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ABSTRACT

Recent efforts to find a complete set of S-matrix axioms are reviewed, emphasis being placed on the impossibility of fitting electromagnetism into the existing framework. It is suggested that a pure S-matrix theory may describe an artificial but recognizable world containing all strongly interacting particles but no photons or leptons. The theory would not be self-sufficient because of its failure to provide a mechanism for the measurement of particle momenta (i.e., for experiments that give a meaning to macroscopic space-time), and therein would lie the necessity for electromagnetism. From this viewpoint, the photon mass and the fine structure constant are linked to the theory of measurement and will not emerge from the dynamical bootstrap that determines the strong interaction parameters.

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INTRODUCTION

About three years ago there occurred a revival of interest in the S matrix as a framework for the formulation of fundamental subatomic laws. The S matrix was defined by Wheeler in 1937,[1] and the possibility of its role being fundamental was suggested already in 1943 by Heisenberg,[2] who recognized a number of the important advantages over conventional quantum theory and who stressed certain properties of the S matrix that remain central features of current work. The property now generally called "maximal analyticity" was not appreciated in the forties, however, and without this notion S-matrix theory lacked dynamical content. Heisenberg and the other S-matrix students of that period eventually lost interest when they realized they had no way to compute interparticle forces, and more than a decade elapsed before the S matrix was resurrected as a competitor with quantum field theory.

The gradual appreciation of the dynamical content in analyticity occurred during the last half of the fifties and involved many names, major figures being Gell-Mann, Goldberger, Low, and Mandelstam. All results at this stage, however, were either motivated by or derived from field theory, and to this day many theorists believe that even if S-matrix axioms can be found they will simply amount to an alternative

statement of field theory. In this view the search for S-matrix axioms is an interesting but academic exercise that is unlikely to increase our understanding of nature. Were I to share such an opinion I should not be presenting this review. I believe that the effort to formulate fundamental laws directly in terms of the S matrix, even if destined to be only partially successful, is opening new avenues of development that will not be found through field theory. This belief is ended in what follows.

Gunson, Stapp, and I independently became impressed by the possibility of adding maximal analyticity to the old Heisenberg scheme and of thereby avoiding the field concept.[3,4,5] During the past three years Stapp, Gunson, and also Olive[6] have made serious efforts to find a minimal set of S-matrix axioms to reproduce all properties conjectured on the basis of perturbation field theory. In contrast, my own chief interest has been in "bootstrap" properties that cannot be motivated by a perturbation approach but which have been suggested by experiment. I have been struck, nevertheless, by difficulties encountered in the work of Stapp, Gunson, and Olive that hint at a connection between their goal and that of the bootstrappers. I propose here to stress these difficulties --rather than the recent successes of S-matrix axiomaticians--because it is only in the difficulties they have uncovered that distinctions from perturbation field theory are to be found.

It must be added that the opinions I shall present concerning the difficulties in S-matrix theory are not all shared by Stapp, Gunson, and Olive. Even among the small clan of S-matrix enthusiasts, there exist serious differences of outlook.

It is a tragedy that Landau is unable to continue his role in the debate. He was perhaps the first unequivocally to reject the field concept, and by 1959 was well aware of the power of combining unitarity with analyticity.[7] Landau at that point, of course, was working with amplitudes both on and off the mass shell, whereas the S matrix is entirely on the shell. Current opinion, which I share, is that taking scattering amplitudes in a meaningful and unique way off the mass shell would be equivalent to field theory; only if such extensions turn out to be meaningless is there likely to be a basic incompatibility between field theory and S-matrix theory. My personal guess is that off-mass-shell continuations are meaningless, but few other physicists share this feeling. Landau's participation in the discussion of such questions would be of enormous value today.

A TENTATIVE SET OF AXIOMS TO REPLACE PERTURBATION FIELD THEORY

It is perhaps premature to speak of a consensus having being reached in the work of Gunson, Olive, and Stapp, but their recent writings contain many common points. They believe that approximately five principles should suffice to achieve all the general properties of the S matrix that are suggested by perturbation field theory. These principles refer only

to the S matrix and its analytic continuation and do not explicitly invoke the full apparatus of quantum mechanics, with its state vectors, complete sets of operators, and commutation rules. Little more than the superposition principle is maintained. The only observables are supposed to be particle momenta and spin orientations, before and after collisions. Actually the usual connection by Fourier transform with macroscopic space-time must be assumed if one is to connect theory with experiment, but localized space-time functions cannot be formed from momenta constrained to the mass shell. The sharpest definition allowable is the particle Compton wavelength, in accord with experimental limitations. By contrast there is no known limit to the accuracy with which momentum can be defined, at least in an infinite universe; the mass-shell momentum-energy continuum is experimentally realizable even though the space-time continuum is not.

The simple framework of S-matrix theory and the restricted set of questions that it presumes to answer constitute its chief advantage over quantum field theory. The latter is burdened by a superstructure, inherited from classical electromagnetic theory, that seems designed to answer a host of experimentally unanswerable questions. Current S-matrix theory goes too far in the other direction, however, because it is not designed to describe experiments in which interparticle forces continue to act while momentum measurements are being performed. The forces that we best understand can behave in this way, namely the long-range interactions of electromagnetism and gravity; in its current form S-matrix theory can

at most describe short-range interactions. I shall have more to say later about the problem of electromagnetism. For the moment, let me remark only that the difficulty here has been obscured by the concentration on S-matrix properties shared with perturbation field theory. In perturbation theory one does not usually consider persistent forces.

The first two of the five Gunson-Olive-Stapp S-matrix principles are clean and noncontroversial: (1) Lorentz invariance and (2) decomposition into connected parts. No comment is required about Lorentz invariance, which was emphasized already by Heisenberg in 1943, but the decomposition law is perhaps less familiar. It represents the obvious physical fact that independent, uncorrelated events can take place concurrently, and it states that any S-matrix element may be broken into sums of products of "connected parts," each depending on a different and nonoverlapping subset of particle momenta and spins and multiplied by the appropriate energy-momentum conservation δ function. Subsequent S-matrix axioms relate to these connected parts, which do not contain δ functions.

The third axiom is that of the correspondence between particles and poles in connected parts--a connection apparently noticed first by Kramers. Here we encounter some division of opinion. In the recent work of Olive[6] the pole-particle correspondence is postulated only in physical regions, where it is directly related to the possibility of a causal sequence of macroscopically spaced collisions between stable particles. Poles in unphysical regions, in particular those associated with unstable particles, are then to be deduced from the two axioms still to come. Such

a sharp distinction between stable and unstable particles at the axiomatic level disturbs me, however. Physically it is clear that the transition between stability and instability is a smooth one; mathematically the dynamical considerations that predict resonances on the basis of the final two axioms can just as well predict bound states.

To my mind it is more satisfactory to treat all poles on a common basis, regardless of their location. As Gunson has argued,[3] once the possibility of analytic continuation is accepted, any part of the complex momentum space is in principle accessible--through sufficiently accurate measurements in the physical region, followed by extrapolation. You may object that the stable particles necessarily play a special role in S-matrix theory, since they define the space in which the S matrix acts. It is unnecessary to speak of such a space, however, if one deals directly with connected parts. It turns out that the residues of all poles in connected parts are factorizable, each factor being itself a connected part for a smaller collection of particles, one of which corresponds to the original pole. As Zwanziger and Stapp have pointed out,[8,9] if the pole in question corresponds to an unstable particle, one can thereby uniquely define a connected part involving this unstable particle. Connected parts for any collection of particles--stable or unstable--may democratically be defined in such a manner.

Factorizability of residues, by the way, as shown by Stapp[9] and others, seems to be a consequence of the final two principles. Were factorizability not to emerge, however, the particle concept itself

would be impossible. Here is an example of "bootstrapping" in axiomatics.

And now a difficulty: If the photon has a strictly zero mass, the infrared phenomenon spoils the simple pole-particle correspondence. Put more simply, the basic notion of an initial or final state with a definite number of particles loses meaning when, regardless of the precision of energy-momentum determinations, the number of low-frequency photons is uncontrollable. This again is a facet of electromagnetism obscured by perturbation field theory, which considers only finite numbers of photons. Some S-matrix theorists believe the infrared problem to be an inessential difficulty because it has been surmounted in field theory and because the photon, after all, is "just another particle." I do not agree. I believe there is vital significance in this mismatch between electromagnetism and current S-matrix axiomatics. I believe the photon to be an aristocrat.

Returning to our catalogue, the fourth principle, roughly speaking, associates branch points in connected parts with channel thresholds and defines the nature of each such isolated singularity by giving a formula for the change in a connected part when a single circuit is made around the branch point. The discontinuity formula, long known in a variety of expressions, has been stated by Gunson[3] and Olive[10] in an elegant general rule:

$$T_{ab}(s) - T_{ab}(s_n) = \int_n T_{an}(s)\, T_{nb}(s_n) ,$$

with $S = 1 + T$. The point s_n lies directly below the point s on

the next Riemann sheet, reached by a single circuit around the singularity in question. The integral runs over all variables of that channel whose threshold lies at the branch point. Note that T_{ab} is in general not a connected part and contains δ functions. These, however, can be shown to appear in a consistent way on the two sides of the equation;[3,6] after cancellation of the δ functions there remains a formula involving connected parts only. A definition of the physical sheet and the physical region must accompany the discontinuity formula to make it complete and to guarantee unitarity. These matters have been discussed with care both by Olive[6] and by Stapp.[11] Olive, in fact, prefers to start with unitarity in physical regions and to derive the discontinuity formula therefrom. However, as Zwanziger has emphasized,[8] threshold branch points for channels containing unstable particles are described by this same discontinuity formula, therefore the democratic character of the axioms probably can be maintained.

It goes without saying that we are in trouble here again with photons. Adding one or several zero-mass particles to a channel fails to displace the threshold, and the unique association of isolated branch points with individual channels is lost. What recipe may replace the discontinuity formula is not known. Unitarity of the S matrix in physical regions is equivalent to the discontinuity formula, so in losing the latter we have lost unitarity. Indeed, if we look back over our catalogue, it appears that only the axiom of Lorentz invariance has failed to clash with electromagnetism; there is no avoiding the conclusion

that the theory presently under consideration describes a world without photons. Fortunately we seem to see a good approximation to such a world if we look only at strongly interacting particles.

One final axiom remains to complete the S-matrix properties guessed on the basis of perturbation field theory. This fifth axiom postulates that, aside from particle-poles and threshold branch points, the only other singularities of connected parts are those implied by the analytic continuation of the set of discontinuity formulas. This postulate, which I shall call maximal analyticity of the first degree, has a marked bootstrap aspect, meriting discussion.

The additional singularities are generated through the integration over products of connected parts in the discontinuity formulas. They arise by the "pinching" of combinations of singularities. The simplest type of Landau singularity, as they are called, arises from the pinching of a pair of particle poles, but a pole also may pinch with a threshold branch point or with a Landau singularity; two Landau singularities may pinch with each other, and so on. Principle #2 starts us off (presumably) with an infinite number of particle poles and certainly principle #3 gives an infinite number of threshold branch points, so the full set of singularities, even with maximal analyticity, is enormously complicated. Now principle #3 requires a definition of the physical sheet. How can this be done before the secondary branch points are understood? In fact the combined set of axioms at this point runs the risk of a contradiction, because we evidently take for granted that analytic continuation in

momenta is possible at least on the physical sheet and in some part of the adjoining sheets. Isolated singularities (poles and branch points) cause no trouble in this respect, but what happens if singularities so multiply through the discontinuity formulas as to become dense?

At present it is a matter of faith that such does not happen and that the physical sheet always can be identified. This faith has a concrete basis, however, in experience with iterative calculations--where a phenomenon has been observed which I shall call the "Mandelstam progression." Mandelstam discovered that with four-line connected parts (two incoming and two outgoing particles), if you start with the physical-sheet particle poles and threshold branch points and generate Landau singularities by an iterative procedure, there is a systematic tendency for the new physical-sheet singularities from each iteration to be located farther from the physical region than the previous set.[12] Recently Hwa has found this same phenomenon in five-line connected parts.[13] Fluctuations may occur in the progression (e. g., anomalous thresholds) but there is an indication that the singularities in a given finite region of the complex momentum space do not continue indefinitely to increase in number. A key requirement of S-matrix theory is to establish that such is really the case.

Recently a quartet of Parisians, Fotiadi, Froissart, Lascoux and Pham, has developed a powerful approach to the Landau singularities that eventually may prove strong enough to answer this question.[14] Alarmingly, the mathematical basis of their new method is homology theory, with which

few physicists are familiar at present; but a multisheeted Riemann surface in several complex variables is undeniably a matter of topology. Advocates of the S-matrix approach cannot evade this circumstance.

At the risk of being tedious I once again call attention to the importance in S-matrix theory of the absence of zero-mass particles. The Mandelstam progression has a chance to operate only because, among strongly interacting particles, there are none with vanishing rest mass. The smallest particle masses necessarily provide the scale for the spacing of singularities.

Although the above tentative list of five principles will require further refinement and study, it is plausible from the work of Gunson, Olive, Stapp, and especially of Mandelstam[15] that all the significant physical content of perturbation field theory is contained therein. In fact, if one wishes to treat a few spin-0 or spin-$\frac{1}{2}$ particle poles as given, with small residues, the same power-series expansions apparently can be developed from these principles as are derived from a Lagrangian with a corresponding set of fields. No further assumptions are needed. We have seen, however, that if the current version of S-matrix theory describes anything it can only be the world of strongly interacting particles. With electromagnetism turned off, not only does the photon disappear but so do the primary interactions of electrons and muons, which are electromagnetic. Not even the residual weak interactions would be tractable if the electron mass, as often conjectured, were to vanish in the absence of electromagnetism; electron-neutrino pairs

would become just as awkward for the S matrix as are photons. Now, to be restricted to strong interactions is not necessarily a fatal flaw of our theory, but perturbation expansions cannot then be trusted. The content of the theory has to be sought by methods other than power series in coupling constants.

MAXIMAL ANALYTICITY OF THE SECOND DEGREE

Perturbation field theory tolerates the arbitrary insertion of elementary particles of spin 0 and 1/2 , and even of spin 1 if coupled to an appropriately conserved current. It has, however, never been established that the perturbation power-series are meaningful, so one cannot infer that our five S-matrix principles necessarily permit poles corresponding to elementary particles. I refer here to poles whose positions and residues can be arbitrarily assigned without violating the axioms. Perhaps no such poles can be tolerated, in which case there may be no need for further principles to complete a theory of strong interactions. Perhaps only one set of poles is consistent, and that is the one we find in nature. The plausibility of such a conjecture is enhanced by the difficulty of fitting photons or leptons into the S matrix. These are the particles that still appear to us as "elementary." None of the strongly interacting particles has such an appearance.

Despite its attractiveness, the conjectured sufficiency of the above five axioms lacks support from the approximation procedures currently used to implement these principles. What is the basis of these procedures? It is that connected parts in a local region of the

complex momentum-space are dominated by "nearby" singularities, the collective effect of distant singularities being representable by boundary conditions. Instead of a series ordered by powers of coupling constants, we have a series of singularities, ordered according to increasing distance from the point of interest. Ignoring all singularities beyond a certain distance leads, through the Cauchy formulae, to an approximate set of integral equations for the connected parts--provided that boundary conditions at infinity are added. These boundary conditions do not seem entirely to be contained in the five axioms.

How are the boundary conditions chosen? If one believes in nuclear democracy, one chooses the solution to any particular approximate set of equations that causes all poles to be dynamically determined--like the bound states of a potential. This is the so-called "bootstrap" dynamics, and it necessarily leads to the property that all poles are continuable in angular momentum. A converse conjecture has been made that an adequate general formulation of the necessary boundary condition is simply to require that all poles be Regge poles.[16] In his recent work Mandelstam has given some support to this conjecture.[17]

Whether or not the uniform requirement of Regge continuation is sufficient, the object of the boundary condition is to eliminate all "unnecessary" poles. For that reason I like to call the sixth requirement "maximal analyticity of the second degree." Let me emphasize the possibility, before leaving this point, that the apparent necessity for a sixth condition may be a consequence of our approximation procedure. In neglecting all

singularities beyond a certain distance, an asymptotic requirement implicitly contained in the first five principles may have been lost.

CONCLUSION

To summarize the current S-matrix picture, which apparently is relevant only to strong interactions, three different although not independent questions can be identified.

1. Can the fifth principle, maximal analyticity of the first degree be solidified? The problem here is the definition of the physical sheet and the propagation of singularities via the discontinuity formula; major progress may require exploitation of homology theory.

2. Can a bootstrap boundary condition, our sixth principle, be found that determines in a democratic fashion all the particle poles? Continuation in angular momentum is a key consideration here.

3. Can an approximation procedure based on nearest singularities plus the boundary condition be made systematic, and then successfully employed to predict the strongly interacting particles?

I should remark parenthetically that my own optimistic feelings about the first two questions are based largely on the qualitative success in the understanding of strong interactions already achieved by crude dynamical applications of the nearest-singularity principle. I can see no reason for this success if a meaning fails to exists for maximal analyticity of first and second degree.

These three questions are tied together by asymptotic considerations. A finite number of Mandelstam-type iterations produces an acceptably finite

density of singularities; the difficulty aspect of question #1 therefore is to show that asymptotically the singularities keep moving to greater and greater distances. If and when the asymptotic behavior of this progression becomes understood, question #2 may disappear; that is, it may turn out to be unnecessary to add a pole-determining boundary condition. In any event an understanding of the most distant singularities should clarify whether dynamical calculations can in fact be based on an ordering of singularities according to distance.

In closing I have three remarks about electromagnetism. First of all, we need not be distraught because the currently defined S matrix is too limited to describe this most familiar of the interactions. All physical theories of the past have been limited to special ranges of phenomena and have been replaced in time by broader theories. It is probably hopeless at present to construct a complete theory; the problem is to identify those areas of nature than can meaningfully be approximated as separate. Strong interactions appear to constitute such a subdivision. Second, it has already developed in practice that, given the strong-interaction S matrix, one can find a recipe for adding electromagnetic perturbations of finite order in the fine-structure constant. What remains obscure is the handling of persistent electromagnetic effects or, if you like, indefinite numbers of soft photons. In fact, Zwanziger[18] and Weinberg[19] have shown that for reactions which can be characterized approximately as involving a finite number of real photons, the special properties of electromagnetism usually associated in field theory with

gauge invariance follow automatically in momentum space from Lorentz invariance and the zero photon mass. Here perhaps is an indication that a concept broader than the S matrix, but still based on the momentum-energy continuum rather than the space-time continuum, eventually will encompass particles of zero mass.

Finally, let me point out the logical incompleteness of current S-matrix theory in its failure to provide the mechanism by which particle momenta are to be experimentally measured. The actual determination of momentum, as well as its definition, requires a coarse-grained macroscopic space-time measurement that never can be described through the present conception of the S matrix. In practice such measurements always depend on electromagnetic interactions; a little thought suggests it may be impossible, in principle, to perform a momentum determination without employing the weak long-range forces characteristic of electromagnetism. The zero-mass photon, together with the small magnitude of the fine-structure constant, makes it feasible for one isolated system to observe another, and thereby plays a role that cannot be filled by any of the strongly interacting particles.

If this view is correct the photon mass and the fine-structure constant are interlocked with the theory of measurement itself, perhaps even with the meaning of macroscopic space-time, and their values never will be explained purely by dynamical considerations. In constrast the parameters of strong interactions, having no connection with the measurement process, have a chance of being determined through dynamics. My

survey today has described the continuing attempt to formulate a purely dynamical theory of the strong interactions.

Anyone looking on from the outside may feel pessimistic about the possibility of developing the consequences of such a theory, but the history of the subject offers encouragement. Progress has been sustained by efforts to understand relatively small and specific aspects of the problem, usually motivated by experiment. Herein lies a secret weapon of S-matrix theory that guarantees its vitality. Because the fundamental element of the theory, the connected part, is susceptible to direct measurement, one is able to cheat by peeking at the solution nature has found for fiercely nonlinear and circular equations. Knowing aspects of the solution, even though the relation to a set of fundamental axioms may be obscure, gives S-matrix theorists an enormous advantage.

A good example is the circumstance that all total cross sections appear to approach constants at high energy. This simple empirical fact has been of little help to field theorists, but it suggests a general constraint on four-line connected parts that has been a powerful stimulant to S-matrix theory, particularly in connection with the bootstrap boundary condition and Regge poles.

An even better example is the Mandelstam representation for connected parts involving especially stable particles,[12] a representation which was motivated to a considerable extent by experimental results on pion-nucleon scattering and which has shed light on many observed aspects of strong interactions. It was the experimental success of Mandelstam's conjecture that encouraged the more general idea of maximal analyticity.

Ironically it has never been shown that maximal analyticity implies the entire detailed content of the Mandelstam representation; fortunately such a demonstration is not essential to the S-matrix program.

Perhaps the most important hint from experiment has been the absence of distinguishing attributes among the observed poles associated with strongly interacting particles. Because there exists in S-matrix theory no concept more primitive than poles, one has thereby been led to the idea of nuclear democracy and the bootstrap boundary condition. Field theory, on the other hand, must contend with the perplexing possibility that, even if all strong poles have an equivalent status, certain of the underlying fields may be more fundamental than others.

You may have been struck by the absence from this survey of symmetry considerations, apart from Lorentz invariance. This was not an oversight, but represents a growing belief that arbitrarily postulated symmetries have no more place in the basic theoretical structure than do arbitrarily postulated particles. The presence in strong interactions of SU_2 and partial SU_3 symmetries, as well as time reversal and parity, cannot be denied; but neither, for example, can the existence of the pion and the nucleon as especially stable particles. Confusion about such questions arises because in special limited applications of S-matrix theory the existence of certain symmetries and particles is often added to the list of basic principles. There is room, however, to hope that all strongly interacting particles and symmetries ultimately will emerge together as bootstrap consequences of the five or six principles we have discussed here today. Many studies of the so-called crossing

matrices, which I have no time to describe here, tend to encourage this hope.

My final remark is directed to a question raised at the beginning: What can the S-matrix approach teach us that cannot just as well be learned from field theory? Perhaps nothing. Perhaps a future field theory will somehow describe a nuclear democracy; but then how will this field theory recognize the distinction between electromagnetic and nuclear interactions? The original idea behind field theory, after all, was that every interaction is like electromagnetism. The absence of a classical limit for quantum fields associated with massive particles is ignored in the properties assigned to these fields. Conversely the assignment of a nonzero mass to the photon seems perfectly allowable in field theory.

S-matrix theory, in contrast, permits no doubt that the zero mass of the photon gives this particle a distinguished status, outside the dynamical bootstrap. Furthermore, with the emphasis on physical observability, one becomes sensitive to a possible connection between the unusual photon properties and the basic requirement underlying all of physics that one isolated system be capable of observing another. We are approaching the time when this requirement must be searchingly examined. I do not see how it <u>can</u> be examined in any framework that fails to rest squarely on physical measurements themselves. The statement is often made that S-matrix theory destroys the unity of physics by placing electromagnetism in a separate category from nuclear interactions; but without such a separation, there would be no physics.

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