

74-21,268

FENELL, Merton Everett, Jr., 1944-  
THE SECONDARY FLOW OF NEWTONIAN FLUIDS  
IN CONE-AND-PLATE VISCOMETERS.

Rice University, Ph.D., 1974  
Engineering, mechanical

University Microfilms, A XEROX Company, Ann Arbor, Michigan

RICE UNIVERSITY

THE SECONDARY FLOW OF NEWTONIAN FLUIDS IN  
CONE-AND-PLATE VISCOMETERS

by

Merton Everett Fewell, Jr.

A THESIS SUBMITTED  
IN PARTIAL FULFILLMENT OF THE  
REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

Thesis Director's Signature:

  
\_\_\_\_\_

Houston, Texas

January 1974

## ACKNOWLEDGMENTS

It is with gratitude that I acknowledge the following persons and organizations for their generous contributions:

My wife, Jonie, and son, Rett, for their patience and confidence as well as their many sacrifices.

My parents, whose encouragement and sacrifices made it possible for me to attend college.

Dr. J. D. Hellums for direction of this work. Not only was his technical guidance invaluable, but his personal interest and friendship are also appreciated and will be remembered.

Drs. A. J. Chapman, L. V. McIntire, and W. F. Walker for serving on the oral examination committee.

Mr. Byron Leverett for many enlightening discussions.

Mrs. Ruby Rost for the typing of this thesis and  
Mr. George Rodenbusch for drafting the figures.

The National Defense Education Act, The National Heart Institute, and Rice University for financial support.

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## NOMENCLATURE

<u>Roman Letter</u>	<u>Definition</u>
$\underline{b}$	Vector which represents the body force per unit volume.
$c_v$	Specific heat at constant specific volume.
$\underline{D}$	Rate-of-deformation tensor.
$\frac{D_{rr}}{Dp}$	Ratio of the $D_{r\theta}$ element of the rate-of-deformation tensor and the primary deformation rate.
$\frac{D_{r\theta}}{Dp}$	Ratio of the $D_{r\theta}$ element of the rate-of-deformation tensor and the primary deformation rate.
$\frac{D_{\phi r}}{Dp}$	Ratio of the $D_{\phi r}$ element of the rate-of-deformation tensor and the primary deformation rate.
$\frac{D_{\theta\theta}}{Dp}$	Ratio of the $D_{\theta\theta}$ element of the rate-of-deformation tensor and the primary deformation rate.
$\frac{D_{\phi\theta}}{Dp}$	Ratio of the $D_{\phi\theta}$ element of the rate-of-deformation tensor and the primary deformation rate.

$\frac{D_{\phi\phi}}{Dp}$	Ratio of the $D_{\phi\phi}$ element of the rate-of-deformation tensor and the primary deformation rate.
$Dp$	Primary deformation rate--the value of the non-zero components of the rate-of-deformation tensor as computed by the primary flow analysis. $Dp = -\frac{\dot{\gamma}}{2}$ .
$e_r$	Unit vector in the $r$ direction (radial).
$e_\theta$	Unit vector in the $\theta$ direction (azimuthial).
$e_\phi$	Unit vector in the $\phi$ direction (Meridian).
$Fr$	Froude number $\equiv \frac{R\Omega^2}{g}$ where $g$ is the magnitude of the acceleration of gravity.
$g_{ij}$	Metric tensor of a curvilinear coordinate system.
$g$	Magnitude of the acceleration of gravity, also the determinate of the matrix of the components of $g_{ij}$ .
$I$	Identity tensor.
$k$	Cartesian unit vector in the vertical direction, i.e., parallel to the axis of the cone.
$knax$	Maximum number of allowed non-linear iterations.
$imax$	Maximum number of allowed iterations to obtain the stream function for each non-linear iteration.

$n_{max}$	Maximum number of time steps.
$P$	Thermodynamic pressure.
$p$	Hydrostatic or mean pressure, which is referred to as simply the pressure.
$P_a$	Pressure of the surrounding air at the free surface.
$Q$	Internal heat generation per unit volume.
$\underline{q}$	Heat flux vector.
$R$	Slant height of the cone.
$r$	Radial measurement from the intersection of the cone and plate.
$Re$	Reynolds number $\equiv R^2\Omega/\nu$ .
$t$	Time.
	Magnitude of the torque exerted by the fluid on the cone and plate.
$p$	Magnitude of the torque exerted by the fluid on the cone and plate as computed by the primary flow analysis.
$\frac{T}{T_p}$	Ratio of the torque at the cone and plate to the torque at the cone and plate as computed by the primary flow analysis.
$\frac{T}{T_p} \Big _{cone}$	Ratio of the torque on the cone to the torque at the cone and plate as computed by the primary flow analysis.



$\frac{T}{T_P} \Big|_{\text{plate}}$

Ratio of the torque on the plate to the torque at the cone and plate as computed by the primary flow analysis.

$\underline{T}$

Stress tensor--represents the stress on the fluid.

$v_r, u$

Radial velocity component (r direction).

$v_\theta, v$

Azimuthial velocity component ( $\theta$  direction).

$v_\phi, w$

Meridian velocity component ( $\phi$  direction).

We

Weber number  $\equiv \rho R^3 \Omega^2 / \sigma$ .

$\dot{\underline{x}}, \underline{v}$

Velocity vector.

$\ddot{\underline{x}}$

Acceleration vector.

### Greek Letter

$\beta$

Angular measurement from the plate.

$\Gamma$

Covariant component of the meridian velocity component.  $\Gamma = r \cos \beta w$ .

$\dot{\gamma}$

Shear rate =  $\frac{\Omega}{\epsilon}$ .

$\Delta\beta$

Step size in the  $\beta$  direction.

$\Delta t$

Time step.

$\Delta z$

Step size in the z direction.

$\epsilon$

Gap angle measured in radians unless otherwise specified.

$\epsilon_g$

A small number which defines a steady state for the numerical solution of  $\Gamma$ .

$\epsilon_p$	A small number which defines a steady state for the numerical solution of $\psi$ .
$\epsilon_z$	A small number which defines convergence of the non-linear iteration as well as a steady state for the numerical solution of $\zeta$ .
$\zeta$	Meridian vorticity variable.
$\eta$	Azimuthal vorticity variable.
$\theta$	Angular measurement from the vertical (Azimuthal direction).
$\mu$	Viscosity of the fluid in the viscometer.
$\nu$	Kinematic viscosity of the fluid in the viscometer.
$\xi$	$\beta/\epsilon$ .
$\rho$	Density of the fluid in the viscometer.
$\rho_a$	Density of the surrounding air at the free surface.
$\sigma$	Surface tension of the fluid in the viscometer.
$\phi$	Angular measurement in the direction of the rotation of the cone (Meridian direction).
$x$	Radial vorticity variable.
$\psi$	Stream function.
$\Omega$	Magnitude of the angular velocity of the cone.

is

Vorticity vector.

NOTE: All of the symbols are not defined here. The remaining symbols are either commonly used or have only localized meaning. The symbols with localized meaning are defined at appropriate places in the text.

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## CHAPTER I

### INTRODUCTION

#### I.1 General Background

Coleman and Noll [29] have defined a certain class of flows, called viscometric flows, which can be used to determine experimentally the rheological properties of simple fluids. Included in this class of flows is the flow produced in a cone-and-plate viscometer--sometimes called a rheogoniometer. The cone-and-plate viscometer consists of an inverted cone with its apex resting on a fixed circular plate. See Figure 1. The angle between the cone and plate, the gap angle, is usually small, ranging from  $1/3^\circ$  to  $4^\circ$ , and the diameter of the cone is typically between 5 and 10 cm. The region between the cone and plate is filled with the test fluid, and the cone is rotated about its axis at a constant angular velocity. The system is allowed to reach a steady state, and then the angular velocity of the cone and the torque exerted on the fixed plate are measured and recorded.

The use of the cone-and-plate viscometer to obtain meaningful rheological properties of simple fluids is dependent upon the assumption of primary flow. Primary flow [17]

is the flow field obtained by solving the governing equations of the flow after making the following assumptions:

- (1) the gap angle  $\epsilon$  is small so that  $\cos \epsilon \approx 1$ , and  $\sin \epsilon = 0$ ,
- (2) inertial effects are negligible, i.e., the only non-zero component of velocity is that in the direction of the rotation of the cone,
- (3) the non-zero velocity component is a linear function of the radial distance measured from the intersection of the cone and plate,
- (4) the flow is rotationally symmetric, and
- (5) edge effects are negligible.

The stress tensor representing the stress on the fluid is uniform throughout the fluid and consists of only two non-zero components; for non-polar materials the stress tensor is symmetric, and these two non-zero components of shear stress are equal. Expressions are derived which provide the shear rate and shear stress as simple functions of the gap angle, the diameter of the cone, the angular velocity of the cone, and the measured torque on the plate. These expressions allow the cone-and-plate viscometer to be especially suited for determining the relationship between the shear stress and shear rate of simple fluids, i.e., used as a rheogoniometer. For Newtonian fluids, this relationship is linear with the viscosity as the proportionality constant.

Hence, the viscosity is easily obtained by computing the ratio of the shear stress and shear rate. Although for non-Newtonian fluids the functional relationship between shear stress and shear rate is more complex, it can be readily obtained by plotting the shear stress vs. shear rate.

Although the primary flow analysis neglects inertial effects, secondary flows (motion in the other two directions) are induced, in real fluids, by inertia at all shear rates [19]. Fortunately, at low shear rates, the effects of the secondary flows are negligible. However, at higher shear rates the effects of the secondary flow become pronounced [5], and; consequently, the primary flow analysis fails to describe the flow. If the cone-and-plate viscometer is to be used with confidence, the shear rates at which the primary flow analysis fails to describe the flow must be predicted, and at higher shear rates the effects of the secondary flow must be determined. This requires that the governing equations of the flow must be solved without making the restrictive assumptions (2), (3), and (5) inherent in the primary flow analysis.

## I.2 Previous Work

Many papers [2,3,5-9,11,13,16,18-21,24,25,27,28,30,31,33,35] have been published concerning secondary flows of both Newtonian and non-Newtonian fluids in viscometric type

flows. However, since this work deals only with the flow of Newtonian fluids in cone-and-plate viscometers, the forthcoming discussion of the published literature is restricted primarily to the cone-and-plate flow of Newtonian fluids.

Cox [6] observed, via flow tracers, radial flows in a cone-and-plate viscometer with both Newtonian and non-Newtonian fluids. The radial flow was observed to be small relative to the cone angular velocity and did not appear to affect the viscosity measurements of the Newtonian fluids tested. The non-Newtonian fluids tested were extruded from the device after one or two rotations of the cone.

Hoppman and Miller [11], using flow visualization techniques, photographed the flow of Newtonian fluids within cone-and-cylinder viscometers (Ferrenti cone-plate viscometer) with gap angles of 1, 10, 35, and 45°. They noted the streamlines of the flow in the instruments with large gap angles were not concentric circles (primary flow) about the axis of the cone, but were toroidal. They were not able to observe the flow in the instruments with a 1° angle, but were able to verify Cox's observation of radial flow. They also measured the torque exerted by the fluid on the cone and the angular velocity of the cone. For a 1° gap angle, the measured torque was within 2% of that predicted by the primary flow analysis. However, at larger gap angles, the discrepancy was much more severe.

Miller and Hoppman [18] extended the experiments of Hoppman and Miller to include more gap angles. The gap angles ranged from 4° to 45°, and the shear rates were

relatively low. The streamlines were also observed in this work to be toroidal, and as the gap angle was decreased, the center of the vortex moved toward the edge of the cone while as the gap angle was increased, the vortex center moved toward the intersection of the cone and plate. From these visual experiments and torque measurements, they outlined a procedure for determining the velocity profiles at these large gap angles. Since these gap angles are larger than those typically used in viscometry, and the shear rates are relatively low, these results have little quantitative value. They are, however, useful in providing a qualitative description of the vortex structure of the flow.

Walters and Waters [35] used a perturbation expansion coupled with a numerical technique to determine the secondary flow of an elastico-viscous liquid in a cone-and-plate viscometer. A polynomial expansion in  $L$ , the square of the Reynolds number, was assumed, and the numerical procedure was used to solve the resulting differential equations. The solution also assumes that edge effects are negligible. Since  $L$  was used as the perturbation parameter, Walters and Waters were forced to restrict their solution to slow flow ( $L \ll 1$ ). The predicted slow flow compared favorably with visual experiments which they conducted. Expressions for the couple exerted by viscous liquids (Newtonian fluids) on both the cone and plate were also derived. At values of  $L$  at which these expressions provide deviations from those of the

primary flow analysis, the slow flow restriction is violated, i.e.,  $L \gg 1$ . Walters and Waters ignored this and erroneously concluded that the secondary flow increases the couple on the moving cone and decreases the couple on the stationary plate. This does not conserve angular momentum, and; therefore, is not physically possible. The conservation of angular or moment of momentum provides that, at steady state, the torque on the cone must be equal in magnitude to that on the plate. See Appendix A. Walters and Waters concluded that the secondary flow effects can be ignored for the slow flow of moderately elastic fluids in cone-and-plate viscometers with gap angles of  $4^\circ$  or less.

Cheng [5] determined experimentally the effects of secondary flow on viscosity measurements. He sheared several Newtonian fluids of known viscosities in eight sets of cones and plates made up of the combinations of three cone diameters--5, 7 1/2, and 10 cm and three gap angles of approximately 1, 2, and  $4^\circ$ . At high shear rates, his data exhibits a strong secondary flow effect on the viscosities determined from the primary flow analysis. Cheng, in attempting to add credibility to his data, compared it with the erroneous torque correction of Walters and Waters [34]. He ignored the predicted negative torque effect on the plate and instead used the positive torque effect predicted on the cone to compare with the torque which he measured on the plate. From this comparison, Cheng erroneously concluded that

although the Walters and Waters theory does not predict the torque effect at high shear rates, it does predict when the secondary flow begins to affect the torque. He suggested that the lack of agreement at high shear rates could be due to Walters and Waters neglect of higher order terms in their asymptotic expansion and to edge effects at the free surface.

King and Waters [13] extended the Walters and Waters theory to include higher order terms in an attempt to make the theory match Cheng's experimental data at high shear rates. Since, as discussed above, the Walters and Waters theory is invalid at high shear rates, agreement with Cheng's data was not attained.

Savins and Metzner [24] used an analytical approximation to conclude that for Newtonian fluids sheared in both parallel plate and cone-and-plate viscometers, the Reynolds number based on the maximum gap clearance, must be less than 0.50 to insure that the effect of the secondary flow on the rate-of-deformation is less than 5%. They compared this Reynolds number criteria with the Reynolds numbers at which Cheng's [5] experimental data indicated a secondary flow effect on viscosity measurements. From this comparison, they concluded that the secondary flow has a 5% effect on the rate-of-deformation at Reynolds numbers approximately  $1/20^{\text{th}}$  of those at which the secondary flow affects the



torque, and; therefore, that the torque is an insensitive measure of the presence of secondary flow. Their analytical approximation was limited to small Reynolds numbers (shear rates), and; consequently, could not be used to predict the effects of secondary flow on torque and on the rate-of-deformation at higher shear rates. They also conducted flow visualization studies in which they observed not one but many small toroidal vortices. This is in contradiction to the observations of Hoppman and Miller [11] and Miller and Hoppman [18] who observed only one toroidal vortex.

Turian [30] used a double perturbation expansion to obtain an outer solution for the steady flow of a Newtonian fluid in an infinite cone-and-plate viscometer. This outer expansion ignores any edge effects of the free surface. He redefined the Reynolds number such that it contained the square of the gap angle, and he used the square of this newly defined Reynolds number  $Re^2$  and the gap angle (in radians) as perturbation parameters to obtain his solution. Since  $Re^2 < L$ , this solution is valid at higher shear rates than is the Walters and Waters theory. However, it is restricted to  $Re^2 \ll 1$ . Turian ignored this restriction on his solution and compared it with the viscosity measurements of Cheng [5], and he claimed very good agreement. However, Turian's "appropriately defined small Reynolds number"  $Re^2$  is approximately equal to 80 before his solution predicts an appreciable torque effect due to the secondary flow, and he claimed good

agreement for values of  $Re^2$  as high as  $3 \times 10^3$ . These values of  $Re^2$  are much larger than unity, and are; therefore, beyond the range of validity of his solution. At values of  $Re^2$  of the order of  $10^3$ , the velocity component in the direction of the rotation of the cone as computed by Turian's solution is negative throughout most of the fluid (a small region near the intersection of the cone and plate is positive) except on the boundaries. This is physically unrealistic as this means that the cone is rotating in one direction and the fluid is rotating in the opposite direction.

Although Savins and Metzner [24] have obtained an approximate criteria which predicts when the secondary flow becomes appreciable, and Cheng [5] has measured the effect of secondary flows on viscosity measurements, the effects of the secondary flow of Newtonian fluids sheared in cone-and-plate viscometers over a wide range of shear rates have not been determined.

### 1.3 Proposed Work

The cone-and-plate viscometer has not only been used to measure the viscosity of Newtonian fluids, but variations of the cone-and-plate viscometer have also been used for shear stress degradation studies [14]. Hence, in addition to the effects of the secondary flow on the torque and the prediction of when the secondary flow becomes appreciable, the effects of the secondary flow on the rate-of-deformation

must also be determined. Although the effects of the secondary flow on the torque can be determined experimentally [5], the effects of the secondary flow on the rate-of-deformation would be extremely difficult if not impossible to measure.

The intent of this work is to solve the governing equations of the flow of a Newtonian fluid in a cone-and-plate viscometer without making the restrictive assumptions inherent in the primary flow analysis. In so doing, the effects of the secondary flow on both the torque and the rate-of-deformation will be determined over a wide range of shear rates.

## CHAPTER II

### ANALYTICAL FORMULATION

#### II.1 Derivation of the Governing Equations

Five physical laws describe the deformation of bodies. They are the conservation of mass, the balance of linear momentum, the balance of the moment of momentum (angular momentum), the balance of energy, and the entropy inequality (the second law of thermodynamics). These laws provide equations, generally referred to as the governing equations, whose solution describes the deformation.

The entropy inequality does not provide a governing equation, but provides information about the behavior of the material. Using the Eulerian or spacial description of motion, the remaining laws can be expressed mathematically as

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \dot{\underline{x}}) = 0, \quad (1.1)$$

$$\rho \ddot{\underline{x}} = \text{div} \underline{T} + \rho \underline{b}, \quad (1.2)$$

$$\underline{T} = \underline{T}^T, \quad (1.3)$$

and

$$\rho c_v \dot{T} = -\text{div} q - T \left( \frac{\partial P}{\partial T} \right)_v \text{div} \dot{\underline{x}} + \text{tr}(\underline{T} \cdot \text{grad} \dot{\underline{x}}) + \rho Q \quad (1.4)$$

where at each point in the material

- $\rho$  is the density,
- $\dot{\underline{x}}$  is the velocity relative to some fixed frame of reference, i.e., material time derivative of the displacement from the reference configuration,
- $\ddot{\underline{x}}$  is the acceleration relative to some fixed frame of reference, i.e., the material time derivative of  $\dot{\underline{x}}$ ,
- $\underline{T}$  is the stress tensor representing the stress on the material--the convention adopted is that the second index denotes the surface on which the stress acts while the first index denotes the component of the stress on that surface. The stress  $\underline{t}$  on any surface is then given by  $\underline{t} = \underline{T} \cdot \underline{n}$  where  $\underline{n}$  is the normal to the surface directed into the material,
- $\underline{b}$  is the body force per unit volume,
- $c_v$  is the specific heat of the material at constant specific volume,
- $\dot{T}$  is the material time derivative of the temperature,
- $\underline{q}$  is the heat flux vector,
- $P$  is the thermodynamic pressure, and
- $Q$  is the internal heat generation per unit volume.

Equation (1.1) is known as the continuity equation; equation (1.2) as Cauchy's first law for non-polar materials (all torques acting on the body are the result of forces acting on the body), and equation (1.4) as the balance of energy equation.

Since a fluid is defined as a material which cannot sustain a shear stress when at rest or in uniform motion, there are infinitely many reference configurations which yield the same stress and density in the deformed configuration. Hence, a particular reference configuration is not required to describe the motion of fluids. For this reason, the Eulerian or spacial description of motion is particularly suited. The Eulerian description of motion fixes attention on a region of space and takes the spacial position  $\underline{x}$  and the time  $t$  as the independent variables, i.e.,  $\dot{\underline{x}} = \underline{x}(\underline{x}, t)$ ,  $\rho(\underline{x}, t)$ , etc. In other words, at each instant of time a particle has the velocity, density, etc. which corresponds to the velocity, density, etc. assigned to the position which it occupies. For general nonsteady flow, the particle paths are not known, and; therefore, the reference configuration at  $t = 0$  is not known. Hence, the material derivative with respect to time is not known, which is essential if the governing equations, equations (1.1) through (1.4) are to be solved. This problem can be circumvented by applying the chain rule of differential calculus. If  $f$  is a continuous and differentiable scalar, vector, or tensor valued function of  $\underline{x}$  and  $t$ , then

the chain rule provides that

$$\dot{f} = \frac{\partial f}{\partial t} + (\text{grad } f) \cdot \underline{x} . \quad (1.5)$$

Hence,

$$\ddot{\underline{x}} = \frac{\partial \dot{\underline{x}}}{\partial t} + (\text{grad } \dot{\underline{x}}) \cdot \dot{\underline{x}} , \quad (1.6)$$

and

$$\dot{T} = \frac{\partial T}{\partial t} + (\text{grad } T) \cdot \dot{\underline{x}} . \quad (1.7)$$

With equations (1.6) and (1.7), equations (1.1) through (1.4) can be solved without any knowledge of the reference configuration at  $t = 0$ . However, the unknowns of equations (1.1) through (1.4) -  $\rho$ ,  $\dot{\underline{x}}$ ,  $\underline{T}$ ,  $c_v$ ,  $T$ ,  $q$ , and  $P$  - number 20 while only 11 independent equations are given. Hence, if the system is to be determinate more equations must be provided. These equations are obtained from knowledge of the behavior of the material which is being deformed and are called constitutive equations. Since the determination of these constitutive equations is beyond the scope of this work, the required constitutive equations will be simply stated and subsequently used without any discussion of their origin. The constitutive equation for the stress for an incompressible Newtonian fluid is

$$\underline{T} = -p\underline{I} + 2\mu\underline{D} \quad (1.9)$$

where

$p$  is the hydrostatic or mean pressure (not thermodynamic pressure),

$\underline{I}$  is the identity tensor,

$\mu$  is the viscosity of the fluid, and

$\underline{D}$  is the rate-of-deformation tensor.

The rate-of-deformation tensor  $\underline{D}$  is defined by

$$\underline{D} \equiv \frac{1}{2} [\text{grad } \dot{\underline{x}} + (\text{grad } \dot{\underline{x}})^T] , \quad (1.10)$$

and the mean pressure, hereafter referred to as simply the pressure, is defined by

$$p \equiv - \frac{1}{3} \text{tr } \underline{T} . \quad (1.11)$$

A constitutive equation for the viscosity  $\mu$  is now needed.

This equation provides that

$$\mu = \text{fn}(T) . \quad (1.12)$$

Since for an incompressible material the density is invariant, the density is known throughout the domain. Furthermore, if the body force  $\underline{b}$  in equation (1.2) is not a function of the temperature, the energy equation, equation (1.4), and the mechanical equations, equations (1.1) through (1.3), are coupled only by the functional dependence of the viscosity on the temperature i.e., equation (1.12). This functional



dependence is generally quite strong. However, for many cases of the deformation (hereafter referred to as the flow) of an incompressible, Newtonian fluid, the temperature change is sufficiently small such that the viscosity can be assumed to be constant. In these cases the mechanical equations and the energy equation become decoupled and the mechanical equations can be solved for the unknowns-- $p$  and  $\dot{\underline{x}}$ . The temperature, if needed, can then be found by substituting the solution of the mechanical equations into the energy equation.

Since the density is constant for an incompressible material, the continuity equation for an incompressible material is simply

$$\text{div } \dot{\underline{x}} = 0 , \quad (1.13)$$

and the combination of equations (1.2), (1.6), (1.9), and (1.10) gives the momentum equation for an incompressible, constant viscosity, Newtonian fluid as

$$\begin{aligned} \frac{\partial \dot{\underline{x}}}{\partial t} + (\text{grad } \dot{\underline{x}}) \cdot \underline{x} = & - \frac{1}{\rho} \text{grad } p \\ & + \nu \text{div}(\text{grad } \dot{\underline{x}}) + \underline{b} \end{aligned} \quad (1.14)$$

where  $\nu$  is the kinematic viscosity and is defined by

$$\nu \equiv \frac{\mu}{\rho} . \quad (1.15)$$

Equations (1.13) and (1.14) are the governing

equations, often called the equations of motion, whose solution yields the pressure and velocity (assumed continuous and differentiable throughout the domain, i.e., no shocks, etc.) for an incompressible, constant viscosity, Newtonian fluid. The balance of the moment of momentum equation, equation (1.3), has been used in determining the constitutive equation for the stress, equation (1.9), and; therefore, is not listed as a governing equation. Equation (1.14) is often called the Navier-Stokes equation.

The flow of an incompressible, Newtonian fluid in the cone-and-plate viscometer is assumed in this work to be modeled by an incompressible, constant viscosity, Newtonian fluid. Hence, equations (1.13) and (1.14) are the governing equations used to obtain the pressure and velocity within the viscometer. The coordinate system used to describe the motion within the cone-and-plate viscometer is a right handed, spherical curvilinear system with the origin at the intersection of the cone and plate. See Figure 2. The radial variable  $r$  is then the radial distance measured from the intersection of the cone-and-plate,  $\phi$  is the angular measurement in the direction parallel to the rotation of the cone (meridian direction), and  $\theta$  is the angle measured from vertical (azimuthal direction). The component form of equations (1.12) and (1.13) in spherical coordinates can be found in tables in reference [26] or they can be derived using tensor analysis. See Appendix B. The component forms of these

equations are written in Appendix B as equations (B.41) through (B.44) where the notation has been changed, for convenience, so that  $y = \hat{x}$ . These equations can be simplified somewhat by invoking the following properties of cone-and-plate flow: the flow is at a steady state, and the flow is rotationally symmetric. These properties provide that all derivatives with respect to time  $t$  and  $\phi$  vanish. In addition, it is also convenient to make the transformation  $\beta = \frac{\pi}{2} - \theta$ . With these simplifications and the indicated transformation, the governing equations can be written in component form as

r - Component of Navier-Stokes Equation

$$\begin{aligned}
 v_r \frac{\partial v_r}{\partial r} - \frac{v_\theta}{r} \frac{\partial v_r}{\partial \beta} - \frac{v_\theta^2 + v_\phi^2}{r} &= - \frac{1}{\rho} \frac{\partial p^*}{\partial r} \\
 + v \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_r}{\partial r}) + \frac{1}{r^2 \cos \beta} \frac{\partial}{\partial \beta} (\cos \beta \frac{\partial v_r}{\partial \beta}) \right. \\
 \left. - \frac{2v_r}{r^2} + \frac{2}{r^2} \frac{\partial v_\theta}{\partial \beta} - \frac{2}{r^2} v_\theta \tan \beta \right] & \quad (1.16)
 \end{aligned}$$

$\theta$  - Component of Navier-Stokes Equation

$$v_r \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \beta} + \frac{v_r v_\theta}{r} - \frac{v_\theta^2 \tan \beta}{r} = \frac{1}{r\rho} \frac{\partial p^*}{\partial \beta}$$

$$\begin{aligned}
& + v \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\theta}{\partial r}) + \frac{1}{r^2 \cos \beta} \frac{\partial}{\partial \beta} (\cos \beta \frac{\partial v_\theta}{\partial \beta}) \right. \\
& \left. - \frac{2}{r^2} \frac{\partial v_r}{\partial \beta} - \frac{v_\theta}{r^2 \cos^2 \beta} \right] \quad (1.17)
\end{aligned}$$

$\phi$  - Component of the Navier-Stokes Equation

$$\begin{aligned}
& v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\phi v_r}{r} - \frac{v_\theta^2 \tan \beta}{r} \\
& v \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\phi}{\partial r}) + \frac{1}{r^2 \cos \beta} (\cos \beta \frac{\partial v_\phi}{\partial \beta}) \right] \quad (1.18)
\end{aligned}$$

Continuity

$$\cos \beta \frac{\partial}{\partial r} (r^2 v_r) - r \frac{\partial}{\partial \beta} (v_\theta \cos \beta) = 0 \quad (1.19)$$

where the brackets  $\langle \rangle$  denoting physical components of the velocities have been discarded. Hereafter, the velocity components are physical components unless otherwise specified. Also,

$$p^* \equiv p + \rho r g \sin \beta . \quad (1.20)$$

The second term on the right hand side of (1.20) comes from the body force in the Navier-Stokes equation. This can be readily seen by realizing that the only body force acting on

the fluid is gravity. Hence

$$\underline{b} = -g\underline{k} = b\langle r \rangle \underline{e}_r + b\langle \theta \rangle \underline{e}_\theta + b\langle \phi \rangle \underline{e}_\phi$$

where  $\underline{e}_r$ ,  $\underline{e}_\theta$ , and  $\underline{e}_\phi$  are the unit base vectors in the  $r$ ,  $\theta$ , and  $\phi$  directions, respectively, and  $\underline{k}$  is the cartesian unit base vector in the vertical direction. See Figure 2. Therefore,

$$b\langle r \rangle = \underline{b} \cdot \underline{e}_r = -g\underline{k} \cdot \underline{e}_r = -g \sin \beta, \quad (1.21)$$

$$b\langle \theta \rangle = \underline{b} \cdot \underline{e}_\theta = -g\underline{k} \cdot \underline{e}_\theta = g \cos \beta, \quad (1.22)$$

and

$$b\langle \phi \rangle = \underline{b} \cdot \underline{e}_\phi = -g\underline{k} \cdot \underline{e}_\phi = 0. \quad (1.23)$$

The solution of the flow within the cone-and-plate viscometer is then the solution of equations (1.16) through (1.19) subject to the boundary conditions of the flow. The governing equations are a set of second order, non-linear, elliptic partial differential equations. Elliptic partial differential equations are properly posed if and only if the domain is closed, i.e., boundary conditions must be provided on the entire boundary.

## II.2 Boundary Conditions

The boundary conditions for velocity along the cone-and-plate are the no slip conditions. Therefore, the

velocity at the cone and the plate is the velocity of the cone and the plate, respectively. Since the plate is fixed, the velocity of the plate is zero. Hence ( $\beta = 0$  on the plate),

$$v_r(r,0) = v_\theta(r,0) = v_\phi(r,0) = 0 . \quad (2.1)$$

The velocity of the cone can be written as

$$\underline{v}(r,\epsilon) = r\underline{e}_r \times \Omega \underline{k} \quad (2.2)$$

where  $\Omega$  is the angular velocity of the cone, and  $\epsilon$  is the gap angle of the viscometer. It then follows that the boundary conditions at the cone are

$$v_r(r,\epsilon) = \underline{e}_r \cdot (r\underline{e}_r \times \Omega \underline{k}) = 0 , \quad (2.3)$$

$$v_\theta(r,\epsilon) = \underline{e}_\theta \cdot (r\underline{e}_r \times \Omega \underline{k}) = 0 , \quad (2.4)$$

and

$$v_\phi(r,\epsilon) = \underline{e}_\phi \cdot (r\underline{e}_r \times \Omega \underline{k}) = r\Omega \cos \epsilon . \quad (2.5)$$

The boundary conditions at a free surface have been formulated by Slattery [25]. Slattery treated free surfaces as singular surfaces at which he neglected: the effect of any gradients in surface tension, the effect of the rate-of-deformation of the free surface upon the surface stress tensor, the effect of mass transfer at the free surface, inertial and gravitational effects in the free surface, and

distortion of the free surface at the solid boundary. Under these restrictions, the equations of motion reduce to a static force balance at the free surface. Also, the free surface is assumed to intersect the cone at its edge, i.e., at  $r = R$ . See Figure 2. For the cone-and-plate viscometer, this provides the following conditions at the free surface:

$$(T_{\langle\theta\theta\rangle}^{(1)} - T_{\langle\theta\theta\rangle}^{(2)})n_{\theta} + (T_{\langle\theta r\rangle}^{(1)} - T_{\langle\theta r\rangle}^{(2)})n_r = 2H\sigma n_{\theta}, \quad (2.6)$$

$$(T_{\langle\phi\theta\rangle}^{(1)} - T_{\langle\phi\theta\rangle}^{(2)})n_{\theta} + (T_{\langle\phi r\rangle}^{(1)} - T_{\langle\phi r\rangle}^{(2)})n_r = 0, \quad (2.7)$$

and

$$(T_{\langle rr\rangle}^{(1)} - T_{\langle rr\rangle}^{(2)})n_r = 2H\sigma n_r \quad (2.8)$$

where

$T_{\langle\theta\theta\rangle}^{(1)}$ ,  $T_{\langle\theta r\rangle}^{(1)}$ ,  $T_{\langle\phi\theta\rangle}^{(1)}$ ,  $T_{\langle\phi r\rangle}^{(1)}$ , and  $T_{\langle rr\rangle}^{(1)}$  are elements of the stress tensor representing the stress on the fluid within the viscometer at the free surface,

$T_{\langle\theta\theta\rangle}^{(2)}$ ,  $T_{\langle\theta r\rangle}^{(2)}$ ,  $T_{\langle\phi\theta\rangle}^{(2)}$ ,  $T_{\langle\phi r\rangle}^{(2)}$ , and  $T_{\langle rr\rangle}^{(2)}$  are elements of the stress tensor representing the stress on the surrounding air at the free surface,

$n_r$ ,  $n_{\theta}$ , and  $n_{\phi}$  are the physical components of the unit normal to the free surface outwardly directed into the surrounding air,

$\sigma$  is the surface tension, and

$H$  is the mean curvature of the free surface.

In addition, the surface is assumed to have no normal component of velocity. Hence,

$$\underline{v} \cdot \underline{n} = v_r n_r + v_\theta n_\theta + v_\phi n_\phi = 0 . \quad (2.9)$$

The mean curvature  $H$  and the outwardly directed normal are given by

$$H = \frac{ff'' - 2(f')^2 - 3(f')^2 + ff' \cot \theta + \cot \theta (f')^3 / f}{2[f^2 + (f')^2]^{3/2}} , \quad (2.10)$$

$$n_r = \frac{f}{[f^2 + (f')^2]^{1/2}} , \quad (2.11)$$

$$r n_\theta = \frac{f}{[f^2 + (f')^2]^{1/2}} , \quad (2.12)$$

and

$$n_\phi = 0 \quad (2.13)$$

where  $r = f(\theta)$  on the free surface. In general,  $f$  is unknown. However, since the gap is very small in the typical cone-and-plate viscometer, the free surface is expected to be essentially spherical in shape and will be so assumed in this work. If the free surface is spherical then equations (2.10) through (2.12) reduce to

$$H = -\frac{1}{R} , \quad (2.14)$$

$$n_r = 1 , \quad (2.15)$$



and

$$n_{\theta} = 0. \quad (2.16)$$

The boundary conditions for a spherical free surface at the edge of the cone can then be written by combining equations (2.14) through (2.16) and equations (2.6) through (2.9) to give

$$T_{\langle\theta r\rangle}^{(1)} = T_{\langle\theta r\rangle}^{(2)}, \quad (2.17)$$

$$T_{\langle\phi r\rangle}^{(1)} = T_{\langle\phi r\rangle}^{(2)}, \quad (2.18)$$

$$T_{\langle rr\rangle}^{(1)} = T_{\langle rr\rangle}^{(2)} - \frac{2\sigma}{R}, \quad (2.19)$$

and

$$v_r = 0. \quad (2.20)$$

If the shear stress of the surrounding air is assumed to be negligible, then

$$\underline{T}^{(2)} = -p_a \underline{I} \quad (2.21)$$

where  $p_a$  is the ambient pressure of the surrounding air.

Equations (2.17) through (2.19) can then be written as

$$T_{\langle\theta r\rangle} = 0, \quad (2.22)$$

$$T_{\langle\phi r\rangle} = 0, \quad (2.23)$$

and

$$T_{\langle rr \rangle} = -p_a - \frac{2\sigma}{R} \quad (2.24)$$

where the superscript (1) has been dropped.

Equations (2.22) through (2.24) can be expressed in terms of pressure and velocity by combining them with equation (1.9) of Section II.1 and equations (B.45), (B.46), (B.51), and (B.52) of Appendix B to give at  $r = R$

$$\frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) = 0, \quad (2.25)$$

$$\frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) = 0, \quad (2.26)$$

and

$$p = p_a + \frac{2\sigma}{R} + 2\mu \frac{\partial v_r}{\partial r} \quad (2.27)$$

where the rotationally symmetric condition  $\frac{\partial}{\partial \phi} = 0$  has been invoked.

Equations (2.20) and (2.28) through (2.29) provide the boundary conditions on the velocities at  $r = R$  while equation (2.30) provides a boundary condition on the pressure at  $r = R$ .

### II.3 Dimensional Analysis

The method used to determine the significant

dimensionless parameters is that published by Hellums and Churchill [10]. First, the dependent variables are non-dimensionalized as follows:

$$\bar{v}_r = v_r/v_{r0} , \quad (3.1)$$

$$\bar{v}_\theta = v_\theta/v_{\theta0} , \quad (3.2)$$

$$\bar{v}_\phi = v_\phi/v_{\phi0} , \quad (3.3)$$

$$\bar{r} = r/r_0 , \quad (3.4)$$

and

$$\bar{p}^* = p^*/p_0^* . \quad (3.5)$$

Equations (3.1) through (3.5) are then substituted into the governing equations, equations (1.16) through (1.20) and the boundary conditions, equations (2.20) and (2.25) through (2.27) to give the following dimensionless equations:

#### r - Component of the Navier-Stokes Equation

$$\begin{aligned} & \left( \frac{v_{r0} r_0}{\nu} \right) \bar{v}_r \frac{\partial \bar{v}_r}{\partial \bar{r}} - \left( \frac{r_0 v_{\theta0}}{\nu} \right) \frac{\bar{v}_\theta}{\bar{r}} \frac{\partial \bar{v}_r}{\partial \beta} - \left( \frac{r_0 v_{\theta0}^2}{\nu v_{r0}} \right) \frac{\bar{v}_\theta^2}{\bar{r}} \\ & - \left( \frac{r_0 v_{\phi0}^2}{\nu v_{r0}} \right) \frac{\bar{v}_\phi^2}{\bar{r}} = - \left( \frac{r_0 p_0^*}{\mu v_{r0}} \right) \frac{\partial \bar{p}^*}{\partial \bar{r}} + \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left( \bar{r}^2 \frac{\partial \bar{v}_r}{\partial \bar{r}} \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{r^2 \cos \beta} \frac{\partial}{\partial \beta} \left( \cos \beta \frac{\partial \bar{v}_r}{\partial \beta} \right) - \frac{2\bar{v}_r}{r^2} \\
& + \left( \frac{v_{\theta 0}}{v_{r 0}} \right) \frac{2}{r^2} \frac{\partial \bar{v}_\theta}{\partial \beta} - 2\bar{v}_\theta \tan \beta
\end{aligned} \tag{3.6}$$

$\theta$  - Component of the Navier-Stokes Equation

$$\begin{aligned}
& \left( \frac{v_{r 0} r_0}{v} \right) \bar{v}_r \frac{\partial \bar{v}_\theta}{\partial r} - \left( \frac{v_{\theta 0} r_0}{v} \right) \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_\theta}{\partial \beta} + \left( \frac{v_{r 0} r_0}{v} \right) \frac{\bar{v}_r \bar{v}_\theta}{r} \\
& - \left( \frac{r_0 v_{\phi 0}^2}{v v_{\theta 0}} \right) \frac{\bar{v}_\phi^2 \tan \beta}{r} = \left( \frac{p_0^* r_0}{\mu v_{\theta 0}} \right) \frac{1}{r} \frac{\partial \bar{p}^*}{\partial \beta} \\
& + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \bar{v}_\theta}{\partial r} \right) + \frac{1}{r^2 \cos \beta} \frac{\partial}{\partial \beta} \left( \cos \beta \frac{\partial \bar{v}_\theta}{\partial \beta} \right) \\
& - \left( \frac{v_{r 0}}{v_{\theta 0}} \right) \frac{2}{r^2} \frac{\partial \bar{v}_r}{\partial \beta} - \frac{\bar{v}_\theta}{r^2 \cos^2 \beta}
\end{aligned} \tag{3.7}$$

$\phi$  - Component of the Navier-Stokes Equation

$$\begin{aligned}
& \left( \frac{r_0 v_{r 0}}{v} \right) \bar{v}_r \frac{\partial \bar{v}_\phi}{\partial r} - \left( \frac{v_{\theta 0} r_0}{v} \right) \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_\phi}{\partial \beta} + \left( \frac{v_{r 0} r_0}{v} \right) \frac{\bar{v}_\phi \bar{v}_r}{r} \\
& + \left( \frac{v_{\theta 0} r_0}{v} \right) \frac{\bar{v}_\theta \bar{v}_\phi}{r} \tan \beta = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \bar{v}_\phi}{\partial r} \right)
\end{aligned}$$

$$+ \frac{1}{\bar{r}^2 \cos \beta} \frac{\partial}{\partial \beta} (\cos \beta \frac{\partial \bar{v}_\phi}{\partial \beta}) - \frac{\bar{v}_\phi}{\bar{r}^2 \cos^2 \beta} \quad (3.8)$$

### Continuity Equation

$$\cos \beta \frac{\partial}{\partial \bar{r}} (\bar{r}^2 \bar{v}_r) - \bar{r} \frac{\partial}{\partial \beta} (\bar{v}_\theta \cos \beta) = 0 \quad (3.9)$$

### Boundary Conditions

$$\bar{v}_r(\bar{r}, 0) = \bar{v}_\theta(\bar{r}, 0) = \bar{v}_\phi(\bar{r}, 0) = 0, \quad (3.10)$$

$$\bar{v}_r(\bar{r}, \epsilon) = \bar{v}_\theta(\bar{r}, \epsilon) = 0, \quad (3.11)$$

$$\bar{v}_\phi(\bar{r}, \epsilon) = \left( \frac{\bar{r}_0 \Omega}{\bar{v}_{\phi 0}} \right) \bar{r} \cos \epsilon, \quad (3.12)$$

$$\left. \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\theta}{\bar{r}} \right) \right|_{\bar{r} = \frac{R}{r_0}} = 0, \quad (3.13)$$

$$\left. \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\phi}{\bar{r}} \right) \right|_{\bar{r} = \frac{R}{r_0}} = 0, \quad (3.14)$$

and

$$\bar{p}^* \left( \frac{R}{r_0}, \beta \right) = \bar{p}_a^* + \left( \frac{\rho g r_0}{p_a^*} \right) \left( \frac{R}{r_0} \right) \sin \beta$$

$$+ 2 \left( \frac{\sigma}{r_0 p_0^*} \right) \left( \frac{r_0}{R} \right) + 2 \left( \frac{\mu v_{r_0}}{r_0 p_0^*} \right) \frac{\partial \bar{v}_r}{\partial \bar{r}} \left( \frac{R}{r_0}, \beta \right) \quad (3.15)$$

where  $\bar{p}_a = p_a/p_0^*$ . Since  $\epsilon$  is dimensionless, it is one of the dimensionless parameters. The remaining dimensionless groups are

$$\left( \frac{v_{r_0} r_0}{v} \right), \left( \frac{r_0 v_{\theta_0}}{v} \right), \left( \frac{r_0 v_{\theta_0}^2}{v v_{r_0}} \right), \left( \frac{r_0 v_{\phi_0}^2}{v v_{r_0}} \right), \left( \frac{r_0 p_0^*}{\mu v_{r_0}} \right),$$

$$\left( \frac{v_{\theta_0}}{v_{r_0}} \right), \left( \frac{r_0 v_{\phi_0}^2}{v v_{\theta_0}} \right), \left( \frac{p_0^* r_0}{\mu v_{\theta_0}} \right), \left( \frac{r_0 \Omega}{v_{\phi_0}} \right), \left( \frac{R}{r_0} \right),$$

$$\left( \frac{\rho g r_0}{p_0^*} \right), \text{ and } \left( \frac{\sigma}{r_0 p_0^*} \right).$$

Setting

$$\left( \frac{R}{r_0} \right) = 1, \left( \frac{r_0 \Omega}{v_{\phi_0}} \right) = \left( \frac{v_{\theta_0}}{v_{r_0}} \right) = 1, \text{ and}$$

$$\left( \frac{v_{r_0} r_0}{v} \right) = \left( \frac{p_0^* r_0}{\mu v_{r_0}} \right) = \text{Re}$$

gives

$$r_0 = R, v_{\phi_0} = v_{\theta_0} = v_{r_0} = R\Omega, p_0^* = \rho R^2 \Omega^2,$$

$$\left( \frac{r_0 v_{\theta_0}}{v} \right) = \left( \frac{r_0 v_{\theta_0}^2}{v v_{\theta_0}} \right) = \left( \frac{r_0 v_{\phi_0}^2}{v v_{r_0}} \right) = \left( \frac{r_0 v_{\phi_0}^2}{\mu v_{\theta_0}} \right)$$

$$= \frac{p_0^* r_0}{\mu v_{\theta_0}} = Re,$$

$$\frac{\rho g r_0}{p_0^*} = \frac{1}{Fr}, \text{ and } \left( \frac{\sigma}{r_0 p_0^*} \right) = \frac{1}{We}$$

where

$$Re \equiv \frac{R^2 \Omega}{v} \equiv \text{Reynolds number}, \quad (3.17)$$

$$Fr \equiv \frac{R \Omega^2}{g} \equiv \text{Froude number, and} \quad (3.18)$$

$$We \equiv \frac{\rho R^3 \Omega^2}{\sigma} \equiv \text{Weber number.} \quad (3.19)$$

Hence, the dimensionless equations are:

### r - Component of the Navier-Stokes Equation

$$Re \left( \bar{v}_r \frac{\partial \bar{v}_r}{\partial \bar{r}} - \frac{\bar{v}_\theta}{\bar{r}} \frac{\partial \bar{v}_r}{\partial \beta} - \frac{\bar{v}_\theta^2}{\bar{r}} - \frac{\bar{v}_\phi^2}{\bar{r}} \right) = -Re \frac{\partial \bar{p}^*}{\partial \bar{r}}$$

$$+ \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left( \bar{r}^2 \frac{\partial \bar{v}_r}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2 \cos \beta} \frac{\partial}{\partial \beta} \left( \cos \beta \frac{\partial \bar{v}_r}{\partial \beta} \right)$$

$$- \frac{2\bar{v}_r}{\bar{r}^2} + \frac{2}{\bar{r}^2} \frac{\partial \bar{v}_\theta}{\partial \beta} \quad (3.20)$$

$\theta$  - Component of the Navier-Stokes Equation

$$\begin{aligned} \operatorname{Re} \left( \bar{v}_r \frac{\partial \bar{v}_\theta}{\partial \bar{r}} - \frac{\bar{v}_\theta}{\bar{r}} \frac{\partial \bar{v}_\theta}{\partial \beta} + \frac{\bar{v}_r \bar{v}_\theta}{\bar{r}} - \frac{\bar{v}_\phi^2 \tan \beta}{\bar{r}} \right) &= \frac{\operatorname{Re}}{\bar{r}} \frac{\partial \bar{p}^*}{\partial \beta} \\ &+ \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left( \bar{r}^2 \frac{\partial \bar{v}_\theta}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2 \cos \beta} \frac{\partial}{\partial \beta} \left( \cos \beta \frac{\partial \bar{v}_\theta}{\partial \beta} \right) \\ &- \frac{2}{\bar{r}^2} \frac{\partial \bar{v}_r}{\partial \beta} - \frac{\bar{v}_\theta}{\bar{r}^2 \cos^2 \beta} \end{aligned} \quad (3.21)$$

$\phi$  - Component of the Navier-Stokes Equation

$$\begin{aligned} \operatorname{Re} \left( \bar{v}_r \frac{\partial \bar{v}_\phi}{\partial \bar{r}} - \frac{\bar{v}_\theta}{\bar{r}} \frac{\partial \bar{v}_\phi}{\partial \beta} + \frac{\bar{v}_\theta \bar{v}_r}{\bar{r}} + \frac{\bar{v}_\theta \bar{v}_\phi}{\bar{r}} \tan \beta \right) \\ = \frac{1}{\bar{r}^2} \frac{\partial}{\partial \bar{r}} \left( \bar{r}^2 \frac{\partial \bar{v}_\phi}{\partial \bar{r}} \right) + \frac{1}{\bar{r}^2 \cos \beta} \frac{\partial}{\partial \beta} \left( \cos \beta \frac{\partial \bar{v}_\phi}{\partial \beta} \right) \\ - \frac{\bar{v}_\phi}{\bar{r}^2 \cos^2 \beta} \end{aligned} \quad (3.22)$$

Continuity Equation

$$\cos \beta \frac{\partial}{\partial \bar{r}} \left( \bar{r}^2 \bar{v}_r \right) - \bar{r} \frac{\partial}{\partial \beta} \left( \bar{v}_\theta \cos \beta \right) = 0 \quad (3.23)$$

Boundary Conditions

$$\bar{v}_r(\bar{r}, 0) = \bar{v}_\theta(\bar{r}, 0) = \bar{v}_\phi(\bar{r}, 0) = 0, \quad (3.24)$$



$$\bar{v}_r(\bar{r}, \epsilon) = \bar{v}_\theta(\bar{r}, \epsilon) = 0, \quad (3.25)$$

$$\bar{v}_\phi(\bar{r}, \epsilon) = \bar{r} \cos \epsilon, \quad (3.26)$$

$$\bar{v}_r(1, \beta) = 0, \quad (3.27)$$

$$\left. \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\theta}{\bar{r}} \right) \right|_{\bar{r}=1} = 0, \quad (3.28)$$

$$\left. \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\phi}{\bar{r}} \right) \right|_{\bar{r}=1} = 0, \quad (3.29)$$

and

$$\begin{aligned} \bar{p}^*(1, \beta) = & \bar{p}_a + \frac{\sin \beta}{Fr} \\ & + \frac{2}{We} + \frac{2}{Re} \frac{\partial \bar{v}_r(1, \beta)}{\partial \bar{r}}. \end{aligned} \quad (3.30)$$

Hence, the flow of a Newtonian fluid in the cone-and-plate viscometer is a 4 parameter problem with  $\epsilon$ ,  $Fr$ ,  $We$ , and  $Re$  as the parameters. However, if the solution for  $\bar{p}^*$  is not required, i.e. only the solution for the velocity profiles is sought,  $\bar{p}^*$  can be eliminated from equations (3.20) and (3.21). The boundary condition (3.30) is then not required. Consequently, the solution for the velocity profiles alone is reduced to a two parameter problem involving the parameters  $\epsilon$  and  $Re$ .

## II.4 Torque and Rate-of-Deformation

The torque measured in the cone-and-plate viscometer is that about the axis of the cone which is exerted on the cone or plate by the fluid. The torque on the plate is given by the following mathematical expression:

$$T|_{\text{plate}} = - \int_A [\underline{k} \cdot \underline{r} \times (\underline{T} \cdot (-\underline{e}_\theta))] dA \quad (4.1)$$

where

- $T|_{\text{plate}}$  is the magnitude of the torque on the plate,
- $\underline{k}$  is the unit vector in the vertical direction,
- $\underline{r}$  is the position vector,
- $\underline{T}$  is the stress tensor,
- $-\underline{e}_\theta$  is the unit normal of the plate directed into the fluid, and
- $A$  is the wetted surface area of the plate.

Equation (4.1) can be rewritten as

$$T|_{\text{plate}} = - \int_0^{2\pi} \int_0^R r^2 T_{\langle \phi \theta \rangle} |_{\beta=0} dr d\phi, \quad (4.2)$$

and since  $T_{\langle \phi \theta \rangle}$  is independent of  $\phi$  (rotationally symmetric)

$$T|_{\text{plate}} = -2\pi R^3 \int_0^1 r^2 \bar{D}_{\langle \phi \theta \rangle} |_{\beta=0} d\bar{r} \quad (4.3)$$

where the variables have been made non-dimensional consistent with Section II.3 ( $\bar{D} = D/\Omega$ ), and the constitutive equation for the stress, equation (1.9), has been substituted. The torque on the plate and cone as given, in reference [17], by the primary flow analysis is  $T_p = \frac{2\pi\mu R^3\Omega}{3\epsilon}$ . Hence, the ratio of the torque on the plate with secondary flow included to that with no secondary flow is

$$\frac{T}{T_p}\bigg|_{\text{plate}} = 3\epsilon \int_0^1 \bar{r} \frac{\partial}{\partial \beta} \left( \frac{\bar{v}_\phi}{\cos \beta} \right) \bigg|_{\beta=0} d\bar{r} \quad (4.4)$$

where equations (B.50) and (B.53) of Appendix B and the rotationally symmetric condition  $\frac{\partial}{\partial \phi} = 0$  have been used. In addition, the transformation  $\beta = \frac{\pi}{2} - \theta$  has also been made.

The corresponding ratio on the cone can be derived in the same manner to obtain

$$\frac{T}{T_p}\bigg|_{\text{cone}} = 3\epsilon \cos^2 \epsilon \int_0^1 \bar{r} \frac{\partial}{\partial \beta} \left( \frac{\bar{v}_\phi}{\cos \beta} \right) \bigg|_{\beta=\epsilon} d\bar{r}. \quad (4.5)$$

As was discussed in Section I.1, the rate-of-deformation tensor, as determined by the primary flow analysis, consists of only two non-zero elements  $D_{\langle \theta \phi \rangle}$  and  $D_{\langle \phi \theta \rangle}$ , which are equal for non-polar materials. The value of these non-zero elements is henceforth referred to as the primary deformation rate  $D_p$  and is given by

$$D_p = -\frac{\dot{\Omega}}{2\epsilon} = -\frac{\dot{\gamma}}{2} \quad (4.6)$$

where  $\dot{\gamma}$  is called the shear rate [17].

Using equation (4.6) and equations (B.45) through (B.53) of Appendix B, the equations which provide the ratio of each of the elements of the rate-of-deformation tensor and the primary deformation rate are derived. These equations are as follows:

$$\frac{D_{\langle rr \rangle}}{D_p} = -2\epsilon \frac{\partial \bar{v}_r}{\partial \bar{r}}, \quad (4.7)$$

$$\frac{D_{\langle \theta r \rangle}}{D_p} = \frac{D_{\langle r \theta \rangle}}{D_p} = \epsilon \left[ -\bar{r} \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\theta}{\bar{r}} \right) + \frac{1}{\bar{r}} \frac{\partial \bar{v}_r}{\partial \beta} \right], \quad (4.8)$$

$$\frac{D_{\langle \phi r \rangle}}{D_p} = \frac{D_{\langle r \phi \rangle}}{D_p} = -\epsilon \bar{r} \frac{\partial}{\partial \bar{r}} \left( \frac{\bar{v}_\phi}{\bar{r}} \right), \quad (4.9)$$

$$\frac{D_{\langle \theta \theta \rangle}}{D_p} = 2\epsilon \left[ \frac{1}{\bar{r}} \frac{\partial \bar{v}_\theta}{\partial \beta} - \frac{\bar{v}_r}{\bar{r}} \right], \quad (4.10)$$

$$\frac{D_{\langle \phi \phi \rangle}}{D_p} = -2\epsilon \left[ \frac{\bar{v}_r}{\bar{r}} + \frac{\bar{v}_\theta \tan \beta}{\bar{r}} \right], \quad (4.11)$$

and

$$\frac{D_{\langle \theta \phi \rangle}}{D_p} = \frac{D_{\langle \phi \theta \rangle}}{D_p} = \frac{\epsilon \cos \beta}{\bar{r}} \frac{\partial}{\partial \beta} \left( \frac{\bar{v}_\phi}{\cos \beta} \right) \quad (4.12)$$

where the rotationally symmetric condition has been invoked

and the transformation  $\beta = \pi/2 - \theta$  was made. Furthermore, the application of the boundary conditions on the velocities from Section II.2 gives

$$\left. \frac{D\langle r r \rangle}{Dp} \right|_{\beta=0} = \left. \frac{D\langle r r \rangle}{Dp} \right|_{\beta=\epsilon} = 0, \quad (4.13)$$

$$\left. \frac{D\langle \theta r \rangle}{Dp} \right|_{r=1} = 0, \quad (4.14)$$

$$\left. \frac{D\langle \phi r \rangle}{Dp} \right|_{r=1} = \left. \frac{D\langle \phi r \rangle}{Dp} \right|_{\beta=0} = \left. \frac{D\langle \phi r \rangle}{Dp} \right|_{\beta=\epsilon} = 0, \quad (4.15)$$

and

$$\left. \frac{D\langle \phi \phi \rangle}{Dp} \right|_{\beta=0} = \left. \frac{D\langle \phi \phi \rangle}{Dp} \right|_{\beta=\epsilon} = 0. \quad (4.16)$$

Since the torque ratios and the ratios of elements of the rate-of-determination tensor and the primary deformation rate are functions of the velocity profiles only, the effect of the secondary flow on the torque and the rate-of-deformation is a two parameter problem with  $\epsilon$  and  $Re$  as the parameters.

## CHAPTER III

### PERTURBATION APPROACH

#### III.1 Derivation of the Outer Solution

Since the gap angle, expressed in radians, is much less than unity for gap angles less than  $30^\circ$ , it seems plausible that an asymptotic expansion using the gap angle as the perturbation parameter could be utilized to obtain a solution. This asymptotic expansion can be obtained by defining a new independent variable  $\xi$  such that

$$\xi = \frac{\beta}{\epsilon} \quad (1.1)$$

where  $\epsilon$  is the gap angle measured in radians. The non-dimensional governing equations and boundary conditions, equations (3.20) through (3.30) of Section II.3 can then be transformed and written as

#### r - Component of the Navier-Stokes Equation

$$\begin{aligned} & \left( v_r \frac{\partial v_r}{\partial r} \epsilon^2 - \frac{v_\theta^2}{r} \epsilon^2 - \frac{v_\phi^2}{r} \epsilon^2 - \frac{v_\theta}{r} \frac{\partial v_r}{\partial \xi} \epsilon \right) \text{Re} = -\text{Re} \epsilon^2 \frac{\partial p^*}{\partial r} \\ & + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v_r}{\partial r} \right) \epsilon^2 + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \xi^2} - \frac{\tan \xi \epsilon}{r^2} \frac{\partial v_r}{\partial \xi} \epsilon \end{aligned}$$

$$-\frac{2v_r \epsilon^2}{r^2} + \frac{2}{r^2} \frac{\partial v_\theta}{\partial \xi} \epsilon - 2 \tan \xi \epsilon \frac{v_\theta}{r^2} \epsilon^2 \quad (1.2)$$

$\theta$  - Component of the Navier-Stokes Equation

$$\begin{aligned} & (v_r \frac{\partial v_\theta}{\partial r} \epsilon^2 \cos^2 \xi \epsilon + \frac{v_r v_\theta}{r} \cos^2 \xi \epsilon \epsilon^2 - \frac{v_\theta^2}{2r} \sin 2\xi \epsilon \epsilon^2 \\ & - \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \xi} \cos^2 \xi \epsilon \epsilon) \text{Re} = \frac{\text{Re}}{r} \epsilon \frac{\partial p^*}{\partial \xi} \cos^2 \xi \epsilon \\ & + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\theta}{\partial r}) \epsilon^2 \cos^2 \xi \epsilon + \frac{1}{r^2} \frac{\partial^2 v_\theta}{\partial \xi^2} \cos^2 \xi \epsilon \\ & - \frac{1}{2r^2} \frac{\partial v_\theta}{\partial \xi} \epsilon \sin 2\xi \epsilon - \frac{2}{r^2} \frac{\partial v_r}{\partial \xi} \epsilon \cos^2 \xi \epsilon - \frac{v_\theta}{r^2} \epsilon^2 \end{aligned} \quad (1.3)$$

$\phi$  - Component of the Navier-Stokes Equation

$$\begin{aligned} & (v_r \frac{\partial v_\phi}{\partial r} \epsilon^2 \cos^2 \xi \epsilon + \frac{v_\phi v_r}{r} \epsilon^2 \cos^2 \xi \epsilon + \frac{v_\theta v_\phi}{2r} \epsilon^2 \sin 2\xi \epsilon \\ & - \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \xi} \epsilon \cos^2 \xi \epsilon) \text{Re} - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\phi}{\partial r}) \epsilon^2 \cos^2 \xi \epsilon \\ & + \frac{1}{r^2} \frac{\partial^2 v_\phi}{\partial \xi^2} \cos^2 \xi \epsilon - \frac{1}{2r^2} \frac{\partial v_\phi}{\partial \xi} \epsilon \sin 2\xi \epsilon - \frac{v_\phi}{r^2} \epsilon^2 \end{aligned} \quad (1.4)$$

Continuity Equation

$$\epsilon \frac{\partial}{\partial r} (r^2 v_r) + r \epsilon \tan \xi \epsilon v_\theta - r \frac{\partial v_\theta}{\partial \xi} = 0 \quad (1.5)$$

Boundary Conditions

$$v_r(r,0) = v_\theta(r,0) = v_\phi(r,0) = 0, \quad (1.6)$$

$$v_r(r,1) = v_\theta(r,1) = 0, \quad (1.7)$$

$$v_\phi(r,1) = r \cos \epsilon, \quad (1.8)$$

$$v_r(1,\xi) = 0, \quad (1.9)$$

$$\frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) \Big|_{r=1} = 0, \quad (1.10)$$

$$\frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) \Big|_{r=1} = 0, \quad (1.11)$$

and

$$p^*(1,\xi) = p_a + \frac{\sin \beta}{Fr} + \frac{2}{We} + \frac{2}{Re} \frac{\partial v_r(1,\xi)}{\partial r} \quad (1.12)$$

where the overbars denoting dimensionless variables have been omitted for convenience.

A closer examination of equations (1.1) through (1.4)



reveals that  $\epsilon^2$  is multiplied by the highest order derivative with respect to  $r$  in each equation. This is the hallmark of a singular perturbation problem [32]. A singular perturbation problem is one in which a straightforward perturbation expansion, called the outer solution, does not satisfy all of the required boundary conditions. An inner solution is then sought in stretched variables to satisfy the remaining boundary conditions. The inner and outer solutions are then matched to obtain a complete solution. The flow of an incompressible, Newtonian fluid along a flat plate is an example of a singular perturbation problem. The inner solution domain is the boundary layer itself where the coordinate normal to the plate is stretched, and the outer solution is the inviscid solution. The inner and outer solutions are then matched at the edge of the boundary layer. Analogously, a complete perturbation solution for the flow of a Newtonian fluid in the cone-and-plate viscometer consists of an outer and inner solution. The inner solution domain is a thin region, a boundary layer, adjacent to the free surface while the outer solution domain is the remaining flow area.

The thickness of the boundary layer at the free surface can be estimated by first defining a stretched variable  $\hat{r}$  such that

$$\hat{r} = \frac{1-r}{\Delta(\epsilon)} \quad (1.13)$$

where  $\hat{r}$  is of the order of unity at the edge of the region influenced by the free surface, i.e., the boundary layer. The second and third terms in the right hand side of equation (1.2) are then transformed to obtain

$$\begin{aligned} \frac{\epsilon^2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v_r}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_r}{\partial \xi^2} &= \frac{1}{r^2(\hat{r})} \frac{\epsilon^2}{\Delta^2(\epsilon)} \frac{\partial}{\partial \hat{r}} \left( r^2(\hat{r}) \frac{\partial \hat{v}_r}{\partial \hat{r}} \right) \\ &+ \frac{1}{r^2(\hat{r})} \frac{\partial^2 \hat{v}_r}{\partial \xi^2} \end{aligned} \quad (1.14)$$

where  $\hat{v}_r \equiv \hat{v}_r(\hat{r}, \beta) = v_r(r, \beta)$ .

Now, if the boundary condition at the free surface is to be satisfied, the second and third terms of equation (1.14) must be of the same order in the domain of influence of the free surface. This is achieved if

$$\Delta(\epsilon) = \epsilon. \quad (1.15)$$

It then follows from equations (1.13) and (1.15) that the boundary layer at the free surface is of the order of  $\epsilon$  thick.

Due to the complexity of the flow within the viscometer, a matching of an inner and outer solution to obtain a complete solution could not be attained. This matching is essential if the perturbation approach is to provide the rate-of-deformation in and near the boundary layer at the free surface. However, since the boundary layer is very thin, the

amount of surface area of the plate within the boundary layer is negligible. Hence, a valid outer solution should provide a good approximation to the torque on the plate. Unfortunately, the Re restrictions on the outer expansion developed, subsequently, prevent it from predicting any torque effect of the secondary flow. Although the outer solution presented here does not provide the rate-of-deformation or the torque at sufficiently high Reynolds numbers, it is useful in interpreting the behavior of the flow near the intersection of the cone-and-plate and is so used in Chapter IV to provide needed numerical boundary conditions.

The outer solution is developed by assuming

$$v_r \equiv u = u_0 + u_1 \epsilon + u_2 \epsilon^2 + u_3 \epsilon^3 + u_4 \epsilon^4 + O(\epsilon^5), \quad (1.16)$$

$$v_\theta \equiv v = v_0 + v_1 \epsilon + v_2 \epsilon^2 + v_3 \epsilon^3 + v_4 \epsilon^4 + O(\epsilon^5) \quad (1.17)$$

$$v_\phi \equiv w = w_0 + w_1 \epsilon + w_2 \epsilon^2 + w_3 \epsilon^3 + w_4 \epsilon^4 + O(\epsilon^5) \quad (1.18)$$

and

$$p^* = p_0^* + p_1^* \epsilon + p_2^* \epsilon^2 + p_3^* \epsilon^3 + p_4^* \epsilon^4 + O(\epsilon^5) \quad (1.19)$$

and expanding the following in Taylor's series:

$$\cos \xi \epsilon = 1 - \frac{\xi^2 \epsilon^2}{2} + \frac{\xi^4 \epsilon^4}{4!} + O(\epsilon^6), \quad (1.20)$$

$$\cos^2 \xi \epsilon = 1 - \xi^2 \epsilon^2 + \frac{\xi^4 \epsilon^4}{3} + O(\epsilon^6) , \quad (1.21)$$

$$\tan \xi \epsilon = \xi \epsilon - \frac{\xi^3 \epsilon^3}{3} + O(\epsilon^5) , \quad (1.22)$$

and

$$\sin 2\xi \epsilon = 2\xi \epsilon - \frac{4\xi^3 \epsilon^3}{3} + O(\epsilon^5) . \quad (1.23)$$

Equations (1.16) through (1.23) are then substituted into equations (1.2) through (1.8) and coefficients of like powers of  $\epsilon$  are equated to provide governing equations and boundary conditions for  $u_0, u_1, \dots, u_n; v_0, v_1, \dots, v_n$ , etc. The solution of these systems of equations involves considerable algebra, and; therefore, only the results are presented here. The details of the solution are given in Appendix C.

The outer perturbation solution is

$$\begin{aligned} u(r, \xi) = & -\frac{r^3}{60} (5\xi^4 - 9\xi^2 + 4\xi) \text{Re } \epsilon^2 \\ & + \frac{r^3}{1260} (56\xi^6 - 273\xi^4 + 350\xi^3 - 159\xi^2 + 26\xi) \text{Re } \epsilon^4 \\ & + O(\epsilon^6) , \end{aligned} \quad (1.24)$$

$$\begin{aligned} v(r, \xi) = & -\frac{r^3}{12} (\xi^5 - 3\xi^3 + 2\xi^2) \text{Re } \epsilon^3 \\ & + \frac{r^3}{252} (5\xi^7 - 42\xi^5 + 77\xi^4 - 53\xi^3 + 13\xi^2) \text{Re } \epsilon^5 \end{aligned}$$

$$+ O(\epsilon^7) , \quad (1.25)$$

$$\begin{aligned} w(r, \xi) = & r\xi - \frac{r}{2} \xi \epsilon^2 + \frac{r}{120} (\xi^5 - 3\xi) \epsilon^4 \\ & - \frac{r^3}{25,200} (50\xi^7 - 63\xi^5 - 70\xi^4 + 83\xi) \text{Re}^2 \epsilon^4 \\ & + O(\epsilon^6) , \end{aligned} \quad (1.26)$$

and

$$\begin{aligned} p^*(r, \xi) = & A_0 + \frac{3r^2}{20} + A_1 \epsilon + A_2 \text{Re} \epsilon^2 \\ & - \frac{r^2}{420} (189\xi^2 - 84\xi + 53) \text{Re} \epsilon^2 + O(\epsilon^3) . \end{aligned} \quad (1.27)$$

Several interesting observations can be made about equations (1.23) through (1.26). As  $\text{Re}$  approaches zero,  $u$  and  $v$  vanish and  $w$  approaches a linear profile, i.e., as  $\text{Re}$  approaches zero then the outer perturbation solution approaches the primary solution (no inertia). Furthermore, in the limit as  $\text{Re}$  approaches zero,  $u$ ,  $v$ , and  $w$  satisfy the boundary conditions at the free surface. Hence, the straightforward perturbation expansion for the velocity becomes singular when inertial effects become significant. In addition, the primary solution for velocity is seen to satisfy the boundary conditions at the free surface. This means there are no edge effects of the free surface on the primary solution for the velocity. It also follows by letting  $\text{Re} \rightarrow 0$ , that

$$A_0 = p_a + \frac{2}{We} - \frac{3}{20},$$

and  $A_1 = \frac{\xi \epsilon}{Fr}$ , which is a contradiction since in the straightforward expression,  $A_1$  was found to be a constant. Hence, the straightforward perturbation expansion for  $p^*$  is singular even when inertial effects are negligible. Using the expression for  $A_0$ , the outer expansion for  $p^*(r, \epsilon)$  can now be written as

$$\begin{aligned} p^*(r, \xi) = & p_a + \frac{2}{We} + \frac{3}{20}(r^2 - 1) + A_1 \epsilon \\ & + A_2 Re \epsilon^2 - \frac{r^2}{420} (189\xi^2 - 84\xi + 53) Re \epsilon^2 + O(\epsilon^3). \end{aligned} \quad (1.28)$$

### III.2 Domain of Validity of the Outer Solution

A Re restriction can be placed on the domain of validity of the outer expansion by comparing the third and fourth terms of equation (1.26). Since in any perturbation expansion, the magnitude of each term in the expansion must be much smaller than the magnitude of the preceding term, the following inequality must hold everywhere within the viscometer if the expansion is to be valid

$$\left| \frac{r^3}{25,200} (50\xi^7 - 63\xi^5 - 70\xi^4 + 83\xi) Re^2 \epsilon^4 \right| \ll \left| -\frac{r}{2} \xi \epsilon^2 \right| \quad (2.1)$$

$$\text{for } 0 < \xi < 1$$

and since the left hand side is a maximum at  $r = 1$ , then

the following must hold

$$\left| \frac{\text{Re} \epsilon^4}{25,200} (50\epsilon^7 - 63\epsilon^5 - 70\epsilon^4 + 83\epsilon) \right| \ll \frac{\epsilon \epsilon^2}{2} \quad (2.2)$$

$$\text{for } 0 < \epsilon < 1 \quad (2.2)$$

The behavior of the left hand side can be predicted by letting

$$A = 50\epsilon^7 - 63\epsilon^5 - 70\epsilon^4 + 83\epsilon, \quad (2.3)$$

then

$$\frac{dA}{d\epsilon} = 350\epsilon^6 - 315\epsilon^4 - 280\epsilon^3 + 83, \quad (2.4)$$

and

$$\frac{d^2A}{d\epsilon^2} = 420\epsilon^2(5\epsilon^3 - 3\epsilon - 2). \quad (2.5)$$

By examining equations (2.3) through (2.5), it can be concluded that

$$\frac{d^2A}{d\epsilon^2} = 0 \quad \text{at} \quad \epsilon = 0 \quad \text{and} \quad \epsilon = 1, \quad (2.6)$$

$$\frac{d^2A}{d\epsilon^2} < 0 \quad \text{for } 0 < \epsilon < 1, \quad (2.7)$$

$$A = 0 \quad \text{at} \quad \epsilon = 0 \quad \text{and} \quad \epsilon = 1, \quad (2.8)$$

$$\frac{dA}{d\xi} > 0 \quad \text{at} \quad \xi = 0, \quad (2.9)$$

and

$$\frac{dA}{d\xi} < 0 \quad \text{at} \quad \xi = 1. \quad (2.10)$$

It follows from equations (2.6) through (2.10) that A is a linear function of  $\xi$  in some neighborhood of  $\xi = 0$ , and that the maximum slope of A occurs at  $\xi = 0$ . Furthermore, since  $\frac{d^2A}{d\xi^2} < 0$  everywhere except at  $\xi = 0$  and  $\xi = 1$ , the slope of A monotonically decreases with  $\xi$ . Hence, if the inequality of equation (2.1) is to hold, then

$$\left| \frac{Re^2 \epsilon^4}{25,200} \frac{dA}{d\xi} \right|_{\xi=0} \ll \left| \frac{d}{d\xi} \left( \frac{\xi \epsilon^2}{2} \right) \right|_{\xi=0} \quad (2.11)$$

which can be expressed as

$$Re \ll \frac{12.3}{\epsilon}. \quad (2.12)$$

If  $\ll$  is arbitrarily chosen to be 1/2 or less then the restriction on the Reynolds number can be written as

$$Re \leq \frac{6.15}{\epsilon}. \quad (2.13)$$

To exemplify, for a gap angle of  $2^\circ$ ,  $\epsilon = .03492$  radians, the outer expansion is restricted to  $Re \leq 176$ .

Since the torque at the plate is a function of  $\frac{\partial W}{\partial \xi}$  at



the plate and the  $Re$  restriction requires that the contribution to  $\frac{\partial w}{\partial \xi}$  of the inertial terms must be negligible, the outer perturbation solution does not predict a torque effect due to the secondary flow.

It should be mentioned that the outer solution is identical to that published by Turian [29]. However, Turian's method of obtaining the solution differs from that presented here. He used the stream function to eliminate the pressure and then expanded the stream function and  $w$  in terms of  $Re^2 \epsilon^4$  as the perturbation parameter. The resulting equations were then expanded using  $\epsilon^2$  as a perturbation parameter and solved. He did not place any restriction on  $Re^2 \epsilon^4$  and let it become large, the order of  $10^3$ . A quick look at equation (1.25) for  $Re^2 \epsilon^4$  of the order  $10^3$  indicates that the fourth term completely dominates the solution. This results in negative values of  $w$  essentially everywhere except on the boundaries, which is physically impossible.

Although the Reynolds number restriction on the outer solution is quite severe for the flow throughout the viscometer, the solution is valid for all finite Reynolds numbers in some neighborhood of the intersection of the cone and plate, i.e., as  $r$  approaches zero. This can readily be seen from equation (1.26). Since the fourth term of equation (1.26) contains  $r^3 Re^2$  and the second term contains  $r$ , for finite Reynolds numbers, a neighborhood of  $r = 0$  can be found such that the fourth term becomes negligible compared to the

second. Hence, the outer solution can be used in interpreting the behavior of the flow as  $r$  approaches zero. As mentioned in the previous section, the outer expansion will be used in Chapter IV to obtain required numerical boundary conditions at  $r = 0$ . The behavior of the stream function defined as

$$u = - \frac{1}{r^2 \cos \beta} \frac{\partial \psi}{\partial \beta} = - \frac{1}{\epsilon r^2 \cos \xi \epsilon} \frac{\partial \psi}{\partial \xi} \quad (2.14)$$

$$v = - \frac{1}{r \cos \beta} \frac{\partial \psi}{\partial r} = - \frac{1}{r \cos \xi \epsilon} \frac{\partial \psi}{\partial r} \quad (2.15)$$

near the origin will be needed. This can be obtained by using the chain rule of differential calculus and equations (2.14) and (2.15) to provide

$$\begin{aligned} \psi(r, \xi) = & - \cos \xi \epsilon \int r v(r, \xi) dr \\ & - \epsilon r^2 \int \cos \xi \epsilon u(r, \xi) d\xi + \psi(0, 0) . \end{aligned} \quad (2.16)$$

Since  $u(0, 0)$  and  $v(0, 0)$  are zero, then  $\psi(0, 0)$  can be assumed to be zero without any loss of generality. Equations (1.19), (1.23), and (1.24) are then substituted into (2.16) to give

$$\begin{aligned} \psi(r, \xi) = & \frac{r^5}{60} (\xi^5 - 3\xi^3 + 2\xi^2) \operatorname{Re} \epsilon^2 \\ & + \frac{r^5}{60} (\xi^5 - 3\xi^3 + 2\xi^2) \operatorname{Re} \epsilon^3 + O(\epsilon^4) . \end{aligned} \quad (2.17)$$

## CHAPTER IV

### NUMERICAL SOLUTION

#### IV.1 Derivation of Continuum Equations

##### a. Governing equations

The most common approach in obtaining numerical solutions of incompressible, Newtonian fluid mechanics problems is to solve the vorticity transport and continuity equations. See Roache [23]. Hence, although the governing equations derived in Chapter II describe the flow in the cone-and-plate viscometer, a different system of governing equations must be derived. In the derivations which follow the tensor analysis notation is consistent with McConnell [15].

The vorticity transport equation can be derived by first defining the vorticity of the flow as

$$\omega = \text{curl } \dot{\mathbf{x}} \quad (1.1)$$

which may be written in general curvilinear coordinates as

$$\omega = - \frac{\epsilon^{ijk}}{\sqrt{g}} \frac{\partial \dot{x}_i}{\partial x^k} g_j \quad (1.2)$$

where  $\epsilon^{ijk}$  is the permutation symbol,  $\sqrt{g}$  is the square root

of the determinant of the metric tensor of the coordinate system,  $\dot{x}_j$  is the  $j^{\text{th}}$  covariant component of velocity,  $x^k$  is the coordinate along the  $k^{\text{th}}$  coordinate surface, and  $\underline{g}_i$  is the base vector parallel to the  $i^{\text{th}}$  coordinate surface. Now, using the well known vector identity

$$(\text{grad } \dot{\underline{x}}) \cdot \dot{\underline{x}} = \frac{1}{2} \text{grad } v^2 - \dot{\underline{x}} \times \underline{\omega} \quad (1.3)$$

where

$$v^2 = \dot{\underline{x}} \cdot \dot{\underline{x}}, \quad (1.4)$$

the Navier-Stokes equation, equation (1.14) of Chapter II, can be written as

$$\frac{\partial \dot{\underline{x}}}{\partial t} = \dot{\underline{x}} \times \underline{\omega} - \text{grad} \left( \frac{p}{\rho} + \frac{v^2}{2} \right) + \nu \text{div}(\text{grad } \dot{\underline{x}}) + \underline{b}. \quad (1.5)$$

If the body force  $\underline{b}$  is conservative, then a scalar potential can be defined such that

$$\underline{b} = - \text{grad } \phi. \quad (1.6)$$

It then follows that the Navier-Stokes equation for conservative body forces can be written as

$$\frac{\partial \dot{\underline{x}}}{\partial t} = \dot{\underline{x}} \times \underline{\omega} - \text{grad} \left( \frac{p}{\rho} + \frac{v^2}{2} + \phi \right) + \nu \text{div}(\text{grad } \dot{\underline{x}}). \quad (1.7)$$

Now, if the curl of equation (1.7) is taken, the following equation is obtained

$$\frac{\partial \underline{\omega}}{\partial t} = \text{curl}(\dot{\underline{x}} \times \underline{\omega}) + \nu \text{curl}(\text{div}(\text{grad } \dot{\underline{x}})) \quad (1.8)$$

where  $\nu$  has been assumed constant, the coordinate system has been assumed fixed in time, and the vector identity that the curl of the gradient of a scalar vanishes has been invoked.

Since equation (1.8) is in direct notation, it must hold for any coordinate system. Hence, any identity proven in cartesian coordinates must hold for any curvilinear coordinate system. Cartesian coordinates will then be used to show that  $\text{curl}(\text{div}(\text{grad } \dot{\underline{x}})) = \text{div}(\text{grad } \underline{\omega})$ . In cartesian coordinates

$$\text{curl}(\text{div}(\text{grad } \dot{\underline{x}})) = - e_{\ell ik} \frac{\partial^3 x_i}{\partial x_k \partial x_j^2} e_{\ell}$$

which is equivalent to

$$\text{curl}(\text{div}(\text{grad } \dot{\underline{x}})) = - e_{\ell ik} \frac{\partial^3 x_i}{\partial x_k \partial x_j^2} e_{\ell} \quad (1.9)$$

where  $e_{\ell jk}$  is the permutation symbol, and  $e_{\ell}$  is the  $\ell^{\text{th}}$  unit vector. The summation convention for cartesian coordinates is that the occurrence of an index twice implies summation of that index. The right hand side of equation (1.9) can be rewritten as

$$\frac{\partial}{\partial x_j^2} \left( \frac{\partial \dot{x}_i}{\partial x^k} e_{kil} e_{\ell} \right)$$

which can be recognized to be equivalent to

$$\operatorname{div}(\operatorname{grad}(\operatorname{curl} \dot{\underline{x}})) = \operatorname{div}(\operatorname{grad} \underline{\omega}) .$$

Hence,

$$\operatorname{curl}(\operatorname{div}(\operatorname{grad} \dot{\underline{x}})) = \operatorname{div}(\operatorname{grad} \underline{\omega}) . \quad (1.10)$$

It then follows from equations (1.8) and (1.10) that

$$\frac{\partial \underline{\omega}}{\partial t} = \operatorname{curl}(\dot{\underline{x}} \times \underline{\omega}) + \nu \operatorname{div}(\operatorname{grad} \underline{\omega}) \quad (1.11)$$

where the first and second terms on the right hand side represent the convection and diffusion of vorticity, respectively. Equation (1.11) is known as the vorticity transport equation.

Tensor algebra and calculus can be used to express equation (1.11) in component form. The convection term can be written as

$$\operatorname{curl}(\dot{\underline{x}} \times \underline{\omega}) = \operatorname{curl}(e_{ijk} \dot{x}^i \omega^j g^k)$$

which can be expanded to give

$$\operatorname{curl}(\dot{\underline{x}} \times \underline{\omega}) = - e^{mk\ell} (e_{ijk} \dot{x}^i \omega^j)_{,\ell} g_m$$

where  $_{,\ell}$  denotes covariant differentiation, and the summation convention is that like indices appearing either above or below but not both above or below are summed, i.e.,

$$x_i^i = \sum_{i=1}^3 x_i^i,$$

but  $x_{ii}$  does not imply summation. Also,

$$e^{mk\ell} \equiv \frac{\epsilon^{mk\ell}}{\sqrt{g}}$$

and

$$e_{ijk} \equiv \sqrt{g} \epsilon_{ijk}.$$

Hence,

$$\text{curl}(\dot{x} \times \omega) = - \frac{\epsilon^{mk\ell}}{\sqrt{g}} (\sqrt{g} \epsilon_{ijk} \dot{x}^i \omega^j)_{,\ell} g_m. \quad (1.12)$$

Exterior algebra also provides that

$$\epsilon^{\ell mk} \epsilon_{ijk} = (3-2)! \begin{vmatrix} \delta_i^\ell & \delta_j^\ell \\ \delta_i^m & \delta_j^m \end{vmatrix} \quad (1.13)$$

where

$$\delta_i^j = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$$

and is called the Kronecker delta. Substitution of equation (1.13) into (1.12) then gives

$$\text{curl}(\dot{x} \times \omega) = \frac{\delta_i^m \delta_i^\ell}{\sqrt{g}} (\dot{x}^i \omega^j \sqrt{g})_{,\ell} g_m$$

$$- \frac{\delta_i^l \delta_j^m}{\sqrt{g}} (\dot{x}^i \omega^j \sqrt{g})_{,l} g_m$$

which can be rewritten as

$$\begin{aligned} \text{curl}(\dot{\underline{x}} \times \underline{\omega}) &= \frac{1}{\sqrt{g}} [(\sqrt{g} \dot{x}^i \omega^j)_{,j} g_i \\ &- (\sqrt{g} \dot{x}^j \omega^i)_{,j} g_i] . \end{aligned} \quad (1.14)$$

Equation (1.14) can be expanded and the continuity equation invoked to obtain a simpler form. However, the corresponding finite difference approximation of the resulting equation is not of conservative form. See Roache [23].

Equation (1.14) can be expanded using Christoffel symbols of the second kind to give

$$\begin{aligned} \text{curl}(\dot{\underline{x}} \times \underline{\omega}) &= \frac{1}{\sqrt{g}} \left[ \frac{\partial}{\partial x^j} (\dot{x}^i \omega^j \sqrt{g}) - \frac{\partial}{\partial x^j} (\dot{x}^j \omega^i \sqrt{g}) \right. \\ &\left. - \{ \begin{smallmatrix} i \\ \mu j \end{smallmatrix} \} \dot{x}^\mu \omega^j \sqrt{g} - \{ \begin{smallmatrix} i \\ j \mu \end{smallmatrix} \} \dot{x}^\mu \omega^j \sqrt{g} \right] g_i , \end{aligned}$$

and since the Christoffel symbols are symmetric, i.e.,

$$\{ \begin{smallmatrix} i \\ \mu j \end{smallmatrix} \} = \{ \begin{smallmatrix} i \\ j \mu \end{smallmatrix} \} ,$$

$$\text{curl}(\dot{\underline{x}} \times \underline{\omega}) = \frac{1}{\sqrt{g}} \left[ \frac{\partial}{\partial x^j} (\dot{x}^i \omega^j \sqrt{g}) - \frac{\partial}{\partial x^j} (\dot{x}^j \omega^i \sqrt{g}) \right] g_i . \quad (1.15)$$



Also, using tensor algebra and calculus, the diffusion term can be written as

$$v \operatorname{div}(\operatorname{grad} \omega) = v \omega_{\ell,ji} g^{ji} g^{\ell k} \underline{g}_k . \quad (1.16)$$

It follows from equations (1.11), (1.15), and (1.16) that the component form of the vorticity transport equation for any curvilinear coordinate system fixed in time can be written as

$$\begin{aligned} \frac{\partial}{\partial t} (\sqrt{g} \omega^k) &= \frac{\partial}{\partial x^j} (\dot{x}^k \omega^j \sqrt{g}) - \frac{\partial}{\partial x^j} (\dot{x}^j \omega^k \sqrt{g}) \\ &+ v \sqrt{g} \omega_{\ell,ji} g^{ji} g^{\ell k} \quad k=1,2,3 . \end{aligned} \quad (1.17)$$

Now, the continuity equation (1.12) of Chapter II, can be written in component form as (see Appendix B)

$$\frac{\partial}{\partial x^i} (\sqrt{g} \dot{x}^i) = 0 . \quad (1.18)$$

For rotationally symmetric flow,  $\frac{\partial}{\partial x^k} (\dot{x}^k) = 0$  (no sum) where  $x^k$  is the coordinate in the direction of rotation. It follows that, for rotationally symmetric flow, the continuity equation can be written as

$$\frac{\partial}{\partial x^j} (\sqrt{g} \dot{x}^j) + \dot{x}^k \frac{\partial \sqrt{g}}{\partial x^k} = 0 \quad (1.19)$$

where  $j \neq k$  and the second term is not summed on  $k$ . For

coordinate systems whose metric tensor is independent of the coordinate in the direction of rotation, i.e.,  $\frac{\partial \sqrt{g}}{\partial x^k} = 0$ , the continuity equation reduces to

$$\frac{\partial}{\partial x^j} (\sqrt{g} \dot{x}^j) = 0 \quad \text{for } j \neq k. \quad (1.20)$$

In which case a scalar can be defined, called the stream function  $\psi$ , such that equation (1.20) is automatically satisfied, i.e.,

$$\dot{x}^j = \frac{\epsilon^{ikj}}{\sqrt{g}} \frac{\partial \psi}{\partial x^i}$$

or

$$\dot{x}_j = \frac{\epsilon^{ikl}}{\sqrt{g}} \frac{\partial \psi}{\partial x^i} g_{jl}. \quad (1.21)$$

The combination of equations (1.2) and (1.21) gives the following relationship between the  $k^{\text{th}}$  component of vorticity and the stream function,

$$\zeta = -\epsilon^{kji} \epsilon^{mk\ell} \frac{\partial}{\partial x^i} \left( \frac{g_{j\ell}}{\sqrt{g}} \frac{\partial \psi}{\partial x^m} \right) \quad (1.22)$$

where

$$\zeta \equiv \sqrt{g} \omega^k. \quad (1.23)$$

The other two components of vorticity are given by equation (1.2) as

$$\sqrt{g} \omega^i = -\epsilon^{ikj} \frac{\partial \dot{x}_k}{\partial x_j} \quad (\text{no sum on } k) \quad (1.24)$$

The unknowns of the problem are the vorticities, velocities, and the stream function which number 7. Hence, for the problem to be determinant, there must be 7 independent equations. If the  $k^{\text{th}}$  component of equation (1.17) and equations (1.21) through (1.24) are chosen as governing equations, then there are 6 equations and 7 unknowns. One more equation must be derived. This remaining equation is provided by choosing one of the three Navier-Stokes equations. Writing the Navier-Stokes equation, equation (1.7), in component form gives

$$\begin{aligned} \frac{\partial \dot{x}_l}{\partial t} = \epsilon_{ijl} \dot{x}^i \omega^j \sqrt{g} - \frac{\partial}{\partial x^l} \left( \frac{p}{\rho} + \frac{v^2}{2} + \phi \right) \\ + \nu \dot{x}_{l,jk} g^{jk} \end{aligned} \quad (1.25)$$

Since the flow is independent of the  $k^{\text{th}}$  coordinate, the second term of (1.25) vanishes for  $l = k$ . Hence, choosing the  $k^{\text{th}}$  component of equation (1.25) gives the remaining required equation to make the system determinant. This equation is

$$\frac{\partial \dot{x}_k}{\partial t} = \epsilon_{ijk} \dot{x}^i \omega^j \sqrt{g} + \nu \dot{x}_{k,jl} g^{jl} \quad (1.26)$$

The  $k^{\text{th}}$  covariant component of the Navier-Stokes equation was chosen in obtaining equation (1.25) so that the additional equation and equation (1.24) would be consistent.

Previously, no mention of a particular coordinate system has been made. The only restrictions placed on the coordinate system is that it must be a curvilinear system fixed in time with its metric tensor independent of the  $k^{\text{th}}$  coordinate. However, the velocities are not consistent, i.e., not all are covariant or contravariant components. To obtain this consistency, one additional restriction must be imposed on the coordinate system--the coordinate system must be orthogonal. The following properties are characteristic of orthogonal curvilinear coordinate systems:

$$g_{ij} = 0 \quad i \neq j$$

therefore,

$$g_{ii} = \frac{1}{g^{ii}},$$

and

$$g = g_{11} g_{22} g_{33}.$$

In addition, physical components can be introduced. These are the components most widely used. The bases of the coordinate system are unit vectors; and; therefore, the components of a vector have the dimensions of the vector, i.e.,

velocity, length, etc. The relationship between physical components and the covariant and contravariant components of a vector  $V$  are (see Appendix B)

$$V_i = \sqrt{g_{ii}} V_{\langle i \rangle} \quad (\text{no sum}), \quad (1.27)$$

and

$$V^i = \frac{V_{\langle i \rangle}}{\sqrt{g_{ii}}} \quad (\text{no sum}). \quad (1.28)$$

where the brackets  $\langle \rangle$  denote physical components. Since  $\sqrt{g_{ii}}$  is a function of the coordinates, and the boundary conditions on velocity are usually expressed in physical components, it is desirable (to avoid singularities in boundary conditions at coordinate surfaces) to express the equations in terms of physical or covariant components. If the covariant component of the  $k^{\text{th}}$  velocity are chosen, then the system of equations can be written as

$$\frac{\partial \zeta}{\partial t} = \frac{\partial}{\partial x^j} \left( r \frac{\omega^j \sqrt{g}}{g_{kk}} \right) - \frac{\partial}{\partial x^j} \left( \frac{\dot{x}_{\langle j \rangle}}{\sqrt{g_{jj}}} \zeta \right) + \frac{v \sqrt{g}}{g_{jj} g_{kk}} \omega_{k,jj},$$

for  $j \neq k$ ,  $j \neq k$  (1.29)

$$\dot{x}_{\langle j \rangle} = \frac{\sqrt{g_{ij}}}{\sqrt{g}} \epsilon^{ijk} \frac{\partial \psi}{\partial x^i}, \quad (1.30)$$

$$\sqrt{g} \omega^j = -\epsilon^{jki} \frac{\partial \dot{x}_k}{\partial x^i} \quad (\text{no sum on } k), \quad (1.31)$$

$$\zeta = -(\epsilon^{kji})^2 \frac{\partial}{\partial x^i} \left( \frac{g_{jj}}{\sqrt{g}} \frac{\partial \psi}{\partial x^i} \right), \quad (1.32)$$

and

$$\frac{\partial \Gamma}{\partial t} = \epsilon_{ijk} \frac{\dot{x}_{<i>}}{\sqrt{g_{ii}}} \sqrt{g} \omega^j + \frac{v}{g_{jj}} \dot{x}_{k,jj} \quad (1.33)$$

where

$$\Gamma \equiv \sqrt{g_{kk}} \dot{x}_{<k>}. \quad (1.34)$$

An expression for  $\dot{x}_{k,jj}$  can be obtained from equation (B.14) of Appendix B by replacing  $k$  by  $j$ . The resulting equation is

$$\begin{aligned} \frac{\dot{x}_{k,jj}}{g_{jj}} &= \frac{1}{g_{jj}} \frac{\partial^2 \dot{x}_k}{\partial x^j \partial x^j} - \frac{\partial}{\partial x^j} \left\{ \begin{matrix} l \\ kj \end{matrix} \right\} \frac{\dot{x}_l}{g_{jj}} - \frac{2}{g_{jj}} \left\{ \begin{matrix} l \\ kj \end{matrix} \right\} \frac{\partial \dot{x}_l}{\partial x^j} \\ &+ \left\{ \begin{matrix} l \\ kj \end{matrix} \right\} \left\{ \begin{matrix} i \\ lj \end{matrix} \right\} \frac{\dot{x}_i}{g_{jj}} - \left\{ \begin{matrix} l \\ jj \end{matrix} \right\} \frac{\partial \dot{x}_k}{\partial x^l} \frac{1}{g_{jj}} \\ &+ \left\{ \begin{matrix} l \\ jj \end{matrix} \right\} \left\{ \begin{matrix} i \\ kl \end{matrix} \right\} \frac{\dot{x}_i}{g_{jj}}. \end{aligned} \quad (1.35)$$

An expanded expression for the last term of equation (1.24) in terms of  $\zeta$  can be obtained by rewriting equation (1.23) as  $\zeta = \sqrt{g} \omega_k / g_{kk}$ . This equation can then be solved for  $\omega_k$  to give  $\omega_k = g_{kk} \zeta / \sqrt{g}$ . It then follows that  $\omega_{k,jj} = (g_{kk} \zeta) / (\sqrt{g})_{,jj}$ , and  $\omega_{k,jj} / g_{jj} = 1/g_{jj} (g_{kk} \zeta / \sqrt{g})_{,jj}$ , which can be expanded by replacing  $\dot{x}_k$ ,  $\dot{x}_l$ , and  $\dot{x}_i$  in equation (1.35)

by  $g_{kk}\zeta/\sqrt{g}$ ,  $g_{ll}\zeta/\sqrt{g}$ , and  $g_{ii}\zeta/\sqrt{g}$ , respectively. The expanded equation can then be multiplied by  $v\sqrt{g}/g_{kk}$  to obtain

$$\begin{aligned} \frac{v\sqrt{g}}{g_{kk}} \frac{\omega_{k,jj}}{g_{jj}} &= v \left[ \frac{\sqrt{g}}{g_{jj}g_{kk}} \frac{\partial^2}{\partial x^j \partial x^j} \left( \frac{g_{kk}}{\sqrt{g}} \zeta \right) \right. \\ &\quad - \frac{\partial}{\partial x^j} \left\{ \frac{l}{kj} \right\} \frac{\zeta}{g_{jj}} - \frac{2\sqrt{g}}{g_{jj}g_{kk}} \left\{ \frac{l}{kj} \right\} \frac{\partial}{\partial x^j} \left( \frac{g_{kk}}{\sqrt{g}} \zeta \right) \\ &\quad + \left\{ \frac{l}{kj} \right\} \left\{ \frac{i}{lj} \right\} \frac{\zeta}{g_{jj}} - \left\{ \frac{l}{jj} \right\} \frac{\partial}{\partial x^l} \left( \frac{g_{kk}\zeta}{\sqrt{g}} \right) \frac{\sqrt{g}}{g_{kk}g_{jj}} \\ &\quad \left. + \left\{ \frac{l}{jj} \right\} \left\{ \frac{i}{kl} \right\} \frac{\zeta}{g_{jj}} \right]. \end{aligned} \quad (1.37)$$

The only restriction placed on equations (1.29) through (1.37) is that the coordinate system be an orthogonal curvilinear system, fixed in time with its metric tensor independent of the  $k^{\text{th}}$  coordinate; the fluid be an incompressible, constant viscosity, Newtonian fluid; the flow be rotationally symmetric; and the body forces be conservative.

The coordinate system natural to the cone-and-plate viscometer is a spherical coordinate system in which  $k = 3$ ,  $X^1 = r$ ,  $X^2 = \theta$ ,  $X^3 = \phi$ ,  $u = \dot{x}_{\langle 1 \rangle}$ ,  $v = \dot{x}_{\langle 2 \rangle}$ , and  $w = \dot{x}_{\langle 3 \rangle}$ . The non-zero Christoffel symbols of the second kind and the metric tensor are given in Appendix B. It then follows that the governing equations for a spherical coordinate system can be written as

$$\begin{aligned} \frac{\partial \zeta}{\partial t} = & \frac{\partial(\Gamma\chi)}{\partial r} - \frac{\partial(\Gamma\eta)}{\partial \beta} - \frac{\partial(u\zeta)}{\partial r} + \frac{1}{r} \frac{\partial(v\zeta)}{\partial \beta} \\ & + \frac{1}{\text{Re}} \left[ \frac{\partial^2 \zeta}{\partial r^2} + \frac{1}{r^2} \frac{\partial}{\partial \beta} \left( \frac{1}{\cos \beta} \frac{\partial}{\partial \beta} (\zeta \cos \beta) \right) \right], \end{aligned} \quad (1.38)$$

$$\chi = \frac{\omega \sqrt{g}}{g_{33}} = - \frac{1}{r^2 \cos^2 \beta} \frac{\partial \Gamma}{\partial \beta}, \quad (1.39)$$

$$\eta = \frac{\omega^2 \sqrt{g}}{g_{33}} = - \frac{1}{r^2 \cos^2 \beta} \frac{\partial \Gamma}{\partial r}, \quad (1.40)$$

$$-\zeta \cos \beta = \frac{\partial^2 \psi}{\partial r^2} + \frac{\cos \beta}{r^2} \frac{\partial}{\partial \beta} \left( \frac{1}{\cos \beta} \frac{\partial \psi}{\partial \beta} \right), \quad (1.41)$$

$$u = - \frac{1}{r^2 \cos \beta} \frac{\partial \psi}{\partial \beta}, \quad (1.42)$$

$$v = - \frac{1}{r \cos \beta} \frac{\partial \psi}{\partial r}, \quad (1.43)$$

and

$$\begin{aligned} \frac{\partial \Gamma}{\partial t} = & -u \frac{\partial \Gamma}{\partial r} + \frac{v}{r} \frac{\partial \Gamma}{\partial \beta} \\ & + \frac{1}{\text{Re}} \left[ \frac{\partial^2 \Gamma}{\partial r^2} + \frac{\cos \beta}{r^2} \frac{\partial}{\partial \beta} \left( \frac{1}{\cos \beta} \frac{\partial \Gamma}{\partial \beta} \right) \right] \end{aligned} \quad (1.44)$$

where the variables have been non-dimensionalized consistent with Chapter II, and the transformation  $\beta = \frac{\pi}{2} - \theta$  was made.



#### IV.1.b Boundary conditions

The physical boundary conditions are the same as those discussed in Section II.2. However, these conditions must be transformed into corresponding boundary conditions for the new dependent variables  $\Gamma$ ,  $\eta$ ,  $\chi$ , and  $\zeta$  as well as the stream function  $\psi$ . In addition, a numerical scheme requires boundary conditions at the origin, i.e., at  $r=0=0$ .

Since  $u$  and  $v$  are the same variables as  $v_r$  and  $v_\theta$ , respectively, in Chapter II, the boundary conditions on  $u$  and  $v$  are those for  $v_r$  and  $v_\theta$  presented in Chapter II. Also, since  $\Gamma$  is the covariant component of the velocity vector in the  $\phi$  direction,  $\Gamma = \sqrt{g_{33}} \dot{x}_{\langle 3 \rangle}$ , which is equivalent to

$$\Gamma = r \cos \beta v_\phi . \quad (1.45)$$

Therefore, the combination of the boundary conditions on  $v_\phi$  from Section II.2 and equation (1.45) gives the following boundary conditions on  $\Gamma$ :

$$\Gamma(r,0) = 0 , \quad (1.46)$$

$$\Gamma(r,\epsilon) = r^2 \cos^2 \epsilon , \quad (1.47)$$

and

$$\left. \frac{\partial}{\partial r} \left( \frac{\Gamma}{r^2} \right) \right|_{r=1} = 0 . \quad (1.48)$$

Equations (1.46) and (1.47) also imply that

$$\Gamma(0, \beta) = 0 . \quad (1.49)$$

From the boundary conditions discussed in Section II.2, the velocity component normal to each boundary is zero, i.e.,  $u = v_r = 0$  at  $r = 1$  and  $v = v_\theta = 0$  at  $\beta = 0, \epsilon$ . It follows from the definition of the stream function, equations (1.42) and (1.43), that the derivative of the stream function with respect to the coordinate along the boundary vanishes. Hence, the stream function can be determined to be a constant on the boundaries by integrating this derivative of the stream function along the boundaries; this constant can be taken to be zero without any loss of generality. Therefore,

$$\psi(0, \beta) = \psi(r, \epsilon) = \psi(r, 0) = \psi(1, \beta) = 0. \quad (1.50)$$

The boundary conditions on  $\zeta$  can be derived by using equations (1.41) through (1.43) and the boundary conditions on  $u$  and  $v$ , to obtain

$$\zeta(0, \beta) = - \frac{1}{\cos \beta} \frac{\partial^2 \psi(0, \beta)}{\partial r^2}, \quad (1.51)$$

and since, as discussed in Section III.2, the outer perturbation solution is valid for all finite Reynolds numbers in some neighborhood of the origin, equation (2.17) of Section III.2. can be used to determine the behavior of  $\zeta$  as  $r \rightarrow 0$ . From equation (2.17) of Section III.2,

$$\psi \sim r^5 \quad \text{as} \quad r \rightarrow 0$$

which means

$$\frac{\partial^2 \psi}{\partial r^2} \sim r^3 \quad \text{as } r \rightarrow 0$$

from which it follows that

$$\zeta(0, \beta) = 0. \quad (1.52)$$

From equations (1.41) and (1.42) and the boundary conditions that  $u = v = 0$  on the plate,

$$\frac{\partial \psi(r, 0)}{\partial r} = \frac{\partial \psi(r, 0)}{\partial \beta} = 0,$$

which, when combined with equation (1.43), gives

$$\zeta(r, 0) = -\frac{1}{r^2} \frac{\partial^2 \psi(r, 0)}{\partial \beta^2} \quad \text{for } r > 0. \quad (1.53)$$

The boundary conditions on  $\zeta$  at the cone and at the free surface can be derived in a similar manner to give

$$\zeta(r, \epsilon) = -\frac{1}{r^2 \cos \epsilon} \frac{\partial^2 \psi(r, \epsilon)}{\partial \beta^2} \quad \text{for } r > 0, \quad (1.54)$$

and

$$\zeta(1, \beta) = -\frac{1}{\cos \beta} \frac{\partial^2 \psi(1, \beta)}{\partial \beta^2} \quad \text{for } 0 < \beta < \epsilon, \quad (1.55)$$

respectively.

Using the equation defining  $\chi(r, \beta)$ , equation (1.39),

and the boundary conditions on  $r$ , equations (1.46) through (1.49), the following boundary conditions on  $\chi(r, \beta)$  can be derived:

$$\chi(r, 0) = -\frac{1}{r^2} \frac{\partial \Gamma(r, 0)}{\partial \beta} \text{ for } r > 0, \quad (1.56)$$

$$\chi(r, \epsilon) = -\frac{1}{r^2 \cos^2 \epsilon} \frac{\partial \Gamma(r, \epsilon)}{\partial \beta} \text{ for } r > 0, \quad (1.57)$$

$$\chi(0, \beta) = -\frac{1}{\cos^2 \beta} \left( \frac{1}{r^2} \frac{\partial \Gamma(r, \beta)}{\partial \beta} \right)_{r=0}, \quad (1.58)$$

and

$$\chi(1, \beta) = -\frac{1}{\cos^2 \beta} \frac{\partial \Gamma(1, \beta)}{\partial \beta}. \quad (1.59)$$

Equation (1.58) suggests that either a singularity exists in  $\chi(r, \beta)$  as  $r$  approaches zero, or that  $\chi(r, \beta)$  becomes intermediate as  $r$  approaches zero. The outer perturbation solution can be used to determine the actual behavior as  $r$  approaches zero. From equation (1.45) of this section and equation (1.25) of Section III.1,

$$r - r^2 \text{ as } r \rightarrow 0 \quad (1.60)$$

which implies that

$$\chi(r, \beta) \rightarrow A \text{ as } r \rightarrow 0 \quad (1.61)$$

where  $A$  is an undetermined function of  $\beta$ .

Utilizing the equation defining  $\eta(r, \beta)$ , equation (1.40), and the boundary conditions on  $\Gamma$ , equations (1.46) through (1.49), the following boundary conditions on  $\eta(r, \beta)$  can be derived:

$$\eta(r, 0) = 0, \quad (1.62)$$

$$\eta(r, \epsilon) = -\frac{2}{r}, \quad (1.63)$$

and

$$\eta(1, \beta) = -\frac{1}{\cos^2 \beta} \frac{\partial \Gamma(1, \beta)}{\partial r} = -\frac{2}{\cos^2 \beta} \Gamma(1, \beta) \quad (1.64)$$

where equation (1.48) was used to obtain equation (1.64). It is easy to see from equations (1.62) and (1.63) that at  $r = 0$ ,  $\eta(r, \beta)$  is zero at the plate and is unbounded on the cone, and; therefore, is multivalued at the origin. Fortunately, the computation of  $\eta$  and  $\chi$  at the origin is not necessary in obtaining a solution, but instead  $r_\eta$  and  $r_\chi$  are required. This can be readily seen from equation (1.38). These required boundary conditions on  $r_\chi$  and  $r_\eta$  at the origin can be derived by combining equations (1.39), (1.40), and (1.60) to obtain

$$r_\chi|_{r=0} = 0, \quad (1.65)$$

and

$$r_\eta|_{r=0} = 0. \quad (1.66)$$

## IV.2 Numerical Approximation

The continuum governing equations and boundary conditions can be approximated by finite differences. This is achieved by approximating derivatives by truncated Taylor's series. For instance, if  $f$  is a function of  $x$  and  $y$ , then by Taylor's series

$$f(x+\Delta x, y) = f(x, y) + \frac{\partial f(x, y)}{\partial x} \Delta x + \frac{\partial^2 f(x, y)}{\partial x^2} \frac{\Delta x^2}{2} + O(\Delta x^3) . \quad (2.1)$$

Equation (2.1) can be solved for  $\frac{\partial f(x, y)}{\partial x}$  to give

$$\frac{\partial f(x, y)}{\partial x} = \frac{f(x+\Delta x, y) - f(x, y)}{\Delta x} + O(\Delta x) . \quad (2.2)$$

Equation (2.2) is then said to approximate  $\frac{\partial f(x, y)}{\partial x}$  to first order and is known as the first order forward difference approximation of  $\frac{\partial f(x, y)}{\partial x}$ . Similarly,

$$f(x-\Delta x, y) = f(x) - \frac{\partial f(x, y)}{\partial x} \Delta x + \frac{\partial^2 f(x, y)}{\partial x^2} \frac{\Delta x^2}{2} + O(\Delta x^3) , \quad (2.3)$$

and

$$\frac{\partial f(x,y)}{\partial x} = \frac{f(x,y) - f(x-\Delta x, y)}{\Delta x} + O(\Delta x) . \quad (2.4)$$

Equation (2.4) is also said to approximate  $\frac{\partial f(x,y)}{\partial x}$  to first order and is known as the first order backward difference approximation of  $\frac{\partial f(x,y)}{\partial x}$ . Still another approximation of  $\frac{\partial f(x,y)}{\partial x}$  can be developed by subtracting equation (2.3) from equation (2.2) and then solving the resulting equation for  $\frac{\partial f(x,y)}{\partial x}$  to give

$$\frac{\partial f(x,y)}{\partial x} = \frac{f(x+\Delta x, y) - f(x-\Delta x, y)}{2\Delta x} + O(\Delta x^2) \quad (2.5)$$

which is a second order approximation of  $\frac{\partial f(x,y)}{\partial x}$  and is known as the centered difference approximation of  $\frac{\partial f(x,y)}{\partial x}$ .

The centered finite difference approximation of  $\frac{\partial^2 f(x,y)}{\partial x^2}$  can be derived by adding equations (2.1) and (2.3) and solving for  $\frac{\partial^2 f(x,y)}{\partial x^2}$  to obtain

$$\frac{\partial^2 f(x,y)}{\partial x^2} = \frac{f(x+\Delta x, y) - 2f(x, y) + f(x-\Delta x, y)}{(\Delta x)^2} + O(\Delta x^2) . \quad (2.6)$$

The centered difference operators are defined as

$$\frac{\delta f(x,y)}{\delta x} = \frac{f(x+\Delta x, y) - f(x-\Delta x, y)}{2\Delta x} , \quad (2.7)$$

and

$$\frac{\delta^2 f(x,y)}{\delta x^2} = \frac{f(x+\Delta x,y) - 2f(x,y) + f(x-\Delta x,y)}{(\Delta x)^2} \quad (2.8)$$

where the error incurred, called the local truncation error, in approximating  $\frac{\partial f}{\partial x}$  and  $\frac{\partial^2 f}{\partial x^2}$  with  $\frac{\delta f}{\delta x}$  and  $\frac{\delta^2 f}{\delta x^2}$ , respectively, is  $O(\Delta x^2)$ . The centered finite difference approximations approach the continuum equations in the limit as  $(\Delta x)^2$  vanishes. Expressions for  $\frac{\delta f}{\delta y}$  and  $\frac{\delta^2 f}{\delta y^2}$  can also be derived in a similar manner.

The finite difference approach to solving partial differential equations is to divide the domain into a mesh of nodes at which all derivatives are approximated by finite differences (referred to as discretizing the equations). The resulting system of algebraic equations are then solved for the values of the dependent variables at each node. As the mesh is refined the solution should approach the continuum solution in the limit.

Since the discretized non-linear partial differential equations yield systems of non-linear algebraic equations, direct methods for inverting these algebraic equations do not exist. Instead these non-linear equations must be inverted, iteratively. It can be shown [23] that these iterative techniques for steady state solutions are identical to approximating  $\frac{\partial}{\partial t}$  by finite differences in  $t$  and obtaining transient solutions at successive time steps until  $\frac{\partial}{\partial t}$  vanishes, i.e., a steady state is reached.



A most important consideration in the selection of a method to solve the discretized equations is the numerical stability of the method. Explicit methods [23] are inherently unstable at large Reynolds numbers unless the convection terms are approximated by first order forward or backward differences. Thus, the accuracy of explicit methods at high Reynolds numbers degenerates to first order. Several implicit methods [23] have been analyzed by the von Neumann method and have been found to be inherently stable with centered difference approximation of the convection terms, provided the time step is less than some critical value. However, implicit methods require the inversion of pentadiagonal matrices, which are difficult to solve. Alternating direction implicit or ADI methods make use of fractional time steps and only require the inversion of tridiagonal matrices which can be solved directly. Since ADI methods also tend to be more stable than explicit methods, and the resulting matrices are easily inverted (see Appendix D), they have become the most widely used methods in obtaining numerical solutions to viscous fluid mechanics problems [23]. For these same reasons, ADI methods are used in this work.

#### IV.2.a Governing equations

To obtain the resolution necessary to determine the flow inside the thin boundary layer at the free surface, which

was predicted in Section III.1 to be of the order of  $\epsilon$  thick, the grid in the  $r$  direction would have to be extremely fine. However, the required resolution can be obtained with a relatively coarse grid in  $z$  by making the following transformation:

$$1 - r = b(e^{az} - 1) \quad (2.9)$$

where the constants  $a$  and  $b$  are determined by requiring that at

$$r = 1, \quad z = 0, \quad (2.10)$$

$$r = 0, \quad z = 1, \quad (2.11)$$

and at

$$r = 1 - \kappa\epsilon, \quad z = z_e \quad (2.12)$$

where  $\kappa$  is a constant which is determined experimentally. Transformations of the type described in equation (2.9) are known as exponential stretches and were first used by Jensen [23] for flow about a sphere.

From equation (2.9) and the chain rule of differential calculus

$$\frac{\partial}{\partial r} = - \frac{e^{-az}}{ab} \frac{\partial}{\partial z}, \quad (2.13)$$

and

$$\frac{\partial^2}{\partial r^2} = \frac{e^{-2az}}{ab^2} \left( \frac{1}{a} \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial z} \right). \quad (2.14)$$

The governing equations, equations (1.38) and (1.44) of Section IV.1.a, can then be transformed by substituting into them equations (2.13) and (2.14). The transformed governing equations are:

$$\begin{aligned} \frac{\partial \zeta}{\partial t} = & - \frac{e^{-az}}{ab} \frac{\partial (r\chi)}{\partial z} - \frac{\partial (r\eta)}{\partial \beta} + \frac{e^{-az}}{ab} \frac{\partial (u\zeta)}{\partial z} \\ & + \frac{1}{r(z)} \frac{\partial (v\zeta)}{\partial \beta} + \frac{e^{-2az}}{\operatorname{Re} a^2 b^2} \frac{\partial^2 \zeta}{\partial z^2} - \frac{e^{-2az}}{\operatorname{Re} ab^2} \frac{\partial \zeta}{\partial z} \\ & + \frac{1}{\operatorname{Re} r(z) \cos \beta} \frac{\partial^2 (\zeta \cos \beta)}{\partial \beta^2} \\ & + \frac{\tan \beta}{\operatorname{Re} r^2(z) \cos \beta} \frac{\partial (\zeta \cos \beta)}{\partial \beta}, \end{aligned} \quad (2.15)$$

$$\begin{aligned} - \zeta \cos \beta = & \frac{e^{-2az}}{a^2 b^2} \frac{\partial^2 \psi}{\partial z^2} - \frac{e^{-2az}}{ab^2} \frac{\partial \psi}{\partial z} \\ & + \frac{1}{r^2(z)} \frac{\partial^2 \psi}{\partial \beta^2} + \frac{\tan \beta}{r^2(z)} \frac{\partial \psi}{\partial \beta}, \end{aligned} \quad (2.16)$$

$$\begin{aligned} \frac{\partial \Gamma}{\partial t} = & \frac{ue^{-az}}{ab} \frac{\partial \Gamma}{\partial z} + \frac{v}{r(z)} \frac{\partial \Gamma}{\partial \beta} + \frac{e^{-2az}}{a^2 b^2 \operatorname{Re}} \frac{\partial^2 \Gamma}{\partial z^2} \\ & - \frac{e^{-2az}}{ab^2 \operatorname{Re}} \frac{\partial \Gamma}{\partial z} + \frac{1}{\operatorname{Re} r^2(z)} \frac{\partial^2 \Gamma}{\partial \beta^2} + \frac{\tan \beta}{\operatorname{Re} r^2(z)} \frac{\partial \Gamma}{\partial \beta}, \end{aligned} \quad (2.17)$$

$$u = - \frac{1}{r^2(z) \cos \beta} \frac{\partial \psi}{\partial \beta}, \quad (2.18)$$

$$v = \frac{e^{-az}}{abr(z) \cos \beta} \frac{\partial \psi}{\partial z}, \quad (2.19)$$

$$\eta = \frac{e^{-az}}{abr^2(z) \cos^2 \beta} \frac{\partial \Gamma}{\partial z}, \quad (2.20)$$

and

$$x = - \frac{1}{r^2(z) \cos^2 \beta} \frac{\partial \Gamma}{\partial \beta} \quad (2.21)$$

where, from equation (2.9),

$$r(z) = 1 - b(e^{az} - 1). \quad (2.22)$$

Equations (2.17) through (2.21) can be discretized by dividing the flow domain into an exponentially stretched spherical grid (see Figure 3) by letting

$$z_i = (i-1)\Delta z \quad i=1,2,\dots,M \quad (2.23)$$

and

$$\beta_j = (j-1)\Delta \beta \quad j=1,2,\dots,N \quad (2.24)$$

Equations (2.9) through (2.12) can then be combined to provide equations for  $a$  and  $b$ . These equations are

$$a = \ln \left[ \frac{1}{\kappa \epsilon} (\exp(a(M\epsilon - 1) \Delta z) - 1) + 1 \right] \quad (2.25)$$

and

$$b = (e^a - 1)^{-1} \quad (2.26)$$

where  $M_e$  is the  $z$  node which corresponds to  $z = z_e$ , and the constants  $M_e$  and  $\kappa$  are determined experimentally. Equations (2.25) must be solved iteratively to obtain  $a$ .

If  $t = n\Delta t$  and  $t_1 = 2\Delta t_1$ , where  $t_1$  is a "time" like variable used in the iterative solution of equation (2.16) for  $\psi$ , then the ADI solution of the governing equations consists of advancing the transient solution in two steps, such that

$$\begin{aligned} \frac{\zeta_{i,j}^{n+1/2} - \zeta_{i,j}^n}{\frac{\Delta t}{2}} = & - \frac{2\phi_{i,j}}{\Delta t} + \frac{e^{-az_i}}{ab} \frac{\delta(u\zeta^{n+1/2})}{\delta z} \\ & + \frac{1}{r(z_i)} \frac{\delta(v\zeta^n)}{\delta \beta} + \frac{e^{-2az_i}}{\text{Re } a^2 b^2} \frac{\delta^2 \zeta^{n+1/2}}{\delta z^2} \\ & - \frac{e^{-2az_i}}{\text{Re } ab^2} \frac{\delta \zeta^{n+1/2}}{\delta z} + \frac{1}{\text{Re } r^2(z_i) \cos \beta_j} \frac{\delta^2 (\zeta^n \cos \beta)}{\delta \beta^2} \\ & + \frac{\tan \beta_j}{\text{Re } r^2(z_i) \cos \beta_j} \frac{\delta (\zeta^n \cos \beta)}{\delta \beta}, \end{aligned} \quad (2.27)$$

$$\frac{\zeta_{i,j}^{n+1} - \zeta_{i,j}^{n+1/2}}{\frac{\Delta t}{2}} = - \frac{2\phi_{i,j}}{\Delta t} + \frac{e^{-az_i}}{ab} \frac{\delta(u\zeta^{n+1/2})}{\delta z}$$

$$\begin{aligned}
& + \frac{1}{r(z_i)} \frac{\delta(v\zeta^{n+1})}{\delta\beta} + \frac{e^{-2az_i}}{\operatorname{Re} a^2 b^2} \frac{\delta^2 \zeta^{m+1/2}}{\delta z^2} \\
& - \frac{e^{-2az_i}}{\operatorname{Re} ab^2} \frac{\delta \zeta^{n+1/2}}{\delta z} + \frac{1}{\operatorname{Re} r^2(z_i) \cos \beta_j} \frac{\delta^2 (\zeta^{n+1} \cos \beta)}{\delta \beta^2} \\
& + \frac{\tan \beta_j}{\operatorname{Re} r^2(z_i) \cos \beta_j} \frac{\delta (\zeta^{n+1} \cos \beta)}{\delta \beta}, \quad (2.28)
\end{aligned}$$

$$\begin{aligned}
\frac{\psi_{i,j}^{\ell+1/2} - \psi_{i,j}^{\ell}}{\frac{\Delta t_1}{2}} & = \frac{e^{-2az_i}}{a^2 b^2} \frac{\delta^2 \psi^{\ell+1/2}}{\delta z^2} - \frac{e^{-2az_i}}{ab^2} \frac{\delta \psi^{\ell+1/2}}{\delta z} \\
& + \frac{1}{r^2(z_i)} \frac{\delta^2 \psi^{\ell}}{\delta \beta^2} + \frac{\tan \beta_j}{r^2(z_i)} \frac{\delta \psi^{\ell}}{\delta \beta} \\
& + \zeta_{i,j}^{n+1} \cos \beta_j, \quad (2.29)
\end{aligned}$$

$$\begin{aligned}
\frac{\psi_{i,j}^{\ell} - \psi_{i,j}^{\ell+1/2}}{\frac{\Delta t_1}{2}} & = \frac{e^{-2az_i}}{a^2 b^2} \frac{\delta^2 \psi^{\ell+1/2}}{\delta z^2} - \frac{e^{-2az_i}}{ab^2} \frac{\delta \psi^{\ell+1/2}}{\delta z} \\
& + \frac{1}{r^2(z_i)} \frac{\delta^2 \psi^{\ell+1}}{\delta \beta^2} + \frac{\tan \beta_j}{r^2(z_i)} \frac{\delta \psi^{\ell+1}}{\delta \beta} \\
& + \zeta_{i,j}^{n+1} \cos \beta_j, \quad (2.30)
\end{aligned}$$

$$\begin{aligned}
\frac{\Gamma_{i,j}^{n+1/2} - \Gamma_{i,j}^n}{\frac{\Delta t}{2}} &= \frac{u_{i,j} e^{-az_i}}{ab} \frac{\delta \Gamma^{n+1/2}}{\delta z} + \frac{v_{i,j}}{r(z_i)} \frac{\delta \Gamma^n}{\delta \beta} \\
&+ \frac{e^{-2az_i}}{a^2 b^2 \operatorname{Re}} \frac{\delta^2 \Gamma^{n+1/2}}{\delta z^2} - \frac{e^{-2az_i}}{ab^2 \operatorname{Re}} \frac{\delta \Gamma^{n+1/2}}{\delta z} \\
&+ \frac{1}{\operatorname{Re} r^2(z_i)} \frac{\delta^2 \Gamma^n}{\delta \beta^2} + \frac{\tan \beta_j}{\operatorname{Re} r^2(z_i)} \frac{\delta \Gamma^n}{\delta \beta}, \quad (2.31)
\end{aligned}$$

$$\begin{aligned}
\frac{\Gamma_{i,j}^n - \Gamma_{i,j}^{n+1/2}}{\frac{\Delta t}{2}} &= \frac{u_{i,j} e^{-az_i}}{ab} \frac{\delta \Gamma^{n+1/2}}{\delta z} + \frac{v_{i,j}}{r(z_i)} \frac{\delta \Gamma^{n+1}}{\delta \beta} \\
&+ \frac{e^{-2az_i}}{a^2 b^2 \operatorname{Re}} \frac{\delta^2 \Gamma^{n+1/2}}{\delta z^2} - \frac{e^{-2az_i}}{ab^2 \operatorname{Re}} \frac{\delta \Gamma^{n+1/2}}{\delta z} \\
&+ \frac{1}{\operatorname{Re} r^2(z_i)} \frac{\delta^2 \Gamma^{n+1}}{\delta \beta^2} + \frac{\tan \beta_j}{\operatorname{Re} r^2(z_i)} \frac{\delta \Gamma^{n+1}}{\delta \beta}, \quad (2.32)
\end{aligned}$$

$$u_{i,j}^{n+1} = - \frac{k}{r^2(z_i) \cos \beta_j} \frac{\delta \psi^{n+1}}{\delta \beta}, \quad (2.33)$$

$$v_{i,j}^{n+1} = \frac{e^{-az_i}}{abr(z_i) \cos \beta_j} \frac{\delta \psi^{n+1}}{\delta z}, \quad (2.34)$$

$$\eta_{i,j}^{n+1} = \frac{e^{-az_i}}{abr^2(z_i) \cos^2 \beta_j} \frac{\delta \Gamma}{\delta z}, \quad (2.35)$$

and

$$x_{i,j}^{n+1} = - \frac{1}{r^2(z_i) \cos^2 \beta_j} \frac{\delta \Gamma}{\delta \beta} \quad (2.36)$$

where

$$\phi_{i,j}^{n+1} = \frac{\Delta t}{2ab} e^{-az_i} \frac{\delta(\Gamma^{n+1} \chi^{n+1})}{\delta z} + \frac{\Delta t}{2} \frac{\delta(\Gamma^{n+1} \eta^{n+1})}{\delta \beta} \quad (2.37)$$

and  $\frac{\delta}{\delta z}$ ,  $\frac{\delta^2}{\delta z^2}$ , etc., indicate centered space differences consistent with equations (2.7) and (2.8). The superscripts  $n$ ,  $n+1/2$ , and  $n+1$  indicate values of the variables at the beginning, middle, and end of the  $n^{\text{th}}$  time step, respectively, and the subscripts,  $i, j$  denote the value of a variable at  $z = z_i$ ,  $\beta = \beta_j$ .

The non-linear terms in equations (2.27) through (2.32) have been written without superscripts indicating that they are not evaluated at either the beginning or at the end of the time step. It would be impossible to evaluate these variables at the end of the time step as their values at the end of the time step depend upon the solution of equations (2.27) through (2.32). Several schemes have been used in linearizing non-linear equations [23]. However, the method used here will be that published by Aziz and Hellums [1]. This method consists of solving equations (2.27) through (2.37) using



$$u_{i,j} = u_{i,j}^n, v_{i,j} = v_{i,j}^n, \text{ and } \phi_{i,j} = \phi_{i,j}^n$$

to obtain

$$(u_{i,j}^{n+1})^{k=1}, (v_{i,j}^{n+1})^{k=1}, \text{ and } (\phi_{i,j}^{n+1})^{k=1}.$$

The equations are then again solved using

$$u_{i,j} = \frac{1}{2}(u_{i,j}^n + (u_{i,j}^{n+1})^{k=1}), v_{i,j} = \frac{1}{2}(v_{i,j}^n + (v_{i,j}^{n+1})^{k=1}),$$

$$\text{and } \phi_{i,j} = \frac{1}{2}(\phi_{i,j}^n + (\phi_{i,j}^{n+1})^{k=1})$$

to obtain

$$(u_{i,j}^{n+1})^{k=2}, (v_{i,j}^{n+1})^{k=2}, \text{ and } (\phi_{i,j}^{n+1})^{k=2}.$$

The iteration is continued until  $(u_{i,j}^{n+1})^{k+1} = (u_{i,j}^{n+1})^k$  or until  $k = k_{\max}$ , where  $k_{\max}$  is the maximum number of non-linear iterations allowed.

If the definitions of the space centered difference operators, equations (2.7) and (2.8), are substituted into equations (2.27) through (2.37), then

$$\begin{aligned} b_2 \zeta_{2,j}^{n+1/2} + c_{2,j} \zeta_{3,j}^{n+1/2} &= d_{2,j} \\ a_{i,j} \zeta_{i-1,j}^{n+1/2} + b_i \zeta_{i,j}^{n+1/2} + c_{i,j} \zeta_{i+1,j}^{n+1/2} &= d_{i,j} \quad 2 < i < M-1 \\ a_{M-1,j} \zeta_{M-2,j}^{n+1/2} + b_{M-1} \zeta_{M-1,j}^{n+1/2} &= d_{M-1,j} \end{aligned}$$

(2.38)

$$\begin{aligned}
 & f_i \zeta_{i,2}^{n+1} + g_{i,2} \zeta_{i,3}^{n+1} = h_{i,2} \\
 e_{i,j} \zeta_{i,j-1}^{n+1} + f_i \zeta_{i,j}^{n+1} + g_{i,j} \zeta_{i,j+1}^{n+1} &= h_{i,j} \quad 2 < j < N-1 \\
 e_{i,N-1} \zeta_{i,N-2}^{n+1} + f_i \zeta_{i,N-1}^{n+1} &= h_{i,N-1} \\
 & (2.39)
 \end{aligned}$$

$$\begin{aligned}
 & b'_2 \psi_{2,j}^{\ell+1/2} + c'_2 \psi_{3,j}^{\ell+1/2} = d'_{2,j} \\
 a'_i \psi_{i-1,j}^{\ell+1/2} + b'_i \psi_{i,j}^{\ell+1/2} + c'_i \psi_{i+1,j}^{\ell+1/2} &= d'_{i,j} \quad 2 < i < M-1 \\
 a'_{M-1} \psi_{M-2,j}^{\ell+1/2} + b'_{M-1} \psi_{M-1,j}^{\ell+1/2} &= d'_{M-1,j} \\
 & (2.40)
 \end{aligned}$$

$$\begin{aligned}
 & f'_i \psi_{i,2}^{\ell+1} + g'_{i,2} \psi_{i,3}^{\ell+1} = h'_{i,2} \\
 e'_{i,j} \psi_{i,j-1}^{\ell+1} + f'_i \psi_{i,j}^{\ell+1} + g'_{i,j} \psi_{i,j+1}^{\ell+1} &= h'_{i,j} \quad 2 < j < N-1 \\
 e'_{i,N-1} \psi_{i,N-2}^{\ell+1} + f'_i \psi_{i,N-1}^{\ell+1} &= h'_{i,N-1} \\
 & (2.41)
 \end{aligned}$$

$$\begin{aligned}
 & b''_2 \Gamma_{2,j}^{n+1/2} + c''_{2,j} \Gamma_{3,j}^{n+1/2} = d''_{2,j} \\
 a''_{i,j} \Gamma_{i-1,j}^{n+1/2} + b''_i \Gamma_{i,j}^{n+1/2} + c''_{i,j} \Gamma_{i+1,j}^{n+1/2} &= d''_{i,j} \quad 2 < i < M-1 \\
 a''_{M-1,j} \Gamma_{M-2,j}^{n+1/2} + b''_{M-1} \Gamma_{M-1,j}^{n+1/2} &= d''_{M-1,j} \\
 & (2.42)
 \end{aligned}$$

$$\begin{aligned}
& f_i'' \Gamma_{i,2}^{n+1} + g_{i,2}'' \Gamma_{i,3}^{n+1} = h_{i,2}'' \\
& e_{i,j}'' \Gamma_{i,j-1}^{n+1} + f_i'' \Gamma_{i,j}^{n+1} + g_{i,j}'' \Gamma_{i,j+1}^{n+1} = h_{i,j}'' \quad 2 < j < N-1 \\
& e_{i,N-1}'' \Gamma_{i,N-2}^{n+1} + f_i'' \Gamma_{i,N-1}^{n+1} = h_{i,N-1}''
\end{aligned} \tag{2.43}$$

$$u_{i,j}^{n+1} = - \frac{(\psi_{i,j+1}^{n+1} - \psi_{i-1,j}^{n+1})}{2r^2(z_i) \cos \beta_j \Delta \beta} \tag{2.44}$$

$$v_{i,j}^{n+1} = \frac{e^{-az_i} (\psi_{i+1,j}^{n+1} - \psi_{i-1,j}^{n+1})}{2ab \Delta z r(z_i) \cos \beta_j} \tag{2.45}$$

$$x_{i,j}^{n+1} = - \frac{(\Gamma_{i,j+1}^{n+1} - \Gamma_{i,j-1}^{n+1})}{2\Delta \beta r^2(z_i) \cos^2 \beta_j} \tag{2.46}$$

$$\eta_{i,j}^{n+1} = \frac{e^{-az_i} (\Gamma_{i+1,j}^{n+1} - \Gamma_{i-1,j}^{n+1})}{2ab \Delta z r^2(z_i) \cos^2 \beta_j} \tag{2.47}$$

$$\begin{aligned}
\phi_{i,j}^{n+1} &= e^{-az_i} \alpha_1 (\Gamma_{i+1,j}^{n+1} x_{i+1,j}^{n+1} - \Gamma_{i-1,j}^{n+1} x_{i-1,j}^{n+1}) \\
&+ \alpha_2 (\Gamma_{i,j+1}^{n+1} \eta_{i,j+1}^{n+1} - \Gamma_{i,j-1}^{n+1} \eta_{i,j-1}^{n+1}) \quad i < M-1
\end{aligned} \tag{2.48}$$

$$\begin{aligned} \phi_{M-1,j}^{n+1} &= e^{-az_{M-1}} \alpha_1 r_{M-2,j}^{n+1} \chi_{n-2,j}^{n+1} \\ &+ \alpha_2 (r_{M-1,j+1}^{n+1} \eta_{M-1,j+1}^{n+1} - r_{M-1,j-1}^{n+1} \eta_{M-1,j-1}^{n+1}) \end{aligned} \quad (2.49)$$

where

$$a_{i,j} = e^{-2az_i} \alpha_3 + e^{-2az_i} \alpha_4 - e^{-az_i} \alpha_1 u_{i-1,j} \quad (2.50)$$

$$b_i = -1 - 2 e^{-2az_i} \alpha_3 \quad (2.51)$$

$$c_{i,j} = e^{-az_i} \alpha_1 u_{i+1,j} + e^{-2az_i} \alpha_3 - \alpha_4 e^{-2az_i} \quad (2.52)$$

$$\begin{aligned} d_{i,j} &= \phi_{i,j} + e_{i,j} \zeta_{i,j-1}^n - (2+f_i) \zeta_{i,j}^n \\ &- g_{i,j} \zeta_{i,j+1}^n \end{aligned} \quad (2.53)$$

$$d_{2,j} = d_{2,j} - a_{2,j} \zeta_{1,j}^{n+1/2} \quad (2.54)$$

$$\begin{aligned} e_{ijj} &= \frac{\alpha_5 \cos \beta_{j-1}}{r^2(z_i) \cos \beta_j} - \frac{\alpha_2 v_{i,j-1}}{r(z_i)} \\ &- \frac{\tan \beta_j \cos \beta_{j-1} \alpha_6}{r^2(z_i) \cos \beta_j} \end{aligned} \quad (2.55)$$

$$f_i = -1 - \frac{2\alpha_5}{r^2(z_i)} \quad (2.56)$$

$$g_{i,j} = \frac{\alpha_2 v_{i,j+1}}{r(z_i)} + \frac{\alpha_5 \cos \beta_{j+1}}{r^2(z_i) \cos \beta_j} + \frac{\tan \beta_j \cos \beta_{j+1} \alpha_6}{r^2(z_i) \cos \beta_j} \quad (2.57)$$

$$h_{i,j} = \phi_{i,j} - a_{i,j} \zeta_{i-1,j}^{n+1/2} - (2+b_i) \zeta_{i,j}^{n+1/2} - c_{i,j} \zeta_{i+1,j}^{n+1/2} \quad (2.58)$$

$$h_{i,2} = h_{i,2} - \zeta_{i,1}^{n+1} e_{i,2} \quad (2.59)$$

$$h_{i,N-1} = h_{i,N-1} - \zeta_{i,N}^{n+1} g_{i,N-1} \quad (2.60)$$

$$a'_{i,j} = e^{-2az_i} \alpha_7 + e^{-2az_i} \alpha_8 \quad (2.61)$$

$$b'_i = -1 - 2 \alpha_7 e^{-2az_i} \quad (2.62)$$

$$c'_i = \alpha_7 e^{-2az_i} - \alpha_8 e^{-2az_i} \quad (2.63)$$

$$d'_{i,j} = -e_{i,j} \psi_{i,j+1}^l - \left( \frac{2+f_i}{1+f_i} \right) \psi_{i,j}^l e_{i,j} - g'_{i,j} \psi_{i,j+1}^l - \zeta_{i,j}^{n+1} \cos \beta_j \frac{\Delta t_1}{2} \quad (2.64)$$

$$e'_{i,j} = \frac{\alpha_9}{r^2(z_i)} - \frac{\alpha_{10} \tan \beta_j}{r^2(z_i)} \quad (2.65)$$

$$f_i' = -1 - \frac{2\alpha_9}{r^2(z_i)} \quad (2.66)$$

$$g_{i,j}' = \frac{\alpha_9}{r^2(z_i)} + \frac{\alpha_{10} \tan \beta_j}{r^2(z_i)} \quad (2.67)$$

$$h_{i,j}' = -a_{i,j}' \psi_{i-1,j}^{l+1/2} - (2+b_i') \psi_{i,j}^{l+1/2} \\ - c_{i,j}' \psi_{i+1,j}^{l+1/2} - \zeta_{i,j}^{n+1} \frac{\Delta t_1}{2} \cos \beta_j \quad (2.68)$$

$$a_{i,j}'' = -u_{i,j} e^{-az_i} \alpha_1 + e^{-az_i} \alpha_3 + \alpha_4 e^{-2az_i} \quad (2.69)$$

$$b_i'' = -1 - 2\alpha_3 e^{-2az_i} \quad (2.70)$$

$$c_{i,j}'' = u_{i,j} e^{-az_i} \alpha_1 + e^{-2az_i} \alpha_3 - e^{-2az_i} \alpha_4 \quad (2.71)$$

$$d_{i,j}'' = -e_{i,j}'' \Gamma_{i,j-1}^n - (2+f_i'') \Gamma_{i,j}^n \\ - g_{i,j}'' \Gamma_{i,j+1}^n \quad (2.72)$$

$$d_{2,j}'' = d_{2,j}'' - \Gamma_{i,j}^{n+1/2} a_{2,j}'' \quad (2.73)$$

$$e_{i,j}'' = -1 - \frac{\alpha_5}{r^2(z_i)} - \frac{v_{i,j} \alpha_2}{r(z_i)} - \frac{\tan \beta_j \alpha_6}{r^2(z_i)} \quad (2.74)$$

$$f_i'' = -1 - \frac{2\alpha_5}{r^2(z_i)} \quad (2.75)$$

$$g_{i,j}'' = \frac{v_{i,j} \alpha_2}{r(z_i)} + \frac{\alpha_5}{r^2(z_i)} + \frac{\tan \beta_j \alpha_6}{r^2(z_i)} \quad (2.76)$$

$$h_{i,j}'' = -a_{i,j}'' y_{i-1,j}^{n+1/2} - (2+b_i'') r_{i,j}^{n+1/2} - c_{i,j}'' r_{i+1,j}^{n+1/2} \quad (2.77)$$

$$h_{i,N-1}'' = h_{i,N-1}'' - g_{i,N-1}'' r_{i,N}^{n+1} \quad (2.78)$$

$$\alpha_1 = \frac{\Delta t}{4ab\Delta z} \quad (2.79)$$

$$\alpha_2 = \frac{\Delta t}{4\Delta\beta} \quad (2.80)$$

$$\alpha_3 = \frac{\Delta t}{2\text{Re } a^2 b^2 (\Delta z)^2} \quad (2.81)$$

$$\alpha_4 = \frac{\Delta t}{4\text{Re } ab^2 \Delta z} \quad (2.82)$$

$$\alpha_5 = \frac{\Delta t}{2\text{Re } \Delta\beta^2} \quad (2.83)$$

$$\alpha_6 = \frac{\Delta t}{4\text{Re } \Delta\beta} \quad (2.84)$$

$$\alpha_7 = \frac{\Delta t_1}{2a^2 b^2 (\Delta z)^2} \quad (2.85)$$

$$\alpha_8 = \frac{\Delta t_1}{4ab^2 \Delta z} \quad (2.86)$$

$$\alpha_9 = \frac{\Delta t_1}{2\Delta\beta^2} \quad (2.87)$$

$$\alpha_{10} = \frac{\Delta t_1}{4\Delta\beta} \quad (2.88)$$

and  $2 \leq i \leq M-1$ ,  $2 \leq j \leq N-1$  unless otherwise denoted. Also, the boundary conditions from Section IV.1 which provide for zero values of the dependent variables have been substituted where applicable to simplify the equations.

Equations (2.38) and (2.39) are the two tridiagonal systems of equations associated with solving for  $\zeta^{n+1}$ ; equations (2.40) and (2.41) are the two tridiagonal systems of equations associated with solving for  $\psi^{n+1}$ ; equations (2.42) and (2.43) are the two tridiagonal systems of equations associated with solving for  $r^{n+1}$ . See Appendix D for the algorithm for solving tridiagonal systems of equations. The stream function at the end of the time step  $\psi^{n+1}$  is found by iterating equations (2.40) and (2.41) until  $\psi^{l+1} = \psi^l$  or until  $l = l_{\max}$ . Equations (2.38) through (2.49) are iterated until a steady state is attained, i.e.,  $\zeta^{n+1} = \zeta^n$  and  $r^{n+1} = r^n$ .

The numerical definition of approximately equal is generally referred to as the convergence criteria. The following convergence criteria are used in this work:



$$u_{i,j}^{k+1} = u_{i,j}^k$$

$$\frac{\text{Max} \left| u_{i,j}^{k+1} - u_{i,j}^k \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| u_{i,j}^{k+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \varepsilon_z, \quad (2.89)$$

$$\zeta_{i,j}^{n+1} = \zeta_{i,j}^n$$

$$\frac{\text{Max} \left| \zeta_{i,j}^{n+1} - \zeta_{i,j}^n \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| \zeta_{i,j}^{n+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \varepsilon_z, \quad (2.90)$$

$$\psi_{i,j}^{\ell+1} = \psi_{i,j}^{\ell}$$

$$\frac{\text{Max} \left| \psi_{i,j}^{\ell+1} - \psi_{i,j}^{\ell} \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| \psi_{i,j}^{\ell+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \varepsilon_p, \quad (2.91)$$

and

$$\Gamma_{i,j}^{n+1} = \Gamma_{i,j}^n$$

$$\frac{\text{Max} \left| \Gamma_{i,j}^{n+1} - \Gamma_{i,j}^n \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| \Gamma_{i,j}^{n+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \varepsilon_g, \quad (2.92)$$

where  $\epsilon_z$ ,  $\epsilon_p$ , and  $\epsilon_g$  are experimentally determined small numbers.

#### IV.2.b Boundary and initial conditions

The additional equations required in the system of numerically approximated equations in the previous section are provided by the remaining boundary conditions. As mentioned in the previous section the boundary conditions derived in Section III.b, which provide for zero values of the dependent variables, have already been invoked, and; consequently, will not be restated here. Of course, the boundary conditions

$$\Gamma(r, \epsilon) = r^2 \cos^2 \epsilon ,$$

$$\eta(r, \epsilon) = - \frac{2}{r} \quad \text{for} \quad r > 0 ,$$

and

$$\eta(1, \beta) = - \frac{2\Gamma(1, \beta)}{\cos^2 \beta} \quad \text{for} \quad 0 < \beta < \epsilon$$

are simply

$$\Gamma_{i,N} = r^2(z_i) \cos^2 \epsilon , \quad (2.93)$$

$$\eta_{i,N} = - \frac{2}{r(z_i)} \quad \text{for} \quad i < M , \quad (2.94)$$

and

$$\eta_{1,j} = - \frac{2r_{1,j}}{\cos^2 \beta_j} \quad \text{for} \quad 1 < j < N, \quad (2.95)$$

respectively. The remaining boundary conditions are approximated by truncated Taylor's series expanded about the boundaries. Expanding  $\psi$  about  $\beta = 0$  in a Taylor's series gives

$$\begin{aligned} \psi_{i,2} = \psi_{i,1} + \frac{\partial \psi_{i,1}}{\partial \beta} \Delta \beta + \frac{\partial^2 \psi_{i,1}}{\partial \beta^2} \frac{\Delta \beta^2}{2} + \frac{\partial^3 \psi_{i,1}}{\partial \beta^3} \frac{\Delta \beta^3}{3!} \\ + O(\Delta \beta^4) \end{aligned} \quad (2.96)$$

and

$$\begin{aligned} \psi_{i,3} = \psi_{i,1} + \frac{2\partial \psi_{i,1}}{\partial \beta} \Delta \beta + \frac{4}{2} \frac{\partial^2 \psi_{i,1}}{\partial \beta^2} \Delta \beta^2 \\ + \frac{8}{3!} \frac{\partial^3 \psi_{i,1}}{\partial \beta^3} \Delta \beta^3 + O(\Delta \beta^4). \end{aligned} \quad (2.97)$$

Solving for  $\partial^2 \psi_{i,1} / \partial \beta^2$  from equations (2.96) and (2.97) and invoking the boundary conditions on  $\psi$  at the plate gives

$$\frac{\partial^2 \psi_{i,1}}{\partial \beta^2} = \frac{8\psi_{i,2} - \psi_{i,3}}{2(\Delta \beta)^2} + O(\Delta \beta^2). \quad (2.98)$$

Equation (1.53) of Section IV.1 and equation (2.98) are then combined to give

$$\zeta_{i,1} = - \frac{(8\psi_{i,2} - \psi_{i,3})}{2r^2(z_i)(\Delta \beta)^2} + O(\Delta \beta^2) \quad \text{for} \quad i < M. \quad (2.99)$$

In a similar manner the numerical approximation of the boundary condition on  $\zeta$  at the cone can be derived to give

$$\zeta_{i,N} = - \frac{(8\psi_{i,N-1} - \psi_{i,N-2})}{2r^2(z_i) \cos \epsilon(\Delta\beta)^2} + O(\Delta\beta^2) \text{ for } i < M \quad (2.100)$$

The numerical boundary condition on  $\zeta$  at the free surface is obtained by expanding  $\psi$  about the free surface in a Taylor's series to give

$$\begin{aligned} \psi_{2,j} = \psi_{1,j} - \frac{\partial \psi_{1,j}}{\partial r} \Delta r_{f.s.} + \frac{\partial^2 \psi_{1,j}}{\partial r^2} \frac{\Delta r_{f.s.}^2}{2} \\ + O(\Delta r_{f.s.}^3), \end{aligned} \quad (2.101)$$

where  $\Delta r_{f.s.} = 1 - r(z_2)$ , and using the boundary condition on  $v$  at the free surface, equation (2.28) of Section II.2.2, and the definition of the stream function, equation (1.43), of Section IV.1.a to obtain

$$\frac{\partial \psi_{1,j}}{\partial r} = \frac{1}{2} \frac{\partial^2 \psi_{1,j}}{\partial r^2}. \quad (2.102)$$

The substitution of equation (2.102) into equation (2.101), and the application of the boundary condition on  $\psi$  at the free surface then gives

$$\frac{\partial^2 \psi_{1,j}}{\partial r^2} = - \frac{2\psi_{2,j}}{[1-r(z_2)]r(z_2)} + O(\Delta r_{f.s.}^2) \quad 1 \leq j < N \quad (2.103)$$

for  $1 < j < N$

where  $1-r(z_2)$  has been substituted for  $\Delta r_{f.s.}$ . The combination of equations (1.55) of Section IV.1.b and equation

(2.103) then provides the following numerical boundary condition on  $\zeta$  at the free surface:

$$\zeta_{1,j} = \frac{2\psi_{2,j}}{\cos \beta_j [1-r(z_2)]r(z_2)} + O(\Delta r_{f.s.}^2) \quad (2.104)$$

for  $1 < j < N$ .

The numerical boundary conditions on  $\chi$  at the plate and cone can be derived by expanding  $r$  about the cone and plate in Taylor's series; invoking the boundary conditions on  $r$  at the cone and plate, and utilizing the continuum boundary conditions on  $\chi$  given in Section IV.1.b to give

$$\chi_{i,1} = \frac{\Gamma_{i,3} - 4\Gamma_{i,2}}{2\Delta\beta r^2(z_i)} + O(\Delta\beta^2) \quad \text{for } i > M \quad (2.105)$$

and

$$\chi_{i,N} = \frac{4\Gamma_{i,N-1} - \Gamma_{i,N-2} + 3\Gamma_{i,N}}{2 \cos^2 \epsilon \Delta\beta r^2(z_i)} + O(\Delta\beta^2) \quad \text{for } i > M. \quad (2.106)$$

Also,  $\frac{v}{r}$  can be expanded in a Taylor's series about  $r = 1$  to give

$$\frac{v_{2,j}}{r(z_2)} = v_{1,j} - \frac{\partial}{\partial r} \left( \frac{v}{r} \right)_{r=1} \Delta r_{f.s.}$$

$$+ \frac{\partial^2}{\partial r^2} \left( \frac{v}{r} \right)_{r=1} \Delta r_{f.s.}^2 + O(\Delta r_{f.s.}^3) \quad (2.107)$$

for  $i < j < N$

and from the continuum boundary condition on  $v$  at  $r = 1$  given by equation (2.28) of Section II.2.2, the following

numerical boundary condition on  $v$  at the free surface can be derived:

$$v_{1,j} = \frac{v_{2,j}}{r(z_2)} + O(\Delta r_{f.s.}^2) . \quad (2.108)$$

Similarly, the expansion of  $r/r^2$  about the free surface in a Taylor's series and the substitution of the continuum boundary condition on  $r$  at the free surface gives

$$r_{1,j} = \frac{r_{2,j}}{r^2(z_2)} + O(\Delta r_{f.s.}^2) . \quad (2.109)$$

In the numerical method described in the previous section  $v_{1,j}$  is not needed to compute  $v_{2,j}$ , and  $v_{1,j}$  can be computed directly from (2.108). However, since values for  $r_{1,j}^{n+1/2}$  are needed to compute  $r_{2,j}^{n+1}$ , this is not the case for  $r_{1,j}$  and  $r_{2,j}$ . The required values for  $r_{1,j}^{n+1/2}$  are obtained by letting

$$(r_{1,j}^{n+1/2})^{k=1} = r_{1,j}^n = \frac{r_{2,j}^n}{r^2(z_2)}$$

and

$$(r_{1,j}^{n+1/2})^{k+1} = \frac{1}{2} ((r_{1,j}^{n+1})^k + r_{1,j}^n)$$

where  $k$  is the non-linear iteration counter. The boundary conditions on  $r$  are also updated within the non-linear

iteration, such that

$$(\zeta_{i,1}^{n+1})^{k=1} = \zeta_{i,1}^n,$$

$$(\zeta_{i,N}^{n+1})^{k=1} = \zeta_{i,N}^n,$$

$$(\zeta_{1,j}^{n+1/2})^{k=1} = \zeta_{1,j}^n,$$

$$(\zeta_{i,1}^{n+1})^{k+1} = (\zeta_{i,1}^{n+1})^k,$$

$$(\zeta_{i,N}^{n+1})^{k+1} = (\zeta_{i,N}^{n+1})^k,$$

and

$$(\zeta_{1,j}^{n+1/2})^{k+1} = \frac{1}{2} ((\zeta_{1,j}^{n+1})^k + \zeta_{1,j}^n).$$

The initial conditions used are those associated with the instantaneous acceleration of the cone to an angular velocity of  $\Omega$  in a fluid which is at rest. These initial conditions can be written mathematically by

$$u_{i,j}^0 = v_{i,j}^0 = \psi_{i,j}^0 = \zeta_{i,j}^0 = 0 \quad \text{for } 1 \leq i \leq M \text{ and } 1 \leq j \leq N, \\ 1 \leq j \leq N, \quad (2.109)$$

$$r_{i,j}^0 = \eta_{i,j}^0 = \chi_{i,j}^0 = 0 \quad \text{for } 1 \leq i \leq M \text{ and } 1 \leq j \leq N \\ 1 \leq j \leq N, \quad (2.110)$$

$$r_{i,N}^0 = r^2(z_i) \cos^2 \epsilon \quad \text{for } 1 \leq i \leq M, \quad (2.111)$$

$$x_{i,N}^0 = -\frac{3}{2\Delta\beta} \quad \text{for} \quad 1 \leq i < M, \quad (2.112)$$

and

$$\eta_{i,N}^0 = -\frac{2}{r(z_i)} \quad \text{for} \quad 1 \leq i < M. \quad (2.113)$$

#### IV.2.c Torque and rate-of-deformation

The torque ratios at the cone and plate, which were derived in Section II.4, can be rewritten by combining equations (1.45), (1.57), and (1.58) of Section IV.1.b with equations (4.4) and (4.5) of Section II.4. The resulting expressions for the torque ratios are

$$\begin{aligned} \left. \frac{T}{T_p} \right|_{\text{cone}} &= -3\epsilon \cos^2 \epsilon \int_0^1 r^2 \chi(r, \epsilon) dr \\ &+ 6\epsilon \tan \epsilon \int_0^1 r \Gamma(r, \epsilon) dr \end{aligned} \quad (2.114)$$

and

$$\left. \frac{T}{T_p} \right|_{\text{plate}} = -3\epsilon \int_0^1 r^2 \chi(r, 0) dr. \quad (2.115)$$

If the boundary condition  $\Gamma(r, \epsilon)$  is substituted into equation (2.107), the second term can be integrated directly to give

$$\frac{T}{T_p} = -3\epsilon \cos^2 \epsilon \int_0^1 r^2 \chi(r, \epsilon) dr + \epsilon \sin 2\epsilon \quad (2.116)$$



The transformation  $r$  to  $z$ , described by equation (2.9), is then used to transform equations (2.115) and (2.116) from  $r$  to  $z$ , and the transformed equations are then discretized to give

$$\left. \frac{T}{T_p} \right|_{\text{cone}} = -3\epsilon \cos^2 \epsilon \int_0^1 r^2(z_i) x_{i,N} dz + \epsilon \sin 2\epsilon \quad (2.117)$$

and

$$\left. \frac{T}{T_p} \right|_{\text{plate}} = -3\epsilon \int_0^1 r^2(z_i) x_{i,1} dz \quad (2.118)$$

where the torque ratios have now been approximated by finite differences. The numerical integration required in equations (2.117) and (2.118) is achieved in this work by a combination of Simpson's and Newton's 3/8 rules, and is provided by the Scientific Subroutine Package subroutine QSF [12].

The elements of the rate-of-deformation tensor are computed by transforming equations (4.7) through (4.12) of Section II.4 from  $r$  to  $z$  and then approximating the derivations of the transformed equations by finite differences. The derivatives within the domain are approximated by centered differences while those on boundaries are approximated by expanding in a Taylor's series about the boundaries in an analogous manner to the development of the numerical

boundary conditions. Hence, the details of the derivation of those approximations are not presented here. Instead only the approximations are listed as follows:

$$\left. \frac{D_{rr}}{Dp} \right|_{i,j} = \frac{\epsilon e^{-az_i}}{ab\Delta z} (u_{i+1,j} - u_{i-1,j}), \quad (2.119)$$

$$\left. \frac{D_{rr}}{Dp} \right|_{1,j} = \frac{\epsilon u_{2,j}}{ab[1-r(z_2)]}, \quad (2.120)$$

$$\begin{aligned} \left. \frac{D_{\theta r}}{Dp} \right|_{i,j} = \epsilon \left[ \frac{r(z_i) e^{-az_i}}{2ab\Delta z} \left( \frac{v_{i+1,j}}{r(z_{i+1})} - \frac{v_{i-1,j}}{r(z_{i-1})} \right) \right. \\ \left. + \frac{(u_{i,j+1} - u_{i,j-1})}{2r(z_i)\Delta\beta} \right], \quad (2.121) \end{aligned}$$

$$\left. \frac{D_{\theta r}}{Dp} \right|_{i,1} = \frac{\epsilon(4u_{i,2} - u_{i,3})}{2r(z_i)\Delta\beta}, \quad (2.122)$$

$$\left. \frac{D_{\theta r}}{Dp} \right|_{i,N} = \frac{\epsilon(u_{i,N-2} - 4u_{i,N-1})}{2r(z_i)\Delta\beta}, \quad (2.123)$$

$$\left. \frac{D_{\phi r}}{Dp} \right|_{i,j} = \frac{\epsilon r(z_i) e^{-az_i}}{2ab \cos \beta_j \Delta z} \left( \frac{\Gamma_{i+1,j}}{r^2(z_{i+1})} - \frac{\Gamma_{i-1,j}}{r^2(z_{i-1})} \right), \quad (2.124)$$

$$\left. \frac{D_{\theta\theta}}{Dp} \right|_{i,j} = \frac{\epsilon}{r(z_i)} \left[ \frac{v_{i,j+1} - v_{i,j-1}}{\Delta\beta} - 2u_{i,j} \right], \quad (2.125)$$

$$\left. \frac{D_{\theta\theta}}{Dp} \right|_{i,1} = \frac{\epsilon}{r(z_i)\Delta\beta} (4v_{i,2} - v_{i,3}), \quad (2.126)$$

$$\left. \frac{D_{\theta\theta}}{Dp} \right|_{i,N} = \frac{\epsilon}{r(z_i)\Delta\beta} (v_{i,N-2} - 4v_{i,N-1}) , \quad (2.127)$$

$$\left. \frac{D_{\phi\phi}}{Dp} \right|_{i,j} = - \frac{2\epsilon}{r(z_i)} (u_{i,j} + v_{i,j} \tan \beta_j) , \quad (2.128)$$

$$\left. \frac{D_{\phi\phi}}{Dp} \right|_{1,j} = -2\epsilon v_{1,j} \tan \beta_j , \quad (2.129)$$

$$\left. \frac{D_{\theta\phi}}{Dp} \right|_{i,j} = \frac{\epsilon \cos \beta_j}{2r^2(z_i)\Delta\beta} \left[ \frac{\Gamma_{i,j+1}}{\cos^2 \beta_{j+1}} - \frac{\Gamma_{i,j-1}}{\cos^2 \beta_{j-1}} \right] , \quad (2.130)$$

$$\left. \frac{D_{\theta\phi}}{Dp} \right|_{i,1} = \frac{\epsilon}{2r^2(z_i)\Delta\beta} \left[ \frac{4\Gamma_{i,2}}{\cos^2 \beta_2} - \frac{\Gamma_{i,3}}{\cos^2 \beta_3} \right] , \quad (2.131)$$

$$\left. \frac{D_{\theta\phi}}{Dp} \right|_{i,N} = \frac{\epsilon \cos \epsilon}{2r^2(z_i)\Delta\beta} \left[ \frac{\Gamma_{i,N-2}}{\cos^2 \beta_{N-2}} - \frac{4\Gamma_{i,N-1}}{\cos^2 \beta_{N-1}} \right] + \frac{3\Gamma_{i,N}}{\cos^2 \epsilon} , \quad (2.132)$$

and

$$\left. \frac{D_{\theta\phi}}{Dp} \right|_{1,j} = \left. \frac{D_{\theta\phi}}{Dp} \right|_{2,j} \quad (2.133)$$

where the brackets denoting physical components have been omitted, and  $1 < i < M$ ,  $1 < j < N$  unless otherwise indicated. Also, the numerical boundary condition for  $\Gamma_{1,j}$ , equation (2.109), has been used to obtain equation (2.133). Of course,

the remaining boundary values of the elements of the rate-of-deformation tensor are the same as those presented in equations (4.14) through (4.17) of Section II.4. In addition, the perturbation solution can be used to obtain the values of the elements of the rate-of-deformation tensor at  $r = 0$ ,  $z = 1$ . These values are

$$\left. \frac{D_{rr}}{Dp} \right|_{r=0} = \left. \frac{D_{\theta r}}{Dp} \right|_{r=0} = \left. \frac{D_{\phi r}}{Dp} \right|_{r=0} = \left. \frac{D_{\theta\theta}}{Dp} \right|_{r=0} = \left. \frac{D_{\phi\phi}}{Dp} \right|_{r=0} = 0 \quad (2.134)$$

and

$$\left. \frac{D_{\theta\phi}}{Dp} \right|_{r=0} = 1.0 . \quad (2.135)$$

### IV.3 Description of Algorithm

The algorithm used to obtain the numerical solution is as outlined below.

1. Input: nprob, ilib
2. Input: gap angle, Re, iprob, M, Me, Npr, kmax, nchc, nmax, kchc, lmax, itorq, iprint, ideo, idsn,  $\Delta t$ ,  $\Delta t_1$ ,  $\epsilon_2$ ,  $\epsilon_p$ ,  $\epsilon_g$ .
3. Convert the gap angle to radians; compute the step sizes in the  $\beta$  and  $z$  directions; compute  $\beta_j$  and  $z_i$ ; iterate to determine a and b; compute  $r_i$ .
4. Initialize  $u_{i,j}$ ,  $v_{i,j}$ ,  $\Gamma_{i,j}$ ,  $\chi_{i,j}$ ,  $\eta_{i,j}$ ,  $\zeta_{i,j}$ ,  $\psi_{i,j}$ ,

and  $(u_{i,j})^{k=1}$ .

- a. If  $nprob = 1$  and  $ilib = 0$ , use the initial conditions presented in the previous section.
  - b. If  $nprob > 1$  and  $ilib = 0$ , use the solution of the previous problem as initial conditions.
  - c. If  $ilib \neq 0$  read the initial conditions from a magnetic disk.
5. Compute  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9$ , and  $\alpha_{10}$ .
  6. Compute the time independent coefficients of the six tridiagonal matrices associated with computing  $\tau, \psi$ , and  $r$ . These coefficients are  $b_{i,j}, f_{i,j}, a_{i,j}, b'_{i,j}, c'_{i,j}, e'_{i,j}, b''_{i,j}$ , and  $f''_{i,j}$ . Also compute the time invariant portion of the tridiagonal algorithm for solving  $\psi^{l+1/2}$  and  $\psi^{l+1}$ .
  7. Set  $n, n_c$ , and  $n_p$  to zero.
  8. Set  $k = 0$ ; linearize the equations by setting  $u_{i,j} = u_{i,j}^n, v_{i,j} = v_{i,j}^n$ , and  $\phi_{i,j} = \phi_{i,j}^n$ ; also set  $r_{i,j}^{n+1/2} = r_{i,j}^n$ ; compute  $n_c = n_c + 1$  and  $n_p = n_p + 1$ .
  9. Compute the boundary conditions on  $\tau_{i,j}$ .
  10. Compute  $\tau_{i,j}^{n+1}$ .
    - a. Compute  $a_{i,j}, c_{i,j}, e_{i,j}$ , and  $g_{i,j}$ .
    - b. Compute  $d_{i,j}$  using the boundary conditions on  $\tau_{i,j}$  and  $\tau_{i,j}^n$ .
    - c. Compute  $\tau_{i,j}^{n+1/2}$  using the tridiagonal algorithm.
    - d. Use  $\tau_{i,j}^{n+1/2}$  and the boundary conditions on  $\tau_{i,j}$

- to compute  $h_{i,j}$ .
- e. Compute  $\zeta_{i,j}^{n+1}$  using the tridiagonal algorithm.
11. Compute  $\psi_{i,j}^{n+1}$ .
- Set  $l = 0$  and  $l_c = 0$ .
  - Equate  $\psi_{i,j}^l$  and  $\psi_{i,j}^n$ .
  - Use  $\psi_{i,j}^l$  and  $\zeta_{i,j}^{n+1}$  to compute  $d_{i,j}'$ .
  - Compute  $\psi_{i,j}^{l+1/2}$  using the tridiagonal algorithm.
  - Use  $\psi_{i,j}^{l+1/2}$  and  $\zeta_{i,j}^{n+1}$  to compute  $h_{i,j}'$ .
  - Compute  $\psi_{i,j}^{l+1}$  using the tridiagonal algorithm.
  - Compute  $l_c = l_c + 1$  and  $l_c = \lfloor l_c + 1 \rfloor$ . If  $l_c < l_{max}$  return to c and iterate.
  - If  $l = l_{max}$  go to 12.
  - Test for convergence - If

$$\frac{\text{Max}_{i=1,M, j=1,N} |\psi_{i,j}^{l+1} - \psi_{i,j}^l|}{\text{Max}_{i=1,M, j=1,N} |\psi_{i,j}^{l+1}|} > \epsilon_p,$$

convergence is not attained, set  $l_c = 0$ ;  $l = l_c + 1$ ;  
return to c, and iterate.

12.  $\psi_{i,j}^{n+1} = \psi_{i,j}^{l+1}$ .
13. Compute  $r_{i,j}^{n+1}$ .
- Compute  $a_{i,j}''$ ,  $c_{i,j}''$ ,  $e_{i,j}''$ , and  $g_{i,j}''$ .
  - Compute  $d_{i,j}''$  using the boundary conditions on  $r_{i,j}$  and  $r_{i,j}^n$ .

- c. Compute  $r_{i,j}^{n+1/2}$  using the tridiagonal algorithm.
- d. Use  $r_{i,j}^{n+1/2}$  and the boundary conditions on  $r_{i,j}$  to compute  $h_{i,j}''$ .
- e. Compute  $r_{i,j}^{n+1}$  using the tridiagonal algorithm.
14. Compute  $(u_{i,j}^{n+1})^{k+1}$ ,  $(v_{i,j}^{n+1})^{k+1}$ ,  $x_{i,j}^{n+1}$ ,  $n_{i,j}^{n+1}$ , and  $(\phi_{i,j}^{n+1})^{k+1}$ .
15. If  $k = k_{\max}$  go to 17.
16. Test for the convergence of the non-linear iteration: If

$$\frac{\text{Max}_{\substack{i=1,M \\ j=1,N}} |(u_{i,j}^{n+1})^{k+1} - (u_{i,j}^{n+1})^k|}{\text{Max}_{\substack{i=1,M \\ j=1,N}} |(u_{i,j}^{n+1})^{k+1}|} \leq \epsilon_z,$$

convergence is attained. Go to 17. Otherwise set

- a.  $(u_{i,j}^{n+1})^k = (u_{i,j}^{n+1})^{k+1}$ .
- b.  $u_{i,j} = 1/2[(u_{i,j}^{n+1})^{k+1} + u_{i,j}^n]$ .
- c.  $v_{i,j} = 1/2[(v_{i,j}^{n+1})^{k+1} + v_{i,j}^n]$ .
- d.  $\phi_{i,j} = 1/2[(\phi_{i,j}^{n+1})^{k+1} + \phi_{i,j}^n]$ .
- e.  $r_{i,j}^{n+1/2} = 1/2[(r_{i,j}^{n+1})^{k+1} + r_{i,j}^n]$ .
- f. Compute  $k = k + 1$  and return to 9.

17. If  $npr = 0$  or if  $np < npr$ , go to 18. Otherwise  
 print  $n, k, \ell_{to}, ii, jj, \psi_{Max}^{n+1}, \zeta_{Me,1}^{n+1}, \zeta_{Me,N}^{n+1}$ ,

$\zeta_{Me, \frac{N+1}{2}}^{n+1}$  where

$$\ell_{to} = \sum_{k=1}^k \ell^k$$

and  $(ii, jj)$  is the location where  $\psi_{Max}$  occurs.

Set  $np = 0$ .

18. If  $nc < nchc$ , compute  $nc = nc + 1$  and  $n = n + 1$ .  
 Return to 8.

19. Test for a steady state

a. If

$$\frac{\text{Max} \left| \zeta_{i,j}^{n+1} - \zeta_{i,j}^n \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| \zeta_{i,j}^{n+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \epsilon_z$$

go to b. Otherwise a steady state is not attained. If  $n \leq nmax$ , set  $nc = 0$ ; compute  $n = n + 1$ , and return to 8. If  $n \geq nmax$ , print "A steady state is not attained" and go to 20.

b. If

$$\frac{\text{Max} \left| \Gamma_{i,j}^{n+1} - \Gamma_{i,j}^n \right|_{\substack{i=1,M \\ j=1,N}}}{\text{Max} \left| \Gamma_{i,j}^{n+1} \right|_{\substack{i=1,M \\ j=1,N}}} \leq \epsilon_g,$$



go to 20. If  $n \leq n_{\max}$ , set  $nc = 0$ ; compute  $n = n + 1$ ; and return to 8. Otherwise, print a "A steady state is not attained." case is not attained."

20. If  $idsn = 0$ , go to 21.

Write the values of  $\psi_{i,j}^{n+1}$ ,  $u_{i,j}^{n+1}$ ,  $v_{i,j}^{n+1}$ ,  $r_{i,j}^{n+1}$ ,  $\xi_{i,j}^{n+1}$ ,  $\eta_{i,j}^{n+1}$  and  $\chi_{i,j}^{n+1}$  on a magnetic disk.

21. If  $itorq = 0$ , go to 22.

If  $itorq = 1$ , compute and print out the value of

$$\frac{T}{T_p} \Big|_{\text{plate}}$$

If  $itorq = 2$ , compute and print out the values of

$$\frac{T}{T_p} \Big|_{\text{plate}} \quad \text{and} \quad \frac{T}{T_p} \Big|_{\text{cone}}$$

22. If  $idef = 0$ , go to 23. Compute and print out the values of the extreme ratios of each of the elements of the rate-of-deformation tensor and the primary deformation rate. If  $idef = 2$ , also print out the values of

$$\frac{D_{rr}}{D_p} \Big|_{1,j}, \quad \frac{D_{rr}}{D_p} \Big|_{Me,j}, \quad \text{and} \quad \frac{D_{r\theta}}{D_p} \Big|_{Me,j}$$

23. If  $i\text{print} = 0$ , go to 24. If  $i\text{print} = 1$ , print out the values of  $\psi_{i,j}$ . If  $i\text{print} = 2$ , print out the values of  $\psi_{i,j}$  and  $\Gamma_{i,j}$ . If  $i\text{print} = 3$ , print out the values of  $u_{i,j}$ ,  $v_{i,j}$ ,  $\psi_{i,j}$ , and  $\Gamma_{i,j}$ . If  $i\text{print} = 4$ , print out the values of  $u_{Me,j}$ ,  $v_{Me,j}$ , and  $\Gamma_{Me,j}$ . If  $i\text{print} > 4$  or  $i\text{print} < 0$ , print out the values of  $u_{i,j}$ ,  $v_{i,j}$ ,  $\psi_{i,j}$ ,  $\Gamma_{i,j}$ ,  $\zeta_{i,j}$ ,  $x_{i,j}$ , and  $h_{i,j}$ .
24. If  $n\text{prob} > i\text{prob}$ , set  $i\text{lib} = 0$ , and go to 2.
25. End.

The listing of the FORTRAN IV coding of the above algorithm is presented in Appendix E. Since the computation of the elements of the rate-of-deformation tensor and the torque ratios requires very little computer time, and considerable storage is required to store their values on disk, they were not written on disk. Instead the torque ratios and the elements of the rate-of-deformation tensor were recomputed from the velocity profiles stored on disk and were then printed in the desired format. This program is straightforward, and; therefore, is not presented.

#### IV.4 Stability and Convergence

As was discussed in Section IV.2, implicit methods have been shown by a von Neumann analysis to be inherently stable provided the time step is less than some critical

value. The equations used in these analyses are much simpler than the equations to be solved in this work. To extend the von Neumann analysis to a complex system of equations such as these would be a subject of considerable research in itself. Instead the stability of the numerical solution in this work was determined experimentally. This was accomplished for given step sizes by choosing a time step such that the solution was unstable, i.e., overflow messages occurred during the execution of the program. The time step was then subsequently decreased until overflows did not occur, and the solution converged to a steady state. This was repeated for several different combinations of step sizes to obtain stability criteria.

It should be noted that other researchers have had difficulty in attaining steady state solutions of fluid mechanics problems at high Reynolds numbers when using second order boundary conditions on vorticity [23]. Many resorted to first order vorticity boundary conditions while others used first order differences of the convective terms [23]. Briley [4] noted an inconsistency in the velocities computed one node inside the boundaries using centered differences of the stream function and the velocities that could be computed at these nodes via the polynomial expansion of the stream function used to compute the second order vorticity boundary conditions. Briley was able to obtain stable solutions at high Reynolds numbers by computing

the velocities at the nodes just inside the boundaries using the stream function polynomial.

Initially, difficulty in attaining a steady state was also encountered in this work. Briley's method was then tried in an attempt to reach steady state, but to no avail, and; therefore, it was not used. Subsequently, it was discovered that if  $\epsilon_p$  was selected such that its value was approximately half that of  $\epsilon_z$  then a steady state was attained. Apparently, if the stream function is not iterated to obtain greater accuracy then the convergence criteria used to define a steady state for the other variables, the inaccuracy is amplified by the nonlinear coefficients computed from the stream function such that the solution becomes unstable.

The solution was found to be stable if  $\epsilon_p = \frac{\epsilon_z}{2}$ ,

$$\frac{\Delta t_1}{\text{Re } \Delta z^2} = \frac{\Delta t}{\text{Re } \Delta z^2} \leq 3 \times 10^{-5}, \quad (2.134)$$

and

$$\frac{\Delta t_1}{\text{Re } \Delta \beta^2} = \frac{\Delta t}{\text{Re } \Delta \beta^2} \leq 3 \times 10^{-2}. \quad (2.135)$$

Suitable steady state solutions were attained if the time steps  $\Delta t$  and  $\Delta t_1$  were computed from equations (2.139) and (2.140) and the following convergence criteria were used:

$$\epsilon_z = \epsilon_y = 10^{-4} \quad (2.136)$$

and

$$\epsilon_p = 0.5 \times 10^{-4} \quad (2.137)$$

The sensitivity of the solution to the local truncation error was also determined. This is also called convergence, but a more accurate name is perhaps truncation convergence or discretization error. Since all partial derivatives and boundary conditions are approximated to second order, then the finite difference solution should approach the exact solution as  $(\Delta z)^2$  and  $(\Delta \beta)^2$  approach zero. To determine the error incurred in using finite values of  $\Delta z$  and  $\Delta \beta$ , a truncation convergence study was conducted at  $\epsilon = 2$  degrees and  $Re = 2 \times 10^4$ . The flow was assumed to possess enough similarity such that an acceptable truncation error at this condition is sufficient to insure accurate results for all combinations of  $\epsilon$  and  $Re$  at which solutions were sought.

This truncation convergence study consisted of obtaining steady state solutions for  $M = 9, 11, \text{ and } 13, N = 11; M = 11 \text{ and } 13, N = 13; \text{ and } M = 11, N = 15$  and plotting the results versus  $(\Delta z)^2$  and  $(\Delta \beta)^2$  as in Figures 4, 5, and 6. In all cases  $Me$  was chosen such that  $Me = \frac{M+1}{2}$ , and  $\kappa$  was chosen to be 3. Since the maximum stream function  $\psi_{\max}$  occurred at the same location for each grid,  $\psi_{\max}$  was

chosen as an acceptable variable to measure the sensitivity of the secondary flow to truncation error. The values of  $\zeta$  on the cone and plate at  $r = 1 - \kappa\epsilon = 0.8953$  and at the center of the free surface were also used to indicate the sensitivity of the secondary flow to truncation error. The values of  $r$  at  $r = 0.8953$ ,  $\beta = \epsilon/2$ , and at the center of the free surface were used to measure the effect of truncation error on  $r$ . The dependence of  $r$ ,  $\psi_{\max}$ , and  $\zeta$  on  $(\Delta z)^2$  is depicted in Figures 4.a, 5.a, and 6.a wherein each figure the dependence is essentially linear over the range of  $(\Delta z)^2$  computed. Also since the lines for  $N = 11$  and  $N = 15$  are parallel, it was assumed that if another solution for  $N = 15$  had been computed that the locus of points for  $N = 15$  would also be parallel to those for  $N = 11$  and  $N = 13$ . With this assumption, the magnitude of the truncation error was estimated by computing the percentage deviation of the value of the variable at a finite  $\Delta z$  from its reference value, which is the value obtained by linearly extrapolating the line for  $N = 11$  to  $(\Delta z)^2 = 0$ . For example, the truncation error incurred by using 11 nodes in the  $z$  direction on  $r(1.0, \frac{\epsilon}{2})$  was estimated by entering Figure 4.a at  $(\Delta z)^2 = .01$ , reading  $r(1.0, \frac{\epsilon}{2}) = 0.871$ , and computing the percentage deviation from 0.900 to be -3.2%. Of course, the magnitude of the truncation is 3.2%.

Of the variables chosen to measure the effect of the truncation error,  $\psi_{\max}$  exhibits the most severe dependence

on  $(\Delta z)^2$ . The magnitudes of the truncation error on  $\psi_{\max}$  due to a finite  $\Delta z$  was estimated to be 5.5% for  $M = 11$  and 4.2% for  $M = 13$ . Since the difference between a 5.5% and 4.2% error is negligible in relation to the increased computer time required, 11 was selected as the optimum of nodes in the  $z$  direction with  $M_e = 6$  and  $\kappa = 3$ .

Figures 4.b, 5.b, and 6.b exhibit the sensitivity of the solution to finite values of  $(\Delta\beta)^2$  for  $M = 11$ , ( $\kappa = 3$ , and  $M_e = 6$ ). The magnitudes of the truncation error due to a finite  $\Delta\beta$  were estimated in the same manner as were those for finite  $\Delta z$ . The total estimated truncation error was found by adding the magnitudes of those for finite  $\Delta z$  to those for finite  $\Delta\beta$ . These values are listed in Table I for  $M = 11$  ( $M_e = 6$ ,  $M = 3$ ) and  $N = 11, 13$ , and  $15$ . The estimated truncation errors listed in Table I are the maximum values, but since Figures 4.5 and 6 indicate that the truncation errors due to finite  $\Delta z$  and  $\Delta\beta$  are compensating errors, it is believed that the magnitudes of the actual truncation errors incurred are somewhat smaller than those presented in Table I. The accuracy obtained from a  $11 \times 15$  ( $M_e = 6$ ,  $M = 3$ ) grid was decided to be sufficient in light of the dramatically increased cost associated with obtaining 1 or 2% better accuracy.

#### IV.5 Discussion of Solution

The IBM 370/155 computer system at Rice University

was used to execute the numerical procedure. This machine has a computational speed of approximately 2  $\mu$  sec. for addition and subtraction and 11  $\mu$  sec. for multiplication and division. To provide greater control over the program, the program was executed via a time sharing remote terminal.

A large portion of the computer time required to advance the solution a time step was consumed in the non-linear iteration and the iteration for the stream function. This was reduced substantially by limiting the number of non-linear iterations and the number of stream function iterations. When this was done, the true transient solution was not obtained, but as the solution approached steady state, the change for each time step became smaller and, near steady state, the number of iterations required was less than the maximum allowed number. Hence, limiting the number of non-linear and stream function iterations did not affect the steady state solution provided the limits were not so severe that the stability of the solution was affected. The maximum number of non-linear iteration  $k_{max}$  was limited to 2 while the maximum number of stream function iterations  $l_{max}$  was limited to 24.

Further savings in computer time were achieved by choosing a relatively crude convergence criteria initially, say  $\epsilon_p = 3 \times 10^{-4}$  and  $\epsilon_z = \epsilon_g = 6 \times 10^{-4}$ ; iterating until these criteria were met, and then writing the results on a magnetic disk. The convergence criteria were then refined



to maybe  $\epsilon_p = 10^{-4}$  and  $\epsilon_z = \epsilon_g = 2 \times 10^{-4}$ ; the results on the magnetic disk were read and used as initial conditions, and the equations were then iterated until these convergence criteria were met. This process was repeated until the convergence criteria used to define a steady state were met, i.e., equations (2.141) and (2.142). Even with these methods of saving execution time, the amount of computer time required to attain a steady state from the initial conditions presented in Section IV.2.b was quite large. For example, for  $\epsilon = 4$  degrees,  $Re = 2 \times 10^1$ , 57 minutes of computer time was required. However, substantial savings in computer time were realized by using a prior steady state solution as the initial conditions. Savings of as much as 75% were achieved. To exemplify, only 14 minutes of computer time were required to obtain the steady state solution for  $\epsilon = 4$  degrees,  $Re = 4 \times 10^1$  using the solution for  $\epsilon = 4$  degrees,  $Re = 2 \times 10^1$  as the initial conditions.

Since the transformation  $r$  to  $z$  is a function of  $\epsilon$ , the steady state solution for one value of  $\epsilon$  can not be used as the initial conditions for another value of  $\epsilon$ . Consequently, for each value of  $\epsilon$ , a steady state solution for a specific  $Re$  was attained using the initial conditions presented in Section IV.2.b, and the solution was written and saved on magnetic disk. The solutions for the other values of  $Re$  were then generated, and written on disk using prior solutions for initial conditions. The solutions

generated from the initial conditions of Section IV.2.b were  $\epsilon = 4$  degrees,  $Re = 2 \times 10^1$ ;  $\epsilon = 2$  degrees,  $Re = 10^2$  and  $2 \times 10^4$ ;  $\epsilon = 1$  degree,  $Re = 8 \times 10^3$ ; and  $\epsilon = 1/3$  degrees,  $Re = 4 \times 10^4$ .

Solutions for two different values of  $Re$  for  $\epsilon = 2$  degrees were computed from the initial conditions of Section IV.2.b to demonstrate that the procedure gives consistent solutions, regardless of the initial conditions. This was achieved by using the solution for  $\epsilon = 2$  degrees,  $Re = 10^2$  as the initial conditions for  $Re = 5 \times 10^1$  and  $2 \times 10^2$ . Initial conditions for  $Re = 4 \times 10^2$  were the solution for  $Re = 2 \times 10^2$ , etc. Until the solution for  $Re = 10^4$  was computed. These solutions were then plotted as in Figures 31, and 37 through 43 to show that they were consistent with the solution for  $Re = 2 \times 10^4$ , which was computed using the initial conditions of the previous section.

The amount of computer time required increased as  $\epsilon$  decreased. This is consistent with the stability conditions, equations (2.134) and (2.135), of the previous section for the stability conditions indicate that the time step must be decreased as  $(\Delta\beta)^2$  decreases, and since  $\Delta\beta = \frac{\epsilon}{N-1}$ , the time step must also be decreased as  $\epsilon$  decreases.

The maximum  $Re$  for which a solution was sought was determined by the lessor of  $Re = 10^5$  or the  $Re$  which corresponds to a torque ratio of approximately 1.5. A torque ratio of approximately 1.5 was achieved at Reynolds numbers

less than  $10^5$  for all gap angles except 1/3 degrees. The solutions for  $\epsilon = 1/3$  degrees,  $Re = 8 \times 10^4$  and  $10^5$  did not converge to a steady state as defined by equations (2.136) and (2.137) of the previous section. However, both solutions converged to a steady state as defined by  $\epsilon_z = \epsilon_g = 4 \times 10^{-4}$  and  $\epsilon_p = 2 \times 10^{-4}$  and were further iterated using  $\epsilon_z = \epsilon_g = 2 \times 10^{-4}$  and  $\epsilon_p = 10^{-4}$  as the convergence criteria until each solution was judged to be at an acceptable steady state. An acceptable steady state was determined by comparing the torque ratios at the cone and plate, the maximum and minimum absolute values of the ratios of the elements of the rate-of-deformation tensor and the primary deformation rate,  $\psi_{max}$ ,  $\xi_{Me,1}$ ,  $\xi_{Me,N}$ ,  $v_{Me,\epsilon/2}$ , and  $\Gamma_{1,\epsilon/2}$  before and after 200 additional iterations. If the values of these variables were not essentially the same before and after these 200 iterations, then the solution was iterated 200 more times, and the values of these variables before and after these additional iterations were compared. This procedure was continued until these variables were essentially invariant through 200 iterations. The stream function and  $\Gamma$  arrays were then inspected for irregularities. The solution for  $Re = 8 \times 10^4$  contained no irregularities, and; therefore, was considered to be an acceptable solution. However, the stream function array for  $Re = 10^5$  contained negative values at  $r = 0.9968$  and  $.00374 \leq \beta \leq .00582$ . Since the resolution in this region is extremely good, and since

the negative values occur at only one value of  $r$ , it is believed that these negative values are not predicting a small counter rotating vortex, but instead constitute an irregularity in the stream function solution. Consequently, the solution for  $\epsilon = 1/3$  degrees and  $Re = 10^5$  was discarded. Due to the amount of computer time required to obtain these solutions, additional experimentation to determine the cause of the failure of the procedure to obtain an acceptable solution at these conditions was not attempted. However, it is felt that if the number of nodes in the  $z$  direction were increased that this procedure would also provide an acceptable solution at  $\epsilon = 1/3$  degrees and  $Re = 10^5$ .

The torque ratios at both the cone and the plate were computed. At steady state these ratios must be equal. See Appendix A. For gap angles of 2, 1, and  $1/3$  degrees, these ratios were essentially equal, but for  $\epsilon = 4$  degrees, and  $Re$  at which the ratios were greater than unity, the ratio at the cone was computed to be larger than that on the plate. The torque ratios were observed to still be changing very slowly as each solution approached a steady state with the ratio at the cone decreasing at approximately the same rate as that at the plate was increasing. Hence, the average of the two ratios was approximately constant and; therefore, had reached a steady state. It was assumed that if the solution had been iterated many, many more times

that the ratios would approach this average. The validity of this assumption and the validity of the computed torque ratios is confirmed by comparing them to the torque ratios computed from the viscosity measurements of Cheng [5]. These comparisons are presented and discussed in the following chapter.

## CHAPTER V

### PRESENTATION AND DISCUSSION OF RESULTS

Figures 7 through 30 present the meridian velocity distribution  $r$  and the streamlines of the secondary flow for selected values of  $Re$  and gap values of 4, 2, 1, and  $1/3$  degrees. The values of  $Re$  were selected such that at each gap angle the smallest value corresponds to approximately a 1% deviation from the rate-of-deformation tensor as computed by the primary flow analysis, the intermediate value corresponds to an incipient torque correction due to the secondary flow, and the largest value is the largest value of  $Re$  at which a solution was obtained. The one exception is the solution depicted in Figures 15 and 16. It corresponds to approximately a 10% torque increase due to the secondary flow. The arrays from which these graphs were plotted are included in Appendix F. If the primary flow solution for  $r$  were plotted versus  $\beta$ , then the graph would be a family of straight lines for constant values of  $r$ . Hence, straight lines on the graphs of the meridian velocity distribution indicate that the solution for  $r$  is identical with that computed from the primary flow analysis. Of course, there is no secondary flow in the primary flow

analysis, and; therefore, no streamlines of the secondary flow. Even though the magnitudes of the stream function for the streamlines of the secondary flow presented in these plots are admittedly small, the effect of secondary flow cannot be neglected. This will be demonstrated subsequently, but first some interesting observations can be made concerning the flow field exhibited in these figures.

The streamlines of the secondary flow consist of a single torisidol vortex, which is consistent with the observations of Hoppman and Miller [11] and Miller and Hoppman [18]. For values of  $Re$  where the secondary flow does not affect the torque, the vortex center moves toward the free surface as the gap angle is decreased. This can readily be seen by comparing Figures 8, 14, 19, and 25. This behavior is also consistent with Miller and Hoppman's [18] observations which were made at relatively low shear rates. For values of  $Re$  at which the secondary flow affects the torque at the cone and plate, the boundary layer thickens as  $Re$  increases until at values of  $Re$  where the torque ratio is approximately 1.5, the boundary layer thickness is approximately the same for all gap angles. This can be seen from Figures 12, 18, and 24, which correspond to torque ratios of approximately 1.5 for gap angles of 4, 2, and 1° degrees, respectively. This behavior suggests that the boundary layer thickness at the free surface is not a simple function of  $\epsilon$  as predicted in Section III.1, but is a more complex

function of  $\epsilon$  and  $Re$ . However, at values of  $Re$  at which the secondary flow does not affect the torque, the dependence of the boundary layer thickness on  $Re$  is negligible, and; therefore, the thickness of the boundary layer as predicted in Section III.1 holds for these values of  $Re$ . The single vortex is in contradiction with the observations reported by Savins and Metzner [24], of many small toroidal vortices.

Since there is no guarantee that the numerical solutions are unique, other solutions could exist which agree with Savins and Metzner's [24] observations. However, since the numerical solutions are consistent with the observations of Hoppman and Miller [11] and Miller and Hoppman [18], it is believed that the numerical solutions presented here depict the actual flow field in the cone-and-plate viscometer.

Since Figures 7, 13, 19, and 25 correspond to a 1% effect of the secondary flow on the rate-of-deformation, and since straight lines on these graphs indicate that the solution for  $r$  is identical to the primary flow solution for  $r$ , it can be concluded from these figures that the secondary flow affects the rate-of-deformation at values of  $Re$  lower than the values at which it affects  $r$ . Also, a close examination of Figures 9, 15, 21, and 37 indicates that the solution for  $r$  deviates from the primary flow solution for  $r$  only near the free surface while an examination of Figures 12, 17, and 23 reveals that the solution for  $r$  deviates from



the primary solution for  $r$  throughout almost all of the outer half of the viscometer. This means that although the region in which the secondary flow affects  $r$  grows at high  $Re$ , a region always exists near the intersection of the cone and plate in which the solution for  $r$  is identical with the primary flow solution for  $r$ . By examining the figures which present the streamlines of the secondary flow, it becomes evident that the stream function becomes vanishingly small in a region near the origin. It then follows that there always exists a region near the intersection of the cone and plate in which the flow is identical to primary flow, and the size of this region decreases with increasing  $Re$ . Also, a comparison of Figures 11, 17, and 23 indicates that, for a given torque increase due to the secondary flow, that the effect of the secondary flow on  $r$  is more severe for smaller gap angles.

It should be pointed out that an anomaly exists in the stream function field for  $\epsilon = 1$  degree,  $Re = 8 \times 10^4$  at the two nodes located at  $r = 0.9970$  and  $\beta = .01496$  and  $.01691$ . These two nodes are the first two nodes interior to the cone and adjacent to the free surface. Since the value of the stream function on the cone and on the free surface is zero, and since the values of the stream function at these nodes are very small negative numbers, the negative values do not indicate reversal of the secondary flow but instead merely mean that the stream function is

nearly zero at these nodes. Although the deviation of the stream function from zero at these nodes represents a very small error in the stream function, the error incurred in the derivatives of the stream function at these nodes and at their immediate neighboring nodes could be significant. Since  $D_{r\theta}$ ,  $D_{rr}$ ,  $D_{\theta\theta}$ , and  $D_{\phi\phi}$  are functions of the derivatives of the stream function with respect to  $\beta$  and  $r$ , their values at these nodes and their immediate neighbors could contain significant errors, and; therefore, are not to be trusted. Also, since the deviation of these small negative values from zero is expected to have a negligible effect on the solution at the remainder of the nodes, the values of the rate-of-deformation at the remaining nodes are expected to be accurate. In view of the fact that the region in question is very small, the lack of accurate values of  $D_{r\theta}$ ,  $D_{rr}$ ,  $D_{\theta\theta}$ , and  $D_{\phi\phi}$  in this region does not seriously affect the usefulness of the solution. It is believed that if the number of nodes in the  $z$  direction were increased, then very small positive values of the stream function would be computed at these nodes. Although negative values of the stream function in the solution at  $\epsilon = 1/3$  degrees,  $Re = 10^5$  were used in the previous chapter to judge that the solution was unacceptable, a contradiction in criteria for judging acceptable solutions does not exist. The negative values of the stream function for  $\epsilon = 1/3$  degrees,  $Re = 10^5$  represent an unacceptable solution because they occurred several

nodes inside the free surface, and; therefore were not nodes adjacent to a boundary where the stream function vanishes.

Figure 31 presents the maximum stream function  $\psi_{\max}$  as a function of  $\epsilon$  and  $Re$ . Since the stream function is zero on the boundaries of the viscometer,  $\psi_{\max}$  is an indication of the volumetric flow rate of the secondary flow. As can be seen from this figure, the volumetric flow rate at a given value of  $Re$  decreases as the gap angle is decreased. This is as expected as the volume within the viscometer decreases as the gap angle decreases.

Figures 32, 33, and 34 present the computed torque ratios at the plate and at the cone as well as the torque ratios which were obtained from Cheng's [5] experimental data vs.  $Re$  for gap angles of 4, 2, and 1 degrees, respectively. As can be seen from these figures, both

$$\frac{L}{T_p} \Big|_{\text{plate}} \quad \text{and} \quad \frac{L}{T_p} \Big|_{\text{cone}}$$

agree very well with the experimental data. As was discussed in the previous chapter, the arithmetic average of

$$\frac{T}{T_p} \Big|_{\text{plate}} \quad \text{and} \quad \frac{T}{T_p} \Big|_{\text{cone}}$$

is believed to yield much more accurate values for  $\frac{T}{T_p}$ . This is verified by imagining this average superimposed on

Figures 32, 33, and 34.

Figure 35 provides  $\frac{T}{T_p}$  vs.  $Re$  for gap angles of 4, 2, 1, and 1/3 degrees. These values are also tabulated in Table II. The values of  $\frac{T}{T_p}$  were obtained by computing the average discussed in the previous paragraph. Figure 36 provides a criteria for determining at what combinations of  $\epsilon$  and  $Re$  the secondary flow begins to affect torque at the plate and cone. Figure 36 is intended to compliment Figure 35 in that the values of  $Re$  at which an incipient torque increase due to the secondary flow is very difficult to ascertain from Figure 35. Figure 36 can be used to design cone-and-plate viscometers to assure that the secondary flow will not affect viscosity measurements of Newtonian fluids whereas Figure 35 can be used to interpret possible existing viscometric data of Newtonian fluids obtained at values of  $Re$  at which the secondary flow affects these measurements.

Figures 37 and 38 present the maximum and minimum absolute values, respectively, of the ratio of the  $D_{\theta\phi}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for gap angles of 4, 2, 1, and 1/3 degrees. Values of 100% on these two graphs indicate that the secondary flow does not affect the  $D_{\theta\phi}$  element of the rate-of-deformation tensor. It is easy to see from these figures that the effect of the secondary flow on the  $D_{\theta\phi}$  element of the rate-of-deformation tensor is to both increase and decrease its absolute value such that it is no

longer uniform throughout the viscometer, and; consequently, the  $\theta\phi$  component of the shear stress is no longer uniform throughout the viscometer. Figures 37 and 38 can be used to determine the magnitude of the deviation from uniformity of the  $\theta\phi$  component of shear stress for a given  $\epsilon$  and  $Re$ .

Figures 39 through 41 present the largest magnitudes of each of the remaining elements of the rate-of-deformation tensor and the primary deformation rate as functions of  $Re$  and  $\epsilon$ . Since the primary flow analysis predicts that  $\frac{D_{r\theta}}{Dp}$ ,  $\frac{D_{rr}}{Dp}$ ,  $\frac{D_{\theta r}}{Dp}$ ,  $\frac{D_{\theta\phi}}{Dp}$ , and  $\frac{D_{\theta\theta}}{Dp}$  are all zero, deviations from zero on the graphs indicate the effect of the secondary flow on these elements of the rate-of-deformation tensor. Deviations from zero in these graphs also indicate that shear stresses other than the  $\theta\phi$  component exist in the fluid. These graphs can be used as criteria in determining when these other shear stresses become important, and what their maximum absolute values are for a given  $\epsilon$  and  $Re$ . The maximum and minimum absolute values and locations of the ratios of each of the elements of the rate-of-deformation tensor and the primary deformation rate along with the arrays of each of these ratios are included in Appendix G. The arrays of the elements of the rate-of-deformation tensor can easily be obtained from Appendix G by multiplying the ratios by  $Dp$ . The arrays of the rate-of-deformation tensor can then be used to determine the real stresses in Newtonian fluids sheared in cone-and-plate viscometers at high shear rates.

By comparing Figure 35 and Figures 37 through 41, it is easy to see that a very dramatic secondary flow effect on the rate-of-deformation can occur with only a modest increase in the torque at the cone and plate.

Figure 44 provides loci of  $Re$  and  $\epsilon$  on which the magnitudes of the effects of the secondary flow on the rate-of-deformation are indicated as percentages of the primary deformation rate. This graph can be used to design cone-and-plate viscometers so that tolerable deviations from a uniform shear stress can be assured. A comparison of Figures 36 and 44 indicates that the secondary flow begins to affect the rate-of-deformation at values of  $Re$  an order of magnitude smaller than the values at which the secondary flow affects the torque at the cone and plate.

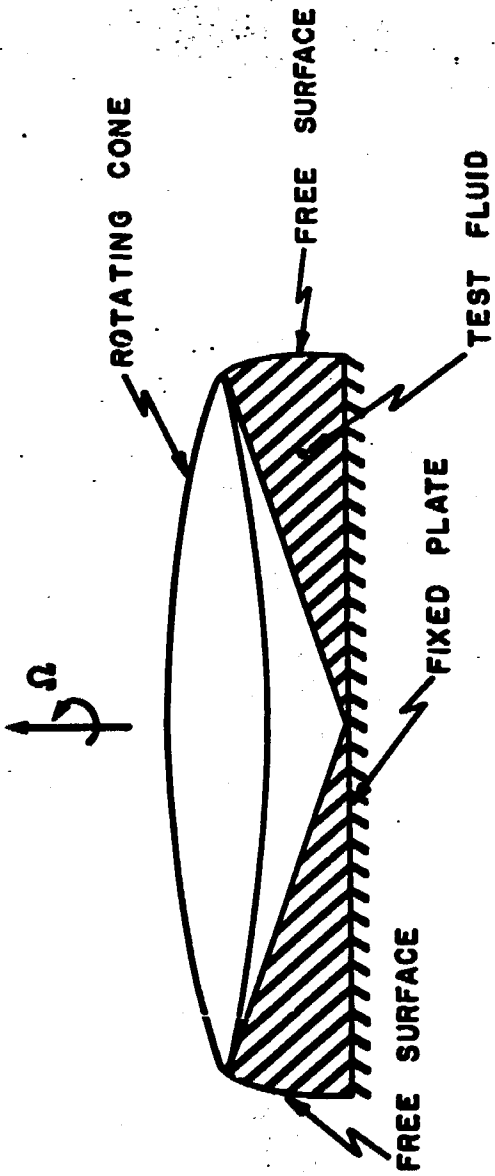


Figure 1. Pictorial description of the cone-and-plate viscometer.

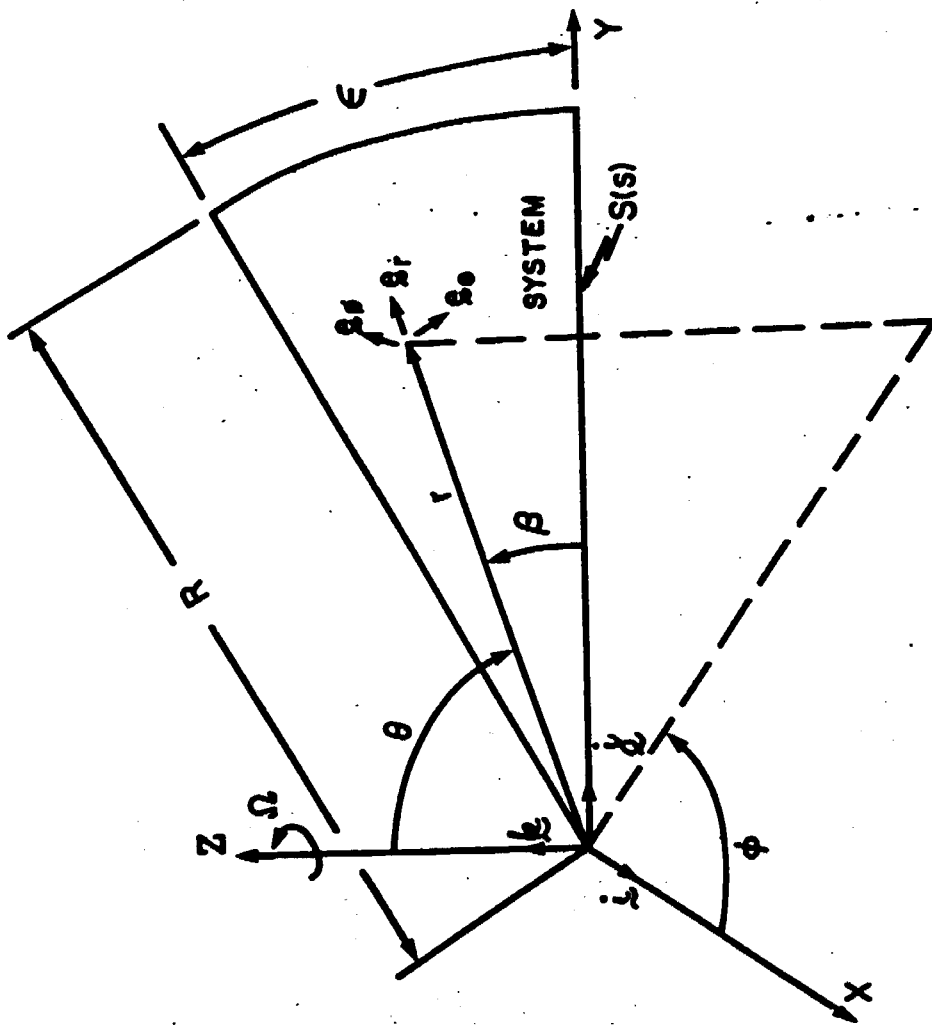


Figure 2. Coordinate orientation on the cone-and-plate viscometer.



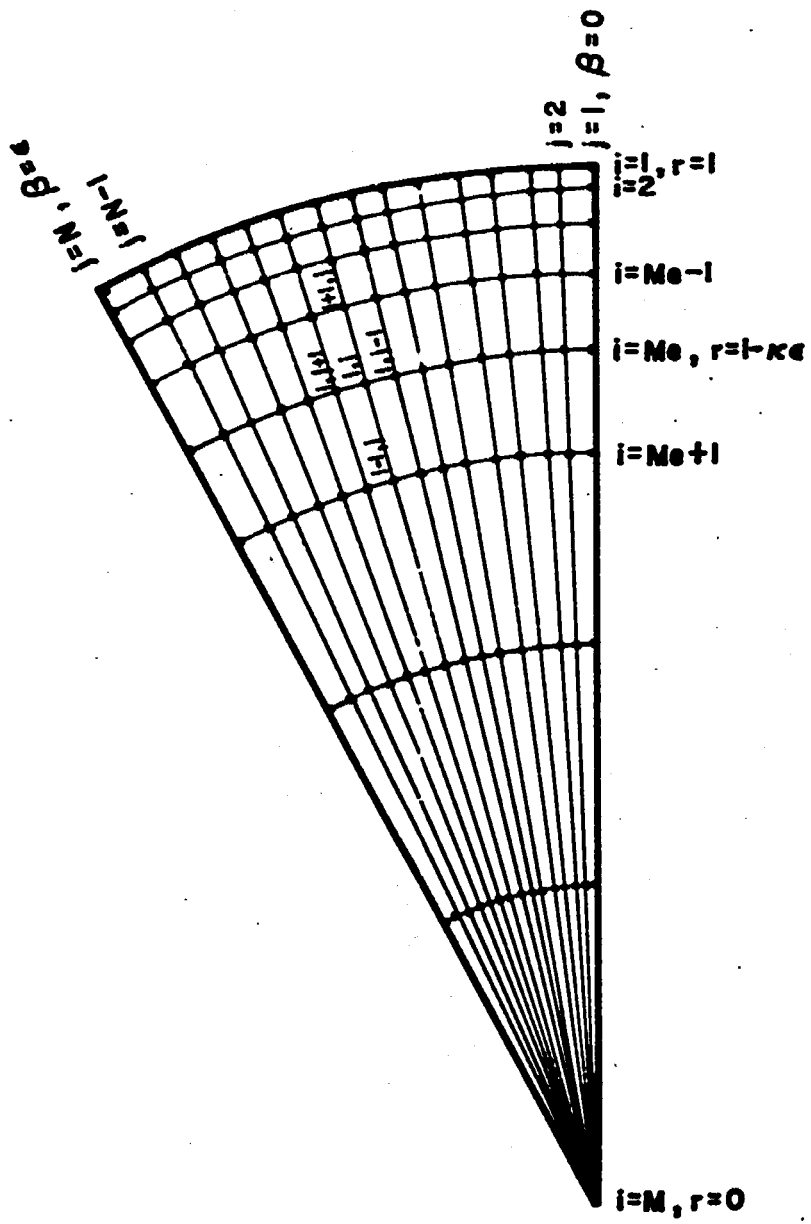


Figure 3. Grid orientation of the finite difference approximation.

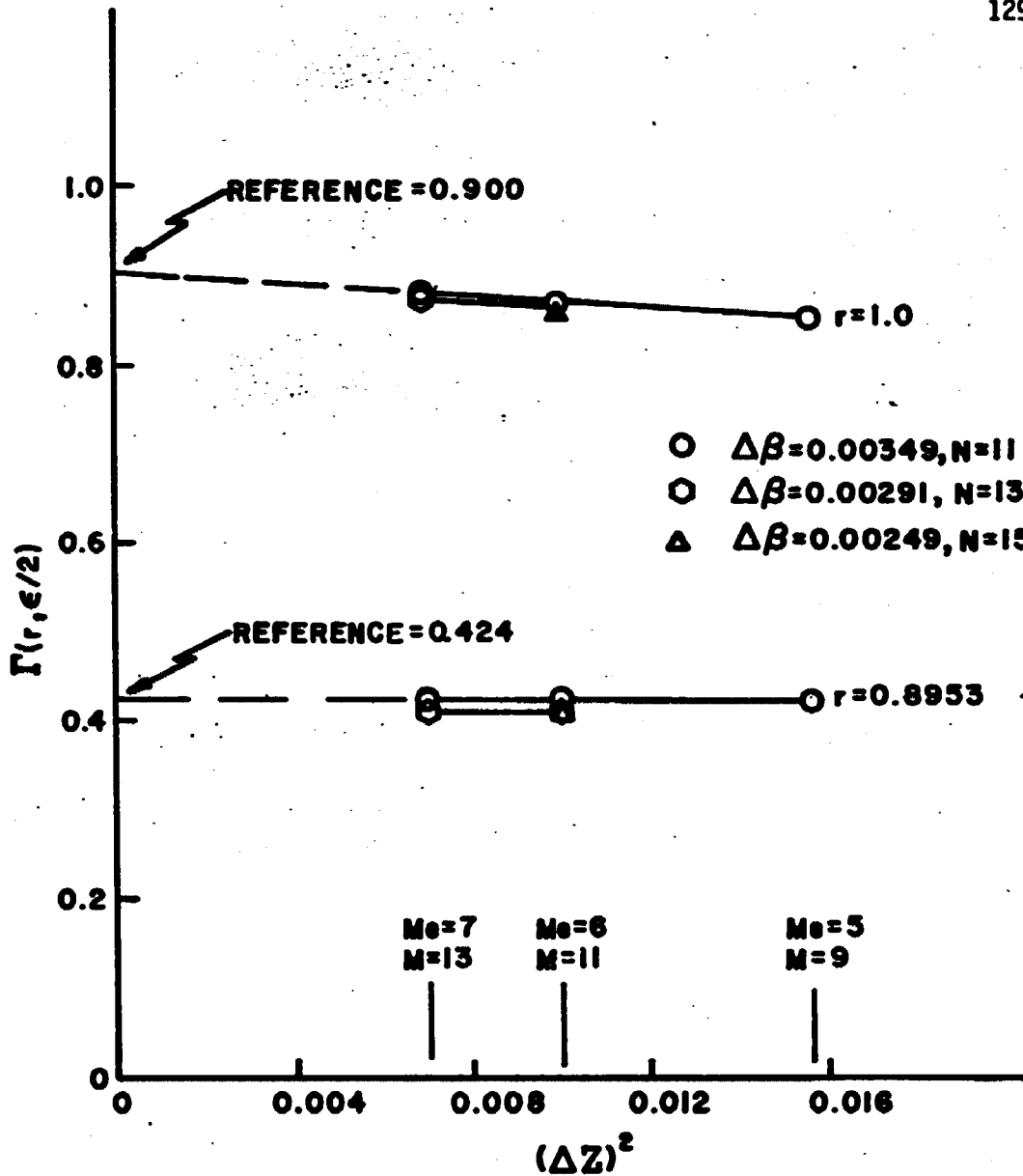


Figure 4. Sensitivity of  $\Gamma$  to the local truncation error:  
 $\epsilon=2$  degrees,  $Re=2 \times 10^4$   
 a.  $\Gamma(0.8953, \epsilon/2)$  and  $\Gamma(1.0, \epsilon/2)$  vs.  $(\Delta Z)^2$  for  
 $N=11, 13$  and  $15$ .

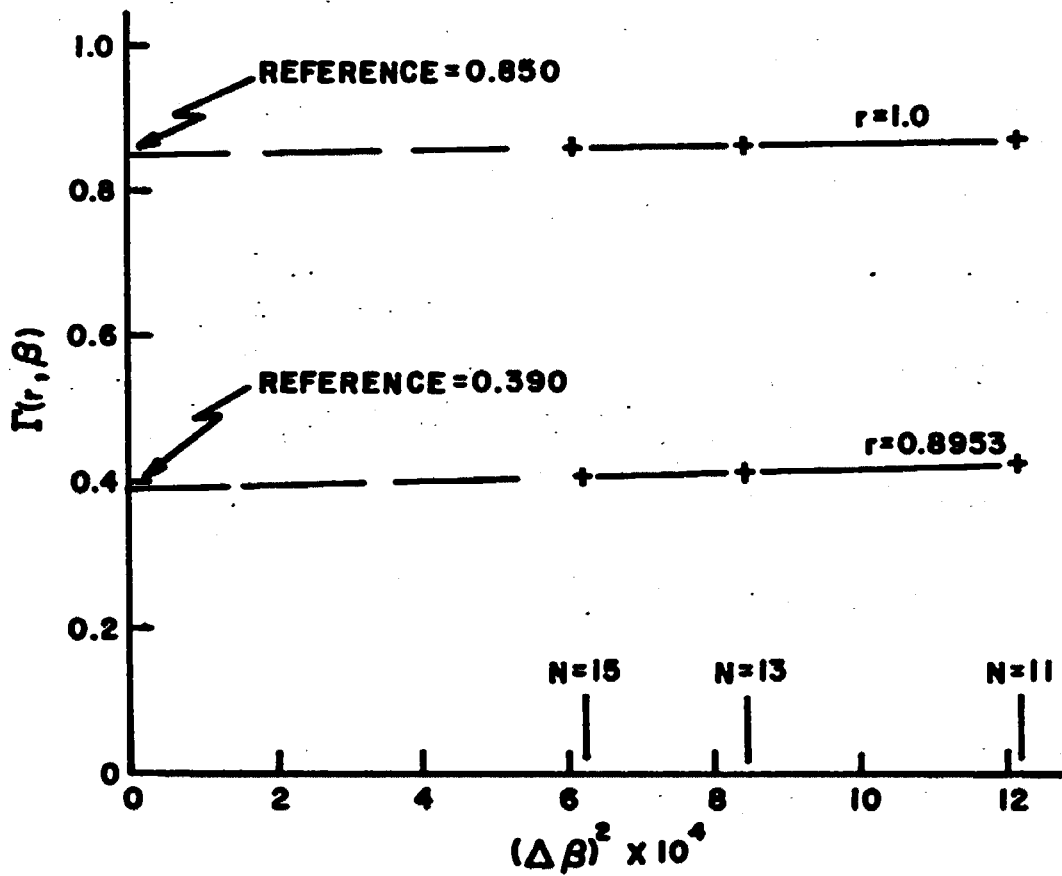


Figure 4.b.  $\Gamma(0.8953, \epsilon/2)$  and  $\Gamma(1.0, \epsilon/2)$  vs.  $(\Delta\beta)^2$   
for  $M=11$ ,  $M_e=6$ .

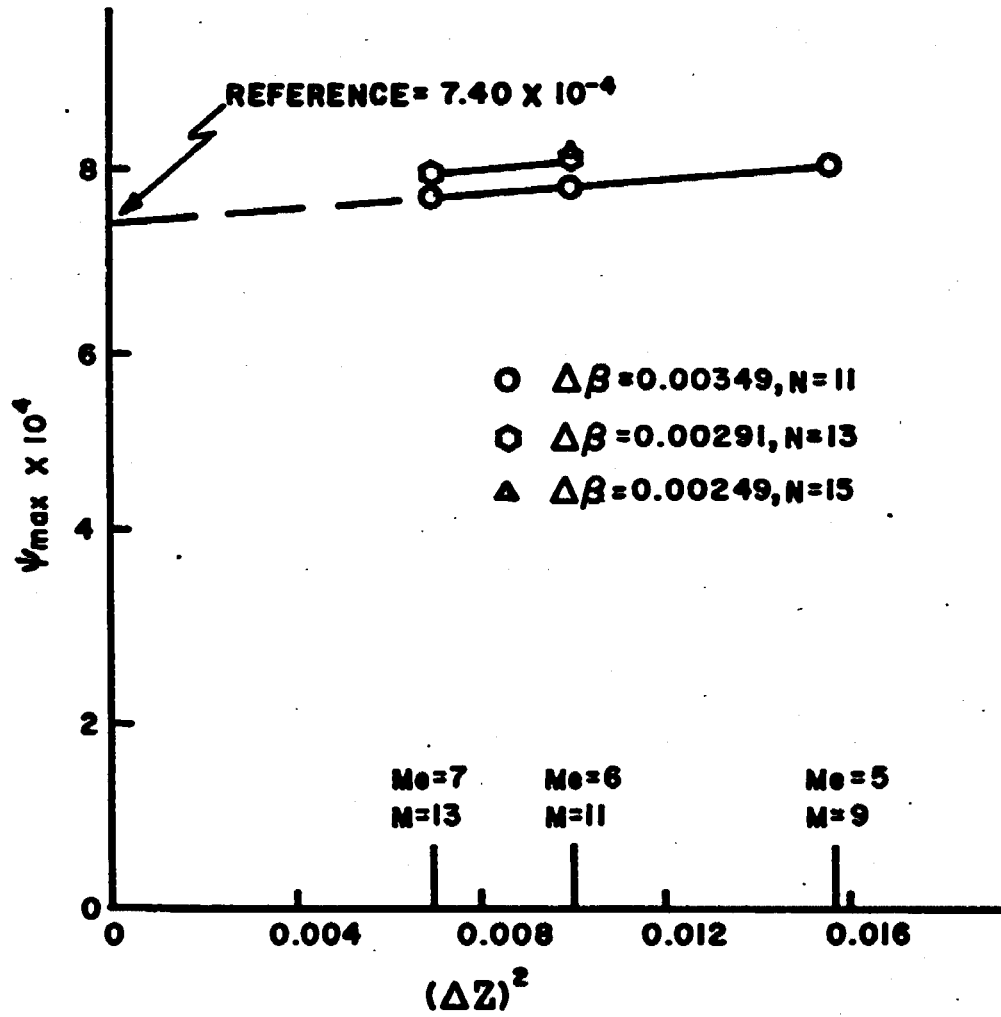


Figure 5. Sensitivity of  $\psi_{max}$  to the local truncation error:  $\epsilon = 2$  degrees,  $Re = 2 \times 10^4$   
 a.  $\psi_{max}$  vs.  $(\Delta Z)^2$  for  $N = 11, 13$  and  $15$ .

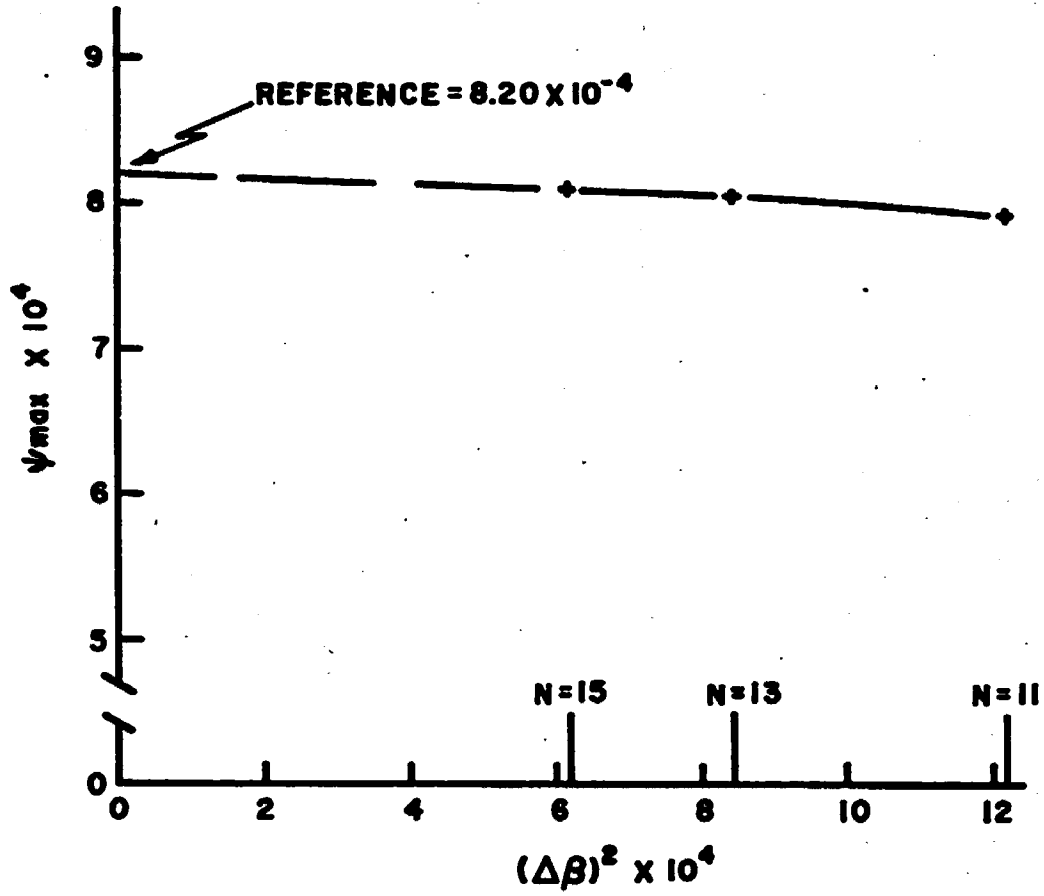


Figure 5.b.  $\psi_{\max}$  vs.  $(\Delta\beta)^2$  for  $M=11$ ,  $M_0=6$ .

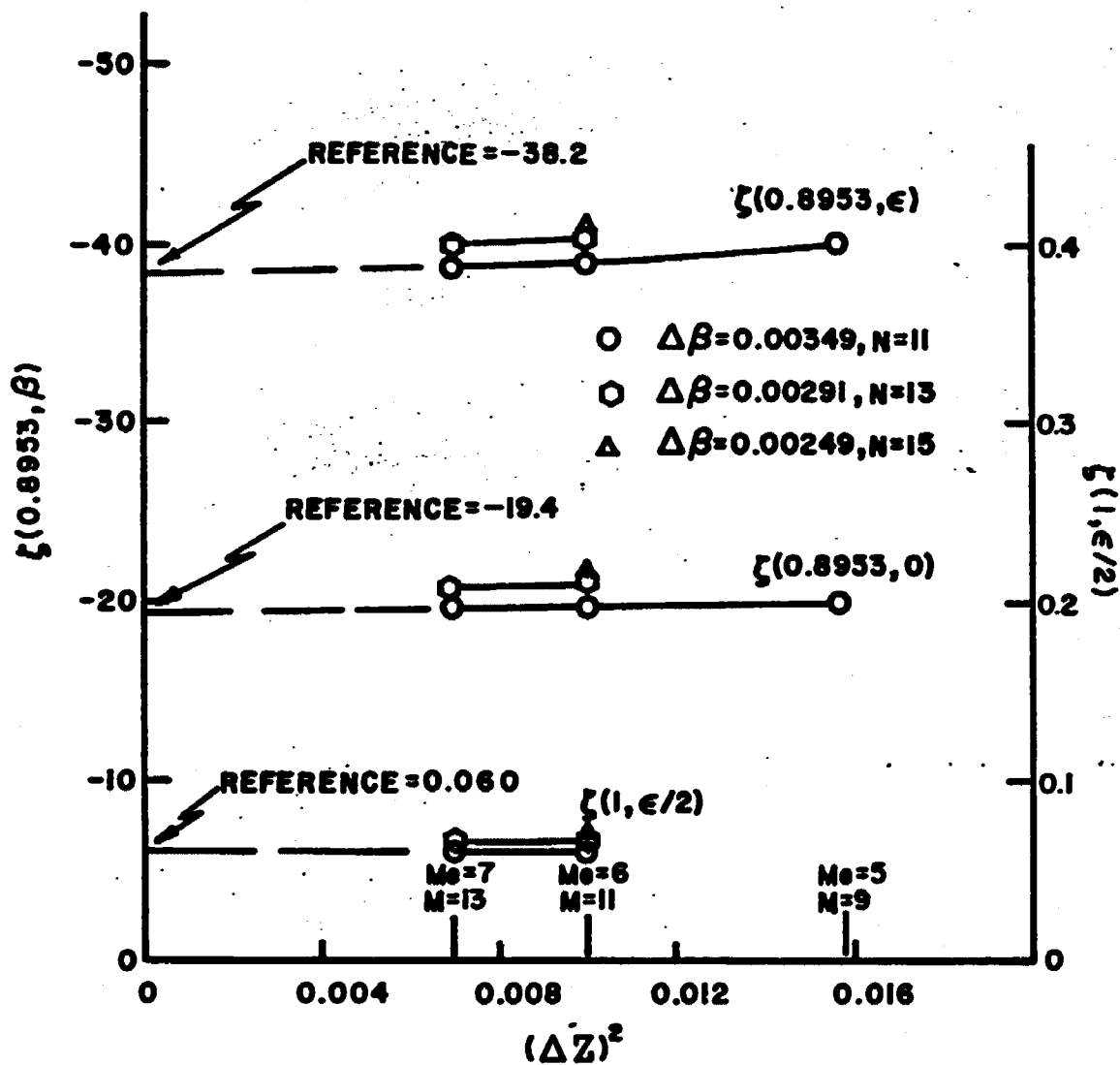


Figure 6. Sensitivity of  $\zeta$  to the local truncation error:

$$\epsilon = 2 \text{ degrees}, Re = 2 \times 10^4$$

- a.  $\zeta(0.8953, 0)$ ,  $\zeta(0.8953, \epsilon)$ , and  $\zeta(1.0, \epsilon/2)$  vs.  $(\Delta Z)^2$  for  $N=11, 13$  and  $15$ .

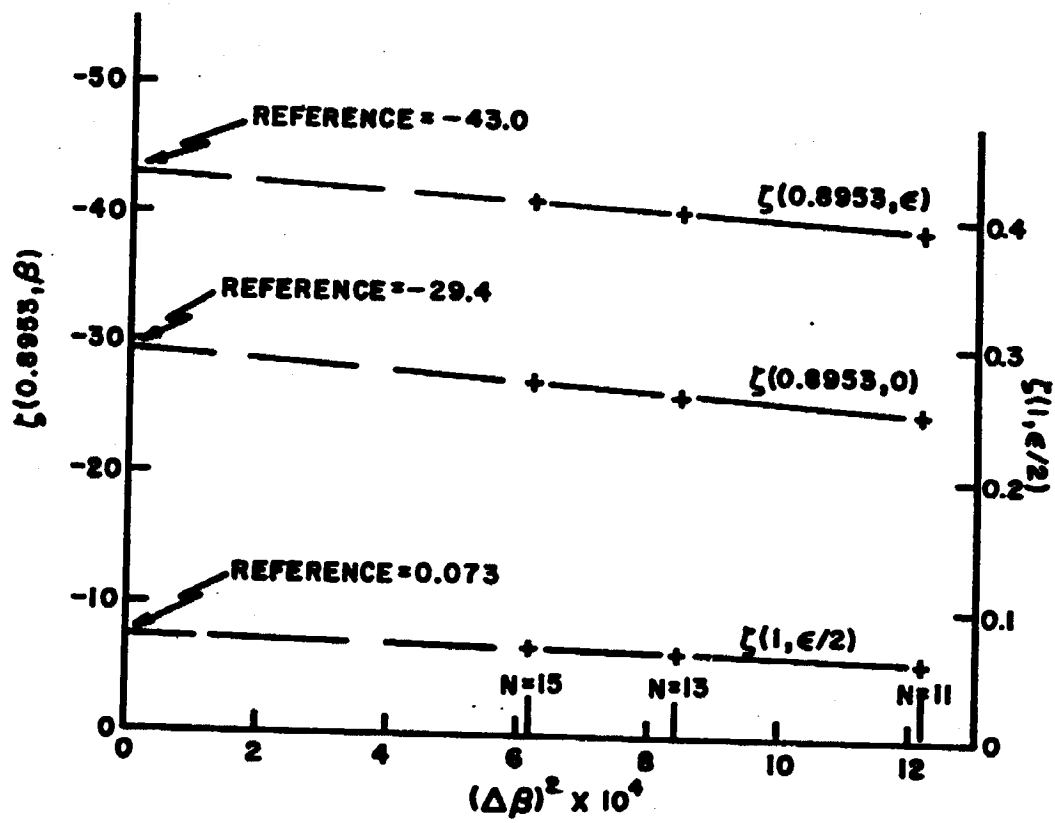


Figure 6.b.  $\zeta(0.8953, 0)$ ,  $\zeta(0.8953, \epsilon)$  and  $\zeta(1.0, \epsilon/2)$  vs.  $(\Delta\beta)^2$  for  $M=11$ ,  $M_0=6$ .

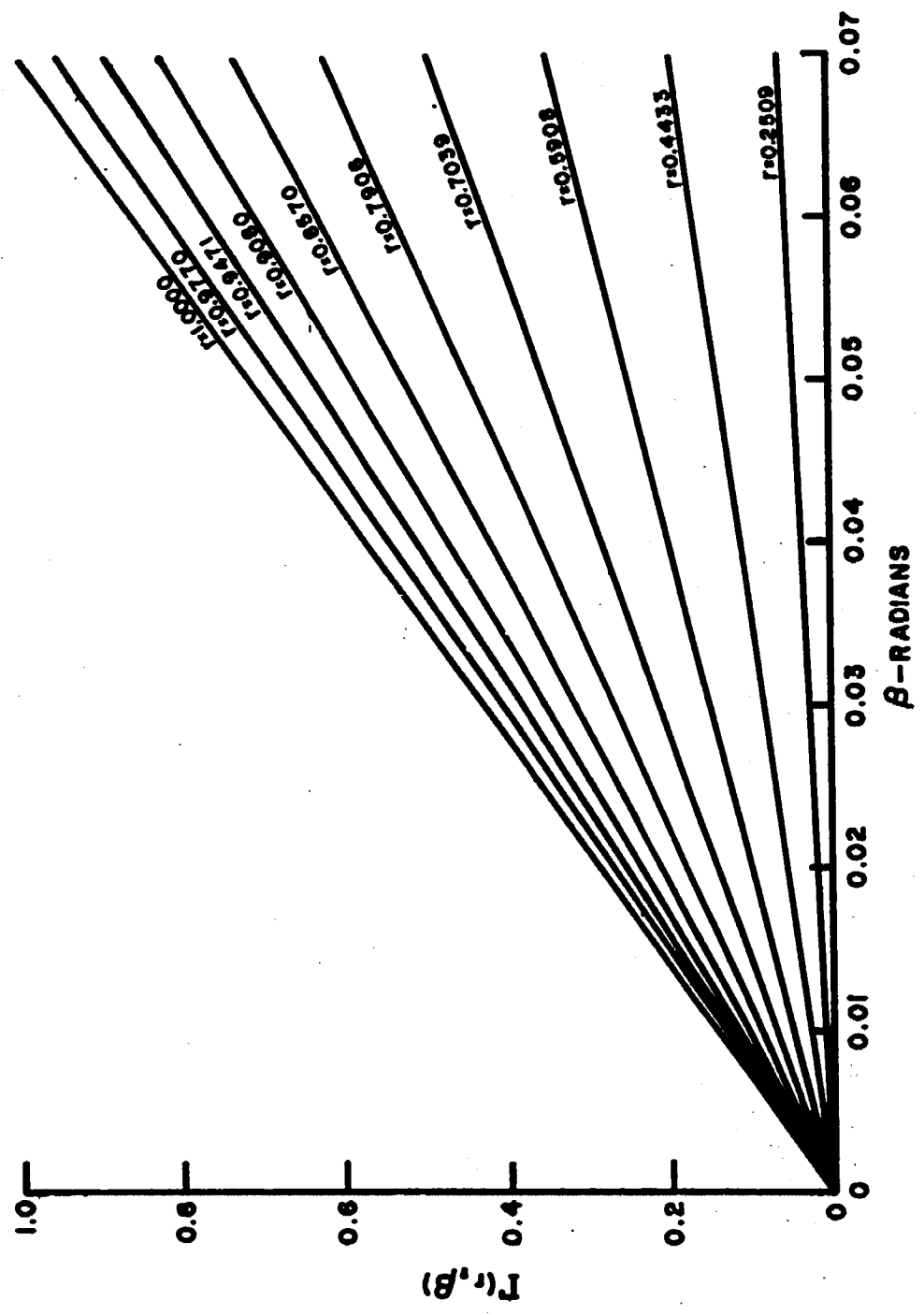


Figure 7. Meridional velocity distributions:  $\epsilon = 4^\circ$ ,  $Re = 4 \times 10^4$ .



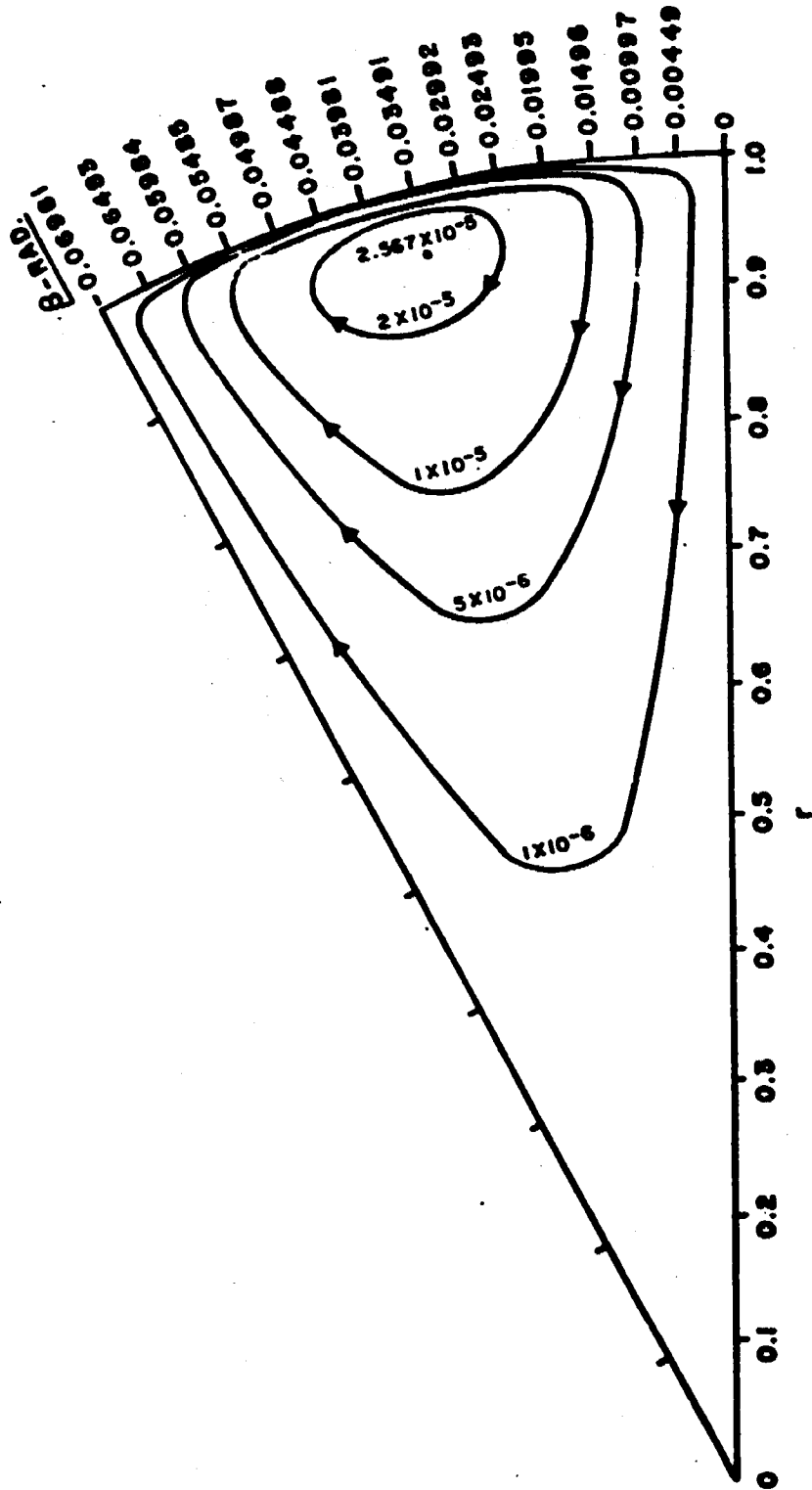


Figure 8. Streamlines of the secondary flow:  $\epsilon = 4^\circ$ ,  $Re = 4 \times 10^4$ .

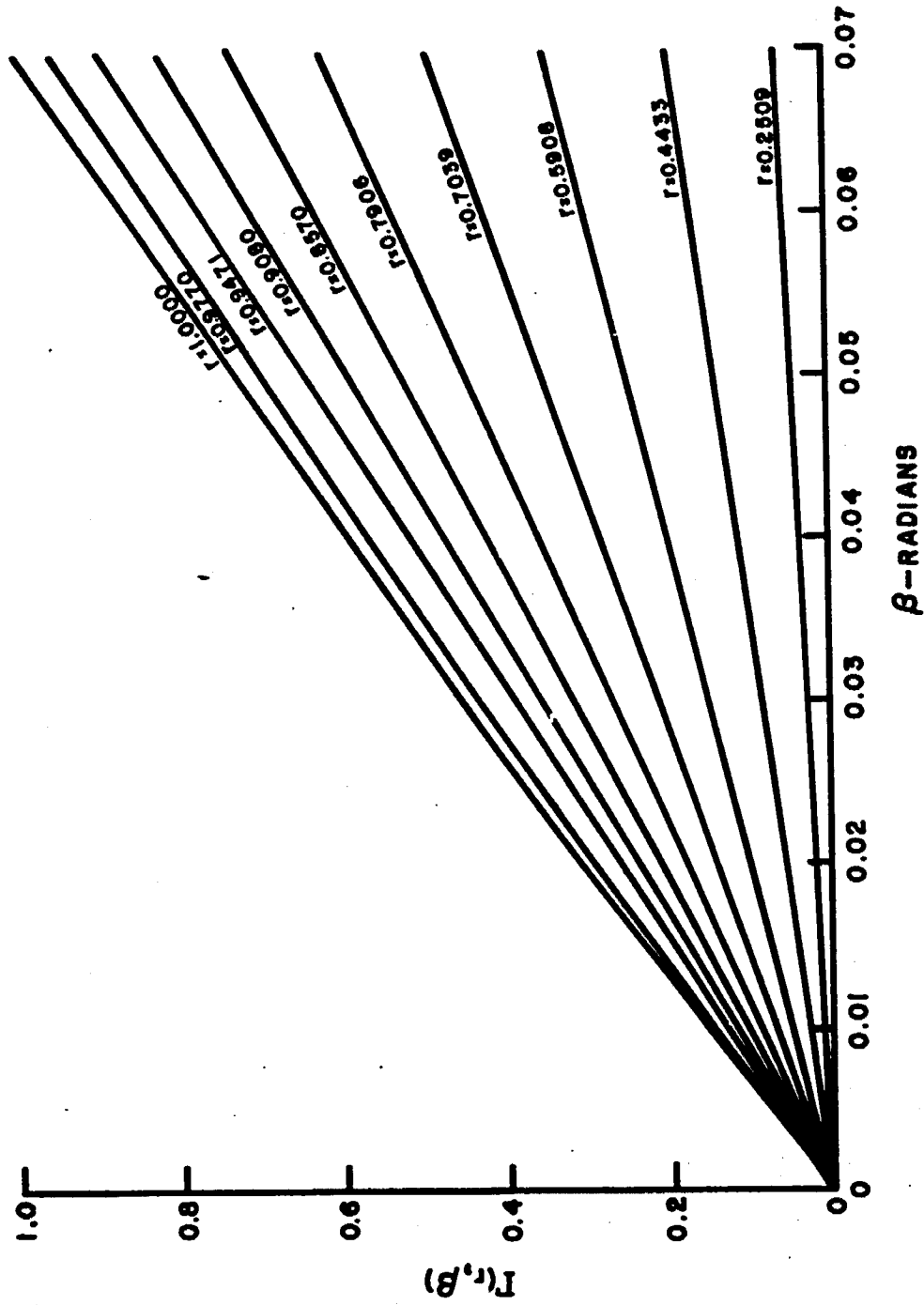


Figure 9. Meridian velocity distribution:  $\epsilon = 4^\circ$ ,  $Re = 8 \times 10^2$ .

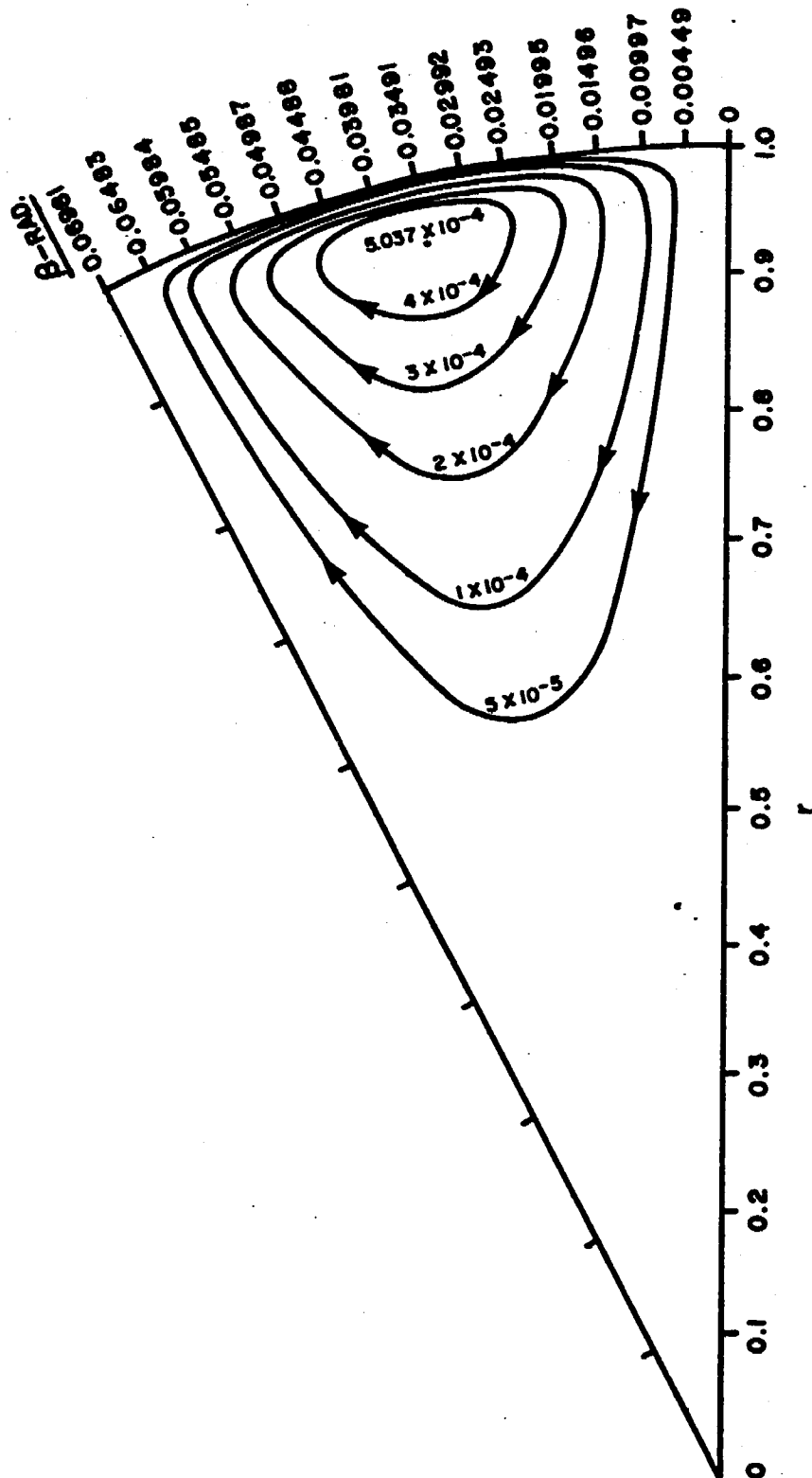


Figure 10. Streamlines of the secondary flow:  $\epsilon = 4^\circ$ ,  $Re = 8 \times 10^2$ .

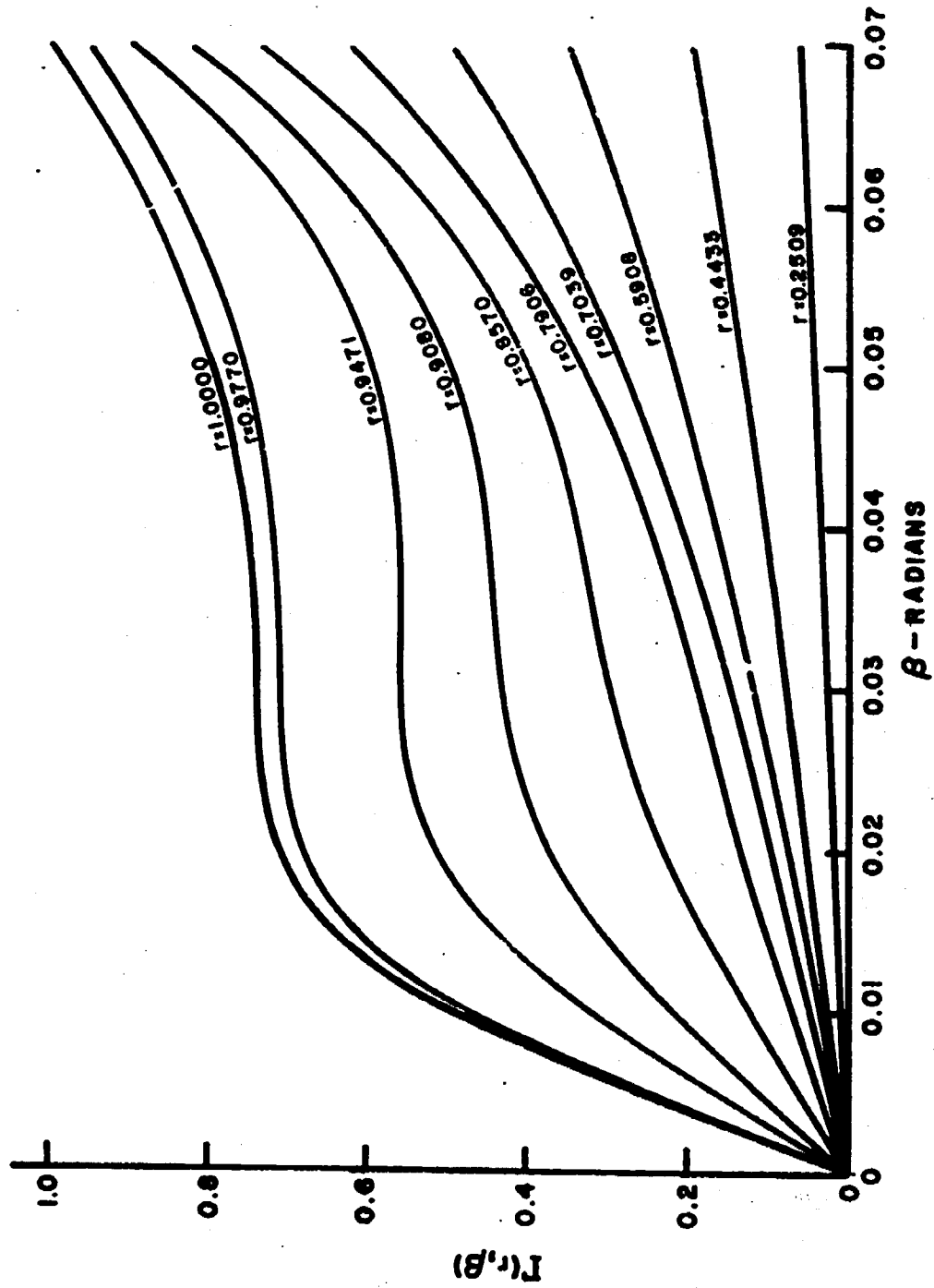


Figure 11. Meridional velocity distribution:  $\epsilon = 4^\circ$ ,  $Re = 6 \times 10^3$ .

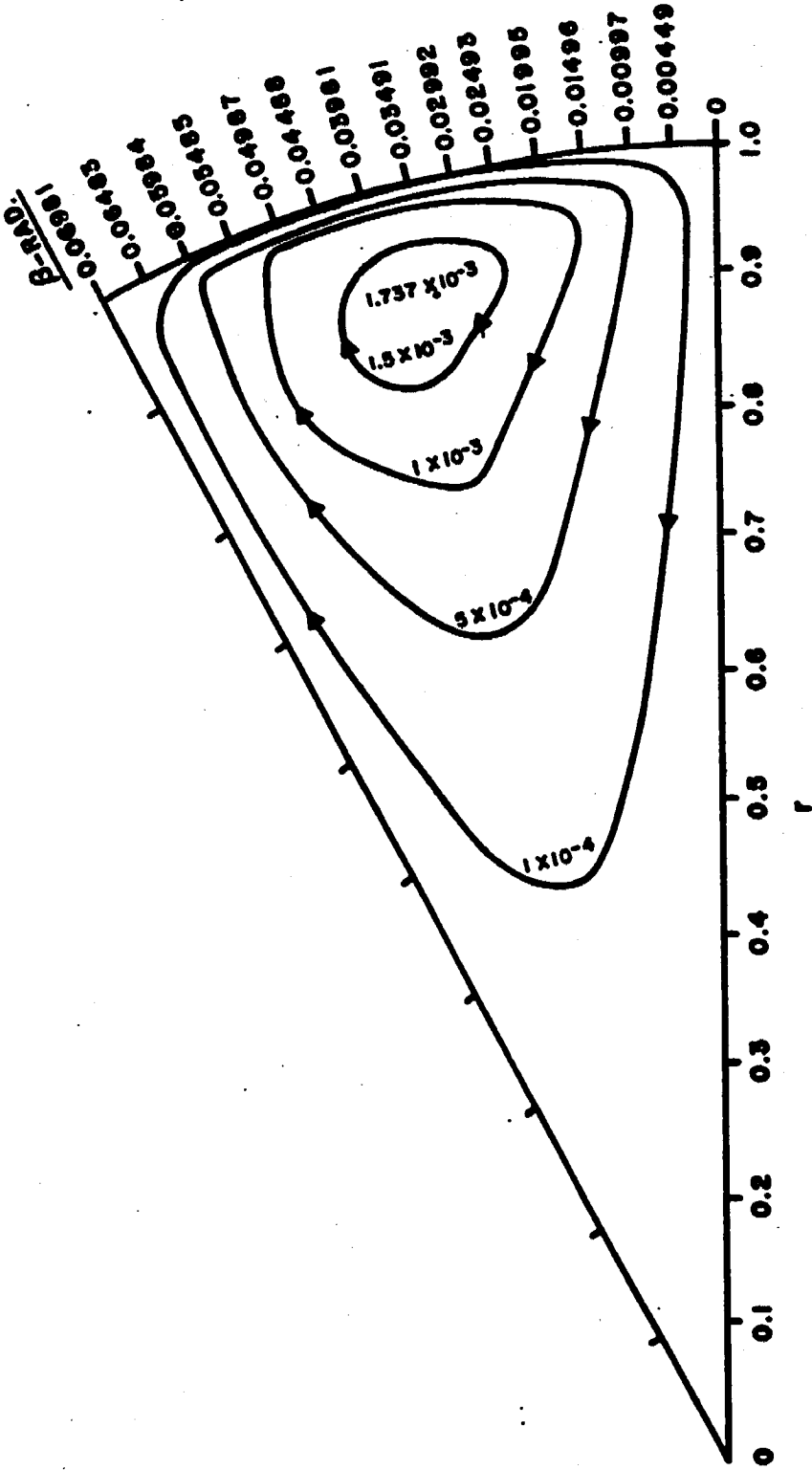


Figure 12. Streamlines of the secondary flow:  $\epsilon = 4^\circ$ ,  $Re = 6 \times 10^3$ .

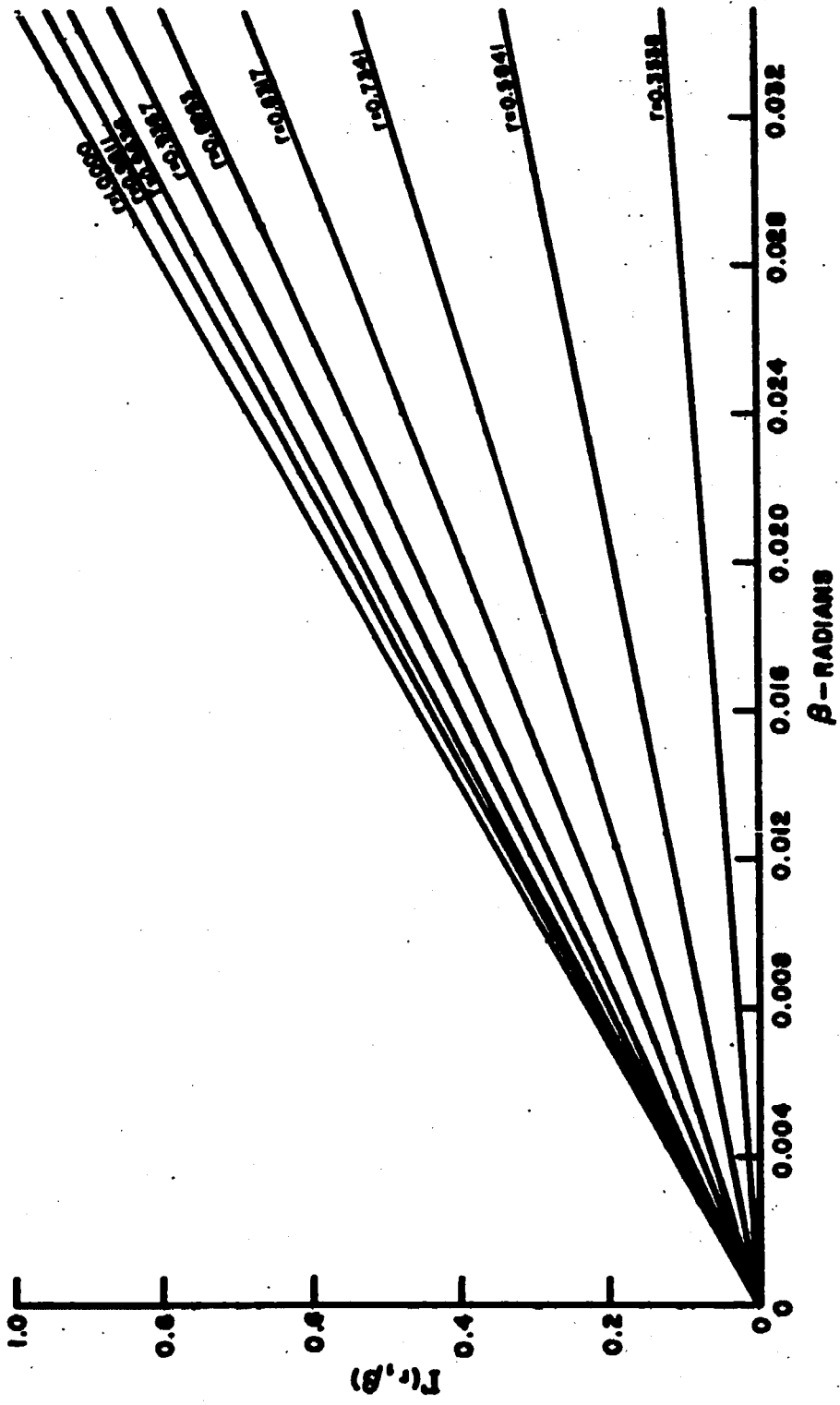


Figure 13. Meridion velocity distribution:  $\epsilon = 2^\circ$ ,  $Re = 1 \times 10^5$ .

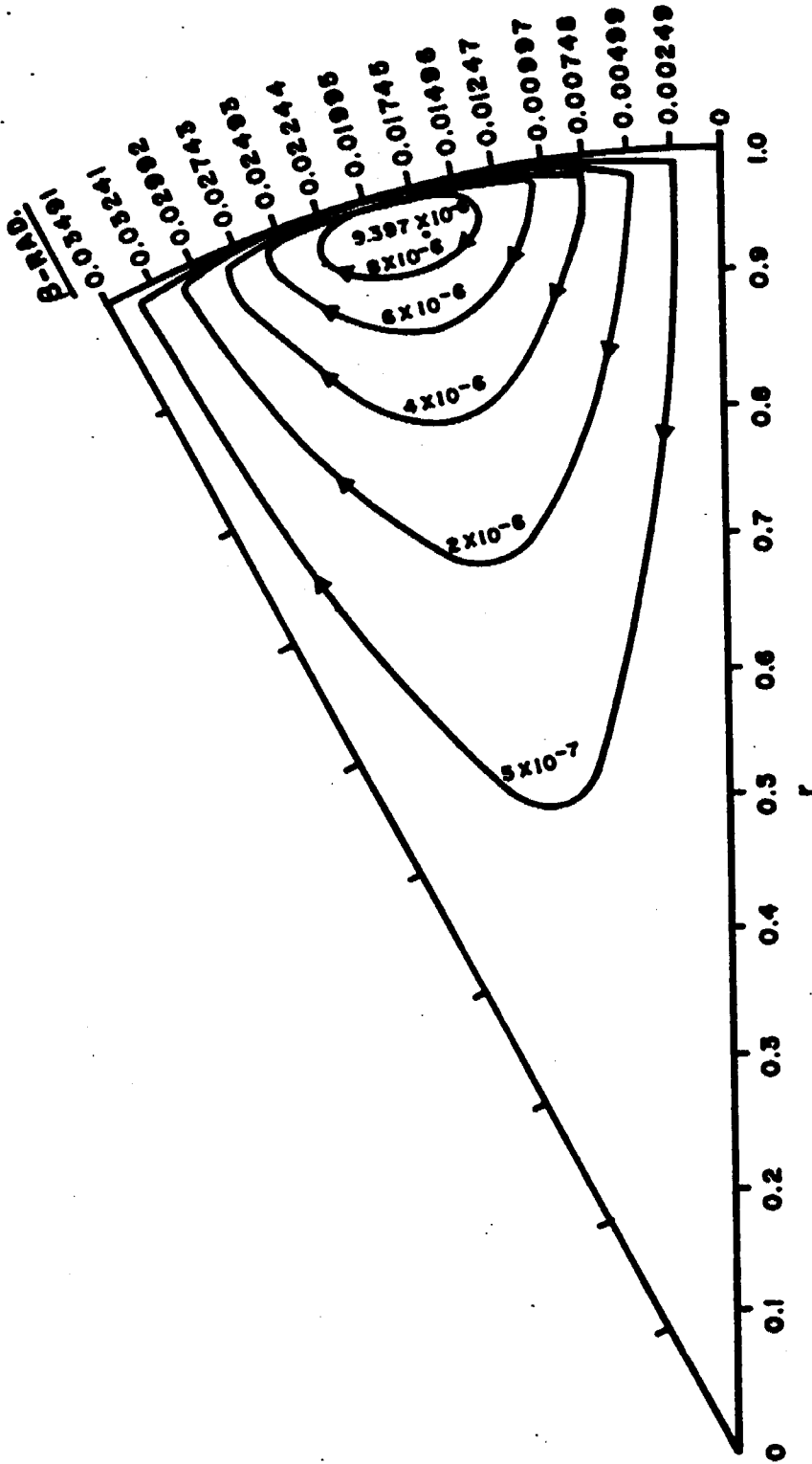


Figure 14. Streamlines of the secondary flow:  $\epsilon = 2^\circ$ ,  $Re = 1 \times 10^2$ .

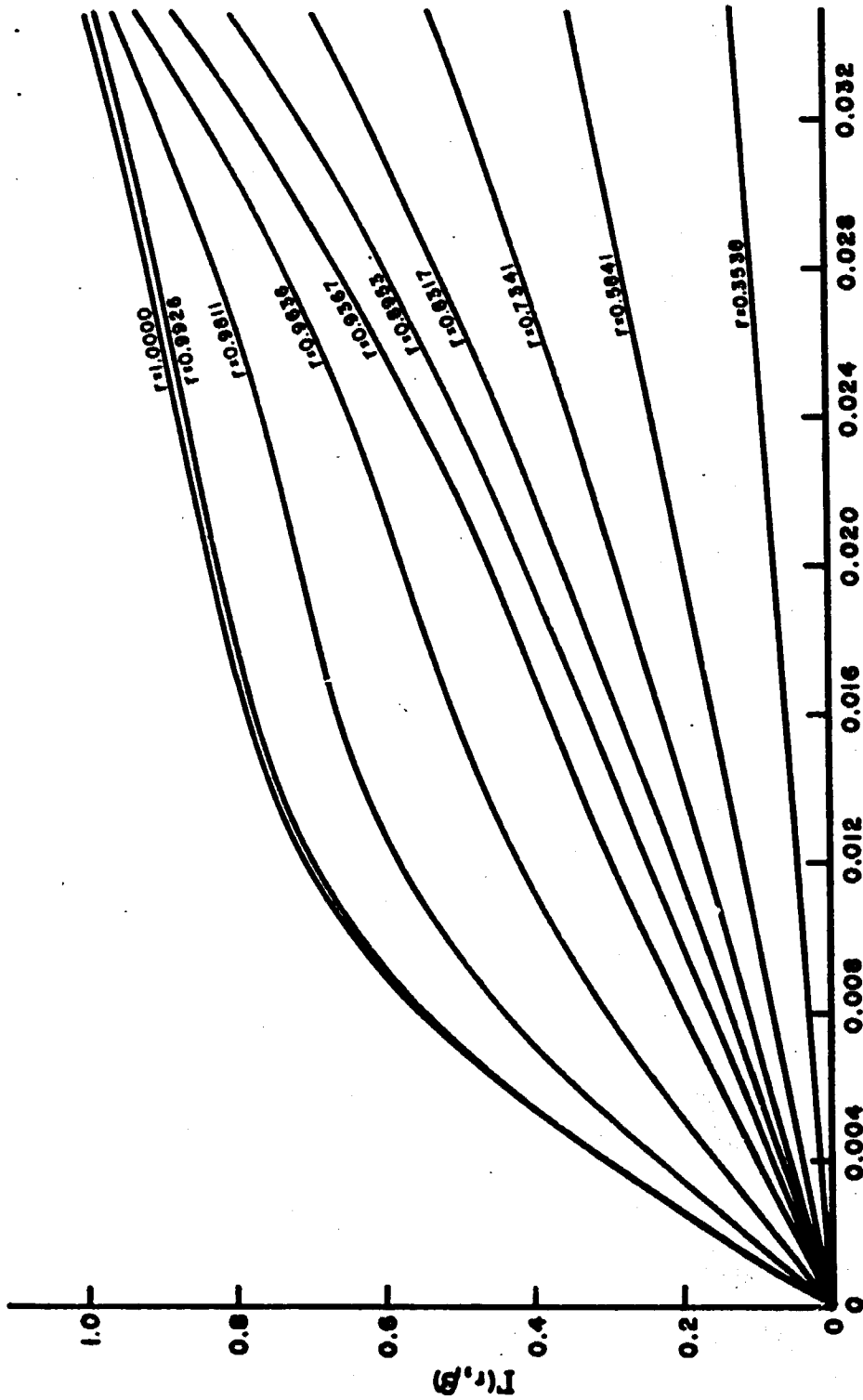


Figure 15. Meridion velocity distribution:  $\epsilon=2^\circ$ ,  $Re=8 \times 10^3$ .



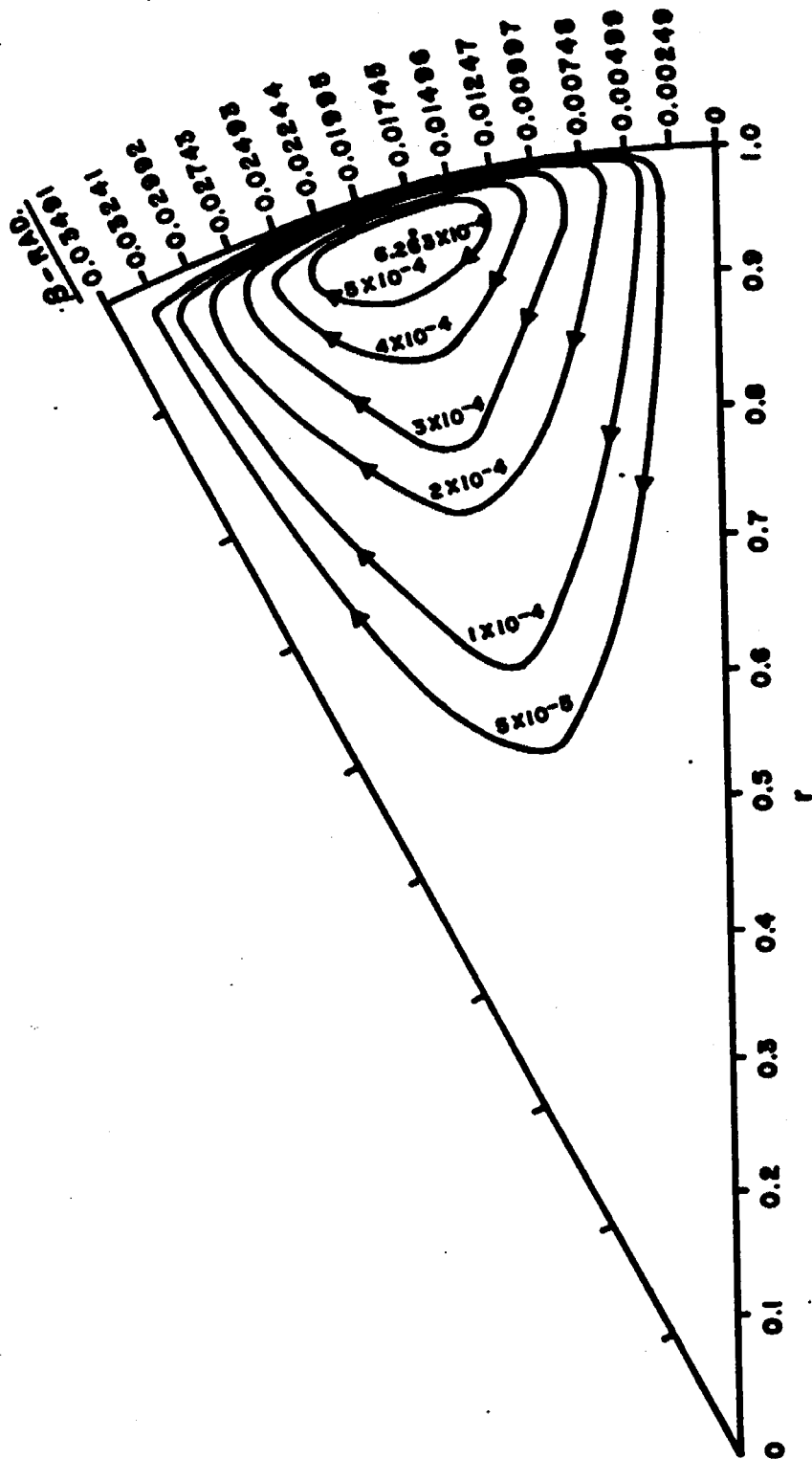


Figure 16. Streamlines of the secondary flow:  $\epsilon = 2^\circ$ ,  $Re = 8 \times 10^3$ .

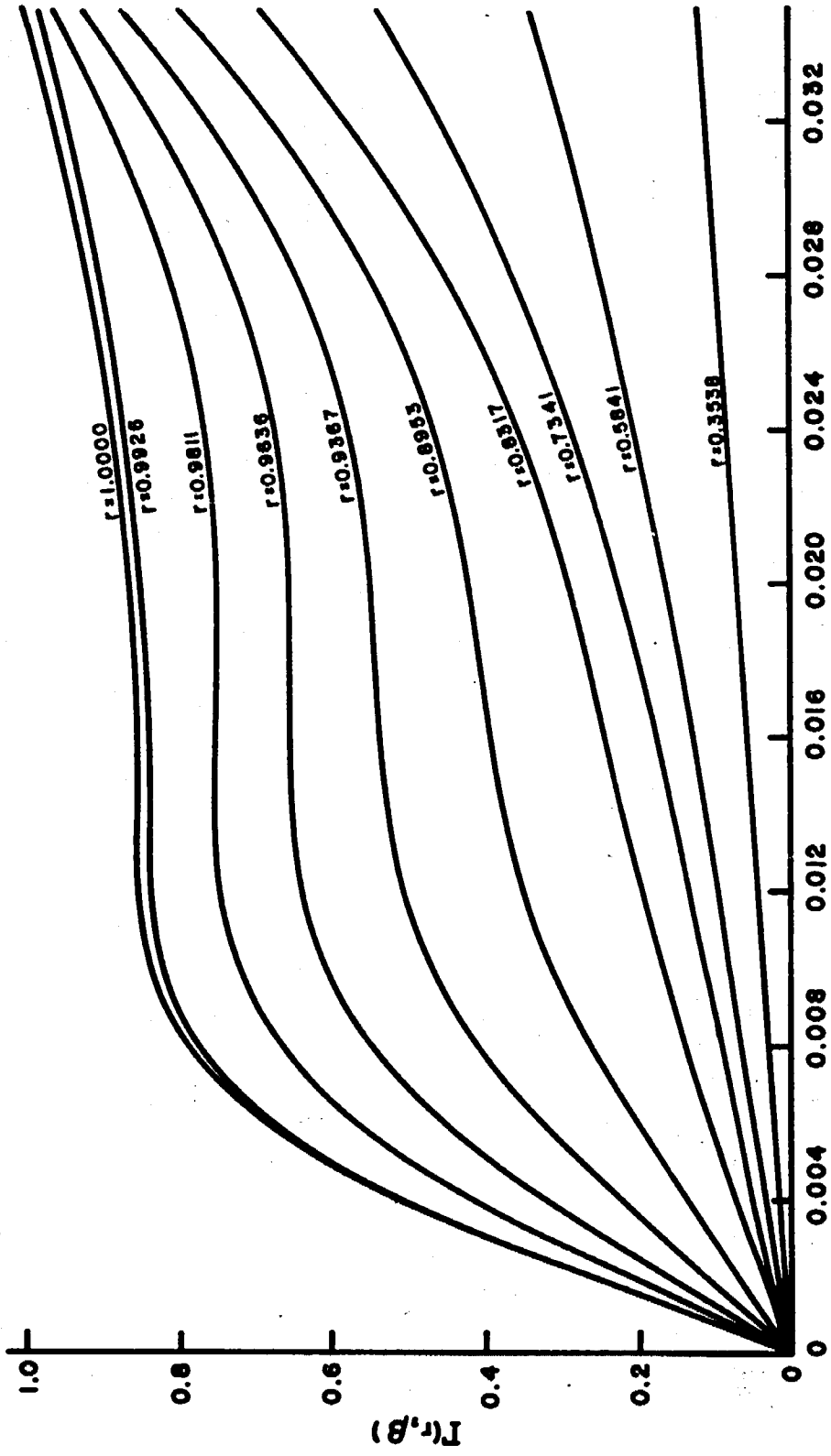


Figure 17. Meridian velocity distribution:  $\epsilon = 2^\circ$ ,  $Re = 2 \times 10^4$ .

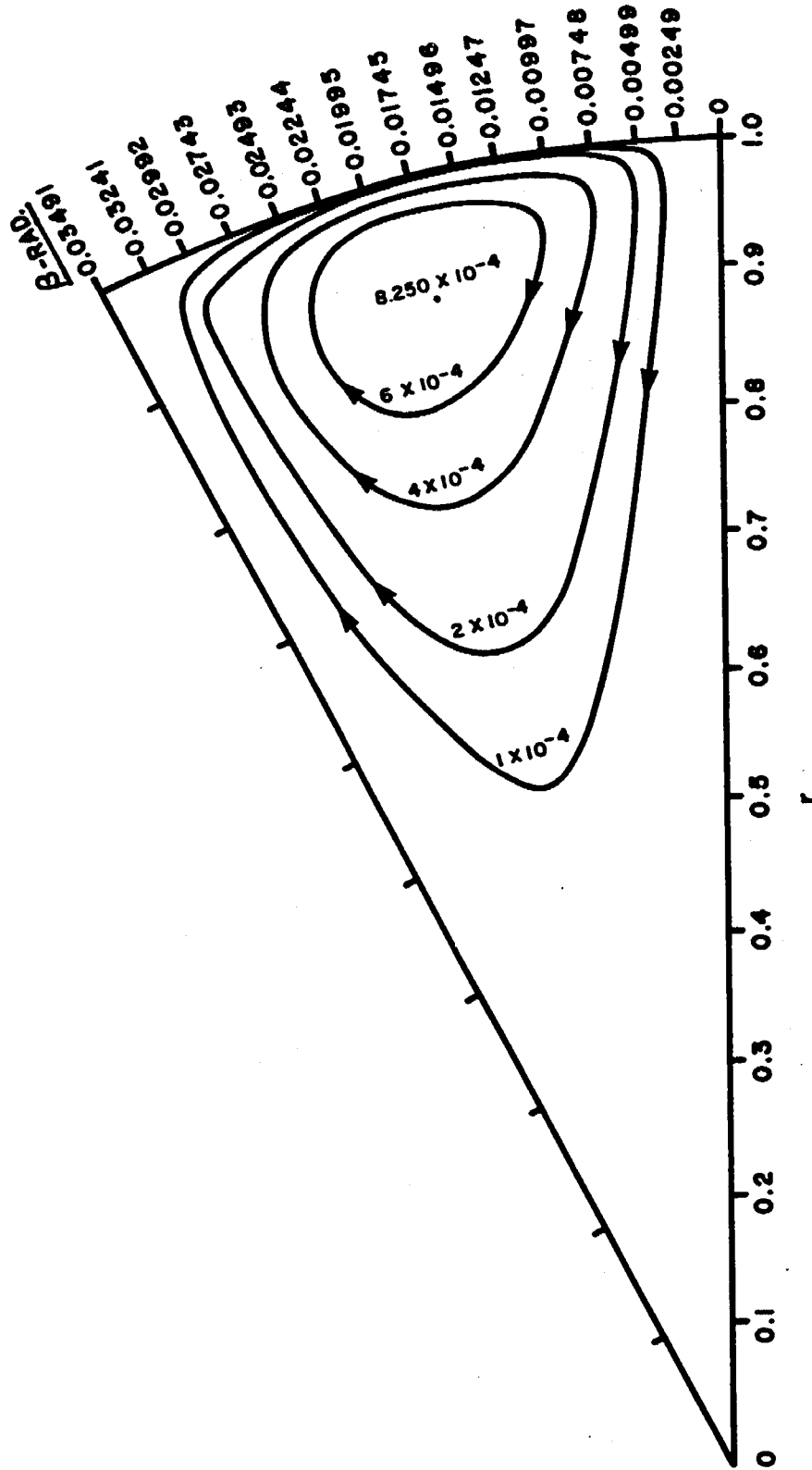


Figure 18. Streamlines of the secondary flow:  $\epsilon = 2^\circ$ ,  $Re = 2 \times 10^4$ .

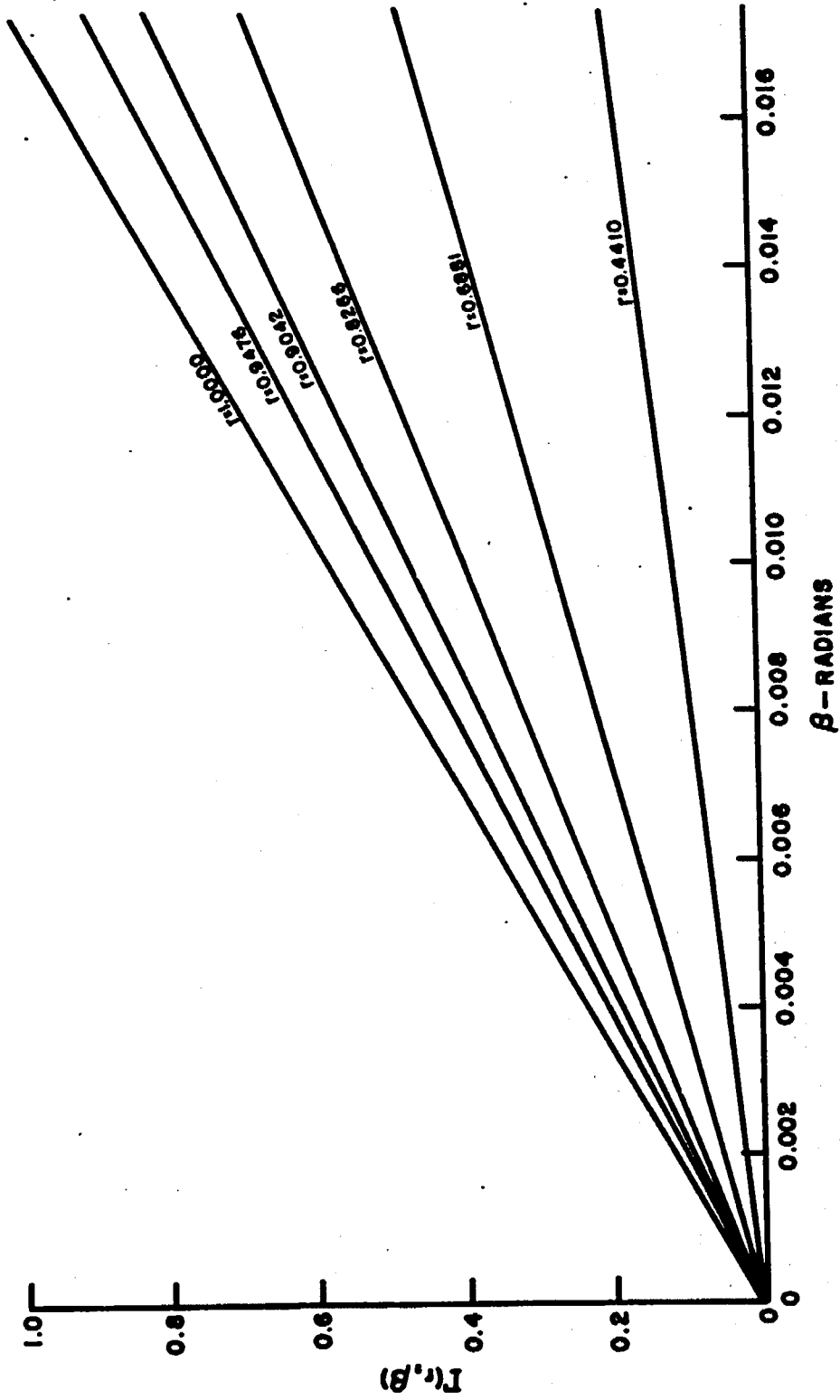


Figure 19. Meridional velocity distribution:  $\epsilon=1^\circ$ ,  $Re=4 \times 10^2$ .

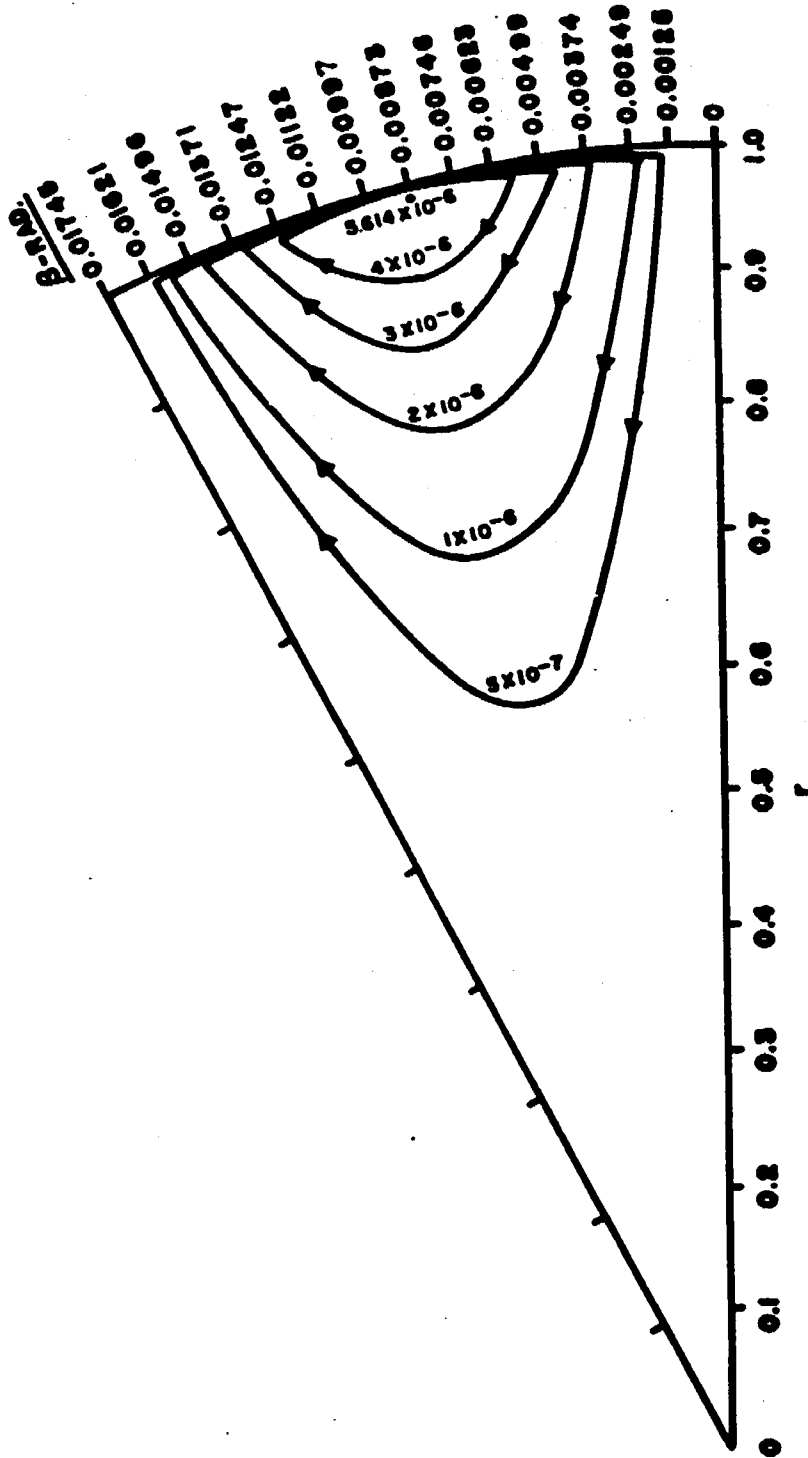


Figure 20. Streamlines of the secondary flow:  $\epsilon=1^\circ$ ,  $Re=4 \times 10^5$ .

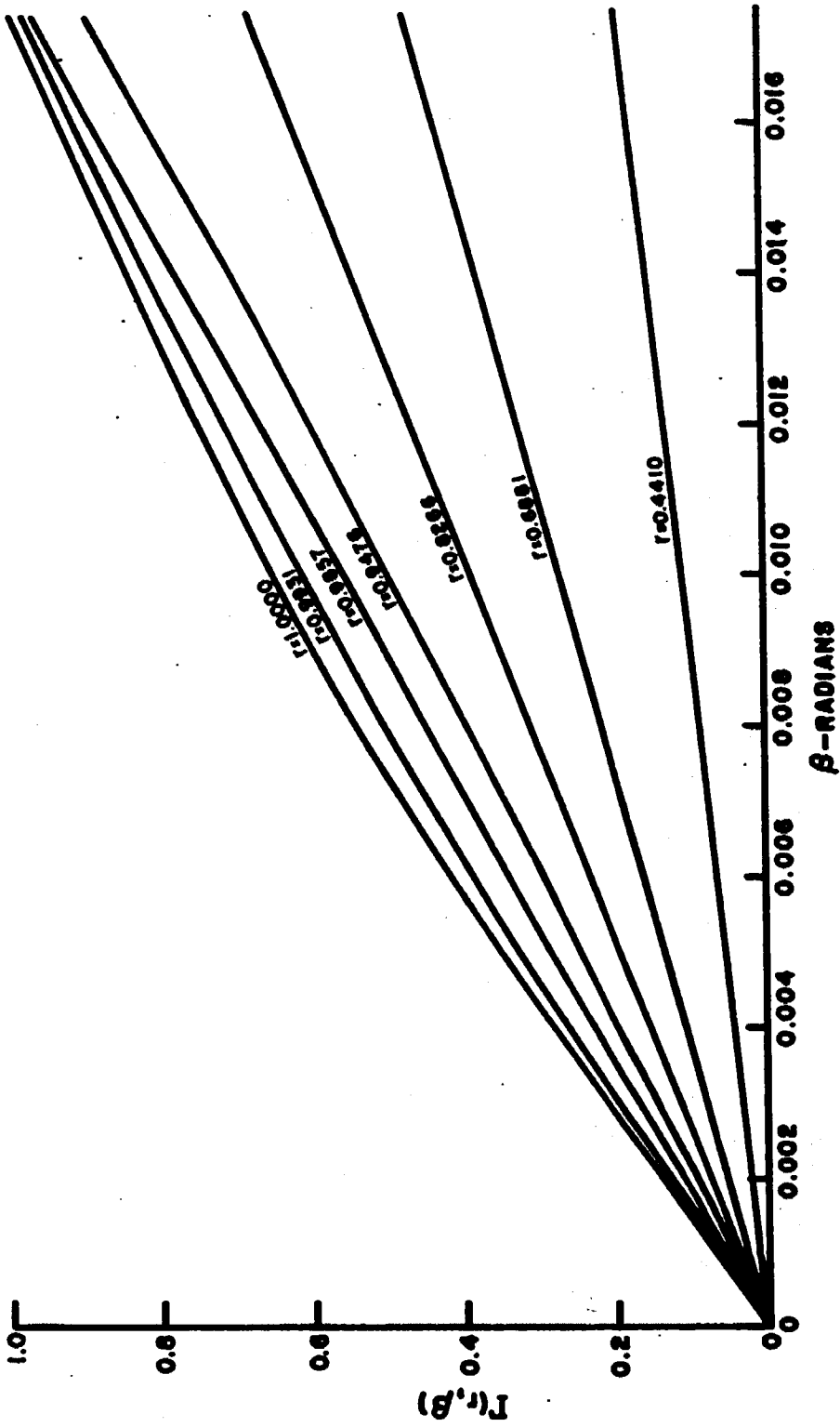


Figure 21. Meridion velocity distribution:  $\epsilon=1^\circ$ ,  $Re=8 \times 10^3$ .

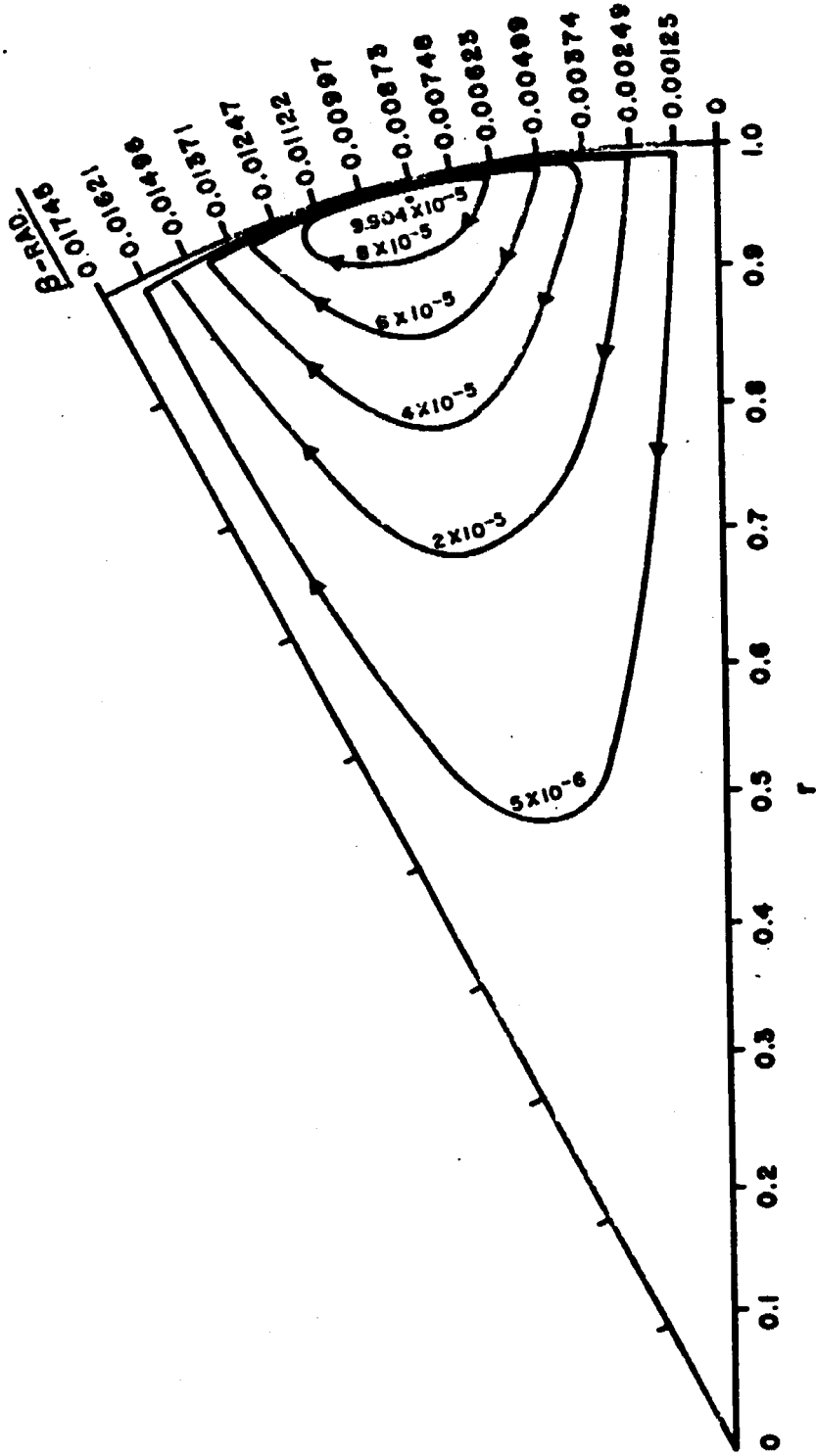


Figure 22. Streamlines of the secondary flow:  $\epsilon=1^\circ$ ,  $Re=8 \times 10^3$ .

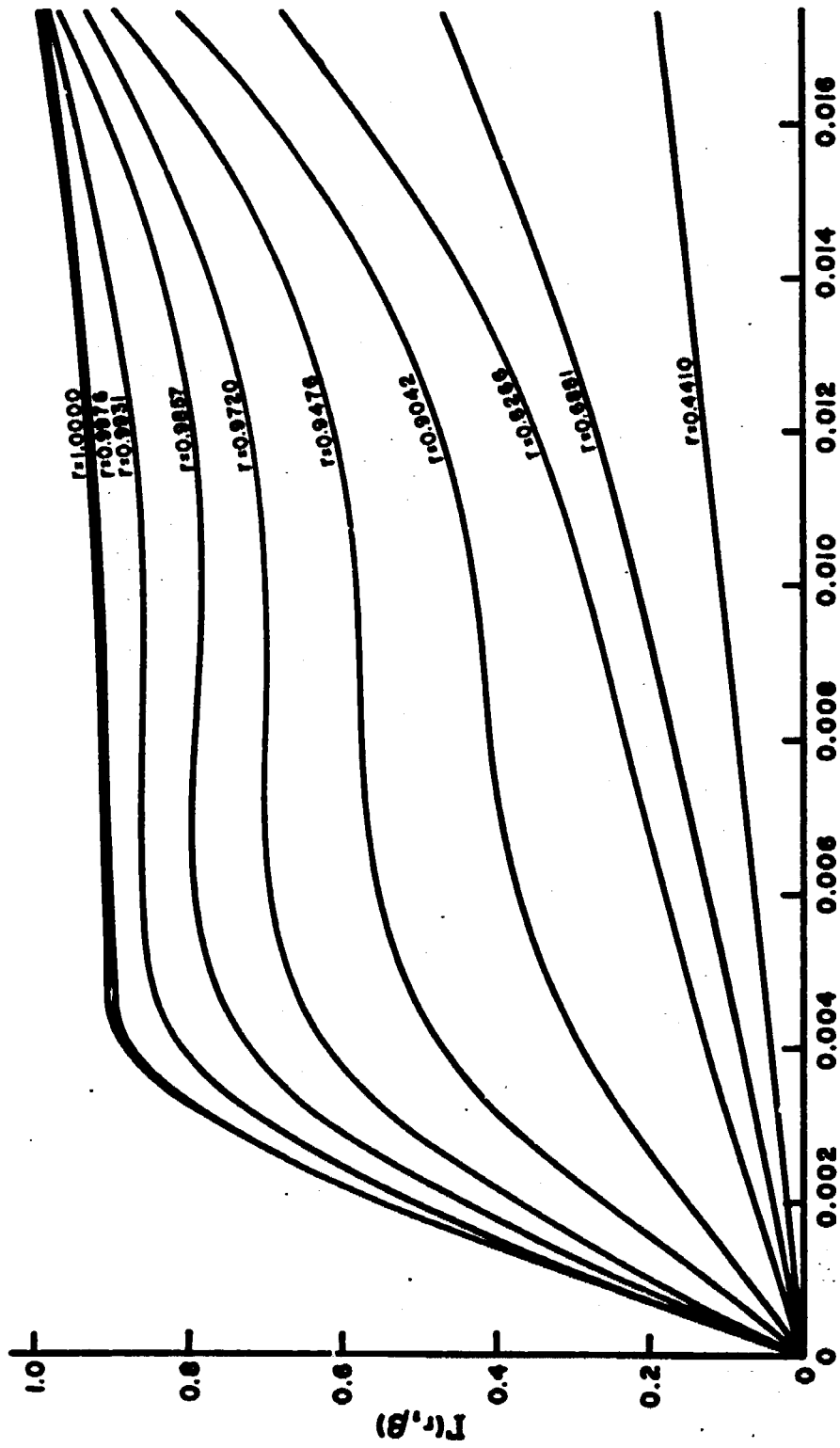


Figure 23. Meridian velocity distribution:  $\epsilon = 1^\circ$ ,  $Re = 8 \times 10^4$ .



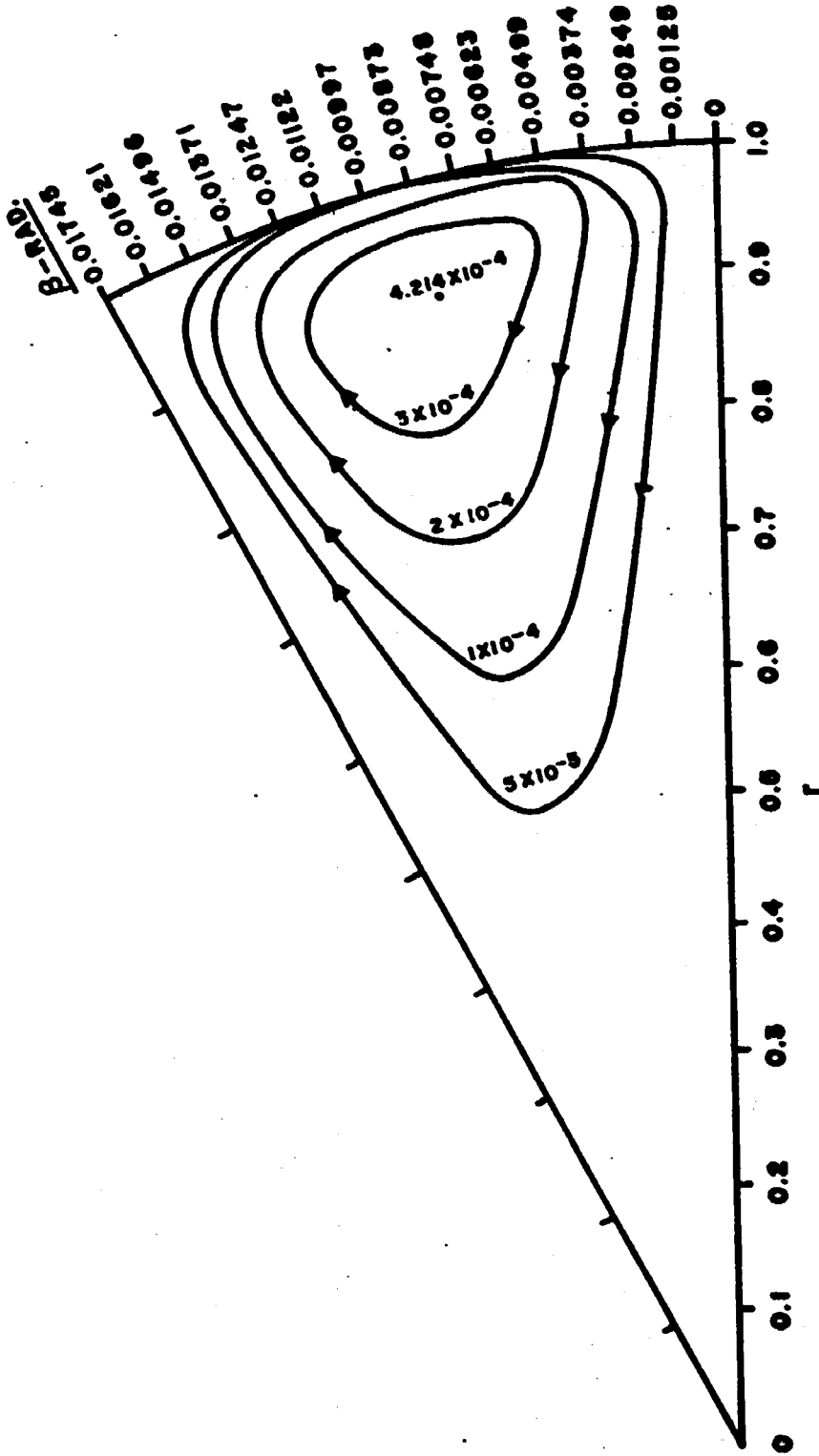


Figure 24. Streamlines of the secondary flow:  $\alpha = 1^\circ$ ,  $Re = 8 \times 10^4$ .

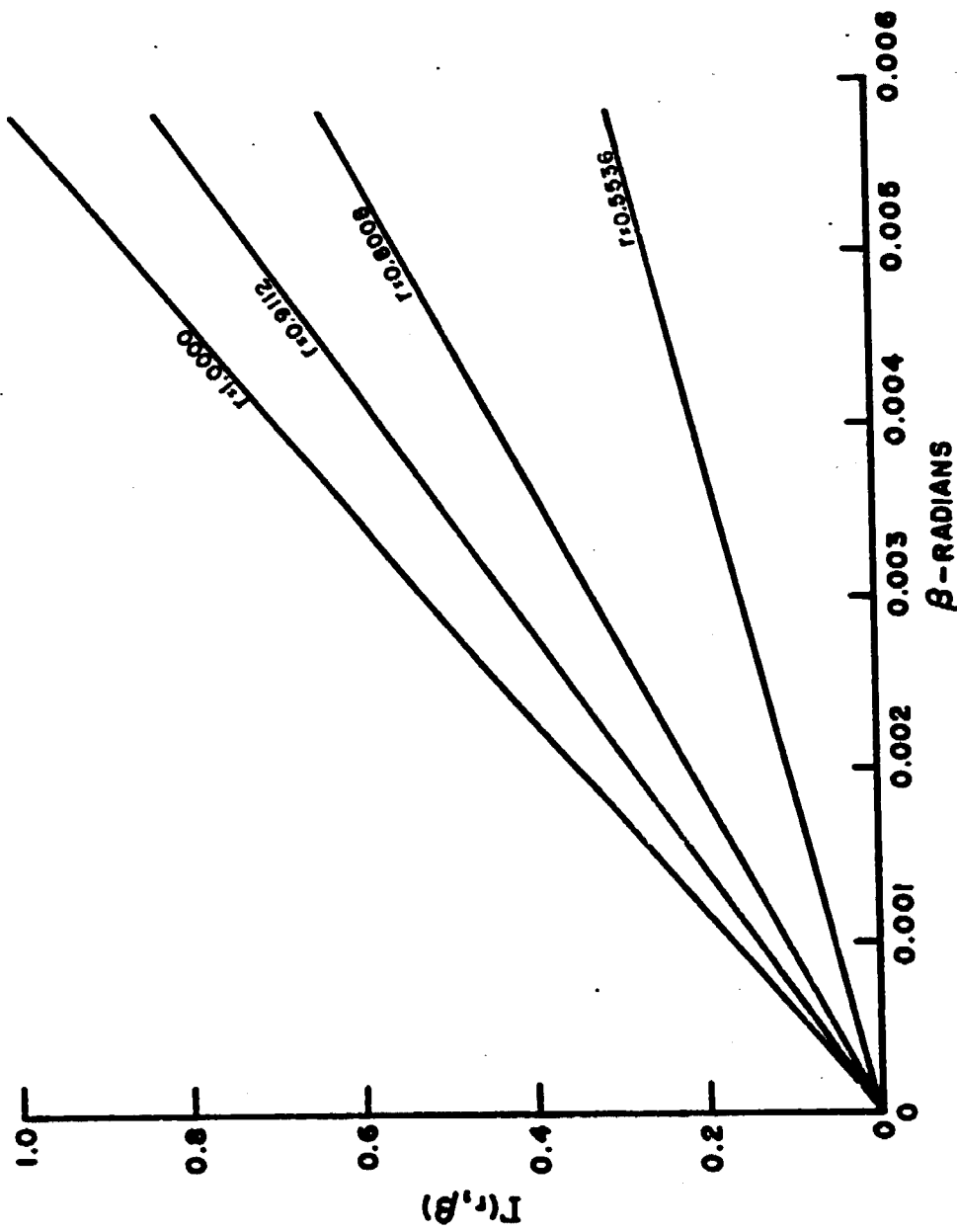


Figure 25. Meridional velocity distributions:  $\epsilon=1/3^\circ$ ,  $Re=4 \times 10^3$ .

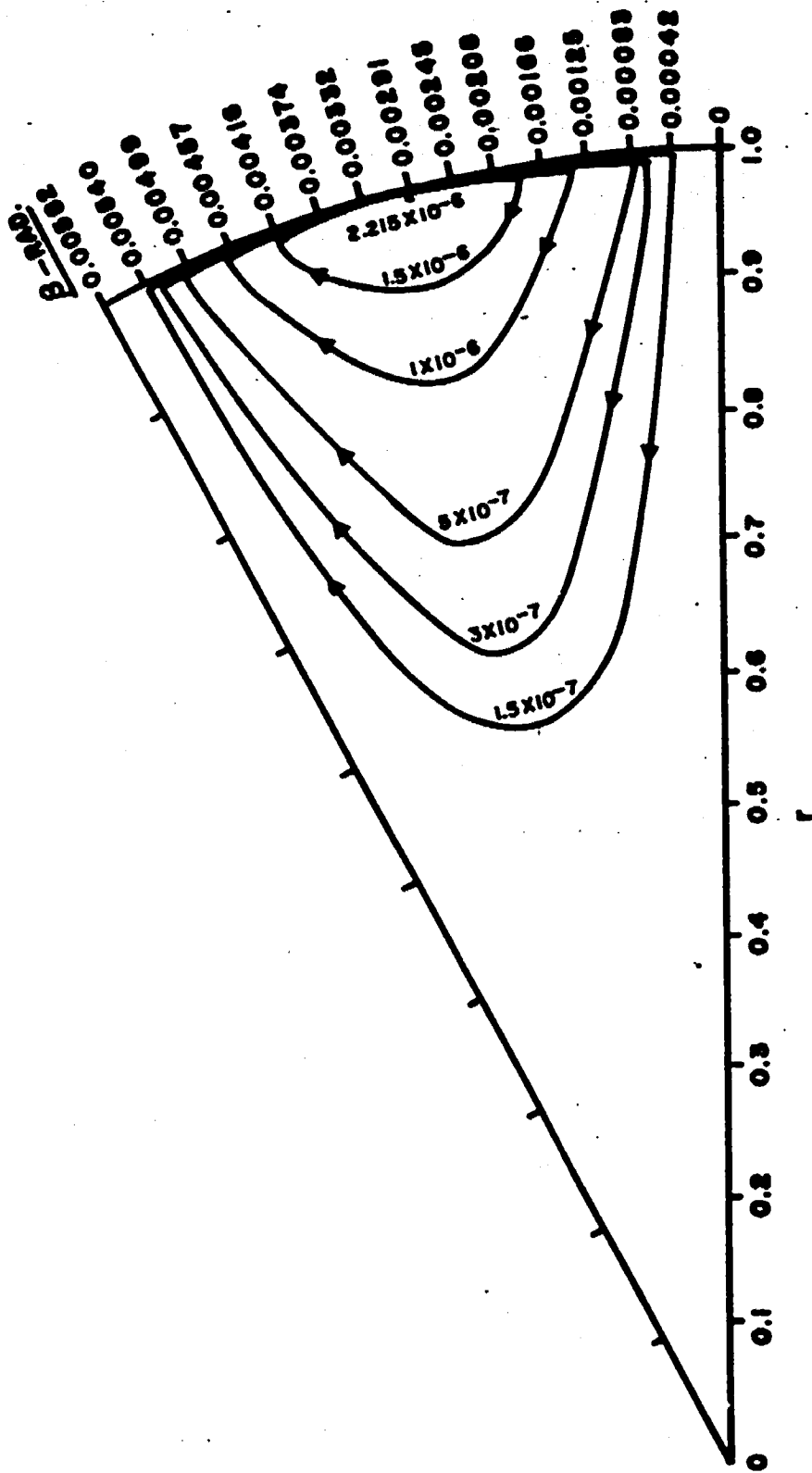


Figure 26. Streamlines of the secondary flow:  $\epsilon = 1/3^\circ$ ,  $Re = 4 \times 10^3$ .

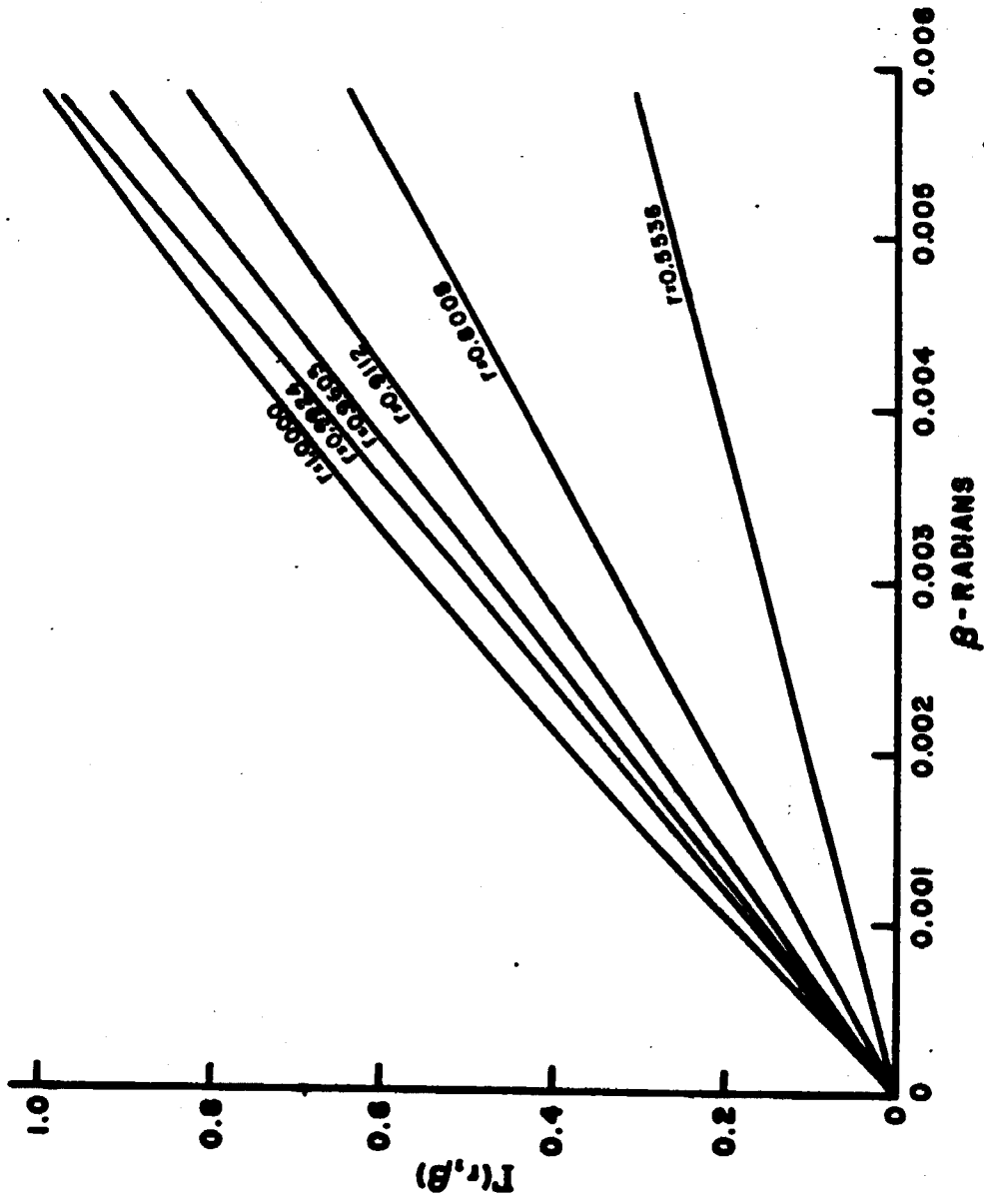


Figure 27. Meridian velocity distribution:  $\epsilon = 1/3^\circ$ ,  $Re = 2 \times 10^4$ .

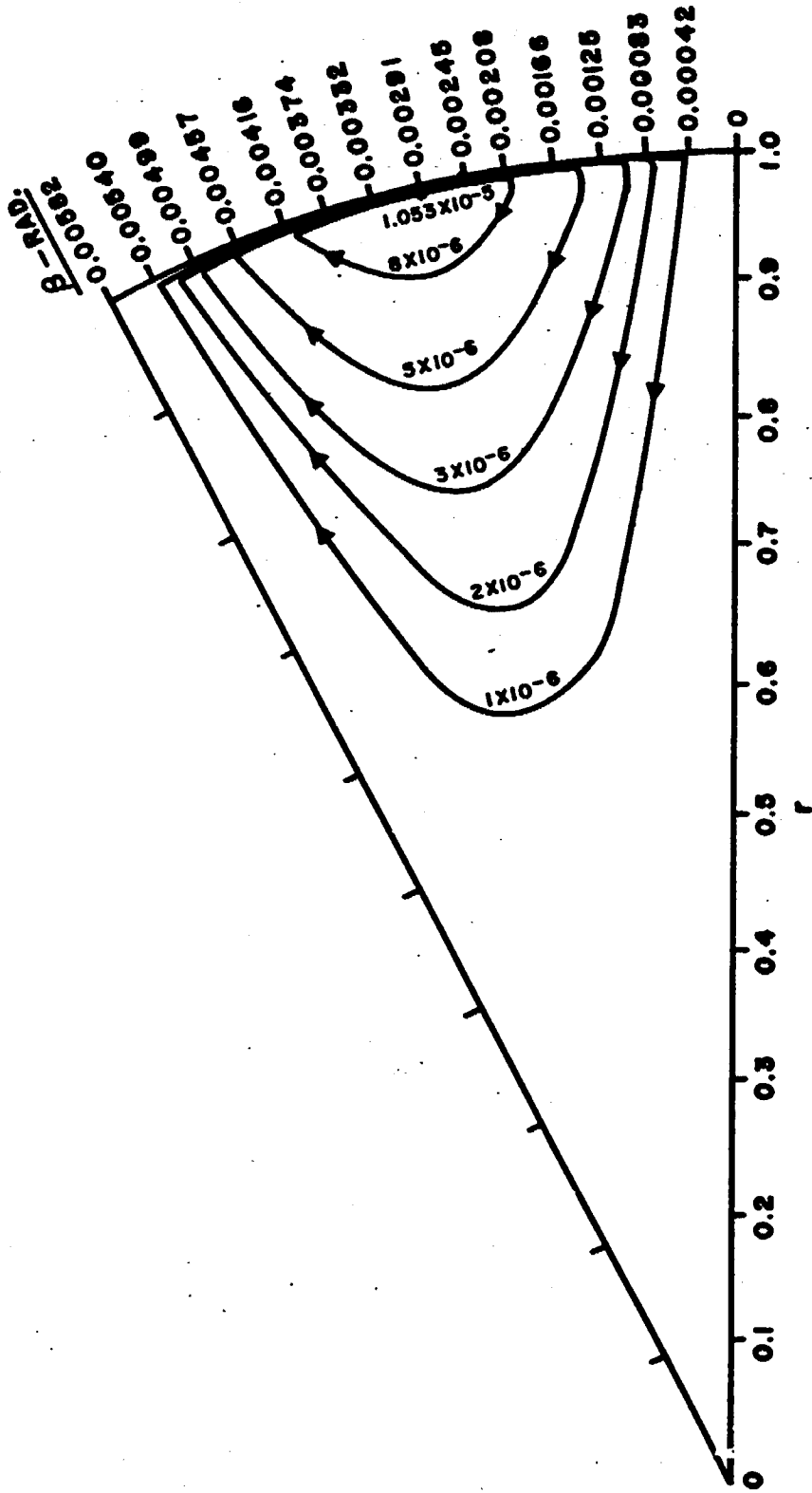


Figure 26. Streamlines of the secondary flow:  $\epsilon = 1/3^\circ$ ,  $Re = 2 \times 10^4$ .

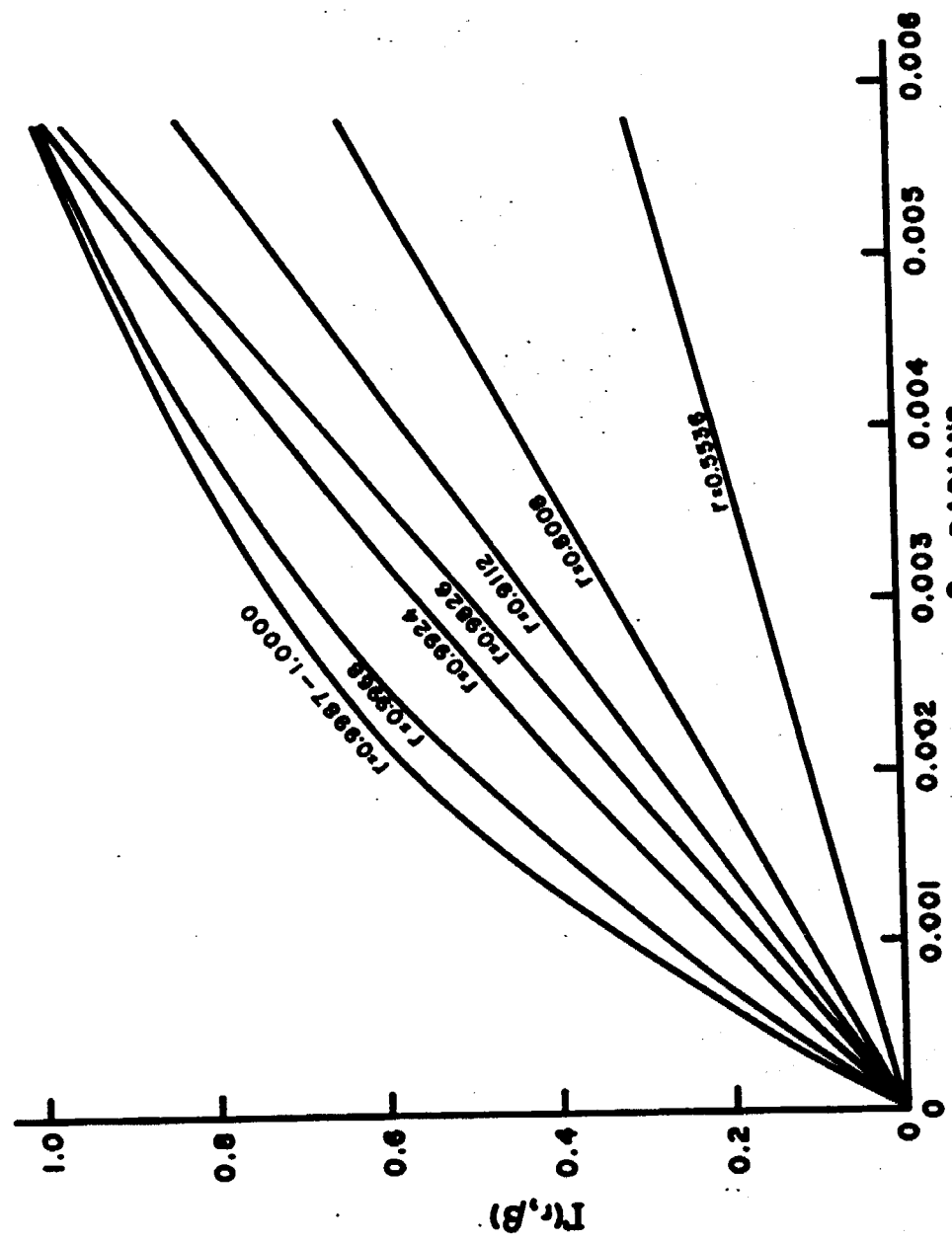


Figure 29. Meridion velocity distribution,  $\epsilon = 1/3^\circ$ ,  $Re = 8 \times 10^4$ .

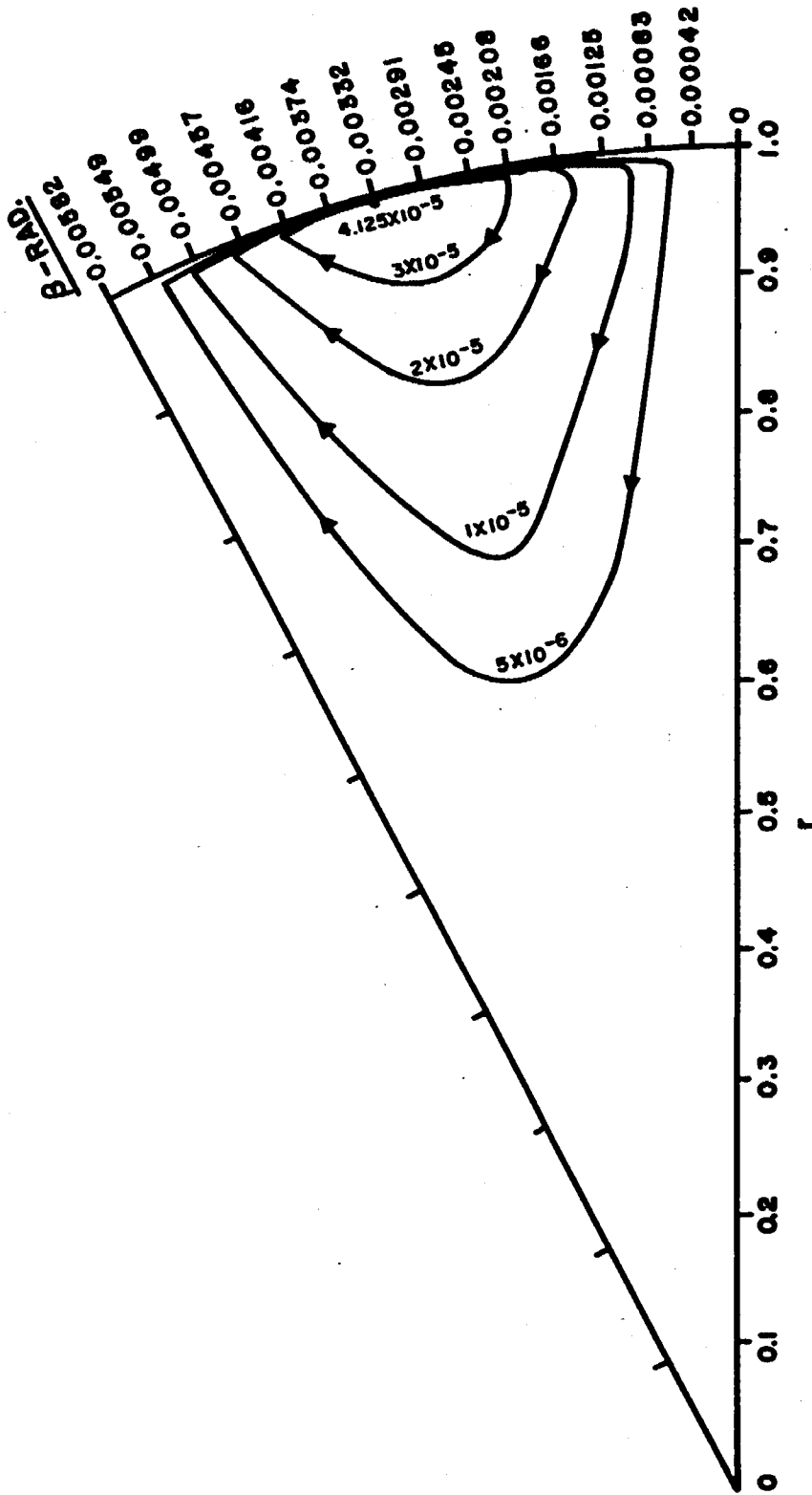


Figure 30. Streamlines of the secondary flow:  $\epsilon = 1/3^\circ$ ,  $Re = 6 \times 10^4$ .

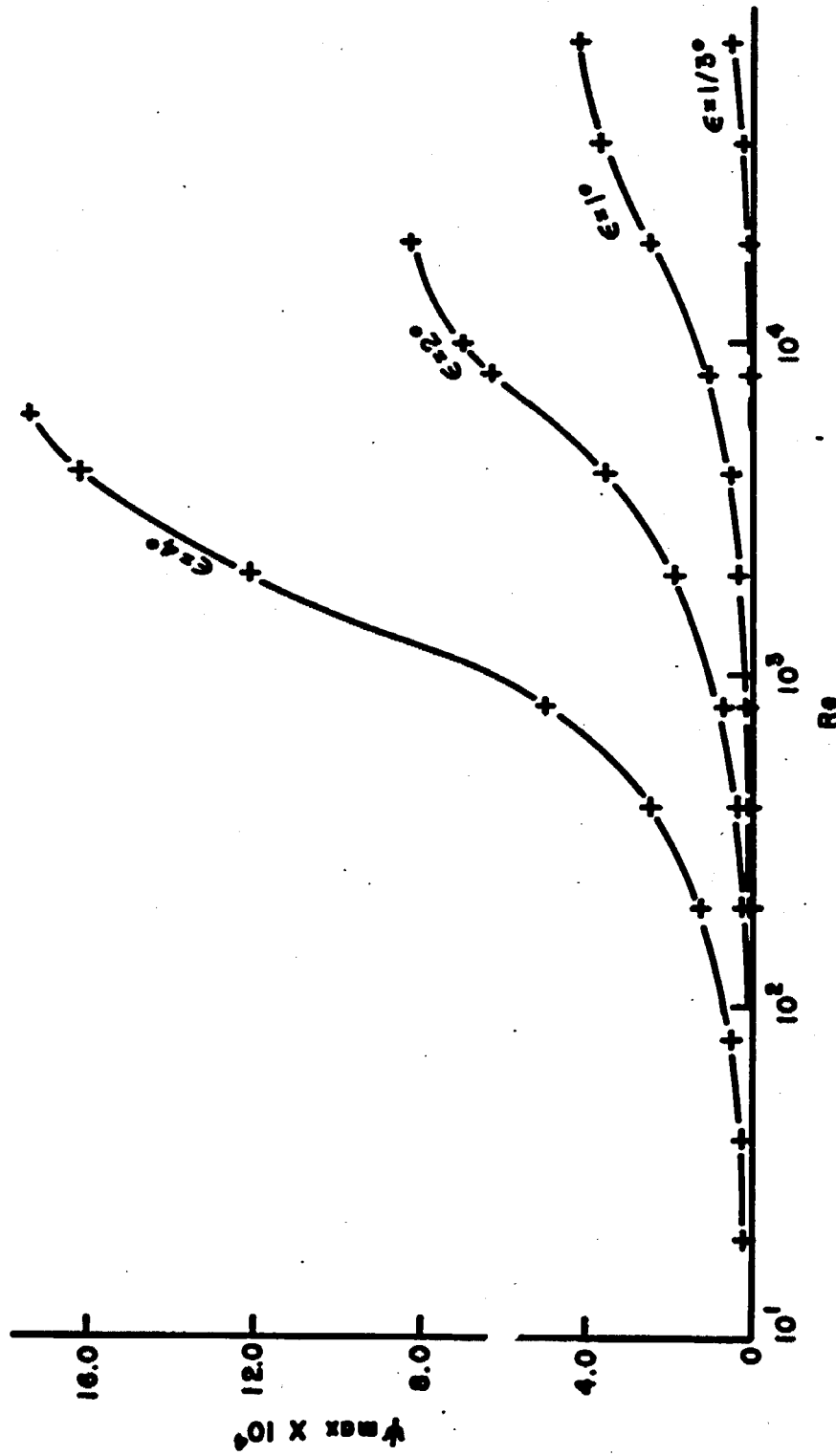


Figure 31.  $\Psi_{\max}$  vs.  $Re$  for  $\epsilon = 4, 2, 1,$  and  $1/3$  degrees.



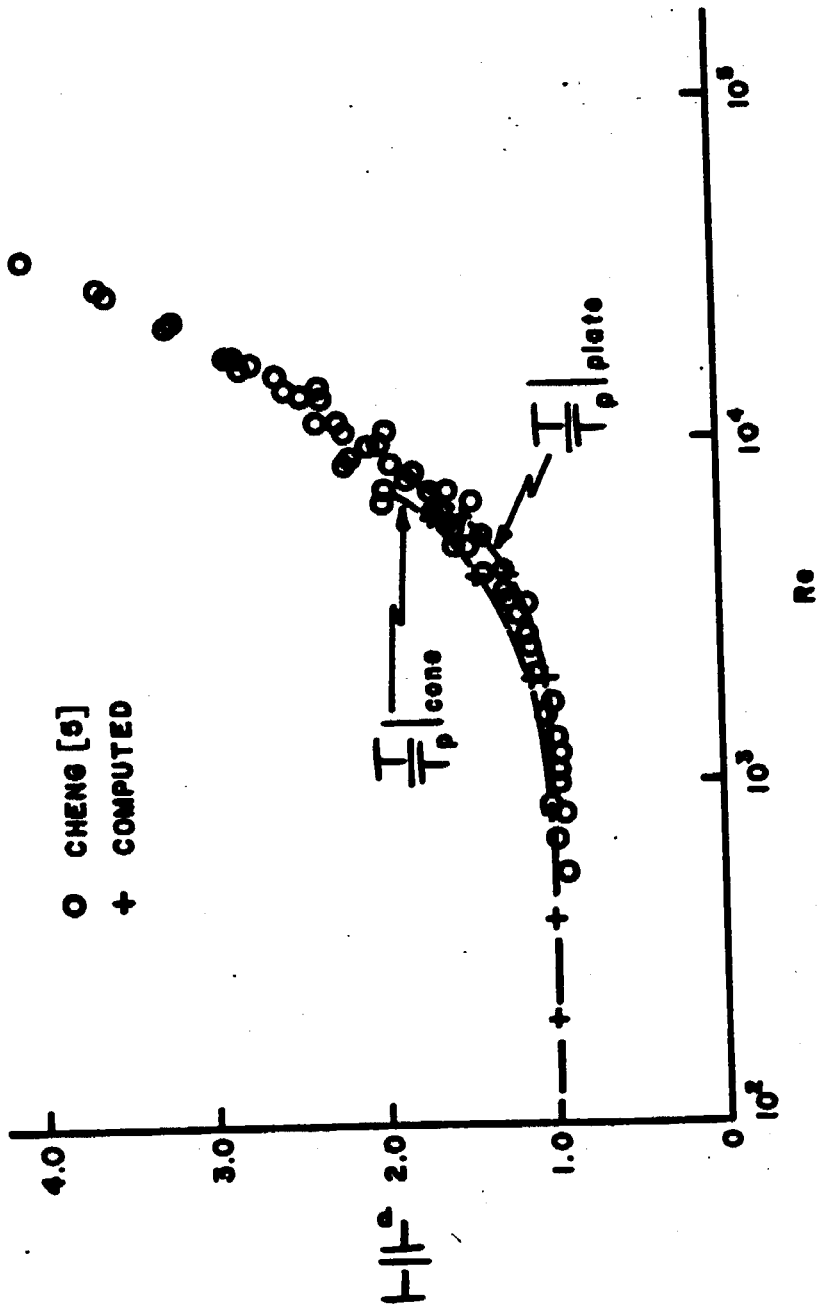


Figure 32. Comparison of the computed torque to primary torque ratios with those obtained from the experimental data of Cheng [5]:  $\epsilon = 4$  degrees.

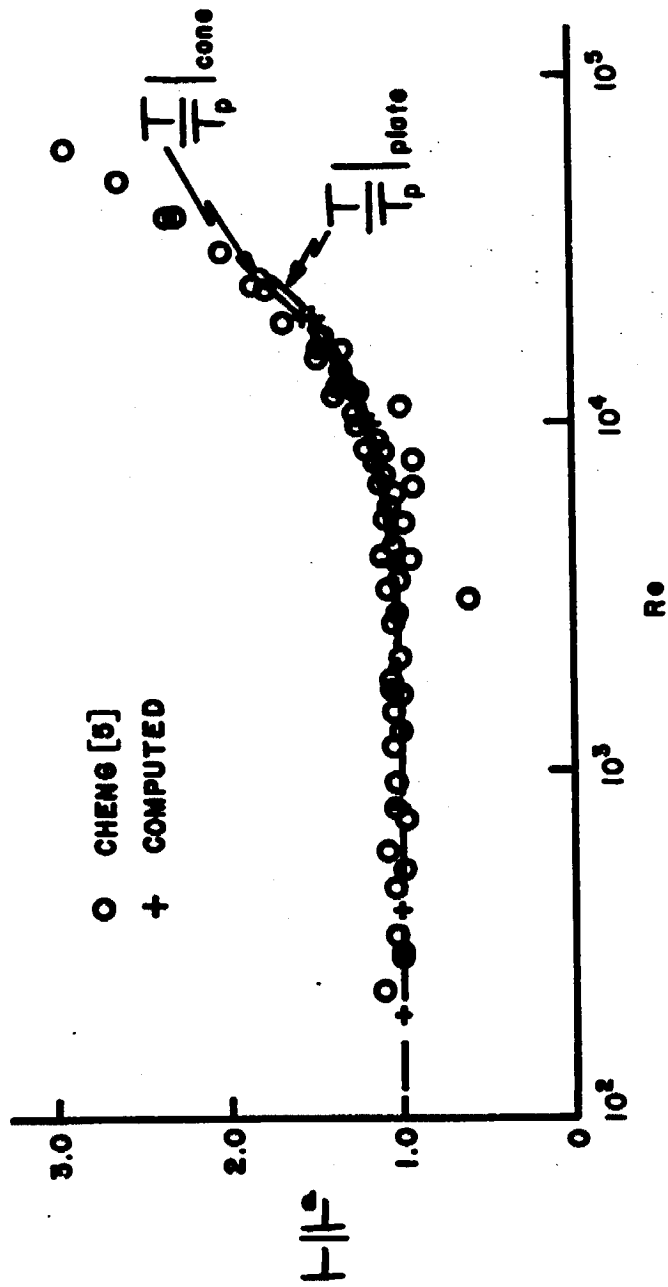


Figure 33. Comparison of the computed torque to the primary torque ratios with those obtained from the experimental data of Cheng [5]:  $\epsilon = 2$  degrees. 161

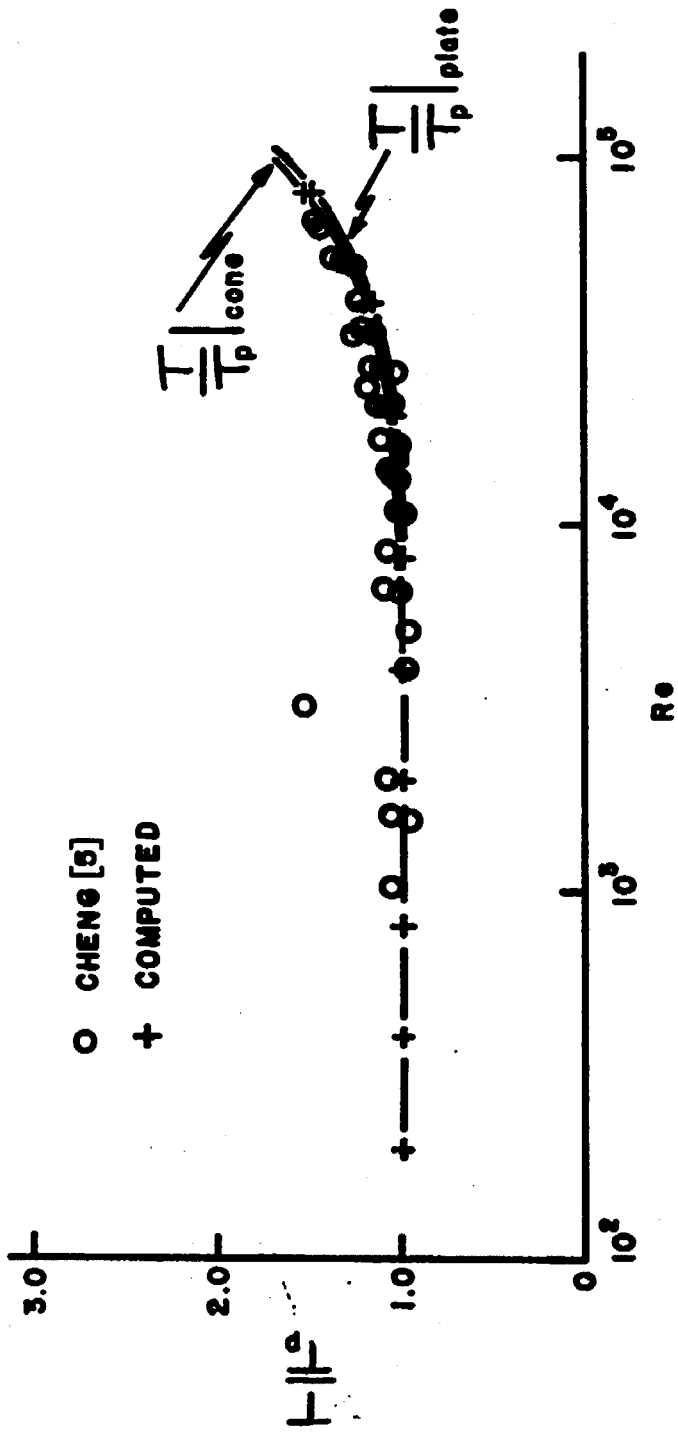


Figure 34. Comparison of the computed torque to primary torque ratios with

those obtained from the experimental data of Cheng [5]:  $\epsilon = 1$  degree.

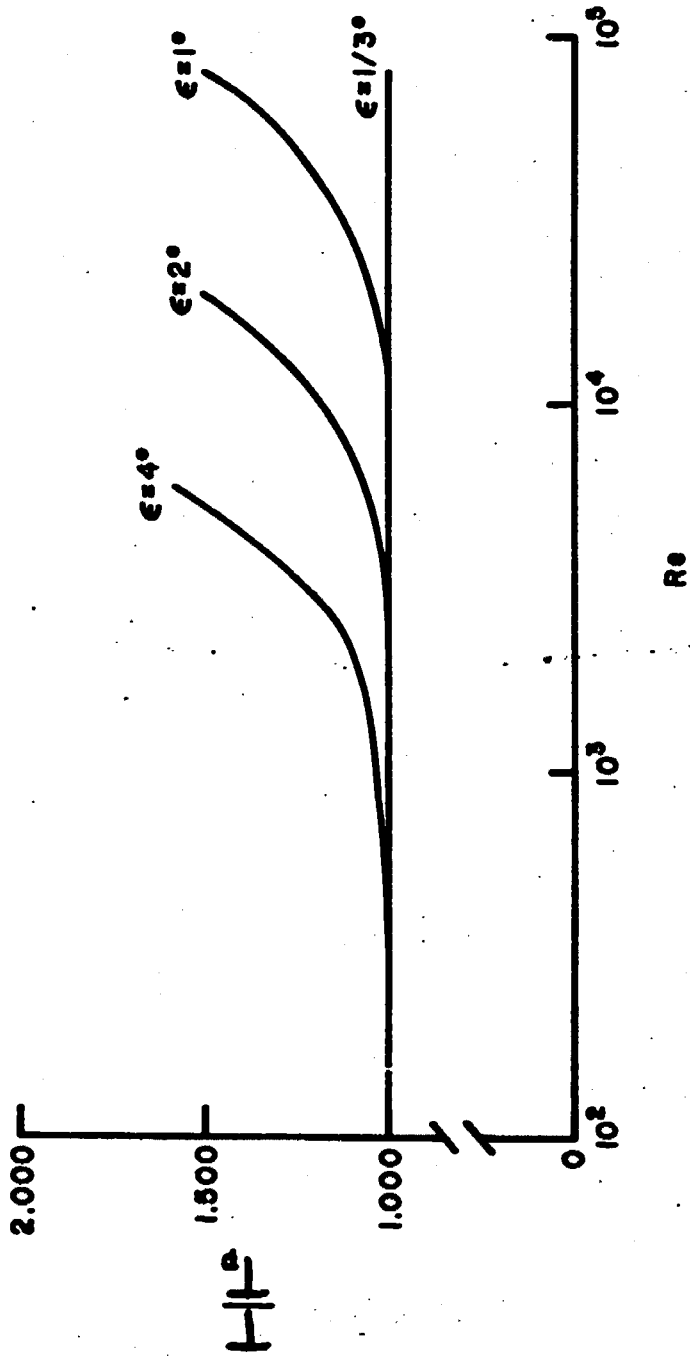


Figure 35. The torque to primary torque ratio at the cone and plate vs.  $Re$  for  $\epsilon = 4, 2, 1,$  and  $1/3$  degrees.

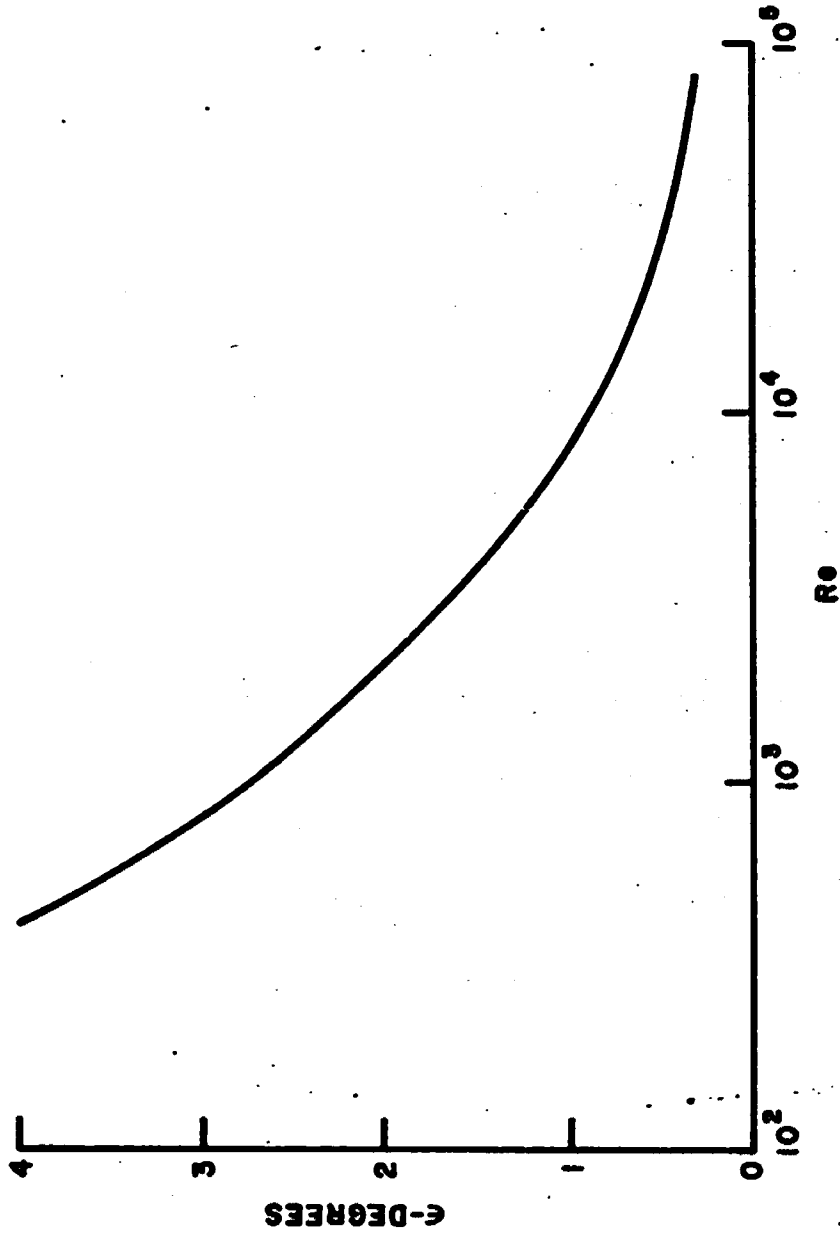


Figure 36. Locus of Reynolds number and gap angles at which there is an incipient torque increase, at the cone and plate, due to the secondary flow.

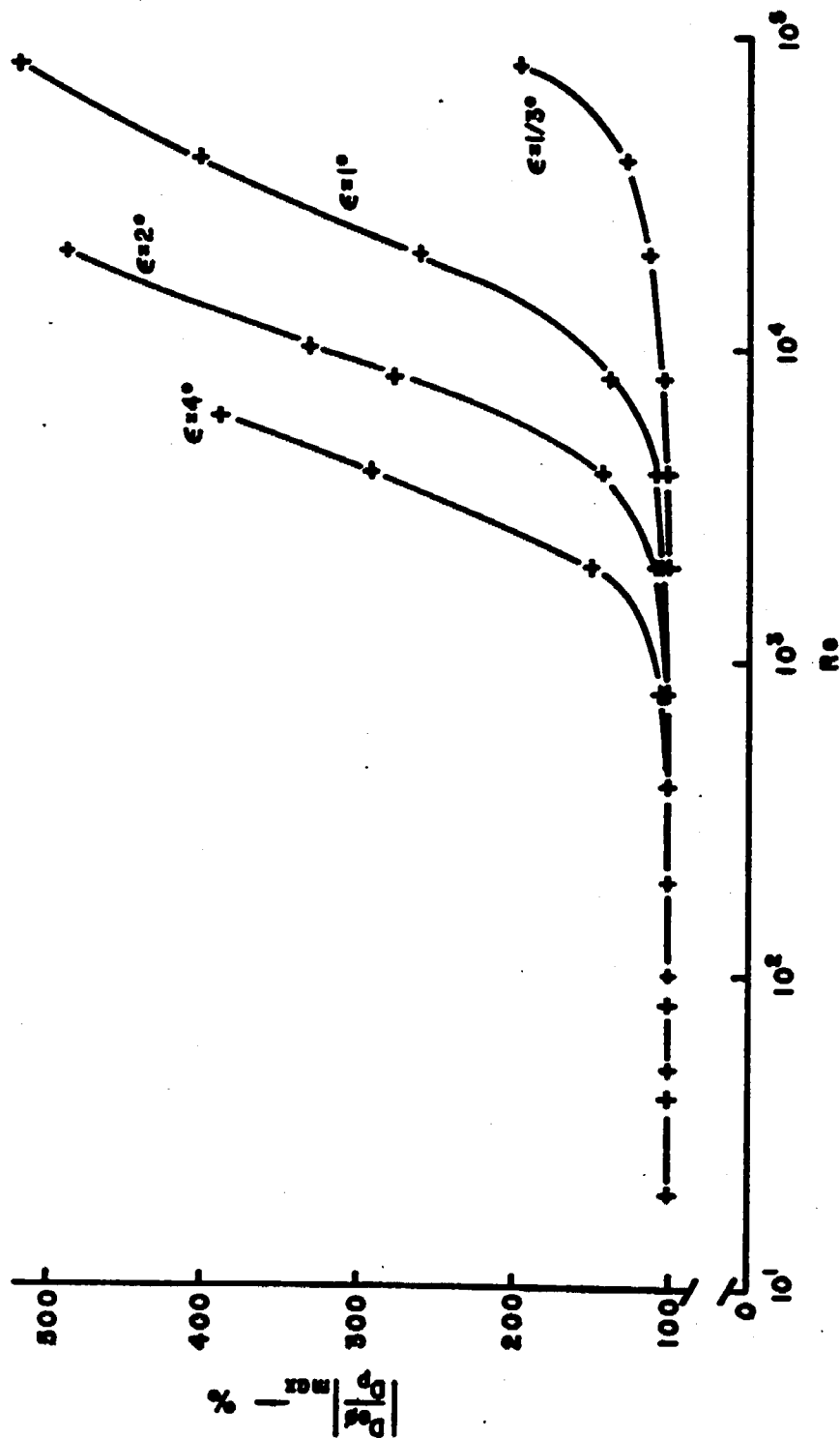


Figure 37. The maximum absolute value of the ratio of the  $D_{01}$  element of the rate-of-deformation tensor and the primary shear rate vs.  $Re$  for  $\epsilon = 4, 2, 1$  and  $1/3$  degrees.

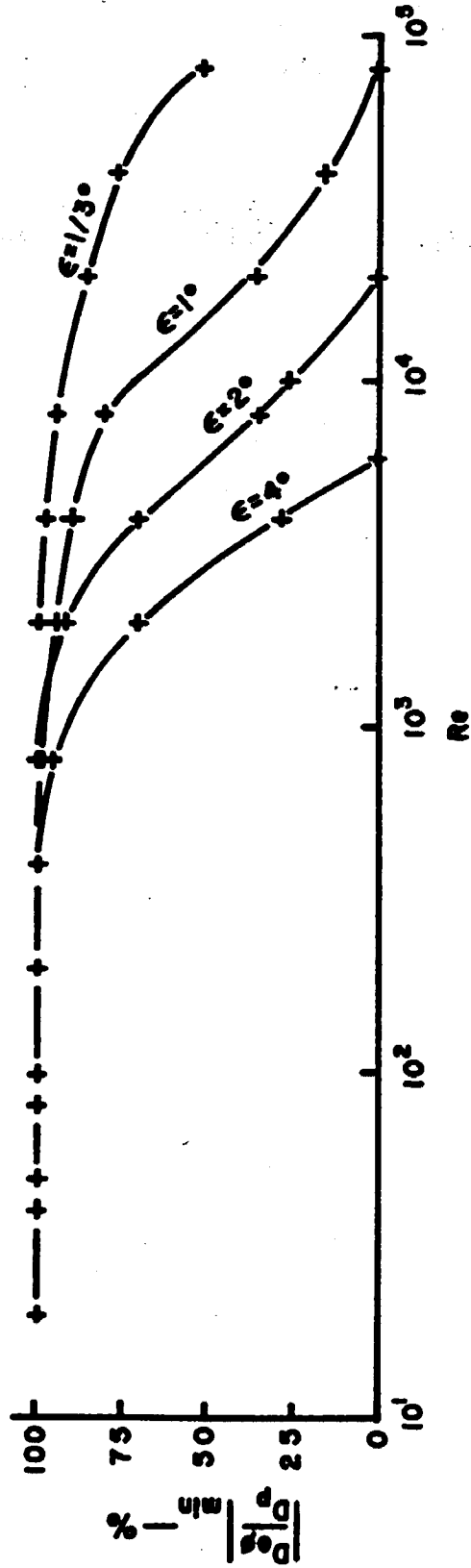


Figure 38. The minimum absolute value of the ratio of the  $D_{\alpha\beta}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for  $\epsilon = 4, 2, 1,$  and  $1/3$  degrees.

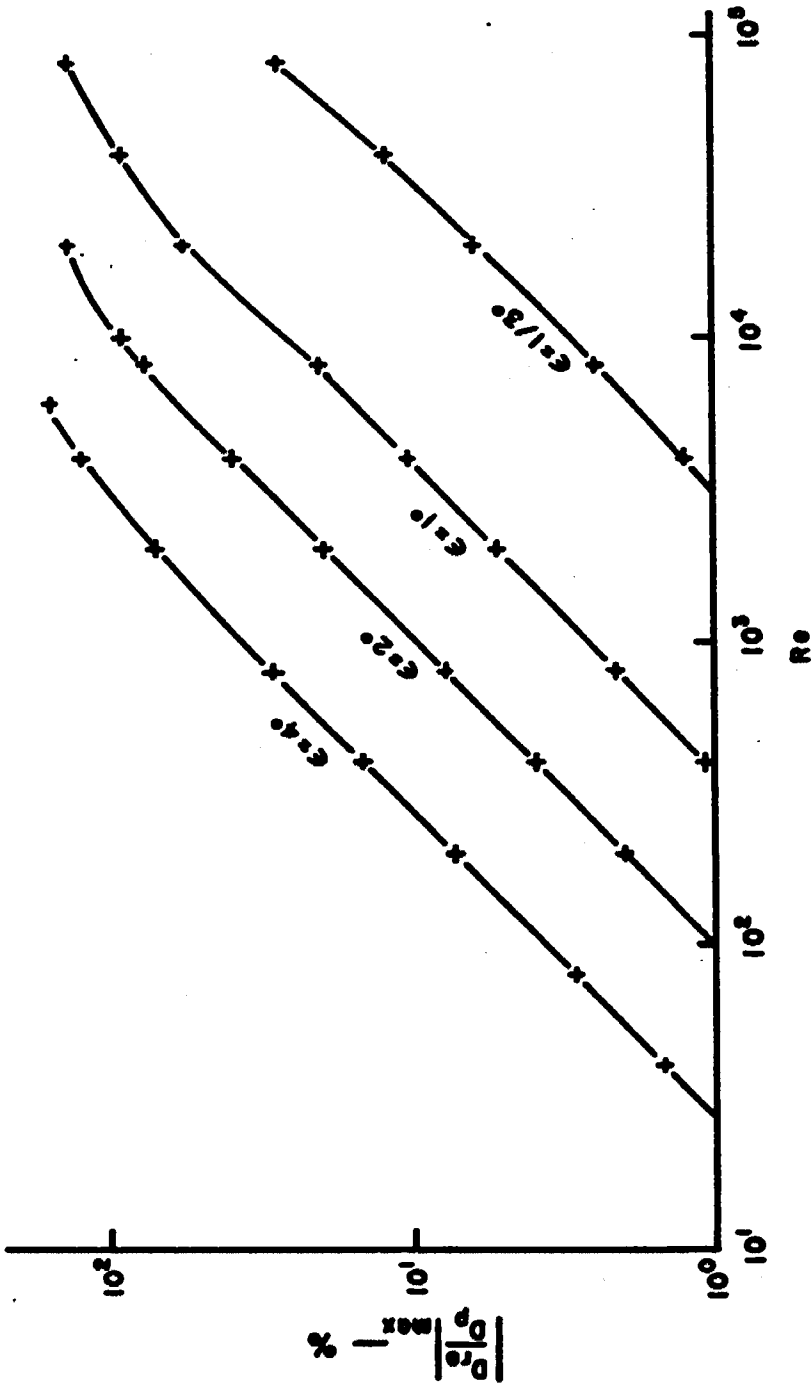


Figure 39. The maximum absolute value of the ratio of the Dre element of the rate-of-deformation tensor and the primary deformation rate vs. Re for  $\epsilon = 4, 2, 1$ , and  $1/3$  degrees.



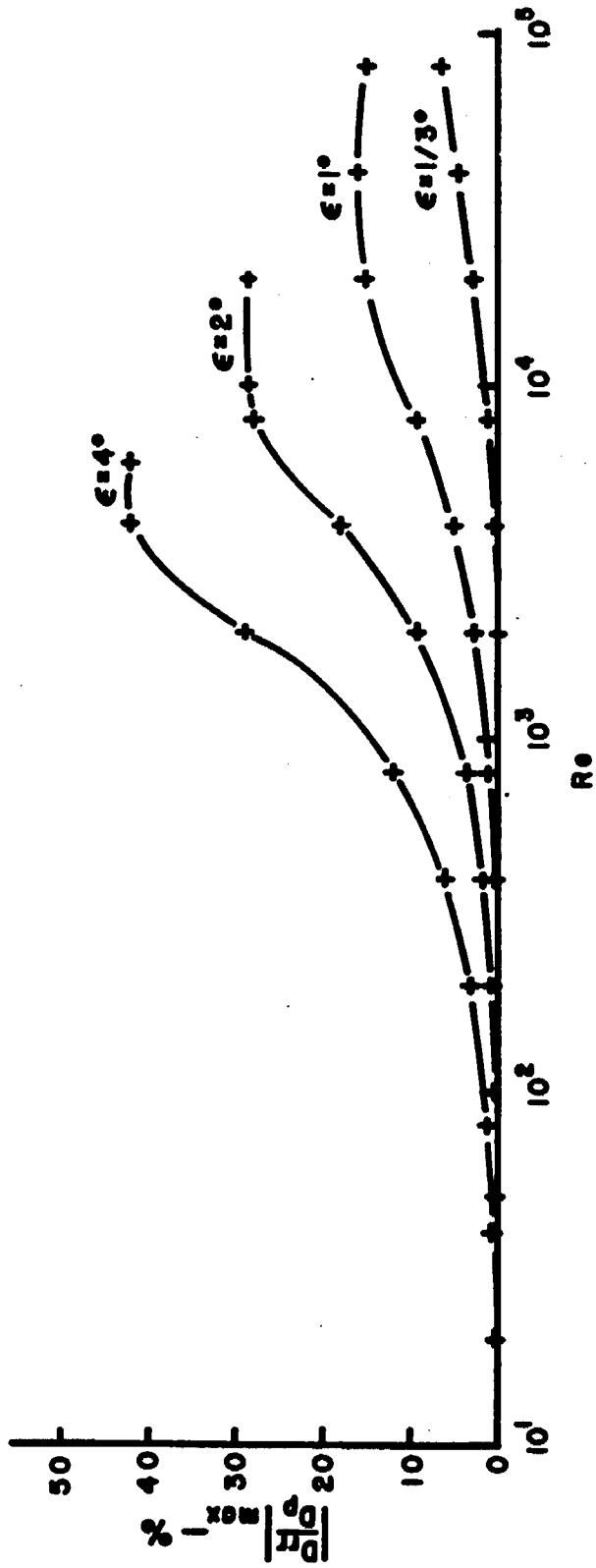


Figure 40. The maximum absolute value of the ratio of the  $D_{rr}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for  $\epsilon = 4, 2, 1$  and  $1/3$  degrees.

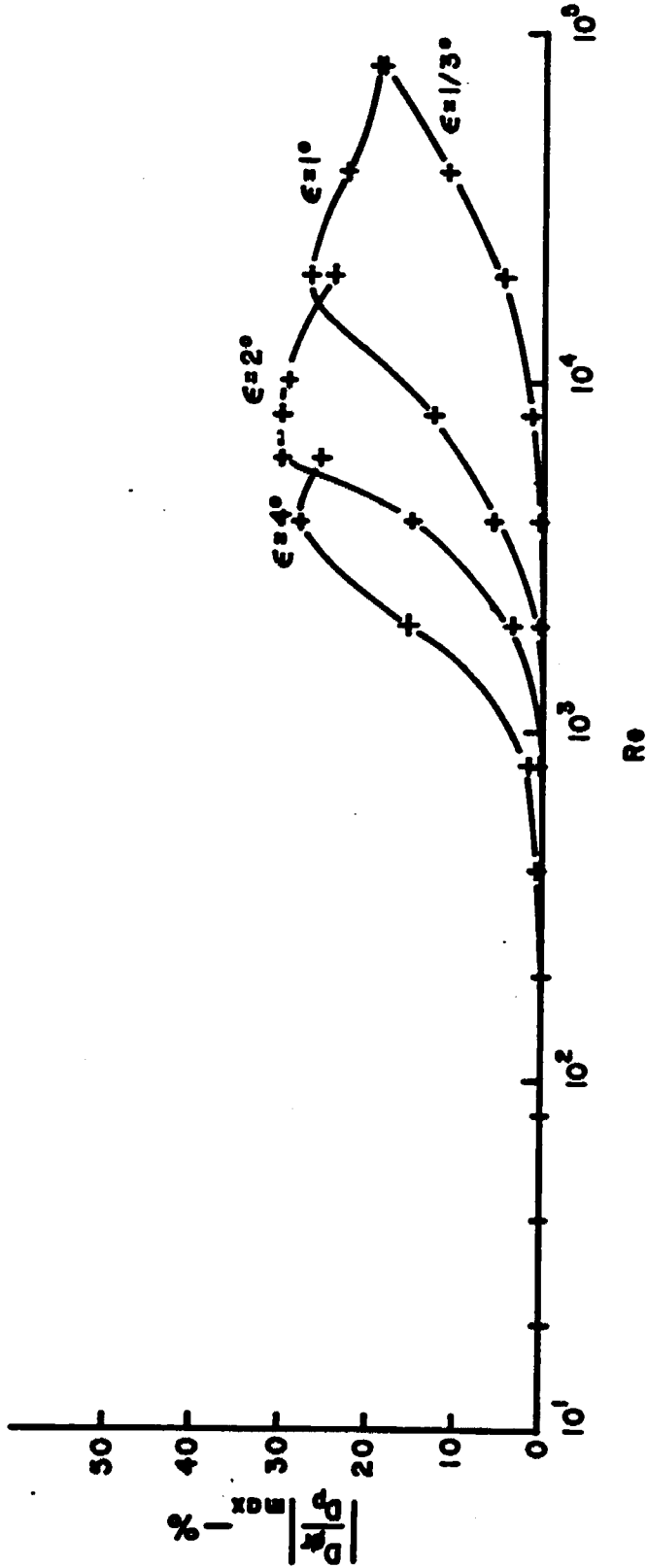


Figure 41. The maximum absolute value of the ratio of the  $D_{gr}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for  $\epsilon=4, 2, 1$  and  $1/3$  degrees.

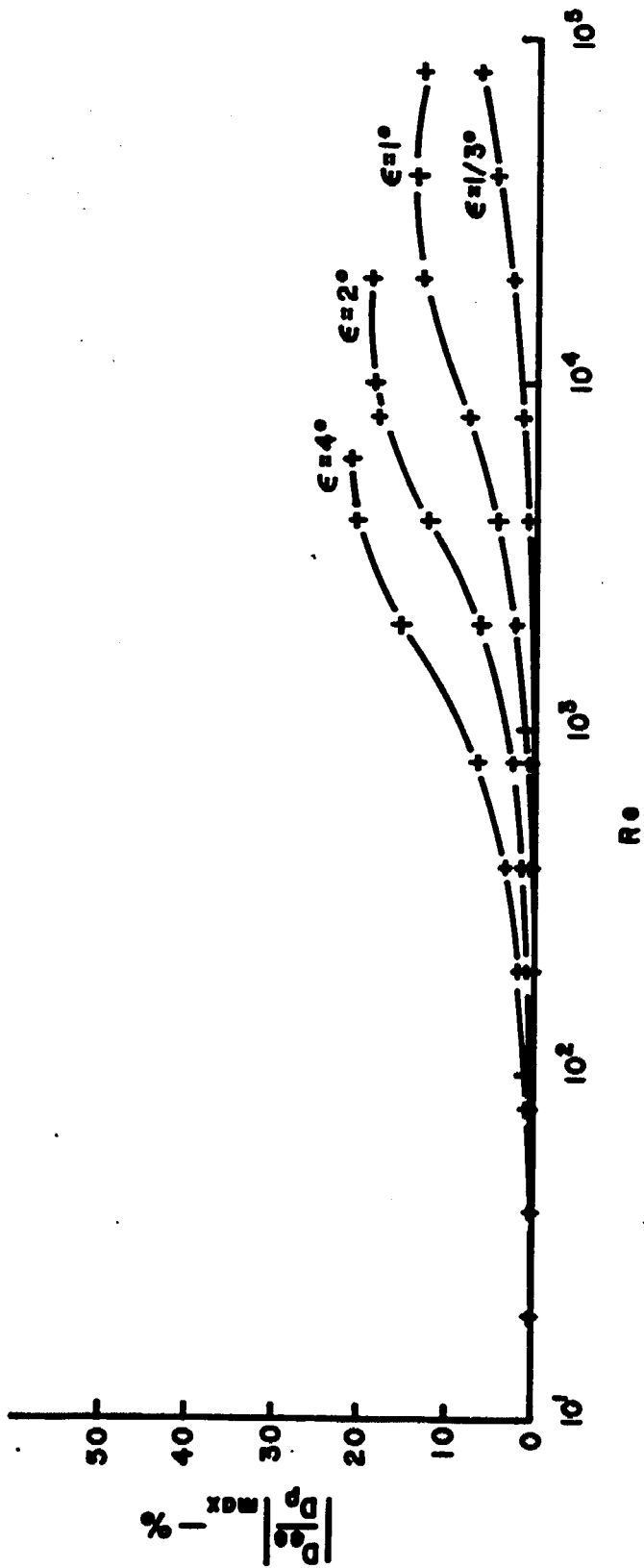


Figure 42. The maximum absolute value of the ratio of the  $D_{ee}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for  $\epsilon = 4, 2, 1$  and  $1/3$  degrees.

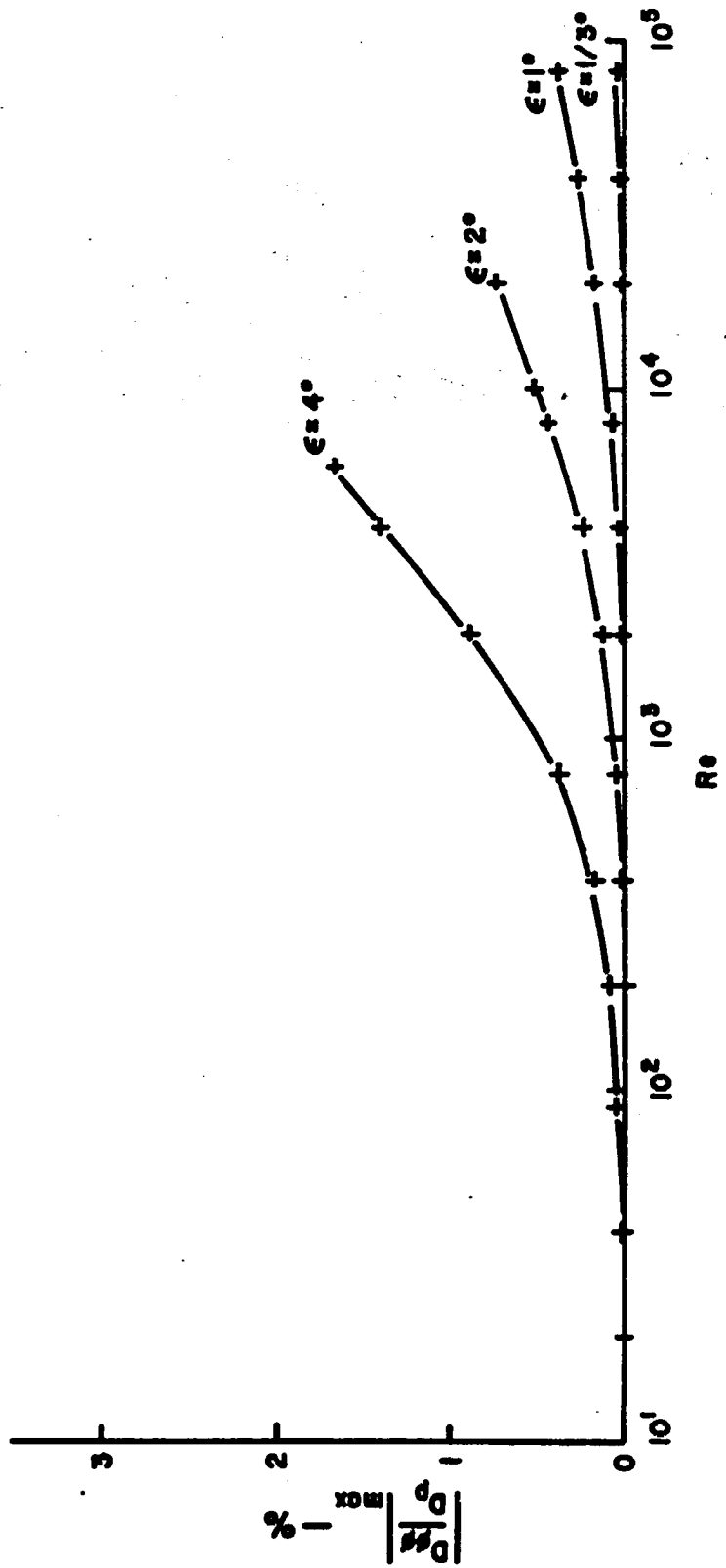


Figure 43. The maximum absolute value of the ratio of the  $D_{ij}$  element of the rate-of-deformation tensor and the primary deformation rate vs.  $Re$  for.  $\epsilon = 4, 2, 1$  and  $1/3$  degrees.

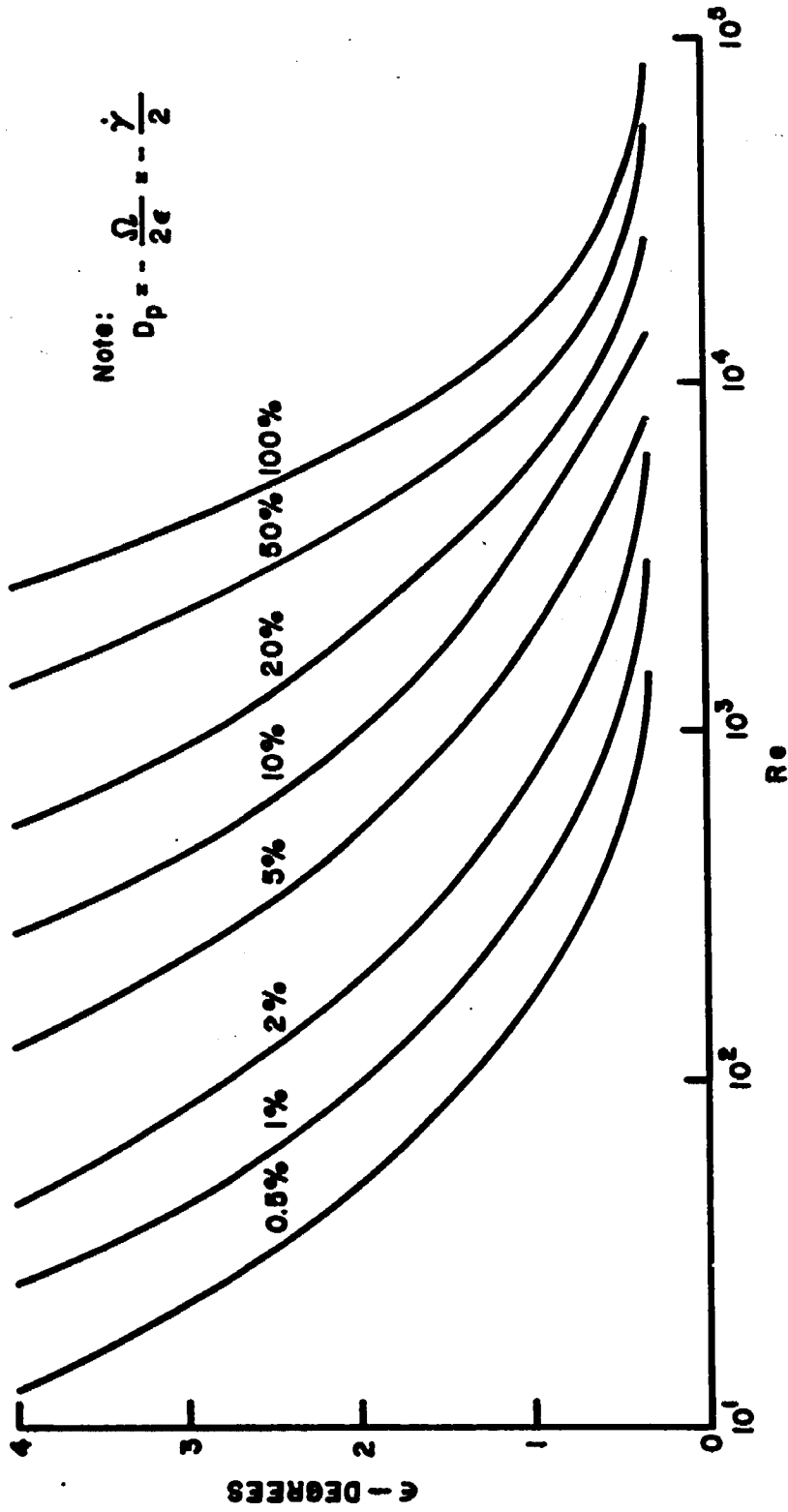


Figure 44. Loci of Reynolds numbers and gap angles on which the magnitudes of the effects of the secondary flow on the rate of deformation are indicated as percentages of the primary deformation rate  $D_p$ .

TABLE I  
ESTIMATED MAXIMUM TRUNCATION ERROR

M	N	$\Gamma(0.8953, \epsilon/2)$	$\Gamma(1, \epsilon/2)$	$\psi_{\max}$	$\xi(0.8953, \epsilon)$	$\zeta(0.8953, 0)$	$\zeta(1, \epsilon/2)$
11	11	7.9%	6.0%	9.0%	11.5%	15.9%	17.7%
11	13	5.3%	5.8%	8.1%	8.6%	11.9%	9.5%
11	15	4.5%	3.5%	6.5%	6.7%	8.1%	5.4%

TABLE II  
 THE TORQUE TO PRIMARY TORQUE RATIO AT THE  
 CONE AND PLATE

Gap Angle	Reynolds Numbers	Torque Ratio
(deg)	(Re)	( $T / T_p$ )
0.3333	$2 \times 10^3$	0.9945
0.3333	$4 \times 10^3$	0.9946
0.3333	$8 \times 10^3$	0.9946
0.3333	$2 \times 10^4$	0.9949
0.3333	$4 \times 10^4$	0.9969
0.3333	$8 \times 10^4$	1.0032
1.0	$2 \times 10^2$	0.9987
1.0	$4 \times 10^2$	0.9987
1.0	$8 \times 10^2$	0.9988
1.0	$2 \times 10^3$	0.9990
1.0	$4 \times 10^3$	0.9994
1.0	$8 \times 10^3$	1.0038
1.0	$2 \times 10^4$	1.0475
1.0	$4 \times 10^4$	1.1860
1.0	$8 \times 10^4$	1.5141
2.0	$5 \times 10^1$	1.0003
2.0	$1 \times 10^2$	1.0006
2.0	$2 \times 10^2$	1.0004
2.0	$4 \times 10^2$	1.0000

TABLE II (Continued)

<u>Gap Angle</u>	<u>Reynolds Numbers</u>	<u>Torque Ratio</u>
(deg)	(Re)	( $\tau / \tau_p$ )
2.0	$8 \times 10^2$	1.0004
2.0	$2 \times 10^3$	1.0036
2.0	$4 \times 10^3$	1.0212
2.0	$8 \times 10^3$	1.1157
2.0	$1 \times 10^4$	1.1786
2.0	$2 \times 10^4$	1.5138
4.0	$2 \times 10^1$	1.0014
4.0	$4 \times 10^1$	1.0006
4.0	$8 \times 10^1$	1.0006
4.0	$2 \times 10^2$	1.0011
4.0	$4 \times 10^2$	1.0027
4.0	$8 \times 10^2$	1.0094
4.0	$2 \times 10^3$	1.0740
4.0	$4 \times 10^3$	1.3268
4.0	$6 \times 10^3$	1.5935



## CHAPTER VI

### CONCLUDING REMARKS

The results presented in this work not only provide the effects of the secondary flow on viscosity measurements (torque on the plate) without the data scatter associated with experimental data, but they also provide, for the first time, the effects of the secondary flow on the rate-of-deformation.

It is interesting to note that although the vortex structure of the secondary flow does not agree with that reported by Savins and Metzner [24], the results do confirm their contention that the torque at the cone and plate is an insensitive measure of the secondary flow. Their analytical approximation which predicted that the secondary flow affects the rate-of-deformation at values of  $Re$  approximately  $1/20$  of those at which it affects the torque at the plate is in good agreement with the results presented in this work. Also, their conclusion that the effect of the secondary flow on the rate-of-deformation is less than 5% if the Reynolds number based on the gap angle width between the cone and plate,  $N_{Re}$ , is less than 0.50, is in excellent agreement with the curve representing a 5% deviation in Figure 44. Note:

$$N_{Re} = Re \epsilon^2.$$

In a straightforward way a slightly different formulation of the Navier-Stokes equation can be derived. This formulation of the Navier-Stokes equation can be used to develop a procedure in which the finite difference approximation of the convective terms conserve momentum as well as vorticity. It is suggested that such a procedure be investigated in future work since it might prove to be more efficient than the method used in this work.

The velocity profiles, torque ratios, and the elements of the rate-of-deformation tensor computed in this work have been stored on a magnetic tape for future reference. A possible extension of this work would be to use the velocity profiles from the magnetic tape to obtain the pressure field throughout the viscometer. From the pressure solution, the normal force on the cone and plate could be computed. Another possible extension would be to use these velocity profiles to solve the energy equation to obtain the temperature rise due to the viscous dissipation of energy.

Another, but more difficult extension of this work would be to compute the effects of the secondary flow on non-Newtonian fluids sheared in cone-and-plate viscometers.

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## APPENDIX A

### TORQUE BALANCE ON THE CONE-AND-PLATE VISCOMETER

This appendix uses the integral form of the balance of the moment of momentum to prove that the torque, at steady state, exerted by the fluid on the plate of a cone-and-plate viscometer is essentially equal in magnitude to that exerted by the fluid on the cone.

The integral form [26] of the balance of the moment of momentum (or angular momentum) for a single phase system can be expressed mathematically as

$$\frac{d}{dt} \int_{V(s)} \underline{x} \times \rho \underline{\dot{x}} dv = \int_{S(s)} (\underline{x} \times \rho \underline{\dot{x}}) (\underline{\dot{x}} - \underline{\dot{x}}(s)) \cdot (-\underline{n}) ds - \underline{T} + \int_{V(s)} \underline{x} \times \rho \underline{b} dv \quad (\text{A.1})$$

where  $V(s)$  is the volume of the system,

$S(s)$  is the closed bounding surface of the system,

$\underline{x}$  is the position vector relative to a fixed frame of reference with respect to the earth. (The difference between using the earth for a fixed frame of reference and the fixed stars as a fixed frame of reference is assumed negligible.),

- $\rho$  is the density of the fluid at any point in the system,
- $\dot{\underline{x}}$  is the velocity of the fluid at any point in the system relative to a fixed frame of reference with respect to earth,
- $\dot{\underline{x}}_{(s)}$  is the velocity of the system relative to a fixed frame of reference with respect to the earth,
- $\underline{n}$  is the inwardly directed unit normal to the boundary surfaces at the system,
- $\underline{T}$  is the torque which the system exerts upon its boundary surfaces,
- $\underline{b}$  is the body force per unit volume exerted on the fluid at any point within the system.

If the system is chosen to be the fluid within the cone-and-plate viscometer, and the frame of reference is the system at rest, i.e., prior to the rotation of the cone, with the origin being the intersection of the cone and plate (Figure 2), then

$$\underline{x} = r \underline{e}_f ,$$

$$\underline{b} = -g \underline{k} ,$$

$$V_{(s)} = \text{volume of the fluid within the viscometer,}$$

$$S_{(s)} \text{ is composed of the wetted surface of the cone and plate and the free surface, and}$$

$$\dot{\underline{x}}_{(s)} = 0.$$

Since there is no mass that crosses the boundary of the system,

$$\int_{S(s)} (\underline{x} \times \rho \dot{\underline{x}}) (\dot{\underline{x}}) \cdot (-\underline{n}) ds = 0.$$

Also, when the system is at a steady state, the term on the left hand side of equation A.1 vanishes. The balance of the moment of momentum for the system at steady state can then be written as

$$\underline{T} = \int_{V(s)} \underline{r} \times \rho \underline{b} dv \quad (\text{A.2})$$

The integrand of the right hand side of (A.2) can be written as

$$\underline{r} \times \underline{b} = r \underline{e}_r \times \rho (-g \underline{k}) = r \rho \sin \theta \underline{e}_\phi \quad (\text{A.3})$$

Hence, equation (A.2) can be written as

$$\underline{T} = \int_{V(s)} r \rho \sin \theta \underline{e}_\phi dv \quad (\text{A.4})$$

Take the scalar product of equation (A.4) and the unit vector in the direction of the axis of cone,  $\underline{k}$ , to obtain

$$\underline{k} \cdot \underline{T} = \underline{k} \cdot \int_{V(s)} r \rho \sin \theta \underline{e}_\phi dv \quad (\text{A.5})$$



Since  $\underline{k}$  is independent of position, it can be brought inside the integral in equation (A.5), and then

$$\underline{k} \cdot \underline{T} = \int_{V(s)} r \rho \sin \theta (\underline{k} \cdot \underline{e}_\phi) dv = 0 . \quad (\text{A.6})$$

Therefore,

$$\underline{k} \cdot \underline{T} = 0 , \quad (\text{A.7})$$

or the algebraic sum of the torques about the axis of the cone exerted by the fluid on its boundaries is zero. This can be written as

$$T_{\text{plate}} + T_{\text{free surface}} - T_{\text{cone}} = 0 . \quad (\text{A.8})$$

Since the shear stresses at the free surface are assumed to be negligible (Section II.2), and the surface area of the free surface is small,  $T_{\text{free surface}}$  is negligible. Hence,

$$T_{\text{plate}} = T_{\text{cone}} . \quad (\text{A.9})$$

## APPENDIX B

### DERIVATION OF THE COMPONENT FORM OF THE EQUATIONS OF MOTION AND THE RATE-OF-DEFORMATION TENSOR IN SPHERICAL COORDINATES

The equations of motion in direct form are equations (1.13) and (1.14) of Section II.1. These equations are

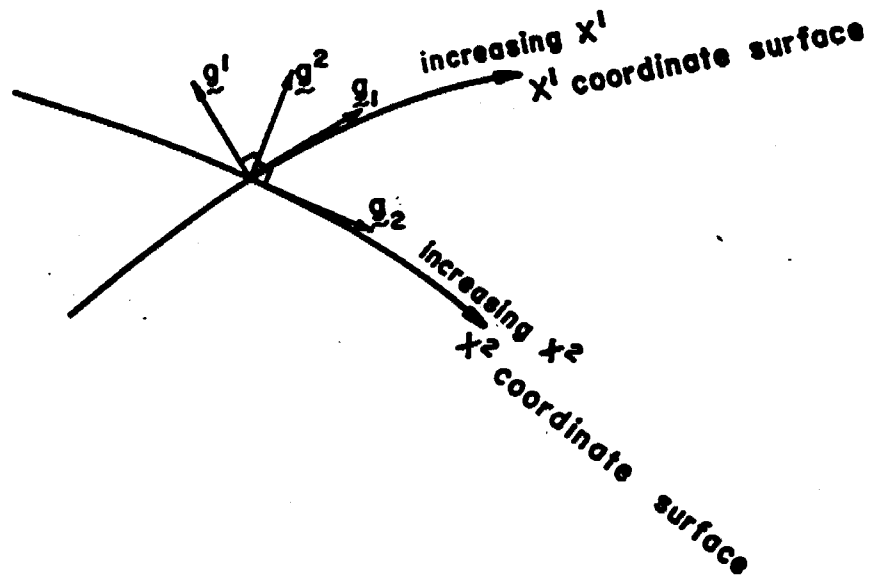
$$\frac{\partial \dot{\underline{x}}}{\partial t} + (\text{grad } \dot{\underline{x}}) \cdot \dot{\underline{x}} = - \frac{1}{\rho} \text{grad } p + \nu \text{div}(\text{grad } \dot{\underline{x}}) + \underline{b} \quad (\text{B.1})$$

and

$$\text{div } \dot{\underline{x}} = 0 . \quad (\text{B.2})$$

The notation and definitions used henceforth are consistent with that of McConnell [15]. The summation convention that like indices appearing above and below but not both above and below are summed will be used throughout. Using the adopted notation, a natural basis  $\{g_1^{(\underline{x})}, g_2^{(\underline{x})}, g_3^{(\underline{x})}\}$  and a reciprocal natural basis  $\{g^{(\underline{x})1}, g^{(\underline{x})2}, g^{(\underline{x})3}\}$  of a 3-dimensional vector space can be defined such that  $g^i$  is pointed normal to the  $i^{\text{th}}$  coordinate surface and  $g_i$  is tangent to the  $i^{\text{th}}$  coordinate surface. The vector  $\underline{x} = \underline{x}(x^1, x^2, x^3)$  is the location of a point in space in terms of the coordinates  $x^1, x^2, x^3$ .

The following sketch indicates the orientation for the  $x^1$  and  $x^2$  coordinates.



The reciprocal basis is defined such that

$$\underline{g}^i \cdot \underline{g}_j = \delta_j^i \quad (\text{B.3})$$

where  $\delta_j^i = \begin{cases} 1 & i=j \\ 0 & i \neq j \end{cases}$  and is called the Kronecker delta. Any vector  $\underline{v}$  in the vector space  $U$  can be written as

$$\underline{v} = v^i \underline{g}_i = v_i \underline{g}^i \quad (\text{B.4})$$

The reciprocal natural base vectors are referred to as covariant vectors while the natural base vectors are referred to as contravariant vectors. Consequently,  $v^i$  is called a

contravariant component and  $v_i$  a covariant component. The gradient is defined as

$$\text{grad } \dot{\underline{x}} = \dot{x}_{i,j} \underline{g}^i \otimes \underline{g}^j \quad (\text{B.5})$$

where  $x_{i,j}$  is the covariant differentiation of the  $i^{\text{th}}$  covariant component of velocity with respect to the  $j^{\text{th}}$  coordinate, and can be written in terms of Christoffel symbols of the second kind as

$$x_{i,j} = \frac{\partial \dot{x}_i}{\partial x^j} - \{ \begin{matrix} l \\ ij \end{matrix} \} \dot{x}_l \quad (\text{B.6})$$

where  $\{ \begin{matrix} l \\ ij \end{matrix} \}$  is the Christoffel symbol of the second kind.

The Christoffel symbols of the second kind are defined as

$$\{ \begin{matrix} l \\ ij \end{matrix} \} = \frac{\partial g_{il}}{\partial x^j} \cdot \underline{g}^l \quad (\text{B.7})$$

Using equations (B.5) and (B.6), the second term on the left hand side of equation (B.1) can be written as

$$(\text{grad } \dot{\underline{x}}) \cdot \dot{\underline{x}} = \left( \frac{\partial \dot{x}_i}{\partial x^j} - \{ \begin{matrix} l \\ ij \end{matrix} \} \dot{x}_l \right) \underline{g}^i (\underline{g}^j \cdot \dot{x}^k \underline{g}_k)$$

or

$$(\text{grad } \dot{\underline{x}}) \cdot \dot{\underline{x}} = \left( \frac{\partial \dot{x}_i}{\partial x^j} \dot{x}^j - \{ \begin{matrix} l \\ ij \end{matrix} \} \dot{x}_l \dot{x}^j \right) \underline{g}^i \quad (\text{B.8})$$

The gradient of a scalar  $p$  is defined as

$$\text{grad } p = \frac{\partial p}{\partial x^i} \underline{g}^i . \quad (\text{B.9})$$

The divergence of a second order tensor  $\underline{T}$  is defined by

$$\text{div}(\underline{T}) = \text{div}(T_{ij} \underline{g}^i \otimes \underline{g}^j) = c_{23} \text{grad}(T_{ij} \underline{g}^i \otimes \underline{g}^j)$$

where  $c_{23}$  is the contraction operator.

Hence,

$$\text{div}(\text{grad } \dot{x}) = \text{div}(\dot{x}_{i,j} \underline{g}^i \otimes \underline{g}^j) = c_{23} \text{grad}(\dot{x}_{i,j} \underline{g}^i \otimes \underline{g}^j)$$

or

$$\begin{aligned} \text{div}(\text{grad } \dot{x}) &= c_{23} \dot{x}_{i,jk} \underline{g}^i \otimes \underline{g}^j \otimes \underline{g}^k \\ &= g^{jk} \dot{x}_{i,jk} \underline{g}^i \end{aligned} \quad (\text{B.10})$$

where  $\dot{x}_{i,jk}$  denotes the covariant differentiation of  $x_{i,j}$  with respect to the  $k^{\text{th}}$  coordinate, and

$$g^{ik} \equiv \underline{g}^j \cdot \underline{g}^k . \quad (\text{B.11})$$

Also,

$$g_{jk} \equiv \underline{g}_j \cdot \underline{g}_k . \quad (\text{B.12})$$

The tensor  $g_{jk}$  is called the metric tensors of the coordinate system. It can be shown that

$$[g^{jk}] = [g_{jk}]^{-1} \quad (\text{B.13})$$

where [ ] denotes a matrix and [ ]<sup>-1</sup> denotes the inverse of a matrix.

Using equation (B.6) and writing  $(x_{i,j})_{,k}$  in terms of Christoffel symbols gives

$$\begin{aligned} \text{div}(\text{grad } \dot{x}) &= g^{jk} \left[ \frac{\partial^2 \dot{x}_i}{\partial x^k \partial x^j} - \frac{\partial}{\partial x^k} \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l \right. \\ &\quad - \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \frac{\partial \dot{x}_l}{\partial x^k} - \{ \begin{smallmatrix} l \\ ik \end{smallmatrix} \} \frac{\partial \dot{x}_l}{\partial x^j} + \{ \begin{smallmatrix} l \\ ik \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ lj \end{smallmatrix} \} \dot{x}_m \\ &\quad \left. - \{ \begin{smallmatrix} l \\ jk \end{smallmatrix} \} \frac{\partial \dot{x}_i}{\partial x^l} + \{ \begin{smallmatrix} l \\ jk \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ il \end{smallmatrix} \} \dot{x}_m \right] g^i. \end{aligned} \quad (\text{B.14})$$

Substituting equations (B.8), (B.9), and (B.14) into equation (B.1) gives the component form of equation (B.1) as

$$\begin{aligned} \frac{\partial \dot{x}_i}{\partial t} + \frac{\partial \dot{x}_i}{\partial x^j} \dot{x}^j - \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l \dot{x}^j &= g^{jk} \left[ \frac{\partial^2 \dot{x}_i}{\partial x^k \partial x^j} \right. \\ &\quad - \frac{\partial}{\partial x^k} \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l - \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \frac{\partial \dot{x}_l}{\partial x^k} - \{ \begin{smallmatrix} l \\ ik \end{smallmatrix} \} \frac{\partial \dot{x}_l}{\partial x^j} \\ &\quad + \{ \begin{smallmatrix} l \\ ik \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ lj \end{smallmatrix} \} \dot{x}_m - \{ \begin{smallmatrix} l \\ jk \end{smallmatrix} \} \frac{\partial \dot{x}_i}{\partial x^l} \\ &\quad \left. + \{ \begin{smallmatrix} l \\ jk \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ il \end{smallmatrix} \} \dot{x}_m \right] - \frac{1}{\rho} \frac{\partial p}{\partial x^i} + b_i \end{aligned} \quad (\text{B.15})$$

where the following equations have been used

$$\underline{\dot{x}} = \dot{x}_i \underline{g}^i$$

and

$$\underline{b} = b_i \underline{g}^i .$$

Since  $i$  is a free index, i.e.,  $i$  does not appear in both the upper and lower index locations in any term of equation (B.15), there are three independent equations corresponding to  $i = 1, 2$  and  $3$ . Equation (B.15) is the component form of the Navier-Stokes equation for any curvilinear coordinate system.

The derivation of the following equation can be found in McConnel [15].

$$\text{div } \underline{\dot{x}} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} (\sqrt{g} \dot{x}^i) \quad (\text{B.16})$$

where

$$g \equiv \det[g_{ij}] . \quad (\text{B.17})$$

Substituting equation (B.16) into equation (B.2) gives the continuity equation in component form for any curvilinear coordinate system as

$$\frac{\partial}{\partial x^i} (\sqrt{g} \dot{x}^i) = 0 \quad (\text{B.18})$$

Since there is not a free index, equation (B.18) provides only one independent equation.

From the definition of the rate-of-deformation tensor  $\underline{D}$ , equation (1.10) of Section II.1,

$$\underline{D} = \frac{1}{2} [\text{grad } \dot{\underline{x}} + (\text{grad } \dot{\underline{x}})^T] \dots \quad (\text{B.20})$$

Substituting equations (B.5) and B.6) into (B.20) gives

$$\underline{D} = \frac{1}{2} \left[ \frac{\partial \dot{x}_i}{\partial x^j} - \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l + \frac{\partial \dot{x}_j}{\partial x^i} - \{ \begin{smallmatrix} l \\ ji \end{smallmatrix} \} \dot{x}_l \right] \underline{g}^i \otimes \underline{g}^j$$

The Christoffel symbols of the second kind can be shown to be symmetric, i.e.,  $\{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} = \{ \begin{smallmatrix} l \\ ji \end{smallmatrix} \}$ . Hence, the last equation can be written as

$$\underline{D} = \frac{1}{2} \left[ \frac{\partial \dot{x}_i}{\partial x^j} + \frac{\partial \dot{x}_j}{\partial x^i} - 2 \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l \right] \underline{g}^i \otimes \underline{g}^j$$

which yields the component form of  $\underline{D}$  in any curvilinear coordinate system as

$$D_{ij} = \frac{1}{2} \left( \frac{\partial \dot{x}_i}{\partial x^j} + \frac{\partial \dot{x}_j}{\partial x^i} \right) - \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_l \dots \quad (\text{B.21})$$

Since  $i$  and  $j$  are free indices, there are nine elements of  $\underline{D}$  corresponding to the combinations of  $i = 1, 2, 3$  and  $j = 1, 2, 3$ .

If the coordinate system is orthogonal, then



$$g^{ij} \equiv \underline{g}^i \cdot \underline{g}^j = 0 \quad \text{if} \quad i \neq j, \quad (\text{B.22})$$

and

$$g_{ij} \equiv \underline{g}_i \cdot \underline{g}_j = 0 \quad \text{if} \quad i \neq j. \quad (\text{B.23})$$

Equations (B.22) and (B.23) together with equation (B.13) provide, for orthogonal coordinates, that

$$g^{ii} = \frac{1}{g_{ii}}. \quad (\text{B.24})$$

The norm or length of a vector  $\underline{g}_i$ , denoted by  $||\underline{g}_i||$ , is defined as

$$||\underline{g}_i|| \equiv (\underline{g}_i \cdot \underline{g}_i)^{1/2} = \sqrt{g_{ii}} \quad (\text{B.25})$$

and likewise

$$||\underline{g}^i|| \equiv (\underline{g}^i \cdot \underline{g}^i)^{1/2} = \sqrt{g^{ii}}. \quad (\text{B.26})$$

Hence, the natural base vectors and the reciprocal base vectors can be converted to a set of orthogonal unit base vectors by

$$\underline{1}_{\langle i \rangle} = \frac{\underline{g}_i}{\sqrt{g_{ii}}} \quad (\text{B.27})$$

and

$$\underline{1}_{\langle i \rangle} = \frac{\underline{g}^i}{\sqrt{g^{ii}}} = \sqrt{g_{ii}} \underline{g}^i. \quad (\text{B.28})$$

It follows that

$$\underline{v} = v_{\langle i \rangle} \underline{l}_{\langle i \rangle} = v^i \underline{g}_i = \sqrt{g_{ii}} v^i \underline{l}_{\langle i \rangle} \quad (\text{B.29})$$

or

$$v^i = \frac{v_{\langle i \rangle}}{\sqrt{g_{ii}}} \quad (\text{no sum}) \quad (\text{B.30})$$

where the brackets  $\langle \rangle$  denote physical components, i.e., the components of a vector written in terms of unit base vectors. The summation convention for physical components is that like indices within  $\langle \rangle$  are summed. Similarly,

$$v_i = v_{\langle i \rangle} \sqrt{g_{ii}} \quad (\text{no sum}). \quad (\text{B.31})$$

Substituting equations (B.27), (B.28), (B.30), and (B.31) into equations (B.15), (B.18), and (B.21), the equations of motion and the rate-of-deformation tensor can be written in terms of physical components (restricted to orthogonal curvilinear coordinates) as

$$\begin{aligned} \sqrt{g_{ii}} \frac{\partial \dot{x}_{\langle i \rangle}}{\partial t} + \frac{\partial}{\partial x^j} (\sqrt{g_{ii}} \dot{x}_{\langle i \rangle}) \frac{\dot{x}_{\langle j \rangle}}{\sqrt{g_{jj}}} - \{ \begin{matrix} l \\ ij \} \frac{\sqrt{g_{ll}}}{\sqrt{g_{jj}}} \\ \\ \dot{x}_{\langle l \rangle} \dot{x}_{\langle j \rangle} = v \left[ \frac{1}{g_{jj}} \frac{\partial^2}{\partial x^j \partial x^j} (\sqrt{g_{ji}} \dot{x}_{\langle i \rangle} \right. \\ \left. - \frac{\sqrt{g_{ll}}}{g_{jj}} \frac{\partial}{\partial x^j} \{ \begin{matrix} l \\ ij \} \dot{x}_{\langle l \rangle} - \frac{2}{g_{jj}} \{ \begin{matrix} l \\ ij \} \frac{\partial}{\partial x^j} (\sqrt{g_{ll}} \dot{x}_{\langle l \rangle}) \right] \end{aligned}$$

$$\begin{aligned}
& + \frac{\sqrt{g_{mm}}}{g_{jj}} \{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ lj \end{smallmatrix} \} \dot{x}_{\langle m \rangle} - \{ \begin{smallmatrix} l \\ jj \end{smallmatrix} \} \frac{1}{g_{jj}} \frac{\partial}{\partial x^l} (\sqrt{g_{ii}} \dot{x}_{\langle i \rangle}) \\
& + \{ \begin{smallmatrix} l \\ jj \end{smallmatrix} \} \{ \begin{smallmatrix} m \\ ll \end{smallmatrix} \} \frac{\sqrt{g_{mm}}}{g_{jj}} \dot{x}_{\langle m \rangle} ] - \frac{1}{\rho} \frac{\partial p}{\partial x^i} \\
& + b_{\langle i \rangle} \sqrt{g_{ii}} , \tag{B.32}
\end{aligned}$$

$$\frac{\partial}{\partial x^i} \left( \frac{\sqrt{g}}{\sqrt{g_{ii}}} \dot{x}_{\langle i \rangle} \right) = 0 , \tag{B.33}$$

and

$$\begin{aligned}
D_{\langle ij \rangle} & = \frac{1}{2\sqrt{g_{ii}g_{jj}}} \left[ \frac{\partial}{\partial x^j} (\dot{x}_{\langle i \rangle} \sqrt{g_{ii}}) \right. \\
& \left. + \frac{\partial}{\partial x^i} (\dot{x}_{\langle i \rangle} \sqrt{g_{jj}}) \right] - \frac{\{ \begin{smallmatrix} l \\ ij \end{smallmatrix} \} \dot{x}_{\langle l \rangle} \sqrt{g_{ll}}}{\sqrt{g_{ii}g_{jj}}} \tag{B.34}
\end{aligned}$$

where the coordinate system has been assumed fixed in time, i.e.,

$$\frac{\partial}{\partial t} (\sqrt{g_{ii}} \dot{x}_{\langle i \rangle}) = \sqrt{g_{ii}} \frac{\partial \dot{x}_{\langle i \rangle}}{\partial t} .$$

For spherical coordinates, the metric tensor is

$$[g_{ij}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & (r \sin \theta)^2 \end{bmatrix} \quad (\text{B.35})$$

where

$$r \equiv x^1, \theta \equiv x^2, \phi \equiv x^3$$

and the non-vanishing Christoffel symbols are

$$\left\{ \begin{matrix} 2 \\ 2 \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 2 \\ 1 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 3 \\ 3 \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 3 \\ 1 \\ 3 \end{matrix} \right\} = \frac{1}{r}, \quad (\text{B.36})$$

$$\left\{ \begin{matrix} 1 \\ 2 \\ 2 \end{matrix} \right\} = -\frac{1}{r}, \quad (\text{B.37})$$

$$\left\{ \begin{matrix} 1 \\ 3 \\ 3 \end{matrix} \right\} = -r \sin^2 \theta, \quad (\text{B.38})$$

$$\left\{ \begin{matrix} 3 \\ 3 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 3 \\ 2 \\ 3 \end{matrix} \right\} = \cot \theta, \quad (\text{B.39})$$

and

$$\left\{ \begin{matrix} 2 \\ 3 \\ 3 \end{matrix} \right\} = -\sin \theta \cos \theta. \quad (\text{B.40})$$

Substituting equations (B.35) through (B.38) into equations

(B.32) through (B.34) and letting  $v_r = \dot{x}_{\langle 1 \rangle}$ ,  $v_\theta = \dot{x}_{\langle 2 \rangle}$ ,

$v_\phi = \dot{x}_{\langle 3 \rangle}$ ,  $\underline{e}_r = \underline{1}_{\langle 1 \rangle}$ ,  $\underline{e}_\theta = \underline{1}_{\langle 2 \rangle}$ , and  $\underline{e}_\phi = \underline{1}_{\langle 3 \rangle}$ , the equations

of motion and the rate-of-deformation tensor in spherical

coordinates can be written as

i=1, r - Component of the Navier-Stokes Equation

$$\begin{aligned}
& \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \\
& = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v_r}{\partial r} \right) \right. \\
& + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial v_r}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_r}{\partial \phi^2} \\
& - \frac{2}{r^2} v_r \frac{\partial v_r}{\partial r} - \frac{2}{r^2} v_r \cot \theta \\
& \left. - \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi} \right] + b_{\langle r \rangle} . \tag{B.41}
\end{aligned}$$

i=2,  $\theta$  - Component of the Navier-Stokes Equation

$$\begin{aligned}
& \frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\phi}{r} - \frac{v_\phi^2 \cot \theta}{r} \\
& = - \frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v_\theta}{\partial r} \right) \right. \\
& + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial v_\theta}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2} \\
& + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi} \left. \right] \\
& + b_{\langle \theta \rangle} . \tag{B.42}
\end{aligned}$$

i=3,  $\phi$  - Component of the Navier-Stokes Equation

$$\begin{aligned}
& \frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi v_r}{r} + \frac{v_\theta v_\phi}{r} \cot \theta \\
& = - \frac{1}{\rho r \sin \theta} \frac{\partial p}{\partial \phi} + \nu \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial v_\phi}{\partial r} \right) \right. \\
& \quad + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial v_\phi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2} \\
& \quad \left. - \frac{v_\phi}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi} \right] \\
& + b_{\langle \phi \rangle} . \tag{B.43}
\end{aligned}$$

Continuity

$$\frac{\partial}{\partial r} (r^2 \sin \theta v_r) + \frac{\partial}{\partial \theta} (r \sin \theta v_\theta) + \frac{\partial}{\partial \phi} (r v_\phi) = 0. \tag{B.44}$$

Rate-of-Deformation Tensor

$$D_{\langle rr \rangle} = \frac{\partial v_r}{\partial r} \tag{B.45}$$

$$D_{\langle r\theta \rangle} = \frac{1}{2} \left[ r \frac{\partial}{\partial r} \left( \frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \tag{B.46}$$

$$D_{\langle r\phi \rangle} = \frac{1}{2} \left[ \frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left( \frac{v_\phi}{r} \right) \right] \tag{B.47}$$

$$D_{\langle\theta\theta\rangle} = \frac{1}{r} \frac{\partial v_{\theta}}{\partial \theta} + \frac{v_r}{r} \quad (\text{B.48})$$

$$D_{\langle\phi\phi\rangle} = \frac{1}{r \sin \theta} \frac{\partial v_{\phi}}{\partial \phi} + \frac{v_r}{r} + \frac{v_{\theta} \cot \theta}{r} \quad (\text{B.49})$$

$$D_{\langle\theta\phi\rangle} = \frac{1}{2} \left[ \frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left( \frac{v_{\phi}}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_{\theta}}{\partial \phi} \right] \quad (\text{B.50})$$

and since  $D$  is symmetric

$$D_{\langle\theta r\rangle} = D_{\langle r\theta\rangle} \quad (\text{B.51})$$

$$D_{\langle\phi r\rangle} = D_{\langle r\phi\rangle} \quad (\text{B.52})$$

$$D_{\langle\phi\theta\rangle} = D_{\langle\theta\phi\rangle} \quad (\text{B.53})$$

## APPENDIX C

### DETAILS OF THE DEVELOPMENT OF THE OUTER PERTURBATION SOLUTION

Equations (1.16) through (1.23) of Chapter II are substituted into equations (1.12) through (1.12) of Chapter III and coefficients of  $\epsilon^0$  are equated to give

#### Governing Equations

$$\frac{\partial^2 u_0}{\partial \xi^2} = 0 \quad (C.1)$$

$$\frac{\partial^2 v_0}{\partial \xi^2} = 0 \quad (C.2)$$

$$\frac{\partial^2 w_0}{\partial \xi^2} = 0 \quad (C.3)$$

$$\frac{\partial v_0}{\partial \xi} = 0 \quad (C.4)$$

#### Boundary Conditions

$$u_0(r,0) = v_0(r,0) = w_0(r,0) = 0 \quad (C.5)$$



$$u_0(r,1) = v_0(r,1) = 0 \quad (C.6)$$

$$w_0(r,1) = r . \quad (C.7)$$

Integrate equation (C.4) to give

$$v_0 = k_0(r) . \quad (C.8)$$

The boundary conditions (C.5), and (C.6) are satisfied if and only if  $k_0(r) = 0$ . Therefore,

$$v_0(r,\xi) = 0 . \quad (C.9)$$

Equation (C.9) also satisfies equation (C.2). Integrate (C.3) twice with respect to  $\xi$  to give

$$w_0(r,\xi) = g_{01}(r)\xi + g_{00}(r) . \quad (C.10)$$

Apply the boundary conditions (C.5) and (C.7) to obtain

$$w_0(r,\xi) = r\xi . \quad (C.11)$$

Integrate equation (C.1) twice with respect to  $\xi$  to give

$$u_0(r,\xi) = f_{01}(r)\xi + f_{00}(r) . \quad (C.12)$$

Apply the boundary conditions (C.5) and (C.6) to give

$$u_0(r,\xi) = 0 . . \quad (C.13)$$

The solution can be summarized as

$$u(r, \xi) = O(\epsilon), \quad (C.14)$$

$$v(r, \xi) = O(\epsilon), \quad (C.15)$$

$$w(r, \xi) = r\xi + O(\epsilon), \quad (C.16)$$

and

$$p^* = p_0^* + O(\epsilon) \quad (C.17)$$

where  $p_0^*$  will come from higher order solutions.

The  $\epsilon^1$  problem can be determined by equating coefficients of  $\epsilon^1$  in the aforementioned substitution. It is

### Governing Equations

$$\frac{\partial^2 u_1}{\partial \xi^2} = 0 \quad (C.18)$$

$$\frac{\partial^2 v_1}{\partial \xi^2} + r \operatorname{Re} \frac{\partial p_0^*}{\partial \xi} = 0 \quad (C.19)$$

$$\frac{\partial^2 w_1}{\partial \xi^2} = 0 \quad (C.20)$$

$$\frac{\partial v_1}{\partial \xi} = 0 \quad (C.21)$$

### Boundary Conditions

$$u_1(r, 0) = v_1(r, 0) = w_1(r, 0) = 0 \quad (C.22)$$

$$u_1(r,1) = v_1(r,1) = w_1(r,1) = 0 \quad (C.23)$$

Since equations (C.21) and (C.18) and the boundary conditions for  $u$  and  $v$ , equations (C.22) and (C.23) are identical to those for the  $\epsilon^0$  problem,

$$u_1(r,\xi) = 0 \quad (C.24)$$

and

$$v_1(r,\xi) = 0 \quad (C.25)$$

Substitute equation (C.25) into equation (C.19) and integrate to obtain

$$p_0^*(r,\xi) = \lambda_0(r) \quad (C.26)$$

where  $\lambda_0(r)$  will be determined by higher order solutions.

Since the governing equation and the boundary conditions for  $w_1(r,\xi)$  are identical to those for  $u_1(r,\xi)$ ,

$$w_1(r,\xi) = 0 \quad (C.27)$$

The  $\epsilon^2$  problem is developed by equating coefficients of the  $\epsilon^2$  terms and substituting the results of the lower order solutions to obtain

### Governing Equations

$$\frac{\partial^2 u_2}{\partial \xi^2} - r^2 \operatorname{Re} \lambda_0'(r) = -r^3 \xi^2 \operatorname{Re} = -r w_0^2 \operatorname{Re} \quad (C.28)$$

$$\frac{\partial^2 v_2}{\partial \xi^2} + r \operatorname{Re} \frac{\partial p_1^*}{\partial \xi} = 0 \quad (\text{C.29})$$

$$\frac{\partial^2 w_2}{\partial \xi^2} = 0 \quad (\text{C.30})$$

$$\frac{\partial v_2}{\partial \xi} = 0 \quad (\text{C.31})$$

### Boundary Conditions

$$u_2(r,0) = v_2(r,0) = w_2(r,0) = 0 \quad (\text{C.32})$$

$$u_2(r,1) = v_2(r,1) = 0 \quad (\text{C.33})$$

$$w_2(r,1) = -\frac{r}{2} \quad (\text{C.34})$$

As in the lower order solutions,

$$v_2(r,\xi) = 0 \quad (\text{C.35})$$

Substitute equation (C.35) into equation (C.29) and integrate with respect to  $\xi$  to obtain

$$p_1^*(r,\xi) = \lambda_1(r) \quad (\text{C.36})$$

where  $\lambda_1(r)$  will be determined by higher order terms.

Integrate (C.30) twice with respect to  $\xi$  and apply the boundary conditions (C.32) and (C.34) to obtain

$$w_2(r, \xi) = -\frac{r}{2} \xi. \quad (\text{C.37})$$

Integrate (C.28) twice with respect to  $\xi$  and apply the boundary conditions (C.32) and (C.33) to obtain

$$u_2(r, \xi) = -\frac{r^3}{12} (\xi^4 - \xi) \text{Re} + \frac{r^2 \cdot \mathfrak{L}_0^1(r)}{2} \text{Re}(\xi^2 - \xi). \quad (\text{C.38})$$

Higher order terms must be obtained to determine  $\mathfrak{L}_0(r)$ .

The third order problem is obtained by equating coefficients of  $\epsilon^3$ . It is

### Governing Equations

$$\frac{\partial^2 u_3}{\partial \xi^2} - r^2 \text{Re} \frac{\partial p_1^*}{\partial r} = 0 \quad (\text{C.39})$$

$$\frac{1}{r^2} \frac{\partial^2 v_3}{\partial \xi^2} - \frac{2}{r^2} \frac{\partial u_2}{\partial \xi} + \frac{\text{Re}}{r} \frac{\partial p_2^*}{\partial \xi} - \frac{\xi^2}{r} \text{Re} \frac{\partial p_0^*}{\partial \xi} = -\frac{w_0 \xi \text{Re}}{r} \quad (\text{C.40})$$

$$\frac{\partial^2 w_3}{\partial \xi^2} = 0 \quad (\text{C.41})$$

$$\frac{\partial}{\partial r} (r^2 u_2) - r \frac{\partial w_3}{\partial \xi} = 0. \quad (\text{C.42})$$

### Boundary Conditions

$$u_3(r, 0) = v_3(r, 0) = w_3(r, 0) = 0 \quad (\text{C.43})$$

$$u_3(r,1) = v_3(v,1) = w_3(r,1) = 0 . \quad (\text{C.44})$$

Substitute the solution for  $u_2$ , equation (C.38) into equation (C.42) and integrate twice with respect to  $\xi$  to give

$$\begin{aligned} v_3(r,\xi) = & -\frac{r^3}{24} (2\xi^5 - 5\xi^7) \text{Re} + \frac{r^2 \ell_0'(r)}{3} (2\xi^3 - 3\xi^2) \text{Re} \\ & + \frac{r^3}{12} \text{Re} \ell_0''(r) (2\xi^3 - 3\xi^2) + k_3(r) . \end{aligned} \quad (\text{C.45})$$

Apply the boundary conditions for  $v_3$  to give

$$k_3(r) = 0 \quad (\text{C.46})$$

and

$$r^2 \ell_0''(r) + 4r \ell_0'(r) = \frac{3}{2} r^2 . \quad (\text{C.47})$$

Equation (C.47) is a linear, non-homogeneous, ordinary differential equation; therefore,

$$\ell_0(r) = \ell_{0H}(r) + \ell_{0p}(r) \quad (\text{C.48})$$

where  $\ell_{0H}(r)$  is the homogeneous solution, and  $\ell_{0p}(r)$  is a particular solution. The homogeneous solution can be obtained by Euler's method. Assume

$$\ell_{0H}(r) = r^{\alpha_0} , \quad (\text{C.49})$$

then substitute (C.49) into (C.48) to obtain:

$$\alpha_0(\alpha_0+3) r^{\alpha_0} = 0. \quad (C.50)$$

Since  $r^{\alpha_0}$  is not in general zero,

$$\alpha_0(\alpha_0+3) = 0$$

and

$$\alpha_0 = 0, -3.$$

So,

$$x_{0H}(r) = A_0 + \frac{B_0}{r^3} \quad (C.51)$$

where  $A_0$  and  $B_0$  are constants. The solution must be bounded at  $r = 0$ ; therefore,  $B_0 = 0$ . Then,

$$x_{0H}(r) = A_0. \quad (C.52)$$

To obtain a particular solution, assume

$$x_{0p}(r) = c_2 r^2 + c_1 r, \quad (C.53)$$

then substitute into equation (C.47) and equate coefficients of like powers of  $r$  to give

$$x_{0p}(r) = \frac{3r^2}{20}. \quad (C.54)$$

Hence,

$$x_0(r) = A_0 + \frac{3r^2}{20}. \quad (C.55)$$

Substitute equation (C.55) into equations (C.26), (C.38), and (C.45) to give

$$p_0^*(r) = A_0 + \frac{3r^2}{20}, \quad (C.56)$$

$$u_2(r, \xi) = -\frac{r^3}{60} (5\xi^4 - 9\xi^2 + 4\xi) \text{Re}, \quad (C.57)$$

and

$$v_3(r, \xi) = -\frac{r^3}{12} (\xi^5 - 3\xi^3 + 2\xi^2) \text{Re}. \quad (C.58)$$

Substitute equations (C.11), (C.56), (C.57), and (C.58) into equation (C.40) and integrate with respect to  $\xi$  to give

$$p_2^*(r, \xi) = -\frac{r^2}{20} (9\xi^2 - 4\xi) \text{Re} + \text{Re } \lambda_2(r) \quad (C.59)$$

where  $\lambda_2(r)$  can be determined by higher order solutions. As in the  $\epsilon^1$  problem, equations (C.41), (C.43), and (C.44) imply

$$w_3(r, \xi) = 0. \quad (C.60)$$

Substitute equation (C.36) into equation (C.39), integrate twice with respect to  $\xi$  and apply the boundary conditions to give

$$u_3(r, \xi) = \frac{r^2 \text{Re } \lambda_1'(r)}{2} (\xi^2 - \xi). \quad (C.61)$$

The current solution can be summarized by



$$u(r, \epsilon) = -\frac{r^3}{60} (5\epsilon^4 - 9\epsilon^2 + 4\epsilon) \text{Re } \epsilon^2 + \frac{r^2 \ell_1'(r)}{2} (\epsilon^2 - \epsilon) \epsilon^3 + O(\epsilon^4), \quad (\text{C.62})$$

$$v(r, \epsilon) = -\frac{r^3}{12} (\epsilon^5 - 3\epsilon^3 + 2\epsilon^2) \text{Re } \epsilon^3 + O(\epsilon^4), \quad (\text{C.63})$$

$$w(r, \epsilon) = r\epsilon - \frac{r}{2} \epsilon \epsilon^2 + O(\epsilon^4), \quad (\text{C.64})$$

and

$$p^*(r, \epsilon) = A_0 + \frac{3r^2}{20} + \ell_1(r)\epsilon + \ell_2(r)\epsilon^2 + O(\epsilon^3). \quad (\text{C.65})$$

The effect of inertia (terms which contain Re) on  $w$  has not as yet been ascertained. Hence, higher order solutions must be obtained.

The  $\epsilon^4$  problem is defined by equating coefficients of  $\epsilon^4$ . It is

### Governing Equations

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial u_2}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_4}{\partial \epsilon^2} - \frac{\epsilon}{r^2} \frac{\partial u_2}{\partial \epsilon} - \frac{2u_2}{r^2} + \frac{2}{r^2} \frac{\partial v_3}{\partial \epsilon} - \frac{\partial p_2^*}{\partial r} \text{Re} = -\frac{2w_0 w_2}{r} \text{Re} \quad (\text{C.66})$$

$$\frac{1}{r^2} \frac{\partial^2 v_4}{\partial \epsilon^2} - \frac{2}{r^2} \frac{\partial u_3}{\partial \epsilon} + \frac{1}{r} \frac{\partial p_3^*}{\partial \epsilon} \text{Re} = 0 \quad (\text{C.67})$$

$$\begin{aligned}
& \frac{1}{r^2} \frac{\partial^2 w_4}{\partial \xi^2} - \frac{\xi^2}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial w_0}{\partial r} \right) - \frac{\xi^2}{r^2} \frac{\partial^2 w_2}{\partial \xi^2} + \frac{\xi^4}{3r^2} \frac{\partial^2 w_0}{\partial \xi^2} \\
& - \frac{\xi}{r^2} \frac{\partial w_2}{\partial \xi} + \frac{2}{3} \frac{\xi^3}{r^2} \frac{\partial w_0}{\partial \xi} - \frac{w_2}{r^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial w_2}{\partial r} \right) \\
& = u_2 \frac{\partial w_0}{\partial r} \operatorname{Re} + \frac{w_0 u_2}{r} \operatorname{Re} - \frac{v_3}{r} \frac{\partial w_0}{\partial \xi} \operatorname{Re} \quad (\text{C.68})
\end{aligned}$$

$$\frac{\partial}{\partial r} (r^2 u_3) - r \frac{\partial v_4}{\partial \xi} = 0 \quad (\text{C.69})$$

### Boundary Conditions

$$u_4(r,0) = v_4(r,0) = w_4(r,0) = 0 \quad (\text{C.70})$$

$$u_4(r,1) = v_4(r,1) = 0 \quad (\text{C.71})$$

$$w_4(r,1) = \frac{r}{4\Gamma} \quad (\text{C.72})$$

Substitute equation (C.61) into equation (C.69) and integrate with respect to  $\xi$  to obtain

$$\begin{aligned}
v_4(r,\xi) &= \frac{r^2}{3} \operatorname{Re} \lambda_1'(r) (2\xi^3 - 3\xi^2) + \frac{r^3}{12} \lambda_1''(r) (2\xi^3 - 3\xi^2) \\
&+ k_4(r) \quad (\text{C.73})
\end{aligned}$$

Application of the boundary conditions for  $v_4(r,\xi)$  provides

$$k_4(r) = 0 \quad (C.74a)$$

and

$$r^2 \ell_1''(r) + 4r \ell_1'(r) = 0 . \quad (C.74b)$$

Equation (C.74b) is a linear, homogeneous ordinary differential equation and can be solved by the same method which was used to solve for  $\ell_{0H}(r)$  to obtain

$$\ell_1(r) = A_1 \quad (C.75)$$

where  $A_1$  is a constant.

It follows from equations (C.56), (C.61), (C.73), (C.74a), and (C.75) that

$$p_1^*(r, \xi) = A_1 , \quad (C.76)$$

$$u_3(r, \xi) = 0 , \quad (C.77)$$

and

$$v_4(r, \xi) = 0 . \quad (C.78)$$

Substitute equations (C.77) and (C.78) into equations (C.66) and integrate with respect to  $\xi$  to give

$$p_3^*(r, \xi) = \ell_3(r) . \quad (C.79)$$

Substitute the solutions for  $w_0$ ,  $w_2$ ,  $w_3$ , and  $v_2$  and  $v_4$  into equation (C.66); integrate twice with respect to  $\xi$ , and apply the boundary conditions for  $w_4(r, \xi)$  to obtain

$$w_4(r, \xi) = \frac{r}{120} (\xi^5 - 3\xi) - \frac{r^3}{25,200} (50\xi^7 - 63\xi^5 - 70\xi^5 + 83\xi) \text{Re}^2. \quad (\text{C.80})$$

In an analogous manner,  $u_4(r, \xi)$  can be obtained from equation (C.66) as

$$u_4(r, \xi) = \frac{r^3}{180} (8\xi^6 - 39\xi^4 + 50\xi^3 - 19\xi) \text{Re} + \frac{\text{Re } \lambda_2'(r)}{2} \frac{r^2}{r^2} (\xi^2 - \xi). \quad (\text{C.81})$$

To determine  $\lambda_2(r)$ , the solution for  $v_5$  must be obtained. The equation for  $v_5$  and the boundary conditions are obtained by equating coefficients of  $\xi^5$  in the continuity equation and the boundary conditions for  $v_5$ . They are

#### Governing Equation

$$\frac{\partial}{\partial r} (r^2 u_4) + r \xi v_3 - r \frac{\partial v_5}{\partial \xi} = 0. \quad (\text{C.82})$$

#### Boundary Conditions

$$v_3(r, 0) = v_3(r, 1) = 0. \quad (\text{C.83})$$

Substituting the equations for  $u_4$  and  $v_3$  into equation (C.82) and integrating with respect to  $\xi$  gives

$$v_5(r, \xi) = \frac{r^3}{504} (10\xi^7 - 84\xi^5 + 154\xi^4 - 133\xi^2) \text{Re}$$

$$\begin{aligned}
& + \frac{\ell_2'(r)}{3} \operatorname{Re} r^2 (2\xi^3 - 3\xi^2) + \frac{\ell_2''(r)}{12} \operatorname{Re} r^3 (2\xi^3 - 3\xi^2) \\
& + k_5(r) . \quad (C.84)
\end{aligned}$$

Application of the boundary conditions gives

$$k_5(r) = 0 \quad (C.85)$$

and

$$r^2 \ell_2''(r) + 4\ell_2''(r) r = -\frac{53r^2}{420} \quad (C.86)$$

which can be solved in the same manner as was  $\ell_0(r)$  to give

$$\ell_2(r) = A_2 - \frac{53r^2}{420} \quad (C.87)$$

where  $A_2$  is a constant. Equations (C.59), (C.81), and (C.85) can now be substituted into equation (C.87) to give

and (C.87) gives

$$p_2^*(r) = A_2 \operatorname{Re} - \frac{r^2}{420} (189\xi^2 - 84\xi + 53) \operatorname{Re} , \quad (C.88)$$

$$u_4(r, \xi) = \frac{r^3}{1260} (56\xi^6 - 273\xi^4 + 350\xi^3 - 159\xi^2 + 26\xi) \operatorname{Re}, \quad (C.89)$$

and

$$v_5(r, \xi) = \frac{r^3}{252} (5\xi^7 - 42\xi^5 + 77\xi^3 + 53\xi^2 + 13\xi^2) . \quad (C.90)$$

The following cyclic behavior of the solution can be observed:

$$v_n(r, \xi) = 0 \quad \text{for even } n$$

$$w_n(r, \xi) = u_n(r, \xi) = 0 \quad \text{for odd } n.$$

and can be approximated by

$$\begin{aligned} u(r, \xi) = & -\frac{r^3}{60} (5\xi^4 - 9\xi^2 + 4\xi) \operatorname{Re} \epsilon^2 + \frac{r^3}{1260} (56\xi^6 - 273\xi^4 \\ & + 350\xi^3 - 159\xi^2 + 26\xi) \operatorname{Re} \epsilon^4 + O(\epsilon^6), \end{aligned} \quad (\text{C.91})$$

$$\begin{aligned} v(r, \xi) = & -\frac{r^3}{12} (\xi^5 - 3\xi^3 + 2\xi^2) \operatorname{Re} \epsilon^3 \\ & + \frac{r^3}{252} (5\xi^7 - 42\xi^5 + 77\xi^4 - 53\xi^3 + 13\xi^2) \operatorname{Re} \epsilon^5 \\ & + O(\epsilon^7), \end{aligned} \quad (\text{C.92})$$

$$\begin{aligned} w(r, \xi) = & r\xi - \frac{r}{2} \xi \epsilon^2 + \frac{r}{120} (\xi^5 - 3\xi) \epsilon^4 \\ & - \frac{r^3}{25,200} (50\xi^7 - 63\xi^5 - 70\xi^4 + 83\xi) \operatorname{Re}^2 \epsilon^4 \\ & + O(\epsilon^6), \end{aligned} \quad (\text{C.93})$$

and

$$\begin{aligned} p^*(r, \xi) = & A_0 + \frac{3r^2}{20} + A_1 \epsilon + A_2 \operatorname{Re} \epsilon^2 \\ & - \frac{r^2}{420} (189\xi^2 - 84\xi + 53) \operatorname{Re} \epsilon^2 + O(\epsilon^3). \end{aligned} \quad (\text{C.94})$$

## APPENDIX D

THE ALGORITHM FOR SOLVING TRIDIAGONAL  
SYSTEMS OF EQUATIONS

The following development is from Peaceman [22].

Consider the general tridiagonal system of equations.

$$\begin{array}{r}
 \left[ \begin{array}{l}
 b_1 x_1 + c_1 x_2 \\
 a_2 x_1 + b_2 x_2 \\
 a_3 x_2 + b_3 x_3 + c_3 x_4 \\
 \vdots \\
 a_{M-1} x_{M-2} + b_{M-1} x_{M-1} + c_{M-1} x_M \\
 a_M x_{M-1} + b_M x_M
 \end{array} \right] = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_{M-1} \\ d_M \end{bmatrix}
 \end{array}$$

which can be written in direct form as

$$Ax = d \tag{D.1}$$

where





and an upper triangular U with ones on the diagonal. Then,

$$A = LU \quad (D.5)$$

where

$$L = \begin{bmatrix} \beta_1 & & & & & \\ a_2 & \beta_2 & & & & \\ & a_3 & \beta_3 & & & \\ & & & \ddots & & \\ & & & & a_M & \beta_M \end{bmatrix} \quad (D.6)$$

and

$$U = \begin{bmatrix} 1 & w_1 & & & & \\ & 1 & w_2 & & & \\ & & 1 & w_3 & & \\ & & & & \ddots & \\ & & & & & 1 & w_{M-1} \\ & & & & & & 1 \end{bmatrix} \quad (D.7)$$

Substituting (D.2), (D.6), and (D.7) into (D.5) and performing the indicated matrix multiplication gives

$$\begin{aligned} b_1 &= \beta_1 & ; & c_1 = \beta_1 w_1 \\ a_i w_{i-1} + \beta_i &= b_i; & c_i &= \beta_i w_i & \quad 2 \leq i \leq M-1 \\ a_M w_{M-1} + \beta_M &= b_M \end{aligned} \quad (D.8)$$

which can be solved for the  $\beta_i$ 's and the  $w_i$ 's. Now, define a vector  $\gamma$  such that

$$UX = \gamma \quad (D.9)$$

where

$$\gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \cdot \\ \cdot \\ \gamma_M \end{bmatrix} \quad (D.10)$$

Substituting (D.5) and (D.9) into (D.1) then gives

$$L\gamma = d \quad (D.11)$$

into which the substitution of (D.6) and (D.10) and the subsequent matrix multiplication yields

$$\begin{aligned} \beta_1 \gamma_1 &= d_1 \\ a_i \gamma_{i-1} + \beta_i \gamma_i &= d_i \quad 2 \leq i \leq M. \end{aligned} \quad (D.12)$$

Since, the  $\beta_i$ 's are known, equations (D.12) can be solved for the  $\gamma_i$ 's. Substituting equations (D.5) and (D.7) into (D.9) and performing the indicated matrix multiplication gives

$$\begin{aligned} x_i &= w_i x_{i+1} = \gamma_i \quad 1 \leq i \leq M-1 \\ x_M &= \gamma_M, \end{aligned} \quad (D.13)$$

and since the  $\gamma_i$ 's and  $w_i$ 's are now known, equation (D.13) can be solved for the  $x_i$ 's by starting from  $i = M$  and proceeding backwards until  $i = 1$ .

The algorithm for the solution of the  $x_i$ 's is as follows:

In order of increasing  $i$  (forward solution)

$$1. \quad \beta_1 = b_1$$

$$w_1 = \frac{c_1}{\beta_1}$$

$$2. \quad \beta_i = b_i - a_i w_{i-1}$$

$$2 \leq i \leq M$$

$$w_i = \frac{c_i}{\beta_i}$$

$$3. \quad \gamma_1 = \frac{d_1}{\beta_1}$$

$$4. \quad \gamma_i = \frac{d_i - a_i \gamma_{i-1}}{\beta_i} \quad 2 \leq i \leq M$$

Then in order of decreasing  $i$  (back solution)

$$5. \quad x_M = \gamma_M$$

$$6. \quad x_i = \gamma_i - w_i x_{i+1} \quad 1 \leq i \leq M-1 .$$

APPENDIX E

LISTING OF FORTRAN IV PROGRAM  
Nomenclature

<u>FORTRAN Name</u>	<u>Variable</u>
A(I,J)	$a_{i,j}$
ALPHA1	$\alpha_1$
ALPHA2	$\alpha_2$
ALPHA3	$\alpha_3$
ALPHA4	$\alpha_4$
ALPHA5	$\alpha_5$
ALPHA6	$\alpha_6$
ALPHA7	$\alpha_7$
ALPHA8	$\alpha_8$
ALPHA9	$\alpha_9$
ALPHA0	$\alpha_{10}$
APRIM(I,J)	$a'_{i,j}$
APRIM2(I,J)	$a''_{i,j}$
AZ	a
B(I,J)	$b_{i,j}$
BETA(J)	$\beta_j$
BPRIM(I)	$b'_i$
BPRIM2(I,J)	$b''_i$
BZ	b

C(I,J)	$c_{i,j}$
CHI(I,J)	$x_{i,j}$
CPRIM(I,J)	$c'_{i,j}$
CPRIM2(I,J)	$c''_{i,j}$
D(I,J)	$d_{i,j}$
DELTAB	$\Delta\beta$
DELTAT	$\Delta t$
DELTAT1	$\Delta t_1$
DELTAZ	$\Delta z$
DPP(I,J)	$\left  \frac{D_{\phi\phi}}{Dp} \right _{i,j}$
DPR(I,J)	$\left  \frac{D_{\phi r}}{Dp} \right _{i,j}$
DPRIM(I,J)	$d'_{i,j}$
DPRIM2(I,J)	$d''_{i,j}$
DRR(I,J)	$\left  \frac{D_{rr}}{Dp} \right _{i,j}$
DRT(I,J)	$\left  \frac{D_{r\theta}}{Dp} \right _{i,j}$
DFF(I,J)	$\left  \frac{D_{\theta\phi}}{Dp} \right _{i,j}$
DTT(I,J)	$\left  \frac{D_{\theta\theta}}{Dp} \right _{i,j}$
E(I,J)	$e_{i,j}$
EPRIM(I,J)	$e'_{i,j}$

EPRIM2(I,J)	$e''_{i,j}$
EPSLON	$\epsilon$ in radians
ETA(I,J)	$\eta_{i,j}$
F(I)	$f_i$
FPRIM(I)	$f'_i$
FPRIM2(I)	$f''_i$
G(I,J)	$g_{i,j}$
GAMMA(I,J)	$\Gamma_{i,j}$
GAMMAN(I,J)	$\Gamma^n_{i,j}$
GAPANG	$\epsilon$ in degrees
GPRIM(I,J)	$g'_{i,j}$
GPRIM2(I,J)	$g''_{i,j}$
H(I,J)	$h_{i,j}$
HPRIM(I,J)	$h'_{i,j}$
HPRIM2(I,J)	$h''_{i,j}$
I	$i$
IDEF	idef
IDSN	idsn
ILIB	ilib
IPRINT	iprint
IPROB	iprob
ITORQ	itorq
J	$j$
J12	$(N+1)/2$
K	$k$
KMAX	kmax

L	$l$
LC	$lc$
LCHC	$lchc$
LMAX	$lmax$
LTO	$lt_0$
M	M
ME	Me
MM1	M-1
MM2	M+2
MP1	M+1
N	N
NM1	N-1
NM2	N-2
NPROB	nprob
NZ	n
NZC	nc
NZCHC	nchc
NZMAX	nmax
NZP	np
NZPR	npr
PHI (I, J)	$\phi_{i,j}$
$\bar{P}HIN(I, J)$	$\phi_{i,j}^n$
PSI (I, J)	$\psi_{i,j}$
PSIL(I, J)	$\psi_{i,j}^l$
PSIMAX	$\psi_{max}$

R(I)	$r_i$
RE	Re
SMALLG	$\epsilon_g$
SMALLP	$\epsilon_p$
SMALLZ	$\epsilon_z$
TORQC	$\frac{T}{T_p} \Big _{\text{cone}}$
TORQP	$\frac{T}{T_p} \Big _{\text{plate}}$
H(I,J)	$u_{i,j}$
UK(I,J)	$(u_{i,j}^{n+1})^k$
UN(I,J)	$u_{i,j}^n$
V(I,J)	$v_{i,j}$
VN(I,J)	$v_{i,j}^n$
Z(I)	$z_i$
ZETA(I,J)	$\zeta_{i,j}$
ZETAN(I,J)	$\zeta_{i,j}^n$



C NUMERICAL SOLUTION OF THE FLOW OF A NEWTONIAN, INCOMPRESSIBLE FLUID  
C SHEARED IN A CONE AND PLATE VISCOMETER USING AN METHOD

C

```

DIMENSION ZETAN(21,21),UN(21,21),U(21,21),VN(21,21),V(21,21),GAMMA
1 N(21,21),CAMP(21,21),PHIN(21,21),PHI(21,21),CHI(21,21),Z(21),R(
2 21),ETA(21,21),UK(21,21),A(21,21),B(21,21),C(21,21),D(21,21),E(2
3 1,21),F(21),G(21,21),H(21,21),APRIM2(21,21),BPRIM2(21,21),CPRIM2
4 (21,21),DPRIM2(21,21),EPRIM2(21,21),FPRIM2(21),GPRIM2(21,21),HPR
5 I42(21,21),HPRIM(21,21),ZT(21),Y(21)
COMMON /PSI1/APRIM(21),BPRIM(21),CPRIM(21),EPRIM(21,21),FPRIM(21),
1 CPHI(21,21),WI(21),BETA(21),WJ(21,21),BETAJ(21,21)/PSI2/DELTA1
2 ,ZETA(21,21),FS(21,21),EETA(21),DPRIM(21,21)/SUB1/XBETA(21,21),
3 ,GAMMA(21,21),X(21,21)/SLB2/M,MM1,MM2,N,MM1,MM2/DEF/DRR(21,
4 21),DTT(21,21),JPH(21,21),DHT(21,21),DTP(21,21),DPR(21,21),EX(21
5 ),AZ,BZ
EQUIVALENCE (A(1),APRIM2(1),DPR(1)),(B(1),EPRIM2(1),DTT(1)),(C(1),
1 CPRIM2(1),DPR(1)),(D(1),CPRIM(1),DPRIM2(1)),(H(1),HPRIM(1),HPRIM2
2 (1)),(F(1),EPRIM2(1),CPT(1)),(F(1),FPRIM2(1)),(G(1),GPRIM2(1),DTP
3 (1)),(ZETAN(1),DPR(1)),(ZT(1),UN(1)),(Y(1),VN(1))

```

C

C READ AND THEN PRINT INFLT

C

```

READ(5,4)NPPROB,ILIE
4 FORMAT(2I5)
5 READ(5,6)GAPANG,RE
6 FORMAT(F5.3,E15.5)
REA(5,4)IPROB,M,ME,N,NZPR,KMAX,NZCHC,NZMAX,LCMC,LMAX,ITORG,IPRINT
1 ,IDFF,IDSN
4 FORMAT(14I5)
REA(5,7)DELTAT,DELTA1,SMALLZ,SMALLP,SMALLG
7 FORMAT(5F10.5)
WRITE(6,1200)IPROB
1200 FORMAT(' IPROB NUMBER = ',I5/)
WRITE(6,1201)
1201 FORMAT(' PHYSICAL PARAMETERS'// '-----'//)
WRITE(6,7)GAPANG,RE
7 FORMAT(' GAP ANGLE = ',F8.4,2X,' DEGREES'// ' REYNOLDS NUMBER = '
1 ,IPE13.5)
WRITE(6,1202)
1202 FORMAT(' NUMERICAL PARAMETERS'// '-----'//)
WRITE(6,1203)M,ME,N,DELTAT,DELTA1,ILIE,SMALLZ,SMALLP,SMALLG,NZPR,
1 NZCHC,NZMAX,LCMC,LMAX,ITORG,KMAX,IDSN,IDEF,IPRINT,NPROB
1203 FORMAT(' M = ',I4,11X,' ME = ',I3,12X,' N = ',I4/
1 ' DELTAT = ',F10.6,5X,' DELTA1 = ',F10.6,5X,' ILIE = ',I3/
2 ' SMALLZ = ',F10.6,5X,' SMALLP = ',F10.6,5X,' SMALLG = ',F10.6/
3 ' NZPR = ',I4,11X,' NZCHC = ',I3,12X,' NZMAX = ',I5/
4 ' LCMC = ',I3,12X,' LMAX = ',I4,11X,' ITORG = ',I3/
5 ' KMAX = ',I3,12X,' IDSN = ',I3,12X,' IDEF = ',I3/
6 ' IPRINT = ',I3,12X,' NPROB = ',I3//)
WRITE(6,1204)
1204 FORMAT(' OUTPUT'// '-----'//)

```

C

C CONVERT GAP ANGLE TO RADIANS; COMPUTE THE STEPSIZES IN THE Z  
C AND BETA DIRECTIONS; IFRATEL TO COMPUTE AZ AND BZ

C

```

MM1 = M + 1
MM2 = M - 1
MM3 = M - 2

```

```

NM1 = N - 1
NM2 = N - 2
DELTAZ = 1./NM1
EPSLON = GAPANC * 3.141593 / 100.
DELTAB = EPSLON/NM1
IAZ = 0
AZ1 = 2.0
10 IAZ = IAZ + 1
AZ = ALOG((EXP(AZ1*(NE-1)*DELTAZ) - 1.)/EPSLON + 1.)
IF(ABS(AZ1-AZ) .LT. .001) GO TO 15
AZ1 = AZ
IF(IAZ .LT. 50) GO TO 10
WRITE(6,12)IAZ
12 FORMAT(/' ***** IAZ =',I2,JX,'THE ITERATION FOR AZ FAILED *****'/)
GO TO 1005
15 BZ = 1./(EXP(AZ) - 1.)
C
C INITIALIZE VELOCITIES, VORTICITIES, AND THE STREAMFUNCTION
C
IF(IP=CB .GT. 1 .AND. NPNDE .GT. 1) GO TO 28
DU 25 I = 1,M
OU 25 J = 1,N
CHI(I,J) = 0.0
FTA(I,J) = 0.0
ZETA(I,J) = 0.0
U(I,J) = 0.0
UK(I,J) = 0.0
V(I,J) = 0.0
GAMMA(I,J) = 0.0
PSI(I,J) = 0.0
PHI(I,J) = 0.0
25 CONTINUE
IF(ILIB .EQ. 0) GO TO 28
26 READ(1)((PSI(I,J),U(I,J),V(I,J),GAMMA(I,J),ZETA(I,J),
1 ZETA(I,J),CHI(I,J),J=1,N),I=1,M)
28 DO JC I = 1,M
Z(I) = (I-1)*DELTAZ
EX(I) = EXP(AZ+Z(I))
R(NP1-I) = 1. - BZ*(EX(I) - 1.)
GAMMA(I,N) = CCS(EPSLON)*R(NP1-I)**2
30 CONTINUE
R(1) = 0.0
GAMMA(M,N) = 0.0
DO 34 J = 1,N
ZETA(J) = (J-1)*DELTAB
34 CONTINUE
C
C COMPUTE THE RATIOS OF THE STEPSIZES
C
ALPHA1 = DELTAT/(4.*AZ*HZ*DELTAZ)
ALPHA2 = DELTAT/(4.*DELTAB)
ALPHA3 = DELTAT/(2.*RE*(AZ*EZ*DELTAZ)**2)
ALPHA4 = DELTAT/(4.*RE*AZ*DELTAZ*BZ**2)
ALPHA5 = DELTAT/(2.*RE*DELTAB**2)
ALPHA6 = DELTAT/(4.*RE*DELTAB)
ALPHA7 = DELTAT/(2.*(AZ*EZ*DELTAZ)**2)
ALPHA8 = DELTAT/(4.*AZ*DELTAZ*HZ**2)
ALPHA9 = DELTAT/(2.*DELTAB**2)

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      ALPHA2 = DELTA1/(4.*CLTAB)
C
C COMPUTE THE INITIAL CONDITIONS ON PHI
C
      IF((IPROB .GT. 1 .AND. NPROE .GT.1) GO TO 38
      IF((ILIP .EQ. 1) GO TO 36
      DO 35 I = 1,MM1
      ETA(I,N) = -2./R(MP1-I)
  35 CONTINUE
  36 DO 37 I = 2,MM1
      DO 37 J = 2,MM1
      PHI(I,J) = -ALPHA1*(GAMMA(I-1,J)*CHI(I-1,J)
  1          - GAMMA(I+1,J)*CHI(I+1,J))/EX(I)
  2          + ALPHA2*(GAMMA(I,J+1)*ETA(I,J+1)
  3          - GAMMA(I,J-1)*ETA(I,J-1))
  37 CONTINUE
C
C COMPUTE THE TIME INDEPENDENT COEFFICIENTS OF THE SIX TRIDIAGONAL
C MATRICES ASSOCIATED WITH COMPUTING ZETA, PSI, AND GAMMA
C AT THE ENDS OF BOTH THE HALF AND FULL TIME STEPS
C
  38 DO 39 I = 2,MM1
      B(I,2) = -1. - 2.*ALPHA3/EX(I)**2
      F(I) = -1. - 2.*ALPHA5/R(MP1-I)**2
      APRIM(I) = (ALPHA7 + ALPHA8)/EX(I)**2
      BPRIM(I) = -1. - 2.*ALPHA7/EX(I)**2
      CPRIM(I) = (ALPHA7 - ALPHA8)/EX(I)**2
      FPRIM(I) = -1. - 2.*ALPHA9/R(MP1-I)**2
      DO 39 J = 2,MM1
      EPRIM(I,J) = (ALPHA9 - ALPHA10*TAN(BETA(J)))/R(MP1-I)**2
      GPRIM(I,J) = (ALPHA5 + ALPHA10*TAN(BETA(J)))/R(MP1-I)**2
      B(I,J) = B(I,2)
  39 CONTINUE
C
C COMPUTE THE TIME INVARIANT PORTIONS OF THE TRIDIAGONAL ALGORITHMS FOR
C COMPUTING PSI AT THE END OF THE TIME STEP
C
      CALL PSIA
      NZ = 0
      NZC = 0
      NZP = 0
C
C BEGIN THE ITERATION TO STEADY STATE
C
  40 NZ = NZ + 1
      NZC = NZC + 1
      NZP = NZP + 1
      K = 0
      LTO = 0
      DO 41 J = 1,N
      DO 41 I = 1,M
      ZETAN(I,J) = ZETA(I,J)
      UN(I,J) = U(I,J)
      VN(I,J) = V(I,J)
      GAMMAN(I,J) = GAMMA(I,J)
      PHIN(I,J) = PHI(I,J)
  41 CONTINUE
  42 K = K + 1

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C
C COMPUTE THE BOUNDARY CONDITIONS ON ZETA ( ZETA AT P = 0 IS EQUAL TO
C ZERO FOR ALL TIMES )
C
  DO 50 I = 1,MM1
    ZETA(I,1) = (PSI(I,2) - 8.*PSI(I,2))/(2.*(R(MP1-I)*DELTA)**2)
    ZETA(I,N) = (PSI(I,MM2) - 8.*PSI(I,MM1))/(2.*COS(EPSLON)*(R(MP1-I)
    *DELTA)**2)
  50 CONTINUE
  DO 60 J = 2,MM1
    ZETA(1,J) = 2.*PSI(2,J)/(COS(BETA(J))*(1. - R(MM1))*R(MM1))
    ZETA(1,J) = (ZETA(1,J) + ZETAN(1,J))/2.
  60 CONTINUE
C
C COMPUTE THE TIME DEPENDENT COEFFICIENTS OF THE TRIDIAGONAL MATRIX
C FOR COMPUTING ZETA AT THE ENDS OF BOTH THE HALF AND FULL TIME STEPS
C
  DO 70 J = 2,MM1
    DO 70 I = 2,MM1
      A(I,J) = (ALPHA3 + ALPHA4)/EX(I)**2 - ALPHA1*U(I-1,J)/EX(I)
      C(I,J) = (ALPHA3 - ALPHA4)/EX(I)**2 + ALPHA1*U(I+1,J)/EX(I)
      E(I,J) = (ALPHA5 - ALPHA6*TAN(BETA(J)))*COS(BETA(J-1))/(R(MP1-I)**
      *COS(BETA(J))) - ALPHA2*V(I,J-1)/R(MP1-I)
      G(I,J) = (ALPHA5 + ALPHA6*TAN(BETA(J)))*COS(BETA(J+1))/(R(MP1-I)**
      *COS(BETA(J))) + ALPHA2*V(I,J+1)/R(MP1-I)
    70 CONTINUE
C
C COMPUTE THE RIGHT HAND VECTOR FOR COMPUTING ZETA
C AT THE END OF THE HALF TIME STEP
C
  DO 90 J = 2,MM1
    DO 80 I = 2,MM1
      D(I,J) = -ZETAN(I,J-1)*E(I,J) - ZETAN(I,J)*(2. + F(I))
      *ZETAN(I,J+1)*G(I,J) + PHI(I,J)
    80 CONTINUE
    D(2,J) = D(2,J) - A(2,J)*ZETA(1,J)
  90 CONTINUE
C
C COMPUTE ZETA AT THE END OF THE HALF TIME STEP
C
  CALL TRIDAZ(A,B,C,D,ZETA)
C
C COMPUTE THE RIGHT HAND VECTOR FOR COMPUTING ZETA
C AT THE END OF THE TIME STEP
C
  DO 110 I = 2,MM1
    DO 100 J = 2,MM1
      H(I,J) = -ZETA(I-1,J)*A(I,J) - ZETA(I,J)*(2. + B(I,J))
      *ZETA(I+1,J)*C(I,J) + PHI(I,J)
    100 CONTINUE
    H(I,2) = H(I,2) - E(I,2)*ZETA(I,1)
    H(I,MM1) = H(I,MM1) - G(I,MM1)*ZETA(I,N)
  110 CONTINUE
  DO 120 J = 2,MM1
    ZETA(1,J) = 2.*ZETA(1,J) - ZETAN(1,J)
  120 CONTINUE
C
C COMPUTE ZETA AT THE END OF THE TIME STEP

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C
C CALL TRICAN(F,F,G,H,ZETA)
C
C COMPUTE PSI AT THE END OF THE TIME STEP
C
C CALL PSI(SMALLF,L,LPAR,LCFC,II,JJ,PSIMAX)
C
C COMPUTE THE TIME DEPENDENT COEFFICIENTS An, Cn, En, AND Gn OF THE TWO
C TRIDIAGONAL MATRICES ASSOCIATED WITH COMPUTING GAMMA AT THE ENDS
C OF BOTH THE HALF AND FULL TIME STEPS
C
C DO 170 J = 2,NM1
C DO 170 I = 2,NM1
  APRIN2(I,J) = (ALPHA7 + ALPHA4)/EX(I)**2 - ALPHA10L(I,J)/EX(I)
  CPRIN2(I,J) = (ALPHA3 - ALPHA6)/EX(I)**2 + ALPHA10U(I,J)/EX(I)
  EPRIN2(I,J) = (ALPHA5 - ALPHACOTAN(ETA(J)))/R(NP1-I)**2 - ALPHA20
  V(I,J)/R(NP1-I)
  GPRIN2(I,J) = (ALPHA5 + ALPHACOTAN(ETA(J)))/R(NP1-I)**2 + ALPHA20
  V(I,J)/R(NP1-I)
170 CONTINUE
C
C COMPUTE THE RIGHT HAND VECTOR FOR COMPUTING
C GAMMA AT THE END OF THE HALF TIME STEP
C
C DO 180 J = 2,NM1
C DO 180 I = 2,NM1
  DPRIN2(I,J) = -EPRIN2(I,J)*GAMMA(I,J-1) - GAMMA(I,J)*(2. + F(I))
  ) - GPRIN2(I,J)*GAMMA(I,J+1)
180 CONTINUE
  DPRIN2(2,J) = DPRIN2(2,J) - APRIN2(2,J)*GAMMA(1,J)
190 CONTINUE
C
C COMPUTE GAMMA AT THE END OF THE HALF TIME STEP
C
C CALL TRICAZ(AFFIN2,PFRIN2,CFRIN2,DPRIN2,GAMMA)
C
C COMPUTE THE RIGHT HAND VECTOR FOR COMPUTING
C GAMMA AT THE END OF THE TIME STEP
C
C DO 210 I = 2,NM1
C DO 200 J = 2,NM1
  HPRIN2(I,J) = -APRIN2(I,J)*GAMMA(I-1,J) - GAMMA(I,J)*(2. +
  ) - CPRIN2(I,J) - DPRIN2(I,J)*GAMMA(I+1,J)
200 CONTINUE
  HPRIN2(I,NM1) = HPRIN2(I,NM1) - GPRIN2(I,NM1)*GAMMA(I,N)
210 CONTINUE
C
C COMPUTE GAMMA AT THE END OF THE TIME STEP
C
C CALL TRICAP(EFRIN2,FPRIN2,CFRIN2,HPRIN2,GAMMA)
C
C DO 215 J = 2,NM1
  GAMMA(1,J) = GAMMA(2,J)/K(NM1)**2
215 CONTINUE
C
C COMPUTE THE COMPONENTS OF VELOCITY IN THE R AND BETA DIRECTIONS
C
C DO 225 J = 2,NM1

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      DO 220 I = 2, NPI
      U(I,J) = (PSI(I,J-1) - PSI(I,J+1))/(2.*DELTA*B*COS(BETA(J))*
1       R(NPI-I)**2)
      V(I,J) = (PSI(I+1,J) - PSI(I-1,J))/(2.*DELTA*AZ*BZ*EX(I)*COS(BETA
1       (J))*R(NPI-I))
220 CONTINUE
      V(I,J) = V(2,J)/R(NPI)
225 CONTINUE
C
C COMPUTE CHI AND ETA ( NOTE NEITHER IS COMPUTED AT R = 0)
C
      DO 240 I = 2, NPI
      DO 230 J = 2, NPI
      CHI(I,J) = -(GAMMA(I,J+1) - GAMMA(I,J-1))/(2.*DELTA*B
1       (COS(BETA(J))*R(NPI-I)**2)
      ETA(I,J) = (GAMMA(I+1,J) - GAMMA(I-1,J))/(2.*AZ*BZ*DELTA*EX(I)*
1       COS(BETA(J))*R(NPI-I)**2)
230 CONTINUE
      CHI(I,1) = -(4.*GAMMA(I,2) - GAMMA(I,3))/(2.*DELTA*B*R(NPI-I)**2)
      CHI(I,N) = -(GAMMA(I,N*2) - 4.*GAMMA(I,N*1) + 3.*GAMMA(I,N))
1       / (2.*DELTA*B*(R(NPI-I)*COS(EPSLON))**2)
240 CONTINUE
      CHI(1,1) = -(4.*GAMMA(1,2) - GAMMA(1,3))/(2.*DELTA*B)
      CHI(1,N) = -(GAMMA(1,N*2) - 4.*GAMMA(1,N*1) + 3.*GAMMA(1,N))/
1       (2.*DELTA*B*COS(EPSLON)**2)
      DO 260 J = 2, NPI
      ETA(1,J) = -2.*GAMMA(1,J)/COS(BETA(J)**2
      CHI(1,J) = -(GAMMA(1,J+1) - GAMMA(1,J-1))/(2.*DELTA*B*COS(BETA(J))
1       **2)
      DO 280 I = 2, NPI
      PHI(I,J) = -ALPHA1*(GAMMA(I-1,J)*CHI(I-1,J)
1       - GAMMA(I+1,J)*CHI(I+1,J))/EX(I)
2       + ALPHA2*(GAMMA(I,J+1)*ETA(I,J+1)
3       - GAMMA(I,J-1)*ETA(I,J-1))
280 CONTINUE
C
C CHECK FOR CONVERGENCE OF THE NONLINEAR ITERATION
C ( IFLAG = 0 INDICATES CONVERGENCE )
C
      LTC = LTC + L
      CALL CONV(NMI,NPI,LK,U,SPALLZ,IFLAG,UMAX,17,J7)
      IF(IFLAG.EQ.0.OR.K.EQ.KMAX) GO TO 32C
300 DO 310 I = 2,N
      GAMMA(I,J) = (GAMMA(I,J) + GAMMAN(I,J))/2.
      DO 310 J = 2,NPI
      UK(I,J) = U(I,J)
      U(I,J) = (U(I,J) + UN(I,J))/2.
      V(I,J) = (V(I,J) + VN(I,J))/2.
      PHI(I,J) = (PHI(I,J) + PHI(I,J))/2.
310 CONTINUE
      GO TO 42
320 IF(NZPR.EQ.0) GO TO 34C
      IF(NZ.GT.1) GO TO 390
C
C PRINT SELECTED VARIABLES AT THE END OF THE TIME STEP
C
      WRITE(6,350)
350 FORMAT(' OPTIONAL INTERMEDIATE PRINTCUT'//3X,'NZ',1X,'K',1X,'LTO',

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11X,'II',1X,'JJ',1X,'PSIMAX',5X,'ZETAEL',5X,'ZETAEN',6X,'VEI/2',4X,
2'GAMMAE1/2'/)
GO TO 3901
3900 IF((NZPR - NZP) .GT. 0) GO TO 340
    NZP = 0
3901 J12 = (N + 1)/2
    WRITE(6,400)NZ,K,LTD,II,JJ,PSIMAX,ZETA(ME,1),ZETA(ME,N),V(ME,J12)
    1,GAMMA(ME,J12)
4000 FORMAT(15,12,14,213,1P5E11.3)
340 IF((NZCNC - NZC) .GT. 0) GO TO 40
C
C CHECK FOR STEADY STATE SOLUTION OF ZETA
C ( IFLAG = 3 INDICATES CONVERGENCE )
C
    NZC = 0
    CALL CONV(M,N,ZFTAN,ZETA,SMALLZ,IFLAG,NMAX,II,JJ)
    IF(IFLAG .EQ. 0) GO TO 342
    IF(NZ .LT. NZMAX) GO TO 40
    WRITE(6,370)NZ
330 FORMAT(/' NZ = ',14,5X,'***** A STEADY STATE IS NOT ATTAINED *****'
1
)
GO TO 340
C
C CHECK FOR STEADY STATE SOLUTION OF GAMMA
C ( IFLAG = 0 INDICATES CONVERGENCE )
C
342 CALL CONV(M,N,GAMMA,GAMMA,SMALLG,IFLAG,GMAX,II,JJ)
    IF(IFLAG .EQ. 0) GO TO 343
    IF(NZ .LT. NZMAX) GO TO 40
    WRITE(6,330)NZ
GO TO 340
C
C THE STEADY STATE SOLUTION IS REACHED
C
343 WRITE(6,345)
345 FORMAT(/' ***** A STEADY STATE IS ATTAINED *****')
C
C IF IDSN IS NOT ZERO THEN THE U,V,GAMMA,PSI,ETA,ZETA,AND CHI
C FIELDS ARE WRITTEN ON DISK
C
348 IF((IDSN .EQ. 0) GO TO 349
    IUNIT = IPRON + 10
    WRITE(IUNIT)((PSI(I,J),U(I,J),V(I,J),GAMMA(I,J),ZETA(I,J),
1
    ETA(I,J),CHI(I,J),J=1,N),I=1,M)
C
C COMPUTE THE TORQUE RATIO ON THE PLATE AND CONE: IF ITCRO = 0, NEITHER
C THE TORQUE RATIO ON THE CONE OR PLATE IS COMPUTED; IF ITORO = 1, ONLY
C THE TORQUE RATIO ON THE PLATE IS COMPUTED AND PRINTED; IF ITORO IS NOT
C 0 OR 1, BOTH THE TORQUE RATIOS ON THE PLATE AND CONE ARE COMPUTED AND
C PRINTED
C
349 IF((ITORO .EQ. 0) GO TO 390
    DO 350 I = 1,M
    V(I) = CHI(I,1)*AZ*PZ*EX(I)*H(MPI-I)*02
350 CONTINUE
    CALL TOROP(M,Y,EPSLON,CLLT,AZ,ZT,TOROPL)
    WRITE(6,360)TOROPL
360 FORMAT(/' THE TORQUE RATIO ON THE PLATE IS',F10.7)

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      IF(I TORQ .EQ. 1) GO TO 390
      DO 370 I = 1,M
      Y(I) = CHI(I,N)*AZ*EZ*EX(I)*R(NP1-I)**2
370 CONTINUE
      CALL TCRCC(M,V,EPSLON,CELT,ZT,TORQCO)
      WRITE(6,38C)TORQCO
380 FORMAT(' THE TORQUE RATIO ON THE CONE IS',F10.7)
C
C IF IDEF IN NOT ZERO, THEN THE RATIO OF EACH ELEMENT OF THE RATE-OF-
C DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS COMPUTED: IF
C = 2, THEN DRR(ME,J), DFR(1,J), AND DRT(ME,J) ARE PRINTED OUT
C
390 IF(IDEF .EQ. 0) GO TO 400
      CALL DEFORM(U,V,GAMMA,EPSLCN,R,BETA,DELTAZ,DELTAB)
      IF(IDEF .NF. 2) GO TO 400
      MER = MPI - ME
      WRITE(6,396)
396 FORMAT('////' DPR(ME,J)'/)
      CALL ARRAY(MER,MER,1,N,R,BETA,DRR)
      WRITE(6,397)
397 FORMAT('////' DRR(1,J)'/)
      CALL ARRAY(M,N,1,N,R,BETA,CRR)
      WRITE(6,398)
398 FORMAT('////' DRT(ME,J)'/)
      CALL ARRAY(MER,MER,1,N,R,BETA,DRT)
      WRITE(6,399)
399 FORMAT('////' DRT(1,J)'/)
      CALL ARRAY(M,N,1,N,R,BETA,CRT)
C
C PRINT OUT THE VELOCITY, VORTICITY, AND STREAM FUNCTION FIELDS: IF
C IPRINT = 0, NC FIELDS ARE PRINTED OUT; IF IPRINT = 1, THE STREAM
C FUNCTION FIELD IS PRINTED OUT; IF IPRINT = 2, THE STREAM FUNCTION AND
C MERIDIAN VELOCITY FIELDS ARE PRINTED OUT; IF IPRINT = 3, THE THREE
C VELOCITY FIELDS AND THE STREAM FUNCTION FIELDS ARE PRINTED OUT; IF
C IPRINT = 4, U(ME,J), V(ME,J), AND GAMMA(ME,J) ARE PRINTED OUT; IF
C IPRINT IS NOT 0,1,2,3, OR 4, THE VELOCITY, VORTICITY, AND STREAM
C FUNCTION FIELDS ARE PRINTED OUT
C
400 IF(IPRINT .EQ. 4) GO TO 470
      IF(IPRINT .EQ. 0) GO TO 500
      WRITE(6,405)
405 FORMAT(' '//////////' THE STREAM FUNCTION FIELD IS'/)
      CALL ARRAY(1,M,1,N,R,BETA,FSI)
      IF(IPRINT .EQ. 1) GO TO 500
      IF(IPRINT .EQ. 2) GO TO 425
      WRITE(6,410)
410 FORMAT(' '//////////' THE RADIAL VELOCITY FIELD IS'/)
      CALL ARRAY(1,M,1,N,R,BETA,U)
      WRITE(6,420)
420 FORMAT(' '//////////' THE AZIMUTHAL VELOCITY FIELD IS'/)
      CALL ARRAY(1,M,1,N,R,BETA,V)
425 WRITE(6,430)
430 FORMAT(' '//////////' THE MERIDIAN VELOCITY FIELD IS'/)
      CALL ARRAY(1,M,1,N,R,BETA,GAMMA)
      IF(IPRINT .EQ. 2 .OR. IPRINT .EQ. 3) GO TO 500
      WRITE(6,440)
440 FORMAT(' '//////////' THE RADIAL VORTICITY FIELD IS'/)
      CALL ARRAY(2,M,1,N,R,BETA,CHI)

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WRITE(6,45C)
450 FORMAT('OTHE AZMUTHIAL VORTICITY FIELD IS')
CALL ARRAY(2,M,1,N,P,BETA,ETA)
WRITE(6,46C)
460 FORMAT('OTHE MERIDIAN VORTICITY FIELD IS')
CALL ARRAY(1,M,1,N,P,BETA,ZETA)
GC TO 500
470 MFR = MP1 - ME
WRITE(6,48C)
480 FORMAT('//// U(ME,J)')
CALL ARRAY(MFR,MFR,1,N,R,BETA,U)
WRITE(6,482)
482 FORMAT('//// V(ME,J)')
CALL ARRAY(MFR,MFR,1,N,R,BETA,V)
WRITE(6,484)
484 FORMAT('//// GAMMA(ME,J)')
CALL ARRAY(MFR,MFR,1,N,P,BETA,GAMMA)
500 IF((IFRCH .GE. NPROE) GC TO 1000)
  ILI3 = 0
  GO TO 5
1000 STOP
  END
  SUBROUTINE PSIA
C
C THIS SUBROUTINE COMPUTES THE U AND WETA ASSOCIATED WITH SOLVING THE
C TWO TRIDIAGONAL MATRICES FOR PSI AT THE ENDS OF BOTH THE HALF AND
C FULL TIME STEPS
C
COMMON/PSIA/A(21),E(21),C(21),E(21,21),F(21),G(21,21),W(21),BETA1
1 (21),WJ(21,21),BETAJ(21,21)/SUB2/M,MP1,MM1,MM2,N,NN1,NN2
C
C COMPUTE THE "TIME" INDEPENDENT VARIABLES OF THE TRIDIAGONAL
C ALGORITHM
C
BETA1(2) = 0(2)
W(2) = C(2)/F(2)
DO 10 I = 3,MM1
  BETA1(I) = P(I) - A(I)*W(I-1)
  W(I) = C(I)/BETA1(I)
10 CONTINUE
DO 15 I = 2,MM1
  BETAJ(I,2) = F(I)
  WJ(I,2) = G(I,2)/F(I)
DO 15 J = 3,NN1
  BETAJ(I,J) = F(I) - E(I,J)*WJ(I,J-1)
  WJ(I,J) = G(I,J)/BETAJ(I,J)
15 CONTINUE
RETURN
END
SUBROUTINE TRICAZ(A,B,C,D,X)
C
C THIS SUBROUTINE SOLVES THE SYSTEM OF LINEAR ALGEBRAIC EQUATIONS
C CONSISTING OF THE TRIDIAGONAL MATRIX OF COEFFICIENTS A,B, AND C
C AND THE RIGHT HAND VECTOR D ASSOCIATED WITH SOLVING FOR X
C AT THE END OF THE HALF TIME STEP
C
DIMENSION A(21,21),B(21,21),C(21,21),D(21,21),X(21,21)
COMMON/SUB1/BETA(21,21),GAMMA(21,21),U(21,21)/SUB2/M,MP1,MM1,MM2,N

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      I ,NM1,NM2
C
C FORWARD SOLUTION FOR W AND GAMMA
C
      DO 20 J = 2,NM1
      BETA(2,J) = B(2,J)
      W(2,J) = C(2,J)/BETA(2,J)
      GAMMA(2,J) = D(2,J)/BETA(2,J)
      DO 10 I = 3,NM1
      BETA(I,J) = B(I,J) - A(I,J)*W(I-1,J)
      W(I,J) = C(I,J)/BETA(I,J)
      GAMMA(I,J) = (D(I,J) - A(I,J)*GAMMA(I-1,J))/BETA(I,J)
10 CONTINUE
C
C EACH SOLUTION FOR X(I,J)
C
      X(NM1,J) = GAMMA(NM1,J)
      DO 20 I = 2,NM2
      X(N-I,J) = GAMMA(N-I,J) - W(N-I,J)*X(N+1-I,J)
20 CONTINUE
      RETURN
      END
      SUBROUTINE TRICAB(E,F,G,H,X)
C
C THIS SUBROUTINE SOLVES THE SYSTEM OF LINEAR ALGEBRAIC EQUATIONS
C CONSISTING OF THE TRI-DIAGONAL MATRIX OF COEFFICIENTS E,F, AND G
C AND THE RIGHT HAND VECTOR H ASSOCIATED WITH SOLVING FOR X
C AT THE END OF THE TIME STEP
C
      DIMENSION E(21,21),F(21),G(21,21),H(21,21),X(21,21)
      COMMON/SUB1/BETA(21,21),GAMMA(21,21),W(21,21)/SUB2/M,NM1,NM2,N
      I ,NM1,NM2
C
C FORWARD SOLUTION FOR W AND GAMMA
C
      DO 20 I = 2,NM1
      BETA(I,2) = F(I)
      W(I,2) = G(I,2) / BETA(I,2)
      GAMMA(I,2) = H(I,2) / BETA(I,2)
      DO 10 J = 3,NM1
      BETA(I,J) = F(I) - E(I,J)*W(I,J-1)
      W(I,J) = G(I,J)/BETA(I,J)
      GAMMA(I,J) = (H(I,J) - E(I,J)*GAMMA(I,J-1))/BETA(I,J)
10 CONTINUE
C
C BACK SOLUTION FOR X(I,J)
C
      X(I,NM1) = GAMMA(I,NM1)
      DO 20 J = 2,NM2
      X(I,N-J) = GAMMA(I,N-J) - W(I,N-J)*X(I,N+1-J)
20 CONTINUE
      RETURN
      END
      SUBROUTINE PSIB (EPSLON,L,LPAX,LCHC,II,JJ,PSIMAX)
C
C THIS SUBROUTINE COMPUTES PSI AT THE END OF EACH TIME STEP
C
      DIMENSION H(21,21),PSIL(21,21)

```

```

COMMON/PSI1/A(21),E(21),C(21),E(21,21),F(21),G(21,21),W(21),BETA
1 (21),WJ(21,21),BETAJ(21,21)/PSI2/DELTAT,ZETA(21,21),PSI(21,21),
2 BETA(21),C(21,21)/SUB1/XETA(21,21),GAMMA(21,21),XW(21,21)/SUB2/
3 N,MM1,MM2,N,MM1,MM2
EQUIVALENCE (D(1),F(1))
C
LC = 0
L = 0
C
C BEGIN THE ITERATION TO STEADY STATE
C
20 L = L + 1
LC = LC + 1
DO 25 I = 1,M
CO 25 J = 1,M
PSIL(I,J) = PSI(I,J)
25 CONTINUE
C
C COMPUTE THE RIGHT HAND VECTOR C ASSOCIATED WITH SOLVING FOR PSI
C AT THE END OF THE "TIME" STEP ( NOTE PSI IS ZERO EVERYWHERE ON THE
C BOUNDARIES, HENCE C(2,J) AND D(N-1,J) ARE COMPUTED BY THE SAME
C EQUATIONS AS ARE D(I,J) )
C
DO 30 I = 2,MM1
DO 30 J = 2,MM1
C(I,J) = -PSIL(I,J-1)*E(I,J) - PSIL(I,J)*(2. + F(I))
1 -PSIL(I,J+1)*G(I,J) - COS(BETA(J))*ZETA(I,J)*DELTAT/2.
30 CONTINUE
C
C COMPUTE PSI AT THE END OF THE HALF "TIME" STEP
C
DO 60 J = 2,MM1
GAMMA(2,J) = D(2,J)/B(2)
C
C FORWARD SOLUTION FOR GAMMA(I,J)
C
DO 50 I = 3,MM1
GAMMA(I,J) = (D(I,J) - A(I)*GAMMA(I-1,J))/BETA(I)
50 CONTINUE
C
C BACK SOLUTION FOR PSI(I,J) AT THE END OF THE HALF "TIME" STEP
C
PSI(MM1,J) = GAMMA(MM1,J)
DO 60 I = 2,MM1
PSI(M-I,J) = GAMMA(M-I,J) - W(M-I)*PSI(M+1-I,J)
60 CONTINUE
C
C COMPUTE THE RIGHT HAND VECTOR F ASSOCIATED WITH SOLVING FOR PSI
C AT THE END OF THE "TIME" STEP ( NOTE PSI IS ZERO EVERYWHERE ON
C THE BOUNDARIES, HENCE F(1,2) AND H(1,N-1) ARE COMPUTED BY
C THE SAME EQUATIONS AS ARE H(I,J) )
C
DO 70 I = 2,MM1
DO 70 J = 2,MM1
H(I,J) = -A(I)*PSI(I-1,J) - PSI(I,J)*(2. + B(I)) - PSI(I+1,J)*C(I)
1 - COS(BETA(J))*DELTAT*ZETA(I,J)/2.
70 CONTINUE
C

```

```

C COMPUTE PSI AT THE END OF THE "TIME" STEP
C
  DO 100 I = 2,NM1
    GAMMA(I,2) = H(I,2)/F(I)
C
C FORWARD SOLUTION FOR GAMMA(I,J)
C
  DO 90 J = 3,NM1
    GAMMA(I,J) = (F(I,J) - E(I,J)*GAMMA(I,J-1))/BETAJ(I,J)
  90 CONTINUE
C
C BACK SOLUTION FOR PSI(I,J) AT THE END OF THE "TIME" STEP
C
  PSI(I,NM1) = GAMMA(I,NM1)
  DO 100 J = 2,NM2
    PSI(I,N-J) = GAMMA(I,N-J) - WJ(I,N-J)*PSI(I,N+1-J)
  100 CONTINUE
C
C CHECK FOR STEADY STATE ( IFLAG = 0 INDICATES CONVERGENCE )
C
  IF(L .EQ. 1) GO TO 105
  IF((LCMC - LC) .GT. 0) GO TO 20
  LC = J
105 CALL CONV(NM1,NM1,PSI,L,PSI,EPSLON,IFLAG,PSIMAX,II,JJ)
  IF(IFLAG .EQ. 0) GO TO 120
  IF(L .LT. LMAX) GO TO 20
120 RETURN
  END
  SUBROUTINE CONV(M,N,A,B,EPSLON,IFLAG,HMAX,II,JJ)
  DIMENSION A(21,21),B(21,21)
C
C THIS SUBROUTINE CHECKS FOR CONVERGENCE USING MAX(ABS(B(I,J) - A(I,J)))
C /MAX(ABS(B(I,J))) AS THE CONVERGENCE CRITERIA
C
  IFLAG = 0
  BMAX = ABS(B(1,1))
  DMAX = ABS(B(1,1) - A(1,1))
  II = 1
  JJ = 1
  DO 20 I = 1,M
    DO 20 J = 1,N
      BMAX1 = ABS(B(I,J))
      IF(BMAX1 .LE. BMAX) GO TO 10
      BMAX = BMAX1
    II = I
    JJ = J
  10 DMAX1 = ABS(B(I,J) - A(I,J))
  IF(DMAX1 .LE. DMAX) GO TO 20
  DMAX = DMAX1
  20 CONTINUE
  IF(DMAX .EQ. 0.0) GO TO 30
  RAT = DMAX/BMAX
  IF(RAT .LE. EPSLON) GO TO 100
  IFLAG = 1
  GO TO 100
  30 IF(DMAX .LE. EPSLON) GO TO 100
  IFLAG = 1
  100 RETURN

```

```

END
SUBROUTINE TORQC(M,Y,EPSLON,DELTAZ,Z,TORQCC)

```

```

C
C THIS SUBROUTINE COMPUTES THE TORQUE ON THE CONE USING SIMPSON'S RULE
C TOGETHER WITH NEWTON'S 3/8 RULE OR A COMBINATION OF THESE TWO RULES
C TO PERFORM THE NUMERICAL INTEGRATION ( USES SSP SUBROUTINE QSF)
C

```

```

    DIMENSION GAMMA(21,21),Z(M),Y(M)
    CALL QSF(DELTAZ,Y,Z,M)
    TORQCC = -3.*EPSLON*COS(EPSLON)**2*Z(M) + EPSLON*SIN(2*EPSLON)
    RETURN
    END
SUBROUTINE TORCF(M,Y,EPSLON,DELTAZ,Z,TORCPL)

```

```

C
C THIS SUBROUTINE COMPUTES THE TORQUE ON THE PLATE USING SIMPSON'S RULE
C TOGETHER WITH NEWTON'S 3/8 RULE OR A COMBINATION OF THESE TWO RULES
C TO PERFORM THE NUMERICAL INTEGRATION ( USES SSP SUBROUTINE QSF)
C

```

```

    DIMENSION GAMMA(21,21),Z(M),Y(M)
    CALL QSF(DELTAZ,Y,Z,M)
    TORCPL = -3.*EPSLON*Z(M)
    RETURN
    END
SUBROUTINE DEFCRP(U,V,GAMMA,EPSLON,R,BETA,DELTAZ,DELTAB,IDEF,
1 GAPANG,RF)

```

```

C
C THIS SUBROUTINE COMPUTES THE RATIOS OF THE SIX INDEPENDENT ELEMENTS
C OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY SHEAR RATE
C

```

```

    DIMENSION U(21,21),V(21,21),GAMMA(21,21),R(2),BETA(21)
    COMMON/DEF/DRR(21,21),DTT(21,21),DPP(21,21),DRT(21,21),DTP(21,21)
1   CPR(21,21),EX(21),AZ,EZ/SUB2/N,MPI,MNI,MM2,N,NMI,MMZ
    DO 1C I = 2,MM1
    EX(I) = AZ*EZ*EX(I)
    DO 1C J = 2,MM1
    DRR(I,J) = EPSLON*(U(I+1,J) - U(I-1,J))/(DELTAZ*EX(I))
    DTT(I,J) = EPSLON*(-2.*U(I,J) + (V(I,J+1) - V(I,J-1))/DELTAB)/
1   F(MPI-I)
    DPP(I,J) = -2.*EPSLON*(U(I,J) + V(I,J)*TAN(BETA(J)))/R(MPI-I)
    DRT(I,J) = EPSLON*(R(MPI-I)*(V(I+1,J)/R(M-I) - V(I-1,J)/R(M+2-I))/
1   (2.*DELTAZ*EX(I)) + (U(I,J+1) - U(I,J-1))/(2.*R(MPI-I)*
2   DELTAB))
    DTP(I,J) = EPSLON*COS(BETA(J))*(GAMMA(I,J+1)/COS(BETA(J+1))**2 -
1   GAMMA(I,J-1)/COS(BETA(J-1))**2)/(2.*DELTAB*R(MPI-I)**2)
10  CONTINUE
    DO 15 I = 2,MM2
    DO 15 J = 2,MM1
    DPR(I,J) = R(MPI-I)*EPSLON*(GAMMA(I+1,J)/R(M-I)**2 - GAMMA(I-1,J)/
1   R(M+2-I)**2)/(2.*EX(I)*COS(BETA(J))*DELTAZ)
15  CONTINUE
    DO 20 I = 1,MM1
    DRR(I,N) = 0.C
    DRR(I,1) = 0.0
    DPP(I,N) = 0.0
    DPP(I,1) = 0.0
    DPR(I,1) = 0.C
    DPR(I,N) = 0.0
    DTT(I,1) = EPSLON*(4.*V(I,2) - V(I,3))/(R(MPI-I)*DELTAB)

```

```

DTT(I,N) = EPSLON*(V(I,NM2) - 4.*V(I,NM1))/(R(NM1-I)*DELTA)
DRT(I,1) = EPSLON*(4.*U(1,2) - U(1,3))/(2.*R(NM1-I)*DELTA)
DRT(I,N) = EPSLON*(L(I,NM2) - 4.*U(I,NM1))/(2.*DELTA*R(NM1-I))
DTP(I,1) = EPSLON*(4.*GAMMA(1,2)/COS(BETA(2))**2 - GAMMA(1,3)/
1 COS(BETA(3))**2)/(2.*DELTA*R(NM1-I)**2)
1 DTP(I,N) = EPSLON*(COS(EPSLON)*(GAMMA(1,NM2)/CCS(BETA(NM2))**2 -
1 4.*GAMMA(1,NM1)/CCS(BETA(NM1))**2 + 3.*GAMMA(1,N)/
2 COS(EPSLON)**2)/(2.*DELTA*R(NM1-I)**2)
20 CONTINUE
DN 30 J = 1,N
DPR(M,J) = 0.0
DPP(M,J) = 0.0
DTT(M,J) = 0.0
DRT(M,J) = 0.0
DTP(M,J) = 1.0
DPR(M,J) = 0.0
30 CONTINUE
DN 40 J = 2,NM1
DPR(1,J) = 0.0
DRT(1,J) = 0.0
DTT(1,J) = EPSLON*(V(1,J+1) - V(1,J-1))/DELTA
DPR(1,J) = 2.*EPSLON*U(2,J)/(1. - R(NM1))
DPP(1,J) = -2.*EPSLON*V(1,J)*TAN(BETA(J))
DTP(1,J) = DTP(2,J)
DPR(NM1,J) = R(2)*BETA(J) - EPSLON*GAMMA(NM2,J)/K(3)**2/
(2.*EX(NM1)*COS(BETA(J))*DELTA)
40 CONTINUE
IF(IDEF.EQ.0) GO TO 500
WRITE(6,900)GAPANG,RE
WRITE(6,1000)
WRITE(6,1100)
CALL MAX(DRR,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DRR,M,N,IMIN,IMINR,JMIN)
WRITE(6,1200)DRR(IMAX,JMAX),R(IMAXR),BETA(JMAX),DPR(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
CALL MAX(DRT,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DRT,M,N,IMIN,IMINR,JMIN)
WRITE(6,1300)DRT(IMAX,JMAX),R(IMAXR),BETA(JMAX),DRT(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
CALL MAX(DPR,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DPR,M,N,IMIN,IMINR,JMIN)
WRITE(6,1400)DPR(IMAX,JMAX),R(IMAXR),BETA(JMAX),DPR(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
CALL MAX(DTT,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DTT,M,N,IMIN,IMINR,JMIN)
WRITE(6,1500)DTT(IMAX,JMAX),R(IMAXR),BETA(JMAX),DTT(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
CALL MAX(DTP,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DTP,M,N,IMIN,IMINR,JMIN)
WRITE(6,1600)DTP(IMAX,JMAX),R(IMAXR),BETA(JMAX),DTP(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
CALL MAX(DPP,M,N,IMAX,IMAXR,JMAX)
CALL MIN(DPP,M,N,IMIN,IMINR,JMIN)
WRITE(6,1700)DPP(IMAX,JMAX),R(IMAXR),BETA(JMAX),DPP(IMIN,JMIN),
1 R(IMINR),BETA(JMIN)
WRITE(6,1800)
900 FORMAT('1'//////////5X,'GAP ANGLE =',F7.4,' DEGREES REYNOLDS N
1UMBER =',1PE7.1//)

```

```

1000 FORMAT(' THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF
1 THE RATE'// ' OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE
2 ARE'//)
1100 FORMAT(11X,'RATIO WHERE',21X,'RATIO WHERE'/11X,'THE ABS VAL',22X,'
1 THE ABS VAL'/11X,'CF THE RATIO',5X,'LOCATION',5X,'CF THE RATIO',5X
2,'LOCATION'/11X,'IS A MAXIMUM',5X,'-----',5X,'IS A MINIMUM',5X
3,'-----'// 'ELEMENT',4X,'(PERCENT)',7X,'N',5X,'BETA',7X,'(PERCENT
4)'.7X,'R',5X,'DETA'// '-----',3X,'-----',3X,'-----'//
5',5X,'-----',3X,'-----'//)
1200 FORMAT(' DR',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1300 FORMAT(' DRT',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1400 FORMAT(' DPP',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1500 FORMAT(' DTT',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1600 FORMAT(' DTP',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1700 FORMAT(' DPF',2PF15.4,OPF11.4,1X,FS.4,2PF15.4,OPF11.4,1X,FS.4)
1800 FORMAT(// ' * EXCLUDES AN ABSOLUTE VALUE OF ZERO'//
1      ' * DRR = 0 AT BETA = 0, EPSILON AND AT R = C'//
2      ' * DPP = 0 AT BETA = 0, EPSILON AND AT R = 0'//
3      ' * DRT = 0 AT BETA = 0, EPSILON AND AT R = C'//
4      ' * DKT = 0 AT R = C, 1'//
5      ' * DTT = 0 AT R = 0'//
6      ' * DTP IS NEVER ZERO'//)
500 RETURN
END
SUBROUTINE MAX(X,N,N,II,IIR,JJ)
C
C THIS SUBROUTINE LOCATES THE ELEMENT OF AN ARRAY WITH THE
C LARGEST ABSOLUTE VALUE
C
  DIMENSION X(21,21)
  XMAX = ABS(X(1,1))
  II = 1
  JJ = 1
  DO 10 I = 1,N
  DO 10 J = 1,N
  XMAX1 = ABS(X(I,J))
  IF(XMAX1 .LT. XMAX) GO TO 10
  XMAX = XMAX1
  II = I
  JJ = J
10 CONTINUE
  IIR = N + 1 - II
  RETURN
  ENC
SUBROUTINE MIN(X,N,N,II,IIR,JJ)
C
C THIS SUBROUTINE LOCATES THE ELEMENT OF AN ARRAY WITH THE
C SMALLEST ABSOLUTE VALUE
C
  DIMENSION X(21,21)
  CC 10 I = 1,N
  DO 10 J = 1,N
  XMIN = ABS(X(I,J))
  II = I
  JJ = J
  IF(XMIN .NE. 0.0) GO TO 20
10 CONTINUE
20 DO 30 I = 1,N

```

```

DO 30 J = 1,N
XMINI = ABS(X(I,J))
IF(XMINI .GE. XMIN .OR. XMINI .EQ. 0.0) GO TO 30
XMIN = XMINI
II = I
JJ = J
30 CONTINUE
IIR = M + 1 - II
RETURN
END
SUBROUTINE ARRAY(J1,J2,I1,I2,R,BETA,X)

```

```

C
C THIS SUBROUTINE PRINTS OUT ARRAYS
C

```

```

DIMENSION X(21,21),R(21),BETA(21)
COMMON/SUB2/M,MP1,MM1,MM2,N,NM1,NM2
J2P1 = J2 + 1
JE = J2
IF((J2P1-J1) .GT. 11) JF = 11
WRITE(6,10)(R(J),J=J1,JE)
10 FORMAT(4X,'**/5X,'**/6X,'* R',F5.1,F10.5,9F11.5)
WRITE(6,15)
15 FORMAT(2X,'BETA **/8X,'**')
JS = J1
DO 20 I = I1,I2
WRITE(6,30)BETA(I),(X(MP1-J,I),J=J1,JE)
20 CONTINUE
30 FORMAT(1X,F6.5,F7.1,1P1'E11.3)
IF(JE .EQ. J2) GO TO 80
40 JS = JS + 1
JF = JE + 1
IF((J2P1-JS) .LT. 11) JF = J2
WRITE(6,50)(R(J),J=JS,JF)
50 FORMAT(//4X,'**/5X,'**/6X,'* R',F5.1,F10.5,9F11.5)
WRITE(6,15)
DO 60 I = I1,I2
WRITE(6,30)BETA(I),(X(MP1-J,I),J=JS,JE)
60 CONTINUE
IF(J2-JE)80,80,40
80 RETURN
END

```



APPENDIX F

MERIDIAN VELOCITY AND STREAM FUNCTION ARRAYS  
FOR SELECTED REYNOLDS NUMBERS AND GAP ANGLES

This appendix provides the arrays of  $\Gamma$  and  $\psi$  from which Figures 7 through 30 are plotted.

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$\Gamma$  and  $\psi$  arrays:

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$\epsilon = 1/3$  degrees,  $Re = 2 \times 10^4$  . . . . . 252

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GAP ANGLE = 4.0330 DEGREES REYNOLDS NUMBER = 4.0E+01

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

BETA	0.0	0.25000	0.44327	0.59077	0.70365	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	4.497E-03	1.404E-02	2.444E-02	3.539E-02	4.424E-02	5.233E-02	5.854E-02	6.390E-02	6.741E-02	7.052E-02
.00997	0.0	8.994E-03	2.807E-02	4.987E-02	7.078E-02	8.848E-02	1.047E-01	1.171E-01	1.270E-01	1.349E-01	1.413E-01
.01496	0.0	1.348E-02	4.210E-02	7.440E-02	1.062E-01	1.339E-01	1.570E-01	1.757E-01	1.900E-01	2.024E-01	2.129E-01
.01995	0.0	1.798E-02	5.613E-02	9.473E-02	1.316E-01	1.755E-01	2.094E-01	2.344E-01	2.543E-01	2.700E-01	2.829E-01
.02493	0.0	2.248E-02	7.018E-02	1.247E-01	1.769E-01	2.231E-01	2.618E-01	2.931E-01	3.181E-01	3.378E-01	3.539E-01
.02992	0.0	2.697E-02	8.417E-02	1.446E-01	2.123E-01	2.677E-01	3.142E-01	3.516E-01	3.820E-01	4.067E-01	4.250E-01
.03491	0.0	3.146E-02	9.819E-02	1.745E-01	2.477E-01	3.133E-01	3.648E-01	4.107E-01	4.455E-01	4.737E-01	4.962E-01
.03989	0.0	3.595E-02	1.122E-01	1.943E-01	2.830E-01	3.569E-01	4.190E-01	4.698E-01	5.100E-01	5.418E-01	5.678E-01
.04488	0.0	4.043E-02	1.262E-01	2.242E-01	3.183E-01	4.014E-01	4.714E-01	5.248E-01	5.741E-01	6.101E-01	6.391E-01
.04987	0.0	4.491E-02	1.402E-01	2.441E-01	3.536E-01	4.402E-01	5.232E-01	5.673E-01	6.243E-01	6.755E-01	7.107E-01
.05485	0.0	4.939E-02	1.542E-01	2.739E-01	3.888E-01	4.904E-01	5.781E-01	6.422E-01	7.023E-01	7.469E-01	7.824E-01
.05984	0.0	5.387E-02	1.681E-01	2.937E-01	4.240E-01	5.168E-01	6.248E-01	7.050E-01	7.666E-01	8.154E-01	8.542E-01
.06483	0.0	5.835E-02	1.821E-01	3.234E-01	4.591E-01	5.792E-01	6.896E-01	7.639E-01	8.304E-01	8.838E-01	9.289E-01
.06981	0.0	6.283E-02	1.960E-01	3.482E-01	4.942E-01	6.244E-01	7.377E-01	8.224E-01	8.948E-01	9.522E-01	9.976E-01

THE STREAM FUNCTION FIELD IS

BETA	0.0	0.25000	0.44327	0.59077	0.70365	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	1.861E-09	3.426E-08	1.472E-07	3.850E-07	6.398E-07	9.869E-07	1.274E-06	1.617E-06	1.890E-06	0.0
.00997	0.0	4.553E-09	1.208E-07	5.191E-07	1.255E-06	2.285E-06	3.371E-06	4.484E-06	5.315E-06	4.443E-06	0.0
.01496	0.0	1.263E-08	3.361E-07	1.014E-06	2.452E-06	4.405E-06	6.584E-06	8.782E-06	1.034E-05	4.553E-06	0.0
.01995	0.0	1.947E-08	3.582E-07	1.530E-06	3.728E-06	6.881E-06	9.984E-06	1.327E-05	1.664E-05	1.288E-05	0.0
.02493	0.0	2.593E-08	4.676E-07	2.009E-06	4.866E-06	8.720E-06	1.303E-05	1.731E-05	2.098E-05	1.671E-05	0.0
.02992	0.0	3.240E-08	5.444E-07	2.355E-06	5.706E-06	1.022E-05	1.528E-05	2.030E-05	2.390E-05	1.960E-05	0.0
.03491	0.0	3.887E-08	6.212E-07	2.627E-06	6.122E-06	1.097E-05	1.640E-05	2.179E-05	2.547E-05	2.114E-05	0.0
.03989	0.0	4.534E-08	6.980E-07	2.898E-06	6.543E-06	1.183E-05	1.820E-05	2.432E-05	2.802E-05	2.104E-05	0.0
.04488	0.0	5.181E-08	7.748E-07	3.169E-06	6.963E-06	1.279E-05	1.948E-05	2.594E-05	2.964E-05	1.930E-05	0.0
.04987	0.0	5.828E-08	8.516E-07	3.440E-06	7.383E-06	1.384E-05	2.094E-05	2.848E-05	3.317E-05	1.898E-05	0.0
.05485	0.0	6.475E-08	9.284E-07	3.711E-06	7.803E-06	1.490E-05	2.240E-05	3.102E-05	3.671E-05	1.418E-05	0.0
.05984	0.0	7.122E-08	1.0052E-06	3.982E-06	8.223E-06	1.606E-05	2.386E-05	3.348E-05	3.912E-05	4.375E-06	0.0
.06483	0.0	7.769E-08	1.0804E-06	4.253E-06	8.643E-06	1.722E-05	2.532E-05	3.594E-05	4.260E-05	1.999E-06	0.0
.06981	0.0	8.416E-08	1.1556E-06	4.524E-06	9.064E-06	1.838E-05	2.678E-05	3.840E-05	4.508E-05	0.0	0.0

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

BETA	0.0	0.25090	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.00489	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00987	0.0	0.496E-03	1.401E-02	2.483E-02	3.815E-02	4.429E-02	5.200E-02	5.654E-02	6.500E-02	7.206E-02	7.849E-02
.01486	0.0	0.991E-03	2.801E-02	4.466E-02	7.030E-02	8.856E-02	1.040E-01	1.170E-01	1.300E-01	1.442E-01	1.511E-01
.01985	0.0	1.349E-02	4.202E-02	7.448E-02	1.094E-01	1.328E-01	1.559E-01	1.754E-01	1.948E-01	2.163E-01	2.286E-01
.02484	0.0	1.708E-02	5.602E-02	9.429E-02	1.406E-01	1.770E-01	2.077E-01	2.337E-01	2.594E-01	2.878E-01	3.015E-01
.02983	0.0	2.247E-02	7.002E-02	1.244E-01	1.757E-01	2.212E-01	2.596E-01	2.919E-01	3.235E-01	3.585E-01	3.756E-01
.03482	0.0	2.696E-02	8.402E-02	1.449E-01	2.108E-01	2.654E-01	3.114E-01	3.600E-01	4.073E-01	4.484E-01	4.841E-01
.03981	0.0	3.145E-02	9.802E-02	1.738E-01	2.460E-01	3.092E-01	3.632E-01	4.061E-01	4.506E-01	4.922E-01	5.194E-01
.04480	0.0	3.594E-02	1.120E-01	1.986E-01	2.811E-01	3.539E-01	4.182E-01	4.663E-01	5.136E-01	5.630E-01	5.885E-01
.04979	0.0	4.042E-02	1.263E-01	2.238E-01	3.164E-01	3.984E-01	4.673E-01	5.247E-01	5.764E-01	6.287E-01	6.586E-01
.05478	0.0	4.491E-02	1.400E-01	2.444E-01	3.518E-01	4.430E-01	5.198E-01	5.844E-01	6.493E-01	7.035E-01	7.265E-01
.05977	0.0	4.939E-02	1.540E-01	2.733E-01	3.872E-01	4.878E-01	5.725E-01	6.425E-01	7.028E-01	7.579E-01	7.939E-01
.06476	0.0	5.388E-02	1.680E-01	2.982E-01	4.226E-01	5.329E-01	6.257E-01	7.021E-01	7.661E-01	8.223E-01	8.614E-01
.06975	0.0	5.837E-02	1.820E-01	3.232E-01	4.585E-01	5.761E-01	6.791E-01	7.622E-01	8.303E-01	8.871E-01	9.293E-01
.07474	0.0	6.286E-02	1.960E-01	3.482E-01	4.942E-01	6.234E-01	7.327E-01	8.224E-01	8.948E-01	9.523E-01	9.976E-01

THE STREAM FUNCTION FIELD IS

BETA	0.0	0.25090	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.00489	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00987	0.0	3.722E-08	6.837E-07	2.927E-06	7.049E-06	1.256E-05	1.869E-05	2.486E-05	3.006E-05	3.677E-05	4.000
.01486	0.0	1.312E-07	2.411E-06	1.032E-05	2.466E-05	4.424E-05	6.887E-05	9.749E-05	1.051E-04	1.159E-04	0.0
.01985	0.0	2.565E-07	4.712E-06	2.017E-05	4.859E-05	8.653E-05	1.267E-04	1.705E-04	2.040E-04	1.750E-04	0.0
.02484	0.0	3.693E-07	7.150E-06	3.061E-05	7.374E-05	1.313E-04	1.953E-04	2.589E-04	3.079E-04	2.611E-04	0.0
.02983	0.0	5.065E-07	9.337E-06	3.478E-05	9.632E-05	1.715E-04	2.551E-04	3.380E-04	4.006E-04	3.366E-04	0.0
.03482	0.0	6.966E-07	1.095E-05	4.640E-05	1.130E-04	2.013E-04	2.994E-04	3.966E-04	4.689E-04	3.922E-04	0.0
.03981	0.0	8.407E-07	1.176E-05	5.039E-05	1.214E-04	2.163E-04	3.216E-04	4.262E-04	5.032E-04	4.195E-04	0.0
.04480	0.0	9.342E-07	1.161E-05	4.974E-05	1.197E-04	2.138E-04	3.183E-04	4.217E-04	4.976E-04	4.155E-04	0.0
.04979	0.0	9.734E-07	1.050E-05	4.498E-05	1.085E-04	1.936E-04	2.883E-04	3.621E-04	4.510E-04	3.782E-04	0.0
.05478	0.0	4.698E-07	8.538E-06	4.056E-05	8.822E-05	1.579E-04	2.347E-04	3.113E-04	3.679E-04	3.106E-04	0.0
.05977	0.0	3.279E-07	5.981E-06	2.550E-05	6.178E-05	1.103E-04	1.646E-04	2.184E-04	2.587E-04	2.211E-04	0.0
.06476	0.0	1.799E-07	3.263E-06	1.394E-05	3.365E-05	6.014E-05	8.976E-05	1.193E-04	1.416E-04	1.230E-04	0.0
.06975	0.0	5.822E-08	9.978E-07	4.243E-06	1.022E-05	1.827E-05	2.728E-05	3.626E-05	4.323E-05	3.840E-05	0.0
.07474	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 6.0E+03

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

BETA	0.0	0.2	0.25090	0.44327	0.59077	0.70388	0.79088	0.88703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	1.370E-02	1.972E-02	2.830E-02	3.842E-02	4.924E-02	6.134E-02	7.412E-01	1.080E-01	2.504E-01	2.428E-01
0.0997	0.0	0.0	2.740E-02	3.944E-02	5.660E-02	7.648E-02	9.896E-02	1.247E-01	2.192E-01	3.804E-01	4.702E-01	4.928E-01
0.1496	0.0	0.0	4.110E-02	5.916E-02	8.490E-02	1.142E-01	1.524E-01	1.986E-01	3.102E-01	4.430E-01	6.132E-01	6.424E-01
0.1993	0.0	0.0	5.480E-02	7.774E-02	1.094E-01	1.484E-01	1.988E-01	2.574E-01	4.184E-01	5.193E-01	6.800E-01	7.123E-01
0.2492	0.0	0.0	6.850E-02	9.938E-02	1.356E-01	1.824E-01	2.368E-01	2.996E-01	4.404E-01	5.282E-01	7.022E-01	7.387E-01
0.2991	0.0	0.0	8.220E-02	1.210E-01	1.588E-01	2.044E-01	2.688E-01	3.392E-01	4.404E-01	5.082E-01	7.022E-01	7.412E-01
0.3489	0.0	0.0	9.590E-02	1.424E-01	1.792E-01	2.264E-01	2.908E-01	3.604E-01	4.404E-01	4.882E-01	7.022E-01	7.402E-01
0.3988	0.0	0.0	1.096E-01	1.638E-01	1.996E-01	2.484E-01	3.124E-01	3.828E-01	4.404E-01	4.722E-01	7.022E-01	7.392E-01
0.4487	0.0	0.0	1.233E-01	1.852E-01	2.196E-01	2.684E-01	3.324E-01	4.048E-01	4.404E-01	4.562E-01	7.022E-01	7.382E-01
0.4986	0.0	0.0	1.370E-01	2.066E-01	2.398E-01	2.884E-01	3.524E-01	4.248E-01	4.404E-01	4.402E-01	7.022E-01	7.372E-01
0.5485	0.0	0.0	1.507E-01	2.280E-01	2.596E-01	3.084E-01	3.724E-01	4.448E-01	4.404E-01	4.242E-01	7.022E-01	7.362E-01
0.5984	0.0	0.0	1.644E-01	2.494E-01	2.794E-01	3.284E-01	3.924E-01	4.648E-01	4.404E-01	4.082E-01	7.022E-01	7.352E-01
0.6483	0.0	0.0	1.781E-01	2.708E-01	2.992E-01	3.484E-01	4.124E-01	4.848E-01	4.404E-01	3.922E-01	7.022E-01	7.342E-01
0.6982	0.0	0.0	1.918E-01	2.922E-01	3.188E-01	3.684E-01	4.324E-01	5.048E-01	4.404E-01	3.762E-01	7.022E-01	7.332E-01

THE STREAM FUNCTION FIELD IS

BETA	0.0	0.2	0.25090	0.44327	0.59077	0.70388	0.79088	0.88703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	4.262E-06	1.648E-05	3.300E-05	4.867E-05	6.435E-05	8.003E-05	1.224E-04	1.385E-04	1.931E-04	0.0
0.0997	0.0	0.0	8.524E-06	3.296E-05	6.600E-05	9.734E-05	1.287E-04	1.673E-04	2.448E-04	2.770E-04	3.862E-04	0.0
0.1496	0.0	0.0	1.278E-05	4.944E-05	9.900E-05	1.480E-04	1.976E-04	2.560E-04	3.696E-04	4.280E-04	5.872E-04	0.0
0.1993	0.0	0.0	1.702E-05	7.266E-05	1.480E-04	2.140E-04	2.840E-04	3.680E-04	5.360E-04	6.160E-04	8.400E-04	0.0
0.2492	0.0	0.0	2.126E-05	1.018E-04	2.140E-04	2.840E-04	3.680E-04	4.800E-04	6.800E-04	7.840E-04	1.064E-03	0.0
0.2991	0.0	0.0	2.550E-05	1.256E-04	2.540E-04	3.280E-04	4.320E-04	5.600E-04	7.840E-04	9.040E-04	1.212E-03	0.0
0.3489	0.0	0.0	2.974E-05	1.494E-04	2.940E-04	3.720E-04	4.860E-04	6.240E-04	8.680E-04	9.920E-04	1.312E-03	0.0
0.3988	0.0	0.0	3.398E-05	1.732E-04	3.340E-04	4.260E-04	5.500E-04	7.040E-04	9.680E-04	1.104E-03	1.412E-03	0.0
0.4487	0.0	0.0	3.822E-05	1.970E-04	3.740E-04	4.760E-04	6.000E-04	7.640E-04	1.032E-03	1.176E-03	1.512E-03	0.0
0.4986	0.0	0.0	4.246E-05	2.208E-04	4.140E-04	5.160E-04	6.400E-04	8.160E-04	1.084E-03	1.240E-03	1.612E-03	0.0
0.5485	0.0	0.0	4.670E-05	2.446E-04	4.520E-04	5.560E-04	6.800E-04	8.640E-04	1.136E-03	1.296E-03	1.684E-03	0.0
0.5984	0.0	0.0	5.094E-05	2.684E-04	4.900E-04	5.960E-04	7.200E-04	9.040E-04	1.192E-03	1.352E-03	1.740E-03	0.0
0.6483	0.0	0.0	5.518E-05	2.922E-04	5.280E-04	6.360E-04	7.600E-04	9.440E-04	1.248E-03	1.408E-03	1.796E-03	0.0
0.6982	0.0	0.0	5.942E-05	3.160E-04	5.660E-04	6.760E-04	8.000E-04	9.840E-04	1.304E-03	1.464E-03	1.852E-03	0.0
0.7481	0.0	0.0	6.366E-05	3.398E-04	5.960E-04	7.120E-04	8.360E-04	1.024E-03	1.360E-03	1.520E-03	1.908E-03	0.0



GAP ANGLE = 2.000 DEGREES PPHOLDS NUMBER = 8.0E+03  
THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

Table with 15 columns of numerical data for the Meridian Velocity field. Values range from approximately 0.0 to 9.994E-01. The title 'THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS' is centered above the data columns.

THE STREAM FUNCTION FIELD IS

Table with 15 columns of numerical data for the Stream Function field. Values range from approximately 0.0 to 9.994E-01. The title 'THE STREAM FUNCTION FIELD IS' is centered above the data columns.





GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 4.0E+02

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

BETA	0.0	0.0	0.44034	0.68509	3.42656	0.90416	0.94764	0.97203	3.48560	0.99711	0.99760	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	1.384E-02	5.338E-02	4.774E-02	5.641E-02	6.411E-02	6.757E-02	6.972E-02	6.972E-02	6.972E-02	6.972E-02	6.972E-02
0.0249	0.0	4.774E-02	1.753E-01	1.608E-01	1.283E-01	1.024E-01	1.332E-01	1.332E-01	1.332E-01	1.332E-01	1.332E-01	1.332E-01
0.0374	0.0	1.015E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01	1.931E-01
0.0499	0.0	5.842E-02	1.453E-01	2.337E-01	2.560E-01	2.560E-01	2.560E-01	2.560E-01	2.560E-01	2.560E-01	2.560E-01	2.560E-01
0.0623	0.0	0.944E-02	1.691E-01	2.439E-01	2.921E-01	3.207E-01	3.379E-01	3.379E-01	3.379E-01	3.379E-01	3.379E-01	3.379E-01
0.0748	0.0	3.338E-02	2.030E-01	2.627E-01	3.095E-01	3.563E-01	4.031E-01	4.031E-01	4.031E-01	4.031E-01	4.031E-01	4.031E-01
0.0873	0.0	7.722E-02	2.368E-01	3.415E-01	4.095E-01	4.911E-01	4.732E-01	4.732E-01	4.732E-01	4.732E-01	4.732E-01	4.732E-01
0.0997	0.0	1.112E-01	2.706E-01	4.074E-01	4.874E-01	5.132E-01	5.403E-01	5.403E-01	5.403E-01	5.403E-01	5.403E-01	5.403E-01
0.1122	0.0	1.250E-01	3.044E-01	4.311E-01	5.238E-01	5.774E-01	6.032E-01	6.032E-01	6.032E-01	6.032E-01	6.032E-01	6.032E-01
0.1247	0.0	1.388E-01	3.382E-01	4.379E-01	5.641E-01	6.416E-01	6.759E-01	6.759E-01	6.759E-01	6.759E-01	6.759E-01	6.759E-01
0.1371	0.0	1.526E-01	3.721E-01	5.107E-01	6.425E-01	7.057E-01	7.426E-01	7.426E-01	7.426E-01	7.426E-01	7.426E-01	7.426E-01
0.1495	0.0	1.667E-01	4.059E-01	5.853E-01	7.026E-01	7.694E-01	8.101E-01	8.101E-01	8.101E-01	8.101E-01	8.101E-01	8.101E-01
0.1621	0.0	1.805E-01	4.398E-01	6.343E-01	7.591E-01	8.339E-01	8.774E-01	8.774E-01	8.774E-01	8.774E-01	8.774E-01	8.774E-01
0.1745	0.0	1.944E-01	4.734E-01	6.831E-01	8.174E-01	8.972E-01	9.440E-01	9.440E-01	9.440E-01	9.440E-01	9.440E-01	9.440E-01

THE STREAM FUNCTION FIELD IS

BETA	0.0	0.0	0.44000	0.68509	3.42656	0.90416	0.94764	0.97203	3.48560	0.99711	0.99760	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	4.501E-04	4.377E-04	1.242E-07	2.064E-07	2.683E-07	3.047E-07	3.047E-07	3.047E-07	3.047E-07	3.047E-07	3.047E-07
0.0249	0.0	1.589E-04	1.720E-07	4.525E-07	7.299E-07	9.382E-07	1.0771E-06	1.0771E-06	1.0771E-06	1.0771E-06	1.0771E-06	1.0771E-06
0.0374	0.0	3.103E-04	3.360E-07	9.619E-07	1.427E-06	1.836E-06	2.139E-06	2.139E-06	2.139E-06	2.139E-06	2.139E-06	2.139E-06
0.0499	0.0	4.711E-04	5.097E-07	1.341E-06	2.160E-06	2.794E-06	3.293E-06	3.293E-06	3.293E-06	3.293E-06	3.293E-06	3.293E-06
0.0623	0.0	6.155E-04	6.694E-07	1.751E-06	2.826E-06	3.641E-06	4.181E-06	4.181E-06	4.181E-06	4.181E-06	4.181E-06	4.181E-06
0.0748	0.0	7.224E-04	7.803E-07	2.032E-06	3.315E-06	4.268E-06	4.924E-06	4.924E-06	4.924E-06	4.924E-06	4.924E-06	4.924E-06
0.0873	0.0	7.767E-04	8.374E-07	2.202E-06	3.654E-06	4.576E-06	5.294E-06	5.294E-06	5.294E-06	5.294E-06	5.294E-06	5.294E-06
0.0997	0.0	7.687E-04	8.270E-07	2.173E-06	3.505E-06	4.313E-06	5.177E-06	5.177E-06	5.177E-06	5.177E-06	5.177E-06	5.177E-06
0.1122	0.0	6.949E-04	7.477E-07	1.942E-06	3.142E-06	4.007E-06	4.665E-06	4.665E-06	4.665E-06	4.665E-06	4.665E-06	4.665E-06
0.1247	0.0	5.886E-04	6.075E-07	1.592E-06	2.582E-06	3.442E-06	3.782E-06	3.782E-06	3.782E-06	3.782E-06	3.782E-06	3.782E-06
0.1371	0.0	4.810E-04	4.253E-07	1.112E-06	1.786E-06	2.293E-06	2.827E-06	2.827E-06	2.827E-06	2.827E-06	2.827E-06	2.827E-06
0.1495	0.0	3.814E-04	2.318E-07	6.031E-07	9.669E-07	1.233E-06	1.611E-06	1.611E-06	1.611E-06	1.611E-06	1.611E-06	1.611E-06
0.1621	0.0	2.936E-04	7.071E-08	1.626E-07	2.692E-07	3.707E-07	4.835E-07	4.835E-07	4.835E-07	4.835E-07	4.835E-07	4.835E-07
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGRS REYNOLDS NUMBER = 8.0E+03

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

Table with columns: X, Y, Z and corresponding velocity components (U, V, W) for various grid points. Includes a 'DATA' label and a '1.00000' value at the end.

THE STREAM FUNCTION FIELD IS

Table with columns: X, Y, Z and corresponding stream function values for various grid points. Includes a 'DATA' label and a '1.00000' value at the end.







GAP ANGLE = 0.333 DEGREES REYNOLDS NUMBER = 0.0004

THE MERIDIAN VELOCITY (COVARIANT COMPONENT) FIELD IS

BETA	0.0	0.58360	0.60082	0.91122	0.96053	0.98255	0.99234	0.99677	0.99873	0.99961	1.00000
0.0002	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0003	0.0	2.164E-02	4.847E-02	8.846E-02	0.473E-02	0.994E-02	0.174E-02	1.128E-01	1.373E-01	1.348E-01	1.366E-01
0.0010	0.0	4.369E-02	9.093E-02	1.189E-01	1.296E-01	1.398E-01	1.638E-01	2.244E-01	2.716E-01	2.746E-01	2.764E-01
0.0020	0.0	6.853E-02	1.364E-01	1.784E-01	1.943E-01	2.092E-01	2.447E-01	3.318E-01	3.951E-01	3.926E-01	3.939E-01
0.0030	0.0	8.738E-02	1.819E-01	2.339E-01	2.592E-01	2.780E-01	3.244E-01	4.303E-01	5.018E-01	4.971E-01	4.978E-01
0.0040	0.0	1.092E-01	2.274E-01	2.924E-01	3.282E-01	3.489E-01	4.015E-01	5.185E-01	5.985E-01	5.837E-01	5.832E-01
0.0050	0.0	1.311E-01	2.724E-01	3.511E-01	3.872E-01	4.132E-01	4.743E-01	5.956E-01	6.966E-01	6.531E-01	6.536E-01
0.0060	0.0	1.530E-01	3.184E-01	4.099E-01	4.538E-01	4.808E-01	5.432E-01	6.632E-01	7.182E-01	7.119E-01	7.128E-01
0.0070	0.0	1.748E-01	3.642E-01	4.599E-01	5.132E-01	5.478E-01	6.184E-01	7.222E-01	7.666E-01	7.623E-01	7.629E-01
0.0080	0.0	1.966E-01	4.100E-01	5.052E-01	5.622E-01	6.142E-01	6.918E-01	7.762E-01	8.032E-01	8.071E-01	8.072E-01
0.0090	0.0	2.187E-01	4.560E-01	5.880E-01	6.518E-01	7.027E-01	7.931E-01	8.269E-01	8.462E-01	8.481E-01	8.482E-01
0.0095	0.0	2.408E-01	5.044E-01	6.681E-01	7.382E-01	7.822E-01	8.874E-01	8.994E-01	8.962E-01	8.970E-01	8.970E-01
0.0098	0.0	2.645E-01	5.548E-01	7.493E-01	8.232E-01	8.232E-01	9.118E-01	9.118E-01	9.118E-01	9.247E-01	9.258E-01
0.0099	0.0	2.888E-01	6.073E-01	8.232E-01	8.532E-01	8.532E-01	9.208E-01	9.208E-01	9.208E-01	9.420E-01	9.420E-01
0.00995	0.0	3.098E-01	6.413E-01	8.303E-01	8.266E-01	8.266E-01	9.298E-01	9.298E-01	9.298E-01	9.592E-01	9.592E-01

THE STREAM FUNCTION FIELD IS

BETA	0.0	0.58360	0.60082	0.91122	0.96053	0.98255	0.99234	0.99677	0.99873	0.99961	1.00000
0.0002	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0003	0.0	1.624E-07	7.843E-07	1.584E-06	2.032E-06	2.181E-06	1.207E-06	2.708E-06	1.377E-06	3.345E-07	0.0
0.0010	0.0	3.977E-07	5.482E-06	5.992E-06	7.342E-06	7.637E-06	4.908E-06	8.824E-06	3.242E-06	1.641E-06	0.0
0.0020	0.0	1.061E-06	8.314E-06	1.059E-05	1.336E-05	1.508E-05	1.093E-05	1.883E-05	5.290E-06	1.762E-06	0.0
0.0030	0.0	1.386E-06	1.086E-05	1.387E-05	1.811E-05	2.304E-05	1.866E-05	2.168E-05	6.016E-06	2.173E-06	0.0
0.0040	0.0	1.628E-06	1.274E-05	1.653E-05	2.042E-05	2.603E-05	2.092E-05	2.495E-05	5.801E-06	2.169E-06	0.0
0.0050	0.0	1.781E-06	1.368E-05	1.791E-05	2.237E-05	2.892E-05	2.266E-05	2.810E-05	5.032E-06	1.892E-06	0.0
0.0060	0.0	1.873E-06	1.382E-05	1.809E-05	2.297E-05	2.992E-05	2.266E-05	2.810E-05	4.070E-06	1.582E-06	0.0
0.0070	0.0	1.917E-06	1.332E-05	1.731E-05	2.297E-05	3.089E-05	2.126E-05	1.799E-05	3.138E-06	1.169E-06	0.0
0.0080	0.0	1.873E-06	1.222E-05	1.641E-05	2.242E-05	3.054E-05	2.126E-05	1.827E-05	2.332E-06	9.204E-07	0.0
0.0090	0.0	1.786E-06	9.932E-06	1.493E-05	2.052E-05	2.907E-05	2.052E-05	1.622E-05	1.642E-06	7.076E-07	0.0
0.0095	0.0	1.660E-06	6.944E-06	1.287E-05	1.827E-05	2.642E-05	2.052E-05	1.373E-05	1.090E-06	5.144E-07	0.0
0.0098	0.0	1.503E-06	5.277E-06	1.060E-05	1.511E-05	2.266E-05	1.511E-05	1.219E-05	8.742E-07	3.042E-07	0.0
0.0099	0.0	1.418E-06	4.144E-06	8.248E-06	1.117E-05	1.817E-05	1.117E-05	9.216E-06	1.390E-07	5.077E-08	0.0
0.00995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

## APPENDIX G

### THE ARRAYS OF THE RATIOS OF THE ELEMENTS OF THE RATE-OF-DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE

#### Nomenclature

<u>Symbol</u>	<u>Definition</u>
DPP	$D_{\phi\phi}$ element of the rate-of-deformation tensor
DPR	$D_{\phi r}$ element of the rate-of-deformation tensor.
DRR	$D_{rr}$ element of the rate-of-deformation tensor.
DRT	$D_{r\theta}$ element of the rate-of-deformation tensor.
DTP	$D_{\theta\phi}$ element of the rate-of-deformation tensor.
DTT	$D_{\theta\theta}$ element of the rate-of-deformation tensor.

For each combination of  $\epsilon$  and  $Re$ , there are four pages of data: the first page contains a table which provides the extreme values and locations of the ratios of each

of the elements of the rate-of-deformation tensor and the primary deformation rate, the second page contains the arrays of the ratios of the  $D_{r\theta}$  and  $D_{rr}$  elements of the rate-of-deformation tensor and the primary deformation rate, the third page contains the arrays of the ratios of the  $D_{\theta\phi}$  and  $D_{\phi r}$  elements of the rate-of-deformation tensor and the primary deformation rate, and the fourth page contains the arrays of the ratios of the  $D_{\theta\theta}$  and  $D_{\phi\phi}$  elements of the rate-of-deformation tensor and the primary deformation rate. The following index indicates the page number at which the first page of data for each combination of  $\epsilon$  and  $Re$  begins.

Index of the Data Contained in this Appendix

<u><math>\epsilon</math></u> (deg.)	<u><math>Re</math></u>	<u>Page</u>
4.0	$2 \times 10^1$	257
4.0	$4 \times 10^1$	261
4.0	$8 \times 10^1$	265
4.0	$2 \times 10^2$	269
4.0	$4 \times 10^2$	273
4.0	$8 \times 10^2$	277
4.0	$2 \times 10^3$	281
4.0	$4 \times 10^3$	285
4.0	$6 \times 10^3$	289
2.0	$5 \times 10^1$	293



$\epsilon$ (deg.)	Re	Page
2.0	$1 \times 10^2$	297
2.0	$2 \times 10^2$	301
2.0	$4 \times 10^2$	305
2.0	$8 \times 10^2$	309
2.0	$2 \times 10^3$	313
2.0	$4 \times 10^3$	317
2.0	$8 \times 10^3$	321
2.0	$1 \times 10^4$	325
2.0	$2 \times 10^4$	329
1.0	$2 \times 10^2$	333
1.0	$4 \times 10^2$	337
1.0	$8 \times 10^2$	341
1.0	$2 \times 10^3$	345
1.0	$4 \times 10^3$	349
1.0	$8 \times 10^3$	353
1.0	$2 \times 10^4$	357
1.0	$4 \times 10^4$	361
1.0	$8 \times 10^4$	365
0.3333	$2 \times 10^3$	369
0.3333	$4 \times 10^3$	373
0.3333	$8 \times 10^3$	377
0.3333	$2 \times 10^4$	381
0.3333	$4 \times 10^4$	385
0.3333	$8 \times 10^4$	389

GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 2.0E+01

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.3115	1.0000	.0549	0.0003	0.2509	.0349
DRT	-0.7501	0.9471	.0698	0.0030	0.2509	.0549
DPR	0.3023	0.9080	.0349	-0.0003	0.2509	.0050
DTT	-0.1720	0.9770	.0549	-0.0004	0.2509	.0
DTP	103.1642	1.0000	.0698	97.2866	1.0000	.0
DPP	-0.0097	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE ORY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70388	0.79085	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0097	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0196	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0195	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0292	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0291	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0398	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0498	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0497	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0594	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0693	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0691	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ORY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70388	0.79085	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0097	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0196	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0195	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0292	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0291	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0398	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0498	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0497	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0594	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0693	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0691	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70365	0.79055	0.85703	0.90709	0.94707	0.97703	1.00000
0.0	0.0	-0.120E-06	-7.304E-06	-1.190E-05	-1.671E-05	-2.107E-05	-2.492E-05	-2.827E-05	-3.127E-05	-3.387E-05	-3.615E-04	1.081E-04
0.0099	0.0	-0.776E-05	-4.462E-05	-7.092E-05	-9.912E-05	-1.251E-04	-1.472E-04	-1.673E-04	-1.847E-04	-1.998E-04	7.989E-04	7.922E-04
0.0097	0.0	-0.637E-05	-7.484E-05	-1.189E-04	-1.696E-04	-2.091E-04	-2.489E-04	-2.823E-04	-3.102E-04	-3.323E-04	1.306E-03	1.291E-03
0.0196	0.0	-0.488E-05	-6.767E-05	-1.393E-04	-1.948E-04	-2.401E-04	-2.810E-04	-3.181E-04	-3.523E-04	-3.838E-04	1.532E-03	1.467E-03
0.3198	0.0	-5.325E-05	-4.682E-05	-1.389E-04	-1.901E-04	-2.403E-04	-2.832E-04	-3.236E-04	-3.614E-04	-3.968E-04	1.493E-03	1.431E-03
0.0293	0.0	-0.374E-05	-7.026E-05	-1.117E-04	-1.664E-04	-2.134E-04	-2.534E-04	-2.894E-04	-3.224E-04	-3.524E-04	1.308E-03	1.179E-03
0.0292	0.0	-2.783E-05	-4.467E-05	-7.098E-05	-9.948E-05	-1.264E-04	-1.497E-04	-1.676E-04	-1.814E-04	-1.914E-04	7.991E-04	7.662E-04
0.3391	0.0	-7.608E-05	-1.214E-05	-1.927E-05	-2.717E-05	-3.511E-05	-4.264E-05	-4.973E-05	-5.643E-05	-6.273E-05	3.673E-04	3.733E-04
0.0488	0.0	1.444E-05	4.330E-05	3.704E-05	5.168E-05	6.466E-05	7.649E-05	8.652E-05	9.517E-05	1.024E-04	9.004E-04	9.418E-04
0.0487	0.0	3.843E-05	8.701E-05	9.047E-05	1.268E-04	1.594E-04	1.871E-04	2.104E-04	2.297E-04	2.467E-04	1.460E-03	1.400E-03
0.0487	0.0	9.206E-05	8.374E-05	1.331E-04	1.693E-04	2.062E-04	2.371E-04	2.637E-04	2.859E-04	3.032E-04	1.720E-03	1.647E-03
0.5884	0.0	5.731E-05	9.205E-05	1.403E-04	1.725E-04	2.048E-04	2.368E-04	2.644E-04	2.879E-04	3.074E-04	1.638E-03	1.566E-03
0.0483	0.0	3.789E-05	6.303E-05	9.426E-05	1.333E-04	1.642E-04	1.937E-04	2.207E-04	2.454E-04	2.674E-04	1.468E-03	1.428E-03
0.0491	0.0	4.267E-05	1.588E-05	2.835E-05	3.535E-05	4.274E-05	5.030E-05	5.806E-05	6.604E-05	7.424E-05	8.266E-04	8.246E-04

THE FIELD OF THE RATIO OF THE DPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70365	0.79055	0.85703	0.90709	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	2.909E-06	9.710E-06	1.742E-05	2.525E-05	3.196E-05	3.789E-05	4.219E-05	4.519E-05	4.722E-05	3.371E-04	3.371E-04
0.0097	0.0	4.862E-06	1.623E-05	2.943E-05	4.214E-05	5.432E-05	6.282E-05	7.043E-05	7.755E-05	8.325E-05	5.918E-04	5.918E-04
0.0196	0.0	3.729E-06	1.904E-05	3.463E-05	4.963E-05	6.287E-05	7.398E-05	8.284E-05	9.004E-05	9.582E-05	6.392E-04	6.392E-04
0.0195	0.0	6.032E-06	1.806E-05	3.380E-05	4.844E-05	6.139E-05	7.249E-05	8.099E-05	8.807E-05	9.385E-05	6.188E-04	6.188E-04
0.293	0.0	0.629E-06	1.335E-05	2.761E-05	3.988E-05	5.031E-05	5.949E-05	6.673E-05	7.247E-05	7.702E-05	5.092E-04	5.092E-04
0.0292	0.0	2.974E-06	4.801E-06	1.774E-05	2.541E-05	3.232E-05	3.801E-05	4.270E-05	4.638E-05	4.938E-05	3.593E-04	3.593E-04
0.3391	0.0	5.669E-07	2.742E-06	4.919E-06	7.032E-06	8.947E-06	1.086E-05	1.264E-05	1.427E-05	1.570E-05	9.924E-04	9.924E-04
0.0399	0.0	-1.434E-06	-4.959E-06	-9.057E-06	-1.300E-05	-1.698E-05	-1.926E-05	-2.147E-05	-2.304E-05	-2.404E-05	-1.568E-04	-1.568E-04
0.0488	0.0	-3.934E-06	-1.230E-05	-2.234E-05	-3.208E-05	-4.044E-05	-4.777E-05	-5.394E-05	-5.844E-05	-6.167E-05	-4.086E-04	-4.086E-04
0.0487	0.0	-5.398E-06	-1.813E-05	-3.293E-05	-4.722E-05	-5.945E-05	-7.044E-05	-7.902E-05	-8.534E-05	-9.002E-05	-6.187E-04	-6.187E-04
0.5884	0.0	-0.386E-06	-2.116E-05	-3.643E-05	-5.092E-05	-6.493E-05	-7.771E-05	-8.923E-05	-9.863E-05	-1.063E-04	-7.426E-04	-7.426E-04
0.0483	0.0	-0.012E-06	-2.003E-05	-3.630E-05	-5.091E-05	-6.494E-05	-7.771E-05	-8.923E-05	-9.863E-05	-1.063E-04	-7.426E-04	-7.426E-04
0.0491	0.0	-3.986E-06	-1.314E-05	-2.371E-05	-3.393E-05	-4.304E-05	-5.092E-05	-5.807E-05	-6.402E-05	-6.892E-05	-4.886E-04	-4.886E-04
0.0091	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 4.0E+01

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.6125	1.0000	.0549	0.0005	0.2509	.0349
DRT	-1.4821	0.9471	.0698	0.0062	0.2509	.0549
DRH	0.1804	0.9471	.0349	-0.0000	0.2509	.0100
DTT	-0.3389	0.9770	.0549	-0.0008	0.2509	.0
DTP	101.5373	1.0000	.0698	98.8324	1.0000	.0
DRP	-3.0191	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE IF ZERO  
 DRR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRP = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRH = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO



GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 4.0E+31

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

X R		0.28090	0.44327	0.59077	0.70398	0.78068	0.88703	0.93799	0.94707	0.97703	1.00000	
YETA												
0.0	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.974E-01	9.941E-01	9.909E-01	9.883E-01	9.863E-01	
.00997	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.994E-01	9.975E-01	9.945E-01	9.916E-01	9.891E-01	9.871E-01	
.01494	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.999E-01	9.979E-01	9.948E-01	9.920E-01	9.894E-01	9.874E-01	
.01998	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.993E-01	9.962E-01	9.935E-01	9.908E-01	9.888E-01	
.02493	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.000E+00	9.969E-01	9.942E-01	9.915E-01	9.895E-01	
.02992	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.974E-01	9.947E-01	9.927E-01	
.03491	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.989E-01	9.962E-01	9.942E-01	
.03989	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	
.04488	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	
.04987	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	
.05486	1.0	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	
.05984	1.0	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	1.003E+00	
.06483	1.0	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	
.06981	1.0	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	1.004E+00	

THE FIELD OF THE RATIO OF THE DPR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

X R		0.0	0.28090	0.44327	0.59077	0.70398	0.78068	0.88703	0.93799	0.94707	0.97703	1.00000
YETA												
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	-1.429E-07	-1.598E-06	-1.460E-06	1.374E-05	7.277E-05	0.0	0.0	3.126E-04	3.643E-04	2.343E-04	0.0
.00997	0.0	-2.134E-08	-2.742E-06	-2.940E-06	2.693E-05	1.409E-04	0.0	0.0	6.048E-04	7.436E-04	4.886E-04	0.0
.01494	0.0	6.881E-07	-3.779E-06	-8.143E-06	3.267E-05	2.046E-04	0.0	0.0	8.888E-04	1.076E-03	6.681E-04	0.0
.01998	0.0	2.164E-06	-6.080E-06	-1.870E-05	3.635E-05	2.878E-04	0.0	0.0	1.075E-03	1.352E-03	8.528E-04	0.0
.02493	0.0	4.798E-06	-7.971E-06	-2.807E-05	4.070E-05	2.993E-04	0.0	0.0	1.235E-03	1.578E-03	1.008E-03	0.0
.02992	0.0	8.090E-06	-1.024E-05	-3.327E-05	4.010E-05	3.184E-04	0.0	0.0	1.394E-03	1.728E-03	1.124E-03	0.0
.03491	0.0	1.240E-05	-1.266E-05	-4.039E-05	3.681E-05	3.167E-04	0.0	0.0	1.562E-03	1.898E-03	1.283E-03	0.0
.03989	0.0	1.820E-05	-1.822E-05	-6.435E-05	3.117E-05	3.040E-04	0.0	0.0	1.747E-03	1.808E-03	1.234E-03	0.0
.04488	0.0	2.587E-05	-2.587E-05	-9.980E-05	2.276E-05	2.749E-04	0.0	0.0	1.818E-03	1.724E-03	1.212E-03	0.0
.04987	0.0	3.633E-05	-3.633E-05	-1.496E-04	1.331E-05	2.400E-04	0.0	0.0	1.818E-03	1.724E-03	1.212E-03	0.0
.05486	0.0	4.683E-05	-4.683E-05	-2.401E-04	3.444E-06	1.733E-04	0.0	0.0	1.818E-03	1.724E-03	1.212E-03	0.0
.05984	0.0	6.095E-05	-6.095E-05	-3.444E-04	-3.748E-06	1.124E-04	0.0	0.0	1.818E-03	1.724E-04	7.936E-04	0.0
.06483	0.0	7.779E-05	-8.939E-05	-5.019E-05	-9.994E-06	8.270E-05	0.0	0.0	1.818E-03	1.724E-04	4.316E-04	0.0
.06981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.818E-03	1.724E-04	0.0	0.0



GAP ANGLE = 4.000C DEGREES REYNOLDS NUMBER = 4.0E+01

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.25000	0.44327	0.59077	0.70385	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.00499	0.0	-8.241E-06	-1.462E-05	-2.394E-05	-3.394E-05	-4.242E-05	-5.039E-05	-5.691E-05	-2.419E-05	2.115E-04	2.115E-04
.00997	0.0	-5.851E-05	-8.930E-05	-1.419E-04	-1.979E-04	-2.498E-04	-2.911E-04	-2.871E-04	9.687E-05	1.557E-03	1.490E-03
.01496	0.0	-4.274E-05	-1.492E-04	-2.370E-04	-3.308E-04	-4.191E-04	-4.847E-04	-4.699E-04	1.046E-04	3.583E-03	2.474E-03
.01993	0.0	-1.091E-04	-1.784E-04	-2.746E-04	-3.888E-04	-4.878E-04	-5.687E-04	-5.439E-04	2.553E-04	3.031E-03	2.695E-03
.02493	0.0	-1.064E-04	-1.711E-04	-2.717E-04	-3.789E-04	-4.757E-04	-5.543E-04	-5.264E-04	2.585E-04	2.960E-03	2.817E-03
.02992	0.0	-8.744E-05	-1.400E-04	-2.222E-04	-3.113E-04	-3.911E-04	-4.590E-04	-4.330E-04	9.028E-04	2.419E-03	2.317E-03
.03491	0.0	-4.565E-05	-8.937E-05	-1.418E-04	-1.979E-04	-2.492E-04	-2.917E-04	-2.830E-04	9.944E-05	1.551E-03	1.485E-03
.03989	0.0	-1.821E-05	-3.427E-05	-5.845E-05	-8.331E-05	-9.875E-05	-9.244E-05	-9.053E-05	-2.819E-05	4.475E-04	4.252E-04
.04488	0.0	2.688E-05	4.666E-05	7.419E-05	1.031E-04	1.283E-04	1.467E-04	1.238E-04	-1.822E-04	-7.658E-04	-7.358E-04
.04987	0.0	7.083E-05	1.141E-04	1.813E-04	2.526E-04	3.163E-04	3.664E-04	3.448E-04	-2.544E-04	-1.930E-03	-1.893E-03
.05486	0.0	1.041E-04	1.676E-04	2.642E-04	3.712E-04	4.661E-04	5.426E-04	5.126E-04	-2.772E-04	-2.078E-03	-2.756E-03
.05984	0.0	1.214E-04	1.952E-04	3.102E-04	4.328E-04	5.443E-04	6.367E-04	6.179E-04	-2.415E-04	-3.369E-03	-3.245E-03
.06483	0.0	1.147E-04	1.842E-04	2.824E-04	4.053E-04	5.148E-04	6.038E-04	6.014E-04	-1.472E-04	-3.232E-03	-3.090E-03
.06981	0.0	7.500E-05	1.201E-04	1.934E-04	2.658E-04	3.393E-04	3.948E-04	4.023E-04	-4.211E-05	-2.125E-03	-2.049E-03
.06981	0.0	1.853E-05	3.1170E-05	5.068E-05	7.058E-05	8.902E-05	1.057E-04	1.109E-04	7.701E-05	-4.457E-04	-4.457E-04

THE FIELD OF THE RATIO OF THE DPT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.25000	0.44327	0.59077	0.70385	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
.00499	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00997	0.0	5.818E-06	1.942E-05	3.525E-05	5.051E-05	6.390E-05	7.498E-05	6.347E-05	8.740E-05	6.646E-05	-2.116E-05
.01496	0.0	3.728E-06	3.248E-05	5.864E-05	8.439E-05	1.087E-04	1.252E-04	1.399E-04	1.534E-04	1.059E-04	-1.482E-07
.01993	0.0	1.144E-05	3.819E-05	6.929E-05	9.928E-05	1.285E-04	1.472E-04	1.644E-04	1.792E-04	1.298E-04	-4.326E-07
.02493	0.0	1.121E-05	3.724E-05	6.764E-05	9.647E-05	1.232E-04	1.437E-04	1.604E-04	1.666E-04	1.216E-04	-8.728E-07
.02992	0.0	9.285E-06	3.070E-05	5.564E-05	7.967E-05	1.008E-04	1.182E-04	1.320E-04	1.361E-04	9.986E-05	-1.422E-06
.03491	0.0	9.947E-06	1.960E-05	3.848E-05	5.078E-05	6.424E-05	7.843E-05	8.431E-05	8.712E-05	6.447E-05	-2.011E-06
.03989	0.0	1.734E-06	5.485E-06	9.834E-06	1.404E-05	1.780E-05	2.102E-05	2.366E-05	2.582E-05	1.447E-05	-2.588E-06
.04488	0.0	-2.673E-06	-8.459E-06	-1.614E-05	-2.602E-05	-3.280E-05	-3.837E-05	-4.261E-05	-4.346E-05	-3.046E-05	-2.837E-06
.04987	0.0	-1.079E-05	-3.628E-05	-6.589E-05	-9.443E-05	-1.194E-04	-1.401E-04	-1.564E-04	-1.622E-04	-1.091E-04	-2.629E-06
.05486	0.0	-1.268E-05	-4.236E-05	-7.689E-05	-1.102E-04	-1.394E-04	-1.630E-04	-1.830E-04	-1.908E-04	-1.460E-04	-2.031E-06
.05984	0.0	-1.202E-05	-4.004E-05	-7.242E-05	-1.030E-04	-1.314E-04	-1.544E-04	-1.732E-04	-1.817E-04	-1.427E-04	-1.213E-06
.06483	0.0	-1.071E-05	-2.828E-05	-4.744E-05	-6.785E-05	-8.533E-05	-1.008E-04	-1.131E-04	-1.193E-04	-9.629E-05	-4.006E-07
.06981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES

REYNOLDS NUMBER = 8.0E+01

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	1.2198	1.0000	.0549	0.0011	0.2509	.0349
DRT	-2.9407	0.9471	.0698	0.0119	0.2509	.0549
DPR	0.1079	0.9471	.0399	-0.0000	0.2509	.0100
DTT	-0.6705	0.9770	.0549	-0.0016	0.2509	.0
DTP	100.8741	0.9770	.0698	99.4955	1.0000	.0
DPP	-0.0378	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.EPSLON AND AT R = 0

DPP = 0 AT BETA = 0.EPSLON AND AT R = 0

DPR = 0 AT BETA = 0.EPSLON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 4.0030 DEGREES REYNOLDS NUMBER = 8.0E+01

THE FIELD OF THE RATIO OF THE DRN ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25090	0.44327	0.59077	0.70388	0.74055	0.88703	0.90799	0.94707	0.97703	1.00000
0.0	1.367E-03	4.538E-03	8.232E-03	-1.178E-02	-1.488E-02	-1.748E-02	-1.950E-02	-2.090E-02	-2.242E-02	-2.405E-02	-1.869E-02	0.0
0.0	7.748E-04	3.253E-03	8.502E-03	-6.439E-03	-1.058E-02	-1.247E-02	-1.378E-02	-1.461E-02	-1.501E-02	-1.541E-02	-1.090E-02	0.0
0.0	9.638E-04	1.877E-03	3.434E-03	-4.463E-03	-6.130E-03	-7.170E-03	-8.018E-03	-8.423E-03	-8.423E-03	-8.423E-03	-6.160E-03	0.0
0.0	1.430E-04	4.769E-04	8.614E-04	1.227E-03	1.550E-03	1.800E-03	2.070E-03	2.264E-03	2.405E-03	2.645E-03	1.830E-03	0.0
0.0	3.270E-04	7.430E-04	1.340E-03	1.990E-03	2.814E-03	3.933E-03	5.143E-03	6.400E-03	7.430E-03	8.600E-03	1.703E-03	0.0
0.0	5.348E-04	1.788E-03	3.280E-03	4.647E-03	6.888E-03	9.710E-03	1.089E-02	1.170E-02	1.242E-02	1.305E-02	6.642E-03	0.0
0.0	7.611E-04	2.843E-03	4.619E-03	6.802E-03	9.730E-03	1.136E-02	1.242E-02	1.305E-02	1.370E-02	1.433E-02	8.007E-03	0.0
0.0	9.908E-04	2.974E-03	5.400E-03	7.748E-03	1.092E-02	1.198E-02	1.242E-02	1.242E-02	1.242E-02	1.242E-02	8.481E-03	0.0
0.0	9.078E-04	3.028E-03	5.494E-03	7.881E-03	9.925E-03	1.198E-02	1.242E-02	1.242E-02	1.242E-02	1.242E-02	7.829E-03	0.0
0.0	7.923E-04	2.648E-03	4.799E-03	6.863E-03	8.668E-03	1.018E-02	1.118E-02	1.068E-02	1.068E-02	1.068E-02	7.829E-03	0.0
0.0	9.374E-04	1.788E-03	3.224E-03	4.810E-03	6.831E-03	8.846E-03	7.837E-03	7.837E-03	7.837E-03	7.837E-03	5.763E-03	0.0
0.0	1.198E-04	3.748E-04	6.658E-04	9.819E-04	1.215E-03	1.448E-03	1.674E-03	1.844E-03	1.944E-03	2.044E-03	1.007E-03	0.0
0.0	-0.749E-04	-1.628E-03	-2.967E-03	-4.231E-03	-5.364E-03	-6.274E-03	-7.020E-03	-7.419E-03	-7.419E-03	-7.419E-03	-5.138E-03	0.0
0.0	-1.210E-03	-4.025E-03	-7.284E-03	-1.043E-02	-1.317E-02	-1.548E-02	-1.728E-02	-1.818E-02	-1.818E-02	-1.818E-02	-1.416E-02	0.0
0.0	-1.998E-03	-6.517E-03	-1.178E-02	-1.677E-02	-2.117E-02	-2.488E-02	-2.782E-02	-2.941E-02	-2.941E-02	-2.941E-02	-2.406E-02	0.0

THE FIELD OF THE RATIO OF THE DRN ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.28090	0.44327	0.59077	0.70388	0.74055	0.88703	0.90799	0.94707	0.97703	1.00000
0.0	1.930E-05	1.153E-04	2.047E-04	2.988E-04	3.789E-04	4.358E-04	4.721E-04	4.988E-04	5.161E-04	5.242E-04	3.149E-03	-5.624E-03
0.0	6.588E-05	1.926E-04	3.484E-04	4.988E-04	6.401E-04	7.748E-04	8.846E-04	9.748E-04	1.048E-04	1.101E-04	-5.714E-03	-9.180E-03
0.0	7.728E-05	2.264E-04	4.095E-04	5.888E-04	7.488E-04	8.846E-04	9.748E-04	1.048E-04	1.101E-04	1.154E-04	-5.094E-03	-1.082E-02
0.0	7.834E-05	2.208E-04	4.095E-04	5.706E-04	7.195E-04	8.292E-04	9.092E-04	9.692E-04	1.019E-04	1.076E-04	-5.928E-03	-1.029E-02
0.0	8.191E-05	1.814E-04	3.282E-04	4.683E-04	6.079E-04	7.488E-04	8.846E-04	9.748E-04	1.048E-04	1.101E-04	-4.869E-03	-8.803E-03
0.0	3.937E-05	1.182E-04	2.082E-04	2.978E-04	3.744E-04	4.358E-04	4.721E-04	4.988E-04	5.161E-04	5.242E-04	-3.118E-03	-6.880E-03
0.0	1.073E-05	3.110E-05	5.616E-05	8.091E-05	1.038E-04	1.233E-04	1.392E-04	1.527E-04	1.638E-04	1.727E-04	-8.919E-04	-1.848E-03
0.0	-2.049E-05	-6.046E-05	-1.094E-04	-1.586E-04	-2.127E-04	-2.717E-04	-3.351E-04	-4.028E-04	-4.748E-04	-5.501E-04	1.849E-03	2.313E-03
0.0	-3.020E-05	-1.474E-04	-2.670E-04	-3.809E-04	-4.789E-04	-5.648E-04	-6.428E-04	-7.128E-04	-7.748E-04	-8.288E-04	6.400E-03	6.442E-03
0.0	-7.374E-05	-2.144E-04	-3.910E-04	-5.600E-04	-7.031E-04	-8.092E-04	-8.992E-04	-9.748E-04	-1.038E-04	-1.091E-04	7.905E-04	8.602E-03
0.0	-8.602E-05	-2.824E-04	-4.564E-04	-6.532E-04	-8.214E-04	-9.514E-04	-1.048E-04	-1.128E-04	-1.198E-04	-1.268E-04	7.681E-04	8.377E-03
0.0	-8.184E-05	-2.379E-04	-4.304E-04	-6.166E-04	-7.772E-04	-9.072E-04	-1.019E-04	-1.101E-04	-1.161E-04	-1.221E-04	8.517E-04	1.191E-02
0.0	-8.319E-05	-1.881E-04	-2.802E-04	-4.018E-04	-5.070E-04	-5.928E-04	-6.633E-04	-7.188E-04	-7.688E-04	-8.133E-04	4.284E-03	8.080E-03
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 4.000 DEGREES REYNOLDS NUMBER = 8.0E+01

THE FIELD OF THE RATIO OF THE ODD ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25090	0.44327	0.59077	0.70388	0.79055	0.85703	0.90709	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	1.648E-05	-2.923E-05	-4.768E-05	-6.704E-05	-8.649E-05	-1.011E-04	-1.139E-04	-1.250E-04	-1.346E-04	-1.428E-04	-1.498E-04
0.0997	0.0	1.110E-04	-1.786E-04	-2.830E-04	-4.336E-04	-6.330E-04	-8.873E-04	-1.191E-03	-1.554E-03	-1.976E-03	-2.458E-03	-2.999E-03
0.1496	0.0	1.033E-04	-2.903E-04	-4.732E-04	-6.548E-04	-8.228E-04	-9.872E-04	-1.141E-03	-1.284E-03	-1.416E-03	-1.538E-03	-1.651E-03
0.1995	0.0	2.182E-04	-3.508E-04	-4.889E-04	-6.282E-04	-7.716E-04	-9.182E-04	-1.064E-03	-1.208E-03	-1.349E-03	-1.487E-03	-1.622E-03
0.2493	0.0	2.129E-04	-3.422E-04	-4.417E-04	-5.822E-04	-7.238E-04	-8.644E-04	-1.002E-03	-1.136E-03	-1.262E-03	-1.381E-03	-1.494E-03
0.2992	0.0	1.178E-04	-2.811E-04	-4.480E-04	-6.131E-04	-7.738E-04	-9.284E-04	-1.072E-03	-1.226E-03	-1.376E-03	-1.522E-03	-1.664E-03
0.3491	0.0	1.113E-04	-1.787E-04	-2.829E-04	-4.317E-04	-6.193E-04	-8.073E-04	-9.917E-04	-1.161E-03	-1.325E-03	-1.485E-03	-1.641E-03
0.3989	0.0	3.043E-05	-4.354E-05	-7.652E-05	-1.074E-04	-1.366E-04	-1.624E-04	-1.871E-04	-2.095E-04	-2.296E-04	-2.474E-04	-2.630E-04
0.4488	0.0	5.778E-05	9.328E-05	1.478E-04	2.041E-04	2.528E-04	2.979E-04	3.395E-04	3.787E-04	4.156E-04	4.503E-04	4.828E-04
0.4987	0.0	1.417E-04	2.282E-04	3.613E-04	5.010E-04	6.249E-04	7.207E-04	7.907E-04	8.419E-04	8.797E-04	9.083E-04	9.297E-04
0.5486	0.0	2.029E-04	3.391E-04	5.307E-04	7.370E-04	9.220E-04	1.059E-03	1.180E-03	1.286E-03	1.371E-03	1.438E-03	1.488E-03
0.5984	0.0	2.429E-04	3.905E-04	6.184E-04	8.392E-04	1.078E-03	1.285E-03	1.451E-03	1.579E-03	1.674E-03	1.741E-03	1.783E-03
0.6483	0.0	2.893E-04	3.683E-04	5.634E-04	8.119E-04	1.020E-03	1.193E-03	1.329E-03	1.431E-03	1.502E-03	1.547E-03	1.577E-03
0.6981	0.0	1.800E-04	2.402E-04	3.891E-04	5.290E-04	6.658E-04	7.807E-04	8.707E-04	9.407E-04	9.938E-04	1.031E-03	1.056E-03
0.7480	0.0	3.727E-05	6.342E-05	1.013E-04	1.410E-04	1.777E-04	2.135E-04	2.372E-04	2.531E-04	2.634E-04	2.700E-04	2.741E-04

THE FIELD OF THE RATIO OF THE ODD ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25090	0.44327	0.59077	0.70388	0.79055	0.85703	0.90709	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	1.163E-05	3.884E-05	7.049E-05	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	1.944E-05	6.491E-05	1.174E-04	1.064E-04	1.232E-04	1.492E-04	1.664E-04	1.737E-04	1.782E-04	1.812E-04	1.832E-04
0.1496	0.0	2.822E-05	7.038E-05	1.380E-04	1.081E-04	2.128E-04	2.496E-04	2.777E-04	2.880E-04	2.902E-04	2.912E-04	2.918E-04
0.1995	0.0	2.241E-05	7.439E-05	1.353E-04	1.932E-04	2.438E-04	2.883E-04	3.174E-04	3.272E-04	3.282E-04	3.282E-04	3.282E-04
0.2493	0.0	1.850E-05	6.140E-05	1.113E-04	1.593E-04	2.005E-04	2.347E-04	2.614E-04	2.802E-04	2.932E-04	3.012E-04	3.062E-04
0.2992	0.0	1.189E-05	3.921E-05	7.998E-05	1.013E-04	1.279E-04	1.498E-04	1.670E-04	1.781E-04	1.842E-04	1.882E-04	1.908E-04
0.3491	0.0	3.464E-06	1.097E-05	1.997E-05	2.806E-05	3.951E-05	4.187E-05	4.697E-05	4.918E-05	5.092E-05	5.224E-05	5.318E-05
0.3989	0.0	5.748E-06	-1.984E-05	-3.628E-05	-2.186E-05	-6.237E-05	-7.602E-05	-8.424E-05	-8.688E-05	-8.999E-05	-9.262E-05	-9.482E-05
0.4488	0.0	1.488E-05	-4.919E-05	-8.950E-05	-1.280E-04	-1.613E-04	-1.888E-04	-2.097E-04	-2.185E-04	-2.248E-04	-2.292E-04	-2.322E-04
0.4987	0.0	2.188E-05	-7.282E-05	-1.318E-04	-1.884E-04	-2.377E-04	-2.782E-04	-3.100E-04	-3.308E-04	-3.408E-04	-3.492E-04	-3.562E-04
0.5486	0.0	2.830E-05	-8.472E-05	-1.838E-04	-2.199E-04	-2.776E-04	-3.282E-04	-3.630E-04	-3.776E-04	-3.858E-04	-3.922E-04	-3.972E-04
0.5984	0.0	2.405E-05	-8.012E-05	-1.442E-04	-2.076E-04	-2.622E-04	-3.074E-04	-3.438E-04	-3.599E-04	-3.688E-04	-3.752E-04	-3.802E-04
0.6483	0.0	1.898E-05	-5.252E-05	-9.488E-05	-1.335E-04	-1.711E-04	-2.007E-04	-2.244E-04	-2.366E-04	-2.442E-04	-2.492E-04	-2.522E-04
0.6981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 2.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	3.0318	1.0000	.0549	0.0014	0.9471	.0349
DRT	-7.3529	0.9471	.0698	0.0299	0.2509	.0549
DPR	-0.0584	0.9471	.0349	-0.0000	0.2509	.0050
DTT	-1.6785	0.9770	.0549	-0.0041	0.2509	.0
DTP	100.4719	0.4433	.0698	100.0000	0.0000	.0
DPP	-0.0945	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO



GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 2.0E+02

THE FIELD OF THE RATIO OF THE OPM ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2500	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94737	0.97703	1.00000
.0	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.001E+00
.00499	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.001E+00
.00997	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.001E+00
.01496	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.000E+00	1.000E+00	1.000E+00	1.001E+00	1.001E+00
.01995	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00
.02493	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00
.02992	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00
.03491	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00
.03989	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00
.04488	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00
.04987	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00
.05486	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00
.05984	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00
.06483	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00
.06981	1.0	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00	1.002E+00

THE FIELD OF THE RATIO OF THE OPM ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2500	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94737	0.97703	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	-3.334E-08	-2.179E-06	-2.205E-06	-1.646E-06	-6.215E-06	1.388E-05	4.800E-06	-7.745E-05	0.0	0.0
.00997	0.0	1.494E-07	-4.141E-06	-3.905E-06	-2.359E-06	-1.293E-05	2.458E-05	-4.840E-06	-1.769E-04	-8.673E-05	0.0
.01496	0.0	9.184E-07	-5.569E-06	-8.105E-06	-7.284E-06	-1.734E-05	3.856E-05	-1.558E-05	-2.949E-04	-2.9557E-04	0.0
.01995	0.0	2.518E-06	-7.624E-06	-1.377E-05	-1.040E-05	-2.204E-05	4.914E-05	-2.637E-05	-4.111E-04	-4.040E-04	0.0
.02493	0.0	5.207E-06	-9.662E-06	-2.090E-05	-1.843E-05	-2.526E-05	6.470E-05	-3.657E-05	-5.092E-04	-4.994E-04	0.0
.02992	0.0	8.634E-06	-1.129E-05	-2.680E-05	-1.650E-05	-2.992E-05	8.347E-05	-4.354E-05	-6.709E-04	-5.956E-04	0.0
.03491	0.0	1.411E-05	-1.284E-05	-3.051E-05	-1.462E-05	-2.400E-05	8.674E-05	-4.622E-05	-8.841E-04	-8.047E-04	0.0
.03989	0.0	1.902E-05	-1.379E-05	-3.292E-05	-1.731E-05	-2.214E-05	8.879E-05	-4.308E-05	-9.417E-04	-8.169E-04	0.0
.04488	0.0	2.648E-05	-1.430E-05	-3.502E-05	-1.648E-05	-2.114E-05	8.400E-05	-4.472E-05	-9.492E-04	-8.173E-04	0.0
.04987	0.0	3.892E-05	-1.412E-05	-3.197E-05	-1.467E-05	-2.982E-05	9.371E-05	-2.298E-05	-3.209E-04	-2.794E-04	0.0
.05486	0.0	4.722E-05	-1.319E-05	-2.609E-05	-1.230E-05	-1.980E-05	3.932E-05	-1.165E-05	-1.701E-04	-1.308E-04	0.0
.05984	0.0	6.116E-05	-1.063E-05	-2.200E-05	-1.120E-05	-1.622E-05	2.432E-05	-8.593E-06	-5.792E-05	-7.785E-05	0.0
.06483	0.0	7.787E-05	-8.659E-06	-1.372E-05	-9.968E-06	-1.182E-05	8.825E-06	-6.361E-06	1.273E-05	4.863E-05	0.0
.06981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 4.0000 DEGREES

REYNOLDS NUMBER = 4.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DDR	6.0162	1.0000	.0549	0.0054	0.2509	.0349
DRT	-14.6130	0.9471	.0698	0.0599	0.2509	.0549
DPR	-0.8274	0.9471	.0349	0.0001	0.2509	.0080
DTT	-3.3291	0.9770	.0549	-0.0082	0.2509	.0
DTP	101.3361	1.0000	.0100	99.0533	1.0000	.0549
DPP	-0.1875	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DDR = 0 AT BETA = 0.EPSLON AND AT R = 0

DPP = 0 AT BETA = 0.EPSLON AND AT R = 0

DPR = 0 AT BETA = 0.EPSLON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO







GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 8.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	12.0099	1.0000	.0549	0.0108	0.2509	.0349
DRT	-29.0720	0.9471	.0698	0.1198	0.2509	.0549
DPR	-2.4025	0.9471	.0349	0.0006	0.2509	.0050
DTT	-6.6305	0.9770	.0549	-0.0164	0.2509	.0
DTP	105.7154	1.0000	.0100	95.2148	1.0000	.0549
DPP	-0.3734	0.9471	.0549	-0.0000	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0, EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0, EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0, EPSILON AND AT R = 0  
 DRT = 0 AT R = 0, 1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO



GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 8.0E+02

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2500	0.4327	0.5977	0.70385	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	1.0	0.9972-01	0.9902-01	0.9815-01	0.9705-01	0.9575-01	0.9425-01	0.9265-01	0.9105-01	0.8945-01	0.8785-01
0.0099	1.0	0.9982-01	0.9912-01	0.9825-01	0.9715-01	0.9585-01	0.9435-01	0.9275-01	0.9115-01	0.8955-01	0.8795-01
0.0099	1.0	0.9992-01	0.9922-01	0.9835-01	0.9725-01	0.9595-01	0.9445-01	0.9285-01	0.9125-01	0.8965-01	0.8805-01
0.0198	1.0	1.0002-00	0.9932-01	0.9845-01	0.9735-01	0.9605-01	0.9455-01	0.9295-01	0.9135-01	0.8975-01	0.8815-01
0.0198	1.0	1.0002+00	0.9932-01	0.9845-01	0.9735-01	0.9605-01	0.9455-01	0.9295-01	0.9135-01	0.8975-01	0.8815-01
0.0297	1.0	1.0002+00	0.9932-01	0.9845-01	0.9735-01	0.9605-01	0.9455-01	0.9295-01	0.9135-01	0.8975-01	0.8815-01
0.0297	1.0	1.0002+00	0.9932-01	0.9845-01	0.9735-01	0.9605-01	0.9455-01	0.9295-01	0.9135-01	0.8975-01	0.8815-01
0.0396	1.0	1.0012+00	0.9942-01	0.9855-01	0.9745-01	0.9615-01	0.9465-01	0.9305-01	0.9145-01	0.8985-01	0.8825-01
0.0396	1.0	1.0012+00	0.9942-01	0.9855-01	0.9745-01	0.9615-01	0.9465-01	0.9305-01	0.9145-01	0.8985-01	0.8825-01
0.0495	1.0	1.0022+00	0.9952-01	0.9865-01	0.9755-01	0.9625-01	0.9475-01	0.9315-01	0.9155-01	0.8995-01	0.8835-01
0.0495	1.0	1.0022+00	0.9952-01	0.9865-01	0.9755-01	0.9625-01	0.9475-01	0.9315-01	0.9155-01	0.8995-01	0.8835-01
0.0594	1.0	1.0032+00	0.9962-01	0.9875-01	0.9765-01	0.9635-01	0.9485-01	0.9325-01	0.9165-01	0.9005-01	0.8845-01
0.0594	1.0	1.0032+00	0.9962-01	0.9875-01	0.9765-01	0.9635-01	0.9485-01	0.9325-01	0.9165-01	0.9005-01	0.8845-01
0.0693	1.0	1.0042+00	0.9972-01	0.9885-01	0.9775-01	0.9645-01	0.9495-01	0.9335-01	0.9175-01	0.9015-01	0.8855-01
0.0693	1.0	1.0042+00	0.9972-01	0.9885-01	0.9775-01	0.9645-01	0.9495-01	0.9335-01	0.9175-01	0.9015-01	0.8855-01
0.0792	1.0	1.0052+00	0.9982-01	0.9895-01	0.9785-01	0.9655-01	0.9505-01	0.9345-01	0.9185-01	0.9025-01	0.8865-01
0.0792	1.0	1.0052+00	0.9982-01	0.9895-01	0.9785-01	0.9655-01	0.9505-01	0.9345-01	0.9185-01	0.9025-01	0.8865-01
0.0891	1.0	1.0062+00	0.9992-01	0.9905-01	0.9795-01	0.9665-01	0.9515-01	0.9355-01	0.9195-01	0.9035-01	0.8875-01
0.0891	1.0	1.0062+00	0.9992-01	0.9905-01	0.9795-01	0.9665-01	0.9515-01	0.9355-01	0.9195-01	0.9035-01	0.8875-01

THE FIELD OF THE RATIO OF THE DMR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2500	0.4327	0.5977	0.70385	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	0.0042-00	0.0084-00	0.0126-00	0.0168-00	0.0210-00	0.0252-00	0.0294-00	0.0336-00	0.0378-00	0.0420-00
0.0099	0.0	0.0042-00	0.0084-00	0.0126-00	0.0168-00	0.0210-00	0.0252-00	0.0294-00	0.0336-00	0.0378-00	0.0420-00
0.0198	0.0	0.0084-00	0.0168-00	0.0252-00	0.0336-00	0.0420-00	0.0504-00	0.0588-00	0.0672-00	0.0756-00	0.0840-00
0.0198	0.0	0.0084-00	0.0168-00	0.0252-00	0.0336-00	0.0420-00	0.0504-00	0.0588-00	0.0672-00	0.0756-00	0.0840-00
0.0297	0.0	0.0126-00	0.0252-00	0.0378-00	0.0504-00	0.0630-00	0.0756-00	0.0882-00	0.1008-00	0.1134-00	0.1260-00
0.0297	0.0	0.0126-00	0.0252-00	0.0378-00	0.0504-00	0.0630-00	0.0756-00	0.0882-00	0.1008-00	0.1134-00	0.1260-00
0.0396	0.0	0.0168-00	0.0336-00	0.0504-00	0.0672-00	0.0840-00	0.1008-00	0.1176-00	0.1344-00	0.1512-00	0.1680-00
0.0396	0.0	0.0168-00	0.0336-00	0.0504-00	0.0672-00	0.0840-00	0.1008-00	0.1176-00	0.1344-00	0.1512-00	0.1680-00
0.0495	0.0	0.0210-00	0.0420-00	0.0630-00	0.0840-00	0.1050-00	0.1260-00	0.1470-00	0.1680-00	0.1890-00	0.2100-00
0.0495	0.0	0.0210-00	0.0420-00	0.0630-00	0.0840-00	0.1050-00	0.1260-00	0.1470-00	0.1680-00	0.1890-00	0.2100-00
0.0594	0.0	0.0252-00	0.0504-00	0.0756-00	0.1008-00	0.1260-00	0.1512-00	0.1764-00	0.2016-00	0.2268-00	0.2520-00
0.0594	0.0	0.0252-00	0.0504-00	0.0756-00	0.1008-00	0.1260-00	0.1512-00	0.1764-00	0.2016-00	0.2268-00	0.2520-00
0.0693	0.0	0.0294-00	0.0588-00	0.0882-00	0.1176-00	0.1470-00	0.1764-00	0.2058-00	0.2352-00	0.2646-00	0.2940-00
0.0693	0.0	0.0294-00	0.0588-00	0.0882-00	0.1176-00	0.1470-00	0.1764-00	0.2058-00	0.2352-00	0.2646-00	0.2940-00
0.0792	0.0	0.0336-00	0.0672-00	0.1008-00	0.1344-00	0.1680-00	0.2016-00	0.2352-00	0.2688-00	0.3024-00	0.3360-00
0.0792	0.0	0.0336-00	0.0672-00	0.1008-00	0.1344-00	0.1680-00	0.2016-00	0.2352-00	0.2688-00	0.3024-00	0.3360-00
0.0891	0.0	0.0378-00	0.0756-00	0.1134-00	0.1512-00	0.1890-00	0.2268-00	0.2646-00	0.3024-00	0.3402-00	0.3780-00
0.0891	0.0	0.0378-00	0.0756-00	0.1134-00	0.1512-00	0.1890-00	0.2268-00	0.2646-00	0.3024-00	0.3402-00	0.3780-00



GAP ANGLE = 0.0000 DEGREES REYNOLDS NUMBER = 0.0E+02

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70385	0.78050	0.85703	0.90709	0.94707	0.97703	1.00000
.0	0.0	0.0	-1.044E-04	-2.001E-04	-4.009E-04	-6.940E-04	-8.226E-04	-9.810E-04	-1.194E-03	-8.249E-04	4.240E-03	4.348E-02
.00499	0.0	1.108E-03	-1.774E-03	-2.790E-03	-4.060E-03	-4.918E-03	-5.631E-03	-6.307E-03	-6.943E-03	0.038E-04	0.038E-04	2.946E-02
.00997	0.0	1.851E-03	-2.908E-03	-4.673E-03	-6.493E-03	-8.035E-03	-9.367E-03	-1.098E-02	-1.281E-02	2.318E-03	9.096E-02	4.676E-02
.01496	0.0	2.177E-03	-3.459E-03	-5.490E-03	-7.692E-03	-9.490E-03	-1.098E-02	-1.281E-02	-1.481E-02	3.777E-03	9.789E-02	6.680E-02
.01995	0.0	2.125E-03	-3.406E-03	-5.372E-03	-7.411E-03	-9.236E-03	-1.070E-02	-1.264E-02	-1.464E-02	4.471E-03	9.789E-02	6.816E-02
.02493	0.0	1.749E-03	-2.402E-03	-4.424E-03	-6.117E-03	-7.628E-03	-8.818E-03	-1.034E-02	-1.202E-02	4.160E-03	4.716E-02	4.919E-02
.02992	0.0	1.113E-03	-1.768E-03	-2.849E-03	-3.920E-03	-4.906E-03	-5.698E-03	-6.505E-03	-7.245E-03	2.874E-04	3.005E-02	2.880E-02
.03491	0.0	3.059E-04	-4.924E-04	-7.842E-04	-1.110E-03	-1.428E-03	-1.654E-03	-1.899E-03	-2.173E-03	8.734E-04	8.427E-03	6.077E-03
.03989	0.0	5.745E-04	9.198E-04	1.438E-03	1.958E-03	2.393E-03	2.786E-03	3.150E-03	3.492E-03	2.733E-03	1.409E-02	-1.658E-02
.04488	0.0	1.013E-03	2.289E-03	3.904E-03	4.920E-03	6.004E-03	7.010E-03	7.946E-03	8.725E-03	-3.438E-03	-3.791E-02	-4.634E-02
.04987	0.0	2.079E-03	3.344E-03	4.259E-03	4.920E-03	5.032E-03	1.046E-02	1.046E-02	1.033E-02	-4.672E-03	-5.629E-02	-6.348E-02
.05486	0.0	2.426E-03	3.891E-03	4.147E-03	4.106E-03	3.003E-03	1.234E-02	1.176E-02	1.192E-02	-4.700E-03	-6.630E-02	-6.080E-02
.05985	0.0	2.302E-03	3.678E-03	3.812E-03	3.004E-03	1.010E-02	1.176E-02	1.176E-02	1.192E-02	-4.700E-03	-6.630E-02	-6.080E-02
.06483	0.0	1.494E-03	2.349E-03	2.792E-03	2.264E-03	1.614E-03	2.725E-03	2.725E-03	2.725E-03	-1.480E-03	-4.163E-02	-3.978E-02
.06981	0.0	3.710E-04	6.350E-04	1.019E-03	1.421E-03	1.795E-03	2.118E-03	2.118E-03	2.247E-03	1.113E-03	-8.800E-03	-8.800E-03

THE FIELD OF THE RATIO OF THE OPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70385	0.78050	0.85703	0.90709	0.94707	0.97703	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	1.163E-04	3.476E-04	7.009E-04	1.098E-04	1.285E-03	1.498E-03	1.498E-03	1.636E-03	1.732E-03	1.374E-03	-4.191E-07
.00997	0.0	1.948E-04	6.478E-04	1.172E-03	1.668E-03	2.097E-03	2.447E-03	2.447E-03	2.739E-03	2.866E-03	2.222E-03	-2.930E-06
.01496	0.0	2.201E-04	7.624E-04	1.379E-03	1.964E-03	2.468E-03	2.860E-03	2.860E-03	3.206E-03	3.143E-03	2.834E-03	-8.933E-06
.01995	0.0	2.241E-04	7.448E-04	1.347E-03	1.920E-03	2.413E-03	2.816E-03	2.816E-03	3.133E-03	3.242E-03	2.913E-03	-1.718E-05
.02493	0.0	1.490E-04	6.133E-04	1.110E-03	1.582E-03	1.993E-03	2.324E-03	2.324E-03	2.684E-03	2.698E-03	1.941E-03	-2.794E-05
.02992	0.0	1.190E-04	3.921E-04	7.092E-04	1.013E-03	1.276E-03	1.494E-03	1.494E-03	1.690E-03	1.691E-03	1.208E-03	-3.926E-05
.03491	0.0	3.470E-05	1.102E-04	1.991E-04	2.666E-04	3.053E-04	4.327E-04	4.327E-04	4.814E-04	4.718E-04	3.012E-04	-4.916E-05
.03989	0.0	5.741E-05	-1.074E-04	-3.580E-04	-5.073E-04	-6.321E-04	-8.114E-04	-8.114E-04	-8.608E-04	-6.817E-04	-1.630E-03	-5.669E-05
.04488	0.0	-1.494E-04	-4.907E-04	-8.893E-04	-1.265E-03	-1.587E-03	-1.847E-03	-1.847E-03	-2.053E-03	-2.140E-03	-1.630E-03	-6.140E-05
.04987	0.0	-2.157E-04	-7.240E-04	-1.312E-03	-1.870E-03	-2.308E-03	-2.742E-03	-2.742E-03	-3.052E-03	-3.173E-03	-2.413E-03	-6.140E-05
.05486	0.0	-2.530E-04	-8.462E-04	-1.533E-03	-2.187E-03	-2.794E-03	-3.219E-03	-3.219E-03	-3.587E-03	-3.734E-03	-2.463E-03	-3.078E-05
.05985	0.0	-2.408E-04	-8.007E-04	-1.440E-03	-2.070E-03	-2.609E-03	-3.058E-03	-3.058E-03	-3.492E-03	-3.698E-03	-2.715E-03	-2.377E-05
.06483	0.0	-1.594E-04	-5.250E-04	-9.478E-04	-1.353E-03	-1.706E-03	-1.998E-03	-1.998E-03	-2.233E-03	-2.333E-03	-1.858E-03	-7.863E-06
.06981	0.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 2.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-29.3949	1.0000	.0150	0.0277	0.2509	.0349
DRT	-71.5365	0.9471	.0698	0.3018	0.2509	.0549
DPR	-15.8092	0.9471	.0299	0.0036	0.2509	.0050
DTT	-15.8668	0.9770	.0549	-0.0404	0.2509	.0
DTP	150.5201	1.0000	.0	70.4762	1.0000	.0449
OPP	-0.8974	0.9471	.0549	-0.0001	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.0150 AND AT R = 0  
 DPP = 0 AT BETA = 0.0150 AND AT R = 0  
 DPR = 0 AT BETA = 0.0150 AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 4.0000 DEGREES KEYHOLES NUMBER = 2.0EY03

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.25090	0.44327	0.59077	0.70385	0.74055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0097	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0199	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0198	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0299	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0298	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0399	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0398	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0498	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0599	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0598	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0699	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0698	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0799	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0798	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0899	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0898	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0999	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0998	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.25090	0.44327	0.59077	0.70385	0.74055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0099	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0097	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0199	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0198	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0299	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0298	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0399	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0398	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0498	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0599	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0598	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0699	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0698	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0799	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0798	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0899	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0898	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0999	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0998	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 4.0000 DEGREE REYNOLDS NUMBER = 2.0E+03

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.44327	0.59077	0.70365	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5485	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6483	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE OPP ELEMENT UP THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.44327	0.59077	0.70365	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5485	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5984	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6483	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 4.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-42.3143	1.0000	.0100	0.0002	0.2500	.0349
DRT	-123.6880	0.9080	.0094	0.0092	0.9471	.0150
DPR	-28.2377	0.9471	.0197	0.0136	0.2500	.0050
DTT	20.7271	0.9770	.0150	0.0275	0.7906	.0399
DTP	293.6500	0.9770	.0	29.7926	0.9471	.0349
DPP	-1.4025	0.9080	.0094	-0.0002	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 4.0030 DEGREES REYNOLDS NUMBER = 4.0E+03

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.4327	0.59077	0.70305	0.79005	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5486	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5985	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6484	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6983	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7482	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.4327	0.59077	0.70305	0.79005	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.4987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5486	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5985	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6484	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.6983	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7482	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.7981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 4.0E+03

THE FIELD OF THE RATIO OF THE ODD ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.04488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.04987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05486	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05985	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.06483	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.06981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ODD ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.25000	0.44327	0.59077	0.70388	0.79055	0.85703	0.90799	0.94707	0.97703	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03989	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.04488	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.04987	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05486	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.05985	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.06483	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.06981	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 4.0000 DEGREES      REYNOLDS NUMBER = 6.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-42.4100	1.0000	.0100	-0.0619	0.5909	.0399
DRT	-155.9452	0.8570	.0698	0.3271	0.9770	.0100
DPR	-25.7923	0.9471	.0150	-0.0045	0.7038	.0050
DTT	21.2907	0.9770	.0150	-0.1056	0.2509	.0
DTP	390.1958	1.0000	.0	-0.2813	0.9471	.0349
DPP	-1.6724	0.8570	.0598	-0.0002	1.0000	.0050

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO





GAP ANGLE = 4.0000 DEGREES REYNOLDS NUMBER = 6.0E+03

THE FIELD OF THE RATIO OF THE ODT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.44327	0.59077	0.70389	0.79086	0.85703	0.90799	0.94737	0.97723	1.00000
0.0	-1.008E-03	-1.496E-03	-1.815E-03	-1.469E-03	-3.561E-03	-6.797E-03	-9.431E-03	-3.567E-02	2.216E-02	2.216E-02	2.216E-02	2.216E-02
0.0499	-7.352E-03	-9.655E-03	-1.191E-02	-1.221E-02	-1.821E-02	-1.821E-02	-1.821E-02	-2.795E-02	-2.123E-02	1.340E-01	1.340E-01	1.298E-01
0.0997	-1.239E-02	-1.644E-02	-2.055E-02	-2.198E-02	-3.134E-02	-4.842E-02	-3.417E-02	1.595E-02	2.114E-01	2.014E-01	2.014E-01	2.014E-01
0.1495	-1.475E-02	-1.990E-02	-2.512E-02	-2.646E-02	-3.744E-02	-4.420E-02	-1.577E-02	6.102E-02	2.129E-01	2.051E-01	2.051E-01	2.051E-01
0.1993	-1.468E-02	-2.021E-02	-2.639E-02	-3.117E-02	-3.744E-02	-2.765E-02	1.454E-02	8.979E-02	1.660E-01	1.610E-01	1.610E-01	1.610E-01
0.2491	-1.235E-02	-1.766E-02	-2.372E-02	-2.979E-02	-3.105E-02	-7.652E-03	3.822E-02	6.071E-02	1.035E-01	1.011E-01	1.011E-01	1.011E-01
0.2989	-9.455E-03	-1.250E-02	-1.818E-02	-2.422E-02	-2.217E-02	8.684E-03	4.524E-02	6.307E-02	4.614E-02	4.579E-02	4.579E-02	4.579E-02
0.3487	-5.516E-03	-9.583E-03	-1.475E-02	-1.475E-02	-1.037E-02	1.859E-02	3.688E-02	4.177E-02	-3.269E-03	-1.555E-03	-1.555E-03	-1.555E-03
0.3985	3.210E-03	2.064E-03	1.221E-03	2.302E-03	3.117E-03	2.316E-02	1.985E-02	1.791E-02	-5.050E-02	-4.674E-02	-4.674E-02	-4.674E-02
0.4483	9.215E-03	1.150E-02	1.317E-02	1.288E-02	1.788E-02	2.492E-02	1.277E-03	-1.374E-02	-1.309E-01	-9.536E-02	-9.536E-02	-9.536E-02
0.4981	1.421E-02	1.934E-02	2.461E-02	2.796E-02	3.272E-02	2.573E-02	-1.310E-02	-4.416E-02	-1.537E-01	-1.464E-01	-1.464E-01	-1.464E-01
0.5479	1.739E-02	2.438E-02	3.298E-02	4.002E-02	4.471E-02	2.583E-02	-2.084E-02	-7.661E-02	-1.979E-01	-1.906E-01	-1.906E-01	-1.906E-01
0.5977	1.633E-02	2.445E-02	3.463E-02	4.398E-02	4.838E-02	2.373E-02	-1.984E-02	-9.456E-02	-2.099E-01	-2.035E-01	-2.035E-01	-2.035E-01
0.6475	1.162E-02	1.674E-02	2.456E-02	3.226E-02	3.534E-02	1.611E-02	-1.207E-02	-7.599E-02	-1.624E-01	-1.483E-01	-1.483E-01	-1.483E-01
0.6973	2.920E-03	5.297E-03	8.924E-03	1.333E-02	1.475E-02	4.280E-03	1.789E-03	-2.678E-02	-4.825E-02	-4.825E-02	-4.825E-02	-4.825E-02

THE FIELD OF THE RATIO OF THE ODT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.2500	0.44327	0.59077	0.70389	0.79086	0.85703	0.90799	0.94737	0.97723	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	4.554E-04	2.599E-03	3.948E-03	4.746E-03	5.022E-03	6.123E-03	7.894E-03	7.894E-03	7.894E-03	7.894E-03	7.894E-03	7.894E-03
0.0997	1.423E-03	4.376E-03	6.775E-03	8.136E-03	8.791E-03	1.056E-02	1.252E-02	1.252E-02	1.252E-02	1.252E-02	1.252E-02	1.252E-02
0.1495	1.622E-03	5.208E-03	8.169E-03	9.936E-03	1.016E-02	1.284E-02	1.377E-02	1.377E-02	1.377E-02	1.377E-02	1.377E-02	1.377E-02
0.1993	1.666E-03	5.166E-03	8.250E-03	1.024E-02	1.165E-02	1.335E-02	1.233E-02	1.233E-02	1.233E-02	1.233E-02	1.233E-02	1.233E-02
0.2491	1.377E-03	4.254E-03	7.159E-03	9.107E-03	1.073E-02	1.142E-02	9.224E-03	9.224E-03	9.224E-03	9.224E-03	9.224E-03	9.224E-03
0.2989	9.929E-04	2.912E-03	5.042E-03	6.725E-03	8.244E-03	8.308E-03	6.379E-03	6.379E-03	6.379E-03	6.379E-03	6.379E-03	6.379E-03
0.3487	2.719E-04	1.309E-03	2.125E-03	3.246E-03	4.474E-03	4.054E-03	1.266E-03	1.266E-03	1.266E-03	1.266E-03	1.266E-03	1.266E-03
0.3985	-4.111E-04	-1.131E-03	-1.291E-03	-9.103E-04	-3.039E-04	-1.018E-03	-2.997E-03	-2.997E-03	-2.997E-03	-2.997E-03	-2.997E-03	-2.997E-03
0.4483	-1.088E-03	-3.239E-03	-4.838E-03	-5.422E-03	-6.095E-03	-6.537E-03	-7.333E-03	-7.333E-03	-7.333E-03	-7.333E-03	-7.333E-03	-7.333E-03
0.4981	-1.596E-03	-4.990E-03	-7.892E-03	-9.601E-03	-1.071E-02	-1.188E-02	-1.188E-02	-1.188E-02	-1.188E-02	-1.188E-02	-1.188E-02	-1.188E-02
0.5479	-1.882E-03	-5.903E-03	-9.691E-03	-1.252E-02	-1.460E-02	-1.584E-02	-1.438E-02	-1.438E-02	-1.438E-02	-1.438E-02	-1.438E-02	-1.438E-02
0.5977	-1.795E-03	-5.294E-03	-9.808E-03	-1.289E-02	-1.534E-02	-1.672E-02	-1.534E-02	-1.534E-02	-1.534E-02	-1.534E-02	-1.534E-02	-1.534E-02
0.6475	-1.194E-03	-3.665E-03	-6.684E-03	-9.618E-03	-1.070E-02	-1.196E-02	-1.094E-02	-1.094E-02	-1.094E-02	-1.094E-02	-1.094E-02	-1.094E-02
0.6973	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES      REYNOLDS NUMBER = 5.0E+01

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.2446	1.0000	.0274	0.0001	0.3533	.0175
DRT	-0.5043	0.9636	.0349	0.0037	0.3538	.0274
DPR	0.0490	0.9636	.0175	0.0000	0.3538	.0025
DTT	-0.1662	0.9926	.0274	-0.0002	0.3538	.0
DTP	100.2230	0.9811	.0299	99.7721	1.0000	.0
DPP	-0.0033	0.9636	.0274	-0.0030	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAM ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 9.3E+01

THE FIELD OF THE RATIO OF THE DUT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.75379	0.58412	0.73428	0.83171	0.89528	0.93866	0.96360	0.98114	0.99254	1.00000
0.0	0.0	-4.019E-04	-1.227E-03	-2.004E-03	-2.614E-03	-3.089E-03	-3.372E-03	-3.583E-03	-3.741E-03	-3.858E-03	-3.938E-03	0.0
0.0	0.0	-2.043E-04	-1.836E-03	-1.674E-03	-2.193E-03	-2.193E-03	-2.570E-03	-2.570E-03	-2.826E-03	-2.826E-03	-3.041E-03	0.0
0.0	0.0	-1.667E-04	-1.576E-03	-1.404E-03	-1.815E-03	-1.815E-03	-2.066E-03	-2.066E-03	-2.237E-03	-2.237E-03	-2.416E-03	0.0
0.0	0.0	-1.310E-05	-1.203E-04	-2.192E-04	-2.737E-04	-3.206E-04	-3.561E-04	-3.888E-04	-4.184E-04	-4.451E-04	-4.692E-04	0.0
0.0	0.0	0.634E-05	2.054E-04	3.375E-04	4.436E-04	5.162E-04	5.594E-04	5.888E-04	6.043E-04	6.163E-04	6.254E-04	0.0
0.0	0.0	2.239E-04	6.855E-04	1.121E-03	1.604E-03	1.716E-03	1.887E-03	1.930E-03	1.930E-03	1.930E-03	1.930E-03	0.0
0.0	0.0	3.623E-04	8.017E-04	1.313E-03	1.711E-03	2.004E-03	2.202E-03	2.262E-03	2.262E-03	2.262E-03	2.262E-03	0.0
0.0	0.0	2.075E-04	6.154E-04	1.033E-03	1.431E-03	1.719E-03	2.004E-03	2.202E-03	2.301E-03	2.301E-03	2.301E-03	0.0
0.0	0.0	2.048E-04	7.136E-04	1.164E-03	1.518E-03	1.776E-03	1.952E-03	2.084E-03	2.181E-03	2.181E-03	2.181E-03	0.0
0.0	0.0	1.546E-04	4.601E-04	7.809E-04	1.016E-03	1.187E-03	1.363E-03	1.539E-03	1.715E-03	1.715E-03	1.715E-03	0.0
0.0	0.0	3.041E-05	1.059E-04	1.594E-04	2.041E-04	2.353E-04	2.539E-04	2.636E-04	2.636E-04	2.636E-04	2.636E-04	0.0
0.0	0.0	-1.381E-04	-4.335E-04	-7.443E-04	-9.446E-04	-1.115E-03	-1.234E-03	-1.322E-03	-1.382E-03	-1.427E-03	-1.457E-04	0.0
0.0	0.0	-3.587E-04	-1.017E-03	-1.772E-03	-2.313E-03	-2.702E-03	-2.977E-03	-3.164E-03	-3.261E-03	-3.313E-03	-3.343E-03	0.0
0.0	0.0	-1.091E-04	-1.762E-03	-2.844E-03	-4.369E-03	-6.318E-03	-8.744E-03	-1.164E-02	-1.504E-02	-1.893E-02	-2.329E-02	0.0

THE FIELD OF THE RATIO OF THE DUT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.75379	0.58412	0.73428	0.83171	0.89528	0.93866	0.96360	0.98114	0.99256	1.00000
0.0	0.0	5.413E-06	1.542E-05	2.598E-05	4.172E-05	5.000E-05	5.000E-05	5.000E-05	5.000E-05	5.000E-05	5.000E-05	0.0
0.0	0.0	9.042E-06	2.575E-05	4.310E-05	5.822E-05	6.925E-05	7.641E-05	8.000E-05	8.000E-05	8.000E-05	8.000E-05	0.0
0.0	0.0	1.043E-05	3.077E-05	5.101E-05	6.822E-05	8.142E-05	8.969E-05	9.300E-05	9.300E-05	9.300E-05	9.300E-05	0.0
0.0	0.0	1.077E-05	2.951E-05	4.972E-05	6.551E-05	7.945E-05	8.714E-05	9.000E-05	9.000E-05	9.000E-05	9.000E-05	0.0
0.0	0.0	8.523E-06	2.423E-05	4.052E-05	5.455E-05	6.516E-05	7.132E-05	7.400E-05	7.400E-05	7.400E-05	7.400E-05	0.0
0.0	0.0	6.824E-06	1.538E-05	2.586E-05	3.454E-05	4.121E-05	4.512E-05	4.700E-05	4.700E-05	4.700E-05	4.700E-05	0.0
0.0	0.0	1.842E-06	4.115E-06	6.890E-06	9.128E-06	1.091E-05	1.199E-05	1.240E-05	1.240E-05	1.240E-05	1.240E-05	0.0
0.0	0.0	-2.014E-06	-6.105E-06	-1.037E-05	-1.859E-05	-2.222E-05	-2.405E-05	-2.405E-05	-2.405E-05	-2.405E-05	-2.405E-05	0.0
0.0	0.0	-6.003E-06	-1.974E-05	-3.334E-05	-4.472E-05	-5.350E-05	-5.836E-05	-6.000E-05	-6.000E-05	-6.000E-05	-6.000E-05	0.0
0.0	0.0	-1.015E-05	-2.892E-05	-4.893E-05	-6.893E-05	-7.811E-05	-8.662E-05	-9.000E-05	-9.000E-05	-9.000E-05	-9.000E-05	0.0
0.0	0.0	-1.183E-05	-3.370E-05	-5.677E-05	-7.595E-05	-9.059E-05	-9.982E-05	-1.000E-04	-1.000E-04	-1.000E-04	-1.000E-04	0.0
0.0	0.0	-1.111E-05	-3.175E-05	-5.341E-05	-7.135E-05	-8.468E-05	-9.410E-05	-9.627E-05	-9.627E-05	-9.627E-05	-9.627E-05	0.0
0.0	0.0	-7.325E-06	-2.066E-05	-3.463E-05	-4.669E-05	-5.484E-05	-6.100E-05	-6.445E-05	-6.445E-05	-6.445E-05	-6.445E-05	0.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 5.0E+01

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	1.0	1.000E+00	9.998E-01	1.000E+00	1.000E+00	1.000E+00	9.997E-01	9.987E-01	9.977E-01	9.977E-01	9.977E-01
.00249	1.0	1.000E+00	9.998E-01	1.000E+00	1.000E+00	1.000E+00	9.998E-01	9.988E-01	9.978E-01	9.978E-01	9.978E-01
.00499	1.0	1.000E+00	9.998E-01	1.000E+00	1.000E+00	1.000E+00	9.999E-01	9.991E-01	9.982E-01	9.982E-01	9.982E-01
.00748	1.0	1.000E+00	9.998E-01	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.994E-01	9.985E-01	9.985E-01	9.985E-01
.00997	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.997E-01	9.991E-01	9.991E-01	9.991E-01
.01247	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.996E-01	9.996E-01	9.996E-01
.01496	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01745	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01995	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.02244	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.02493	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.02743	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.02992	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.03241	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.998E-01	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.03491	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.998E-01	9.994E-01	1.000E+00	1.000E+00	1.000E+00	1.000E+00

THE FIELD OF THE RATIO OF THE DSR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	3.478E-07	-5.370E-07	-4.280E-06	-2.169E-06	1.378E-05	5.434E-05	1.040E-04	6.204E-05	2.484E-06	0.0
.00499	0.0	7.211E-07	-1.031E-06	-6.468E-06	-4.527E-06	2.531E-05	1.035E-04	2.022E-04	1.597E-04	4.630E-06	0.0
.00748	0.0	1.152E-06	-1.842E-06	-1.292E-05	-6.751E-06	3.387E-05	1.463E-04	3.929E-04	2.311E-04	4.517E-06	0.0
.00997	0.0	1.535E-06	-2.774E-06	-1.698E-05	-9.632E-06	3.989E-05	1.810E-04	3.701E-04	2.926E-04	2.371E-06	0.0
.01247	0.0	1.929E-06	-3.682E-06	-2.057E-05	-1.281E-05	4.233E-05	2.054E-04	4.307E-04	3.419E-04	-1.355E-06	0.0
.01496	0.0	2.322E-06	-4.618E-06	-2.343E-05	-1.601E-05	4.090E-05	2.178E-04	4.719E-04	3.767E-04	-6.437E-06	0.0
.01745	0.0	2.929E-06	-5.621E-06	-2.527E-05	-1.907E-05	3.576E-05	2.187E-04	4.901E-04	3.941E-04	-1.254E-05	0.0
.01995	0.0	3.599E-06	-6.626E-06	-2.692E-05	-2.164E-05	2.793E-05	2.078E-04	4.858E-04	3.932E-04	-1.963E-05	0.0
.02244	0.0	4.442E-06	-7.629E-06	-2.839E-05	-2.319E-05	1.786E-05	1.867E-04	4.582E-04	3.718E-04	-2.841E-05	0.0
.02493	0.0	5.501E-06	-8.675E-06	-2.937E-05	-2.346E-05	7.377E-06	1.567E-04	4.083E-04	3.305E-04	-2.982E-05	0.0
.02743	0.0	6.870E-06	-9.667E-06	-1.988E-05	-2.176E-05	-2.178E-06	1.208E-04	3.370E-04	2.690E-04	-3.072E-05	0.0
.02992	0.0	8.562E-06	-1.056E-05	-1.488E-05	-1.775E-05	-8.275E-06	8.128E-05	2.457E-04	1.924E-04	-2.677E-05	0.0
.03241	0.0	1.067E-05	-2.068E-06	-8.183E-06	-1.067E-05	-8.580E-06	4.072E-05	1.344E-04	1.007E-04	-1.562E-05	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 5.0E+01

THE FIELD OF THE RATIO OF THE OTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES

REYNOLDS NUMBER = 1.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.4835	1.0000	.0299	-0.0001	0.9636	.0224
DRT	-1.0024	0.9636	.0349	0.0074	0.3538	.0274
DPR	0.0955	0.9636	.0175	0.0000	0.3538	.0025
DTT	-0.3279	0.9926	.0274	0.0001	0.9636	.0199
DTP	101.1104	0.9926	.0349	98.9543	1.0000	.0
DPP	-0.0066	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO







GAP ANGLE = 2.0000 DEGREES

REYNOLDS NUMBER = 2.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.9672	1.0000	.0299	-0.0002	0.9636	.0224
DRT	-2.0050	0.9636	.0349	0.0147	0.3538	.0274
DPR	0.0786	0.9636	.0175	0.0000	0.3538	.0100
DTT	-0.6560	0.9926	.0274	0.0002	0.9636	.0199
DTP	100.9345	0.9811	.0349	99.0978	0.9811	.0
DPP	-0.0132	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO







GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 2.0E+02

THE FIELD OF THE RATIO OF THE OY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
0.00249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE OY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
0.00249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.01995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.02992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES      REYNOLDS NUMBER = 4.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	1.8778	1.0000	.0299	0.0012	0.3538	.0175
DRT	-3.9223	0.9636	.0349	0.0295	0.3538	.0274
DPR	-0.1220	0.9926	.0193	0.0000	0.5841	.0075
DTT	-1.2623	0.9926	.0274	-0.0010	0.3538	.00
DTP	100.3056	0.9811	.0324	99.6776	0.9811	.00
DPP	-0.0256	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0. EPSILON AND AT R = 0

DPP = 0 AT BETA = 0. EPSILON AND AT R = 0

DPR = 0 AT BETA = 0. EPSILON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO





GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 4.0E+02

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35370	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	1.589E-05	-2.813E-05	-4.462E-05	-5.901E-05	-7.004E-05	-7.965E-05	-7.923E-05	1.268E-04	1.268E-04	1.675E-04	8.675E-04
.00249	0.0	-1.042E-04	-1.678E-04	-2.595E-04	-3.405E-04	-4.007E-04	-4.347E-04	-1.631E-04	1.632E-03	1.632E-03	5.841E-03	5.776E-03
.00499	0.0	-1.741E-04	-2.801E-04	-4.327E-04	-5.669E-04	-6.600E-04	-7.174E-04	-2.6074E-04	2.622E-03	2.622E-03	9.628E-03	9.519E-03
.00748	0.0	-2.047E-04	-3.291E-04	-5.080E-04	-6.647E-04	-7.800E-04	-8.380E-04	-2.727E-04	3.390E-03	3.390E-03	1.120E-02	1.108E-02
.00997	0.0	-1.997E-04	-3.209E-04	-4.985E-04	-6.473E-04	-7.592E-04	-8.096E-04	-2.456E-04	3.325E-03	3.325E-03	1.086E-02	1.075E-02
.01247	0.0	-1.640E-04	-2.638E-04	-4.065E-04	-5.318E-04	-6.240E-04	-6.850E-04	-2.009E-04	2.694E-03	2.694E-03	8.937E-03	8.839E-03
.01496	0.0	-1.443E-04	-2.175E-04	-3.258E-04	-4.175E-04	-4.873E-04	-5.144E-04	-1.614E-04	1.619E-03	1.619E-03	5.764E-03	5.698E-03
.01745	0.0	-2.842E-05	-4.544E-05	-7.063E-05	-9.402E-05	-1.134E-04	-1.286E-04	-1.032E-04	2.642E-04	2.642E-04	1.730E-03	1.706E-03
.01995	0.0	5.428E-05	8.754E-05	1.344E-04	1.738E-04	2.009E-04	2.165E-04	-2.228E-05	-1.148E-03	-1.148E-03	-2.717E-03	-2.653E-03
.02244	0.0	1.330E-04	2.141E-04	3.299E-04	4.302E-04	5.021E-04	5.288E-04	8.916E-05	-2.453E-03	-2.453E-03	-7.044E-03	-6.973E-03
.02493	0.0	1.954E-04	3.148E-04	4.854E-04	6.349E-04	7.449E-04	7.929E-04	2.226E-04	-3.355E-03	-3.355E-03	-1.061E-02	-1.049E-02
.02743	0.0	2.278E-04	3.666E-04	5.665E-04	7.432E-04	8.742E-04	9.399E-04	3.538E-04	-3.651E-03	-3.651E-03	-1.262E-02	-1.248E-02
.02992	0.0	2.181E-04	3.458E-04	5.359E-04	7.032E-04	8.298E-04	9.000E-04	4.228E-04	-3.178E-03	-3.178E-03	-1.219E-02	-1.200E-02
.03241	0.0	1.406E-04	2.251E-04	3.480E-04	4.572E-04	5.408E-04	5.914E-04	3.288E-04	-0.676E-03	-0.676E-03	-1.840E-03	-1.824E-03
.03491	0.0	3.880E-05	6.024E-05	9.181E-05	1.199E-04	1.408E-04	1.591E-04	1.588E-04	-1.399E-04	-1.399E-04	-1.784E-03	-1.744E-03

THE FIELD OF THE RATIO OF THE OPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35370	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	1.378E-05	4.198E-05	6.828E-05	8.627E-05	1.023E-04	1.117E-04	1.175E-04	1.175E-04	1.102E-04	6.542E-05	-2.068E-05
.00499	0.0	2.300E-05	7.013E-05	1.140E-04	1.473E-04	1.758E-04	1.962E-04	1.984E-04	1.984E-04	1.816E-04	1.056E-04	-1.436E-07
.00748	0.0	2.709E-05	8.247E-05	1.340E-04	1.739E-04	2.091E-04	2.184E-04	2.290E-04	2.290E-04	2.112E-04	1.212E-04	-4.170E-07
.00997	0.0	2.648E-05	8.048E-05	1.307E-04	1.686E-04	1.981E-04	2.128E-04	2.228E-04	2.228E-04	2.047E-04	1.168E-04	-8.368E-07
.01247	0.0	2.181E-05	6.616E-05	1.074E-04	1.385E-04	1.603E-04	1.749E-04	1.831E-04	1.831E-04	1.682E-04	9.628E-05	-1.363E-06
.01496	0.0	1.397E-05	4.214E-05	6.831E-05	8.819E-05	1.051E-04	1.118E-04	1.168E-04	1.168E-04	1.091E-04	6.317E-05	-1.917E-06
.01745	0.0	3.989E-06	1.159E-05	1.667E-05	2.419E-05	3.019E-05	3.096E-05	3.270E-05	3.270E-05	3.184E-05	2.102E-05	-2.404E-06
.01995	0.0	-6.916E-06	-2.171E-05	-3.545E-05	-4.868E-05	-5.281E-05	-5.703E-05	-5.933E-05	-5.933E-05	-5.217E-05	-2.623E-05	-2.756E-06
.02244	0.0	-1.733E-05	-5.742E-05	-9.696E-05	-1.122E-04	-1.299E-04	-1.412E-04	-1.475E-04	-1.475E-04	-1.338E-04	-7.373E-05	-2.790E-06
.02493	0.0	-2.564E-05	-7.859E-05	-1.279E-04	-1.651E-04	-1.910E-04	-2.084E-04	-2.116E-04	-2.116E-04	-2.009E-04	-1.182E-04	-2.840E-06
.02743	0.0	-3.006E-05	-9.171E-05	-1.491E-04	-1.927E-04	-2.232E-04	-2.438E-04	-2.560E-04	-2.560E-04	-2.368E-04	-1.419E-04	-1.978E-06
.02992	0.0	-2.860E-05	-8.668E-05	-1.408E-04	-1.829E-04	-2.110E-04	-2.307E-04	-2.427E-04	-2.427E-04	-2.293E-04	-1.419E-04	-1.168E-06
.03241	0.0	-1.907E-05	-5.680E-05	-9.183E-05	-1.186E-04	-1.378E-04	-1.504E-04	-1.586E-04	-1.586E-04	-1.516E-04	-9.778E-05	-3.932E-07
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGRFFS

REYNOLDS NUMBER = 8.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	3.7119	1.0000	.0299	0.0024	0.3538	.0175
DRT	-7.7787	0.9636	.0349	0.0589	0.3538	.0274
DPR	-0.4364	0.9811	.0175	0.0000	0.5841	.0025
DTT	-2.4915	0.9926	.0274	-0.0032	0.3538	.0
DTP	100.8609	1.0000	.0075	99.0633	0.9926	.0349
DPP	-0.3500	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 0.0E+02

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	-0.422E-03	-1.901E-02	-3.183E-02	-4.106E-02	-4.752E-02	-5.181E-02	-5.454E-02	-5.513E-02	-5.117E-02	0.0	0.0
.00249	0.0	-4.612E-03	-1.406E-02	-2.260E-02	-2.935E-02	-3.391E-02	-3.693E-02	-3.860E-02	-3.832E-02	-2.140E-02	0.0	0.0
.00499	0.0	-2.672E-03	-8.114E-03	-1.313E-02	-1.682E-02	-1.943E-02	-2.112E-02	-2.224E-02	-2.201E-02	-1.224E-02	0.0	0.0
.00748	0.0	-6.893E-04	-2.047E-03	-3.318E-03	-4.227E-03	-4.835E-03	-5.263E-03	-5.403E-03	-5.451E-03	-4.427E-03	0.0	0.0
.00997	0.0	1.064E-03	3.263E-03	5.343E-03	6.869E-03	7.965E-03	8.610E-03	8.411E-03	5.499E-03	1.591E-03	0.0	0.0
.01247	0.0	2.511E-03	7.703E-03	1.249E-02	1.605E-02	1.849E-02	2.009E-02	2.009E-02	1.514E-02	6.254E-03	0.0	0.0
.01496	0.0	3.582E-03	1.094E-02	1.775E-02	2.279E-02	2.625E-02	2.841E-02	2.866E-02	2.228E-02	9.787E-03	0.0	0.0
.01745	0.0	4.197E-03	1.282E-02	2.076E-02	2.667E-02	3.074E-02	3.325E-02	3.369E-02	2.665E-02	1.221E-02	0.0	0.0
.01995	0.0	4.280E-03	1.305E-02	2.114E-02	2.720E-02	3.138E-02	3.403E-02	3.455E-02	2.777E-02	1.334E-02	0.0	0.0
.02244	0.0	3.757E-03	1.141E-02	1.850E-02	2.385E-02	2.759E-02	2.995E-02	3.033E-02	2.493E-02	1.272E-02	0.0	0.0
.02493	0.0	2.554E-03	7.680E-03	1.245E-02	1.611E-02	1.871E-02	2.041E-02	2.080E-02	1.714E-02	9.556E-03	0.0	0.0
.02743	0.0	5.889E-04	1.616E-03	2.599E-03	3.427E-03	4.066E-03	4.509E-03	4.434E-03	3.154E-03	2.606E-03	0.0	0.0
.02992	0.0	-2.212E-03	-7.011E-03	-1.146E-02	-1.476E-02	-1.702E-02	-1.850E-02	-1.952E-02	-1.854E-02	-1.010E-02	0.0	0.0
.03241	0.0	-5.740E-03	-1.739E-02	-2.818E-02	-3.634E-02	-4.209E-02	-4.587E-02	-4.855E-02	-4.582E-02	-2.641E-02	0.0	0.0
.03491	0.0	-9.504E-03	-2.816E-02	-4.453E-02	-5.843E-02	-7.386E-02	-7.778E-02	-7.778E-02	-7.481E-02	-4.929E-02	0.0	0.0

THE FIELD OF THE RATIO OF THE ORP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	6.657E-05	2.447E-04	4.038E-04	5.230E-04	6.013E-04	6.389E-04	6.389E-04	1.668E-04	-3.377E-03	-1.171E-02	-1.738E-02
.00499	0.0	1.446E-04	4.004E-04	6.726E-04	8.693E-04	9.967E-04	1.048E-03	1.048E-03	1.734E-04	-6.930E-03	-1.927E-02	-2.804E-02
.00748	0.0	1.700E-04	4.798E-04	7.893E-04	1.018E-03	1.165E-03	1.213E-03	1.165E-03	1.165E-04	-6.990E-03	-2.239E-02	-3.216E-02
.00997	0.0	1.659E-04	4.677E-04	7.868E-04	9.902E-04	1.132E-03	1.172E-03	6.878E-05	6.878E-05	-6.871E-03	-2.169E-02	-3.103E-02
.01247	0.0	1.363E-04	3.841E-04	6.314E-04	8.138E-04	9.304E-04	9.607E-04	8.437E-05	8.437E-05	-6.984E-03	-1.782E-02	-2.670E-02
.01496	0.0	6.674E-05	2.441E-04	4.019E-04	5.195E-04	5.967E-04	6.182E-04	6.083E-05	-3.384E-03	-1.146E-02	-1.709E-02	-2.670E-02
.01745	0.0	2.372E-05	6.654E-05	1.104E-04	1.400E-04	1.724E-04	1.887E-04	1.201E-04	-6.071E-04	-3.427E-03	-6.093E-03	-6.093E-03
.01995	0.0	-4.498E-05	-1.276E-04	-2.079E-04	-2.670E-04	-2.938E-04	-2.938E-04	1.347E-04	2.344E-03	5.440E-03	6.271E-03	6.271E-03
.02244	0.0	-1.104E-04	-3.122E-04	-5.119E-04	-6.587E-04	-7.440E-04	-7.569E-04	6.932E-05	5.012E-03	1.407E-02	1.676E-02	1.676E-02
.02493	0.0	-1.623E-04	-4.587E-04	-7.544E-04	-9.716E-04	-1.110E-03	-1.167E-03	-3.205E-05	6.914E-03	2.118E-02	2.973E-02	2.973E-02
.02743	0.0	-1.893E-04	-5.347E-04	-8.817E-04	-1.140E-03	-1.309E-03	-1.369E-03	-7.688E-04	7.688E-03	2.822E-02	3.692E-02	3.692E-02
.02992	0.0	-1.788E-04	-5.044E-04	-8.333E-04	-1.082E-03	-1.248E-03	-1.302E-03	-3.595E-04	6.985E-03	2.428E-02	3.712E-02	3.712E-02
.03241	0.0	-1.172E-04	-3.283E-04	-5.420E-04	-7.051E-04	-8.162E-04	-8.715E-04	-3.333E-04	3.974E-03	1.608E-02	2.569E-02	2.569E-02
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	1.0	1.000E+00	9.999E-01	1.000E+00	1.000E+00	1.000E+00	9.986E-01	9.993E-01	1.003E+00	1.007E+00	1.007E+00
.00249	1.0	1.000E+00	9.999E-01	1.000E+00	1.000E+00	1.000E+00	9.987E-01	9.995E-01	1.003E+00	1.008E+00	1.008E+00
.00499	1.0	1.000E+00	9.999E-01	1.000E+00	1.000E+00	1.000E+00	9.989E-01	9.996E-01	1.003E+00	1.009E+00	1.009E+00
.00748	1.0	1.000E+00	9.999E-01	1.000E+00	1.000E+00	1.000E+00	9.990E-01	9.998E-01	1.003E+00	1.009E+00	1.009E+00
.00997	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.991E-01	9.999E-01	1.003E+00	1.009E+00	1.009E+00
.01247	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.992E-01	1.000E+00	1.003E+00	1.009E+00	1.009E+00
.01496	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.993E-01	1.000E+00	1.003E+00	1.009E+00	1.009E+00
.01745	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	9.994E-01	1.000E+00	1.003E+00	1.009E+00	1.009E+00
.01994	1.0	1.000E+00	1.001E+00	1.000E+00	1.000E+00	1.000E+00	9.995E-01	1.000E+00	1.003E+00	1.009E+00	1.009E+00
.02244	1.0	1.000E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01
.02493	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01
.02743	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01
.02992	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01
.03241	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01
.03491	1.0	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	1.001E+00	9.995E-01	9.995E-01	9.995E-01

THE FIELD OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	2.617E-07	1.021E-07	1.622E-06	5.246E-06	3.047E-05	3.555E-05	-2.338E-04	-7.271E-04	0.0	0.0
.00499	0.0	5.663E-07	3.361E-07	-3.002E-06	-1.014E-05	9.740E-05	6.309E-05	-4.784E-04	-1.495E-03	-6.595E-04	0.0
.00748	0.0	1.079E-06	-1.816E-07	-5.374E-06	-1.310E-05	8.290E-05	8.651E-05	-7.267E-04	-2.282E-03	-2.091E-03	0.0
.00997	0.0	1.452E-06	-8.666E-07	-6.575E-06	-1.511E-05	1.075E-04	1.042E-04	-9.649E-04	-3.028E-03	-2.787E-03	0.0
.01247	0.0	1.698E-06	-1.572E-06	-7.200E-06	-1.767E-05	1.303E-04	1.212E-04	-1.169E-03	-3.668E-03	-3.391E-03	0.0
.01496	0.0	1.987E-06	-2.227E-06	-6.479E-06	-1.981E-05	1.475E-04	1.343E-04	-1.317E-03	-4.131E-03	-3.840E-03	0.0
.01745	0.0	2.375E-06	-2.768E-06	-5.241E-06	-2.225E-05	1.591E-04	1.454E-04	-1.392E-03	-4.364E-03	-4.079E-03	0.0
.01994	0.0	2.987E-06	-3.170E-06	-3.957E-06	-2.394E-05	1.653E-04	1.591E-04	-1.366E-03	-4.334E-03	-4.074E-03	0.0
.02244	0.0	3.837E-06	-3.638E-06	-3.168E-06	-2.496E-05	1.654E-04	1.591E-04	-1.298E-03	-4.031E-03	-3.814E-03	0.0
.02493	0.0	4.901E-06	-3.826E-06	-2.138E-06	-2.457E-05	1.552E-04	1.550E-04	-1.135E-03	-3.481E-03	-3.311E-03	0.0
.02743	0.0	6.181E-06	-3.939E-06	-1.158E-06	-2.254E-05	1.343E-04	1.397E-04	-9.091E-04	-2.731E-03	-2.610E-03	0.0
.02992	0.0	7.969E-06	-3.812E-06	-1.203E-06	-1.939E-05	1.023E-04	1.085E-04	-6.395E-04	-1.654E-03	-1.776E-03	0.0
.03241	0.0	1.025E-05	-2.235E-06	-1.044E-06	-1.307E-05	5.751E-05	6.140E-05	-3.369E-04	-9.255E-04	-8.682E-04	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 2.0000 DEGREES HEYNOLDS NUMBER = 0.0E+02

THE FIELD OF THE MATRIX OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

DATA

	0.0	0.1	0.53370	0.53412	0.73408	0.93171	0.94928	0.93464	0.96360	0.98114	0.99254	1.00000
0.0	0.0	0.0	-1.179E-05	-9.661E-05	-6.900E-04	-1.177E-04	-1.393E-04	-1.568E-04	-1.533E-04	2.447E-04	1.736E-03	1.736E-03
0.0249	0.0	-7.009E-04	-3.342E-04	-5.192E-04	-6.733E-04	-7.871E-04	-8.520E-04	-9.220E-04	-3.755E-04	3.204E-03	1.187E-02	1.184E-02
0.06949	0.0	-3.492E-04	-5.822E-04	-4.592E-04	-1.120E-03	-1.307E-03	-1.402E-03	-1.482E-03	-5.280E-04	5.505E-03	1.404E-02	1.603E-02
0.0748	0.0	-4.044E-04	-6.559E-04	-1.007E-03	-1.312E-03	-1.528E-03	-1.629E-03	-1.709E-03	-8.299E-04	6.667E-03	2.212E-02	2.186E-02
0.0997	0.0	-1.993E-04	-6.394E-04	-4.811E-04	-1.272E-03	-1.486E-03	-1.676E-03	-1.842E-03	-6.672E-04	6.548E-03	2.143E-02	2.119E-02
0.1247	0.0	-1.241E-04	-5.272E-04	-4.059E-04	-1.059E-03	-1.222E-03	-1.393E-03	-1.543E-03	-3.843E-04	5.318E-03	1.761E-02	1.742E-02
0.1496	0.0	-7.007E-04	-3.339E-04	-5.132E-04	-6.677E-04	-7.825E-04	-8.520E-04	-9.220E-04	-2.630E-04	3.213E-03	1.139E-02	1.122E-02
0.1745	0.0	-5.667E-04	-9.049E-04	-1.413E-04	-1.876E-04	-2.449E-04	-2.449E-04	-1.624E-04	5.558E-04	3.399E-03	3.399E-03	3.393E-03
0.1995	0.0	1.086E-04	1.739E-04	2.647E-04	3.800E-04	5.866E-04	3.943E-04	-1.232E-04	-1.232E-04	-2.263E-03	-2.358E-03	-3.311E-03
0.2244	0.0	2.660E-04	4.262E-04	6.526E-04	6.459E-04	4.749E-04	1.026E-03	1.728E-04	1.728E-04	-4.603E-03	-1.388E-02	-1.374E-02
0.2493	0.0	1.909E-04	4.248E-04	9.619E-04	1.242E-03	1.436E-03	1.342E-03	4.241E-04	6.600E-03	-2.092E-02	-2.070E-02	-2.070E-02
0.2742	0.0	4.557E-04	7.108E-04	1.124E-03	1.688E-03	1.714E-03	1.633E-03	6.723E-04	7.811E-03	-2.491E-02	-2.464E-02	-2.464E-02
0.2991	0.0	4.327E-04	6.077E-04	1.263E-03	1.392E-03	1.612E-03	1.754E-03	8.046E-04	8.046E-04	-2.400E-02	-2.400E-02	-2.372E-02
0.3241	0.0	2.812E-04	4.403E-04	6.420E-04	4.074E-04	1.647E-03	1.154E-03	6.270E-04	6.270E-04	-3.732E-03	-1.590E-02	-1.571E-02
0.3491	0.0	7.162E-04	1.204E-04	1.839E-04	2.387E-04	2.811E-04	3.135E-04	3.014E-04	3.014E-04	-2.935E-04	-3.543E-03	-3.543E-03

THE FIELD OF THE MATRIX OF THE SECOND MOMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

DATA

	0.0	0.1	0.53370	0.53412	0.73408	0.93171	0.94928	0.93464	0.96360	0.98114	0.99254	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	2.751E-04	8.397E-05	1.342E-04	1.754E-04	2.030E-04	2.212E-04	2.326E-04	2.326E-04	2.163E-04	1.302E-04	1.302E-04
0.06949	0.0	4.401E-05	1.403E-04	2.274E-04	2.928E-04	3.383E-04	3.683E-04	3.864E-04	3.864E-04	3.591E-04	2.040E-04	2.040E-04
0.0748	0.0	5.418E-05	1.650E-04	2.672E-04	3.438E-04	3.969E-04	4.319E-04	4.527E-04	4.527E-04	4.171E-04	2.401E-04	2.401E-04
0.0997	0.0	5.792E-05	1.610E-04	2.605E-04	3.351E-04	3.867E-04	4.206E-04	4.398E-04	4.398E-04	4.038E-04	2.304E-04	2.304E-04
0.1247	0.0	4.163E-05	1.323E-04	2.142E-04	2.754E-04	3.178E-04	3.487E-04	3.612E-04	3.612E-04	3.314E-04	1.894E-04	1.894E-04
0.1496	0.0	2.795E-05	8.428E-05	1.303E-04	1.754E-04	2.026E-04	2.207E-04	2.307E-04	2.307E-04	2.191E-04	1.242E-04	1.242E-04
0.1745	0.0	7.979E-04	2.310E-05	3.733E-05	4.839E-05	5.632E-05	6.163E-05	6.443E-05	6.443E-05	6.255E-05	4.089E-05	4.089E-05
0.1995	0.0	-1.384E-05	-6.343E-05	-7.051E-05	-9.026E-05	-1.035E-04	-1.119E-04	-1.165E-04	-1.165E-04	-1.029E-04	-5.236E-05	-5.381E-05
0.2244	0.0	-3.467E-05	-1.066E-04	-1.733E-04	-2.220E-04	-2.564E-04	-2.783E-04	-2.966E-04	-2.966E-04	-2.637E-04	-1.460E-04	-1.460E-04
0.2493	0.0	-4.124E-04	-1.872E-04	-2.950E-04	-3.240E-04	-3.785E-04	-4.117E-04	-4.307E-04	-4.307E-04	-3.942E-04	-2.277E-04	-2.277E-04
0.2742	0.0	-4.018E-04	-1.634E-04	-2.673E-04	-3.032E-04	-3.429E-04	-3.825E-04	-4.025E-04	-4.025E-04	-4.710E-04	-2.805E-04	-2.805E-04
0.2991	0.0	-5.720E-05	-1.714E-04	-2.809E-04	-3.023E-04	-3.429E-04	-4.537E-04	-4.537E-04	-4.537E-04	-4.530E-04	-2.804E-04	-2.804E-04
0.3241	0.0	-3.618E-05	-1.130E-04	-1.631E-04	-2.362E-04	-2.734E-04	-2.984E-04	-3.140E-04	-3.140E-04	-2.998E-04	-1.932E-04	-1.932E-04
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES

REYNOLDS NUMBER = 2.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	9.4994	1.0000	.0274	-0.0017	0.9636	.0150
DRT	-19.8232	0.9636	.0349	0.1462	0.3538	.0274
DPR	-3.7896	0.9811	.0175	0.0002	0.3538	.0025
DTT	-6.4374	0.9926	.0274	-0.0003	0.9636	.0175
DTP	108.6403	1.0000	.0025	91.9100	1.0000	.0299
DPP	-0.1304	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0, EPSILON AND AT R = 0

DPP = 0 AT BETA = 0, EPSILON AND AT R = 0

DPR = 0 AT BETA = 0, EPSILON AND AT R = 0

DRT = 0 AT R = 0, 1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 2.00JC DEGREES      RYKOLDS NUMBER = 2.0E+03

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.58412	0.73408	0.83171	0.89828	0.93846	0.96360	0.98114	0.99284	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1994	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.58412	0.73408	0.83171	0.89828	0.93846	0.96360	0.98114	0.99284	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1994	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.73408	0.93171	0.99528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	1.0	0.9995E-01	0.9990E-01	0.9984E-01	0.9978E-01	0.9972E-01	0.9966E-01	0.9960E-01	0.9954E-01	0.9948E-01
.00249	1.0	0.9996E-01	0.9991E-01	0.9985E-01	0.9979E-01	0.9973E-01	0.9967E-01	0.9961E-01	0.9955E-01	0.9949E-01
.00499	1.0	0.9996E-01	0.9991E-01	0.9985E-01	0.9979E-01	0.9973E-01	0.9967E-01	0.9961E-01	0.9955E-01	0.9949E-01
.00748	1.0	0.9996E-01	0.9991E-01	0.9985E-01	0.9979E-01	0.9973E-01	0.9967E-01	0.9961E-01	0.9955E-01	0.9949E-01
.00997	1.0	0.9997E-01	0.9992E-01	0.9986E-01	0.9980E-01	0.9974E-01	0.9968E-01	0.9962E-01	0.9956E-01	0.9950E-01
.01247	1.0	0.9998E-01	0.9993E-01	0.9987E-01	0.9981E-01	0.9975E-01	0.9969E-01	0.9963E-01	0.9957E-01	0.9951E-01
.01496	1.0	0.9999E-01	0.9994E-01	0.9988E-01	0.9982E-01	0.9976E-01	0.9970E-01	0.9964E-01	0.9958E-01	0.9952E-01
.01745	1.0	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
.01993	1.0	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00	1.0000E+00
.02244	1.0	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00	1.0001E+00
.02493	1.0	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00
.02742	1.0	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00
.02992	1.0	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00
.03241	1.0	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00
.03491	1.0	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00	1.0002E+00

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.35379	0.73408	0.93171	0.99528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	1.635E-06	1.387E-05	1.701E-05	1.237E-05	3.412E-04	-2.690E-03	-7.074E-03	-5.823E-03	0.0
.00499	0.0	3.261E-06	2.807E-05	3.672E-05	2.899E-05	-6.812E-04	-5.369E-03	-1.430E-02	-1.186E-02	0.0
.00748	0.0	5.250E-06	4.167E-05	6.053E-05	5.099E-05	-9.881E-04	-7.968E-03	-2.140E-02	-1.790E-02	0.0
.00997	0.0	7.102E-06	6.479E-05	8.911E-05	8.121E-05	-1.236E-03	-1.723E-02	-2.785E-02	-2.348E-02	0.0
.01247	0.0	8.896E-06	6.690E-05	1.223E-04	1.209E-04	-1.404E-03	-1.198E-02	-3.306E-02	-2.810E-02	0.0
.01496	0.0	1.067E-05	7.690E-05	1.581E-04	1.658E-04	-1.475E-03	-1.100E-02	-3.653E-02	-3.128E-02	0.0
.01745	0.0	1.203E-05	8.440E-05	1.920E-04	2.111E-04	-1.443E-03	-1.336E-02	-3.790E-02	-3.270E-02	0.0
.01993	0.0	1.314E-05	8.887E-05	2.186E-04	2.513E-04	-1.320E-03	-1.278E-02	-3.703E-02	-3.282E-02	0.0
.02244	0.0	1.382E-05	8.905E-05	2.319E-04	2.772E-04	-1.187E-03	-1.167E-02	-3.402E-02	-2.979E-02	0.0
.02493	0.0	1.412E-05	8.951E-05	2.366E-04	2.792E-04	-8.925E-04	-9.585E-03	-2.911E-02	-2.566E-02	0.0
.02742	0.0	1.394E-05	7.144E-05	1.985E-04	2.818E-04	-6.464E-04	-7.629E-03	-2.273E-02	-2.014E-02	0.0
.02992	0.0	1.360E-05	5.235E-05	1.432E-04	1.911E-04	-4.138E-04	-5.148E-03	-1.541E-02	-1.371E-02	0.0
.03241	0.0	1.319E-05	2.774E-05	7.760E-05	1.014E-04	-2.014E-04	-2.873E-03	-7.703E-03	-6.885E-03	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 4.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSION AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		F	BETA		R	BETA
DRP	-18.1061	1.0220	.0075	-0.0117	0.9911	.0199
DRT	-39.5643	0.9636	.0349	0.0588	0.9926	.0075
DRR	-15.6900	0.9811	.0150	0.0005	0.8953	.0025
DTT	-12.1929	0.9976	.0274	-0.0161	0.3538	.0
DTP	144.3444	0.9926	.0	70.1399	1.0000	.0274
DPP	-0.2557	0.9636	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.0000 AND AT R = 0  
 DPP = 0 AT BETA = 0.0000 AND AT R = 0  
 DRR = 0 AT BETA = 0.0000 AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 2.0030 DEGREES REYNOLDS NUMBER = 0.5E+03

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with 14 columns: x, y, z, u, v, w, p, q, r, s, t, u, v, w. Contains numerical data for Reynolds number 0.5E+03.

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with 14 columns: x, y, z, u, v, w, p, q, r, s, t, u, v, w. Contains numerical data for Reynolds number 0.5E+03.

GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 4.0E+03

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.38379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	1.0	0.983E-01	0.924E-01	0.816E-01	0.744E-01	0.677E-01	0.626E-01	0.579E-01	0.536E-01	0.497E-01	0.462E-01	0.430E-01
.00249	1.0	0.983E-01	0.924E-01	0.816E-01	0.744E-01	0.677E-01	0.626E-01	0.579E-01	0.536E-01	0.497E-01	0.462E-01	0.430E-01
.00499	1.0	0.984E-01	0.924E-01	0.817E-01	0.745E-01	0.678E-01	0.627E-01	0.580E-01	0.537E-01	0.498E-01	0.463E-01	0.431E-01
.00748	1.0	0.985E-01	0.927E-01	0.822E-01	0.750E-01	0.683E-01	0.632E-01	0.585E-01	0.542E-01	0.503E-01	0.468E-01	0.435E-01
.00997	1.0	0.987E-01	0.934E-01	0.832E-01	0.762E-01	0.695E-01	0.644E-01	0.597E-01	0.554E-01	0.515E-01	0.480E-01	0.447E-01
.01247	1.0	0.990E-01	0.940E-01	0.850E-01	0.782E-01	0.715E-01	0.664E-01	0.617E-01	0.574E-01	0.535E-01	0.500E-01	0.467E-01
.01496	1.0	0.993E-01	0.944E-01	0.870E-01	0.804E-01	0.747E-01	0.696E-01	0.649E-01	0.606E-01	0.567E-01	0.532E-01	0.500E-01
.01745	1.0	0.998E-01	0.973E-01	0.921E-01	0.866E-01	0.819E-01	0.776E-01	0.736E-01	0.698E-01	0.663E-01	0.630E-01	0.600E-01
.01995	1.0	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01	1.000E-01
.02244	1.0	1.002E-01	1.003E-01	1.005E-01	1.008E-01	1.012E-01	1.017E-01	1.023E-01	1.030E-01	1.038E-01	1.046E-01	1.055E-01
.02493	1.0	1.004E-01	1.006E-01	1.010E-01	1.015E-01	1.021E-01	1.028E-01	1.036E-01	1.044E-01	1.053E-01	1.062E-01	1.072E-01
.02743	1.0	1.006E-01	1.009E-01	1.014E-01	1.020E-01	1.027E-01	1.035E-01	1.043E-01	1.052E-01	1.061E-01	1.070E-01	1.080E-01
.02992	1.0	1.008E-01	1.012E-01	1.018E-01	1.025E-01	1.033E-01	1.041E-01	1.050E-01	1.059E-01	1.068E-01	1.077E-01	1.087E-01
.03241	1.0	1.010E-01	1.015E-01	1.022E-01	1.030E-01	1.038E-01	1.047E-01	1.056E-01	1.065E-01	1.074E-01	1.083E-01	1.093E-01
.03491	1.0	1.012E-01	1.017E-01	1.025E-01	1.034E-01	1.043E-01	1.052E-01	1.061E-01	1.070E-01	1.079E-01	1.088E-01	1.098E-01

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.38379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	1.108E-05	6.501E-05	1.528E-04	1.872E-04	5.270E-06	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	2.379E-05	1.321E-04	3.074E-04	3.806E-04	3.014E-05	-2.486E-03	-4.641E-03	-2.910E-02	-3.518E-02	-2.776E-02	0.0
.00748	0.0	3.507E-05	1.975E-04	4.619E-04	5.839E-04	1.034E-04	-6.903E-03	-4.224E-02	-1.026E-01	-7.018E-02	-5.875E-02	0.0
.00997	0.0	4.712E-05	2.810E-04	6.151E-04	7.947E-04	2.402E-04	-8.409E-03	-5.287E-02	-1.293E-01	-1.026E-01	-8.123E-02	0.0
.01247	0.0	5.774E-05	3.801E-04	7.611E-04	1.008E-03	4.691E-04	-9.177E-03	-5.995E-02	-1.476E-01	-1.163E-01	-1.033E-01	0.0
.01496	0.0	6.691E-05	3.713E-04	6.911E-04	1.208E-03	7.529E-04	-9.147E-03	-6.294E-02	-1.569E-01	-1.259E-01	-1.163E-01	0.0
.01745	0.0	7.384E-05	4.124E-04	6.946E-04	1.378E-03	1.059E-03	-8.307E-03	-6.189E-02	-1.567E-01	-1.264E-01	-1.264E-01	0.0
.01995	0.0	7.784E-05	4.326E-04	1.058E-03	1.492E-03	1.331E-03	-7.128E-03	-5.734E-02	-1.482E-01	-1.264E-01	-1.264E-01	0.0
.02244	0.0	7.811E-05	4.322E-04	1.066E-03	1.526E-03	1.513E-03	-5.594E-03	-5.018E-02	-1.327E-01	-1.088E-01	-1.088E-01	0.0
.02493	0.0	7.393E-05	4.048E-04	1.006E-03	1.456E-03	1.583E-03	-4.056E-03	-4.126E-02	-1.117E-01	-9.240E-02	-9.240E-02	0.0
.02743	0.0	6.497E-05	3.466E-04	8.680E-04	1.265E-03	1.419E-03	-2.695E-03	-3.136E-02	-8.653E-02	-7.205E-02	-7.205E-02	0.0
.02992	0.0	5.119E-05	2.853E-04	6.408E-04	5.430E-04	1.093E-03	-1.590E-03	-2.101E-02	-5.846E-02	-4.697E-02	-4.697E-02	0.0
.03241	0.0	3.339E-05	1.364E-04	3.432E-04	5.061E-04	5.988E-04	-7.244E-04	-1.097E-02	-2.948E-02	-2.452E-02	-2.452E-02	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 2.0000 DEGREES

REYNOLDS NUMBER = 8.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM (PERCENT)	LOCATION	
		P	BETA		R	BETA
DRR	-28.1932	1.0000	.0050	-0.0024	0.8953	.0199
DRT	-76.7589	0.9307	.0349	0.6039	0.3538	.0274
DRP	-30.7841	0.9811	.0125	0.0051	0.3538	.0025
DTT	18.2480	0.9926	.0075	-0.0302	0.3538	.0
DTP	279.1027	0.9926	.0	35.0585	1.0000	.0274
DPP	-0.4645	0.9307	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.0050 AND AT R = 0

DPP = 0 AT BETA = 0.0050 AND AT R = 0

DRP = 0 AT BETA = 0.0050 AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 8.0E+03

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89526	0.93666	0.96360	0.98114	0.99256	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89526	0.93666	0.96360	0.98114	0.99256	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 2.5000 DEGREES

REYNOLDS NUMBER = 1.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-28.7891	1.0000	.0050	0.0333	0.3538	.0175
DRT	-91.1141	0.9367	.0349	-0.1019	0.5926	.0050
DPR	-29.8455	0.9811	.0100	0.0000	0.8317	.0025
DTT	18.9368	0.9926	.0050	-0.0367	0.3538	.00
DTP	334.0452	0.9926	.00	26.7455	1.0000	.0199
DPP	-0.5471	0.9367	.0274	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.0050 AND AT R = 0

DPP = 0 AT BETA = 0.0050 AND AT R = 0

DPR = 0 AT BETA = 0.0100 AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO







GAP ANGLE = 8.000 DEGREES

REYNOLDS NUMBER = 1.02E4

THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

DATA	0.0	0.0	0.30379	0.58842	0.73408	0.83171	0.89828	0.93666	0.96360	0.98114	0.99286	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

DATA	0.0	0.0	0.30379	0.58842	0.73408	0.83171	0.89828	0.93666	0.96360	0.98114	0.99286	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2600	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.2800	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3200	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.3400	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES      REYNOLDS NUMBER = 2.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		P	BETA		R	BETA
DRR	-28.7804	1.0000	.0053	0.0464	0.7341	.0199
DRT	-137.3945	0.8317	.0349	-0.8873	0.9926	.0249
DPR	-24.2036	0.9811	.0075	0.0247	0.3538	.0025
DTT	19.2205	0.9926	.0053	-0.0082	0.7341	.0199
DTP	490.5284	0.9926	.0	0.7358	0.9636	.0175
DPP	-0.7564	0.8317	.0249	-0.0000	1.0000	.0025

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0. EPSILON AND AT P = 0  
 DPR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT P = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 3.0000 DEGREES REYNOLDS NUMBER = 2.0E+04

THE FIELD OF THE RATIO OF THE OXY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	-1.836E-01	-3.978E-01	-5.127E-01	-5.426E-01	-7.937E-01	-9.366E-01	-9.366E-01	-9.366E-01	-9.366E-01	-9.366E-01	-9.366E-01	0.0
.00249	0.0	-1.109E-01	-2.915E-01	-3.839E-01	-4.378E-01	-6.879E-01	-8.672E-01	-8.672E-01	-8.672E-01	-8.672E-01	-8.672E-01	-8.672E-01	0.0
.00499	0.0	-6.537E-02	-1.768E-01	-2.441E-01	-3.072E-01	-5.174E-01	-7.350E-01	-7.350E-01	-7.350E-01	-7.350E-01	-7.350E-01	-7.350E-01	0.0
.00748	0.0	-1.812E-02	-5.823E-02	-9.639E-02	-1.478E-01	-2.822E-01	-5.222E-01	-5.222E-01	-5.222E-01	-5.222E-01	-5.222E-01	-5.222E-01	0.0
.00997	0.0	2.385E-02	9.012E-02	4.291E-02	1.915E-01	1.802E-01	2.548E-01	2.548E-01	2.548E-01	2.548E-01	2.548E-01	2.548E-01	0.0
.01247	0.0	9.956E-02	1.448E-01	1.703E-01	1.817E-01	3.028E-01	3.358E-01	3.358E-01	3.358E-01	3.358E-01	3.358E-01	3.358E-01	0.0
.01496	0.0	9.584E-02	2.321E-01	2.739E-01	3.272E-01	3.988E-01	4.466E-01	4.466E-01	4.466E-01	4.466E-01	4.466E-01	4.466E-01	0.0
.01745	0.0	1.017E-01	2.719E-01	3.031E-01	3.266E-01	4.487E-01	4.487E-01	4.487E-01	4.487E-01	4.487E-01	4.487E-01	4.487E-01	0.0
.01994	0.0	1.050E-01	2.916E-01	4.093E-01	5.135E-01	6.975E-01	7.368E-01	7.368E-01	7.368E-01	7.368E-01	7.368E-01	7.368E-01	0.0
.02244	0.0	9.357E-02	2.716E-01	4.029E-01	5.182E-01	6.813E-01	7.189E-01	7.189E-01	7.189E-01	7.189E-01	7.189E-01	7.189E-01	0.0
.02493	0.0	6.584E-02	2.011E-01	3.231E-01	4.291E-01	5.635E-01	6.817E-01	6.817E-01	6.817E-01	6.817E-01	6.817E-01	6.817E-01	0.0
.02743	0.0	1.709E-02	6.841E-02	1.428E-01	2.067E-01	3.633E-01	5.641E-01	5.641E-01	5.641E-01	5.641E-01	5.641E-01	5.641E-01	0.0
.02992	0.0	-1.272E-02	-1.392E-01	-1.708E-01	-1.972E-01	-2.143E-01	-2.143E-01	-2.143E-01	-2.143E-01	-2.143E-01	-2.143E-01	-2.143E-01	0.0
.03241	0.0	-1.431E-01	-4.059E-01	-5.975E-01	-7.987E-01	-9.424E-01	-9.424E-01	-9.424E-01	-9.424E-01	-9.424E-01	-9.424E-01	-9.424E-01	0.0
.03491	0.0	-2.398E-01	-6.957E-01	-1.097E-01	-1.374E-01	-1.324E-01	-1.039E-01	-1.039E-01	-1.039E-01	-1.039E-01	-1.039E-01	-1.039E-01	0.0

THE FIELD OF THE RATIO OF THE URU ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0	0.35379	0.58412	0.73408	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	1.772E-03	3.616E-03	4.510E-03	4.510E-03	8.912E-03	1.439E-02	1.439E-02	1.439E-02	1.439E-02	1.439E-02	1.439E-02	0.0
.00499	0.0	2.979E-03	6.582E-03	8.257E-03	1.433E-02	1.766E-02	2.736E-02	2.736E-02	2.736E-02	2.736E-02	2.736E-02	2.736E-02	0.0
.00748	0.0	3.980E-03	6.046E-03	1.084E-02	1.568E-02	1.877E-02	2.085E-02	2.085E-02	2.085E-02	2.085E-02	2.085E-02	2.085E-02	0.0
.00997	0.0	3.986E-03	6.369E-03	1.195E-02	1.338E-02	1.781E-02	2.847E-02	2.847E-02	2.847E-02	2.847E-02	2.847E-02	2.847E-02	0.0
.01247	0.0	3.074E-03	7.419E-03	1.140E-02	1.140E-02	2.948E-02	3.608E-02	3.608E-02	3.608E-02	3.608E-02	3.608E-02	3.608E-02	0.0
.01496	0.0	2.112E-03	9.492E-03	9.179E-03	3.965E-02	1.723E-02	2.994E-02	2.994E-02	2.994E-02	2.994E-02	2.994E-02	2.994E-02	0.0
.01745	0.0	3.134E-04	2.689E-03	3.430E-03	6.964E-04	-1.723E-02	-1.784E-02	-1.784E-02	-1.784E-02	-1.784E-02	-1.784E-02	-1.784E-02	0.0
.01994	0.0	-6.791E-04	-7.401E-04	4.639E-04	-4.792E-03	-1.311E-02	1.612E-02	1.612E-02	1.612E-02	1.612E-02	1.612E-02	1.612E-02	0.0
.02244	0.0	-2.184E-03	-4.927E-03	-5.230E-03	-4.999E-03	1.222E-02	3.671E-02	3.671E-02	3.671E-02	3.671E-02	3.671E-02	3.671E-02	0.0
.02493	0.0	-3.471E-03	-7.836E-03	-1.086E-02	-1.183E-02	6.335E-04	2.951E-02	2.951E-02	2.951E-02	2.951E-02	2.951E-02	2.951E-02	0.0
.02743	0.0	-4.251E-03	-1.027E-02	-1.519E-02	-1.424E-02	4.837E-03	3.498E-02	3.498E-02	3.498E-02	3.498E-02	3.498E-02	3.498E-02	0.0
.02992	0.0	-4.174E-03	-1.051E-02	-1.638E-02	-1.438E-02	7.267E-03	3.699E-02	3.699E-02	3.699E-02	3.699E-02	3.699E-02	3.699E-02	0.0
.03241	0.0	-2.819E-03	-7.330E-03	-1.176E-02	-1.008E-02	5.977E-03	2.634E-02	2.634E-02	2.634E-02	2.634E-02	2.634E-02	2.634E-02	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.000 DEGREES REYNOLDS NUMBER = 2.0E+04

THE FIELD OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.68412	0.73408	0.83171	0.89528	0.93666	0.96363	0.98114	0.99256	1.00000
.0	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.00249	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.00499	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.00748	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.00997	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.01247	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.01496	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.01745	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.01995	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.02244	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.02493	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.02743	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.02992	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.03241	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00
.03491	1.0	0.602E-01	0.445E-01	7.224E-01	9.176E-01	1.882E+00	2.413E+00	3.240E+00	3.018E+00	4.905E+00	4.905E+00	4.905E+00

THE FIELD OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	0.68412	0.73408	0.83171	0.89528	0.93666	0.96363	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01995	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.02244	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.02493	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.02743	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.02992	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.03241	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 2.0000 DEGREES REYNOLDS NUMBER = 2.CE+04

THE FIELD OF THE RATIO OF THE ODT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	C.5841P	0.73428	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	-6.058E-04	-7.858E-04	-6.748E-04	-6.748E-04	-2.050E-03	-5.884E-03	-6.455E-03	6.683E-03	-6.563E-03	1.907E-02	1.902E-02
.00249	0.0	-4.179E-03	-5.078E-03	-6.873E-03	-1.186E-02	-3.715E-02	-1.711E-02	-3.718E-02	2.596E-02	4.183E-02	1.242E-01	1.224E-01
.00499	0.0	-7.065E-03	-8.739E-03	-1.137E-02	-1.900E-02	-2.806E-02	-2.806E-02	2.614E-02	3.924E-02	6.234E-02	1.922E-01	1.900E-01
.00748	0.0	-8.479E-03	-1.074E-02	-1.675E-02	-2.682E-02	-4.579E-02	-1.510E-01	1.510E-01	4.424E-02	9.984E-02	1.817E-01	1.801E-01
.00997	0.0	-9.516E-03	-1.113E-02	-1.600E-02	-1.812E-02	-3.830E-02	2.728E-02	2.728E-02	5.346E-02	8.744E-02	1.195E-01	1.186E-01
.01247	0.0	-7.314E-03	-9.942E-03	-1.511E-02	-1.257E-02	7.558E-03	3.184E-02	4.813E-02	6.588E-02	8.688E-02	1.178E-02	5.849E-02
.01496	0.0	-5.688E-03	-7.400E-03	-1.193E-02	-6.030E-03	1.436E-02	2.717E-02	2.928E-02	3.567E-02	1.706E-02	1.582E-02	1.521E-02
.01745	0.0	-2.003E-03	-3.714E-03	-6.767E-03	3.260E-04	1.592E-02	1.653E-02	6.882E-03	1.449E-02	-2.435E-03	-4.978E-02	-4.874E-02
.01995	0.0	1.534E-03	6.723E-04	-8.202E-03	6.151E-03	9.371E-02	3.651E-03	-1.464E-02	-2.835E-03	-2.835E-03	-4.978E-02	-4.874E-02
.02244	0.0	5.122E-03	5.834E-03	7.037E-03	1.152E-02	9.629E-03	3.377E-02	3.377E-02	3.647E-02	8.765E-02	8.629E-02	8.629E-02
.02493	0.0	8.212E-03	1.046E-02	1.871E-02	1.630E-02	5.200E-02	2.061E-02	4.918E-02	-6.824E-02	-1.256E-01	-1.241E-01	-1.241E-01
.02743	0.0	1.011E-02	1.373E-02	2.017E-02	1.964E-02	1.419E-02	-2.874E-02	-5.821E-02	-9.701E-02	-1.535E-01	-1.521E-01	-1.521E-01
.02992	0.0	9.985E-03	1.430E-02	2.144E-02	1.973E-02	-1.042E-02	-3.059E-02	-5.057E-02	-1.093E-01	-1.592E-01	-1.542E-01	-1.542E-01
.03241	0.0	6.732E-03	9.948E-03	1.824E-02	1.371E-02	-1.634E-02	-2.188E-02	-3.825E-02	-8.296E-02	-1.078E-01	-1.078E-01	-1.078E-01
.03491	0.0	1.943E-03	3.459E-03	5.638E-03	5.308E-03	2.666E-04	-5.953E-03	-9.161E-03	-2.981E-02	-2.981E-02	-2.911E-02	-2.911E-02

THE FIELD OF THE RATIO OF THE DPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.35379	C.5841P	0.73428	0.83171	0.89528	0.93666	0.96360	0.98114	0.99256	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	6.596E-04	1.719E-03	2.236E-03	2.800E-03	3.389E-03	3.792E-03	3.792E-03	3.206E-03	2.334E-03	1.841E-03	-4.394E-07
.00499	0.0	1.104E-03	2.907E-03	3.629E-03	4.426E-03	5.642E-03	5.642E-03	4.735E-03	4.735E-03	3.629E-03	2.183E-03	-3.043E-06
.00748	0.0	1.392E-03	3.483E-03	4.672E-03	5.876E-03	6.555E-03	6.121E-03	4.667E-03	4.667E-03	3.483E-03	1.606E-03	-8.407E-06
.00997	0.0	1.768E-03	3.489E-03	4.795E-03	5.901E-03	6.264E-03	5.132E-03	3.582E-03	3.582E-03	2.255E-03	9.100E-04	-1.504E-05
.01247	0.0	1.072E-03	2.985E-03	4.237E-03	5.390E-03	5.049E-03	3.520E-03	2.201E-03	2.201E-03	1.168E-03	3.539E-04	-2.138E-05
.01496	0.0	7.004E-04	2.051E-03	3.190E-03	4.042E-03	3.234E-03	1.759E-03	9.316E-04	9.316E-04	3.127E-04	-1.439E-04	-2.683E-05
.01745	0.0	2.197E-04	7.922E-04	1.461E-03	2.127E-03	1.075E-03	7.640E-04	-2.237E-04	-2.237E-04	-3.089E-04	-6.346E-04	-3.143E-05
.01995	0.0	-3.120E-04	-6.549E-04	-5.179E-04	-3.226E-04	-1.264E-03	-1.722E-03	-1.376E-03	-1.376E-03	-9.513E-04	-1.074E-03	-3.439E-05
.02244	0.0	-6.266E-04	-2.114E-03	-2.618E-03	-2.992E-03	-3.634E-03	-3.421E-03	-2.581E-03	-2.581E-03	-1.669E-03	-1.395E-03	-3.496E-05
.02493	0.0	-1.245E-03	-3.363E-03	-4.534E-03	-5.495E-03	-5.788E-03	-4.948E-03	-3.632E-03	-3.632E-03	-2.389E-03	-1.526E-03	-3.226E-05
.02743	0.0	-1.474E-03	-4.120E-03	-5.891E-03	-7.278E-03	-7.278E-03	-5.966E-03	-4.367E-03	-4.367E-03	-2.921E-03	-1.611E-03	-2.876E-05
.02992	0.0	-1.416E-03	-4.046E-03	-5.961E-03	-7.564E-03	-7.401E-03	-5.932E-03	-4.338E-03	-4.338E-03	-2.958E-03	-1.603E-03	-1.601E-05
.03241	0.0	-9.511E-04	-2.734E-03	-4.141E-03	-5.316E-03	-5.151E-03	-4.073E-03	-2.975E-03	-2.975E-03	-2.031E-03	-4.542E-04	-8.513E-06
.03491	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES      REYNOLDS NUMBER = 2.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.2811	1.0000	.3150	0.0001	0.4410	.3087
DRT	-0.5464	0.9857	.0175	-0.0047	0.9976	.0137
DPR	-0.1463	0.9857	.0100	-0.0000	0.4410	.0012
DTT	-0.2325	0.9976	.0137	-0.0001	0.4410	.0
DTP	100.6453	0.9976	.0	98.7717	0.9857	.3175
OPP	-0.0018	0.9857	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.EPSLON AND AT R = 0

DPP = 0 AT BETA = 0.EPSLON AND AT R = 0

DPR = 0 AT BETA = 0.EPSLON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 2.00E+02

THE FIELD OF THE RATIO OF THE DRX ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DRX ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE UP DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.4409M	0.68809	3.82656	Q.90416	0.94766	0.97200	J.85860	0.99331	0.99760	1.00000
.0	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00125	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00249	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00374	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00499	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00623	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00748	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00873	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.00997	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01122	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01247	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01371	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01496	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01621	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00
.01745	1.0	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00	1.000E+00

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE UP DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.4409M	0.68809	3.82656	Q.90416	0.94766	0.97200	J.85860	0.99331	0.99760	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06	1.112E-06
.00249	0.0	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06	2.213E-06
.00374	0.0	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06	3.314E-06
.00499	0.0	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06	4.415E-06
.00623	0.0	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06	5.516E-06
.00748	0.0	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06	6.617E-06
.00873	0.0	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06	7.718E-06
.00997	0.0	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06	8.819E-06
.01122	0.0	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06	9.920E-06
.01247	0.0	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05	1.0921E-05
.01371	0.0	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05	1.1922E-05
.01496	0.0	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05	1.2923E-05
.01621	0.0	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05	1.3924E-05
.01745	0.0	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05	1.4925E-05



GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 2.0E+02

THE FIELD OF THE RATIO OF THE DTY ELEMENT OF THE RATE OF DEFORMATION TENSION AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1122	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1371	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DMP ELEMENT OF THE RATE OF DEFORMATION TENSION AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1122	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1371	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES

REYNOLDS NUMBER = 4.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.5617	1.0000	.0150	0.0002	0.4410	.0087
DRT	-1.0917	0.9857	.0175	-0.0098	0.9976	.0137
DPR	-0.2528	0.9857	.0087	-0.0000	0.4410	.0012
DTT	-0.4648	0.9976	.0137	-0.0003	0.4410	.0
DTP	101.0798	0.9976	.0	98.5047	0.9933	.0175
DPP	-0.0037	0.9857	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.EPSILON AND AT R = 0

DPP = 0 AT BETA = 0.EPSILON AND AT R = 0

DPR = 0 AT BETA = 0.EPSILON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 4.0E+02

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4000	0.6000	0.8000	0.9200	0.9400	0.9700	0.9800	0.9900	1.0000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1122	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1371	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4000	0.6000	0.8000	0.9200	0.9400	0.9700	0.9800	0.9900	1.0000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1122	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1371	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 1.0000 DEGREES      REYNOLDS NUMBER = 8.0E+02

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	1.1140	1.0000	.0150	0.0004	0.4410	.0087
DRT	-2.1662	0.9857	.0175	-0.0196	0.9976	.0137
DPR	-0.5237	0.9933	.0087	0.0000	0.4410	.0012
DTT	-0.9207	0.9976	.0137	-0.0005	0.4410	.0
DTP	102.1354	0.9976	.0	97.5495	1.0000	.0175
DPP	-0.0073	0.9857	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0. EPSLON AND AT R = 0  
 DPP = 0 AT BETA = 0. EPSLON AND AT R = 0  
 DPR = 0 AT BETA = 0. EPSLON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO



THE FIELD OF THE RATIO OF THE DTI ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	1.0	0.9992E-01	0.9974E-01	0.9956E-01	0.9938E-01	0.9920E-01	0.9902E-01	0.9884E-01	0.9866E-01	0.9848E-01	0.9830E-01
.0025	1.0	0.9984E-01	0.9966E-01	0.9948E-01	0.9930E-01	0.9912E-01	0.9894E-01	0.9876E-01	0.9858E-01	0.9840E-01	0.9822E-01
.00375	1.0	0.9976E-01	0.9958E-01	0.9940E-01	0.9922E-01	0.9904E-01	0.9886E-01	0.9868E-01	0.9850E-01	0.9832E-01	0.9814E-01
.005	1.0	0.9968E-01	0.9950E-01	0.9932E-01	0.9914E-01	0.9896E-01	0.9878E-01	0.9860E-01	0.9842E-01	0.9824E-01	0.9806E-01
.00625	1.0	0.9960E-01	0.9942E-01	0.9924E-01	0.9906E-01	0.9888E-01	0.9870E-01	0.9852E-01	0.9834E-01	0.9816E-01	0.9798E-01
.0075	1.0	0.9952E-01	0.9934E-01	0.9916E-01	0.9898E-01	0.9880E-01	0.9862E-01	0.9844E-01	0.9826E-01	0.9808E-01	0.9790E-01
.00875	1.0	0.9944E-01	0.9926E-01	0.9908E-01	0.9890E-01	0.9872E-01	0.9854E-01	0.9836E-01	0.9818E-01	0.9800E-01	0.9782E-01
.01	1.0	0.9936E-01	0.9918E-01	0.9900E-01	0.9882E-01	0.9864E-01	0.9846E-01	0.9828E-01	0.9810E-01	0.9792E-01	0.9774E-01
.01125	1.0	0.9928E-01	0.9910E-01	0.9892E-01	0.9874E-01	0.9856E-01	0.9838E-01	0.9820E-01	0.9802E-01	0.9784E-01	0.9766E-01
.0125	1.0	0.9920E-01	0.9902E-01	0.9884E-01	0.9866E-01	0.9848E-01	0.9830E-01	0.9812E-01	0.9794E-01	0.9776E-01	0.9758E-01
.01375	1.0	0.9912E-01	0.9894E-01	0.9876E-01	0.9858E-01	0.9840E-01	0.9822E-01	0.9804E-01	0.9786E-01	0.9768E-01	0.9750E-01
.015	1.0	0.9904E-01	0.9886E-01	0.9868E-01	0.9850E-01	0.9832E-01	0.9814E-01	0.9796E-01	0.9778E-01	0.9760E-01	0.9742E-01
.01625	1.0	0.9896E-01	0.9878E-01	0.9860E-01	0.9842E-01	0.9824E-01	0.9806E-01	0.9788E-01	0.9770E-01	0.9752E-01	0.9734E-01
.0175	1.0	0.9888E-01	0.9870E-01	0.9852E-01	0.9834E-01	0.9816E-01	0.9798E-01	0.9780E-01	0.9762E-01	0.9744E-01	0.9726E-01

THE FIELD OF THE RATIO OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	1.794E-07	6.121E-07	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.0025	0.0	3.733E-07	1.154E-06	1.574E-06	2.012E-06	2.450E-06	2.888E-06	3.326E-06	3.764E-06	4.202E-06	4.640E-06
.00375	0.0	5.648E-07	1.822E-06	2.489E-06	3.156E-06	3.823E-06	4.490E-06	5.157E-06	5.824E-06	6.491E-06	7.158E-06
.005	0.0	7.475E-07	2.546E-06	3.374E-06	4.202E-06	5.030E-06	5.858E-06	6.686E-06	7.514E-06	8.342E-06	9.170E-06
.00625	0.0	9.302E-07	3.264E-06	4.292E-06	5.320E-06	6.348E-06	7.376E-06	8.404E-06	9.432E-06	1.0460E-05	1.1488E-05
.0075	0.0	1.1129E-06	3.982E-06	5.210E-06	6.438E-06	7.666E-06	8.894E-06	1.0122E-05	1.1350E-05	1.2578E-05	1.3806E-05
.00875	0.0	1.2956E-06	4.700E-06	6.128E-06	7.556E-06	8.984E-06	1.0412E-05	1.1840E-05	1.3268E-05	1.4696E-05	1.6124E-05
.01	0.0	1.4783E-06	5.418E-06	7.046E-06	8.674E-06	1.0302E-05	1.1892E-05	1.3482E-05	1.5072E-05	1.6662E-05	1.8252E-05
.01125	0.0	1.6610E-06	6.136E-06	7.964E-06	9.792E-06	1.1682E-05	1.3272E-05	1.4862E-05	1.6452E-05	1.8042E-05	1.9632E-05
.0125	0.0	1.8437E-06	6.854E-06	8.882E-06	1.0912E-05	1.2902E-05	1.4492E-05	1.6082E-05	1.7672E-05	1.9262E-05	2.0852E-05
.01375	0.0	2.0264E-06	7.572E-06	9.810E-06	1.2142E-05	1.4132E-05	1.5722E-05	1.7312E-05	1.8902E-05	2.0492E-05	2.2082E-05
.015	0.0	2.2091E-06	8.290E-06	1.0748E-05	1.3132E-05	1.5122E-05	1.6712E-05	1.8302E-05	1.9892E-05	2.1482E-05	2.3072E-05
.01625	0.0	2.3918E-06	9.008E-06	1.1786E-05	1.4372E-05	1.6362E-05	1.7952E-05	1.9542E-05	2.1132E-05	2.2722E-05	2.4312E-05
.0175	0.0	2.5745E-06	9.726E-06	1.2824E-05	1.5612E-05	1.7602E-05	1.9192E-05	2.0782E-05	2.2372E-05	2.3962E-05	2.5552E-05



GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 8.0E+02

THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.4098	0.68809	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
.0	0.0	-3.115E-04	-9.464E-06	-1.476E-05	-1.883E-05	-2.159E-05	-2.643E-05	-1.044E-05	1.795E-04	1.881E-04	6.607E-04	6.607E-04
.00125	0.0	-3.261E-05	-5.876E-05	-6.892E-05	-1.163E-04	-1.319E-04	-1.487E-04	-1.795E-04	1.795E-04	1.795E-04	4.309E-03	4.296E-03
.00250	0.0	-5.478E-05	-9.3E-05	-1.494E-04	-1.949E-04	-2.282E-04	-2.392E-04	-3.425E-04	3.425E-04	3.047E-03	7.081E-03	7.060E-03
.00375	0.0	-6.641E-05	-1.094E-04	-1.747E-04	-2.398E-04	-2.693E-04	-2.762E-04	-4.417E-04	4.417E-04	3.859E-03	6.214E-03	6.195E-03
.00500	0.0	-6.281E-05	-1.049E-04	-1.734E-04	-2.343E-04	-2.629E-04	-2.644E-04	-4.526E-04	4.526E-04	3.843E-03	7.949E-03	7.927E-03
.00625	0.0	-5.157E-05	-8.769E-05	-1.397E-04	-1.838E-04	-2.118E-04	-2.112E-04	-3.725E-04	3.725E-04	2.880E-03	6.613E-03	6.494E-03
.00750	0.0	-3.275E-05	-5.554E-05	-8.618E-05	-1.158E-04	-1.350E-04	-1.308E-04	-2.178E-04	2.178E-04	1.762E-03	4.166E-03	4.154E-03
.00875	0.0	-8.841E-06	-1.473E-05	-2.255E-05	-2.937E-05	-3.130E-05	-2.969E-05	-1.713E-05	1.713E-05	3.510E-04	1.202E-03	1.197E-03
.00997	0.0	1.719E-05	2.969E-05	4.839E-05	6.419E-05	7.652E-05	7.888E-05	-1.901E-04	-1.901E-04	-1.155E-03	-2.047E-03	-2.042E-03
.01122	0.0	4.195E-05	7.176E-05	1.151E-04	1.523E-04	1.794E-04	1.804E-04	-3.622E-04	-3.622E-04	-2.833E-03	-5.189E-03	-5.175E-03
.01247	0.0	6.155E-05	1.050E-04	1.676E-04	2.208E-04	2.590E-04	2.604E-04	-4.614E-04	-4.614E-04	-3.549E-03	-7.761E-03	-7.740E-03
.01371	0.0	7.168E-05	1.219E-04	1.940E-04	2.547E-04	2.976E-04	3.013E-04	-4.626E-04	-4.626E-04	-3.930E-03	-9.207E-03	-9.181E-03
.01496	0.0	6.757E-05	1.146E-04	1.816E-04	2.377E-04	2.767E-04	2.829E-04	-3.606E-04	-3.606E-04	-3.535E-03	-8.847E-03	-8.821E-03
.01621	0.0	4.400E-05	7.412E-05	1.108E-04	1.523E-04	1.766E-04	1.826E-04	-1.868E-04	-1.868E-04	-2.162E-03	-5.845E-03	-5.826E-03
.01745	0.0	1.129E-05	1.922E-05	2.832E-05	3.993E-05	4.380E-05	4.380E-05	1.953E-05	1.953E-05	-2.793E-04	-1.285E-03	-1.285E-03

THE FIELD OF THE RATIO OF THE DMP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.4098	0.68809	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	5.184E-06	1.476E-05	2.236E-05	2.749E-05	3.066E-05	3.259E-05	3.358E-05	3.358E-05	2.762E-05	1.261E-05	1.261E-05
.00250	0.0	6.665E-06	2.465E-05	3.737E-05	4.977E-05	5.131E-05	5.455E-05	5.602E-05	5.602E-05	4.531E-05	2.031E-05	2.031E-05
.00375	0.0	1.020E-05	3.900E-05	4.345E-05	5.410E-05	5.694E-05	6.424E-05	6.879E-05	6.879E-05	5.264E-05	2.322E-05	2.322E-05
.00500	0.0	9.970E-06	2.829E-05	4.286E-05	5.274E-05	5.694E-05	6.267E-05	6.395E-05	6.395E-05	5.089E-05	2.235E-05	2.235E-05
.00625	0.0	8.220E-06	2.324E-05	3.519E-05	4.359E-05	4.835E-05	5.139E-05	5.232E-05	5.232E-05	4.164E-05	1.836E-05	1.836E-05
.00750	0.0	5.269E-06	1.476E-05	2.322E-05	2.740E-05	3.087E-05	3.248E-05	3.293E-05	3.293E-05	2.655E-05	1.195E-05	1.195E-05
.00875	0.0	1.508E-06	4.027E-06	5.973E-06	7.259E-06	7.998E-06	8.418E-06	8.424E-06	8.424E-06	7.509E-06	3.646E-06	3.646E-06
.00997	0.0	-2.601E-06	-7.688E-06	-1.181E-05	-1.469E-05	-1.651E-05	-1.766E-05	-1.813E-05	-1.813E-05	-1.334E-05	-5.150E-06	-5.150E-06
.01122	0.0	-6.524E-06	-1.883E-05	-2.862E-05	-3.566E-05	-4.229E-05	-4.229E-05	-4.322E-05	-4.322E-05	-3.349E-05	-1.411E-05	-1.411E-05
.01247	0.0	-9.662E-06	-2.767E-05	-4.203E-05	-5.180E-05	-5.789E-05	-6.158E-05	-6.291E-05	-6.291E-05	-4.994E-05	-2.189E-05	-2.189E-05
.01371	0.0	-1.134E-05	-3.228E-05	-4.889E-05	-6.010E-05	-6.706E-05	-7.127E-05	-7.286E-05	-7.286E-05	-5.416E-05	-2.694E-05	-2.694E-05
.01496	0.0	-1.080E-05	-3.046E-05	-4.600E-05	-5.643E-05	-6.289E-05	-6.674E-05	-6.831E-05	-6.831E-05	-5.678E-05	-2.705E-05	-2.705E-05
.01621	0.0	-7.239E-06	-1.991E-05	-2.986E-05	-3.650E-05	-4.089E-05	-4.301E-05	-4.404E-05	-4.404E-05	-3.744E-05	-1.874E-05	-1.874E-05
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES

REYNOLDS NUMBER = 2.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	2.7496	1.0000	.0150	0.0011	0.4410	.0087
DRT	-5.3682	0.9857	.0175	0.0520	0.9933	.0137
DPR	-1.7101	0.9933	.0087	0.0000	0.4410	.0012
DTT	-2.2783	0.9976	.0137	-0.0013	0.4410	.0
DTP	105.2589	1.0000	.0	94.3750	1.0000	.0175
OPP	-0.0181	0.9857	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.0000 AND AT R = 0  
 OPP = 0 AT BETA = 0.0000 AND AT R = 0  
 DPR = 0 AT BETA = 0.0000 AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE DERIVATIVE OF THE RATE OF DEFORMATION TENSORS AND THE PRIMARY DEFORMATION RATE IS

BETA

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
0.0125	0.0	0.0033E-03	-1.0724E-02	-2.0011E-02	-3.2672E-02	-3.8573E-02	-2.8540E-02	-2.7230E-02	-1.7745E-02	-3.4995E-02	-3.0715E-02	-1.9348E-02
0.0250	0.0	0.4195E-03	-1.0247E-02	-2.3046E-02	-2.3046E-02	-2.5602E-02	-2.7230E-02	-2.7230E-02	-2.7230E-02	-2.5602E-02	-2.3046E-02	-1.0247E-02
0.0375	0.0	2.0915E-03	-7.1622E-03	-1.0342E-02	-1.3344E-02	-1.0342E-02	-1.0342E-02	-1.0342E-02	-1.0342E-02	-1.0342E-02	-1.0342E-02	-7.1622E-03
0.0500	0.0	0.4531E-04	-1.0140E-03	-2.7030E-03	-5.4172E-03	-3.0410E-03	-0.8138E-03	-0.2041E-03	0.2041E-03	0.8138E-03	2.7030E-03	1.0140E-03
0.0625	0.0	0.3545E-04	2.0915E-04	4.5020E-03	9.0100E-03	6.0210E-03	0.2041E-03	0.2041E-03	0.2041E-03	0.2041E-03	0.2041E-03	2.0915E-04
0.0750	0.0	3.1895E-03	0.7482E-03	1.0311E-02	1.2722E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.0311E-02
0.0875	0.0	3.0670E-03	0.0670E-03	1.0604E-02	1.0604E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	1.0604E-02
0.1000	0.0	3.0977E-03	1.0701E-02	1.7132E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	1.7132E-02
0.1125	0.0	0.3049E-03	1.0701E-02	2.1620E-02	2.1620E-02	2.1620E-02	2.1620E-02	2.1620E-02	2.1620E-02	2.1620E-02	2.1620E-02	1.0701E-02
0.1250	0.0	2.0125E-03	0.7355E-03	1.0311E-02	1.2722E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	0.7355E-03
0.1375	0.0	2.0701E-04	1.0701E-02	1.0701E-02	1.0701E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	2.0212E-02	1.0701E-02
0.1500	0.0	2.0701E-03	0.7355E-03	1.0701E-02	1.2722E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	1.4232E-02	0.7355E-03
0.1625	0.0	0.4195E-03	-1.0247E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-2.3046E-02	-1.0247E-02
0.1750	0.0	0.7105E-03	-2.0466E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-4.0931E-02	-2.0466E-02

THE FIELD OF THE RATIO OF THE DERIVATIVE OF THE RATE OF DEFORMATION TENSORS AND THE PRIMARY DEFORMATION RATE IS

BETA

	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
0.0125	0.0	3.0909E-05	1.1184E-04	1.7955E-04	2.4204E-04	3.0909E-04	3.0909E-04	2.4204E-04	1.7955E-04	1.1184E-04	3.0909E-05	
0.0250	0.0	0.5212E-05	1.0552E-04	3.3522E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	1.0552E-04	
0.0375	0.0	7.0642E-05	2.1827E-04	3.6542E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	2.1827E-04	
0.0500	0.0	7.4820E-05	2.1280E-04	3.6542E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	4.5212E-04	2.1280E-04	
0.0625	0.0	0.1492E-05	1.7465E-04	3.0445E-04	3.0445E-04	3.0445E-04	3.0445E-04	3.0445E-04	3.0445E-04	3.0445E-04	1.7465E-04	
0.0750	0.0	0.9072E-05	1.1082E-04	1.7465E-04	2.5905E-04	2.5905E-04	2.5905E-04	2.5905E-04	2.5905E-04	2.5905E-04	1.1082E-04	
0.0875	0.0	1.0028E-05	2.9322E-05	0.0240E-04	0.0240E-05	0.0240E-05	0.0240E-05	0.0240E-05	0.0240E-05	0.0240E-05	2.9322E-05	
0.1000	0.0	2.0372E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	0.0909E-05	
0.1125	0.0	0.0944E-05	-1.4307E-04	-2.3335E-04	-3.0109E-04	-3.0109E-04	-3.0109E-04	-3.0109E-04	-3.0109E-04	-3.0109E-04	-3.0109E-04	-1.4307E-04
0.1250	0.0	-7.3202E-05	-2.0912E-04	-3.1945E-04	-4.3785E-04	-4.3785E-04	-4.3785E-04	-4.3785E-04	-4.3785E-04	-4.3785E-04	-4.3785E-04	-2.0912E-04
0.1375	0.0	-8.5330E-05	-2.4204E-04	-3.6542E-04	-5.0470E-04	-5.0470E-04	-5.0470E-04	-5.0470E-04	-5.0470E-04	-5.0470E-04	-5.0470E-04	-2.4204E-04
0.1500	0.0	-8.0946E-05	-2.2815E-04	-3.6542E-04	-4.7399E-04	-4.7399E-04	-4.7399E-04	-4.7399E-04	-4.7399E-04	-4.7399E-04	-4.7399E-04	-2.2815E-04
0.1625	0.0	-8.2088E-05	-1.4742E-04	-2.4204E-04	-3.0445E-04	-3.0445E-04	-3.0445E-04	-3.0445E-04	-3.0445E-04	-3.0445E-04	-3.0445E-04	-1.4742E-04
0.1750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	



GAP ANGLE = 1.3000 DEGREES REYNOLDS NUMBER = 2.0E+03

THE FIELD OF THE RATIO OF THE UTT ELEMENT OF THE RATE OF DEFORMATION TENSION AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44708	0.66804	0.82056	0.94016	0.94704	0.97200	0.98560	0.99331	0.99760	1.00000
.00125	0.0	-1.274E-05	-2.302E-05	-3.681E-05	-5.688E-05	-8.302E-05	-1.072E-04	-1.307E-04	-1.528E-04	1.673E-03	1.673E-03
.00250	0.0	-9.209E-05	-1.394E-04	-2.215E-04	-3.459E-04	-5.330E-04	-7.810E-04	-1.101E-03	-1.508E-03	1.076E-02	1.076E-02
.00375	0.0	-1.571E-04	-2.330E-04	-3.737E-04	-5.658E-04	-8.511E-04	-1.241E-03	-1.801E-03	-2.506E-03	2.041E-02	2.041E-02
.00500	0.0	-1.613E-04	-2.742E-04	-4.365E-04	-6.572E-04	-9.800E-04	-1.415E-03	-2.041E-03	-2.806E-03	1.973E-02	1.973E-02
.00625	0.0	-1.572E-04	-2.674E-04	-4.260E-04	-6.450E-04	-9.493E-04	-1.385E-03	-1.972E-03	-2.727E-03	1.604E-02	1.604E-02
.00750	0.0	-1.241E-04	-2.195E-04	-3.444E-04	-5.248E-04	-7.641E-04	-1.097E-03	-1.501E-03	-2.041E-03	1.076E-02	1.076E-02
.00875	0.0	-8.196E-05	-1.311E-04	-2.207E-04	-3.412E-04	-5.049E-04	-7.241E-04	-1.041E-03	-1.415E-03	7.241E-03	7.241E-03
.00997	0.0	-2.212E-05	-3.681E-05	-5.745E-05	-8.511E-05	-1.241E-04	-1.801E-04	-2.506E-04	-3.412E-04	2.041E-03	2.041E-03
.01122	0.0	4.306E-05	7.411E-05	1.194E-04	1.801E-04	2.741E-04	4.151E-04	6.041E-04	8.511E-04	1.241E-03	1.241E-03
.01247	0.0	1.030E-04	2.974E-04	5.071E-04	7.411E-04	1.041E-03	1.415E-03	1.972E-03	2.727E-03	1.972E-02	1.972E-02
.01371	0.0	1.841E-04	5.071E-04	8.511E-04	1.241E-03	1.801E-03	2.506E-03	3.412E-03	4.506E-03	2.041E-02	2.041E-02
.01496	0.0	1.794E-04	3.050E-04	4.811E-04	6.937E-04	9.493E-04	1.307E-03	1.801E-03	2.415E-03	1.430E-02	1.430E-02
.01621	0.0	1.931E-04	2.822E-04	4.365E-04	6.572E-04	9.493E-04	1.307E-03	1.801E-03	2.415E-03	1.430E-02	1.430E-02
.01745	0.0	2.822E-05	4.811E-05	7.141E-05	1.041E-04	1.501E-04	2.101E-04	2.974E-04	4.151E-04	5.741E-04	7.810E-04

THE FIELD OF THE RATIO OF THE UTT ELEMENT OF THE RATE OF DEFORMATION TENSION AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.66109	0.82556	0.94616	0.94704	0.97200	0.98560	0.99311	0.99760	1.00000
.00125	0.0	1.296E-05	3.694E-05	5.578E-05	8.070E-05	1.101E-04	1.501E-04	2.041E-04	2.727E-04	3.681E-04	4.937E-04
.00250	0.0	2.167E-05	6.170E-05	9.348E-05	1.341E-04	1.801E-04	2.415E-04	3.206E-04	4.151E-04	5.406E-04	7.141E-04
.00375	0.0	2.881E-05	7.250E-05	1.093E-04	1.533E-04	2.041E-04	2.727E-04	3.681E-04	4.937E-04	6.406E-04	8.511E-04
.00500	0.0	2.843E-05	7.078E-05	1.072E-04	1.430E-04	1.801E-04	2.415E-04	3.206E-04	4.151E-04	5.406E-04	7.141E-04
.00625	0.0	2.053E-05	5.816E-05	8.681E-05	1.241E-04	1.601E-04	2.101E-04	2.727E-04	3.681E-04	4.937E-04	6.406E-04
.00750	0.0	1.317E-05	3.694E-05	5.578E-05	8.070E-05	1.101E-04	1.501E-04	2.041E-04	2.727E-04	3.681E-04	4.937E-04
.00875	0.0	3.772E-06	1.072E-05	1.493E-05	2.041E-05	2.727E-05	3.681E-05	4.937E-05	6.406E-05	8.511E-05	1.101E-04
.00997	0.0	-6.502E-06	-1.425E-05	-2.041E-05	-2.727E-05	-3.681E-05	-4.937E-05	-6.406E-05	-8.511E-05	-1.101E-04	-1.425E-04
.01122	0.0	-1.631E-05	-4.714E-05	-7.141E-05	-1.041E-04	-1.415E-04	-1.801E-04	-2.415E-04	-3.206E-04	-4.151E-04	-5.406E-04
.01247	0.0	-2.416E-05	-6.937E-05	-1.041E-04	-1.415E-04	-1.801E-04	-2.415E-04	-3.206E-04	-4.151E-04	-5.406E-04	-7.141E-04
.01371	0.0	-2.844E-05	-8.071E-05	-1.241E-04	-1.601E-04	-2.101E-04	-2.727E-04	-3.681E-04	-4.937E-04	-6.406E-04	-8.511E-04
.01496	0.0	-2.738E-05	-7.620E-05	-1.101E-04	-1.430E-04	-1.801E-04	-2.415E-04	-3.206E-04	-4.151E-04	-5.406E-04	-7.141E-04
.01621	0.0	-1.813E-05	-4.942E-05	-7.141E-05	-1.041E-04	-1.415E-04	-1.801E-04	-2.415E-04	-3.206E-04	-4.151E-04	-5.406E-04
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES      REYNOLDS NUMBER = 4.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	5.2148	1.0000	.0150	0.0021	0.4410	.0097
DRT	-10.3626	0.9857	.0175	0.0086	0.9933	.0137
DPR	-3.9688	0.9933	.0087	0.0001	0.4410	.0012
DTT	-4.3104	0.9976	.0137	-0.0025	0.4410	.0
DTP	109.9234	0.9976	.0	89.9817	1.0000	.0175
OPP	-0.0344	0.9857	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 4.0E+03

THE FIELD OF THE RATIO UP THE DUT ELEMENT UP THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.44038	0.88076	1.32114	0.54764	0.97223	2.46556	0.99331	0.94760	1.00500
0.0	0.0	-1.143E-02	-3.445E-02	-5.186E-02	-6.294E-02	-6.933E-02	-7.273E-02	-7.434E-02	-6.366E-02	-3.031E-02	0.0
0.0125	0.0	-9.731E-03	-2.970E-02	-4.506E-02	-5.292E-02	-5.975E-02	-6.214E-02	-5.326E-02	-4.423E-02	-2.073E-02	0.0
0.0250	0.0	-5.373E-03	-1.826E-02	-2.138E-02	-2.603E-02	-2.878E-02	-3.022E-02	-3.084E-02	-2.587E-02	-1.249E-02	0.0
0.0375	0.0	-1.261E-03	-3.629E-03	-5.483E-03	-6.892E-03	-7.376E-03	-7.682E-03	-8.337E-03	-6.914E-03	-3.125E-03	0.0
0.0500	0.0	1.497E-03	8.764E-03	1.351E-02	1.751E-02	1.819E-02	1.819E-02	1.134E-02	1.807E-02	1.694E-03	0.0
0.0625	0.0	4.723E-03	2.031E-02	2.469E-02	2.723E-02	2.830E-02	2.830E-02	1.683E-02	2.107E-02	3.727E-03	0.0
0.0750	0.0	6.736E-03	1.925E-02	2.832E-02	3.115E-02	3.168E-02	2.742E-02	1.410E-02	2.740E-02	5.247E-03	0.0
0.0875	0.0	7.698E-03	2.281E-02	3.331E-02	3.632E-02	3.632E-02	2.632E-02	1.596E-02	2.824E-02	8.979E-03	0.0
0.0999	0.0	8.065E-03	2.231E-02	3.041E-02	3.199E-02	3.031E-02	2.237E-02	1.016E-02	2.689E-02	5.655E-03	0.0
0.1122	0.0	7.094E-03	2.003E-02	2.666E-02	2.666E-02	2.666E-02	2.666E-02	1.032E-02	2.689E-02	3.694E-03	0.0
0.1247	0.0	4.876E-03	1.344E-02	2.016E-02	2.459E-02	2.724E-02	2.672E-02	1.032E-02	2.689E-02	1.177E-03	0.0
0.1371	0.0	1.409E-03	2.624E-03	4.110E-03	4.971E-03	5.524E-03	5.731E-03	4.574E-03	9.803E-04	-1.177E-03	0.0
0.1496	0.0	4.431E-03	1.234E-02	1.879E-02	2.289E-02	2.584E-02	2.678E-02	2.745E-02	-2.431E-02	-1.101E-02	0.0
0.1621	0.0	-1.113E-02	-3.565E-02	-4.578E-02	-5.371E-02	-6.018E-02	-6.476E-02	-6.940E-02	-5.600E-02	-2.623E-02	0.0
0.1745	0.0	-1.736E-02	-4.331E-02	-5.327E-02	-6.164E-02	-6.813E-02	-7.201E-02	-7.605E-02	-6.713E-02	-4.456E-02	0.0

THE FIELD OF THE RATIO OF THE DUT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	0.0	0.44038	0.88076	1.32114	0.54764	0.97223	2.46556	0.99331	0.94760	1.00500
0.0	0.0	7.773E-05	2.196E-04	3.467E-04	4.331E-04	4.631E-04	4.456E-04	3.0	0.0	0.0	0.0
0.0125	0.0	1.302E-04	3.660E-04	5.791E-04	7.189E-04	8.031E-04	6.948E-04	6.948E-04	-1.637E-03	-3.423E-02	-4.603E-02
0.0250	0.0	1.631E-04	4.403E-04	6.813E-04	8.465E-04	9.143E-04	7.776E-04	2.643E-03	-1.703E-03	-3.423E-02	-4.603E-02
0.0375	0.0	1.423E-04	4.199E-04	6.444E-04	8.212E-04	8.959E-04	7.221E-04	1.624E-03	-1.077E-02	-3.760E-03	-4.430E-02
0.0500	0.0	1.227E-04	3.444E-04	5.464E-04	6.813E-04	7.413E-04	6.837E-04	-2.326E-03	-1.361E-02	-3.037E-02	-3.590E-02
0.0625	0.0	7.606E-05	2.189E-04	3.474E-04	4.349E-04	4.769E-04	3.939E-04	1.659E-03	-8.678E-03	-1.603E-02	-2.301E-02
0.0750	0.0	2.124E-05	5.999E-05	9.414E-05	1.231E-04	1.376E-04	6.154E-05	5.183E-04	-2.108E-03	-4.975E-03	-7.078E-03
0.0875	0.0	4.057E-05	-1.153E-04	-1.822E-04	-2.239E-04	-2.394E-04	-2.251E-04	6.913E-04	6.017E-03	1.024E-02	1.036E-02
0.0999	0.0	-9.943E-05	-2.813E-04	-4.433E-04	-5.325E-04	-5.942E-04	-4.691E-04	1.617E-03	1.617E-02	2.483E-02	2.756E-02
0.1122	0.0	-1.461E-04	-4.123E-04	-6.539E-04	-8.128E-04	-8.811E-04	-7.119E-04	2.624E-03	1.670E-02	3.673E-02	4.243E-02
0.1247	0.0	-1.704E-04	-4.796E-04	-7.614E-04	-9.495E-04	-1.032E-03	-8.293E-04	2.479E-03	1.892E-02	4.335E-02	5.202E-02
0.1371	0.0	-1.610E-04	-4.501E-04	-7.154E-04	-8.941E-04	-9.783E-04	-7.783E-04	2.510E-03	1.714E-02	4.194E-02	5.218E-02
0.1496	0.0	-1.052E-04	-2.933E-04	-4.614E-04	-5.765E-04	-6.209E-04	-5.614E-04	1.474E-03	1.266E-02	2.743E-02	3.610E-02
0.1621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 1.0000 DEGREE REYNOLDS NUMBER = 4.0E+73

THE FIELD OF THE RATIO OF THE OFF ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	Q	P	S	T	U	V	W	X	Y	Z
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE OFF ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	R	Q	P	S	T	U	V	W	X	Y	Z
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES      REYNOLDS NUMBER = 8.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	9.4176	1.0000	.0137	-0.0006	0.9720	.0100
DRT	-20.5055	0.9720	.0175	-0.0222	0.9976	.0050
DPR	-11.4833	0.9933	.0087	0.0004	0.4410	.0012
DTT	-7.8756	0.9976	.0137	-0.0051	0.4410	.0
DTP	123.8382	1.0000	.0012	80.0892	1.0000	.0150
DPP	-0.0669	0.9720	.0137	-0.0000	1.0000	.0612

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.EPSLON AND AT R = 0

DPP = 0 AT BETA = 0.EPSLON AND AT R = 0

DPR = 0 AT BETA = 0.EPSLON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 0.0E+00

THE FIELD OF THE RATIO OF THE URT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4000	0.6000	0.8000	1.0000	1.2000	1.4000	1.6000	1.8000	2.0000	0.99700	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DMR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4000	0.6000	0.8000	1.0000	1.2000	1.4000	1.6000	1.8000	2.0000	0.99700	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0875	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1375	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1500	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1625	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.1750	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0



GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 8.0E+23

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.4408	0.6809	0.8286	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
.00125	0.0	-8.099E-03	-0.242E-05	-1.397E-04	-1.714E-04	-1.754E-04	-1.350E-04	-5.291E-04	6.488E-04	7.372E-03	7.372E-03
.00249	0.0	-3.261E-04	-8.42E-04	-2.294E-04	-1.034E-03	-1.075E-03	-7.004E-04	6.614E-04	1.348E-02	4.167E-02	4.164E-02
.00374	0.0	-8.047E-04	0.048E-04	-1.361E-03	-1.734E-03	-1.814E-03	-1.180E-03	2.474E-03	2.421E-02	6.663E-02	6.642E-02
.00499	0.0	-6.407E-04	-1.680E-03	-1.640E-03	-2.092E-03	-2.167E-03	-1.424E-03	4.220E-03	3.043E-02	7.473E-02	7.481E-02
.00623	0.0	-6.283E-04	-1.944E-03	-1.608E-03	-2.081E-03	-2.162E-03	-1.441E-03	5.486E-03	3.184E-02	6.942E-02	6.923E-02
.00748	0.0	-8.140E-04	-8.628E-04	-1.331E-03	-1.684E-03	-1.834E-03	-1.397E-03	8.731E-03	2.770E-02	9.400E-02	9.382E-02
.00873	0.0	-8.982E-03	-1.842E-04	-8.591E-04	-1.100E-03	-1.234E-03	-9.788E-04	4.866E-03	1.933E-02	3.187E-02	3.146E-02
.00997	0.0	1.697E-04	2.791E-04	4.164E-04	4.994E-04	4.884E-04	4.879E-04	2.929E-03	7.800E-03	5.089E-02	5.079E-02
.01122	0.0	4.166E-04	6.934E-04	1.040E-03	1.417E-03	1.369E-03	8.260E-04	2.302E-04	-6.249E-03	-2.249E-02	-2.243E-02
.01247	0.0	6.124E-04	1.024E-03	1.577E-03	1.961E-03	2.116E-03	1.437E-03	-5.282E-03	-3.103E-02	-4.610E-02	-4.794E-02
.01371	0.0	7.144E-04	1.166E-03	1.682E-03	2.343E-03	2.544E-03	1.834E-03	-6.772E-03	-3.692E-02	-4.876E-02	-4.883E-02
.01496	0.0	6.743E-04	1.110E-03	1.789E-03	2.331E-03	2.447E-03	1.839E-03	-6.378E-03	-3.481E-02	-7.442E-02	-7.420E-02
.01621	0.0	4.394E-04	7.342E-04	1.140E-03	1.483E-03	1.604E-03	1.239E-03	-4.294E-03	-2.228E-02	-4.643E-02	-4.640E-02
.01745	0.0	1.127E-04	1.934E-04	2.904E-04	4.685E-04	4.047E-04	3.621E-04	-7.146E-04	-3.985E-03	-1.023E-02	-1.023E-02

THE FIELD OF THE RATIO OF THE OPP ELEMENT UP THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.4408	0.6809	0.8286	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
.00125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00249	0.0	3.188E-05	1.469E-04	2.180E-04	2.629E-04	2.874E-04	2.971E-04	2.912E-04	2.670E-04	1.225E-04	-3.801E-08
.00374	0.0	1.020E-04	2.644E-04	4.342E-04	4.392E-04	4.817E-04	4.980E-04	4.877E-04	4.269E-04	2.031E-04	-2.542E-07
.00499	0.0	9.972E-05	2.818E-04	4.206E-04	5.075E-04	5.685E-04	5.893E-04	5.766E-04	4.787E-04	2.192E-04	-7.334E-07
.00623	0.0	8.234E-05	2.318E-04	3.466E-04	4.192E-04	4.874E-04	5.192E-04	5.077E-04	4.448E-04	1.968E-04	-1.443E-06
.00748	0.0	8.273E-05	1.977E-04	2.218E-04	2.694E-04	3.078E-04	3.132E-04	4.734E-04	3.423E-04	1.672E-04	-2.296E-06
.00873	0.0	1.815E-05	4.074E-05	6.202E-05	7.732E-05	8.740E-05	9.831E-05	9.731E-05	3.094E-05	6.122E-05	-3.189E-06
.00997	0.0	-2.894E-05	-7.397E-05	-1.128E-04	-1.336E-04	-1.442E-04	-1.459E-04	-1.394E-04	-3.094E-05	-4.374E-05	-3.693E-06
.01122	0.0	-6.830E-05	-1.672E-04	-2.792E-04	-3.396E-04	-3.671E-04	-3.791E-04	-3.698E-04	-1.441E-04	-1.990E-05	-4.381E-06
.01247	0.0	-9.660E-05	-2.786E-04	-4.180E-04	-4.972E-04	-5.461E-04	-5.695E-04	-5.695E-04	-3.103E-04	-1.049E-04	-4.399E-06
.01371	0.0	-1.134E-04	-3.217E-04	-4.614E-04	-5.428E-04	-6.414E-04	-6.690E-04	-6.690E-04	-4.302E-04	-1.972E-04	-3.942E-06
.01496	0.0	-1.082E-04	-3.041E-04	-4.531E-04	-5.510E-04	-6.082E-04	-6.358E-04	-6.358E-04	-4.042E-04	-2.898E-04	-3.039E-06
.01621	0.0	-7.280E-05	-1.989E-04	-2.964E-04	-3.941E-04	-4.146E-04	-4.146E-04	-4.106E-04	-3.119E-04	-1.001E-04	-1.688E-06
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES

REYNOLDS NUMBER = 2.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-15.4312	1.0000	.0025	-3.0084	0.9476	.0012
DRT	-56.4898	0.9720	.0175	3.1472	0.9720	.0050
DPR	-26.9841	0.9933	.0050	0.0019	0.4410	.0012
DTT	13.4037	0.9976	.0025	-0.0123	0.4410	.0
DTP	262.9375	0.9976	.0	36.4952	1.0000	.0137
DPP	-0.1752	0.9720	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with columns for X, Y, Z, U, V, W and a final column with value 1.00000. Includes a 'BETA' label and a series of numerical data points.

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with columns for X, Y, Z, U, V, W and a final column with value 1.00000. Includes a 'BETA' label and a series of numerical data points.

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 2.0E+04

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4009	0.6809	0.8286	0.90416	0.94764	0.97200	0.98866	0.99331	0.99760	1.00000
.0	1.0	0.944E-01	0.774E-01	0.530E-01	0.386E-01	0.282E-01	0.200E-01	1.147E+00	1.693E+00	2.290E+00	2.629E+00	2.629E+00
.00125	1.0	0.945E-01	0.775E-01	0.532E-01	0.387E-01	0.283E-01	0.201E-01	1.148E+00	1.694E+00	2.291E+00	2.630E+00	2.630E+00
.00250	1.0	0.946E-01	0.776E-01	0.533E-01	0.388E-01	0.284E-01	0.202E-01	1.149E+00	1.695E+00	2.292E+00	2.631E+00	2.631E+00
.00374	1.0	0.947E-01	0.777E-01	0.534E-01	0.389E-01	0.285E-01	0.203E-01	1.150E+00	1.696E+00	2.293E+00	2.632E+00	2.632E+00
.00499	1.0	0.948E-01	0.778E-01	0.535E-01	0.390E-01	0.286E-01	0.204E-01	1.151E+00	1.697E+00	2.294E+00	2.633E+00	2.633E+00
.00623	1.0	0.949E-01	0.779E-01	0.536E-01	0.391E-01	0.287E-01	0.205E-01	1.152E+00	1.698E+00	2.295E+00	2.634E+00	2.634E+00
.00748	1.0	0.950E-01	0.780E-01	0.537E-01	0.392E-01	0.288E-01	0.206E-01	1.153E+00	1.699E+00	2.296E+00	2.635E+00	2.635E+00
.00873	1.0	0.951E-01	0.781E-01	0.538E-01	0.393E-01	0.289E-01	0.207E-01	1.154E+00	1.700E+00	2.297E+00	2.636E+00	2.636E+00
.00997	1.0	0.952E-01	0.782E-01	0.539E-01	0.394E-01	0.290E-01	0.208E-01	1.155E+00	1.701E+00	2.298E+00	2.637E+00	2.637E+00
.01122	1.0	0.953E-01	0.783E-01	0.540E-01	0.395E-01	0.291E-01	0.209E-01	1.156E+00	1.702E+00	2.299E+00	2.638E+00	2.638E+00
.01247	1.0	0.954E-01	0.784E-01	0.541E-01	0.396E-01	0.292E-01	0.210E-01	1.157E+00	1.703E+00	2.300E+00	2.639E+00	2.639E+00
.01371	1.0	0.955E-01	0.785E-01	0.542E-01	0.397E-01	0.293E-01	0.211E-01	1.158E+00	1.704E+00	2.301E+00	2.640E+00	2.640E+00
.01496	1.0	0.956E-01	0.786E-01	0.543E-01	0.398E-01	0.294E-01	0.212E-01	1.159E+00	1.705E+00	2.302E+00	2.641E+00	2.641E+00
.01621	1.0	0.957E-01	0.787E-01	0.544E-01	0.399E-01	0.295E-01	0.213E-01	1.160E+00	1.706E+00	2.303E+00	2.642E+00	2.642E+00
.01745	1.0	0.958E-01	0.788E-01	0.545E-01	0.400E-01	0.296E-01	0.214E-01	1.161E+00	1.707E+00	2.304E+00	2.643E+00	2.643E+00

THE FIELD OF THE RATIO OF THE DPK ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.4009	0.6809	0.8286	0.90416	0.94764	0.97200	0.98866	0.99331	0.99760	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00250	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00374	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00499	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00623	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00748	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00873	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00997	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01122	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01247	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01371	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01496	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01621	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





CAP ANGLE = 1.0000 DEGREES      REYNOLDS NUMBER = 4.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-16.1212	1.0000	.0025	0.0256	0.4410	.0087
DRT	-90.4259	0.9476	.0175	0.1811	0.9976	.0100
DPR	-22.8912	0.9933	.0037	-0.0059	0.9042	.0150
DTT	13.3987	0.9976	.0025	-0.0223	0.4410	.00
DTP	405.1126	0.9976	.00	15.9648	1.0000	.0100
DPP	-0.2770	0.9476	.0137	-0.0000	1.0000	.0012

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0.EPSILON AND AT R = 0

DPP = 0 AT BETA = 0.EPSILON AND AT R = 0

DPR = 0 AT BETA = 0.EPSILON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000	
.0	0.0	-1.184E-01	-3.110E-01	-4.213E-01	-5.439E-01	-6.319E-01	-4.679E-01	-2.754E-01	-1.749E-01	0.0	
.00125	0.0	-8.922E-02	-2.834E-01	-3.085E-01	-3.427E-01	-3.693E-01	-2.642E-01	-1.740E-01	-8.028E-02	0.0	
.00250	0.0	-4.968E-02	-1.869E-01	-2.290E-01	-2.849E-01	-3.197E-01	-1.870E-01	-8.028E-02	-5.030E-03	0.0	
.00375	0.0	-1.340E-02	-3.906E-02	-6.058E-02	-9.993E-02	-9.031E-02	6.608E-02	6.632E-02	3.477E-02	0.0	
.00500	0.0	1.688E-02	4.885E-02	8.408E-02	3.186E-02	7.244E-02	1.948E-01	1.521E-01	1.170E-01	4.443E-02	0.0
.00750	0.0	4.577E-02	1.179E-01	1.840E-01	1.883E-01	2.109E-01	2.846E-01	1.753E-01	1.816E-01	3.340E-02	0.0
.00875	0.0	6.586E-02	1.723E-01	2.674E-01	3.109E-01	2.894E-01	1.948E-01	1.948E-01	9.103E-02	1.881E-02	0.0
.00997	0.0	7.962E-02	2.079E-01	2.872E-01	3.482E-01	3.694E-01	2.398E-01	1.308E-01	6.009E-02	8.877E-03	0.0
.01122	0.0	4.067E-02	1.963E-01	2.855E-01	3.628E-01	3.694E-01	2.101E-01	1.010E-01	3.034E-02	1.811E-03	0.0
.01247	0.0	4.669E-02	1.384E-01	2.102E-01	3.748E-01	3.604E-01	1.668E-01	6.764E-02	1.532E-02	-4.333E-03	0.0
.01371	0.0	1.230E-02	3.828E-02	6.931E-02	1.010E-01	8.407E-02	-2.017E-03	-2.928E-02	-3.068E-02	-9.914E-03	0.0
.01496	0.0	-4.015E-02	-1.109E-01	-1.805E-01	-1.786E-01	-1.943E-01	-1.878E-01	-9.424E-02	-5.822E-02	-2.819E-02	0.0
.01621	0.0	-1.078E-01	-2.994E-01	-4.268E-01	-3.268E-01	-3.296E-01	-1.695E-01	-8.289E-02	-2.566E-02	0.0	0.0
.01745	0.0	-1.822E-01	-4.891E-01	-7.166E-01	-8.948E-01	-8.043E-01	-5.004E-01	-2.357E-01	-1.007E-01	-2.410E-02	0.0

THE FIELD OF THE RATIO OF THE DRR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
.0	0.0	7.067E-04	1.754E-03	2.048E-03	2.777E-03	3.600E-03	0.0	0.0	0.0	0.0
.00125	0.0	1.186E-03	2.971E-03	3.775E-03	5.130E-03	5.600E-03	-5.112E-03	-3.444E-02	-5.040E-02	-6.716E-02
.00250	0.0	1.410E-03	3.566E-03	4.646E-03	6.957E-03	1.277E-02	-2.789E-02	-5.704E-02	-6.530E-02	-1.349E-01
.00375	0.0	1.194E-03	3.579E-03	5.324E-03	6.645E-03	-5.877E-03	-3.728E-02	-6.294E-02	-7.047E-02	-1.343E-01
.00500	0.0	1.169E-03	3.069E-03	4.963E-03	5.471E-03	-1.166E-02	-3.993E-02	-4.174E-02	-4.344E-02	-9.014E-02
.00750	0.0	7.728E-04	2.116E-03	3.809E-03	3.434E-03	-1.398E-02	-2.429E-02	-2.440E-02	-1.346E-02	-3.136E-02
.00875	0.0	2.596E-04	8.245E-04	1.970E-03	9.941E-04	-1.256E-02	-2.151E-02	-4.038E-03	1.379E-02	1.702E-02
.00997	0.0	-3.229E-04	-6.645E-04	-3.418E-04	-1.837E-03	-4.082E-03	1.689E-02	3.700E-02	5.053E-02	7.016E-02
.01122	0.0	-6.692E-04	-2.171E-03	-2.819E-03	-3.950E-03	-1.905E-03	1.809E-02	5.961E-02	6.498E-02	8.966E-02
.01247	0.0	-1.386E-03	-3.464E-03	-5.061E-03	-4.843E-03	3.301E-02	5.295E-02	6.793E-02	8.013E-02	8.900E-02
.01371	0.0	-1.618E-03	-4.251E-03	-6.855E-03	-7.206E-03	4.674E-03	6.197E-02	7.119E-02	8.182E-02	7.871E-02
.01496	0.0	-1.537E-03	-4.172E-03	-6.631E-03	-7.010E-03	1.211E-02	4.808E-02	5.889E-02	6.232E-02	6.959E-02
.01621	0.0	-1.034E-03	-2.793E-03	-4.532E-03	-4.632E-03	9.254E-03	3.386E-02	3.456E-02	3.559E-02	2.592E-02
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.68600	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
0.0	1.0	0.787E-01	0.195E-01	0.582E-01	0.048E-C1	1.425E+00	2.276E+00	3.058E+00	3.597E+00	4.081E+00	4.051E+00
0.0125	1.0	0.786E-01	0.196E-01	0.571E-01	0.509E-01	1.402E+00	2.182E+00	2.851E+00	3.382E+00	3.712E+00	3.712E+00
0.0249	1.0	0.793E-01	0.201E-01	0.560E-01	0.509E-01	1.344E+00	1.983E+00	2.486E+00	2.842E+00	3.049E+00	3.049E+00
0.0374	1.0	0.804E-01	0.219E-01	0.557E-01	0.519E-01	1.212E+00	1.804E+00	1.942E+00	1.942E+00	2.029E+00	2.029E+00
0.0499	1.0	0.823E-01	0.261E-01	0.549E-01	0.529E-01	1.021E+00	1.591E+00	1.662E+00	1.662E+00	1.671E+00	1.671E+00
0.0623	1.0	0.854E-01	0.337E-01	0.683E-01	0.617E-01	8.188E-01	7.879E-01	6.636E-01	5.784E-01	5.029E-01	5.029E-01
0.0748	1.0	0.896E-01	0.459E-01	0.792E-01	0.707E-01	6.884E-01	6.037E-01	5.041E-01	4.073E-01	3.288E-01	2.589E-01
0.0873	1.0	0.948E-01	0.614E-01	0.875E-01	0.792E-01	5.773E-01	5.037E-01	4.073E-01	3.288E-01	2.589E-01	1.778E-01
0.0997	1.0	1.001E+00	0.871E-01	0.939E-01	0.792E-01	5.678E-01	3.968E-01	2.779E-01	2.011E-01	1.594E-01	1.594E-01
0.1122	1.0	1.008E+00	1.017E+00	1.028E+00	0.683E-01	6.792E-01	4.698E-01	3.265E-01	2.302E-01	1.647E-01	1.647E-01
0.1247	1.0	1.015E+00	1.052E+00	1.086E+00	0.828E-01	8.098E-01	5.808E-01	3.948E-01	2.667E-01	1.776E-01	1.776E-01
0.1371	1.0	1.022E+00	1.040E+00	1.170E+00	1.215E+00	1.005E+00	7.024E-01	4.841E-01	2.989E-01	1.933E-01	1.933E-01
0.1496	1.0	1.028E+00	1.127E+00	1.237E+00	1.341E+00	1.178E+00	8.130E-01	5.234E-01	3.249E-01	2.094E-01	2.094E-01
0.1621	1.0	1.032E+00	1.155E+00	1.308E+00	1.438E+00	1.308E+00	8.906E-01	5.613E-01	3.444E-01	2.227E-01	2.227E-01
0.1745	1.0	1.035E+00	1.177E+00	1.356E+00	1.513E+00	1.400E+00	9.466E-01	5.877E-01	3.521E-01	2.333E-01	2.333E-01

THE FIELD OF THE RATIO OF THE DTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.68600	0.82656	0.90416	0.94764	0.97200	0.98566	0.99331	0.99760	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0125	0.0	0.712E-05	2.848E-04	0.971E-03	-5.471E-03	-2.443E-02	-5.210E-02	-7.56CE-02	-1.010E-01	0.0	0.0
0.0249	0.0	1.218E-04	5.726E-04	2.379E-04	-1.072E-02	-4.722E-02	-9.839E-02	-1.414E-01	-1.863E-01	-1.485E-01	0.0
0.0374	0.0	2.079E-04	8.647E-04	4.292E-04	-1.807E-02	-6.477E-02	-1.250E-01	-1.803E-01	-2.289E-01	-1.703E-01	0.0
0.0499	0.0	2.677E-04	1.154E-03	6.130E-04	-1.773E-02	-7.654E-02	-1.408E-01	-1.862E-01	-2.271E-01	-1.600E-01	0.0
0.0623	0.0	3.274E-04	1.446E-03	1.401E-03	-1.827E-02	-7.659E-02	-1.384E-01	-1.793E-01	-2.080E-01	-1.418E-01	0.0
0.0748	0.0	3.798E-04	1.713E-03	2.145E-03	-1.676E-02	-7.293E-02	-1.304E-01	-1.667E-01	-1.917E-01	-1.308E-01	0.0
0.0873	0.0	4.195E-04	1.926E-03	2.735E-03	-1.376E-02	-6.892E-02	-1.204E-01	-1.555E-01	-1.803E-01	-1.228E-01	0.0
0.0997	0.0	4.419E-04	2.060E-03	3.622E-03	-1.004E-02	-5.736E-02	-1.104E-01	-1.430E-01	-1.691E-01	-1.180E-01	0.0
0.1122	0.0	4.417E-04	2.100E-03	4.346E-03	-6.354E-03	-4.824E-02	-9.987E-02	-1.326E-01	-1.543E-01	-1.037E-01	0.0
0.1247	0.0	4.139E-04	1.997E-03	4.112E-03	-3.285E-03	-3.901E-02	-8.662E-02	-1.164E-01	-1.337E-01	-8.613E-02	0.0
0.1371	0.0	3.946E-04	1.737E-03	3.716E-03	-1.162E-03	-2.967E-02	-7.603E-02	-9.436E-02	-1.067E-01	-6.871E-02	0.0
0.1496	0.0	2.627E-04	1.297E-03	2.847E-03	-0.789E-03	-2.013E-02	-5.909E-02	-6.698E-02	-7.429E-02	-4.668E-02	0.0
0.1621	0.0	1.416E-04	7.017E-04	1.538E-03	2.025E-04	-1.024E-02	-2.621E-02	-3.486E-02	-3.709E-02	-2.333E-02	0.0
0.1745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 4.0E+04

THE FIELD OF THE RATIO OF THE ODT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.68809	0.90416	0.94764	0.97200	0.98506	0.99331	0.99760	1.00000
.00125	0.0	-2.233E-04	-3.558E-04	-3.208E-04	-3.186E-04	-2.730E-03	-3.844E-03	5.833E-03	6.502E-03	1.849E-02
.00250	0.0	-1.467E-03	-2.179E-03	-2.720E-03	-3.789E-03	-6.729E-03	4.225E-03	3.321E-02	4.951E-02	8.673E-02
.00375	0.0	-2.467E-03	-3.694E-03	-4.803E-03	-6.803E-03	-7.619E-03	5.102E-02	5.102E-02	7.909E-02	1.340E-01
.00500	0.0	-2.432E-03	-4.437E-03	-6.278E-03	-8.961E-03	-2.172E-03	2.803E-02	3.804E-02	5.124E-02	8.420E-02
.00625	0.0	-2.592E-03	-4.660E-03	-6.703E-03	-8.589E-03	3.079E-02	3.804E-02	5.124E-02	6.948E-02	9.937E-02
.00750	0.0	-2.438E-03	-3.832E-03	-6.241E-03	-7.027E-03	1.016E-02	3.797E-02	4.047E-02	4.208E-02	3.110E-02
.00875	0.0	-1.618E-03	-2.681E-03	-4.732E-03	-4.394E-03	1.323E-02	3.288E-02	2.601E-02	1.337E-02	-1.697E-02
.00997	0.0	-5.433E-04	-1.047E-03	-1.233E-03	-1.233E-03	2.091E-02	2.091E-02	1.608E-02	-1.384E-02	-5.011E-02
.01122	0.0	6.822E-04	6.075E-04	5.699E-04	3.090E-03	6.032E-03	1.608E-02	-1.608E-02	-3.656E-02	-7.189E-02
.01247	0.0	1.846E-03	2.089E-03	3.670E-03	3.093E-03	-1.360E-02	-3.138E-02	-6.440E-02	-8.440E-02	-8.440E-02
.01371	0.0	2.824E-03	4.309E-03	6.466E-03	7.838E-03	-2.688E-03	-3.004E-02	-6.124E-02	-6.701E-02	-8.737E-02
.01496	0.0	3.779E-03	5.304E-03	4.262E-03	4.398E-03	-2.783E-03	-4.318E-02	-6.018E-02	-7.049E-02	-6.100E-02
.01621	0.0	3.253E-03	5.218E-03	4.377E-03	9.110E-03	-1.020E-02	-4.883E-02	-5.724E-02	-6.150E-02	-6.416E-02
.01745	0.0	2.155E-03	3.902E-03	5.657E-03	6.019E-03	-0.048E-03	-3.195E-02	-3.788E-02	-3.406E-02	-3.889E-02
.01869	0.0	5.905E-04	1.032E-03	1.715E-03	1.696E-03	-2.599E-03	-6.820E-03	-6.833E-03	-6.686E-03	-5.947E-04

THE FIELD OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.44098	0.68809	0.90416	0.94764	0.97200	0.98506	0.99331	0.99760	1.00000
.00125	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00250	0.0	2.538E-04	6.607E-04	9.037E-04	9.805E-04	1.129E-03	1.262E-03	6.991E-04	5.884E-04	3.178E-04
.00375	0.0	2.250E-04	1.124E-03	1.638E-03	1.709E-03	1.962E-03	1.977E-03	1.312E-03	5.885E-04	3.877E-04
.00500	0.0	5.016E-04	1.344E-03	1.842E-03	2.123E-03	2.398E-03	2.088E-03	1.267E-03	5.896E-04	3.044E-04
.00625	0.0	4.919E-04	1.319E-03	1.842E-03	2.208E-03	2.410E-03	1.686E-03	9.669E-04	5.728E-04	1.486E-04
.00750	0.0	4.078E-04	1.166E-03	1.572E-03	1.944E-03	2.033E-03	1.069E-03	4.991E-04	1.974E-04	-4.937E-04
.00875	0.0	2.638E-04	7.315E-04	1.075E-03	1.418E-03	1.352E-03	3.941E-04	4.644E-04	-1.117E-04	-1.089E-04
.00997	0.0	1.236E-04	3.821E-04	4.068E-04	6.309E-04	4.744E-04	-2.472E-04	-3.320E-04	-3.260E-04	-1.740E-04
.01122	0.0	-3.187E-04	-6.410E-04	-1.186E-03	-3.092E-04	-4.995E-04	-6.348E-04	-6.612E-04	-2.100E-04	-4.988E-04
.01247	0.0	-4.759E-04	-1.262E-03	-1.782E-03	-1.277E-03	-1.462E-03	-8.093E-04	-9.484E-04	-2.804E-04	-4.488E-04
.01371	0.0	-5.615E-04	-1.531E-03	-2.176E-03	-2.612E-03	-2.279E-03	-1.675E-03	-9.884E-04	-8.650E-04	-2.083E-04
.01496	0.0	-5.373E-04	-1.473E-03	-2.128E-03	-2.647E-03	-1.818E-03	-9.458E-04	-8.232E-04	-1.776E-04	-2.498E-04
.01621	0.0	-3.614E-04	-9.740E-04	-1.425E-03	-2.628E-03	-1.843E-03	-8.466E-04	-6.136E-04	-1.308E-04	-1.325E-04
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 1.0000 DEGREES REYNOLDS NUMBER = 8.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-15.4242	1.0000	.0025	0.0201	0.8266	.0087
DRT	-139.7909	0.8266	.0175	-0.0415	0.9976	.0112
DPR	-19.4956	0.9933	.0337	0.0176	0.6881	.0012
DTT	13.4440	0.9976	.0037	-0.0162	0.8266	.0087
DTP	521.7708	0.9976	.0	0.1786	0.9933	.0087
DPP	-0.3890	0.8266	.0150	0.0000	0.9976	.0162

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO

DRR = 0 AT BETA = 0, EPSILON AND AT R = C

DPP = 0 AT BETA = 0, EPSILON AND AT R = 0

DPR = 0 AT BETA = 0, EPSILON AND AT R = 0

DRT = 0 AT R = 0.1

DTT = 0 AT R = 0

DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.04098	0.08099	0.08285	0.00416	0.04764	0.07200	0.09331	0.099760	1.00000	
.00125	0.0	-2.217E-01	-4.096E-01	5.757E-01	-9.046E-01	-9.100E-01	-9.806E-01	-3.897E-01	-1.778E-01	-1.333E-01	0.0
.00249	0.0	-1.606E-01	-2.632E-01	4.888E-01	-8.713E-01	-5.948E-01	-8.696E-01	-2.333E-01	-1.561E-01	-7.849E-02	0.0
.00374	0.0	-9.881E-02	-2.257E-01	3.222E-01	-3.141E-01	-1.748E-01	-8.219E-02	-9.280E-02	-2.409E-02	-2.409E-02	0.0
.00499	0.0	-8.200E-02	-1.986E-01	2.883E-01	-2.800E-01	-1.902E-01	-1.173E-01	-8.886E-02	-3.647E-02	-2.042E-02	0.0
.00623	0.0	-3.227E-02	-1.134E-01	1.670E-01	-1.084E-01	-5.877E-02	-3.902E-02	-1.004E-01	-1.134E-01	-4.118E-02	0.0
.00748	0.0	8.389E-02	1.700E-01	1.044E-01	3.101E-01	2.844E-01	1.828E-01	1.834E-01	8.783E-02	3.444E-02	0.0
.00873	0.0	1.282E-01	2.668E-01	3.661E-01	3.922E-01	2.794E-01	1.430E-01	1.044E-01	5.893E-02	2.964E-02	0.0
.00997	0.0	1.400E-01	3.400E-01	4.675E-01	4.362E-01	2.713E-01	1.636E-01	9.479E-02	5.067E-02	2.621E-02	0.0
.01122	0.0	1.400E-01	3.749E-01	5.376E-01	4.871E-01	2.732E-01	1.744E-01	1.007E-01	5.432E-02	1.966E-02	0.0
.01247	0.0	9.667E-02	2.766E-01	4.376E-01	3.644E-01	2.232E-01	1.377E-01	1.030E-01	4.224E-02	-6.194E-04	0.0
.01371	0.0	2.946E-02	1.189E-01	2.823E-01	1.648E-01	9.044E-02	4.874E-02	3.444E-02	-1.108E-02	-1.919E-02	0.0
.01496	0.0	-7.472E-02	-1.640E-01	-2.124E-01	-2.022E-01	-1.634E-01	-1.184E-02	-4.922E-02	-4.644E-02	-3.822E-02	0.0
.01621	0.0	-2.119E-01	-3.373E-01	-7.802E-01	-7.304E-01	-5.147E-01	-3.266E-01	-2.117E-01	-1.110E-01	-2.046E-02	0.0
.01746	0.0	-3.637E-01	-9.396E-01	-1.396E+00	-1.321E+00	-8.949E-01	-5.499E-01	-3.632E-01	-1.396E-01	-2.271E-02	0.0

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.04098	0.08099	0.08285	0.00416	0.04764	0.07200	0.09331	0.099760	1.00000	
.00125	0.0	0.0	0.0	0.014E-03	8.882E-03	-4.381E-03	-2.649E-02	-2.893E-02	-4.442E-02	-8.816E-02	0.0
.00249	0.0	1.919E-03	4.211E-03	6.802E-03	8.798E-03	-1.226E-02	-3.914E-02	-8.080E-02	-8.885E-02	-1.218E-01	-1.842E-01
.00374	0.0	2.318E-03	6.102E-03	7.181E-03	1.242E-02	-2.033E-02	-3.792E-02	-4.004E-02	-7.211E-02	-1.282E-01	-1.687E-01
.00499	0.0	2.347E-03	6.431E-03	8.238E-03	-4.824E-03	-2.475E-02	-3.104E-02	-3.271E-02	-9.724E-02	-8.723E-02	-8.792E-02
.00623	0.0	2.044E-03	6.418E-03	6.357E-03	-9.173E-03	-2.384E-02	-2.249E-02	-2.466E-02	-3.804E-02	-3.411E-02	-2.883E-02
.00748	0.0	1.451E-03	4.188E-03	2.277E-03	1.023E-02	-1.699E-02	-1.103E-02	-1.340E-02	-1.917E-02	-7.241E-03	1.982E-02
.00873	0.0	6.766E-04	2.248E-03	2.012E-04	-8.319E-03	-8.142E-03	-1.604E-04	-1.644E-03	-8.132E-03	1.472E-02	6.780E-02
.00997	0.0	-1.346E-04	-2.003E-04	-1.709E-03	-4.689E-03	1.891E-03	1.964E-02	1.086E-02	9.724E-03	4.234E-02	1.087E-01
.01122	0.0	-2.299E-03	-6.429E-03	-3.604E-03	-4.890E-04	1.688E-02	2.178E-02	2.306E-02	2.589E-02	7.064E-02	1.283E-01
.01247	0.0	-8.221E-03	-7.236E-03	-6.943E-03	3.336E-04	1.934E-02	3.197E-02	3.729E-02	4.942E-02	9.289E-02	1.211E-01
.01371	0.0	-2.632E-03	-7.839E-03	-6.943E-03	3.001E-03	2.690E-02	3.091E-02	4.810E-02	7.109E-02	1.001E-01	9.219E-02
.01496	0.0	-2.632E-03	-7.839E-03	-6.943E-03	3.001E-03	2.690E-02	3.091E-02	4.810E-02	7.109E-02	1.001E-01	9.219E-02
.01621	0.0	-1.944E-03	-8.268E-03	-8.189E-03	6.087E-03	1.944E-02	2.671E-02	3.721E-02	6.236E-02	4.909E-02	-2.216E-04
.01746	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0





GAP ANGLE = 1.3000 DEGREES REYNOLDS NUMBER = 8.0E+04

THE FIELD OF THE RATIO OF THE DTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2	0.4000	0.6000	0.8000	0.90416	0.94764	0.97200	0.98560	0.99331	0.99760	1.00000
0.0	0.0	-3.310E-04	-3.296E-04	-9.324E-04	-3.000E-03	-3.067E-03	3.538E-03	1.940E-02	6.624E-03	7.505E-03	7.506E-03	7.506E-03
.00125	0.0	-3.242E-03	-2.914E-03	-5.318E-03	-6.935E-03	-2.967E-03	2.444E-02	4.138E-02	4.361E-02	4.496E-02	4.496E-02	4.496E-02
.00250	0.0	-3.605E-03	-5.265E-03	-8.564E-03	-7.841E-03	9.903E-03	3.658E-02	6.643E-02	6.748E-02	1.211E-01	1.211E-01	1.211E-01
.00375	0.0	-4.682E-03	-6.805E-03	-9.372E-03	-3.572E-03	1.781E-02	3.528E-02	3.832E-02	3.832E-02	1.344E-01	1.344E-01	1.344E-01
.00500	0.0	-4.771E-03	-7.371E-03	-8.134E-03	2.568E-03	2.196E-02	2.428E-02	3.150E-02	3.653E-02	3.653E-02	3.653E-02	3.653E-02
.00625	0.0	-4.182E-03	-6.669E-03	-5.679E-03	7.200E-03	2.091E-02	2.112E-02	2.410E-02	3.489E-02	3.506E-02	3.506E-02	3.506E-02
.00750	0.0	-3.007E-03	-5.322E-03	-2.664E-03	6.797E-03	1.530E-02	1.102E-02	1.310E-02	1.902E-02	1.902E-02	1.902E-02	1.902E-02
.00875	0.0	-1.393E-03	-2.071E-03	-1.625E-04	7.575E-03	7.514E-03	5.182E-04	1.776E-03	8.250E-03	1.673E-02	1.673E-02	1.673E-02
.00997	0.0	6.177E-04	2.387E-04	2.387E-03	4.872E-03	-8.387E-04	-9.929E-03	-1.002E-02	-5.477E-03	-4.321E-02	-4.321E-02	-4.321E-02
.01122	0.0	2.644E-03	3.648E-03	6.877E-03	1.745E-03	-9.119E-03	-2.039E-02	-2.264E-02	-2.441E-02	-2.441E-02	-2.441E-02	-2.441E-02
.01247	0.0	4.542E-03	6.642E-03	7.242E-03	1.053E-03	-1.676E-02	2.993E-02	3.887E-02	4.862E-02	4.862E-02	4.862E-02	4.862E-02
.01371	0.0	5.777E-03	9.132E-03	9.075E-03	-3.039E-03	-2.281E-02	-3.640E-02	-4.647E-02	-7.091E-02	-9.444E-02	-9.444E-02	-9.444E-02
.01496	0.0	5.836E-03	9.318E-03	9.349E-03	-3.828E-03	-2.293E-02	-3.624E-02	-4.646E-02	-8.094E-02	-8.094E-02	-8.094E-02	-8.094E-02
.01621	0.0	4.020E-03	6.579E-03	6.579E-03	-2.974E-03	-1.729E-02	-2.494E-02	-3.523E-02	-5.174E-02	-4.867E-02	-4.867E-02	-4.867E-02
.01745	0.0	1.249E-03	2.333E-03	2.589E-03	-4.053E-04	-5.096E-03	-6.653E-03	-1.210E-02	-2.272E-02	1.338E-03	1.338E-03	1.338E-03

THE FIELD OF THE RATIO OF THE DPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.2	0.4000	0.6000	0.8000	0.90416	0.94764	0.97200	0.98560	0.99331	0.99760	1.00000
0.0	0.0	0.0	0.0	3.0	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00125	0.0	4.770E-04	1.069E-03	1.289E-03	1.718E-03	1.800E-03	1.309E-03	7.712E-04	4.152E-04	2.589E-04	2.589E-04	2.589E-04
.00250	0.0	8.024E-04	1.812E-03	2.267E-03	2.649E-03	2.694E-03	1.840E-03	1.141E-03	7.758E-04	3.711E-04	3.711E-04	3.711E-04
.00375	0.0	9.534E-04	2.190E-03	2.895E-03	3.284E-03	2.676E-03	1.711E-03	1.148E-03	6.611E-04	3.869E-04	3.869E-04	3.869E-04
.00500	0.0	9.336E-04	2.221E-03	3.077E-03	3.098E-03	2.100E-03	1.296E-03	8.705E-04	5.945E-04	2.093E-04	2.093E-04	2.093E-04
.00625	0.0	7.926E-04	1.9334E-03	2.813E-03	2.457E-03	1.344E-03	7.488E-04	4.705E-04	2.289E-04	6.903E-05	6.903E-05	6.903E-05
.00750	0.0	5.266E-04	1.373E-03	2.129E-03	1.849E-03	6.191E-04	3.226E-04	1.698E-04	4.339E-05	-5.211E-05	-5.211E-05	-5.211E-05
.00875	0.0	1.785E-04	5.953E-04	1.086E-03	4.983E-04	-5.839E-05	-7.608E-05	-7.635E-05	-1.099E-04	-1.083E-04	-1.083E-04	-1.083E-04
.00997	0.0	-2.112E-04	-3.210E-04	-2.031E-04	-6.308E-04	-7.421E-04	-6.039E-04	-3.246E-04	-2.801E-04	-2.602E-04	-2.602E-04	-2.602E-04
.01122	0.0	-5.033E-04	-1.273E-03	-1.592E-03	-1.782E-03	-1.449E-03	-9.949E-04	-6.004E-04	-4.844E-04	-3.071E-04	-3.071E-04	-3.071E-04
.01247	0.0	-9.090E-04	-2.118E-03	-2.881E-03	-2.951E-03	-2.111E-03	-1.376E-03	-8.641E-04	-5.923E-04	-2.963E-04	-2.963E-04	-2.963E-04
.01371	0.0	-1.090E-03	-2.668E-03	-3.775E-03	-3.605E-03	-1.831E-03	-1.031E-03	-1.050E-03	-6.413E-04	-2.284E-04	-2.284E-04	-2.284E-04
.01496	0.0	-1.056E-03	-2.679E-03	-3.892E-03	-3.682E-03	-2.942E-03	-1.424E-03	-1.053E-03	-5.940E-04	-1.129E-04	-1.129E-04	-1.129E-04
.01621	0.0	-7.174E-04	-1.641E-03	-2.718E-03	-2.668E-03	-1.757E-03	-1.092E-03	-1.092E-03	-3.149E-04	5.532E-05	5.532E-05	5.532E-05
.01745	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 0.3333 DEGREES      REYNOLDS NUMBER = 2.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.3398	0.9996	.0046	0.0000	0.9924	.0029
DRT	-0.6367	0.5024	.0058	0.0100	0.5536	.0046
DPR	-0.2458	0.9968	.0029	-0.0000	0.5536	.0004
DTT	-0.3394	0.9996	.0046	0.0000	0.9924	.0004
DTP	101.2222	0.9996	.00	98.3169	0.9987	.0058
OPP	-0.0007	0.9974	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO







GAP ANGLE = 0.3333 DEGREES      REYNOLDS NUMBER = 4.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	0.6636	0.9996	.0046	0.0001	0.5536	.0029
DRT	-1.2545	0.9924	.0058	0.0196	0.5536	.0046
DPR	-0.5755	0.9968	.0029	-0.0000	0.5536	.0004
DTT	-0.6627	0.9996	.0046	0.0000	0.9924	.0029
DTP	102.4397	0.9996	.00	97.1518	0.9987	.0058
DPP	-0.0014	0.9924	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO









GAP ANGLE = 0.3333 DEGREES      REYNOLDS NUMBER = 8.0E+03

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		P	BETA		R	BETA
DRR	1.290	0.9996	.0046	0.0002	0.5536	.0029
DRT	-2.493	0.9994	.0046	0.0001	0.5536	.0046
DPR	-1.5417	0.9998	.0046	-0.0001	0.5536	.0046
DTT	-1.2882	0.9996	.0046	-0.0002	0.5536	.0
DTP	105.8339	0.9996	.0	93.7922	0.9996	.0058
DPP	-0.0028	0.9924	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT P = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 0.3333 DEGREES REYNOLDS NUMBER = 8.68E+02

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.00340	0.00682	0.01122	0.01603	0.02168	0.02834	0.03617	0.04523	0.05561	1.00000
0.0	-1.139E-03	-1.085E-02	-1.149E-02	-1.702E-02	-1.792E-02	-1.804E-02	-1.709E-02	-1.012E-02	-1.012E-02	-3.368E-03	0.0
0.0	-2.972E-03	-7.70E-03	-1.072E-02	-1.822E-02	-1.832E-02	-1.832E-02	-1.832E-02	-1.832E-02	-1.832E-02	-1.832E-02	0.0
0.0	-1.721E-03	-4.484E-03	-6.197E-03	-7.821E-03	-7.494E-03	-7.809E-03	-7.693E-03	-4.432E-03	-4.432E-03	-1.995E-03	0.0
0.0128	0.0	-1.39E-03	-1.073E-03	-1.006E-03	-1.010E-03	-1.010E-03	-1.010E-03	-1.010E-03	-1.010E-03	-1.010E-03	0.0
0.0166	0.0	0.790E-04	1.087E-03	2.032E-03	3.002E-03	3.992E-03	4.992E-03	5.992E-03	6.992E-03	7.992E-03	0.0
0.0209	0.0	1.611E-03	2.874E-03	4.722E-03	7.102E-03	1.002E-02	1.392E-02	1.782E-02	2.172E-02	2.562E-02	0.0
0.0249	0.0	2.301E-03	4.877E-03	8.411E-03	1.202E-02	1.572E-02	1.942E-02	2.312E-02	2.682E-02	3.052E-02	0.0
0.0291	0.0	2.999E-03	7.097E-03	1.232E-02	1.822E-02	2.412E-02	3.002E-02	3.592E-02	4.182E-02	4.772E-02	0.0
0.0332	0.0	2.708E-03	7.812E-03	1.372E-02	2.022E-02	2.672E-02	3.322E-02	3.972E-02	4.622E-02	5.272E-02	0.0
0.0374	0.0	2.424E-03	6.292E-03	1.051E-02	1.502E-02	1.952E-02	2.402E-02	2.852E-02	3.302E-02	3.752E-02	0.0
0.0416	0.0	1.694E-03	4.292E-03	7.792E-03	1.129E-02	1.479E-02	1.829E-02	2.179E-02	2.529E-02	2.879E-02	0.0
0.0457	0.0	4.012E-04	9.399E-04	1.684E-03	2.592E-03	3.502E-03	4.412E-03	5.322E-03	6.232E-03	7.142E-03	0.0
0.0499	0.0	-1.342E-03	-2.692E-03	-4.042E-03	-5.392E-03	-6.742E-03	-8.092E-03	-9.442E-03	-1.079E-02	-1.214E-02	0.0
0.0540	0.0	-3.712E-03	-6.992E-03	-1.021E-02	-1.371E-02	-1.721E-02	-2.071E-02	-2.421E-02	-2.771E-02	-3.121E-02	0.0
0.0582	0.0	-6.294E-03	-1.231E-02	-1.872E-02	-2.522E-02	-3.172E-02	-3.822E-02	-4.472E-02	-5.122E-02	-5.772E-02	0.0

THE FIELD OF THE RATIO OF THE ORT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.00380	0.00760	0.01140	0.01520	0.01900	0.02280	0.02660	0.03040	0.03420	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0042	0.0	2.408E-04	4.816E-04	7.224E-04	9.632E-04	1.204E-03	1.445E-03	1.686E-03	1.927E-03	2.168E-03	0.0
0.0083	0.0	1.428E-04	2.856E-04	4.284E-04	5.712E-04	7.140E-04	8.568E-04	1.000E-03	1.143E-03	1.286E-03	0.0
0.0128	0.0	1.499E-04	2.998E-04	4.497E-04	5.996E-04	7.495E-04	8.994E-04	1.049E-03	1.198E-03	1.347E-03	0.0
0.0166	0.0	1.048E-04	2.096E-04	3.144E-04	4.192E-04	5.240E-04	6.288E-04	7.336E-04	8.384E-04	9.432E-04	0.0
0.0209	0.0	8.802E-05	1.760E-04	2.640E-04	3.520E-04	4.400E-04	5.280E-04	6.160E-04	7.040E-04	7.920E-04	0.0
0.0249	0.0	2.798E-05	5.596E-05	8.394E-05	1.1192E-04	1.3984E-04	1.6776E-04	1.9568E-04	2.2360E-04	2.5152E-04	0.0
0.0291	0.0	-4.127E-05	-8.254E-05	-1.2381E-04	-1.6472E-04	-2.0563E-04	-2.4654E-04	-2.8745E-04	-3.2836E-04	-3.6927E-04	0.0
0.0332	0.0	-1.032E-05	-2.064E-05	-3.096E-05	-4.130E-05	-5.164E-05	-6.198E-05	-7.232E-05	-8.266E-05	-9.300E-05	0.0
0.0374	0.0	-1.017E-05	-2.034E-05	-3.051E-05	-4.068E-05	-5.085E-05	-6.102E-05	-7.119E-05	-8.136E-05	-9.153E-05	0.0
0.0416	0.0	-1.081E-05	-2.162E-05	-3.243E-05	-4.324E-05	-5.405E-05	-6.486E-05	-7.567E-05	-8.648E-05	-9.729E-05	0.0
0.0457	0.0	-1.081E-05	-2.162E-05	-3.243E-05	-4.324E-05	-5.405E-05	-6.486E-05	-7.567E-05	-8.648E-05	-9.729E-05	0.0
0.0499	0.0	-1.081E-05	-2.162E-05	-3.243E-05	-4.324E-05	-5.405E-05	-6.486E-05	-7.567E-05	-8.648E-05	-9.729E-05	0.0
0.0540	0.0	-1.081E-05	-2.162E-05	-3.243E-05	-4.324E-05	-5.405E-05	-6.486E-05	-7.567E-05	-8.648E-05	-9.729E-05	0.0
0.0582	0.0	-1.081E-05	-2.162E-05	-3.243E-05	-4.324E-05	-5.405E-05	-6.486E-05	-7.567E-05	-8.648E-05	-9.729E-05	0.0

THE FIELD OF THE RATIO OF THE CTP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0	0.00082	0.01122	0.04083	0.05255	0.09232	0.09677	0.09873	0.09961	1.00000
0.00042	1.0	0.9998-01	1.0002+00	0.9998-01	0.9798-01	0.9598-01	0.9398-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.00083	1.0	0.9996-01	1.0004+00	0.9996-01	0.9796-01	0.9596-01	0.9396-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.00166	1.0	0.9992-01	1.0008+00	0.9992-01	0.9792-01	0.9592-01	0.9392-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.00332	1.0	1.0000+00	1.0016+00	1.0000+00	0.9998-01	0.9996-01	0.9994-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.00664	1.0	1.0000+00	1.0032+00	1.0000+00	0.9996-01	0.9992-01	0.9988-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.01328	1.0	1.0000+00	1.0064+00	1.0000+00	0.9992-01	0.9984-01	0.9976-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.02656	1.0	1.0000+00	1.0128+00	1.0000+00	0.9984-01	0.9972-01	0.9960-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.05312	1.0	1.0000+00	1.0256+00	1.0000+00	0.9972-01	0.9956-01	0.9940-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.10624	1.0	1.0000+00	1.0512+00	1.0000+00	0.9956-01	0.9936-01	0.9916-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.21248	1.0	1.0000+00	1.1024+00	1.0000+00	0.9936-01	0.9912-01	0.9888-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.42496	1.0	1.0000+00	1.2048+00	1.0000+00	0.9912-01	0.9872-01	0.9832-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00
0.84992	1.0	1.0000+00	1.4096+00	1.0000+00	0.9872-01	0.9824-01	0.9776-01	1.0000+00	1.0000+00	1.0000+00	1.0000+00	1.0000+00

THE FIELD OF THE RATIO OF THE CTR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0	0.00082	0.01122	0.04083	0.05255	0.09232	0.09677	0.09873	0.09961	1.00000
0.00042	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00083	0.0	-1.3198-07	-2.4178-08	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00166	0.0	-1.9448-07	-3.6408-08	1.2838-08	0.4498-08	-1.6078-08	-1.3498-08	-1.3498-08	-3.3238-08	-2.8798-08	-1.8298-08	0.0
0.00332	0.0	-2.8898-07	-5.2818-08	1.8638-08	-3.3178-08	-3.7948-08	-3.7948-08	-3.7948-08	-6.4348-08	-6.4348-08	-6.4348-08	0.0
0.00664	0.0	-4.3348-07	-7.9228-08	2.3088-08	-4.8168-08	-5.6448-08	-5.6448-08	-5.6448-08	-1.1918-08	-1.1918-08	-1.1918-08	0.0
0.01328	0.0	-6.2348-07	-1.2318-07	2.9748-08	-6.3448-08	-7.8448-08	-7.8448-08	-7.8448-08	-1.2808-08	-1.2808-08	-1.2808-08	0.0
0.02656	0.0	-1.0018-06	-1.7738-08	3.7608-08	-8.1738-08	-1.0018-06	-1.0018-06	-1.0018-06	-1.8008-08	-1.8008-08	-1.8008-08	0.0
0.05312	0.0	-1.1868-06	-2.4188-08	4.8888-08	-1.1868-06	-1.1868-06	-1.1868-06	-1.1868-06	-2.4188-08	-2.4188-08	-2.4188-08	0.0
0.10624	0.0	-1.3338-06	-3.1278-08	6.2888-08	-1.3338-06	-1.3338-06	-1.3338-06	-1.3338-06	-3.1278-08	-3.1278-08	-3.1278-08	0.0
0.21248	0.0	-1.3118-06	-4.2218-08	8.1738-08	-1.3118-06	-1.3118-06	-1.3118-06	-1.3118-06	-4.2218-08	-4.2218-08	-4.2218-08	0.0
0.42496	0.0	-1.1898-06	-5.2728-08	1.2448-06	-1.1898-06	-1.1898-06	-1.1898-06	-1.1898-06	-5.2728-08	-5.2728-08	-5.2728-08	0.0
0.84992	0.0	-8.2408-07	-8.2408-07	3.2448-06	-8.2408-07	-8.2408-07	-8.2408-07	-8.2408-07	-8.2408-07	-8.2408-07	-8.2408-07	0.0

THE FIELD OF THE RATIO OF THE CTT ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0002	0.0112	0.0405	0.0928	0.1922	0.3967	0.6973	0.9961	1.0000
0.0042	0.0	-2.420E-04	-4.016E-06	-7.821E-08	-0.049E-04	-2.688E-04	-1.618E-03	7.632E-03	2.052E-04	1.087E-03	1.087E-03
0.0083	0.0	-1.804E-05	-2.892E-08	-4.632E-09	-8.732E-09	-5.308E-08	2.839E-08	6.941E-04	2.892E-03	6.187E-03	6.187E-03
0.0125	0.0	-2.814E-05	-4.832E-08	-7.752E-09	-1.412E-08	-2.904E-08	6.441E-08	1.237E-03	6.348E-03	9.940E-03	9.940E-03
0.0166	0.0	-2.988E-05	-8.092E-08	-1.322E-08	-2.132E-08	-1.084E-04	9.940E-08	1.132E-03	7.424E-03	1.139E-02	1.139E-02
0.0209	0.0	-2.890E-05	-8.842E-08	-1.982E-08	-3.072E-08	-1.638E-04	1.099E-04	2.099E-03	7.189E-03	1.099E-02	1.099E-02
0.0249	0.0	-2.342E-05	-4.848E-08	-7.832E-09	-2.610E-08	-1.728E-04	1.728E-03	3.831E-03	9.848E-03	9.848E-03	9.848E-03
0.0291	0.0	-1.492E-05	-2.847E-08	-4.848E-09	-8.692E-09	-8.848E-08	7.138E-04	1.044E-03	3.412E-03	9.848E-03	9.848E-03
0.0332	0.0	-3.044E-04	-7.322E-06	-1.182E-06	-1.412E-06	-1.618E-05	2.182E-05	2.182E-04	6.282E-04	1.642E-03	1.642E-03
0.0374	0.0	8.021E-05	1.872E-08	3.812E-08	2.742E-08	3.442E-08	-3.442E-08	-6.942E-04	-1.182E-03	-2.832E-03	-2.832E-03
0.0416	0.0	1.337E-05	3.762E-08	6.062E-08	7.872E-08	6.927E-08	6.927E-08	-1.820E-03	-4.914E-03	-8.682E-03	-8.682E-03
0.0457	0.0	2.832E-05	5.492E-08	8.792E-08	1.092E-04	1.024E-04	-1.024E-04	-2.182E-03	-7.127E-03	-1.087E-02	-1.087E-02
0.0499	0.0	3.289E-05	6.322E-08	1.014E-04	1.252E-04	1.198E-04	-1.198E-04	-3.182E-03	-6.171E-03	-1.280E-02	-1.280E-02
0.0540	0.0	3.088E-05	5.812E-08	9.442E-08	1.162E-04	1.188E-04	-1.188E-04	-2.031E-03	-7.832E-03	-1.874E-02	-1.874E-02
0.0582	0.0	1.992E-05	3.787E-08	6.032E-08	7.402E-08	7.172E-08	-6.874E-08	-1.218E-03	-4.774E-03	-8.608E-03	-8.608E-03
0.0622	0.0	4.991E-04	1.272E-06	1.482E-06	1.476E-06	1.476E-06	2.022E-06	1.309E-04	-7.947E-04	-2.667E-03	-2.667E-03

THE FIELD OF THE RATIO OF THE CPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.0002	0.0112	0.0405	0.0928	0.1922	0.3967	0.6973	0.9961	1.0000
0.0042	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0083	0.0	2.386E-06	7.742E-06	1.037E-05	1.218E-05	1.284E-05	1.288E-05	1.210E-05	7.042E-06	2.322E-06	-6.282E-06
0.0125	0.0	4.945E-06	1.592E-05	1.782E-05	2.032E-05	2.148E-05	2.184E-05	2.001E-05	1.126E-05	3.692E-06	-4.287E-06
0.0166	0.0	8.817E-06	1.820E-05	2.092E-05	2.392E-05	2.820E-05	2.839E-05	2.322E-05	1.308E-05	4.194E-06	-1.214E-06
0.0209	0.0	9.667E-06	1.422E-05	2.042E-05	2.332E-05	2.467E-05	2.479E-05	2.209E-05	1.247E-05	4.027E-06	-2.424E-06
0.0249	0.0	4.690E-06	1.217E-05	1.676E-05	1.912E-05	2.022E-05	2.032E-05	1.829E-05	1.030E-05	3.276E-06	-2.018E-06
0.0291	0.0	3.009E-06	7.717E-06	1.089E-05	1.207E-05	1.274E-05	1.288E-05	1.147E-05	6.991E-06	2.130E-06	-2.484E-06
0.0332	0.0	8.059E-07	2.047E-06	2.771E-06	3.124E-06	3.289E-06	3.284E-06	2.906E-06	2.041E-06	6.928E-07	-4.684E-06
0.0374	0.0	-1.477E-06	-4.082E-06	-5.728E-06	-6.882E-06	-6.882E-06	-6.942E-06	-4.392E-06	-2.442E-06	-6.991E-07	-7.827E-06
0.0416	0.0	-3.718E-06	-9.821E-06	-1.378E-05	-1.677E-05	-1.670E-05	-1.673E-05	-1.818E-05	-7.807E-06	-2.489E-06	-2.664E-06
0.0457	0.0	-6.809E-06	-1.484E-05	-2.011E-05	-2.297E-05	-2.430E-05	-2.430E-05	-2.214E-05	-1.211E-05	-2.888E-06	-7.416E-06
0.0499	0.0	-6.463E-06	-1.481E-05	-2.331E-05	-2.699E-05	-2.811E-05	-2.830E-05	-2.478E-05	-1.475E-05	-4.819E-06	-9.848E-06
0.0540	0.0	-6.172E-06	-1.892E-05	-2.184E-05	-2.486E-05	-2.628E-05	-2.642E-05	-2.331E-05	-1.452E-05	-4.928E-06	-2.664E-06
0.0582	0.0	-4.190E-06	-1.024E-05	-1.407E-05	-1.887E-05	-1.884E-05	-1.896E-05	-1.870E-05	-1.023E-05	-2.489E-06	-1.190E-06
0.0622	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 0.7337 DEGREES      REYNOLDS NUMBER = 2.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	2.8603	0.9996	.0045	0.0002	0.9825	.0033
DRT	-6.1328	0.9825	.0058	-0.0767	0.9987	.0017
DPR	-4.6528	0.9987	.0023	0.0000	0.5536	.0054
DTT	-2.8564	0.9996	.0046	-0.0006	0.5536	.0
DTP	114.5872	0.9996	.0	85.6474	1.0000	.0058
DPP	-0.0069	0.9825	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 0.3333 DEGREES REYNOLDS NUMBER = 2.08464

THE FIELD OF THE RATIO OF THE DRG ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.00081	0.01182	0.04083	0.09258	0.09677	0.09873	0.99961	1.00000
0.0	0.0	-1.0315-02	-2.7075-02	-3.7892-02	-4.8204-02	-4.4182-02	-3.4932-02	-2.4448-02	-8.2795-02	0.0
0.00042	0.0	-7.0485-03	-1.9415-02	-2.6322-02	-3.0222-02	-3.1742-02	-3.7375-02	-4.6702-02	-6.0944-02	0.0
0.00083	0.0	-4.3148-03	-1.1802-02	-1.6392-02	-1.7902-02	-1.8441-02	-1.8122-02	-1.6142-02	-4.6512-02	0.0
0.0125	0.0	-1.1111-03	-8.8432-03	-3.0102-02	-4.4082-02	-4.7441-02	-4.1902-02	-4.5018-02	-4.1818-02	0.0
0.0164	0.0	1.0992-03	4.8802-03	6.2532-02	7.8802-02	7.3841-02	6.4222-02	6.4722-02	-4.1092-02	0.0
0.0208	0.0	4.8282-03	1.0682-02	1.4372-02	1.6682-02	1.7452-02	1.6222-02	1.3142-02	-4.2172-02	0.0
0.0249	0.0	8.7832-03	1.8182-02	2.0602-02	2.3722-02	2.4482-02	2.3522-02	1.8442-02	-4.3292-02	0.0
0.0291	0.0	6.7472-03	1.7712-02	2.4722-02	2.7722-02	2.9122-02	2.8922-02	2.1682-02	-4.3292-02	0.0
0.0332	0.0	6.0902-03	1.8022-02	2.4722-02	2.8142-02	2.9692-02	2.9072-02	2.1682-02	-4.3292-02	0.0
0.0374	0.0	6.0602-03	1.8712-02	2.8422-02	2.8422-02	2.8702-02	2.8702-02	2.1682-02	-4.3292-02	0.0
0.0416	0.0	4.1332-03	1.8822-02	1.8342-02	1.8282-02	1.7112-02	1.7112-02	1.4102-02	-4.3292-02	0.0
0.0457	0.0	9.8602-04	2.0902-02	2.7422-02	3.0102-02	3.1802-02	3.4222-02	3.4222-02	-4.3292-02	0.0
0.0499	0.0	-2.4702-03	-0.8382-02	-1.3772-02	-1.6812-02	-1.6812-02	-1.6812-02	-1.6812-02	-4.3292-02	0.0
0.0540	0.0	-9.1882-03	-2.3922-02	-3.2742-02	-3.7172-02	-3.7172-02	-3.7172-02	-3.7172-02	-4.3292-02	0.0
0.0582	0.0	-1.8732-02	-3.8262-02	-5.1472-02	-5.8322-02	-6.1322-02	-6.1322-02	-6.1322-02	-4.3292-02	0.0

THE FIELD OF THE RATIO OF THE DRG ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0	0.00081	0.01182	0.04083	0.09258	0.09677	0.09873	0.99961	1.00000
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.00042	0.0	2.1472-02	6.2312-02	9.3182-02	1.0492-04	4.6472-02	-3.5922-02	-4.7842-02	-1.4662-02	0.0
0.00083	0.0	3.8672-02	1.0412-04	1.8082-04	1.7982-04	8.2722-02	-4.5322-02	-1.4402-02	-2.3222-02	-1.4412-02
0.0125	0.0	4.2172-02	1.2242-04	1.8392-04	1.8222-04	1.0922-04	-6.1942-04	-1.7012-02	-2.6222-02	-2.4892-02
0.0164	0.0	4.1122-02	1.1932-04	1.7982-04	2.0812-04	1.2242-04	-6.0672-04	-1.6362-02	-2.4422-02	-2.3102-02
0.0208	0.0	3.3762-02	4.7922-02	1.4722-04	1.7182-04	1.1772-04	-6.2392-04	-1.3192-02	-1.9422-02	-1.2422-02
0.0249	0.0	2.1432-02	8.1602-02	9.3022-02	1.0922-04	9.8042-02	-4.4212-04	-8.4942-02	-1.1492-02	-1.1482-02
0.0291	0.0	8.7642-02	1.6072-02	2.4192-02	2.9242-02	2.7822-02	-1.9202-02	-1.7722-02	-5.8222-02	-2.1222-02
0.0332	0.0	-1.1302-02	-3.3622-02	-8.0822-02	-8.7942-02	-1.8642-02	1.2392-02	4.9422-02	6.8722-02	8.8022-02
0.0374	0.0	-2.7512-02	-8.0722-02	-1.2182-04	-1.4072-04	-6.2122-02	4.3722-02	1.1312-02	1.8112-02	1.4942-02
0.0416	0.0	-4.0332-02	-1.1772-04	-1.7742-04	-2.0822-04	-1.2142-04	6.3922-02	1.4202-02	2.3922-02	2.8102-02
0.0457	0.0	-4.0942-02	-1.3642-04	-2.0922-04	-2.3842-04	-1.6182-04	7.9142-04	1.0572-02	2.6422-02	2.7142-02
0.0499	0.0	-4.4212-02	-1.8772-04	-1.9142-04	-2.2282-04	-1.6372-04	5.8222-02	1.7102-02	1.8602-02	2.7922-02
0.0540	0.0	-2.8732-02	-2.1492-02	-1.2192-04	-1.4102-04	-1.6842-04	3.4722-02	1.0982-02	1.8772-02	1.8162-02
0.0582	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0







GAP ANGLE = 0.3333 DEGREES      REYNOLDS NUMBER = 4.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-4.6370	0.9996	.0012	0.0012	0.5536	.0029
DRT	-12.0875	0.9825	.0053	0.2023	0.5536	.0046
DPR	-10.8573	0.9987	.0027	0.0002	0.5536	.0004
DTT	4.6306	0.9996	.0012	0.0005	0.9825	.0
DTP	127.7109	0.9996	.0	76.7968	1.0000	.0054
DPP	-0.0135	0.9825	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPP = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DPR = 0 AT BETA = 0.EPSLON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

THE FIELD OF THE RATIO OF THE DRY ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0536C	0.6008E	0.9112E	0.9608E	0.9828E	0.9927E	0.9987E	0.9996E	1.0000E
.0000E	0.0	-2.087E-02	-8.377E-02	-7.279E-02	-6.189E-02	-5.817E-02	-7.495E-02	-6.989E-02	-5.802E-02	-1.702E-02
.0001E	0.0	-1.488E-02	-3.899E-02	-5.823E-02	-5.679E-02	-5.182E-02	-5.639E-02	-4.875E-02	-3.177E-02	-1.178E-02
.0002E	0.0	-8.680E-03	-3.230E-02	-3.402E-02	-3.402E-02	-2.833E-02	-3.426E-02	-2.733E-02	-1.879E-02	-8.930E-03
.0003E	0.0	-2.273E-03	-8.687E-03	-7.762E-03	-8.826E-03	-9.374E-03	-1.124E-02	-7.987E-03	-4.274E-03	-7.793E-03
.0004E	0.0	3.382E-03	9.068E-03	1.518E-02	1.263E-02	1.384E-02	9.828E-03	5.823E-03	3.823E-03	-7.487E-03
.0005E	0.0	8.684E-03	3.113E-02	2.889E-02	3.208E-02	3.328E-02	2.828E-02	2.372E-02	6.143E-03	-7.148E-03
.0006E	0.0	1.810E-02	3.009E-02	4.076E-02	4.898E-02	4.768E-02	4.202E-02	3.802E-02	7.489E-03	-6.488E-03
.0007E	0.0	1.249E-02	3.881E-02	4.776E-02	5.388E-02	5.608E-02	3.719E-02	3.709E-02	7.809E-03	-6.878E-03
.0008E	0.0	1.377E-02	3.882E-02	4.776E-02	5.388E-02	5.608E-02	3.719E-02	3.709E-02	7.809E-03	-6.878E-03
.0009E	0.0	1.812E-02	3.123E-02	4.828E-02	4.818E-02	5.048E-02	5.029E-02	3.731E-02	2.197E-02	-3.810E-03
.0010E	0.0	2.023E-02	4.891E-02	5.771E-02	6.897E-02	7.049E-02	6.882E-02	3.819E-02	3.909E-02	-3.884E-03
.0011E	0.0	-1.884E-02	-4.782E-02	-2.460E-02	-3.028E-02	-3.177E-02	-3.229E-02	-3.819E-02	-2.982E-02	-1.883E-02
.0012E	0.0	-3.149E-02	-7.041E-02	-1.026E-01	-1.108E-01	-1.208E-01	-1.208E-01	-6.288E-02	-4.819E-02	-1.683E-02

THE FIELD OF THE RATIO OF THE CUR ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.0536C	0.6008E	0.9112E	0.9608E	0.9828E	0.9927E	0.9987E	0.9996E	1.0000E
.0000E	0.0	4.247E-05	1.518E-04	1.749E-04	1.936E-04	2.018E-04	-6.473E-04	-3.744E-03	-1.471E-03	0.0
.0001E	0.0	7.131E-05	2.035E-04	2.988E-04	3.247E-04	-3.082E-05	-1.842E-03	-7.433E-03	-2.497E-02	-2.428E-02
.0002E	0.0	8.387E-05	2.393E-04	3.486E-04	3.868E-04	2.829E-05	-1.972E-03	-1.023E-02	-2.937E-02	-4.378E-02
.0003E	0.0	8.182E-05	2.338E-04	3.411E-04	3.793E-04	1.123E-04	-2.074E-03	-1.181E-02	-2.900E-02	-3.982E-02
.0004E	0.0	4.272E-05	1.918E-04	2.816E-04	3.166E-04	1.904E-04	-1.834E-03	-1.093E-02	-2.398E-02	-2.688E-02
.0005E	0.0	4.272E-05	1.918E-04	2.816E-04	3.166E-04	1.904E-04	-1.834E-03	-1.093E-02	-2.398E-02	-2.688E-02
.0006E	0.0	1.162E-05	3.198E-05	5.881E-05	6.482E-05	2.222E-04	-4.918E-04	-4.283E-03	-4.017E-03	1.988E-03
.0007E	0.0	-8.231E-05	-6.427E-05	-9.190E-05	-9.422E-05	1.491E-04	4.232E-04	1.021E-03	7.876E-03	1.677E-02
.0008E	0.0	-8.460E-05	-1.649E-04	-2.879E-04	-2.490E-04	2.789E-05	1.281E-03	6.891E-03	1.930E-02	2.782E-02
.0009E	0.0	-8.020E-05	-2.202E-04	-3.387E-04	-3.744E-04	-1.129E-04	2.034E-03	1.123E-02	2.868E-02	3.987E-02
.0010E	0.0	-9.340E-05	-2.680E-04	-3.935E-04	-4.419E-04	-2.302E-04	2.398E-03	1.391E-02	2.330E-02	4.474E-02
.0011E	0.0	-8.813E-05	-2.817E-04	-3.793E-04	-4.182E-04	-2.789E-04	2.834E-03	1.340E-02	3.334E-02	4.812E-02
.0012E	0.0	-8.733E-05	-1.612E-04	-2.366E-04	-2.482E-04	-1.979E-04	1.429E-03	6.711E-03	2.808E-02	2.831E-02

THE FILED OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.05360	0.00082	0.01122	0.00083	0.00088	0.00077	0.00073	0.00061	1.00000
.00042	1.0	0.9995-01	0.9918-01	0.9318-01	0.7392-01	0.7280-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00083	1.0	0.9992-01	0.9832-01	0.8378-01	0.7500-01	0.7390-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00125	1.0	0.9988-01	0.9682-01	0.8432-01	0.7666-01	0.7500-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00166	1.0	0.9984-01	0.9608-01	0.8508-01	0.7900-01	0.7800-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00208	1.0	0.9978-01	0.9532-01	0.8582-01	0.8134-01	0.8000-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00249	1.0	0.9972-01	0.9458-01	0.8658-01	0.8366-01	0.8200-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00291	1.0	0.9968-01	0.9382-01	0.8732-01	0.8598-01	0.8400-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00332	1.0	0.9962-01	0.9308-01	0.8808-01	0.8866-01	0.8600-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00374	1.0	0.9958-01	0.9232-01	0.8882-01	0.9034-01	0.8700-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00416	1.0	0.9952-01	0.9158-01	0.8958-01	0.9182-01	0.8800-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00457	1.0	0.9948-01	0.9082-01	0.9032-01	0.9330-01	0.8900-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00499	1.0	0.9942-01	0.9008-01	0.9108-01	0.9578-01	0.9100-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00540	1.0	0.9938-01	0.8932-01	0.9182-01	0.9826-01	0.9200-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00
.00582	1.0	0.9932-01	0.8858-01	0.9258-01	1.0074-01	0.9300-01	1.0078+00	1.1308+00	1.2488+00	1.8778+00

THE FILED OF THE RATIO OF THE ODP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.05360	0.00082	0.01122	0.00083	0.00088	0.00077	0.00073	0.00061	1.00000
.00042	0.0	1.0018+00	0.9932-01	0.9332-01	0.7418-01	0.7306-01	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00083	0.0	0.9988-04	0.9828-04	0.8378-04	0.7500-04	0.7390-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00125	0.0	0.9984-04	0.9752-04	0.8432-04	0.7666-04	0.7500-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00166	0.0	0.9978-04	0.9678-04	0.8508-04	0.7900-04	0.7800-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00208	0.0	0.9972-04	0.9602-04	0.8582-04	0.8134-04	0.8000-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00249	0.0	0.9968-04	0.9526-04	0.8658-04	0.8366-04	0.8200-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00291	0.0	0.9962-04	0.9450-04	0.8732-04	0.8598-04	0.8400-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00332	0.0	0.9958-04	0.9374-04	0.8808-04	0.8866-04	0.8600-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00374	0.0	0.9952-04	0.9298-04	0.8882-04	0.9034-04	0.8700-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00416	0.0	0.9948-04	0.9222-04	0.8958-04	0.9182-04	0.8800-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00457	0.0	0.9942-04	0.9146-04	0.9032-04	0.9330-04	0.8900-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00499	0.0	0.9938-04	0.9070-04	0.9108-04	0.9578-04	0.9100-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00540	0.0	0.9932-04	0.9000-04	0.9182-04	0.9826-04	0.9200-04	0.0013-03	-1.7148-03	-3.4008-03	0.0
.00582	0.0	0.9928-04	0.8930-04	0.9258-04	1.0074-04	0.9300-04	0.0013-03	-1.7148-03	-3.4008-03	0.0

GAP ANGLE = 0.3333 DEGREES REYNOLDS NUMBER = 4.CE+CA  
THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA

	0.0	0.0	0.55360	0.60082	0.91122	0.95053	0.98222	0.99232	0.99677	0.99873	0.99961	1.00000
.00042	0.0	-1.1912E-05	-2.3992E-05	-2.6902E-05	-2.9171E-05	-3.1495E-05	-3.4855E-05	1.3852E-04	-2.1244E-04	1.7346E-03	7.0272E-03	7.0272E-03
.00043	0.0	-7.6382E-05	-1.4012E-04	-2.1712E-04	-3.0222E-04	-2.3833E-05	7.3877E-04	1.0862E-03	3.0962E-03	1.4742E-02	3.5432E-02	2.9432E-02
.00044	0.0	-1.2462E-04	-2.3402E-04	-3.6302E-04	-5.2222E-04	-7.7965E-05	1.8282E-03	7.3362E-03	1.0122E-02	3.4892E-02	4.482E-02	4.412E-02
.00122	0.0	-1.6442E-04	-2.7832E-04	-4.2762E-04	-6.0992E-04	-1.8782E-04	1.8382E-03	1.0382E-02	1.0122E-02	3.0472E-02	4.6712E-02	4.6232E-02
.00123	0.0	-1.8282E-04	-2.9882E-04	-4.1832E-04	-6.0192E-04	-2.4012E-04	1.9472E-03	1.142E-02	1.142E-02	3.8302E-02	5.572E-02	2.6742E-02
.00124	0.0	-1.1722E-04	-2.2102E-04	-3.4512E-04	-4.6932E-04	-2.6782E-04	1.7332E-03	1.0832E-02	1.0832E-02	3.3902E-02	2.7542E-02	1.3392E-02
.00249	0.0	-7.9392E-05	-1.4072E-04	-2.2092E-04	-3.6832E-04	-2.4482E-04	4.8002E-03	4.2312E-02	4.2312E-02	1.2562E-02	1.342E-02	-1.6182E-02
.00291	0.0	-1.9462E-04	-3.2372E-04	-6.1932E-04	-8.082E-04	-2.4482E-04	4.8002E-03	4.2312E-02	4.2312E-02	1.2562E-02	1.342E-02	-1.6182E-02
.00332	0.0	3.9772E-05	7.2482E-05	1.1242E-04	1.2922E-04	1.1772E-04	3.0382E-04	-6.9482E-04	-6.9482E-04	-7.082E-03	-1.6782E-02	-1.6742E-02
.00416	0.0	9.3772E-05	1.8012E-04	2.7932E-04	3.2412E-04	5.9912E-05	1.122E-02	1.122E-02	1.122E-02	3.6062E-02	2.5612E-02	-3.465E-02
.00457	0.0	1.0332E-04	3.0632E-04	4.8172E-04	6.7102E-04	3.7262E-04	4.7812E-03	1.372E-02	1.372E-02	-3.327E-02	-4.4682E-02	-4.4682E-02
.00499	0.0	1.0332E-04	3.0632E-04	4.8172E-04	6.7102E-04	4.1642E-04	4.7812E-03	1.372E-02	1.372E-02	-3.327E-02	-4.4682E-02	-4.4682E-02
.00540	0.0	9.9182E-05	1.8842E-04	2.8072E-04	3.4702E-04	2.8902E-04	-1.3432E-03	-8.6282E-03	-8.6282E-03	-3.002E-02	-3.732E-02	-3.732E-02
.00582	0.0	2.8132E-05	4.4912E-05	6.4262E-05	7.4892E-05	7.4892E-05	3.325E-04	-1.0062E-03	-1.0062E-03	-3.616E-02	-8.116E-02	-8.116E-02

THE FIELD OF THE RATIO OF THE CPP ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

BETA

	0.0	0.0	0.55360	0.60082	0.91122	0.95053	0.98222	0.99232	0.99677	0.99873	0.99961	1.00000
.00042	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.00043	0.0	1.872E-05	3.832E-05	5.199E-05	6.648E-05	6.082E-05	6.082E-05	5.339E-05	4.908E-05	3.735E-05	1.115E-05	-3.014E-05
.00044	0.0	2.467E-05	6.414E-05	8.681E-05	9.772E-05	1.017E-04	1.017E-04	9.368E-05	8.048E-05	6.082E-05	1.648E-05	-2.033E-05
.00122	0.0	2.849E-05	7.835E-05	1.021E-04	1.180E-04	1.198E-04	1.198E-04	1.131E-04	1.131E-04	1.287E-04	1.763E-04	-8.827E-04
.00123	0.0	2.849E-05	7.835E-05	1.021E-04	1.180E-04	1.172E-04	1.172E-04	1.120E-04	1.120E-04	1.692E-04	1.422E-04	-1.646E-04
.00249	0.0	2.849E-05	7.835E-05	1.021E-04	1.180E-04	9.232E-05	9.673E-05	9.562E-05	7.178E-05	3.183E-05	1.023E-05	-1.608E-05
.00291	0.0	1.872E-05	3.832E-05	5.199E-05	6.648E-05	6.193E-05	6.193E-05	4.390E-05	4.487E-05	1.212E-05	4.532E-05	-3.147E-05
.00332	0.0	4.386E-05	1.042E-04	1.426E-04	1.632E-04	1.748E-04	1.748E-04	2.219E-04	1.982E-04	-1.441E-04	-6.478E-04	-2.809E-04
.00416	0.0	-7.350E-06	-2.054E-05	-2.719E-05	-3.041E-05	-3.135E-05	-3.135E-05	-2.812E-05	-1.882E-05	-1.532E-05	-6.072E-05	-2.745E-05
.00457	0.0	-1.984E-05	-4.910E-05	-6.632E-05	-7.801E-05	-7.801E-05	-7.801E-05	-7.803E-05	-6.007E-05	-3.412E-05	-1.682E-05	-2.488E-05
.00499	0.0	-2.749E-05	-7.213E-05	-9.792E-05	-1.105E-04	-1.193E-04	-1.193E-04	-1.102E-04	-8.762E-05	-6.844E-05	-1.480E-05	-1.908E-05
.00540	0.0	-3.230E-05	-8.408E-05	-1.142E-04	-1.288E-04	-1.348E-04	-1.348E-04	-1.314E-04	-1.022E-04	-9.112E-05	-1.021E-05	-1.908E-05
.00582	0.0	-2.060E-05	-5.157E-05	-6.933E-05	-7.637E-05	-8.206E-05	-8.206E-05	-6.638E-05	-4.663E-05	-4.815E-05	-1.445E-05	-1.633E-05
.00583	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

GAP ANGLE = 0.3333 DEGREES

REYNOLDS NUMBER = 8.0E+04

THE EXTREME RATIOS IN PERCENT OF EACH OF THE ELEMENTS OF THE RATE  
OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE ARE

ELEMENT	RATIO WHERE THE ABS VAL OF THE RATIO IS A MAXIMUM (PERCENT)	LOCATION		RATIO WHERE THE ABS VAL OF THE RATIO IS A MINIMUM* (PERCENT)	LOCATION	
		R	BETA		R	BETA
DRR	-6.4916	0.9987	.0012	0.0023	0.5536	.0029
DRT	-27.7205	0.9974	.0058	-0.0541	0.9987	.0050
DPR	-18.6340	0.9988	.0021	0.0002	0.5536	.0004
DTT	6.4662	0.9987	.0012	-0.0024	0.5536	.0
DTP	197.3247	0.9996	.0	51.9341	0.9987	.0050
DPP	-0.0283	0.9924	.0046	-0.0000	1.0000	.0004

\* EXCLUDES AN ABSOLUTE VALUE OF ZERO  
 DRR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPP = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DPR = 0 AT BETA = 0. EPSILON AND AT R = 0  
 DRT = 0 AT R = 0.1  
 DTT = 0 AT R = 0  
 DTP IS NEVER ZERO

GAP ANGLE = 0.3332 DEGREES REYNOLDS NUMBER = 6.0E+04

THE FIELD OF THE RATIO OF THE CRT ELEMENT OF THE RATE OF DEFORMATION TEASER AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.05360	0.08082	0.09128	0.09883	0.09285	0.09238	0.09677	0.09873	0.09961	1.00000
.0	0.0	-1.143E-02	-1.072E-01	-1.449E-01	-1.629E-01	-1.982E-01	-6.599E-02	-1.044E-01	-2.526E-02	-2.362E-02	0.0
.0042	0.0	-2.961E-02	-7.682E-02	-1.040E-01	-1.167E-01	-1.188E-01	6.270E-02	-1.188E-01	-3.691E-02	-1.312E-02	0.0
.0083	0.0	-1.718E-02	-4.440E-02	-6.020E-02	-6.772E-02	-6.894E-02	-7.672E-02	-3.782E-02	2.229E-03	-6.492E-04	0.0
.0126	0.0	-4.936E-03	-1.143E-02	-1.889E-02	-1.777E-02	-2.178E-02	-3.682E-02	2.830E-02	2.128E-02	1.332E-02	0.0
.0164	0.0	6.729E-03	1.781E-02	3.395E-02	2.661E-02	2.140E-02	-1.801E-02	1.766E-02	2.144E-02	2.349E-02	0.0
.0209	0.0	1.408E-02	4.500E-02	5.479E-02	6.382E-02	6.928E-02	7.432E-02	6.143E-02	1.680E-02	2.641E-02	0.0
.0249	0.0	2.294E-02	5.988E-02	8.118E-02	9.099E-02	9.822E-02	1.032E-01	1.043E-01	5.392E-02	2.785E-02	0.0
.0291	0.0	2.693E-02	7.019E-02	9.019E-02	1.070E-01	1.082E-01	1.107E-01	1.048E-01	6.132E-02	2.342E-02	0.0
.0332	0.0	2.781E-02	7.149E-02	9.712E-02	1.132E-01	1.198E-01	1.298E-01	1.178E-01	6.475E-02	1.719E-02	0.0
.0374	0.0	2.421E-02	6.260E-02	8.519E-02	9.614E-02	1.034E-01	1.292E-01	1.292E-01	6.821E-02	1.149E-02	0.0
.0416	0.0	1.683E-02	4.212E-02	6.736E-02	6.482E-02	7.118E-02	9.389E-02	3.768E-02	6.832E-02	9.981E-02	0.0
.0457	0.0	3.468E-03	8.721E-03	1.179E-02	1.381E-02	1.792E-02	3.283E-02	4.462E-02	4.312E-02	1.621E-02	0.0
.0499	0.0	-1.783E-02	-3.692E-02	-6.037E-02	-6.037E-02	-6.037E-02	-6.037E-02	-4.462E-02	-4.412E-02	-1.876E-02	0.0
.0540	0.0	-3.701E-02	-8.821E-02	-1.292E-01	-1.457E-01	-1.820E-01	-1.685E-01	-2.109E-02	-6.614E-02	-4.276E-02	0.0
.0582	0.0	-6.290E-02	-1.677E-01	-2.082E-01	-2.306E-01	-2.428E-01	-2.172E-01	-1.012E-02	-1.139E-02	-6.619E-02	0.0

THE FIELD OF THE RATIO OF THE CRT ELEMENT OF THE RATE OF DEFORMATION TEASER AND THE PRIMARY DEFORMATION RATE IS

BETA	0.0	0.05360	0.08082	0.09122	0.09683	0.09285	0.09238	0.09677	0.09873	0.09961	1.00000
.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
.0042	0.0	2.422E-04	3.496E-04	2.971E-04	-1.456E-03	-1.062E-03	-1.062E-03	-3.848E-03	-4.261E-02	-4.341E-02	0.0
.0083	0.0	1.420E-04	4.051E-04	4.992E-04	-1.679E-03	-1.679E-03	-1.679E-03	-1.434E-02	-6.397E-02	-4.943E-02	0.0
.0126	0.0	1.670E-04	4.767E-04	5.911E-04	-2.194E-03	-2.194E-03	-2.194E-03	-1.810E-02	-6.492E-02	-6.492E-02	0.0
.0164	0.0	1.631E-04	4.652E-04	6.749E-04	-2.870E-04	-2.870E-04	-2.870E-04	-2.870E-04	-4.821E-02	-4.821E-02	0.0
.0209	0.0	1.341E-04	3.830E-04	5.978E-04	-1.142E-04	-1.142E-04	-1.142E-04	-1.142E-04	-2.040E-02	-2.040E-02	0.0
.0249	0.0	6.549E-05	2.440E-04	3.800E-04	4.006E-04	4.006E-04	4.006E-04	4.006E-04	1.082E-02	1.082E-02	0.0
.0291	0.0	2.748E-05	6.674E-05	1.032E-04	2.872E-04	2.872E-04	2.872E-04	2.872E-04	3.889E-02	3.889E-02	0.0
.0332	0.0	-4.417E-05	-1.270E-04	-1.792E-04	-3.888E-04	-3.888E-04	-3.888E-04	-3.888E-04	3.532E-02	3.532E-02	0.0
.0374	0.0	-1.096E-04	-3.119E-04	-4.506E-04	-3.987E-04	-3.987E-04	-3.987E-04	-3.987E-04	2.140E-02	2.140E-02	0.0
.0416	0.0	-1.898E-04	-4.567E-04	-6.482E-04	-7.443E-04	-7.443E-04	-7.443E-04	-7.443E-04	2.358E-02	2.358E-02	0.0
.0457	0.0	-1.898E-04	-6.346E-04	-7.816E-04	-3.213E-04	-3.213E-04	-3.213E-04	-3.213E-04	4.713E-02	4.713E-02	0.0
.0499	0.0	-1.740E-04	-8.027E-04	-7.369E-04	-8.227E-04	-8.227E-04	-8.227E-04	-8.227E-04	3.374E-02	3.374E-02	0.0
.0540	0.0	-1.146E-04	-3.222E-04	-4.713E-04	-2.178E-04	-2.178E-04	-2.178E-04	-2.178E-04	1.742E-02	1.742E-02	0.0
.0582	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	5.041E-03	7.073E-03	0.0





GMP ANGLE = 0.3333 DEGREES REYNOLDS NUMBER = 8.0E+04

THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with columns: BETA, G, R, C, and numerical values. Includes a vertical column of values on the right side.

THE FIELD OF THE RATIO OF THE DIV ELEMENT OF THE RATE OF DEFORMATION TENSOR AND THE PRIMARY DEFORMATION RATE IS

Table with columns: BETA, G, R, C, and numerical values. Includes a vertical column of values on the right side.