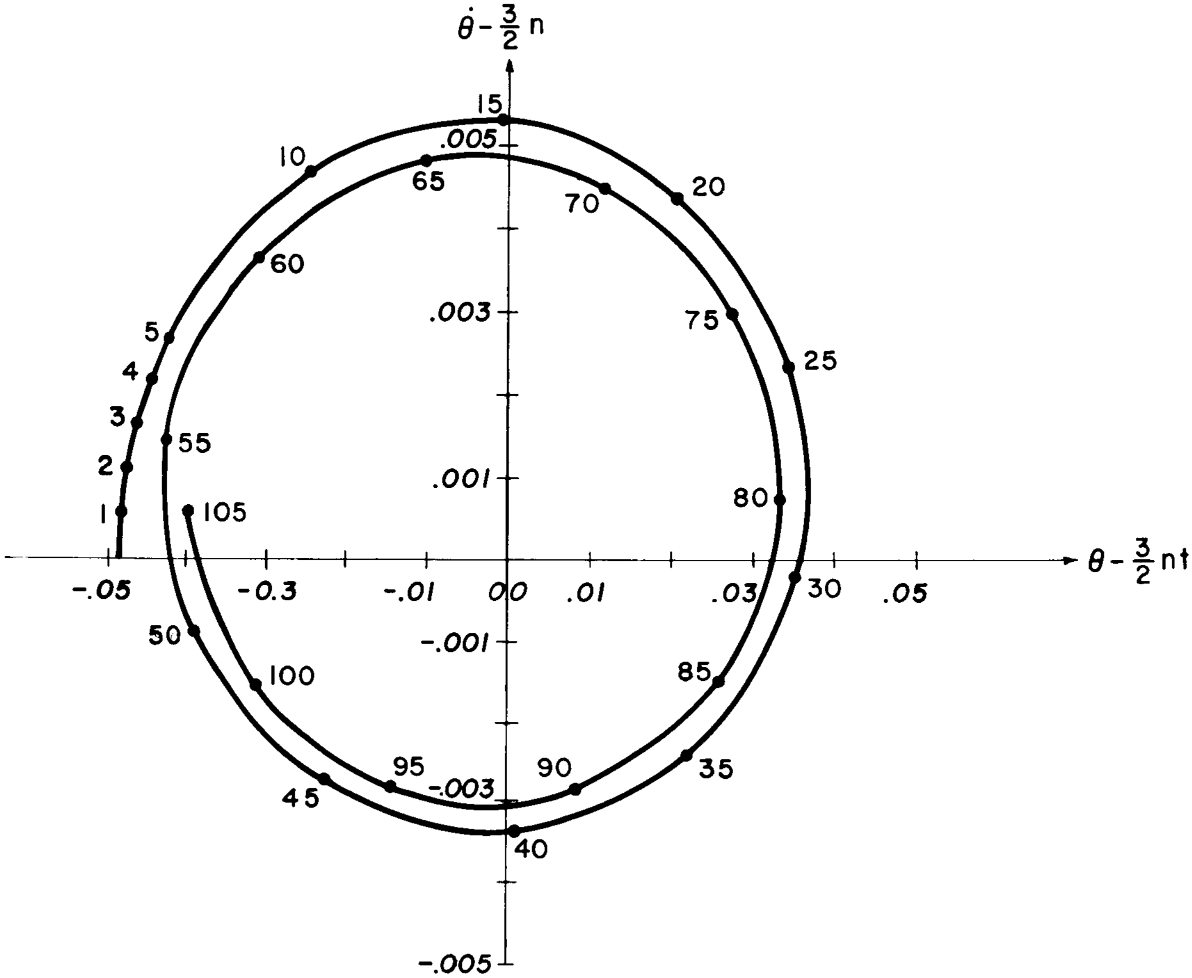

Figure 4.--An indication of the stability of Mercury's rotation for $\frac{\dot{\theta}}{n} = \frac{3}{2}$ obtained from a numerical integration of eq. (5). The points indicate the coordinates in phase space of the axis of minimum moment of inertia at the particular perihelion passage indicated by the accompanying number. A continuous curve was drawn through these points solely to emphasize the trend toward convergence.

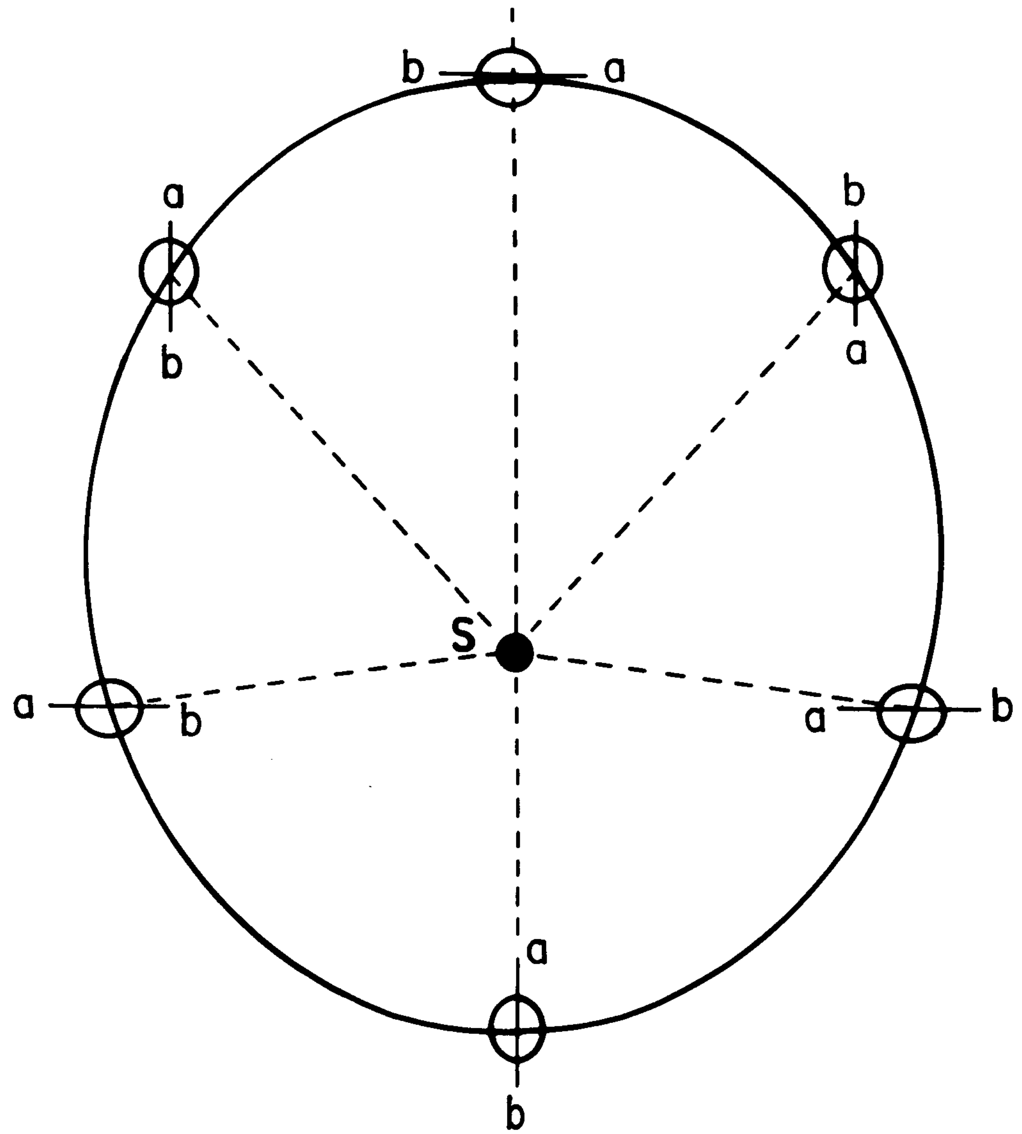

Figure 5.--Orientation of Mercury's axis of minimum moment of inertia at various points along its orbit for $\dot{\theta} = \frac{3}{2} n$, $\theta_0 = 0$.

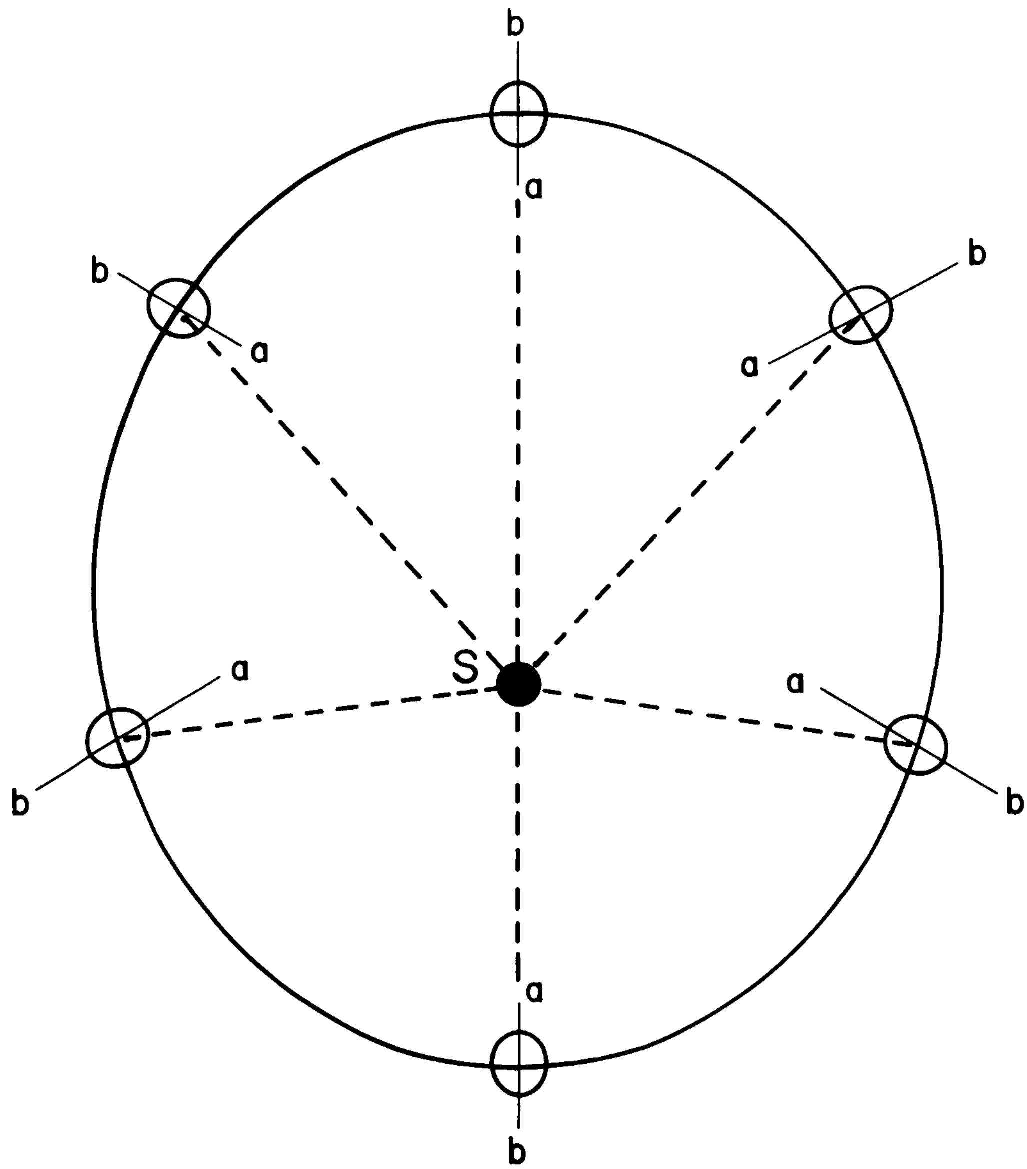

Figure 6.--Orientation of Mercury's axis of minimum moment of inertia at various points along its orbit for $\dot{\theta} = n$, $\theta_0 = 0$.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

First issued to ensure the immediate dissemination of data for satellite tracking, the reports have continued to provide a rapid distribution of catalogs of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals. The Reports are also used extensively for the rapid publication of preliminary or special results in other fields of astrophysics.

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