THE SECOND LAW OF THERMODYNAMICS AND THE GLOBAL CLIMATE SYSTEM: A REVIEW OF THE MAXIMUM ENTROPY PRODUCTION PRINCIPLE

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[1] The long-term mean properties of the global climate system and those of turbulent fluid systems are reviewed from a thermodynamic viewpoint. Two general expressions are derived for a rate of entropy production due to thermal and viscous dissipation (turbulent dissipation) in a fluid system. It is shown with these expressions that maximum entropy production in the Earth's climate system suggested by Paltridge, as well as maximum transport properties of heat or momentum in a turbulent system suggested by Malkus and Busse, correspond to a state in which the rate of entropy production due to the turbulent dissipation is at a maximum. Entropy production due to absorption of solar radiation in the climate system is found to be irrelevant to the maximized properties associated with turbulence. The hypothesis of maximum entropy production also seems to be applicable to the planetary atmospheres of Mars and Titan and perhaps to mantle convection. Lorenz's conjecture on maximum generation of available potential energy is shown to be akin to this hypothesis with a few minor approximations. A possible mechanism by which turbulent fluid systems adjust themselves to the states of maximum entropy production is presented as a self-feedback mechanism for the generation of available potential energy. These results tend to support the hypothesis of maximum entropy production that underlies a wide variety of nonlinear fluid systems, including our planet as well as other planets and stars. *Index Terms:* 3220 Mathematical Geophysics: Nonlinear dynamics; 3309 Meteorology and Atmospheric Dynamics: Climatology (1620); 3379 Meteorology and Atmospheric Dynamics: Turbulence; 9820 General or Miscellaneous: Techniques applicable in three or more fields; *Keywords:* thermodynamics, global climate, maximum entropy production, energetics
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> We must attribute to heat the great movements that we observe all about us on the Earth. Heat is the cause of currents in the atmosphere, of the rising motion of clouds, of the falling of rain and of other atmospheric phenomena ... .
>
> Sadi Carnot (1824)

1. INTRODUCTION

[2] The opening words of Carnot's original treatise on thermodynamics provide a good starting point for this review paper. We consider that Carnot's view contains invaluable insight into the subject, which seems to have been lost from our contemporary view of the world. Carnot regarded the Earth as a sort of heat engine, in which a fluid like the atmosphere acts as working substance transporting heat from hot to cold places, thereby producing the kinetic energy of the fluid itself. His general conclusion about heat engines is that there is a certain limit for the conversion rate of the heat energy into the kinetic energy and that this limit is inevitable for any natural systems including, among others, the Earth's atmosphere. His suggestion on the atmospheric heat engine has been rather ignored. It is the purpose of this paper to reexamine Carnot's view, as far as possible, by reviewing works so far published in the fields of fluid dynamics, Earth sciences, and nonequilibrium thermodynamics.

[3] Figure 1 shows a schematic of energy transport processes in a planetary system composed of the Earth, the Sun, and outer space. Shortwave radiation emitted from the Sun with a brightness temperature of about 5800 K is absorbed by the Earth, mainly in the equatorial region. This energy is transported poleward through direct motions of the atmosphere and oceans (the gen-

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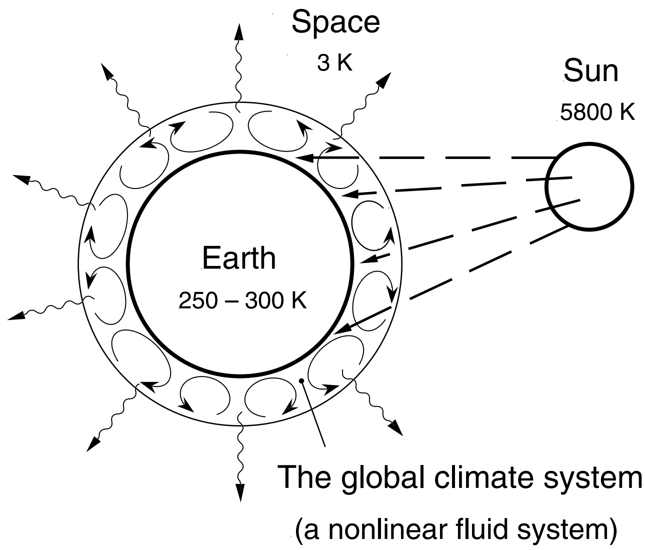


Figure 1. A schematic of energy transport processes in the planetary system of the Earth, the Sun, and space. The Earth receives the shortwave radiation from the hot Sun and emits longwave radiation into space. The atmosphere and oceans act as a fluid system that transports heat from the hot region to cold regions via general circulation.

eral circulation). The energy is finally reemitted to space via longwave radiation. Thus there is a flow of energy from the hot Sun to cold space through the Earth. In the Earth's system the energy is transported from the warm equatorial region to the cool polar regions by the atmosphere and oceans. Then, according to Carnot, a part of the heat energy is converted into the potential energy which is the source of the kinetic energy of the atmosphere and oceans. In this respect, the Earth's system can be regarded as a heat engine operating between thermal reservoirs with different temperatures (equator and poles). The determination of the strength of the circulation, and hence the rate of heat transport, constitutes a fundamental problem in thermodynamics of the general circulation [e.g., *Lorenz*, 1967].

[4] *Lorenz* [1960] suspected that the Earth's atmosphere operates in such a manner as to generate available potential energy at a possible maximum rate. The available potential energy is defined as the amount of potential energy that can be converted into kinetic energy. Independently, *Paltridge* [1975, 1978] suggested that the mean state of the present climate is reproducible as a state with a maximum rate of entropy production due to horizontal heat transport in the atmosphere and oceans. Figure 2 shows such an example [*Paltridge*, 1975]. Without considering the detailed dynamics of the system, the predicted distributions (air temperature, cloud amount, and meridional heat transport) show remarkable agreement with observations. Later on, several researchers investigated Paltridge's work and obtained essentially the same result [*Grassl*, 1981; *Shutts*, 1981; *Mobbs*, 1982; *Noda and Tokioka*, 1983; *Sohn and Smith*, 1993, 1994; *Ozawa and Ohmura*, 1997; *Pujol and Llebot*, 1999a, 1999b]. His suggestion was criticized by *Essex* [1984], however, since a predominant amount of entropy production is due to direct absorption of solar radiation at the Earth's surface, which was a missing factor in Paltridge's work. Since then, the radiation problem has been a central objection to Paltridge's work [e.g., *Lesins*, 1990; *Stephens and O'Brien*, 1993; *Li et al*., 1994; *Li and Chylek*, 1994]. As we shall discuss in section 3, the large background radiative down-conversion of energy from solar to terrestrial temperatures is essentially a linear process which is irrelevant to the maximized process related to nonlinear turbulence. In fact, *Ozawa and Ohmura* [1997] applied the maximum condition specifically to the entropy production associated with the turbulent heat transport in the atmosphere and reproduced vertical distributions of air temperature and heat fluxes that resemble those of the present Earth. Thus it is likely that the global climate system is regulated at a state with a maximum rate of entropy production by the turbulent heat transport, regardless of the entropy production by the absorption of solar radiation [*Shimokawa and Ozawa*, 2001; *Paltridge*, 2001]. This result is also consistent with a conjecture that entropy of a whole system connected through a nonlinear system will increase along a path of evolution, with a maximum rate of entropy production among a manifold of possible paths [*Sawada*, 1981]. We shall resolve this radiation problem in this paper by providing a complete view of dissipation processes in the climate system in the framework of an entropy budget for the globe.

[5] The hypothesis of the maximum entropy production (MEP) thus far seems to have been dismissed by some as coincidence. The fact that the Earth's climate system transports heat to the same extent as a system in a MEP state does not prove that the Earth's climate system is necessarily seeking such a state. However, the coincidence argument has become harder to sustain now that *Lorenz et al*. [2001] have shown that the same condition can reproduce the observed distributions of temperatures and meridional heat fluxes in the atmospheres of Mars and Titan, two celestial bodies with atmospheric conditions and radiative settings very different from those of the Earth. A popular account of this work is given by *Lorenz* [2001a] and *Lorenz* [2003].

[6] Similar suggestions have been proposed in the general field of fluid dynamics. For thermal convection of a fluid layer heated from below (i.e., *Bénard* [1901] convection), *Malkus* [1954] suggested that the observed mean state represents a state of maximum convective heat transport. For turbulent flow of a fluid layer under a simple shear, *Malkus* [1956] and *Busse* [1970] suggested that the realized state corresponds to a state with a maximum rate of momentum transport. Their approach is now called the "optimum theory" or "upper bound theory" and is well known in the field [e.g., *Howard*, 1972; *Busse*, 1978]. Their suggestions were recently shown to be unified into a single condition in which the rate of entropy production by the turbulent

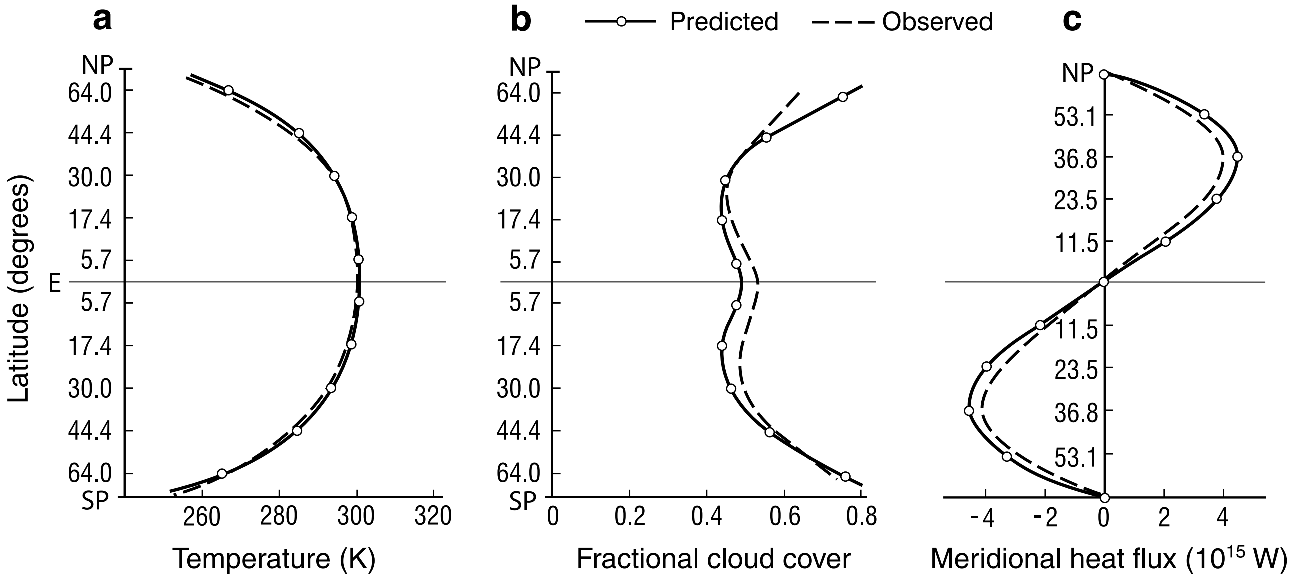


Figure 2. Latitudinal distributions of (a) mean air temperature, (b) cloud cover, and (c) meridional heat transport in the Earth. Solid line curves indicate those predicted with the constraint of maximum entropy production (equation (9)), and dashed lines indicate those observed. Reprinted from *Paltridge* [1975] with permission from the Royal Meteorological Society.

dissipation (thermal and viscous dissipation) is at a maximum: $\dot{S}_{\text{turb}}$ = max [*Ozawa et al.*, 2001]. Thus the maximum transport properties of heat and momentum hitherto suggested, as well as the maximum entropy production in the climate system, can be seen to be a manifestation of the same state of $\dot{S}_{\text{turb}}$ = max.

[7] Despite the seeming plausibility of the maximum entropy production (MEP) hypothesis and its potential importance to a wide variety of nonlinear systems including our planet, as far as we know, there is no review paper on this subject. We shall therefore start this paper with a definition of thermodynamic entropy so that a nonspecialist can follow the basic concepts in this subject (section 2). Then we shall see how the MEP hypothesis can explain the mean properties of various kinds of fluid systems, e.g., the Earth's climate (section 3), climates on other planets (section 4), mantle convection (section 5), and transport properties of turbulence (section 6). In section 7 we shall discuss conditions to be satisfied for fully developed turbulence, i.e., stability criteria for turbulence and time constants of the fluid system and the surrounding system. Prigogine's principle of minimum entropy production [*Prigogine*, 1947] is shown to be a different case in this respect. In section 8 we shall examine a generation rate of available potential energy in the atmosphere proposed by *Lorenz* [1955]. It will be shown that the available potential energy is dissipated by thermal and viscous dissipation (the turbulent dissipation) in the fluid system. In a steady state the generation rate is balanced by the dissipation rate, and Lorenz's conjecture on maximum generation of the available potential energy [*Lorenz*, 1960] is shown to be akin to MEP due to the turbulent dissipation. Finally, we shall present a possible mechanism by which a turbulent fluid system adjusts itself to the MEP state on the basis of a feedback process for the generation of the available potential energy. It is hoped that the present attempt to unify the thermodynamic properties and the maximum principles will be an apt starting point toward a general understanding of the nature of the forced-dissipative systems in general, including our planet.

2. BASIC CONCEPTS

2.1. Thermodynamic Entropy

[8] Entropy of a system is defined as a summation of "heat supplied" divided by its "temperature" [*Clausius*, 1865]. If a certain small amount of heat δQ is supplied quasi-statically to a system with an absolute temperature of T, then the entropy of the system will increase by

$$dS \equiv \frac{\delta Q}{T}, \tag{1}$$

where S is the entropy of the system, d represents an infinitesimal small change of a state function, and δ represents that of a path function. Heat can be supplied by conduction, by convection, or by radiation. The entropy of the system will increase by equation (1) no matter which way we may choose. When we extract the heat from the system, the entropy of the system will decrease by the same amount. Thus the entropy of a diabatic system, which exchanges heat with its surrounding system, can either increase or decrease, depending on the direction of the heat exchange. This is not a violation of the second law of thermodynamics since the entropy increase in the surrounding system is larger. The

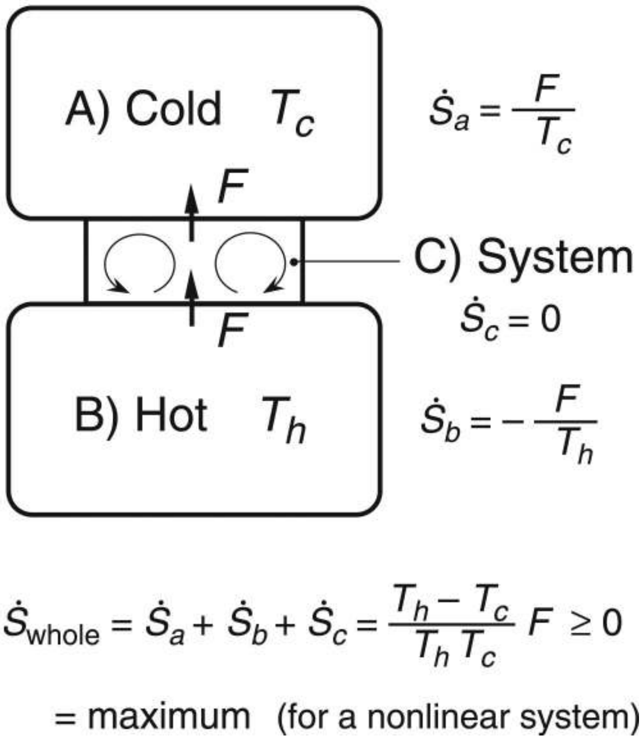


Figure 3. A schematic of heat transport through a small system (C) between two thermal reservoirs with different temperatures (A, cold and B, hot). By the heat transport from hot to cold, entropy of the whole system increases. In the case of a fluid system in a supercritical condition, the rate of entropy production tends to be a maximum among all possible states.

second law (the law of entropy increase) is valid for a whole (isolated) system. When we sum up all the changes of entropy of interacting subsystems, the total change must be nonnegative. This is a statement of the second law of thermodynamics.

[9] In this paper we use "diabatic" for systems that exchange heat (and/or work) with their surroundings. Such systems have been called "closed" in some cases [e.g., *De Groot and Mazur*, 1962, chapter 3], while they are regarded as "open" in some textbooks [e.g., *Landau and Lifshitz*, 1937, section 2; *Kittel and Kroemer*, 1980, chapter 2]. To avoid any confusions in terminology, we use "diabatic" for systems with thermal and mechanical interactions, following a definition by *Kuiken* [1994].

2.2. Heat Flow and Entropy Production

[10] Let us consider two large thermal reservoirs with different temperatures: T_c for the cold reservoir (designated A) and T_h for the hot one (B), as shown in Figure 3. Let us then connect the thermal reservoirs with a small system (C) so that heat can flow from the hot (B) to the cold reservoir (A). The small system (C) can be a fluid system or a solid system, but let us first consider a solid system, such as a metal block. In this case, heat is transported by heat conduction through the metal block. In a steady state the flow rate is known to show a linear relationship with the temperature difference; it is proportional to the applied temperature gradient.

[11] Let us then calculate the rate of increase of entropy in the whole system (A, B, and C) by a steady heat flow through the small system. Let F be the flux of heat through the system per unit time. Then, according to equation (1), the entropy of the cold reservoir will increase by F/T_c. On the other hand, since the heat is flowing out from the hot reservoir (B), its entropy will decrease by $-F/T_h$. The entropy of the small system (C) remains unchanged so long as a steady state can be assumed for this system. Then, the change rate of entropy of the whole system by the heat flow is given by

$$\dot{S}_{\text{whole}} = \dot{S}_a + \dot{S}_b + \dot{S}_c = \frac{F}{T_c} - \frac{F}{T_h} = \frac{T_h - T_c}{T_h T_c} F \geq 0, \tag{2}$$

where $\dot{S}_{\text{whole}} = dS_{\text{whole}}/dt$ is the change rate of the entropy in the whole system, and $\dot{S}_a$, $\dot{S}_b$, and $\dot{S}_c$ are those in the subsystems, respectively. The inequality in equation (2) corresponds to the fact that heat flows from hot to cold ($F \geq 0$) and is a consequence of the second law of thermodynamics.

[12] Equation (2) represents the increase rate of entropy in the whole system by the irreversible heat transport from hot to cold and can thus be seen as the rate of entropy production inside the small system [see, e.g., *Zemansky and Dittman*, 1981, section 8–13]. It should be borne in mind, however, that the rate of entropy production is related to the increase rate of entropy in the whole system (system and the surroundings); it is related not to the state of the specific small system (C), but to that of the whole system. If the small system is in a steady state, then the produced entropy as in equation (2) is completely discharged into the surrounding system, thereby increasing the entropy (a state function) of the surrounding system. In other words, we can equally say that the state of the surrounding system is approaching its equilibrium state by the heat flow from hot to cold through the small system. In Boltzmann's statistical interpretation of entropy [*Boltzmann*, 1896, section 8], the probability of the macroscopic state of the surrounding system is increasing by equation (2) as a result of the heat flow from hot to cold. The same is true for any heat transport processes from hot to cold, provided that no part of the heat is stored as mechanical energy (work) in the system. If we observe the composed system for a considerably long period of time ($t \to \infty$), then the heat transport will make the temperature difference negligible. This final state is called thermodynamic equilibrium, in which the entropy of the whole system is at a maximum. In this respect, heat is transported from hot to cold so as to recover the equilibrium of the surrounding system that has been kept in a nonequilibrium state (Figure 3).

2.3. Generation and Dissipation of Work

[13] Consider now that the heat transport from the hot to the cold reservoir is carried out in a reversible manner, e.g., by a Carnot cycle, rather than the irreversible heat conduction. Then, a part of the heat energy flown into the small system ($F_{\rm in}$) can be converted into mechanical energy (or work W) in the system. In this case, the outflow rate of heat from the system ($F_{\rm out}$) can be less than the inflow rate: $F_{\rm out} = F_{\rm in} - W$. The second law of thermodynamics requires that the total change rate of entropy in the whole system by this conversion process must be larger than zero:

$$\dot{S}_{\rm whole} = \frac{F_{\rm out}}{T_c} - \frac{F_{\rm in}}{T_h} = \frac{F_{\rm in} - W}{T_c} - \frac{F_{\rm in}}{T_h} \geq 0. \tag{3}$$

From inequality (3), we will get the maximum possible work that can be generated during this heat transport process:

$$W \leq \frac{T_h - T_c}{T_h} F_{\rm in} = \eta_C F_{\rm in}, \tag{4}$$

where $\eta_C = 1 - T_c/T_h$ is called the Carnot efficiency [*Carnot*, 1824]. One can see from equation (4) that the generation rate of maximum possible work is proportional to the flow rate of heat and the temperature difference.

[14] It should be noted that the maximum work (equation (4)) is not in general attainable for natural systems where irreversible processes (e.g., heat conduction, frictional dissipation) are inevitable. For example, in pure heat conduction discussed earlier, there is no generation of work and therefore no efficiency. The same thing happens in natural systems: A part of the heat is conducted directly to the cold reservoir without doing any work. This "leakage" of heat results in the reduction of W and the enlargement of the rate of entropy production. In addition, there is a natural tendency of dissipation of mechanical energy into heat energy by various kinds of irreversible processes with, e.g., viscosity in fluids, friction at material surfaces, and plasticity of solids [e.g., *Ozawa*, 1997]. These irreversible conversions of mechanical energy into heat energy (δQ) lead to additional contributions to the entropy production ($\delta Q/T$) in equation (3). When all mechanical energy (W) returns to heat energy, there can be no reduction in the outflow rate of heat in a steady state ($F_{\rm out} = F_{\rm in}$), and the rate of entropy production becomes identical to equation (2). In short, both thermal and mechanical dissipation lead to the entropy production in the whole system.

2.4. Entropy Production in Fluid Systems

[15] Let us consider a fluid system for the small system (C) in Figure 3. Then the system is identical to a convection system investigated by *Bénard* [1901]. In this system, fluid is heated at the bottom and cooled at the top, and the resultant expansion and contraction lead to a "top-heavy" density distribution that is gravitationally unstable. The potential energy in this top-heavy density distribution is generated by the differential heating and results from the conversion of the heat energy into the mechanical energy (work). When the temperature difference (or the potential energy) becomes larger than a certain critical value, the fluid is no longer stable against small perturbations, and convective motions tend to develop. *Rayleigh* [1916] investigated the critical condition at which convection starts and showed that the critical condition is related to a dimensionless parameter called a Rayleigh number. The details will be discussed in section 6.1. It should be noted here that once convection starts, the dynamic equation and conservation equations that govern the dynamics of the system become nonlinear, and this nonlinearity makes the analytical solution difficult to obtain.

[16] Once convection starts, the fluid motion itself transports the heat energy, and thereby the total heat flux F increases. The generated potential energy in this case is converted into the kinetic energy of the fluid and then dissipated into heat energy by viscous dissipation. The conversion process is related to the nonlinear dynamic equation and is therefore intricate. However, when the system can be seen to be in a steady state in a statistical sense, the generation rate of the potential energy has to be balanced by the viscous dissipation rate, so long as no part of the kinetic energy is stored in the system, e.g., by a water wheel. Then, the inflow rate of heat should be equal to the outflow rate. In this steady state the rate of entropy production is again expressed by equation (2) no matter what happens in the system.

[17] We can also derive a general expression for the rate of entropy production in a fluid system with an arbitrary shape. The derivation can be found in some textbooks [*Landau and Lifshitz*, 1944; *De Groot and Mazur*, 1962] and some publications [*Shimokawa and Ozawa*, 2001; *Ozawa et al.*, 2001]. So let us just explain the results of the derivation as follows. In principle, the rate of entropy production due to some irreversible processes associated with turbulence in a fluid system can be given by a sum of the change rate of entropy in the system and its surrounding system that interacts heat with the system as

$$\begin{aligned} \dot{S}_{\rm turb} &= \dot{S}_{\rm whole} \\ &= \int_V \frac{1}{T}\left[\frac{\partial(\rho c T)}{\partial t} + {\rm div}(\rho c T \mathbf{v}) + p\, {\rm div}\, \mathbf{v}\right] dV \\ &\quad + \int_A \frac{F}{T}\, dA, \end{aligned} \tag{5}$$

where $\dot{S}_{\rm turb}$ is the rate of entropy production due to turbulence, ρ is the density of the fluid, c is the specific heat at constant volume, T is the absolute temperature, $\mathbf{v}$ is the velocity of the fluid, p is the pressure, V is the volume of the fluid system, A is the surface surrounding the system, and F is the diabatic heat flux at the surface, defined as positive outward. (Material fluxes can also be taken into account by means of chemical potential [see, e.g., *Shimokawa and Ozawa*, 2001].) The first volume integration on the right-hand side is taken over the volume of the system V, and the second surface integration is taken over the boundary surface A. The first term represents the change rate of entropy of the fluid system, and the second term represents that of the surrounding system. If the concerned fluid system is in a steady state in a statistical sense, then the entropy, a state function of the fluid system, should remain unchanged. In this case, equation (5) becomes simply

$$\dot{S}_{\text{turb}} = \int_A \frac{F}{T} dA. \qquad \text{(in a steady state)} \qquad (5')$$

Equation (5′) represents the fact that the entropy produced by some irreversible processes associated with turbulence is completely discharged into the surrounding system through the boundary heat flux (F). In the case of heat conduction discussed in section 2.2, the surface integral of equation (5′) leads to $F/T_c - F/T_h = F\,(T_h - T_c)/(T_h\,T_c)$, which is indeed identical to equation (2).

[18] General equation (5) can be rewritten in a different form. Because of the first law of thermodynamics, the terms in brackets in the volume integral in equation (5) are related to the convergence of diabatic heat flux and the heating rate due to viscous dissipation [*Chandrasekhar*, 1961, section 7; *Ozawa et al.*, 2001] as

$$\frac{\partial(\rho c T)}{\partial t} + \text{div}(\rho c T\,\mathbf{v}) + p\,\text{div}\,\mathbf{v} = -\,\text{div}\,\mathbf{F} + \Phi, \qquad (6)$$

where $\mathbf{F}$ is the diabatic heat flux density due to turbulence (i.e., heat conduction and latent heat transport), and Φ is the dissipation function, representing the rate of viscous dissipation of kinetic energy into heat energy per unit time per unit volume of the fluid. Substituting equation (6) into (5), and transforming the surface integral of F/T into the volume integral of div ($\mathbf{F}/T$) by Gauss's theorem, we obtain

$$\dot{S}_{\text{turb}} = \int_V \mathbf{F} \cdot \text{grad}\left(\frac{1}{T}\right) dV + \int_V \frac{\Phi}{T} dV. \qquad (7)$$

The first term on the right-hand side represents the rate of entropy production by the diabatic heat flux from hot to cold, and the second term represents that by viscous dissipation of the kinetic energy; both terms should be nonnegative. The first term may be called thermal dissipation, and the second one may be called viscous dissipation. (If there is diffusion of material particles, e.g., solute molecules, there will also be a material diffusion term [*Shimokawa and Ozawa*, 2001].) Each term depends on the small-scale gradient of temperature or velocity and is therefore determined by the way of turbulent mixing in the fluid system. For this reason, we shall call the sum of these terms turbulent dissipation. Notice here that the diabatic heat flux $\mathbf{F}$ in equations (6) and (7) does not in principle include the advective heat flux. The advective tansport of heat is caused by a movement of internal energy of fluid from one place to another, but it is essentially a reversible process; one can reverse the heat transport by reversing the movement. However, the advective heat transport, say, a movement of hot water into cold water, leads to a large local temperature gradient at the very front of the advecting fluid, resulting in a considerable amount of entropy production by the heat conduction at the front. This contribution, however, is quite difficult to assess in a large-scale fluid model of which scale of resolution is larger than the dissipation scale [e.g., *Nicolis*, 1999]. For this reason, the alternative expression (equation (5)) has been used to assess the entropy production in conventional fluid models [e.g., *Shimokawa and Ozawa*, 2001, 2002].

[19] A significant consequence of our mathematical manipulation is that even though entropy production is caused by the small-scale dissipation processes associated with turbulence (equation (7)), the total rate is described by the rate of entropy discharge from the system (equation (5′)) so long as the fluid system is in a steady state. As we shall see in section 8, the rate of entropy discharge (equation (5′)) is related to a total generation rate of available energy (i.e., maximum possible work), which in turn cascades down to the smallest scale, and dissipates and produces entropy (equation (7)).

2.5. Maximum Entropy Production by Turbulent Dissipation

[20] The overall physical hypothesis to be discussed in this paper is the notion that a nonlinear system with many degrees of freedom for dynamic motions tends to be in a state with maximum entropy production, among all other possible states. Although the hypothesis of this sort has been hinted at already by several authors [e.g., *Onsager*, 1931; *Félici*, 1974; *Jaynes*, 1980; *Sawada*, 1981], there is only a recent attempt that tries to justify this MEP hypothesis [*Dewar*, 2003]. We shall therefore simply describe this hypothesis in the case of turbulent fluid systems, leaving its possible justification in section 8.2. According to equations (5′) and (7), we will have

$$\dot{S}_{\text{NL}} = \dot{S}_{\text{turb}} = \int_V \mathbf{F} \cdot \text{grad}\left(\frac{1}{T}\right) dV + \int_V \frac{\Phi}{T} dV = \text{maximum}, \qquad (8a)$$

$$\dot{S}_{\mathrm{NL}} = \int_A \frac{F}{T}\, dA = \text{maximum}, \qquad \text{(in a steady state)} \tag{8b}$$

where $\dot{S}_{\mathrm{NL}}$ is the rate of entropy production due to nonlinear processes in the system and corresponds to that by turbulent dissipation in our case. The validity of the MEP hypothesis (equation (8)) should be found in the agreement with observational and experimental evidences. In what follows, we shall examine this hypothesis in the light of various examples and aspects of turbulent fluid systems including our climate system.

3. GLOBAL CLIMATE

3.1. Paltridge's Work and Its Implications

[21] It was *Paltridge* [1975, 1978] who first suggested that the global state of the present climate is reproducible, as a long-term mean, by a state of maximum entropy production. (To be more precise, a minimum exchange rate of entropy was suggested in his 1975 paper, where the exchange rate was defined positive inward. This condition is identical to the maximum entropy discharge into the surrounding system (8b), which corresponds to the maximum entropy production due to the turbulent dissipation (8a) [*Paltridge*, 1978].) He made a simple 10-box model for the entire globe and assumed an energy balance condition for each box (the steady state for each box). Vertical energy transports by shortwave and longwave radiation are represented by empirical functions of surface temperature T and cloud cover θ in each box. There are basically three unknown variables (T, θ, and meridional heat flux F_m by the atmosphere and oceans), and two energy balance equations for the atmosphere and the ocean in each box. Thus, in principle, the problem cannot be solved. In fact, the heat flux F_m is composed of atmospheric and oceanic parts. These two components are separated by observed data [*Paltridge*, 1975] and later by a condition of equal thermodynamic dissipation [*Paltridge*, 1978]. Paltridge sought one more constraint by which realistic distributions of the present climate (T, θ, and F_m) could be reproduced. He found that the constraint is to maximize the following quantity:

$$\sum_i^N \frac{F_{\mathrm{long},i}(\mathrm{TOA}) - F_{\mathrm{short},i}(\mathrm{TOA})}{T_{a,i}} = \text{maximum}, \tag{9}$$

where $F_{\mathrm{long},i}(\mathrm{TOA})$ (>0) is the net rate of emission of longwave radiation from the ith box at the top of the atmosphere (TOA), $F_{\mathrm{short},i}(\mathrm{TOA})$ (>0) is that of absorption (input $-$ output) of shortwave radiation at TOA, $T_{a,i}$ is the mean emission temperature of the ith box (i.e., a characteristic atmospheric temperature), and the summation is taken over all boxes (N). In general, the numerator is negative (input) in the hot equatorial regions and is positive (output) and similar in magnitude in the cold polar regions [e.g., *Peixoto and Oort*, 1992]. Since the mean emission temperature is higher in equatorial regions than in polar ones, the summation in equation (9) should have a positive value. Paltridge suggested that this value is not only positive but also a maximum among all other possible states, and the maximum state corresponds well with the observed mean state of the present climate.

[22] Figure 2 shows the example of latitudinal distributions of the surface temperature T, cloud cover θ, and the meridional heat flux F_m; solid lines are predicted with equation (9), and dashed lines are from observations [*Paltridge*, 1975]. Agreement of the two profiles is remarkable, despite the simple treatments used in his model. Later on, several researchers checked his work and obtained essentially the same results [e.g., *Nicolis and Nicolis*, 1980; *Grassl*, 1981; *Mobbs*, 1982; *Noda and Tokioka*, 1983; *Sohn and Smith*, 1993; *O'Brien and Stephens*, 1995; *Ozawa and Ohmura*, 1997; *Pujol and Llebot*, 1999a, 1999b, 2000a; *Pujol and Fort*, 2002; *Pujol*, 2003].

[23] It is possible to show that equation (9) corresponds to the rate of entropy production due to turbulent dissipation (equation (8)). Strictly speaking, Paltridge's box model is not equivalent to the two thermal reservoirs system with a metal block in stationary states described in section 2.2. Indeed, the atmosphere (or the ocean) may be identified like the small system connecting the two large reservoirs used in section 2.2. The difference with that example is that now the flux coming from one reservoir (e.g., outer space) differs from the flux injected to the other (e.g., ground), since both absorption and scattering processes modify the flux (mainly absorption for the longwave radiation). In a steady state the rate of entropy production due to turbulence is equal to the rate of entropy discharge into the immediate surrounding system, so from equation (8b) we get

$$\dot{S}_{\mathrm{turb}} = \dot{S}_{\mathrm{turb},a} + \dot{S}_{\mathrm{turb},o} = \int_a \frac{F}{T}\, dA + \int_o \frac{F}{T}\, dA$$

$$= \int_A \frac{F_{\mathrm{long}}(\mathrm{TOA}) - F_{\mathrm{short}}(\mathrm{TOA}) - F_{\mathrm{long}}(0) + F_{\mathrm{short}}(0)}{T_a}\, dA$$

$$+ \int_A \frac{F_{\mathrm{long}}(0) - F_{\mathrm{short}}(0)}{T_s}\, dA$$

$$= \int_A \frac{F_{\mathrm{long}}(\mathrm{TOA}) - F_{\mathrm{short}}(\mathrm{TOA})}{T_a}\, dA$$

$$+\int_A [F_{\mathrm{short}}(0) - F_{\mathrm{long}}(0)]\left(\frac{1}{T_a} - \frac{1}{T_s}\right)dA, \quad (10)$$

where $\dot{S}_{\mathrm{turb},a}$ and $\dot{S}_{\mathrm{turb},o}$ are the discharge rate of entropy from the atmosphere and from the ocean (or ground), respectively. Notice that both systems have different characteristic temperatures (T_a for the atmosphere, T_s for the surface of the ocean-ground). $F_{\mathrm{long}}(0)$ (>0) and $F_{\mathrm{short}}(0)$ (>0) are net longwave and shortwave energy fluxes at the surface. In equation (10) the first integral on the right-hand side of the third equality represents the entropy discharge rate from the atmosphere into the surrounding system (i.e., outer space and ocean-ground), whereas the second integral corresponds to that from the ocean-ground system into the surrounding system (i.e., atmosphere). The minus signs in equation (10) indicate that the flux is inward to the system analyzed (e.g., $F_{\mathrm{short}}(\mathrm{TOA})$ and $F_{\mathrm{long}}(0)$ for the atmosphere and $F_{\mathrm{short}}(0)$ for the ocean-ground system). Note that the radiation fluxes in the atmosphere and oceans used in equation (10) may be exclusively described in terms of turbulent fluxes (the meridional heat flux F_m and the vertical convective heat flux F_c) if the heat energy is assumed to be conserved and does not change with production and dissipation of mechanical energy (section 2.3) in the first approximation.

[24] The first term on the right-hand side of the final identity in equation (10) is identical to equation (9), which represents the rate of entropy production due to horizontal heat transport, and the second term is that due to vertical heat transport (vertical convection). Thus Paltridge's condition is identical to a state with the maximum rate of the entropy production due to the horizontal heat transport. The vertical term was also made maximum by *Paltridge* [1978], who assumed the convective heat transport ($F_c \approx F_{\mathrm{short}}(0) - F_{\mathrm{long}}(0)$) to be a maximum. Strictly speaking, this convective hypothesis differs from that of maximum entropy production if the temperature difference ($T_s - T_a$) changes with changing the convective heat flux. However, a detailed study by *Noda and Tokioka* [1983] showed that the single maximum condition ($\dot{S}_{\mathrm{turb}} = \max$) can reproduce both vertical and horizontal structures of the atmosphere. Thus Paltridge's result (Figure 2) can be seen to be the horizontal aspect of the total maximum field of equation (8).

[25] Paltridge's hypothesis has also been used to study several practical problems, e.g., global warming by an increase of carbon dioxide [*Grassl*, 1981; *Pujol and Llebot*, 2000b], quasi-geostrophic ocean circulation [*Shutts*, 1981], faint Sun paradox in the early stage of the Earth [*Gerard et al.*, 1990], and application to planetary atmospheres other than that of the Earth [*Lorenz et al.*, 2001]. The reasonable results obtained from these studies also support the hypothesis of equation (8) as well as (9). The MEP hypothesis has also been analyzed in several studies by parameterizing the turbulent fluxes with the eddy diffusivity approach [*Golitsyn and Mokhov*, 1978; *Wyant et al.*, 1988; *Lorenz et al.*, 2001; *Pujol and Fort*, 2002; *Pujol*, 2003]. In essence, their approach consists of expressing the turbulent fluxes as a function of the temperature gradient by using a diffusivity coefficient, which is finally tuned to maximize equation (8). This procedure is slightly different from Paltridge's in the sense that the former assumes the same functional dependence between the turbulent heat flux and the temperature gradient everywhere in the system, while the latter allows a variable dependence so as to produce a best fit maximum in $\dot{S}_{\mathrm{turb}}$. Recent studies show, however, that the two approaches produce virtually identical distributions of temperature and heat flux in the atmosphere [*Ozawa and Ohmura*, 1997; *Pujol*, 2003]. In addition, it is worth noting that in a simple two-box model with constant surface areas [*Lorenz et al.*, 2001], the results from the diffusivity approach are, of course, identical to those obtained by maximizing the general expression of $\dot{S}_{\mathrm{turb}}$ (see section 4).

[26] *Paltridge* [1978] also went to a two-dimensional model. In this work the entire globe was divided into 400 boxes of equal surface area. The energy balance requirements and the radiation treatments were the same as those assumed in his model of 1975. In addition, the model took into account that there can be no oceanic heat flux in continental regions. Figure 4 shows global distributions of the mean surface temperature, the cloud cover, and the convergence of horizontal heat flux by the atmosphere and oceans, estimated by the maximum condition of equation (9). Again, the estimates show a reasonable agreement with observations. Later on, *Sohn and Smith* [1994] and *Pujol and Llebot* [2000a] examined this two-dimensional approach and succeeded in reproducing a realistic distribution of the present climate.

[27] Thus, despite the ambiguity remaining in arbitrary assumptions used in different models, the basic concept of the maximization of the rate of entropy production seems to be valid, at least for the long-term mean state of the global climate.

3.2. Radiation Entropy and a Global Entropy Budget

[28] In the previous section we are concerned with the rate of entropy production due to the turbulent dissipation in the atmosphere and oceans. *Paltridge* [1975, 1978] suggested that this rate should be a maximum among all other possible steady states. However, he did not specify the surface temperature where absorption of shortwave (solar) radiation takes place, as we have seen in equation (9). This leads to an ambiguity when a corresponding radiation temperature was introduced [*Essex*, 1984]. Moreover, *Paltridge* [1978] considered equation (9) as "the total rate of entropy production in the planet." This was somewhat misleading and was questioned by *Essex* [1984], who showed that a predominant contribution to the entropy production in the Earth's system is due to direct absorption of solar radiation at the Earth's sur-

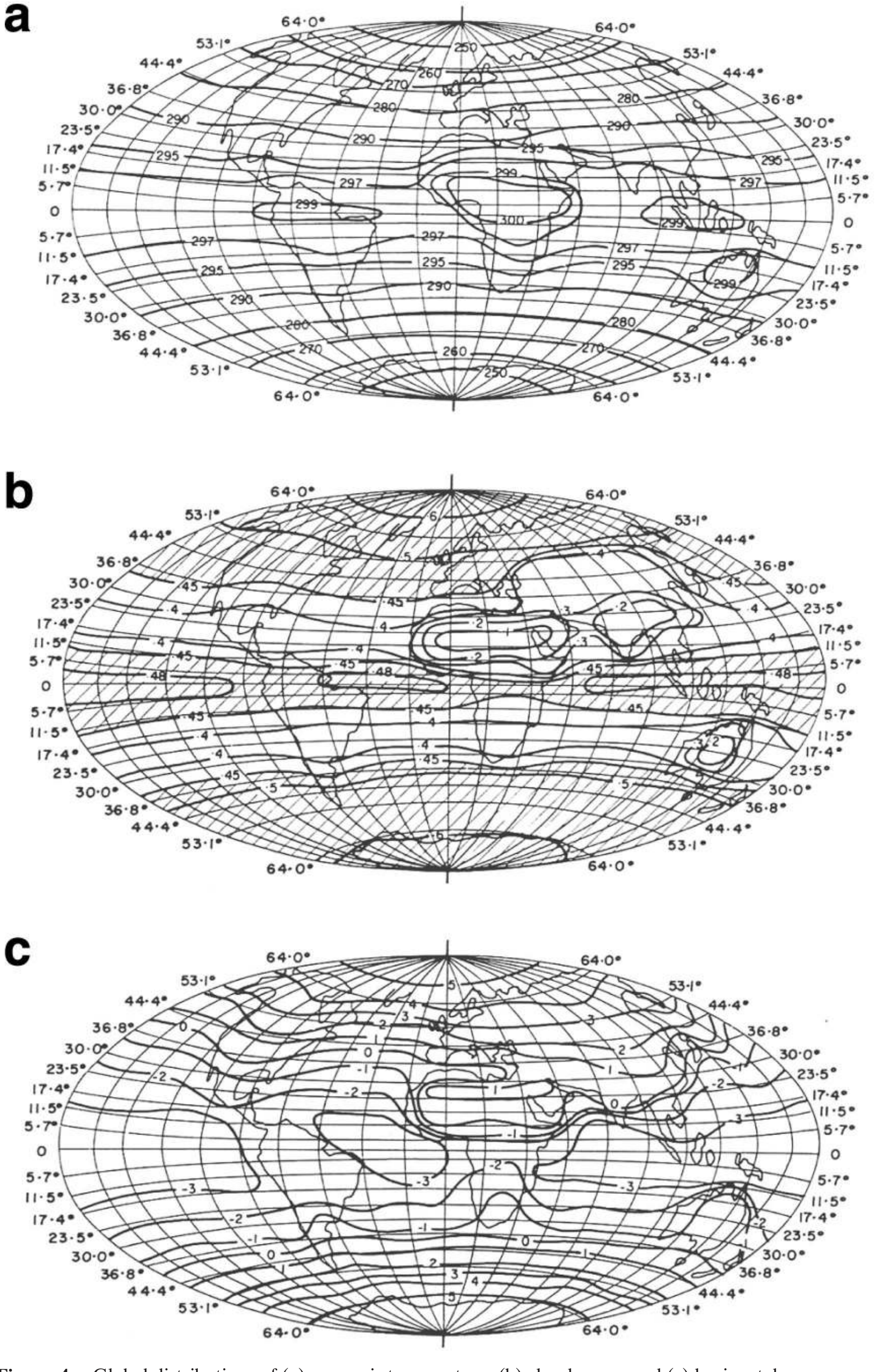


Figure 4. Global distributions of (a) mean air temperature, (b) cloud cover, and (c) horizontal convergence of heat flux in the Earth, predicted with the constraint of maximum entropy production (equation (9)). Reprinted from *Paltridge* [1978] with permission from the Royal Meteorological Society.

face. Since then, the radiation problem has been a central objection to Paltridge's work [*Lesins*, 1990; *Peixoto et al.*, 1991; *Stephens and O'Brien*, 1993; *Li et al.*, 1994; *Li and Chylek*, 1994; *O'Brien and Stephens*, 1995; *O'Brien*, 1997].

[29] In this section we shall discuss entropy of radiation and put rough values on the various components of the total entropy production in the Earth's climate system. We shall also point out that because of the essential linearity of radiative conversions in the climate system, it should not be expected that the entropy production associated with radiation is included in the maximization process.

[30] Suppose that a certain amount of radiant energy, $\delta Q_{\rm rad}$, is emitted from a surface of a black body, such as the Sun (Figure 1). Then, according to equation (1), the entropy of the Sun will decrease by $-\delta Q_{\rm rad}/T_{\rm Sun}$, where $T_{\rm Sun} \approx 5800$ K is the surface temperature of the Sun. The emission of solar radiation into surrounding space (vacuum) is essentially a reversible process through which entropy of the whole system remains unchanged [*Landau and Lifshitz*, 1937, section 63]. Then, the radiation itself should have a certain amount of entropy expressed by

$$dS_{\rm rad} = \frac{\delta Q_{\rm rad}}{T_{\rm br}}. \tag{11}$$

Here $T_{\rm br} = T_{\rm Sun}$ is the brightness temperature of the Sun and is, more generally, defined by the radiation density in a certain direction and certain frequency [*Landau and Lifshitz*, 1937, equation (63.26)]. In the case of emission of solar radiation into space, the radiation density per unit solid angle remains unchanged, and therefore the corresponding brightness temperature remains the same as that of the emitting blackbody [*Landau and Lifshitz*, 1937, p. 190]. In fact, we can focus the solar radiation with spherical mirrors or lenses, thereby producing the radiation temperature up to the Sun's surface temperature. In short, we can define the amount of "radiation entropy" by the flux of radiant energy divided by its brightness temperature [e.g., *Wildt*, 1956].

[31] It should be noted that equation (11) does not include a numerical factor of 4/3, which appears in some of the literature [e.g., *Fortak*, 1979; *Landsberg and Tonge*, 1979; *Essex*, 1984]. This factor would be needed if the emitted radiation were absorbed and scattered and changed into isotropic radiation that can be supposed to be in thermodynamic equilibrium [*Planck*, 1913, sections 61–65]. It is not the case for the radiation of solar radiation, however, since it is a nonequilibrium beam radiation lying in a specific solid angle and possessing the brightness temperature of $T_{\rm Sun}$ [*Landau and Lifshitz*, 1937, p. 190]. The entropy of radiation emitted freely into space is simply given by equation (11), so long as the radiation is not absorbed or scattered by material bodies. For this reason, *Wildt* [1956] once suggested that the numerical factor of 4/3 is needed for "fictitious entropy" that would result from transformation of the original (nonequilibrium) radiation into equilibrium one. Some relevant arguments are given by *Herbert and Pelkowski* [1990], *Peixoto et al.* [1991], *Goody and Abdou* [1996], and *Goody* [2000].

[32] It is possible to show an entire view of the entropy budget for the Earth with the expression of radiation entropy of equation (11). Since the Earth is receiving solar radiation with a brightness temperature of $T_{\rm Sun}$ and it emits longwave radiation with a brightness temperature of the atmosphere (T_a), the entropy of the surrounding system (space and Sun) is increasing by

$$\dot{S}_{\rm surr} = \int_A \left(\frac{F_{\rm long}({\rm TOA})}{T_a} - \frac{F_{\rm short}({\rm TOA})}{T_{\rm Sun}} \right) dA, \tag{12}$$

where $F_{\rm long}$(TOA) is the net rate of emission of longwave radiation per unit surface at the top of the atmosphere (TOA), and $F_{\rm short}$(TOA) is that of absorption (input − output) of shortwave radiation per unit surface at TOA, and the integration is taken over the whole global surface. On the contrary, the entropy of the Earth's system itself should remain constant so long as a steady state can be assumed for the long-term mean. Therefore

$$\dot{S}_{\rm sys} = 0. \qquad \text{(in a steady state)} \tag{13}$$

The rate of entropy increase in the whole universe (i.e., entropy production) due to all irreversible processes in the Earth's system is then given by the sum of equations (12) and (13):

$$\dot{S}_{\rm whole\ (univ)} = \dot{S}_{\rm surr} + \dot{S}_{\rm sys} = \int_A \left(\frac{F_{\rm long}({\rm TOA})}{T_a} - \frac{F_{\rm short}({\rm TOA})}{T_{\rm Sun}} \right) dA. \tag{14}$$

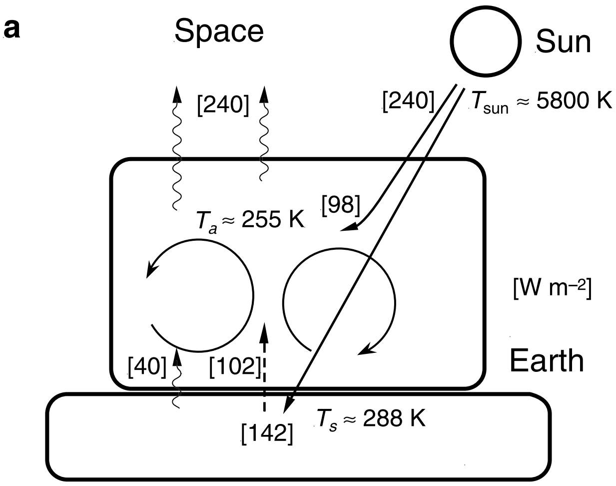
Since $T_{\rm Sun} \approx 5800$ K is much higher than T_a, equation (14) should be much larger than equation (9) or (10). In fact, a mathematical manipulation can show that

$$\dot{S}_{\rm whole\ (univ)} = \dot{S}_{\rm turb} + \dot{S}_{{\rm abs\ (short},s)} + \dot{S}_{{\rm abs\ (short},a)} + \dot{S}_{{\rm abs\ (long},a)}, \tag{15}$$

where $\dot{S}_{\rm turb}$ is given by equation (10), and the rest of the terms are

$$\dot{S}_{{\rm abs\ (short},s)} = \int_A \left(\frac{1}{T_s} - \frac{1}{T_{\rm Sun}} \right) F_{\rm short}(0)\, dA, \tag{16a}$$

$$\dot{S}_{{\rm abs\ (short},a)} = \int_A \left(\frac{1}{T_a} - \frac{1}{T_{\rm Sun}} \right) [F_{\rm short}({\rm TOA}) - F_{\rm short}(0)] dA, \tag{16b}$$



b

$$\dot{S}_{\text{whole (univ)}} \approx \dot{S}_{\text{surr}} = \left(\frac{1}{T_a} - \frac{1}{T_{\text{sun}}}\right) 240 \approx 0.90 \ (\text{W K}^{-1}\ \text{m}^{-2})$$

$$= \dot{S}_{\text{turb}} + \dot{S}_{\text{abs (short,s)}} + \dot{S}_{\text{abs (short,a)}} + \dot{S}_{\text{abs (long,a)}}$$

$$= \left(\frac{1}{T_a} - \frac{1}{T_s}\right)102 + \left(\frac{1}{T_s} - \frac{1}{T_{\text{sun}}}\right)142 + \left(\frac{1}{T_a} - \frac{1}{T_{\text{sun}}}\right)98 + \left(\frac{1}{T_a} - \frac{1}{T_s}\right)40$$

$$\approx 0.046 \quad + \quad 0.469 \quad + \quad 0.367 \quad + \quad 0.018$$

$$(5\ \%) \qquad (52\ \%) \qquad (41\ \%) \qquad (2\ \%)$$

Figure 5. Energy and entropy budgets for the Earth. (a) Global-mean (surface-area mean) energy flux components (i.e., shortwave radiation, longwave radiation, vertical turbulent heat transport), in W m^{-2}. (b) Corresponding rates of entropy production in the whole system (universe) due to the irreversible processes (absorption of radiation, turbulent convection, etc.) in the climate system. The total rate of entropy production is 0.90 (W K^{-1} m^{-2}).

$$\dot{S}_{\text{abs (long},a)} = \int_A \left(\frac{1}{T_a} - \frac{1}{T_s}\right) F_{\text{long}}(0)\,dA, \qquad (16c)$$

respectively, where $\dot{S}_{\text{abs (short},s)}$ is the rate of entropy production due to absorption of the solar radiation at the surface of the Earth (down-conversion of the solar radiation from T_{Sun} to T_s), $\dot{S}_{\text{abs (short},a)}$ is that of the solar radiation in the atmosphere (down-conversion of the solar radiation from T_{Sun} to T_a), and $\dot{S}_{\text{abs (long},a)}$ is that of the surface longwave radiation in the atmosphere (down-conversion of the longwave radiation from T_s to T_a). Intuitively, they are understandable with a fact that when down-conversion of energy (δQ) from a higher corresponding temperature (T_h) to a lower corresponding temperature (T_l) takes place, the corresponding amount of entropy production is $(1/T_l - 1/T_h)\delta Q$.

[33] A schematic of energy and entropy budgets of the Earth's climate system is shown in Figure 5. For simplicity, a global-mean (surface-area mean) state is shown, and thereby the representation is vertically one-dimensional. The values in the brackets represent the global-mean energy fluxes (W m^{-2}) based on global surface radiation measurements [*Ohmura and Gilgen*, 1993] and satellite measurements [*Barkstrom et al.*, 1990]. We can see that 40% of the solar radiation ($F_{\text{short}}(\text{TOA}) = 240$ W m^{-2}) is absorbed in the atmosphere (98 W m^{-2}), and the rest of it is absorbed at the surface ($F_{\text{short}}(0) = 142$ W m^{-2}). The energy gain at the surface is transported to the atmosphere by convective transport ($F_c = 102$ W m^{-2}) of latent heat and sensible heat (internal energy of the atmosphere) and by net longwave radiation ($F_{\text{long}}(0) = 40$ W m^{-2}). Strictly speaking, the convective transport should include a small amount of energy that is converted into the kinetic energy of the atmosphere. This contribution, however, is small ($\sim$2 W m^{-2}) in comparison with other components and is usually neglected in the first approximation. All these energies are finally emitted back to space via longwave radiation. Figure 5b shows the corresponding rates of entropy production due to the irreversible energy transport processes, e.g., the turbulent convection ($\dot{S}_{\text{turb}}$), absorption of solar radiation ($\dot{S}_{\text{abs (short},s)}$ and $\dot{S}_{\text{abs (short},a)}$) and that of longwave radiation ($\dot{S}_{\text{abs (long},a)}$), calculated with equations (10) and (16a)–(16c). Note here that in the one-dimensional (1-D) vertical atmosphere, the first term in equation (10) (the horizontal contribution) is zero and $F_{\text{short}}(0) - F_{\text{long}}(0) = F_c$ in the second term. The temperatures are assumed as $T_{\text{Sun}} = 5800$ K for the Sun, $T_s = 288$ K for the Earth's surface, and $T_a = 255$ K for the atmosphere, respectively. Figure 5b shows that the turbulent contribution ($\dot{S}_{\text{turb}}$) is only about 5% of the total rate and more than 90% is due to direct absorption of the solar radiation at the surface (52%) and that in the atmosphere (41%). Notice also that the estimate of $\dot{S}_{\text{whole (univ)}} = 0.90$ (W K^{-1} m^{-2}) is about 25% smaller than previous estimates [e.g., *Fortak*, 1979; *Aoki*, 1983; *Stephens and O'Brien*, 1993; *Weiss*, 1996] since we have excluded the numerical factor of 4/3 from the radiation entropy of equation (11).

[34] It should be noted that although the rate of entropy production by turbulent dissipation ($\dot{S}_{\text{turb}}$) is small in comparison with that by absorption of radiation ($\dot{S}_{\text{abs}}$), it is this small rate that tends to be a maximum in the climate system. As we shall discuss in section 8.2, a nonlinear feedback mechanism in the turbulent fluid system will adjust the transport process so as to generate the available energy (i.e., maximum possible work) at a possible maximum rate, and hence the maximum entropy production. On the contrary, absorption of radiation is essentially a linear process; its rate is given by the flux of radiation multiplied by the absorptivity of the material under consideration. There can be no feedback mechanism for the strength of the flux or the absorptivity in this process. Radiation can therefore be seen to be just an energy source for the climate system (Figure 5). For this reason, the rate of entropy production by the turbulent dissipation alone tends to be a maximum,

regardless of entropy production by the absorption of solar radiation.

[35] To be more precise, plants extract available energy (i.e., free energy) from solar radiation through photosynthesis. The reproduction process of plants can form a feedback loop that will change the absorptivity (albedo) of the planet in the long timescale. The long-term albedo regulation by plants will cast new light on the Gaia hypothesis suggested by *Lovelock* [1972] and was recently investigated by A. Kleidon (Beyond Gaia: Thermodynamics of life and Earth system functioning, submitted to *Climatic Change*, 2003).

4. CLIMATES ON OTHER PLANETS

[36] Zonal energy-balance models have been applied in the past to other planets, in particular to study the past climate of Mars, which is of particular interest with regard to the question of the origin of life and whether that body was habitable in the past. Zonal energy balance models were widely used in the 1970s to study terrestrial climate [e.g., *North et al.*, 1981], before general circulation models (GCMs) and the computing power required to support them became more widespread. Such models have fewer free parameters than GCMs and are therefore still useful tools where there is relatively little data available to constrain the climate state. The radiative input F_{short} of a planet is determined by its orbit around the Sun and its obliquity (the tilt of the equator to the orbital plane), and by the optical and thermal opacity of the atmosphere. In the absence of atmospheric convection, each latitude zone in the model (which may have as few as two boxes) can be assumed to be in radiative equilibrium, usually by linearizing the outgoing longwave radiation with respect to surface temperature, i.e., $F_{\text{long}} = a + bT$, with a and b constants; b depends on the typical surface temperature and on opacity. When a climate system is present, heat may be transferred between boxes, usually also expressed in a linearized way, with the flux from one latitude to another (arcsin (x) to arcsin ($x + dx$)) proportional to the temperature gradient dT/dx and some constant D, which has dimensions of diffusivity and describes how "diffusive" the atmosphere is. With the functions $F_{\text{short}}(x)$, $F_{\text{long}}(T)$ and constant D specified, the climate system can be solved, either in a steady state annual average sense, or (with appropriate heat capacities set for the surface) in a time-marching seasonally resolved sense. An electrical analogue for a two-box climate model is shown in Figure 6.

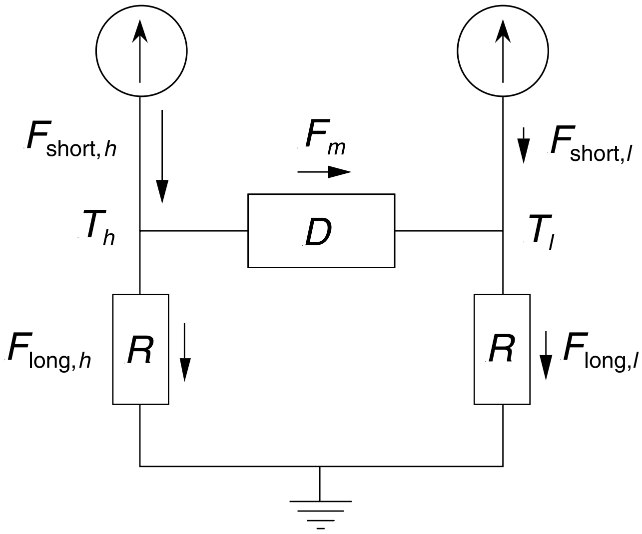


Figure 6. Equivalent electrical circuit for a simple two-box climate system. Currents $F_{\text{short},h}$ and $F_{\text{short},l}$ correspond to the solar flux input; current F_m depends on the diffusivity parameter of D. Potentials T_h and T_l (corresponding to temperature) adjust to ensure currents into each node sum to zero (i.e., energy balance). "Losses" R correspond to the radiative loss (i.e., longwave emission) to space. The optimum property of the climate system corresponds to component D adjusting itself such that the rate of entropy production $[(1/T_l - 1/T_h)F_m]$ is maximized.

[37] The problem is that the parameter D was determined empirically for the Earth, with typical values of 0.6 W m^{-2} K^{-1} [*North et al.*, 1981]. If the physics of heat transport is fully understood, physically reasonable parameterizations for D could be developed; for example, conventional theories suggest that D may be proportional to pressure and inversely proportional to the square of the planetary rotation rate. However, as discussed by *Lorenz et al.* [2001], such parameterizations fail in the case of Saturn's satellite Titan, which is a small, slowly rotating body with an atmosphere rather thicker than Earth's. Conventional parameterizations suggest D similar to or much larger than Earth, and yet the observed temperature contrast between low and high latitudes is several degrees, requiring a D value 2 or more orders of magnitude smaller than Earth (Figure 7b; P and P, Ω).

[38] *Lorenz et al.* [2001] found that the D values required by both Earth and Titan are in fact quite consistent with the climates adjusting to maxima in the rates of entropy production by latitudinal (meridional) heat transport: $\dot{S}_{\text{turb}} = (1/T_l - 1/T_h)\, D(T_h - T_l) = \max$ (see Figure 7). Specifically, D should relate simply to the radiative parameter b; if the radiative inputs to the low and high latitudes in a two-box model are within a modest factor of each other, $D_{\text{MEP}} = b/4$, where the suffix MEP represents the state of maximum entropy production. This contrasts dramatically with previous work on Mars, which has used D values much smaller than Earth, since the Martian atmosphere is thin. However, such models require "correction" by another physical process, namely, the pinning of polar cap temperatures by carbon dioxide condensation during polar night. When the latent heat transported by this process is calculated, the net heat transport is in fact in close agreement with that predicted by the maximum condi-

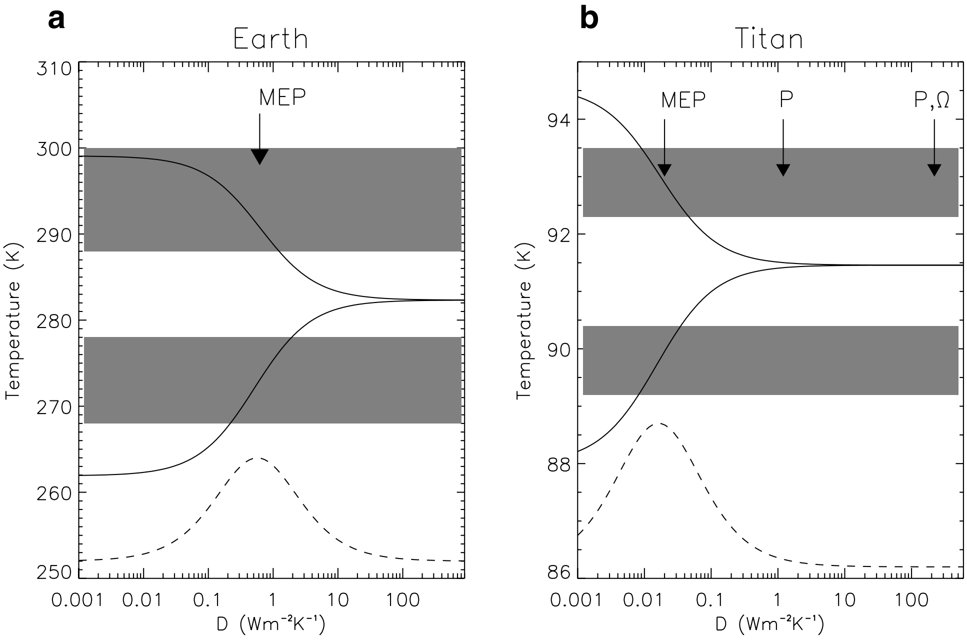


Figure 7. Low- and high-latitude surface temperatures on (a) Earth and (b) Titan as a function of the diffusivity parameter D. Shaded areas denote approximate observed temperatures. The dashed curves at bottom are the entropy production; the observed states correspond to the maximum entropy production (MEP). Reprinted from *Lorenz et al*. [2001].

tion, which (since Martian temperatures, and hence parameter b, are only modestly lower than those of Earth) requires rather similar values of D.

[39] Clearly there are some planetary settings where the maximum cannot hold; for example, Mercury has an atmosphere too thin to physically transport the heat required by the maximum since there are physical limits, such as the speed of sound, which prevent such transport [*Lorenz*, 2002b].

5. MANTLE CONVECTION IN PLANETS

[40] Consider a simple model of the Earth made purely of mantle material (the core and crust do not significantly affect the results) with thermal conductivity $k = 3$ W m^{-1} K^{-1} and a radiogenic heat production $H = 2 \times 10^{-8}$ W m^{-3}. The temperature distribution $T(r)$, r = radius, in steady state is defined by energy balance as $dT/dr = -Hr/(3k\ Nu)$, where the Nusselt number Nu is the ratio of actual (convective) heat transport to purely conductive transport. On a purely conductive Earth ($Nu = 1$) with a surface temperature of 300 K, the central temperature would be around 48,000 K; a vigorously convecting interior ($Nu \sim$ infinity) would have temperatures everywhere close to 300 K. Clearly, both of these cases are unphysical; rocks do not churn and roil at 300 K, nor do they stay rigid at 45,000 K. The usual modeling approach relies on parameterizations of Nu as a function of Rayleigh number, which in turn relies on estimates of viscosity as a function of T, derived from laboratory experiments or estimates of postglacial rebound. Such models yield central temperatures of between 4400 and 7000 K.

[41] This same result may be obtained more simply if the mantle is assumed to convect at a rate (e.g., at a constant Nusselt number throughout) which maximizes the rate of entropy production. It is found by simple numerical calculation for the parameters above that the maximum occurs at $Nu_{\mathrm{MEP}} \approx 7.6$; this profile yields a temperature profile with a central temperature of 5600 K [*Lorenz*, 2001b, 2002a]. Varying the heat production and/or the thermal conductivity by 50% yields a range similar to that above. Given the extreme simplicity of the model, this result is prima facie very encouraging. Clearly, the principle could be applied in more sophisticated models (e.g., compositionally layered, $Nu = f(z)$, and where the system evolves through time). In fact, *Vanyo and Paltridge* [1981] applied the MEP condition to a mantle-core model and obtained a dissipation rate that is consistent with a dynamo theory.

[42] The hypothesis can be expressed even more simply where the radiogenic heat production is separated from the convecting system, as on the icy satellites of the outer solar system. The situation resembles the optimization of nuclear power plants [e.g., *Bejan*, 1996], where the reactor (of power Q) warms to a temperature T_h higher than the ambient T_l, and the designer must choose the effective thermal conductance k_e of the power converter which is "shorted" by an unavoidable heat leak conductance k_l. Too low a value of k_e, and most of the heat flows through the leak and is wasted; too high a value, and $T_h = T_l + Q/(k_e + k_l)$ falls, lowering the Carnot efficiency $(1 - T_l/T_h)$ such that the work output

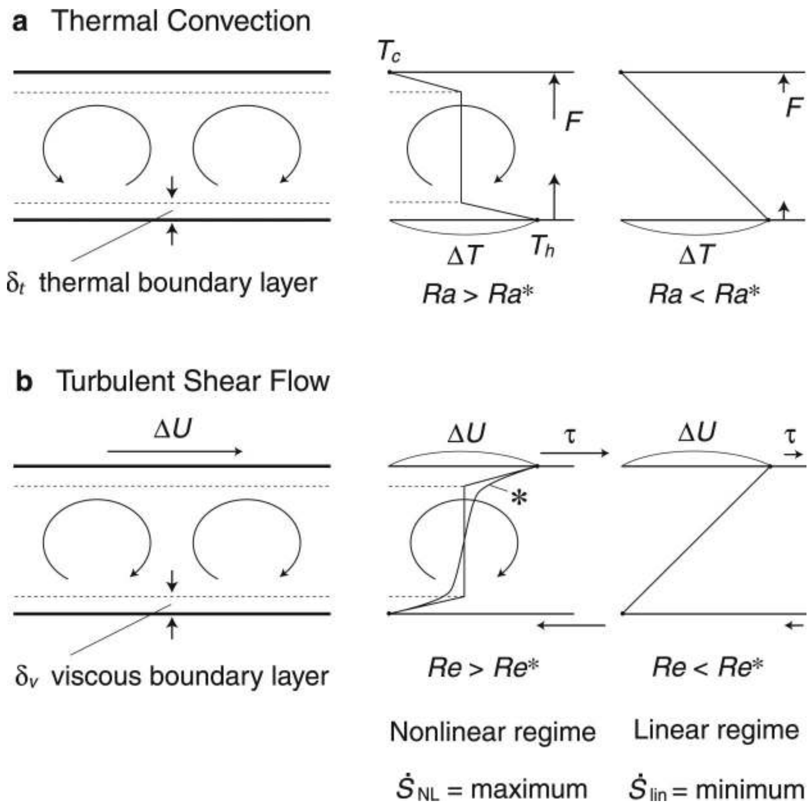


Figure 8. Schematic illustrations of (a) thermal convection and (b) turbulent shear flow. In a supercritical condition (*Ra* > *Ra** or *Re* > *Re**), turbulent motions develop, and the system is in a nonlinear regime with maximum entropy production by turbulent dissipation ($\dot{S}_{\mathrm{NL}} = \dot{S}_{\mathrm{turb}}$ = max). In contrast, in a subcritical condition (*Ra* < *Ra** or *Re* < *Re**), no turbulent motion can occur, and the system is in a linear regime with minimum entropy production ($\dot{S}_{\mathrm{lin}}$ = min).

of (and the dissipation into) the converter is small, even though the heat flow increases. A simple mathematical analysis can show that the maximum in $\dot{S}_{\mathrm{turb}} = (1/T_l - 1/T_h)\, k_e(T_h - T_l)$ occurs at $Nu_{\mathrm{MEP}} = 1 + T_h/T_l$ since the maximum condition $d\dot{S}_{\mathrm{turb}}/dk_e = 0$ with the energy balance equation $Q = (k_e + k_l)(T_h - T_l)$ yields a relation $k_e = k_l T_h/T_l$ so that $Nu_{\mathrm{MEP}} = Q/[k_l\,(T_h - T_l)] = 1 + k_e/k_l = 1 + T_h/T_l$. As with the two-box climate model, this system lends itself to electrical analogy; the problem is for us to choose a conductance (i.e., a resistor) to dissipate the maximum power when a constant current is supplied to it while it is shorted by an imposed conductance. For cases where the supplied power is small, the optimum Nu_{MEP} is very close to 2; only when the supplied power Q can increase the hot end temperature significantly (i.e., $Q \gg k_l T_l$) does the optimum Nu_{MEP} increase.

[43] As discussed by *Lorenz* [2001b], applying this relation to the Jovian satellite Europa, with a core heating of 3.2–3.4 $\times$ 10^{11} W beneath a 100–200 km thick ice/water layer of thermal conductivity 3 W m^{-2} K^{-1} and surface temperature T_l = 100 K, yields T_h = 220–300 K and Nu_{MEP} = 3.2–4; it seems likely then that Europa has a liquid layer (even without tidal heating in the soft ice crust), and indeed the magnetic signature of this layer has been observed [*Kivelson et al.*, 2000]. A water layer seems unavoidable for larger Ganymede and Callisto (which has also been observed to have the magnetic signature of a liquid layer, to the initial surprise of many modelers) and also for Saturn's satellite Titan.

6. FLUID TURBULENCE

[44] In this section we shall discuss phenomena of fluid turbulence. There are a variety of aspects of fluid turbulence, and there are indeed numbers of phenomenological theories of turbulence. We shall therefore consider a simple theory of turbulence, called the "optimum theory" or "upper bound theory" [*Malkus*, 1954, 1956], which was recently shown to be consistent with the hypothesis of MEP [*Ozawa et al.*, 2001]. As two typical examples, we shall discuss thermal convection and shear turbulence as follows.

6.1. Thermal Convection

[45] Let us consider thermal convection of a fluid layer which is in contact with thermal reservoirs with different temperatures; hot at the bottom and cold at the top (Figure 8a). Then, as is mentioned in section 2.4, resultant expansion of the fluid at the bottom and contraction at the top will produce a "top-heavy" density

distribution that is gravitationally unstable. When the temperature difference (or, equivalently, potential energy) exceeds a certain critical value, a convective motion tends to start. This phenomenon was first observed by *Bénard* [1901] and then investigated theoretically by *Rayleigh* [1916]. Rayleigh showed that the stability of the fluid layer is related to the following dimensionless parameter:

$$Ra \equiv \frac{g\alpha\Delta T d^3}{\kappa\nu}, \tag{17}$$

where Ra is the Rayleigh number, g is the acceleration due to gravity, d is the depth of the layer, ΔT is the temperature difference between the two boundaries, and α, κ, and ν are the coefficients of volume expansion, thermal diffusivity, and kinematic viscosity, respectively. The numerator represents the potential energy released by the fluid motion, and the denominator represents the dissipation of the energy due to thermal and viscous dissipation. When this Rayleigh number exceeds a certain critical value Ra^*, the fluid layer is no longer stable against small perturbations, and the convective motion tends to develop.

[46] Once convection occurs, the heat flow rate F tend to increase. As we have discussed in section 2.4, when the system is in a steady state, the rate of entropy production can be given by the discharge rate of entropy into the surrounding system. The hypothesis of MEP is then expressed by equation (8b) as

$$\dot{S}_{\mathrm{turb}} = \frac{T_h - T_c}{T_h T_c} F = \text{ maximum}. \tag{18}$$

Equation (18) says that when the boundary temperatures are kept constant, as in the example of Bénard convection, the condition of MEP is identical to that of maximum convective heat transport (F = max). In other words, the suggestion by *Malkus* [1954], *Howard* [1963], and *Busse* [1969] that Bénard convection involves maximum heat transport is equivalent to saying that such convection involves MEP.

[47] As a simplest case, let us follow the boundary layer approach originally proposed by *Malkus* [1954]. Malkus suggested that the maximum F is attained by the largest temperature gradient at a thermal boundary layer δ_t adjacent to the boundary where heat is mainly transported by heat conduction (Figure 8a). On the contrary, in the interior between the boundary layers, the heat transport by macroscopic eddies is so efficient that the temperature gradient in the interior is virtually negligible. In this case, the maximum heat transport will be attained by the largest temperature gradient at the boundary layer with its minimum thickness $\delta_{t,\min}$ as

$$F_{\max} = k\frac{\Delta T/2}{\delta_{t,\min}}, \tag{19}$$

where k is the thermal conductivity. In general, with decreasing the thickness of the boundary layer, the corresponding Rayleigh number (equation (17)) decreases. If the Rayleigh number of this layer becomes less than the critical value, then the layer becomes stable against perturbations, and, as a result, convective motions cannot penetrate into the thermal boundary layer (Figure 8a). Thus there is a certain minimum limit for the thickness of the boundary layer, $\delta_{t,\min}$, which will be given by the critical value of the Rayleigh number Ra^* as

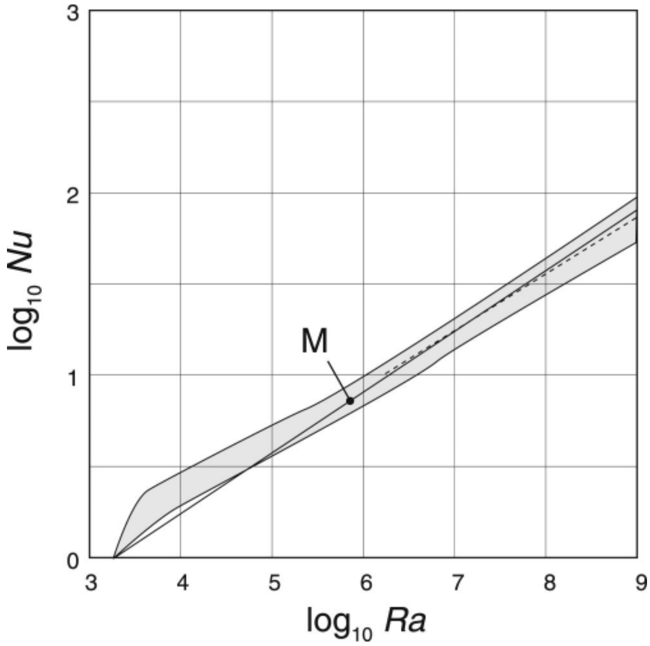


Figure 9. Relation between the Nusselt number Nu and the Rayleigh number Ra. Solid line M indicates the maximum estimate by equation (21), and the shaded region indicates experimental results [*Chandrasekhar*, 1961; *Howard*, 1963]. Dotted line shows experiments by *Niemela et al.* [2000]. Reprinted from *Ozawa et al.* [2001] with permission from the American Physical Society.

$$Ra^* = \frac{g\alpha\Delta T(2\,\delta_{t,\min})^3}{\kappa\nu}, \tag{20}$$

where $Ra^* = 1708$ is the critical Rayleigh number for a fluid layer between two rigid boundaries [*Chandrasekhar*, 1961]. Here a factor 2 is added since the sum of the two boundary layers constitutes a fluid layer with a temperature difference of ΔT (see Figure 8a). Substituting equation (20) into equation (19) and eliminating $\delta_{t,\min}$, we obtain

$$F_{\max} = \frac{k\Delta T}{d}\left(\frac{Ra}{Ra^*}\right)^{1/3}. \tag{21}$$

Equation (21) shows the maximum rate of convective heat transport permitted by the thermal boundary layer.

[48] Figure 9 shows the maximum heat flux (line M) estimated with equation (21) and the experimental results (shaded region). The ordinate is the Rayleigh number, and the abscissa is the dimensionless heat flux, the Nusselt number $Nu \equiv F/(k\Delta T/d)$, shown on a common

logarithmic scale. The experimental data are compiled from *Chandrasekhar* [1961] and *Howard* [1963]. Recent experimental results [*Niemela et al.*, 2000] are also shown with a dotted line. The agreement between the estimate and the experiments is remarkable, despite the simple treatments applied. This boundary layer approach gives an upper bound estimate; it may become invalid at very large Rayleigh numbers. However, the general agreement of the estimate with the experiments for a wide range of Rayleigh numbers provides an empirical support for the hypothesis of MEP. The maximum heat transport hitherto suggested can therefore be seen to be a manifestation of MEP under the fixed temperature condition at the boundary [*Ozawa et al.*, 2001].

6.2. Shear Turbulence

[49] Let us next consider turbulent shear flow of a fluid layer in contact with two reservoirs with different velocities; the relative velocity of the upper reservoir to the lower one is ΔU (Figure 8b). When the relative velocity is low, a laminar Couette flow will be realized in the system. In this case, velocity distribution is a linear function with depth, and momentum is transported by the viscosity of the fluid. When the relative velocity exceeds a certain critical value, a turbulent motion tends to develop. *Reynolds* [1894] investigated this phenomenon and found that the fluid layer becomes unstable when the following dimensionless parameter exceeds a certain critical value:

$$Re \equiv \frac{\Delta U d}{\nu}, \tag{22}$$

where Re is the Reynolds number and d is the thickness of the layer under consideration.

[50] Like thermal convection, once turbulence happens, the fluid motion itself transports the momentum, and the surface shear stress at the boundary τ tends to increase (Figure 8b). In this case, work is done on the system through the upper surface by a rate $\tau\Delta U$ per unit surface per unit time, and this energy is dissipated in the system by viscous dissipation. The dissipation process is again related to a nonlinear dynamic equation, and it is difficult to solve. However, if the system can be seen to be in a steady state, the work done on the system has to be balanced by the total rate of viscous dissipation in the fluid system. In this case, the hypothesis of MEP can be expressed by equation (8a) as

$$\dot{S}_{\mathrm{turb}} = \int_V \frac{\Phi}{T}\, dV \approx \frac{\Delta U}{T}\tau = \text{maximum}. \tag{23}$$

Here we have assumed that the temperature is almost uniform in the fluid layer. (In a real steady state, however, the rate of viscous heating has to be balanced by the rate of heat discharge through the boundary via heat conduction: $\int \Phi\, dV = \int F\, dA$. Thus expression (8b) is also valid in this case. It is impractical, though, to estimate F by the small temperature gradient at the boundary [*Ozawa et al.*, 2001].) Equation (23) says that when the velocity difference is kept constant, the condition of MEP is identical to that of maximum shear stress (or, equivalently, maximum momentum transport). In other words, the maximum momentum transport suggested by *Malkus* [1956] and *Busse* [1968, 1970] is identical to MEP.

[51] As before, the maximum shear stress (or the maximum momentum transport) will be attained by the maximum velocity gradient at a viscous boundary layer δ_ν adjacent to the boundary where the momentum is mainly transported by viscosity (Figure 8b). In the interior between the boundary layers, the momentum transport by the turbulent eddies is so efficient that the velocity gradient may be virtually negligible. In this case, the maximum shear stress will be attained by the maximum velocity gradient at the boundary layer with its minimum thickness $\delta_{\nu,\mathrm{min}}$ as

$$\tau_{\max} = \mu \frac{\Delta U/2}{\delta_{\nu,\min}}, \tag{24}$$

where $\mu = \rho\nu$ is the viscosity and ρ is the density. Like thermal convection, the minimum thickness will be given by a critical value of the Reynolds number, above which turbulence would occur, as

$$Re^* = \frac{\Delta U(2\delta_{\nu,\min})}{\nu}, \tag{25}$$

where Re^* is the critical Reynolds number. Substituting equation (25) into equation (24), and eliminating $\delta_{\nu,\mathrm{min}}$, one obtains

$$\tau_{\max} = \frac{\mu\Delta U}{d}\frac{Re}{Re^*}. \tag{26}$$

Equation (26) shows the maximum shear stress permitted by the viscous boundary layer.

[52] Figure 10 shows the maximum shear stress (line M) estimated with equation (26) and the experimental results (dots). The ordinate is the Reynolds number, and the abscissa is a dimensionless shear stress $\Gamma \equiv \tau/(\mu\Delta U/d)$, shown on a common logarithmic scale. The experimental results are plotted from *Reichardt* [1959], and the critical Reynolds number is set to be $Re^* \approx 500$ in reference to the experiments. The agreement is again reasonable, despite the simple treatments in the estimate. As before, this boundary layer approach gives an upper bound estimate without any dynamic constraint in the interior; it may become invalid at large Reynolds numbers. A rigorous analysis based on the dynamic equation and the continuity equation [*Busse*, 1970, 1978] showed that a velocity profile with maximum momentum transport is in close agreement with the observed one, as shown by an asterisk in Figure 8b. These results provide additional support for the hypothesis of MEP. The maximum shear stress (or maximum momentum transport) can therefore be seen to be a manifestation of MEP

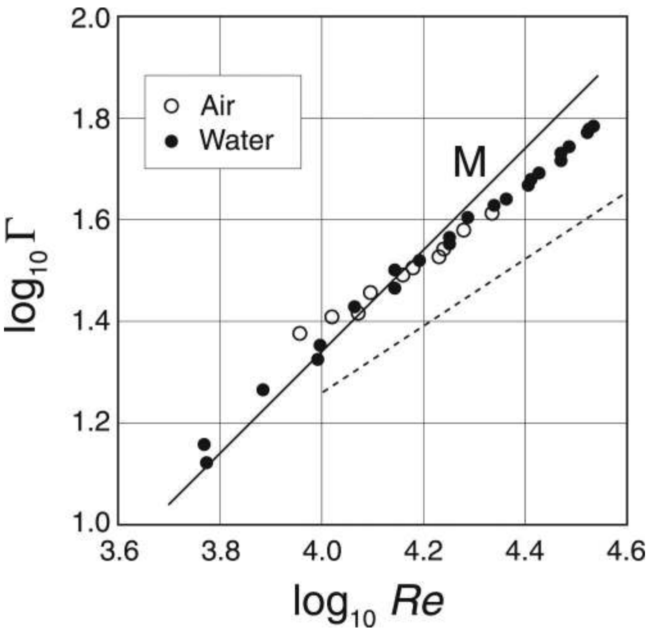


Figure 10. Relation between dimensionless shear stress $\Gamma \equiv \tau(\mu \Delta U/d)^{-1}$ and the Reynolds number Re. Solid line M indicates the maximum estimate by equation (26), and dots indicate laboratory experiment [*Reichardt*, 1959]. Dotted line shows results from Couette-Taylor flow experiment by *Lathrop et al*. [1992]. Reprinted from *Ozawa et al*. [2001] with permission from the American Physical Society.

under the fixed relative velocity at the boundary [*Ozawa et al*., 2001].

7. CONDITIONS FOR FULLY DEVELOPED TURBULENCE

[53] So far we have examined the applicability of the MEP hypothesis (equation (8)) to a variety of natural phenomena. It turns out that a key factor is related to nonlinearity in transport processes in turbulent media. In this section we shall examine the conditions in which the fluid system can be seen to be in a state of fully developed turbulence. We shall see in due course that a well-known principle of minimum entropy production [*Prigogine*, 1947] is not applicable to the turbulent systems in this respect.

7.1. Stability Condition and Timescales

[54] As we have seen in the previous section, the stability of a fluid system can be measured in terms of a certain dimensionless parameter such as the Rayleigh number or the Reynolds number (Figure 8). When such a parameter exceeds a certain critical value, the fluid system is no longer stable against small perturbations, and turbulent motions tend to develop. The condition for a fluid system to be in a turbulent state is then given by

$$N > N^*, \tag{27}$$

where N is a dimensionless parameter that describes the stability of a system (it becomes unstable as N increases) and N^* is the critical value at which turbulence would start. This condition (27) may be called a supercritical condition.

[55] The second condition is related to the timescale of observation. For instance, even if the supercritical condition is met ($N > N^*$), the turbulent motion cannot develop fully, unless we observe the system for a certain period of time. Let Δt denote the timescale of observation, and let τ_{NL} denote a characteristic time constant for the formation of a turbulent structure in the nonlinear system. The second condition is then given by

$$\Delta t > \tau_{NL}. \tag{28}$$

When the two conditions (inequalities (27) and (28)) are satisfied, turbulence will develop fully in the system, and the corresponding rate of entropy production ($\dot{S}_{NL}$) would be a maximum.

[56] If we observe the composed system for a considerably long period of time ($\Delta t \to \infty$), then the temperature (or velocity) difference in the surrounding system will eventually become negligible (section 2.2). In this state the entropy of the whole system is at a maximum, and there can be no difference in temperature or velocity ($N = 0$). The additional condition for the system to be in an "active" turbulent state ($N > N^*$) is therefore

$$\Delta t < \tau_{surr}, \tag{29}$$

where τ_{surr} is the relaxation time constant of the surrounding system (i.e., the reservoirs) to be in thermodynamic equilibrium. If this condition is not satisfied ($\Delta t \approx \tau_{surr}$), then the entire system is in the thermodynamic equilibrium ($N = 0$), and there is no energy available for the system. This state is called "heat death" [*Boltzmann*, 1898, section 90], which is the "final stage" of the nonlinear system. For this reason, the "lifetime" of a nonlinear system can be divided into "initial stage" ($\Delta t < \tau_{NL}$), "developed stage" ($\tau_{NL} < \Delta t < \tau_{surr}$), and "final stage" ($\Delta t \approx \tau_{surr}$), depending on the timescale of the system and that of the surrounding system, as listed in Table 1.

7.2. Prigogine's Minimum Principle for Linear Systems

[57] If the supercritical condition of (27) is not satisfied ($N < N^*$), no turbulent motion can develop in the fluid system, and heat or momentum is transported only by molecular diffusion. Then the heat flux (or momentum flux) is given by a linear function of temperature (or velocity) gradient. This sort of system is called a linear system. In a steady state the temperature (or velocity) distribution shows a linear distribution with depth in the one-dimensional case (see Figure 8, right-hand side). This steady state is known as the one with minimum entropy production. This minimum entropy production state was shown to be a final steady state of a linear

Table 1. Conditions for Maximum and Minimum Entropy Production

Stability Condition (Equation (27))	*Timescale (Equations (28) and (29))*	*Entropy Production Rate*
$0 < N < N^*$ (subcritical)	$\Delta t < \tau_{\mathrm{surr}}$	$\dot{S}_{\mathrm{lin}}$ = minimum (linear regime [*Prigogine*, 1947])
$N > N^*$ (supercritical)	$\Delta t < \tau_{\mathrm{NL}}$; "initial stage"	between minimum and maximum
$N > N^*$ (supercritical)	$\tau_{\mathrm{NL}} < \Delta t < \tau_{\mathrm{surr}}$; "developed stage"	$\dot{S}_{\mathrm{NL}}$ = maximum (nonlinear regime)
$N = 0$ (equilibrium)	$\Delta t \approx \tau_{\mathrm{surr}}$; "final stage"	0; "heat death"

system by *Prigogine* [1947]. Later on, several attempts have been made to extend his principle to a nonlinear regime [e.g., *Glansdorff and Prigogine*, 1954, 1964; *De Groot and Mazur*, 1962]. Recent studies, however, show that no such extension is possible for a nonlinear system [*Sawada*, 1981; *Kondepudi and Prigogine*, 1998]. In fact, it is possible to show that this minimum principle is valid only for a linear regime of a fluid system without any turbulent motion.

[58] Let us consider a linear heat conduction system without any turbulent motion ($\mathbf{v} = 0$). The general expression for the rate of entropy production (equation (7)) is valid also in the case, and is given by

$$\dot{S}_{\mathrm{lin}} = \int_V \mathbf{F} \cdot \mathrm{grad}\left(\frac{1}{T}\right) dV, \tag{30}$$

where $\dot{S}_{\mathrm{lin}}$ is the rate of entropy production by the pure heat conduction. The dissipation function is zero ($\Phi = 0$) since there is no turbulent motion. A simple mathematical manipulation [e.g., *De Groot and Mazur*, 1962, chapter 5] can show that the change rate of this rate per unit time is given by

$$\frac{d\dot{S}_{\mathrm{lin}}}{dt} = \int_V \frac{2}{T^2} \frac{\partial T}{\partial t} \,\mathrm{div}\, \mathbf{F}\, dV. \tag{31}$$

Here we have assumed that the conductive heat flux is a linear function of the temperature gradient and that the temperature at the boundary is fixed to a prescribed distribution. Equation (31) can be rewritten by using the conservation law for energy (equation (6)) with the condition of no turbulent motion ($\mathbf{v} = 0$):

$$\frac{\partial(\rho c T)}{\partial t} = -\,\mathrm{div}\, \mathbf{F}. \tag{32}$$

Note that equation (32) is valid only for a system without heat advection. Substituting equation (32) into (31) and assuming that ρc is constant with respect to time, we get

$$\frac{d\dot{S}_{\mathrm{lin}}}{dt} \approx -2\int_V \frac{\rho c}{T^2}\left(\frac{\partial T}{\partial t}\right)^2 dV \leq 0. \tag{33}$$

Inequality (33) shows that the rate of entropy production in a pure heat conduction system always decreases or remains constant with time; starting from any arbitrary temperature distribution, the rate will decrease and finally arrive at its minimum value in the final steady state ($\partial T/\partial t = 0$) so long as the boundary temperature is kept to a prescribed distribution. This is the well-known "minimum entropy production principle" for a pure conduction system [e.g., *De Groot and Mazur*, 1962, p. 46]. This minimum state corresponds to a linear temperature distribution in the one-dimensional heat conduction case (Figure 8a). The same result can be obtained for a laminar flow without turbulent motion; the final steady state is one with a linear velocity distribution (Figure 8b). It should be borne in mind, however, that this minimum principle (equation (33)) is based on the assumption of no advective transport of heat or momentum (equation (32)), which is by no means justified for a turbulent system [see also *Woo*, 2002]. Prigogine's minimum principle is therefore a principle for a linear system, e.g., pure heat conduction or a laminar flow, namely, a linear regime of a fluid system under the subcritical condition (Table 1).

[59] A few remarks may be in order about the difference between the linear and nonlinear systems. In a linear system the time evolution can be described by linear governing equations. There is only one solution, and the system is completely predictable. Prigogine's minimum entropy production principle for a linear system is then trivial since the behavior of the system is soluble without any other principles. On the other hand, time evolution of a nonlinear system is, in general, not predictable by the nonlinear governing equations alone since a negligibly small change in initial conditions can grow into a large difference in the final state [*Lorenz*, 1963]. There are, in fact, a set of possible steady states for a turbulent fluid system under the same boundary conditions. The maximum entropy production principle, then, acts as a guiding principle for choosing the most probable state among all other possible states allowed by the nonlinear system. The MEP principle is therefore fundamental to the nonlinear systems and should not be confused with Prigogine's one for linear systems.

8. ENERGETICS OF LORENZ

[60] In this final section we shall discuss the relationship between entropy production and a concept of en-

ergetics developed by *Lorenz* [1955, 1960, 1967, 1978]. It will be shown that the energetics is related to Carnot's concept of available energy generation discussed in section 2.3, whereas entropy production is a concept of dissipation of this energy. In a steady state the generation rate is balanced by the dissipation rate, and consequently, the hypothesis of MEP becomes identical to what *Lorenz* [1960] suggested as the maximum generation of available potential energy. A simple mechanism by which a turbulent fluid system adjusts itself to a state of maximum generation of available potential energy, and hence MEP, is discussed in this respect.

8.1. Generation and Dissipation of Available Potential Energy

[61] *Lorenz* [1955] investigated an adiabatic expansion process of a fluid through which a part of internal energy of the fluid is converted into mechanical energy that is "available" for conversion to kinetic energy. During a transport process of the atmosphere from a real state to a reference state, he found a maximum possible amount of the energy that is available for kinetic energy, and named it available potential energy. The maximum amount is attainable if the transport takes place in a reversible manner, in other words, in a quasi-static process. His thought experiment is quite similar to that of *Carnot* [1824] despite the fact that Lorenz considered a reversible adiabatic process only, while Carnot considered a reversible diabatic process in addition to the reversible adiabatic one, thereby combining them to form a cycle.

[62] According to a general expression of *Lorenz* [1960, 1967], the generation rate of the available potential energy G is given by

$$G = \int_V \dot{q}\left[1 - \left(\frac{p_r}{p}\right)^{\gamma}\right] dV = \int_V \dot{q}\left(1 - \frac{T_r}{T}\right) dV, \tag{34}$$

where $\dot{q}$ is the rate of diabatic heating due to radiation and viscous dissipation per unit volume of the fluid, p is the pressure, p_r is the pressure of the fluid at a reference state, $\gamma = 1 - c_v/c_p$ (c_v and c_p are the specific heats of the fluid at constant volume and pressure, respectively), T is the temperature, and T_r is the temperature at the reference state. In this manipulation, we have used a relation between temperature and pressure [$T_r/T = (p_r/p)^{\gamma}$] for an adiabatic transport from the real state to the reference state. One can see from equation (34) that the generation rate G is essentially the same as the generation rate of maximum possible work found by Carnot (equation (4)).

[63] If we can assume that the rate of viscous heating is negligible in comparison with that of radiative heating or cooling, and that the entire atmosphere is in a steady state, then

$$\int_V \dot{q} dV = 0. \tag{35}$$

In addition, we may assume that the reference temperature is virtually constant in comparison with that of the real atmosphere, then $T_r = \overline{T}_r =$ const. This assumption seems to be justified if we consider the fact that the reference state is defined as the most stable state after such reversible replacement processes as reversible diabatic [*Carnot*, 1824] as well as reversible adiabatic [*Lorenz*, 1955]. Substituting equation (35) into (34), and replacing T_r with $\overline{T}_r$, we get

$$G = \overline{T}_r \int_V \frac{-\dot{q}}{T} dV. \tag{36}$$

The volume integral in equation (36) is identical to the rate of entropy discharge into the immediate surroundings via radiation: $\int (-\dot{q})/T\, dV = \int F/T\, dA = \dot{S}_{\text{surr}}$, so that

$$G = \overline{T}_r \dot{S}_{\text{surr}}. \tag{37}$$

Equation (37) shows that the generation rate of available potential energy is proportional to the rate of entropy discharge into the surrounding system. This equation give us a thermodynamic view that is consistent with the one by *Carnot* [1824]. As we have seen in section 1, Carnot regarded the Earth as a heat engine, in which the fluid like the atmosphere transports heat from hot to cold regions (Figure 1). This transport leads to entropy increase in the surrounding system ($\dot{S}_{\text{surr}} > 0$). Along this process, a part of the heat energy can be converted into the potential energy that is "available" for kinetic energy of the fluid [e.g., *Carnot*, 1824; *Dutton*, 1973; *Ozawa*, 1997].

[64] In a steady state, entropy of the fluid system should remain constant, so that the rate of entropy discharge should be balanced by the internal entropy production processes associated with turbulence (thermal and viscous dissipation) (equation (7)). Thus

$$G = \overline{T}_r \dot{S}_{\text{turb}} \approx D_{\text{therm}} + D_{\text{vis}}, \qquad \text{(in a steady state)} \tag{38}$$

where $D_{\text{therm}} = \overline{T}_r \int \mathbf{F} \cdot \text{grad}\,(1/T)\, dV$ is the dissipation rate of available potential energy by thermal dissipation, and $D_{\text{vis}} = \int \Phi\, dV$ is that by viscous dissipation (kinetic dissipation). Here we have assumed $\overline{T}_r \int \Phi/T\, dV \approx \int \Phi\, dV$ within the limits of an approximation of $T - \overline{T}_r \ll T$. It should be noted that *Lorenz* [1960] once suggested that the present atmosphere is operated at a state with maximum generation of available potential energy, i.e., $G =$ max. This hypothesis was confirmed to some extent by *Schulman* [1977] and *Lin* [1982]. *Lorenz* [1967], however, questioned this hypothesis since the estimated G was much larger than that of D_{vis}. In fact, the generation rate

G is about 10–14 W m^{-2} [*Schulman*, 1977; *Pauluis and Held*, 2002a], while the viscous dissipation rate D_{vis} is estimated to be 2–3 W m^{-2} from the wind speed [*Oort and Peixóto*, 1983]. (To be precise, there is dissipation due to the drag of falling rain [e.g., *Pauluis et al.*, 2000; *Lorenz and Rennó*, 2002]. This contribution can be up to 2–3 W m^{-2} but cannot fill the gap of 10 W m^{-2}.) Equation (38) clearly shows that the discrepancy is caused by the thermal dissipation term (D_{therm}). A recent study [*Pauluis and Held*, 2002a, 2002b] shows that the thermal dissipation is caused mainly by irreversible transport of latent heat in the moist atmosphere, and it can be about 8 W m^{-2}. This thermal dissipation leads to direct waste of the "available" potential energy, which was a missing factor in the framework of Lorenz's treatment of the adiabatic atmosphere [*Lorenz*, 1955, 1967]. When we consider both two terms (thermal and viscous dissipation), it is, in fact, possible to show that Lorenz's hypothesis of maximum generation of available potential energy (G = max) is identical to the hypothesis of MEP by the turbulent dissipation ($\dot{S}_{\mathrm{turb}}$ = max). The two hypotheses can therefore be unified into the single condition of maximum G.

8.2. A Mechanism for Maximum Entropy Production

[65] Finally, let us discuss a possible mechanism by which a turbulent fluid system adjusts itself to a state of maximum generation of available potential energy or, equivalently, MEP.

[66] As a simplest case, let us consider the Earth composed of two regions: the tropics and poles. The average temperature in the tropical region is T_t, and that in the polar region is T_p (Figure 11a). In the present state, there is a net gain of radiation in the tropical region and a net loss in the polar regions. The energy imbalance is compensated by energy transport F due to the direct motion of the atmosphere and oceans. Suppose an extreme case with no motion (i.e., static state) with negligible amount of heat transport ($F \approx 0$). Then, the tropical region will be heated up, and the polar region will be cooled down. Then, according to the Stefan-Boltzmann law of radiation (or an equivalent linear function in section 4), this leads to an increase in longwave emission from the tropical region and a decrease in that from the polar region, thereby compensating the energy imbalance in each region. Thus, in the static state, the temperature difference will be the largest. With increasing F from zero, the temperature difference will decrease. At very large F with extreme mixing, the temperature difference will become negligible. Thus the temperature difference $\Delta T = T_t - T_p$ is a decreasing function of F (Figure 11b).

[67] As we have seen in the previous section, when heat energy is transported from hot to cold regions, a part of the energy can be converted into potential energy that is available for kinetic energy of the fluid. The generation rate of the available potential energy is given by equation (34) as

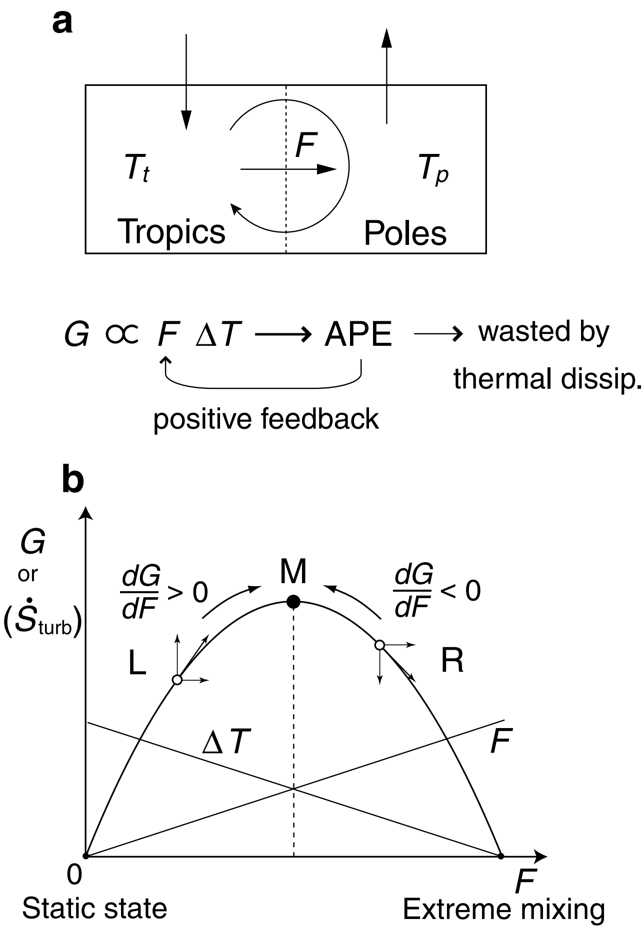


Figure 11. (a) Schematic illustration of the Earth consisting of two regions: tropics and poles. F denotes horizontal energy transport by direct motion of the atmosphere and oceans. $G \propto F\Delta T$ is the generation rate of available potential energy that is the source of the kinetic energy of the fluids. When the dynamic motion is accelerated, F increases, and it leads to an excess generation of G, resulting in positive feedback to the dynamic motion. (b) Generation rate of available potential energy G as a function of F. A positive fluctuation at L leads to an acceleration of the fluctuation since $dG/dF > 0$, while that at R leads to a deceleration since $dG/dF < 0$. The net effect is therefore toward the maximum M.

$$G = \int_V \dot{q}\left(1 - \frac{T_r}{T}\right) dV = \overline{T}_r \frac{F\Delta T}{T_t T_p} \tag{39}$$

where $\overline{T}_r$ is the reference temperature and approximates the mean temperature of the system. Since G is proportional to a product of F and ΔT, it should have a maximum between the two extreme states: the static state ($F \approx 0$) and the extreme mixing ($\Delta T \approx 0$), as shown in Figure 11b.

[68] The basic question is whether there is any reason why the actual state of such a turbulent system should be in a state at (or near) its maximum possible value in G or $\dot{S}_{\mathrm{turb}}$. One can see a feedback loop in this system: If a dynamic motion is accelerated, the heat transport (F)

increases, and it leads to an excess generation of available potential energy, resulting in positive feedback to the dynamic motion. This "self-feedback mechanism" can be a possible cause for the MEP. Suppose, for instance, that the system is in a steady state that lies on the left-hand side of the G curve (Figure 11b, point L). While the system is in the steady state for a certain period of time, the system is subject to fluctuations induced by variations of turbulent eddies [e.g., *Paltridge*, 2001]. A positive fluctuation in F ($+\Delta F$ at point L) caused by velocity fluctuations leads to a positive gain in the generation rate G since $dG/dF > 0$. Then, the fluctuation tends to develop by the positive feedback from G. This development can continue until the maximum point (M) where no further gain in G is expected (i.e., $dG/dF = 0$). On the other hand, if the system is in a steady state that lies on the right-hand side of the G curve (Figure 11b, point R), then the positive fluctuation ($+\Delta F$) cannot develop, but tends to be suppressed since $dG/dF < 0$. In contrast, a negative fluctuation ($-\Delta F$ at R) tends to develop by a positive gain in G. This development can again continue until the maximum point (M) by the positive feedback from G. The net drift from anywhere on the G curve is therefore toward the single maximum point M. The maximum in G corresponds to a maximum in $\dot{S}_{\mathrm{turb}}$ since G is proportional to the rate of entropy production (see equation (37)). Notice that only a part of G contributes to the actual kinetic energy of the system. However, even this small part can constitute a feedback loop to maximize G (Figure 11a).

[69] The above explanation is a qualitative one. Further theoretical and experimental studies are therefore needed to quantify this explanation. It should be noted that the outline of this explanation was speculated about by *Lorenz* [1960] although the feedback loop for G was not clearly written. In addition, thermal dissipation (direct waste of G) was not properly concerned in his treatment (see section 8.1). On the other hand, a regulation mechanism by turbulent fluctuations has been suggested by *Paltridge* [1979, 1981, 2001]. The above explanation can therefore be seen to be a specific feedback mechanism applied to a setting inspired by Lorenz and Paltridge.

[70] The above explanation can easily be extended for shear turbulence discussed in section 6.2. The generation rate of available potential energy in this case is the real supply of mechanical energy by external work ($G = \tau\Delta U$). The same "self-feedback mechanism" can work to maximize G in this system. This approach shows a way toward a theory of turbulence [*Ozawa et al.*, 2001], and the details will be dealt with in future publications.

9. CONCLUDING REMARKS

[71] In this paper we have reviewed the thermodynamical properties of various kinds of turbulent fluid systems in nature. It is shown that the long-term mean states of the climate system of the Earth, those of other planets, and those of thermal convection and shear turbulence correspond to a certain extent to a unique state in which the rate of entropy production due to thermal and viscous dissipation is at a maximum. Lorenz's conjecture on maximum generation of available potential energy [*Lorenz*, 1960] is shown to be akin to this state with a few minor approximations. A possible mechanism by which a turbulent fluid system adjusts itself to a state of MEP is suggested based on the energetic concept of Lorenz. It is hoped that the present attempt will provide an apt starting point for future developments in the studies of thermodynamics and energetics of forced-dissipative systems in general, including our planet.

[72] Two developments should be mentioned here. One is a theoretical investigation of MEP based on statistical interpretation of entropy by *Dewar* [2003]. Following Jaynes's information theory [*Jaynes*, 1957], he showed that the most probable macroscopic steady state is the one with MEP among all other possible states, given the boundary conditions and mass and energy conservation laws. This statistical approach will broaden the horizons between MEP and information theory [*Lorenz*, 2002b, 2003; *Delsole*, 2002]. It may also be a theoretical basis for the energetic explanation shown in section 8.2 since the difference between the heat energy and the kinetic energy is only of statistical significance; that is, spontaneous conversion of the heat energy into the kinetic energy is in principle possible, but is just extremely improbable.

[73] Another development has been made with numerical model simulations. *Suzuki and Sawada* [1983] and *Chen and Wang* [1983] carried out numerical experiments on Bénard-type convection and obtained multiple steady states under the same boundary condition. They found that these states are not equally stable against perturbations, and the state tends to shift to the one with a higher rate of entropy production by perturbations. *Rennó* [1997] found two stable steady states in a radiative-convective model of the atmosphere and suggested that the state with a higher rate of dissipation is selected. *Minobe et al*. [2000] carried out numerical experiments of thermal convection in a rotating fluid system and found a kink in the rate of entropy production at a boundary between two different convection regimes. They suggested that the kink results from a preferred selection of a regime with a higher rate of entropy production. More direct evidence was recently obtained from numerical simulations of oceanic general circulation [*Shimokawa and Ozawa*, 2002]. They found that irreversible changes always occur in the direction of the increase of entropy production. The numerical investigation is the subject of future studies, and the details will be reported on other occasions.

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