The Scalp Distribution of the Fractal Dimension of the EEG and Its Variation with Mental Tasks

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Summary: The insights gained by the concept of deterministic chaos for the EEG is that this seemingly disordered process may be governed by relatively few simple laws which could be determined. One of the quantitative measures of a complex dynamical system is that of its dimension. The term 'dimension' refers to the ability of a space to contain a set of points. We estimated the correlational dimension of the EEG and compared the outcome to traditional Fourier analyses. In addition, we tested the hypothesis that the EEG can be described as filtered noise. Data from 15 electrode sites and 31 subjects are reported in the present study. We have utilized a variety of tasks that cut across sensory modalities including touch, vision, and imagery which reflect neuropsychological processes that differentially engage areas of the cortex in the first part of the study. In the second part, the differences between the perception of an object and the imagination of the same object were evaluated. The outcome shows variations between scalp sites for all measures and also variations between tasks in terms of dimensionality of the EEG. The hypothesis of a higher dimensionality ("complexity") of imagery compared to actual perceptual processing was confirmed. A statistical comparison between the maps generated by means of the various measures shows that different informations are extracted when using the different measures. There is also statistical evidence that the EEG cannot completely be described by the model of filtered noise.

Key words: Fractal dimension of the EEG; Chaos; EEG mapping; Dimensionality; Cortical connectivity; Imagery.

Introduction

EEG activity essentially results from the summation of postsynaptic potentials which originates primarily in cerebral cortex and reflects summed electrical activity of billion of neurons. Their interconnections form an extremely complex system. Given the high number of neurons with their infinite number of firing patterns, it is not obvious why systematic EEG activity can be observed at all when recorded from the scalp. The common explanation is that large numbers/populations of neurons are synchronized through thalamic afferent input so that their activity becomes superimposed. A simple examination of the waning and waxing of irregular waves might still lead one to infer that an infinite number of degrees of freedom contribute to the temporal development of the recorded voltage. However, this intuitive conclusion may be misleading.

It has been demonstrated that a simple system with as few as three differential equations can generate totally irregular fluctuations of the system's variables - a phenomenon presently referred to as deterministic chaos. When a system produces irregularity in one of its variables, it is possible that this behaviour results from randomness (meaning that the number of degrees of freedom is infinite) or, that a finite, and possibly small number of degrees of freedom has produced the chaos (meaning that the system is deterministic). The prominent features of chaos are unpredictability over extended time periods, and sensitive dependence on initial conditions. Once started with specific initial values, the system's future can develop totally differently, if it had been started under slightly different initial conditions. The importance of this finding for the EEG is that this seemingly disordered process may be governed by relatively few simple laws which could be determined.

One of the classic quantitative measures of a complex dynamical system is that of dimension. The classic definition of a dimension refers to the number of independent directions in a set which have been generalized to include both higher dimensions and fractal dimensions (Rapp et al. 1990). In the present context, the term dimension refers to the ability of a space to contain a set of points. Although beyond the scope of the present discussion, it should be pointed out that in this newly

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developing literature, the term dimension does not always have an absolute technical meaning (see Rapp et al. 1990 for a further discussion of this point). For example, even when the various mathematical procedures used to compute the fractal dimension are applied to the same data set, different values may result. Furthermore, we have to make a tradeoff between an appropriate length of the time series required for the estimation of the dimension and some kind of stationarity of the EEG: a good estimation of the relatively high dimensions of the EEG requires extended time series, maybe even in the order of days (Smith 1988), but we cannot assume that the brain will remain in the same state over an extended time. Therefore, relative differences in dimension between conditions, or within the same condition are of more interest than the absolute values. It seems mathematically reasonable to assume that with an increase of the number of independent processes generating the EEG the dimension increases too. The procedure used here for calculating dimensions of the EEG-attractor combines the 'singular value decomposition' introduced by Broomhead and King (1986; described by Albano et al. 1987) with the method of 'averaged pointwise dimension' developed by Farmer et al. (1983).

Since dimensional analysis of the EEG has not been applied to a large variety of cognitive and sensory tasks previously nor different electrode sites, it is difficult to make exact predictions of how sensory and cognitive activity would be reflected in such a nonlinear analysis. Initial EEG studies using deterministic chaos offer some suggestions. For example, Babloyantz and her colleagues (Babloyantz et al. 1985; Babloyantz and Destexhe 1986) have made dimensional calculations between states such as sleep stages, waking, and epilepsy with higher dimension being found in the awake condition and lower dimensions in the sleep and epilepsy condition. Others (Mayer-Kress and Layne 1987; Albano et al. 1986; Dvorak and Siska 1986; Rapp et al. 1986) reported dimensions lower than 10 during the relaxed waking state with eyes either closed or open. Mayer-Kress and Holzfuß (1987) examined subjects who were either awake or under anesthesia and found higher dimensions in the anesthesized state. Rapp and his colleagues used two mathematical tasks (serial 7s, and serial 2s) and compared these with rest (Rapp et al. 1990). In general this last group found larger dimensions (approx. 4.8) on the math tasks as compared to rest. Lower dimensionalities have generally been reported during resting periods, particularly when eyes were closed, than during active engagement (e.g., Basar et al. 1989; Graf and Elbert 1989; Mayer-Kress et al. 1988; Nan and Jinghua 1988; Pritchard and Duke 1990; Rapp et al. 1986).

Pijn, Van Neerven, Noest and Lopes da Silva (1991) investigated the EEG of rats with epileptic foci. During epileptic attacks, they found good evidence for low-dimensional attractors, but the normal resting EEG did not differ in its dimension from a filtered noise series with identical power spectrum. They concluded, that the concepts of attractors and dimension are not very useful to describe the EEG, and they prefer to describe the EEG as filtered noise.

In the present study, we sought to extend the previous work in several ways. First, we have utilized a variety of tasks that cut across sensory modalities including touch, vision, imagery, and verbal processing which reflect neuropsychological processes that differentially activate both the frontal and more posterior areas of the cortex. Second, we compared these results to measures from conventional Fourier analysis. Third, we compared the dimension of the EEG with dimension of time series obtained by filtered noise with power spectra identical to the EEG power spectra. Fourth, we tested the reliability of the finding that mental imagery produces higher dimensional complexity than perception in a second experiment. Whereas previous studies have been limited to only one or two electrode sites, we collected data from 15 electrode sites in the present study. Thus, the study was designed to answer four major questions: (1) are there variations between tasks in terms of dimensionality, (2) are there variations between cortical sites in terms of dimensionality, (3) if so, are these variations qualitatively different from those provided by Fourier analyses of the EEG-traces, and (4) is there some evidence that the normal waking EEG can be distinguished from filtered noise.

Study I

Methods

Subjects. Twelve subjects (3 women and 9 men), students or members of the faculty, participated in the experiment. The age ranged from 26 to 47 years. Two men were excluded from the sample, one due to frequent artefacts, and the other because of demonstrated psychopathology.

Apparatus. Physiological signals were acquired using AT class personal computers and digitized at 256 Hz. Data were uploaded to a DEC VAX-station 2000 and digitally filtered to 128 points per second for subsequent data analysis. Two Nihon-Kohden electroencephalographs (models 4300) were used for the amplification, analog filtering, and display of EEG and EOG data.

Physiological recording. Zak Ag/AgCl electrodes were attached for recording from 37 sites according to the international 10-20 system. For the present study data were analyzed for the following electrode placements:

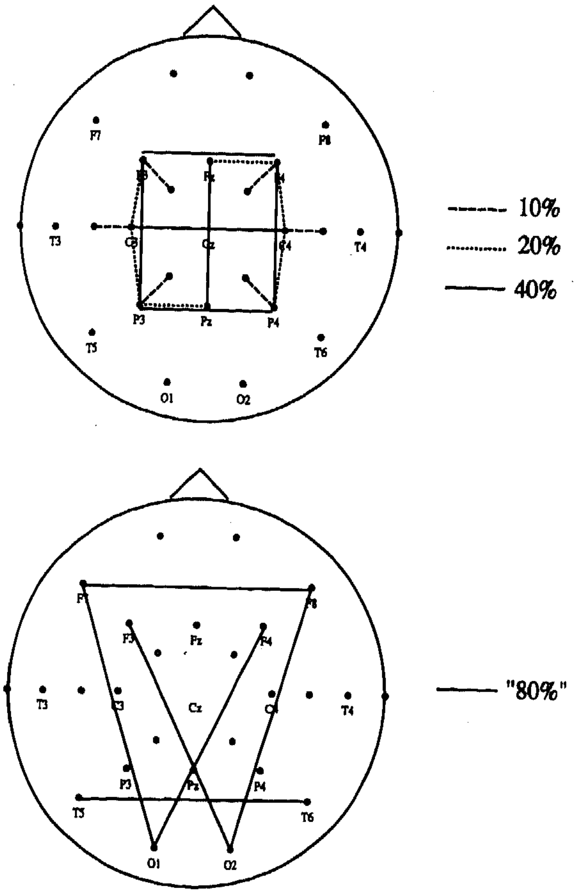

Figure 1: Distribution of the 24 bipolar recordings used for the extended analysis. Six bipolar recordings were analysed for each of the four different electrode distances. The choice was guided by the available electrodes and by an attempt to minimize asymmetries.

F7, F3, Fz, F4, F8, T3,C3, Cz, C4, T4, P3, Pz, P4, T5, T6. Sites were rubbed with alcohol and gently scraped with a sterile lancet to reduce impedances to less than 5KΩ, using Grass cream as the conducting medium. All leads were referenced to the vertex electrode for recording purposes and off-line corrected for time lags in the sampling procedure (Lutzenberger and Elbert 1991) and changed to an average reference giving all electrodes equal weight. The average reference was calculated using only the 15 sites discussed in this paper. Three eye movement measures were recorded, all with Ag-AgCl electrodes. A single ground electrode was attached to the left wrist.

The average reference used here includes only the upper half of the head like in most studies. As this can lead to problems of interpretation of EEG topography (e.g., Desmedt and Tomberg 1990), 24 bipolar recordings were evaluated to test the influence of the reference: figure 1 shows the four groups of 6 sites spanning 10%, 20%, 40%, and approximately 80% of the 10-20 system.

All channels were amplified with a bandwidth from 0.016 Hz to 70 Hz which was subsequently reduced by means of digital filtering to a range from 0.8 Hz to 40 Hz.

Design. Each subject received the six conditions in a different order which was not completely counterbalanced. The imagery tasks always were the last two tasks, but were counterbalanced across subjects. The other tasks were presented in a predetermined order which was incompletely counterbalanced due to the limited number of subjects, but no two subjects received the same order for the tasks.

The six tasks were as follows: 1) observation of a swinging double pendulum; 2) to name aloud nouns which started with the letter M (e.g., mouse, man, milk, morning); 3) to determine which of six pieces of sandpaper was the smoothest with the right index finger; 4) the same task with the left index finger; 5) to image and experience an extremely positive time in their past in which they had felt in love (without actual sexual imagery) and 6) to image the same type of extremely positive experience which included a sexual experience.

Procedure. Subjects were instructed that throughout the entire procedure, they were to keep their eyes open. During the tasks, they were instructed to maintain a fixation point which was either the pendulum or a white cross in the center of the monitor screen. Both were placed about 2m in front of the subject, in order to minimize disruptive effects of eye movements on the EEG. Instructions for each tasks was presented in a written form prior to each task.

Data reduction and analysis. For each task an interval of 16 s in duration was selected for the computation of different brain maps. Thus the length of each EEG trace was 2048 points. The EEG was corrected for ocular artefacts with the regression method including the vertical and horizontal EOG. The following measures were calculated for every EEG trace:

(i) The **EEG alpha power** was obtained from the average log power in the range from 8 to 12 Hz. The power spectrum was calculated by averaging the Fourier transforms of 15 overlapping 2 s segments (256 points), using Parzen windows on the 2 s segments.

(ii) **EEG beta power** was calculated as the average log power in the range from 14 to 30 Hz.

(iii) The **phase space dimension** of the EEG: The singular value decomposition was based on the autocovariation function with time-lags ranging from 0 to 32 points. A symmetrical 32x32 matrix was constructed with the covariances as elements. The first row was the autocovariation function itself; in the second row, elements were shifted by one column to the right; in the

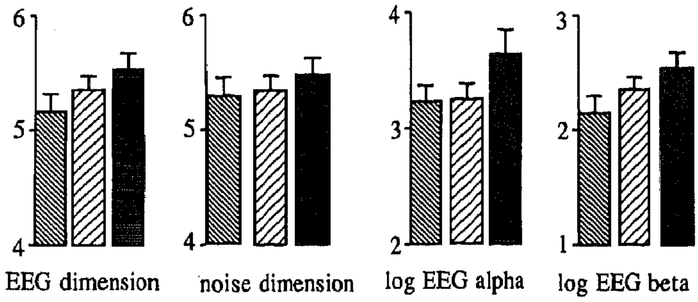

Figure 2: Main effects of tasks and standard errors of the means for EEG dimension, noise dimension, alpha, and beta power.

third row by two columns etc. so that the diagonal element was always the covariance with time lag zero. Then the Eigenvectors and Eigenvalues were obtained. A subset of the Eigenvectors was used to reconstruct the state space. Only Eigenvectors were selected with Eigenvalues larger than twice the smallest of all 32 Eigenvalues. This selection was used to separate the signal from the noise. The criterion chosen seems somewhat arbitrary but has two advantages: It is independent of the particular gains used and it is easily reproducible among different laboratories. We will refer to the number of eigenvectors used as **'embedding dimension'**.

(iv) The **filtered noise dimension** was calculated with the same procedure. The Fourier-transformed EEG series were randomly phase-shifted and transformed back to the time domain as in Pijn et al. (1991).

A calculation of the dimension was done separately for 32 equidistant points using the method of 'pointwise dimension' as proposed by Farmer et al. (1983). Given a distinct reference point, the number of points N(r) which lie in a hypercube with radius r around this chosen point is counted. This counting is performed for subsequently larger radii until ultimately all points of the time series lie within this hypercube. For the present computations, 20 different radii r_i were chosen. The distance between subsequent radii was selected such that each enlargement of the radius increased the total count by an equal number of points, i.e., $N(r_i) - N(r_i\text{-}1) = 2048/20$. The counts are plotted against r using a double logarithmic scale. The resulting function starts with a straight line of a certain slope, but then declines parallel to the abscissa. (Usually, the radii are chosen such that there are equidistant differences on a logarithmic scale. This results in a S-like shape for the double logarithmic representation. The present method has the advantage that the lower curvature does not show up). A linear fit is performed on the straight segment, the slope of which is used for further calculation of the dimension. In order to obtain an estimation of the straight segment, only the lowest ten values are chosen first. If the highest of these ten values has the largest distance to the straight line, the linear fit is recalculated for nine values only. If again the highest of these has the largest distance from the straight line, it is omitted from the next calculation. The process is repeated until the highest point has no more the largest distance from the estimated straight line. Typically, this procedure results in an estimate out of the five to seven lowest radii for the slope. After a slope has been determined for each of the reference points, the median determines the desired fractal dimension.

These measures were analyzed by means of an analysis of variance. For visual display, brain maps were constructed using the Akima smooth surface fit (Akima 1978). The statistical comparison of the different measures was achieved by nonparametric transformations to normal distribution (McCall 1939) and inclusion of the additional factor 'measure' in the ANOVA.

Results

A first analysis of the data showed no statistical differences between the two tactile task. The alliteration task, which required overt speech, produced some artefacts. The two imagery tasks gave significant different dimensions with the average reference, but the bipolar recordings could not replicate this finding. Therefore we dismissed the alliteration task from the final analysis and collapsed the results of the two tactile task and the two imagery tasks to the conditions 'tactile task' and 'image task'. According to this procedure, the following results are based on three conditions (visual observation, tactile task, and imagery).

ANOVAS with the factors electrode (15) and tasks (3) gave significant effects of electrodes for the dimension ($F(14,126) = 7.0$, $p < 0.01$) and for the beta power ($F = 10.0$). The main effect of task was significant in all measures (dimension: $F(2,18) = 8.8$, alpha: $F = 7.5$, beta: $F = 13.1$, $p < 0.01$). The interaction of electrodes X tasks did not reach significance in any of the measures. Figure 2 shows the main effects of tasks. Post-hoc tests demonstrate significant higher values of dimension, alpha, and beta in the imagery task than in the observation task.

The analysis of the dimension of the filtered noise showed a significant effect of electrode ($F(14,126) = 5.9$, $p < 0.01$), but only a borderline effect of tasks ($F(2,18) = 3.9$, $p = 0.07$ after Greenhouse Geisser correction). An

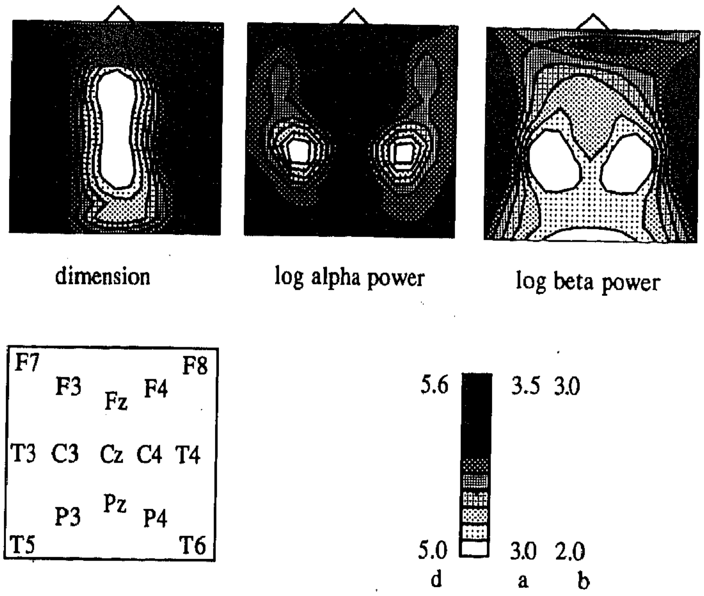

Figure 3: Brain maps for EEG dimension, alpha, and beta averaged across subjects and tasks. Isopotential maps are based on the Akima smooth surface fitting algorithm.

ANOVA including the EEG dimension and the noise dimension showed an interaction of task X measure ($F(2,18) = 10.0$, $p < 0.01$): the EEG dimension is lower than the noise dimension in the visual task ($t(9) = 3.59$, $p < 0.01$), but there is no significant difference for the other tasks ($t < 1$).

A statistical comparison of the dimension with EEG alpha and beta was performed with T-transformed values by expanding the ANOVA with the repeated factor measure. The main effects of task were highly significant ($F(2,18) = 20.5$ and 14.2), but only weak effects of task X measure were found ($p < 0.20$). These data confirm the parallel variation of the three measures over the tasks. Significant interactions of electrode X measure found for the comparison dimension vs. alpha ($F(14,126) = 5.1$, $p < 0.01$) and for dimension vs. beta ($F = 7.0$, $p < 0.01$) indicate to different topographical distributions shown in figure 3.

The analysis of the bipolar recording measures was based on the factors distance (4) and tasks (3). For each level of distance the mean of the measures of six electrode combinations was used. For the dimension, a significant main effect of task was found ($F(2,18) = 6.4$, $p < 0.02$) while the main effect of distance ($F < 1$) and the interaction of distance X task ($F = 1.7$) were far from significance. EEG alpha and beta also showed significant effects of task ($F = 8.3$ and $F = 15.8$), but in contrast to the dimension, both showed a highly significant main effect of distance ($F(3,27) = 114.7$ and $F = 78.1$). Figure 4 demonstrates the effects: the dimension is nearly independent of the electrode distance while EEG alpha power shows a

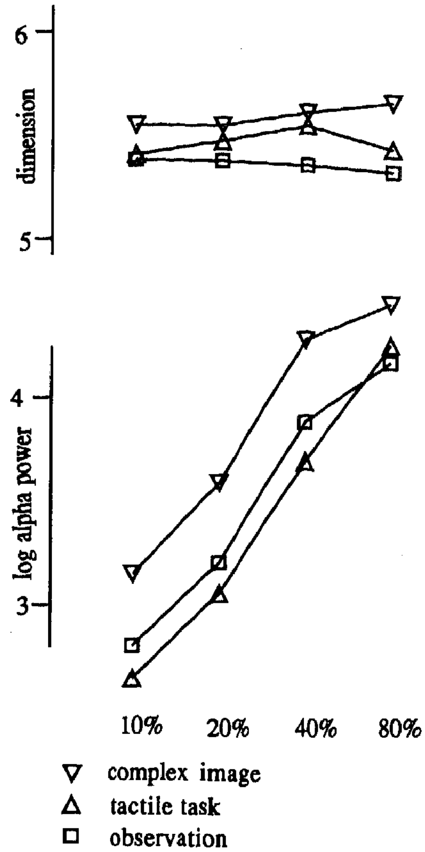

Figure 4: Effect of task and electrode distance on the dimension and EEG alpha power.

pronounced increase with the distance between the electrodes.

Discussion

In this study we present comparisons of traditional EEG Fourier measures with measures from non-linear dynamics using a variety of sensory, motor, and mental tasks. Overall, the brain maps of dimensionality demonstrate that the dimensional measures can extract information not available from conventional EEG-analyses based on power estimates of EEG bands. The topological patterns of dimensionality suggest the possibility that information may be contained within the EEG yet to be decoded.

In terms of alpha activity, there is a decreased alpha activity on the sensory tactile and visual attention tasks and an increase in the imagery tasks. The finding is consistent with earlier research which suggest increased alpha activity on tasks which require attention to internal processing and decreased alpha on tasks requiring external attention (Ray and Cole 1985). The similarity of alpha and beta activity is also consistent with previous reports from our lab (cf. McCarthy & Ray, 1988).

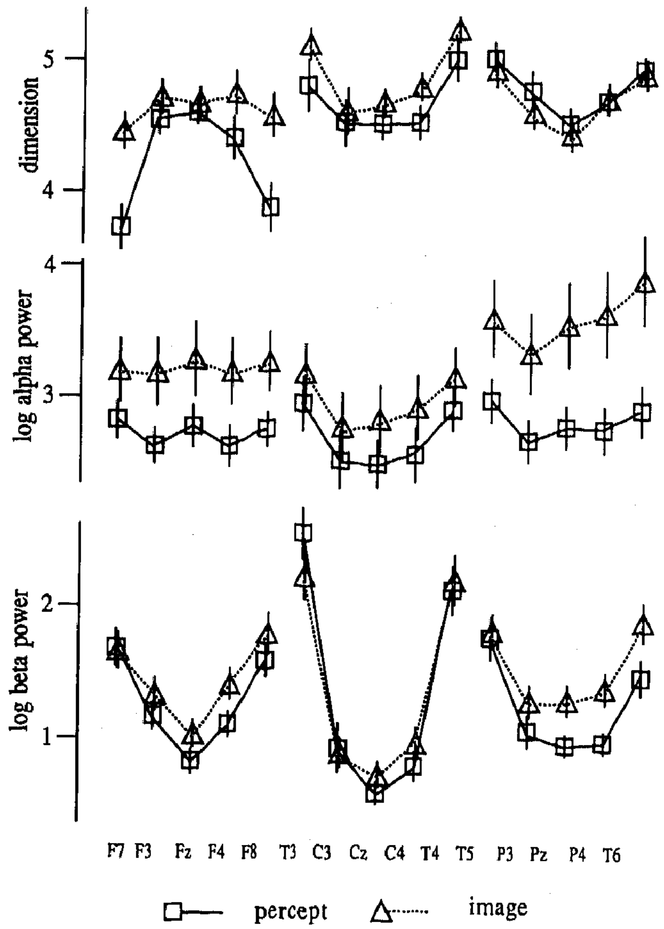

Figure 5: The effects of the two tasks of study 2 on EEG dimension, alpha and beta power. The bars indicate between standard errors of the means.

In terms of the dimensions derived from the reconstructed phase space, we reported the highest dimensions for the imagery tasks, followed by the sensory touching tasks and observational task. Previous researchers (e.g., Graf and Elbert,1989, Mayer-Kress et al. 1988; Rapp et al. 1986; Rapp et al. 1990) have reported lower dimensionality during rest than more active engagement. This would lead to the conclusion that the imagery tasks were of a more complex nature. We are convinced that 'rest' is not a well chosen control condition, because a very large number of more or less complex brain processes may happen during such an unstructured situation. A simple task like our observation task defines the stimulus - response requirements much more precise and may therefore constitute a more preferable control condition.

Study 2

The goal of study 2 was to replicate and specify the differences between perception and image found in study 1, specifically the result that imagery seems to be associated with a more complex dynamics than sensory processing: the perception was more structured by imposing a memory element and the imagery was matched in content to the percept.

Methods

Subjects. Twenty-one subjects (10 women and 11 men), all students, participated in the experiment. The age ranged from 20 to 24 years.

Apparatus and physiological recording were similar to study 1 with the following modifications: EEG was recorded with a HF cutoff of 35 Hz and sampled by a PDP 11/73 with a rate of 100 Hz. Analysis was based on epochs of 20.48 sec duration. In addition to a computed average reference, we used a computed linked ears reference. Bipolar recordings were not evaluated.

Design and Procedure. The experiment had two conditions: first, the subjects observed the double-pendulum used in study 1 with the instruction to memorize its movements for the second task. Then the pendulum was removed and the subjects had to imagine the pendulum with eyes open.

Data reduction and analysis was identical to study 1.

Results

ANOVAs with the factors electrodes (15) and task (2) gave highly significant main effects and interactions ($p < 0.001$) for the dimension, the EEG alpha and the EEG beta for both the average reference and the linked ears reference except the main effect of task for EEG beta with the linked ears reference.

The influence of the reference was investigated by including the within-factor of reference (2). All measures were highly significant larger for the linked ears reference (dimension: $F(1,20) = 48$, alpha and beta: $F > 360$). For alpha and beta, significant interactions of electrode X reference ($F > 89$, $p < 0.001$) and interactions of electrode x task x reference ($F > 3.9$, $p < 0.01$) demonstrated a high sensitivity of both the topography and its interaction with tasks on the chosen reference. For the dimension, only a significant interaction of electrode X reference ($F(14,280) = 3.3$, $p < 0.01$) was found. The following analysis is based on the average reference.

The statistical comparison of the dimension with alpha and beta gave highly significant effects of electrode X measure (alpha: $F(14, 280) = 6.9$, beta: $F = 18.8$, both $p < 0.001$), confirming the results of study 1. In addition, significant interactions of electrode X task X measure (alpha: $F = 7.0$, beta: $F = 9.0$, $p < 0.001$) were found which support the effects shown in figure 5: during imagery, there is an increase of the dimension mainly in the frontal areas and a slight decrease in the parietal areas, while alpha and beta show an increase both frontal but most pronounced in the parietal region.

Discussion

Study 2 confirms the increased dimension of the EEG during imagery in comparison to perceptual processing. If the two tasks are matched for content, the differences are confined to frontal sites. This seems to be in accordance with earlier PET and EEG studies indicating to an increased frontal metabolism during 'thinking' of an event compared to perceptual processing (Phelps et al. 1981; Roland 1982). Our own studies with slow brain potentials also alluded to increased utilization of attentional resources during imagery than during actual processing and abstract 'thinking' (Birbaumer et al. 1988; Birbaumer et al. in press). As EEG alpha and beta did not show this pattern, we speculate that changes in EEG dimension are more correlated with local metabolic changes and slow brain potentials than with EEG power measures.

Overall, we suggest that measures from non-linear systems theory are potentially important and worthy of further investigation; especially since traditional EEG measures have more historical than theoretical basis for their inclusion and are primarily descriptive in nature. Our data, like those reported in the literature, do not lead to a clear decision whether the concept of attractors is appropriate to describe the normal EEG. The differences found between our EEG dimension and the filtered noise dimension seem to indicate dynamical aspects of the EEG which are not explainable by power spectrum analysis. Although traditional measures of EEG are useful in certain situations, there exists no a priori justification for choosing these measures over non-linear dynamics alone. Of course, traditional measures in combination with measures developed from non-linear analysis may offer a rich source of important information.

References

Akima, H. Bivariate interpolation and smooth surface fitting for irregularly distributed data points. ACM Transactions on Mathematical Software. 1987, 4: 160-164.

Albano, A.M., Abraham, N.B., de Guzman, G.C., Tarroja, M.F.H., Bandy, D.K., Gioggia, R.S., Rapp, P.E., Zimmerman, I.D., Greenbaum, N.N. and Bashore, T.R. Lasers and brains: complex systems with low-dimensional attractors. In: G. Mayer-Kress (Ed.), Dimension and Entropies in Chaotic Systems. Berlin, Heidelberg, New York, Tokyo, Springer, 1986: 231-252.

Babloyantz, A. Strange attractors in the dynamics of brain activity. In: H. Haken (Ed.), Complex Systems - Operational Approaches in Neurobiology, Physics, and Computers. Berlin, Springer, 1985: 116-122.

Babloyantz, A., Salazar, J.M. and Nicolis, C. Evidence of chaotic dynamics of brain activity during the sleep cycle. Phys. Lett., 1985, Vol.111A, 3: 152-156.

Babloyantz, A. and Destexhe, A. Low dimensional chaos in an instance of epilepsy. Proc. Natl. Acad. Sci., 1986, 83: 3513-3517.

Basar, E., Basar-Eroglu, Röschke, J. and Schult, J. Chaos and alpha-preparation in brain function. In: R.M.J. Corterill (Ed.), Models of Brain Function. Cambridge University Press, Cambridge, 1989: 365-395.

Birbaumer, N., Lang, P.J., Cook, E., Elbert, T., Lutzenberger, W. and Rockstroh, B. Slow brain potentials, imagery, and hemispheric differences. Int. J. Neuroscience, 1988, 39: 101-116.

Birbaumer, N., Elbert, T., Canavan, A. and Rockstroh, B. Slow Potentials of the Cerebral Cortex and Behavior. Physiological Reviews, 1990, 70: 1-41.

Birbaumer, N., Lutzenberger, W., Flor, H., Elbert, T. and Rockstroh, B. Imagery and brain processes. In: N. Birbaumer and A. Öhman (Eds), The Organization of Emotions. Hogrefe, Bern, 1992.

Broomhead, D.S. and King, G.P. Extracting qualitative dynamics from experimental data. Physica, 1986, 20D: 217-236.

Desmedt, J.E. and Tomberg, C. Topographic analysis in brain mapping can be compromised by the average reference. Brain Topography, 1990, 3: 35-42.

Farmer, J.D., Ott, E. and York, J.A. Dimension of chaotic attractors, Physica, 1983, 7D: 153-180.

Graf, K.E. and Elbert, T. Dimensional analysis of the waking EEG. In: E. Basar, T.H. Bullock (Eds.), Brain Dynamics. Springer-Verlag, Berlin, Heidelberg, 1989: 174-191.

Lutzenberger, W. and Elbert, T. A/D conversion in DMA-mode - problems and solutions. Psychophysiology, 1991, 28: 607-608.

Mayer-Kress, G. and Holzfuß, J. Analysis of the human electroencephalogram with methods from nonlinear dynamics. In: L. Rensing, U. an der Heiden, M.C. Mackey (Eds.), Temporal Disorder in Human Oscillatory Systems. Berlin, Heidelberg, New York, Tokyo, Springer, 1987.

Mayer-Kress, G., Yates, F.E., Benton, L., Keidel, M., Tirsch, W., Pöppl, S.J. and Geist, K. Dimensional analysis of nonlinear oscillations in brain, heart, and muscle. Math. Biosciences, 1988, 90: 155-182.

McCall, W.A. Measurement. Wiley, New York, 1939.

McCarthy, P. and Ray, W.J. Interactions between alpha and beta EEG frequencies. Psychophysiology, 1988, 25: 468.

Nan, X. and Jinghua, X. The fractal dimension of EEG as a physical measure of conscious human brain activities. Bulletin of Mathematical Biology, 1988, 50: 559-565.

Phelps, M.E., Mazziotta, S.C., Kuhl, D.E., Nuwer, M., Packwood, J., Metter, J. and Engel, J. Tomographic mapping of human cerebral metabolism: Visual stimulation and deprivation. Neurology, 1981, 31: 517-529.

Pijn, J.P., Van Neerven, J., Noest, A. and Lopes da Silva, F.H. Chaos or noise in EEG signals; dependence on state and brain site. Electroencephalography and Clinical Neurophysiology, 1991, 79: 371-381.

Pritchard, W.S. and Duke, D.W. Deterministic chaos and the human EEG. Psychophysiology, 1990, 27: 56.

Rapp, P.E. Oscillations and chaos in cellular metabolism and

physiological systems. In: A.V. Holden, (Ed.), Chaos - Non-linear Science: Theory and Application. Manchester University Press, 1986: 179-208.
Rapp, P.E., Bashore, T., Martinerie, J., Albano, A., Zimmerman, I., and Mees, A. Dynamics of Brain Electrical Activity. Brain Topography, 1990, 2: 99-118.
Rapp, P.E., Zimmerman, I.D., Albano, A.M., deGuzman, G.C. and Greenbaum, N.N. Dynamics of spontaneous neuronal activity in the simian motor cortex: The dimension of chaotic neurons. Phys. Lett. 110A, 1985, 6: 335.
Rapp, P.E., Zimmerman, I.D., Albano, A.M., deGuzman, G.C., Greenbaum, N.N. and Bashore, T.S. Experimental studies of chaotic neuronal behaviour: cellular activity and electroencephalographic signals. In: H.G. Othmer (Ed.), Nonlinear Oscillation in Biology and Chemistry. Berlin, Heidelberg, New York, Tokyo, Springer, 1986: 175-205.
Ray, W.J. and Cole, H. EEG alpha reflects attentional demands, beta reflects emotional and cognitive processes. Science, 1985, 228: 750-752.
Roland, P.E. Cortical regulation of selective attention in man. J. of Neurophysiology, 1982, 48: 1059 -1078.
Smith, L.A. Intrinsic limits on dimension calculations. Physics Letters A, 1988, 133: 283- 288.