KOCHI UNIVERSITY OF TECHNOLOGY

Social Design Engineering Series SDES-2015-13

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19th March, 2015

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The Sandwich Property in the Voluntary Contribution Mechanism:
The Instability Approach

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March 11, 2015

Abstract

This paper proposes an alternative explanation for the sandwich property in voluntary contribution mechanism experiments. This property refers to the phenomenon of experimental data being "sandwiched" between a Nash equilibrium above the midpoint of the endowment and a Nash equilibrium below this midpoint. The explanation is in terms of the instability of the system with best response dynamics, i.e., "pulsing" behaviors, in nonlinear environments rather than the quantal response equilibrium analysis. Since most experimental models are unstable in quasilinear environments (where the utility function is linear in a private good and nonlinear in a public good), and Cobb–Douglas environments, using equilibrium analysis is problematic.

JEL classification codes: C62, C72, C92, H41

* The author would like to thank participants at the Wednesday workshop at the Research Center for Social Design Engineering, Kochitech, on August 6, 2014. Of course, all remaining errors are the author's.

1. Introduction

A system in which players voluntarily contribute a part of their endowments for the provision of a public good is called a voluntary contribution mechanism (VCM). Even if non-contribution is the dominant strategy when all players have linear utility functions, researchers have observed significantly positive contributions, which gradually decrease toward the end of sessions (for a survey, see Ledyard (1995)). On the other hand, even if contributing the entire endowment is the dominant strategy, this is rarely observed.[1] The deviations from "rational" strategies have been one of the central themes in the provision of public goods. Experimental economists have been attributed these observations to characteristics such as altruism, warm-glow, equity, and reciprocity (for a survey, see Chaudhuri (2011)).

A few experimental economists started investigating properties of the VCM with *nonlinear* utility functions. With nonlinearity, rational strategies are in the interior rather than on the boundaries (for a survey, see Laury and Holt (2008)). Innovators such as Sefton and Steinberg (1996), Isaac and Walker (1998) and Laury, Walker, and Williams (1999) used quasilinear utility functions that are linear in player *i*'s private good consumption x_i and nonlinear in a public good *y*, that is, $u(x_i, y) = x_i + t(y)$.[2] The rationale behind this formulation is that the private good is money, and hence its marginal utility is constant, while the marginal utility of the public good decreases, so that the $t(y)$ part is nonlinear.

One of their findings is the *sandwich* property named by Anderson, Goeree, and Holt (1998). Isaac and Walker (1998) ingeniously designed three treatments using quasilinear utility functions: one in which the symmetric Nash equilibrium is below the midpoint of the endowment, a second in which it is exactly at the midpoint, and a third in which it is above the midpoint. If the equilibrium is below the midpoint, over investment occurs relative to the Nash equilibrium; if it is at the midpoint, the data are also clustered around the midpoint; and if it is above the midpoint, under investment occurs. That is, the Nash equilibria sandwich the data. The data from Sefton and Steinberg (1996), and Laury, Walker, and Williams (1999) also support part of the sandwich property (that over investment occurs when the Nash equilibrium is below the midpoint of the endowment). While the sandwich result was originally found for a linear environment, these authors' findings confirmed that the result holds for a quasilinear environment as well.

Anderson et al. (1998) explained the sandwich property using "errors." They showed that each player has a normal density function which peaks at the symmetric Nash equilibrium. If

[1] See Saijo and Nakamura (1995), Brunton, Hasan, and Mestelman (2001), and Chun, Kim, and Saijo (2011) for experiments in which contributing the entire endowment is the dominant strategy.

[2] Although Isaac, McCue, and Plott (1985) and Isaac and Walker (1991) used quasilinear utility functions, zero contribution was the dominant strategy due to their choice of parameters.

the equilibrium is below the midpoint, the weight on bigger than equilibrium contributions is higher than the weight on contributions smaller than the equilibrium amount, since the density function should be truncated at zero contributions. That is, the average contribution should be more than the equilibrium contribution. The same argument can be applied to cases where the equilibrium is "at" or "above" the midpoint.

There has been related work using Cobb–Douglas (CD) utility functions. This includes Andreoni (1993), Chan, Mestelman, Moir, and Muller (1996), Cason, Saijo, and Yamato (2002), and Sutter and Weck-Hannemann (2004). An advantage of using this type of utility function is the avoidance of multiple Nash equilibria in the VCM under quasilinear utility functions. That is, the Nash equilibrium is unique with CD utility functions. The design of all these papers falls into the "below" category (the Nash equilibrium was below the midpoint of the endowment), but the data are very close to the Nash equilibrium. This difference between the quasilinear and the Cobb–Douglas environments has created a new puzzle.[3]

Recently, Saijo (2014) has shown, using best response dynamics, that the VCM is unstable when utility functions are quasilinear[4]. It is also unstable with Cobb–Douglas utility functions, given some mild conditions on the slope of the best response function and the number of players. This is quite a contrast to the linear utility case, where the existence of a dominant strategy guarantees stability. In this respect, the designs of all the studies mentioned above except for Cason, Saijo, and Yamato (2002) fall into this unstable category. If they are unstable, using equilibrium analysis is problematic.

The current paper uses this instability approach to explain the puzzling facts noted above. If the system is unstable, players' choices of strategies tend to pulse. The average contribution with pulsing behavior is a key factor in explaining the data. Consider the quasilinear case. If the symmetric Nash equilibrium is below the midpoint of the endowment, the average contribution with pulsing behavior is always larger than the equilibrium. If it is at the midpoint, the average contribution is the same as the equilibrium contribution. If it is above the midpoint, the average contribution is smaller than the equilibrium contribution. On the other hand, in all experiments with Cobb–Douglas utility functions, the average contribution is the same as the equilibrium contribution. That is, the instability approach provides an answer to the puzzle of why the sandwich property may not hold for Cobb–Douglas environments, while holding for quasilinear environments.

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[3] Another type of nonlinear utility function is nonlinear with respect to the public good and linear with respect to the private good. I will consider the role of this type in the conclusion.

[4] See also Saijo and Kobayashi (2014).

The organization of the paper is as follows. I consider the model with best response dynamics and quasilinear utility functions in section 2. Section 3 considers Cobb–Douglas utility functions. Section 4 suggests avenues for further research.

2. The model with best responses: the quasilinear case

Let x be a private good and y be a public good. The production function of the public good is $y = f(x) = x$. That is, for example, one hour of labor input produces one meter of road. All players have the same endowment w, and each must decide to divide w into their own consumption of private good x_i and contribution s_i to the public good. That is, $y = \sum_1^n s_j$ where n is the number of players and n is at least two. This system is called the voluntary contribution mechanism (VCM).

Following Isaac and Walker (IW) (1998), I suppose that each player has the same quasilinear utility function $u(x_i, y) = x_i + t(y)$. Let $s_{-i} = \sum_{j \neq i} s_j$ and $u(w - s_i, s_i + s_{-i}) = v(s_i, s_{-i})$. Then a list of contributions $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_n)$ is a *Nash equilibrium* if for all i, $v(\hat{s}_i, \hat{s}_{-i}) \geq v(s_i, \hat{s}_{-i})$ for all $s_i \in [0, w]$. Define the best response function as

$$r(s_{-i}) = \arg\max_{s_i} \{ v(s_i, s_{-i}) \mid s_i \in [0, w] \}$$

. Since v and w are the same for all players, all players have the same best response function. Since $v(s_i, s_{-i}) = w - s_i + t(s_i + s_{-i})$, the first order condition is $\partial v / \partial s_i = -1 + t'(s_i + s_{-i}) = 0$. That is, $s_i = -s_{-i} + t'^{-1}(1)$ for all i which shows that $\left\{ (\hat{s}_1, \ldots, \hat{s}_n) : t'^{-1}(1) = \sum s_j \text{ and } w \geq s_j \geq 0 \right\}$ is the set of Nash equilibria. IW prepared the following three best response functions.

(1-1) $s_i = -s_{-i} + 48$, (1-2) $s_i = -s_{-i} + 124$, and (1-3) $s_i = -s_{-i} + 200$.

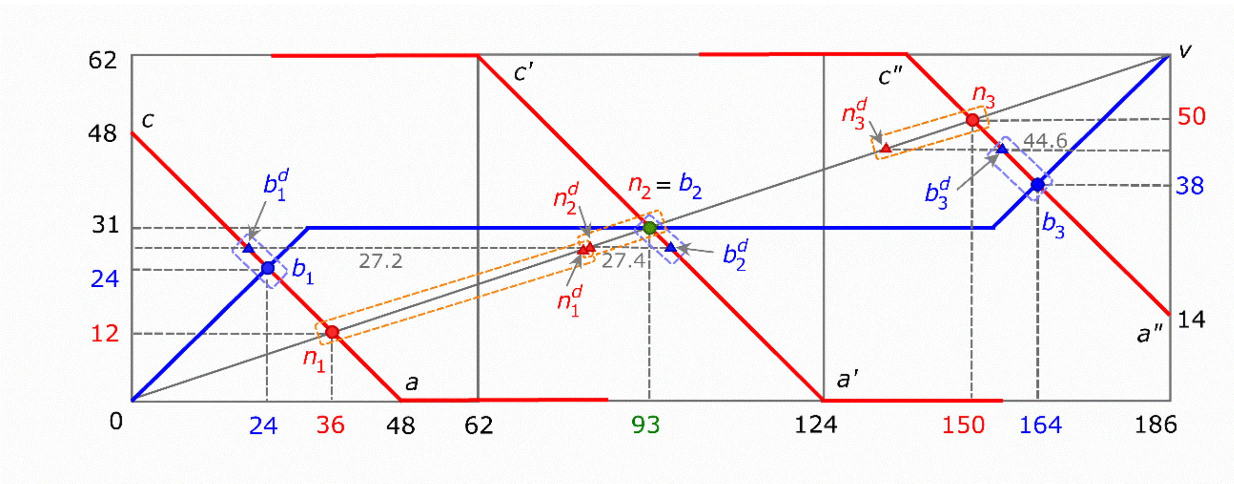

Figure 1. Best response functions from Isaac and Walker (1998).

Consider Figure 1. The horizontal axis represents the sum of the other players' contributions and the vertical axis represents the contribution of player *i*'s response to s_{-i}. Since w = 62, and n = 4 in IW, the three best response functions in Figure 1 are *a-c, a′-c′*, and *a″-c″*. Consider (1-1). If $s_{-i} > 48$, player *i*'s contribution s_i should be zero, so that the best response function has a kink at *a* on the horizontal axis. Consider (1-2). Since player *i* cannot contribute more than w, the best response function has a kink at *c′*. If $s_{-i} > 124$, player *i*'s contribution s_i should be zero, so that the best response function has a kink at *a′*. Consider (1-3). Since player *i* cannot contribute more than w, the best response function has a kink at *c″*. Since $s_{-i} > 186 = 62 \times 3$ is impossible, player *i*'s best response function stops at *a″*.

Although multiple Nash equilibria exist in these cases, there is a symmetric Nash equilibrium for each case. The slope of line 0-v is 1/3 and hence the symmetric Nash equilibrium is at the intersection of the best response line and 0-v. That is, n_1, n_2, and n_3 are symmetric Nash equilibria for (1-1), (1-2), and (1-3) respectively.

Let us consider a simple dynamic process for the VCM. At time *t*, let player *i*'s choice of contribution be s_i^t. Suppose that player *i* chooses $r(s_{-i}^t)$ at time *t*+1, where *t* = 1, 2,… That is, I assume that every player chooses the best response to the sum of strategies chosen by the other players at time *t*. Then the system of simultaneous difference equations $s_i^{t+1} = r(s_{-i}^t)$ ($i = 1,2,\cdots,n$) is *locally stable* at the Nash equilibrium $\hat{s}$ if the system $s_i^{t+1} = r'(\hat{s}_{-i})s_{-i}^t + k_i$ ($i = 1,2,\cdots,n$) is stable, where $k_i = \hat{s}_i - r'(\hat{s}_{-i})\hat{s}_{-i}$. That is, the system is a linear approximation to the original system at the Nash equilibrium.[5] Then, Saijo (2014) showed the following property.[6]

Property 1. *The system* $s_i^{t+1} = r(s_{-i}^t)$ *($i = 1,2,\cdots,n$) is unstable.*

Choose any s^1. For example, choose case (1-2) and let $s^1 = (20, 30, 40, 50)$. Then $s_-^1 = (s_{-1}^1, s_{-2}^1, s_{-3}^1, s_{-4}^1)$ = (120, 110, 100, 90). The best response to s_-^1 is s^2 = (4, 14, 24, 34) and hence s_-^2 = (72, 62, 52, 42). The best response to s_-^2 is s^2 = (52, 62, 72, 82). Then s_-^3 = (216, 206, 196, 186). The best response to s_-^3 is s^3 = (0, 0, 0, 0). Clearly, s^4 = (62, 62, 62, 62). After s^4, the system cycles between (0, 0, 0, 0) and (62, 62, 62, 62). Since 0 and 62 are the boundaries of the possible contributions, I call this cycle between the two boundaries a *boundary cycle*. I conduct a simple simulation using Mathematica with random initial values for period 1. From the data, I find the first period in which either (0, 0, 0, 0) or (62, 62, 62, 62) is realized. This is period 3 in the above example. Among 100 simulation sessions, the relevant period was period 2 for 16 sessions,

[5] As (1-1), (1-2), and (1-3) show, the best response functions are linear and hence no linear approximation is necessary.
[6] This property assumes that the utility function is quasilinear, of the form $u(x_i, y) = x_i + t(y)$. See Saijo (2014) for other cases.

period 3 for 38 sessions, period 4 for 26 sessions, period 5 for 15 sessions, period 3 for 3 sessions, period 7 for 1 session, and period 1 for the randomized first session that is a Nash equilibrium, i.e., (5, 11, 62, 46). Figure 2 is an example when the relevant period is period five. The horizontal axis denotes periods, and the vertical axis denotes best responses where the numbers in period 1 are chosen randomly, in this case (0, 24, 51, 54).

Consider the prediction of the best response dynamics and consider case (1-1). Conducting an analysis similar to that for case (1-2), I find that the contribution vector reaches either (0, 0, 0, 0) or (48, 48, 48, 48) within a few periods. Thus, the average contribution of each player should be 24 = (0+48)/2. That is, this is point b_1, the midpoint between *a* and *c* in Figure 1. Although both *a* and *c* represent best responses to the sum of the other players' contributions, the *average* best response point b_1 with *instability* does not imply that the player responds to 24. This is not the sum of the other players' contributions, which should be 72 (=24x3). That is, a player who follows *pulsing* dynamics behaves on average as if choosing a point between *a* and*c.* The deviation from b_1 can be attributed to the distribution of the weights of *a* and *c.* That is, the average from experimental data should be a convex combination of *a* and *c*. For this reason, I put b_1 on [*a, c*]. Similarly, it is b_2 for case (1-2) that is the midpoint between *a′* and *c′* and it is b_3 for case (1-3) that is the midpoint between *a″* and *c″*. The average contributions are 31 for case (1-2) and 38 for case (1-3). That is, the piecewise linear line 0-b_1-b_2-b_3-v represents the prediction of the pulsing dynamics. Notice that both Nash and pulsing dynamics predictions coincide for case (1-2), i.e., $n_2 = b_2$.

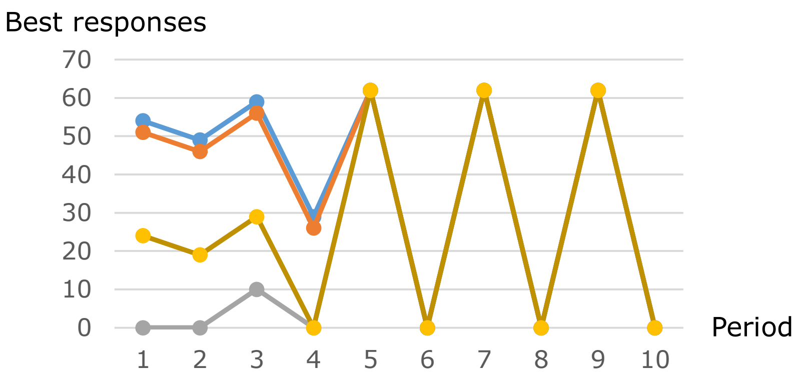

Figure 2. Best response dynamics starting from (0, 24, 51, 54) for Case (1-2).

Comparing the two lines 0-v and 0-b_1-b_2-b_3-v in Figure 1, the vertical height of b_1 is larger than that of n_1, the vertical height of b_2 is equal to the height of n_2, and the vertical height of b_3 is larger than that of n_3. That is, I have a version of the sandwich property from the viewpoint of pulsing dynamics.

Property 2. *Suppose that all players have identical quasilinear utility functions and endowments. Then* (*i*) *if the symmetric Nash equilibrium contribution is less than half of the endowment, the average contribution with pulsing dynamics is always larger than the symmetric Nash equilibrium contribution;* (*ii*) *if the symmetric Nash equilibrium contribution is equal to half of the endowment, the average contribution with pulsing dynamics is equal to the symmetric Nash equilibrium contribution; and* (*iii*) *if the symmetric Nash equilibrium contribution is more than half of the endowment, the average contribution with pulsing dynamics is always smaller than the symmetric Nash equilibrium contribution.*

Consider the data from IW. The average contributions are 27.2 for case (1-1), 27.4 for case (1-2), and 44.6 for case (1-3). Consider case (1-1) in Figure 1. The prediction of pulsing dynamics should be between *a* and *c*, and the average contribution is 27.2; this datum should be b_1^d on line *a-c* where the superscript *d* stands for datum. On the other hand, the prediction for the symmetric Nash equilibrium should be on line *0-v*, and hence n_1^d represents the Nash prediction. Similarly, b_2^d, n_2^d, b_3^d, and n_3^d are shown in Figure 1.

	$d(b_i^d, b_i)$	$d(n_i^d, n_i)$
Case (1-1)	4.5	48.1
Case (1-2)	5.1	11.4
Case (1-3)	9.3	10.2
Sum	18.9	69.7

Table 1. The distance between data and predictions.

It is useful to use the Euclidean distance to measure the gap between the data and the predictions. Let $d(x, y)$ be the Euclidean distance between *x* and *y*. Then Table 1 shows the distances in the three cases. Comparing the distances, I have the following property.

Property 3. *The average in the IW experiment is closer to the prediction using the pulsing dynamics approach than to the prediction of the symmetric Nash equilibrium, for all three cases.*

An interesting difference between the two predictions is illustrated by case (1-3). Since the average of experimental data (44.6) is smaller than the Nash equilibrium prediction (50), this suggests *under* contribution if the symmetric Nash equilibrium contribution is more than the midpoint of the endowment. On the other hand, the average (44.6) is larger than the pulsing dynamics prediction (38), which can be interpreted as *over* contribution. Although the average of the data in case (1-1), i.e., 27.2, is larger than both predictions, i.e., 12 for Nash and 24 for pulsing dynamics, the same reversal would happen if b_1^d were between n_1 and b_1.

The data of Sefton and Steinberg (1996) (who first used quasilinear utility functions in a VCM experiment) and Laury, Walker, and Williams (1999) (who confirmed the absence of an endowment effect in quasilinear environments) also support Property 3. Sefton and Steinberg (1996) use the following environment. The best response function is $s_i = -s_{-i} + 8$, $n = 4$, $w = 8$, the symmetric Nash equilibrium contribution is 2, and the symmetric Pareto efficient contribution is 7. They use ten rounds of stranger matching. Laury, Walker, and Williams (1999) use the following environment. The best response function is $s_i = -s_{-i} + 100$, $n = 5$, w is either 125 or 200, and the symmetric Nash equilibrium contribution is 20. Table 2 summarizes these experiments. All symmetric Nash equilibrium contributions are less than half of the endowments, and the average contribution is always larger than the Nash contribution. The pulsing dynamics contribution is 4 for Sefton and Steinberg (1996) and it is 50 for Laury, Walker, and Williams (1999). The distances between the data and the Nash equilibrium contributions are from seven to thirteen times larger than the distances between the data and the pulsing dynamics contributions.

	n	w	Sym. Nash	Pulsing dynamics	Average Cont.	$d(b^d, b)$	$d(n^d, n)$
Sefton-Steinberg (96)	4	8	2	4	4.42	0.59	7.66
Laury-Walker-Williams (99)*	5	125s	20	50	62.47	17.64	175.10
		200s			71.28	30.09	211.43
		125d			43.31	9.46	96.11
		200d			44.06	8.40	99.20

* w is either 125 or 200. "s" stands for summary information on the payoff structure and "d" stands for detailed information on the payoff structure.

Table 2. Experiments by Sefton and Steinberg and Laury, Walker, and Williams.

3. The Cobb–Douglas case

Andreoni (1993), Chan, Mestelman, Moir, and Muller (CMMM) (1996), Sutter and Weck-Hannemann (2004), and others used Cobb-Douglas utility functions in VCM experiments. The basic reason is that these models have a unique Nash equilibrium, which is quite a contrast to the quasilinear environment. Suppose that players have identical utility functions $u(x_i, y) = x_i^{\alpha} y^{1-\alpha}$, $\alpha \in (0,1)$ and identical endowments. Then the best response function is $s_i = -\alpha s_{-i} + (1-\alpha)w$.[7] Saijo (2014) shows the following property.

[7] Since $v(s_i, s_{-i}) = (w - s_i)^{\alpha}(s_i + s_{-i})^{1-\alpha}$, the first order condition is $\partial v / \partial s_i = -\alpha((s_i + s_{-i})/(w - s_i))^{1-\alpha}$ $+(1-\alpha)((w - s_i)/(s_i + s_{-i}))^{\alpha} = 0$. Then $(s_i + s_{-i})/(w - s_i) = (1-\alpha)/\alpha$, and hence $s_i = -\alpha s_{-i} + (1-\alpha)w$.

Property 4. *Suppose that the utility function is a Cobb–Douglas type. Then*
(*i*) *if n = 2, the system* $s_i^{t+1} = r(s_{-i}^t)$ (*i* = 1, 2,⋯,*n*) *is asymptotically stable;*[8] *and*
(*ii*) *if* $n \geq 3$ *and* $\alpha < 1/(n-1)$, *the system is asymptotically stable.*

As Table 3 shows, Andreoni (1993), CMMM (1996), and Sutter and Weck-Hannemann (2004) used *n* = 3 and $\alpha = 1/2$, and hence $\alpha = 1/(n-1)$. That is, their systems are not stable. The instability property differs from the quasilinear utility case. Consider the case with CMMM. Since *w* = 20, the best response function is $s_i = -(1/2)s_{-i} + 10$ which is *a-c* in Figure 3. The Nash equilibrium is unique and it is $(s_1, s_2, s_3) = (5,5,5)$ solving the three best response functions simultaneously. It is *n* in Figure 3, that is at the intersection of *a-c* and 0-*v*. Choose any s^1. For example, let $s^1 = (3,7,9)$. Then $s_-^1 = (s_{-1}^1, s_{-2}^1, s_{-3}^1)$ = (16, 12, 10). Since the slope of the best response is 1/2, s^2 = (2, 4, 5) and hence s_-^2 = (9, 7, 6). The best response to s_-^2 is s^2 = (5.5, 6.5, 7). Since CMMM used only integer announcements, let s^2 = (6, 7, 7), rounding the numbers. Then s_-^3 = (14, 13, 13) and s^3 = (4, 4, 4). Clearly, s^4 = (6, 6, 6). After s^4, the system cycles between (4, 4, 4) and (6, 6, 6). Since 4 and 6 are the interior points of the possible contributions, I call this cycle between two interior points an *interior cycle*. Figure 4 shows an interior cycle starting from (2, 8, 10). I conduct 100 simulation sessions using Mathematica with random initial values for period 1. Ninety-seven sessions got into interior cycles, one session got into the boundary cycle, i.e., between 0 and 10, one session was a Nash equilibrium from the beginning, and one session got into a Nash equilibrium starting from non-Nash initial values.[9]

Consider predictions using pulsing dynamics. Since the contribution vector reaches either an interior or a boundary cycle within a few periods, the average contribution of each player should be 5. The interior cycle between (4, 4, 4) and (6, 6, 6) is the cycle between b^a and b^c in Figure 3. The midpoint between b^a and b^c is *b* which is the prediction of pulsing dynamics. That is, the Nash and pulsing dynamics predictions coincide. The average contribution in CMMM is 5.3, and n^d and b^d show the location of the data based upon the Nash and pulsing dynamics approaches respectively. Then, $d(n^d, n) = d(b^d, b) = 0.58$ and the data are very close to

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Although CMMM (1996) used $v(s_i, s_{-i}) = (w - s_i) + (s_i + s_{-i}) + s_i(s_i + s_{-i})$, the best response function is the same as $s_i = -\alpha s_{-i} + (1-\alpha)w$ with $\alpha = 1/2$.

8 Hirsch and Smale (1974) give an intuitive interpretation of asymptotical stability of differential equations. "An equilibrium $\bar{x}$ is *stable* if all nearby solutions stay nearby. It is *asymptotically stable* if all nearby solutions not only stay nearby, but also tend to $\bar{x}$" (p.180).

9 Due to the integer announcements, CMMM found that (4, 5, 6) is also a Nash equilibrium. If announcements are real numbers, it is easy to show that only two cases occur. One is the case where the sequence $\{s^i\}$ converges to the Nash equilibrium (5, 5, 5) if $s_1 + s_2 + s_3 = 15$. Otherwise, the sequence gets into either interior or boundary cycles.

n and b. Other best response functions such as a'-c' and a''-c'' show that both Nash and pulsing dynamics yield the same prediction. That is, the discrepancy between 0-v and 0-b_1-b_2-b_3-v in Figure 1 is not replicated in Figure 3.

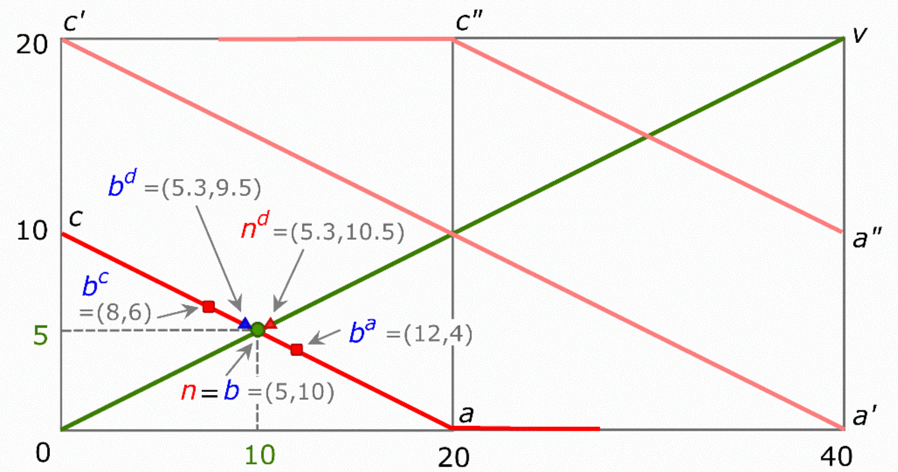

Figure 3. Best response functions from Chan, Mestelman, Moir, and Muller (1996).

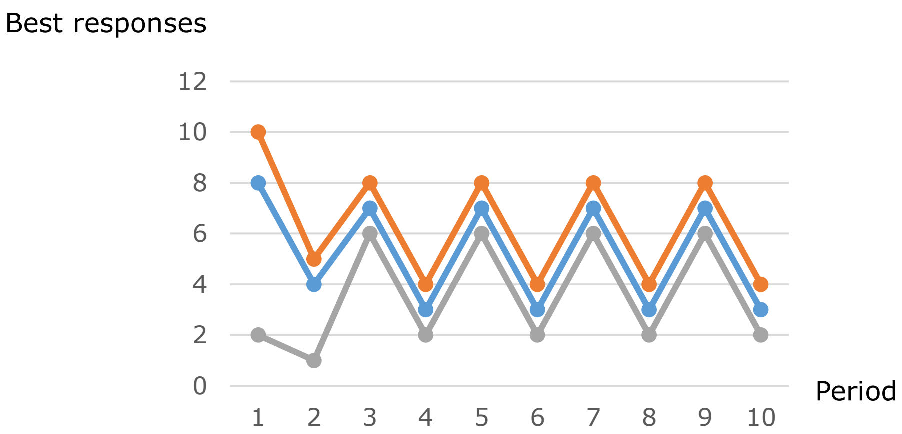

Figure 4. Best response dynamics starting from (2, 8, 10).

	n	w	Nash= Pulsing d.	Average Cont.	$d(b^d,b) = d(n^d,n)$
Andreoni (93)	3	7	3	2.78	0.49
CMMM (96)		20	5	5.3	0.58
Sutter and Weck-Hannemann (04)		7	3	3.18	0.40

Table 3. Experiments using Cobb-Douglas utility functions.

Although the Nash equilibrium contribution is less than half of the endowment for every case in Table 3, the average contribution is close to the Nash equilibrium. Cason, Saijo, and

Yamato (2002) used $n = 2$, $w = 24$, and $\alpha = 0.47$ and hence the system is stable. They observed that the average contribution is 7.37, close to 8, which is the Nash equilibrium contribution. Since their system is asymptotically stable, the Nash and the best response dynamics predictions coincide. These observations lead to the following property.

Property 5. *If the Nash and the best response dynamics predictions coincide, experimental data are close to the prediction.*

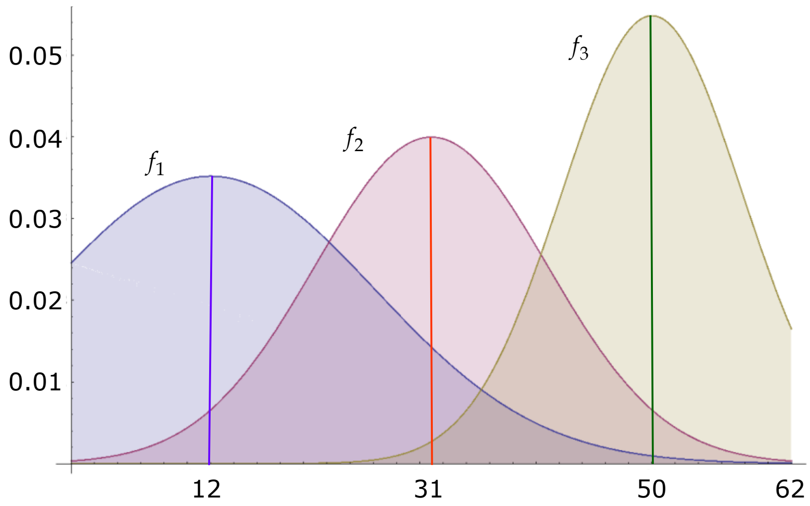

Figure 5. Truncated normal distributions using quantal response equilibrium analysis.

In order to understand the sandwich property and altruism, Anderson, Goeree, and Holt (1998) introduced errors in their quasilinear utility functions, in which the public good term is quadratic, following quantal response equilibrium analysis introduced by McKelvey and Palfrey (1995). This process involves first setting up player *i*'s expected payoff when others' strategies are stochastic. Second, it requires finding the density of player *i*'s strategies given the densities of other players' strategies. Then, this process yields a truncated normal distribution whose mean is the symmetric Nash equilibrium strategy.[10] Consider Figure 5. The horizontal axis shows the best responses from 0 to 62. 12 is for n_1, 31 is for n_2, and 50 is for n_3. The propensities to make errors go down from f_1 to f_3. Since the area on the right hand side of f_1 is larger than that on the left hand side of f_1, a player tends to contribute more than 12. If the symmetric Nash equilibrium is at 31, then the areas on both sides are the same, so a player

[10] For further details, see pp. 312–3 and appendix A in Anderson, Goeree, and Holt (1998).

chooses around 31. If it is at 50, then the area on the left hand side is larger than that on the right hand side; thus, a player tends to choose less than 50.

Applying the same procedure to the log linear form of the Cobb–Douglas utility function, I can obtain truncated normal distributions corresponding to Figure 5. That is, the average contribution in every case with a Cobb–Douglas utility function should have been more than the Nash equilibrium. However, as Property 5 shows, it is very close to the Nash equilibrium. As Anderson, Goeree, and Holt (1998) noticed, their approach is suitable for static equilibrium analysis, but not for dynamic analysis. In order for the static equilibrium analysis to be meaningful, the system must be stable.

4. Concluding Remarks

I showed the sandwich property using the instability of best response dynamics, rather than through equilibrium analysis, since the VCM system is not necessarily stable. Previous experimental data are consistent with theoretical average data incorporating pulsing behavior.

The data of Sefton and Steinberg (1996), Isaac and Walker (1998), and Laury, Walker, and Williams (1999) covered all three cases, i.e., with the Nash equilibrium "below," "at," and "above" the midpoint of the endowment in a quasilinear environment. However, no experiments have looked at cases where the equilibrium contribution is "at" or "above" the midpoint of the endowment in the Cobb-Douglas utility function environment.

Although the instability approach explains experimental data with nonlinear utility functions, this approach cannot be applied to a linear environment. This is because the VCM is always stable in a linear environment due to the existence of a dominant strategy. Altruism, warm-glow, reciprocity, equity, and so on are the central foci explaining deviations from "rational" choices in the linear environment, but it seems that they do not play major roles and some of them might partially disappear due to instability in nonlinear environments. Further research on these issues is needed.

There is another type of nonlinear utility function that many experimental economists have been using. These functions are of the form $u(x_i, y) = h(x_i) + y$, that is, nonlinear with respect to x_i and linear with respect to *y*. They have been used by Sefton and Steinberg (1996), Keser (1996), Falkinger, Fehr, Gächter, and Winter-Ebmer (2000), van Dijk, Sonnemans, and van Winden (2002), Uler (2011), Maurice, Rouaix, and Willinger (2013) and Cason and Gangadharan (2014), among others.[11] This utility function incorporates the peculiar assumption that the marginal utility of the private good decreases, but the marginal utility of the public good is constant. An advantage of this utility function is that each player has a dominant strategy in the

[11] There are many other experimental papers in this category and the number has been growing.

VCM, as in a linear environment. Since a dominant strategy exists, the stability of the best response dynamics is guaranteed. The observations in this category are similar to the observations in a linear environment. From the viewpoint of stability analysis, an important research topic is to learn what type of utility functions players really have. That is, *external* validity of utility functions in the provision of public goods should be explored.

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