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The Segregated Approach to Predicting Viscous Compressible Fluid Flows

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ABSTRACT

The SIMPLE method of Patankar and Spalding and its variants such as SIMPLER, SIMPLEC and SIMPLEX are segregated methods for solving the discrete algebraic equations representing the equations of motion for an incompressible fluid flow. The present paper presents the extension of these methods to the solution of compressible fluid flows within the context of a generalized segregated approach. To provide a framework for better understanding the segregated approach to solving viscous compressible fluid flows an interpretation of the role of pressure in the numerical method is presented. With this interpretation it becomes evident that the linearization of the equation for mass conservation and the approach used to solve the linearized algebraic equations representing the equations of motion are important in determining the performance of the numerical method. The relative performance of the various segregated methods are compared for several subsonic and supersonic compressible fluid flows.

NOMENCLATURE

a_p, a_e, a_w, a_n, a_s	coefficients of pressure equation and algebraic representation of momentum and energy conservation
$\mathbf{A}$	matrix of a coefficients
$\bar{\mathbf{A}}$	approximation of $\mathbf{A}$
b	coefficient of pressure equation and algebraic representation of momentum and energy conservation
$\mathbf{b}$	vector of b coefficients
c	pressure coefficient
$\mathbf{c}$	vector of c coefficients
C_p	specific heat at constant pressure
$\mathbf{C}$	matrix of c coefficients
$\mathbf{C}_p$	matrix of pressure coefficients
d	coefficient of pressure difference influence
$\bar{d}$	approximation of d
$\bar{\mathbf{d}}$	vector of $\bar{d}$ coefficients
$\mathbf{D}_p$	matrix of pressure influence coefficients
$\bar{\mathbf{D}}_p$	approximation of $\mathbf{D}_p$
k	thermal conductivity
L	length
m_e, m_w, m_n, m_s	coefficients of algebraic representation of mass conservation
M	mass contained in control volume
$\dot{M}$	mass flux through control volume face
$\mathbf{M}$	matrix of m coefficients
N	number of control volumes in length L
p	pressure
p^*	estimate of pressure
$\bar{p}$	improved estimate of pressure
$\bar{\bar{p}}$	improved estimate of pressure of PUP
p'	pressure correction
p''	pressure correction of PUP
Re	Reynold's number
u, v	velocity components in x and y directions
u^*, v^*	velocities based on p^*
u^{**}, v^{**}	intermediate velocities of PUP
$\bar{u}$, $\bar{v}$	improved estimate of velocities
$\bar{\bar{u}}$	improved estimate of velocity of PUP
u', v'	velocity corrections
u''	velocity corrections of PUP
$\hat{u}$	pseudo-velocity of PUP
V	characteristic velocity
w	width of one-dimensional duct
x, y	Cartesian coordinates
Δt	time step
α_p	under-relaxation factor for pressure correction
ρ	density
ρ^*	density based on p^*
ρ^{**}	intermediate density of PUP
$\bar{\rho}$	improved estimate of density
$\bar{\bar{\rho}}$	improved estimate of density of PUP
ρ'	density correction
ρ''	density correction of PUP
$\hat{\rho}$	pseudo-density of PUP
μ	viscosity

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Subscripts

E, W, N, S, P	grid points
nb	neighbour grid point

Acronyms

PUP	Pressure Update of Patankar
SIMPLE	Semi-IMplicit Pressure Linked Equations
SIMPLER	SIMPLE-Revised
SIMPLEC	SIMPLE-Consistent approximation
SIMPLEX	SIMPLE-eXtrapolated pressure gradients

INTRODUCTION

Many problems of practical interest, such as the analysis of gas turbine flows, require the solution of the equations of motion for a viscous compressible fluid flow. Over the last two decades the numerical methods for treating such problems have evolved rapidly. However, many of the methods which can be used for the prediction of compressible flows, such as those found in references [1-4], are not appropriate in the low Mach number limit and not at all applicable to incompressible flows. Another class of methods used extensively for the prediction of viscous incompressible fluid flows and which has been extended to compressible fluid flows is based on the SIMPLE algorithm of Patankar and Spalding [5]. The extension of SIMPLE-based methods to compressible flows was first proposed by Patankar [6] and later by Issa and Lockwood [7] and Hah [8]. While these segregated methods are used, they are based on the extension of the original SIMPLE algorithm which has been, since it was first introduced, the subject of considerable study and enhancement. As a result a number of improved variants of the original SIMPLE algorithm including SIMPLER [9], SIMPLEC [10] and SIMPLEX [11] have been developed. These enhanced variants have not been extended to compressible flows. In the present paper, the extension of these methods to compressible flows is advanced and an understanding of their limitations and knowledge of their relative computational performance is developed.

To provide a framework for understanding the segregated approach to solving viscous compressible fluid flows an interpretation of the role of pressure in the numerical method is presented. With this interpretation of the role of pressure it becomes evident that the linearization of the equation representing mass conservation and the solution method used to solve the linear algebraic equations representing the equations of motion are important in determining the applicability and the computational performance of the method.

To provide a clear understanding of the segregated approach to solving compressible fluid flows, a generalized segregated approach is advanced and the extension of the segregated methods for incompressible flows to compressible flows is presented within this generalized framework. Also, in the interest of clarity, the concepts outlined above are presented within the context of one-dimensional duct flow. Fortunately all of the concepts developed within this one-dimensional context are readily extended to two or three dimensions. To demonstrate the latter and to evaluate the relative performance of the various segregated methods, the results of a number of numerical experiments including one-dimensional flow in a duct and the steady state two-dimensional supersonic flow around a flat plate obstacle are reported. The numerical experiments were designed to determine the sensitivity of the computational requirements of each of the segregated methods to the size of time step chosen. A simple and stable representation of the equations of motion was used throughout this study; the question of discretization accuracy was not addressed.

EQUATIONS OF MOTION

Differential Equations

The differential equations expressing the conservation of mass, momentum and energy for a laminar one-dimensional viscous compressible flow of a perfect gas can be expressed as

$$w\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u w)=0 \tag{1}$$

$$w\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u w u)=\frac{\partial}{\partial x}(w\mu\frac{\partial u}{\partial x}) -w\frac{\partial p}{\partial x}+\frac{\mu}{3}\frac{\partial}{\partial x}(\frac{\partial}{\partial x}(uw)) \tag{2}$$

$$w\frac{\partial}{\partial t}(\rho T)+\frac{\partial}{\partial x}(\rho u w T)=\frac{\partial}{\partial x}(w\frac{k}{C_p}\frac{\partial T}{\partial x}) +\frac{1}{C_p}(w\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}(uwp)-p\frac{\partial}{\partial x}(uw)) \tag{3}$$

where the dependent variables, u, p, ρ and T are the velocity, pressure, density and temperature, respectively, μ is the viscosity, C_p is the specific heat at constant pressure, k is the thermal conductivity and w is the the width of the duct.

To close the set of equations an equation of state, relating pressure, density and temperature, is required. For an ideal perfect gas this relation is given by

$$\rho=\frac{p}{RT} \tag{4}$$

where R is the gas constant.

Algebraic Equations

To solve the equations of motion a grid is generated to cover the domain of interest and any one of a number of discretization methods applied. In this paper the staggered grid of Harlow and Welch [12], shown in Fig. 1, and the control volume based discretization described by Patankar [9] with the upstream weighted variant of Raithby and Torrance [13] are applied. Also, the recommendation of Issa and Lockwood [7] is adopted whereby densities are upwinded in the discretization of the mass conservation equation. As a result the algebraic equations expressing the conservation of mass momentum and energy for the control volumes shown in Fig. 1 can be expressed as

$$\frac{M-M^o}{\Delta t}=\dot{M}_e-\dot{M}_w \tag{5}$$

$$a_p^u u_P=\sum a_{nb}^u u_{nb}-c^u(p_E-p_P)+b^u \tag{6}$$

$$a_p^T T_P=\sum a_{nb}^T T_{nb}+b^T \tag{7}$$

where

$$\sum a_{nb}^u u_{nb}=a_e^u u_E+a_w^u u_W$$
$$\sum a_{nb}^T T_{nb}=a_e^T T_E+a_w^T T_W$$

and where M is the mass contained in the control volume, the superscript o is to denote the value at the beginning of the time step and $\dot{M}$ is the mass flux through the control volume face indicated by the corresponding subscript. The lower case subscript e refers to the location of the control volume face that lies between P and E, and w to the face between P and W. To close the algebraic set of equations the equation of state can be expressed as

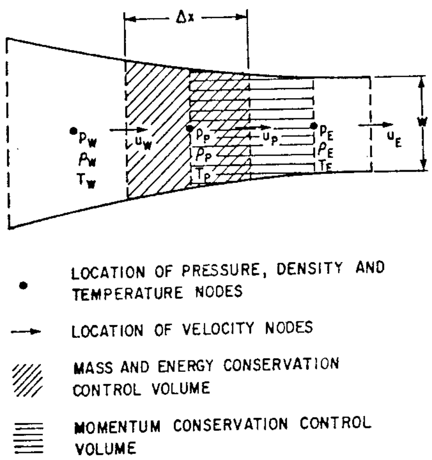

Fig. 1 Geometry and staggered arrangement of mesh nodes and control volumes for flow in a one-dimensional variable area duct.

$$\rho_P = c^{\rho} p_P + b^{\rho} \tag{8}$$

The solution of Eqs. (5)-(8), together with the algebraic representation of the prescribed boundary conditions, advances the solution over a time step Δt. The remainder of this paper is addressed to methods used to obtain this solution.

An Interpretation of the Role of Pressure

Before examing in detail the segregated approach to solving compressible fluid flows it is instructive to consider one interpretation of the role of pressure in a segregated approach.

The treatment of the coupling between pressure and velocity in the solution of *incompressible* fluid flows has in the past been particularly troublesome. This difficulty arises because only velocity and not pressure appears in the mass conservation equation. The mass conservation equation can be interpreted in this case as an indirect constraint equation for pressure whereby the correct pressure distribution is identified as that which, when used in the momentum equations, results in velocities which conserve mass.

For *compressible* fluid flows both velocity and density appear as dependent variables in the mass conservation equation. Nevertheless, the algebraic representation of mass conservation can still be interpreted as a constraint equation for pressure and the modified interpretation of the role of pressure for the segregated approach to solving compressible flows becomes that the pressure must influence the velocity through the momentum conservation and the density through the equation of state such that together the resulting velocities and the resulting densities conserve mass.

Nonlinearity

The form of Eqs. (5)-(8) implies that these equations are linear. In fact the coefficients of these equations themselves depend on the dependent variables. To obtain solutions to these nonlinear equations, linearization and iteration are required.

For Eqs. (6)-(8) the linearization adopted is the one implied by the form of these equations whereby the current available estimates of dependent variables are used to evaluate the coefficients. However, the linearized algebraic representation of mass conservation given by Eq. (5) is not appropriate because the desired dependent variables do not explicitly appear. To remedy this, it is necessary that the coefficients of Eq. (5) be linearized in terms of density and the components of velocity. This choice is motivated by the interpretation of the role of pressure described previously, that pressure must influence velocity and density such that both velocity and density together conserve mass.

To accomplish this the mass flux term, $\dot{M}_e$, is approximated by

$$\dot{M}_e = \left\{ \rho u w \right\}_e \approx (\rho u^* w)_e + [\rho^* u w]_e - \left\{ \rho^* u^* w \right\}_e \tag{9}$$

where the superscript * is used to denote that current estimates are to be used to evaluate the corresponding dependent variable. As a result, the mass flux term enclosed in the ( ) brackets is linearized in terms of density and the mass flux term enclosed in the [ ] brackets is linearized in terms of velocity. With ρ_e taken as the upstream nodal value of density, with $\dot{M}_w$ linearized in a similar fashion and upon expanding the control volume mass, M, in terms of density, Eq. (5) can be represented by

$$m_p^{\rho} \rho_P + m_e^{\rho} \rho_E + m_w^{\rho} \rho_W + m_e^{u} u_P + m_w^{u} u_W = b^c \tag{10}$$

where current estimates of velocity and density are used to evaluate the coefficients of Eq. (10).

The linearization of the mass conservation equation described above, in terms of both velocity and density, has been adopted to ensure that the linearization is applicable to both incompressible as well as compressible flows. To linearize mass conservation in terms of velocity alone restricts the application of the resulting solution method to low Mach number and incompressible flows and linearization in terms of density alone restricts applications to compressible flows with very small time steps required in the low Mach number limit to maintain stability. The linearization of mass conservation resulting in Eq. (10) is also a generalization of the linearization of mass conservation suggested by Harlow and Amsden [16] for the ICE method as well as the linearization implied by Patankar [6] in the extension of the SIMPLE approach to compressible flows.

With regards to the linearized algebraic representation of the equation of state, again, the form of Eq. (8) is motivated by the interpretation of the role of pressure already discussed. For the linearization of the algebraic representation of energy conservation given by Eq. (7), the assumption is made that the couplings between temperature and velocity as well as between temperature and pressure are not dominant so that in the linear algebraic set of equations the energy equation is decoupled from the mass, momentum and state equations. This allows the focus of the present work to centre on the remaining linear equations for pressure, velocity and density. There are, of course, flows for which such a segregation of the energy equation is inappropriate.

SOLUTION OF LINEAR EQUATIONS FOR PRESSURE, VELOCITY AND DENSITY

To advance the solution of the equations for mass, momentum and energy conservation and the equation of state over a time step, Eqs. (6)-(8) and (10) are solved, the coefficients updated, and the sequence repeated until the effect of the non-linearities have been adequately treated. Each repetition of the sequence is defined here as a cycle. For small time steps one cycle is adequate while for large time steps several cycles may be required. Because the coupling with temperature has been assumed to be weak only the solution of the linearized algebraic representations of mass conservation, Eq. (10), momentum conservation, Eq. (6) and the equation of state, Eq. (8), for pressure, velocity and density are examined in detail further.

The development of a clear understanding of the segregated approach to solving Eq. (6), (8) and (10) for compressible flows begins by considering a direct non-iterative solution method. Based on this direct method, the framework of a generalized segregated approach is advanced. Different approximations introduced into the generalized approach lead directly to segregated methods for compressible flows which are extensions of methods developed originally for incompressible flows.

Direct Solution

For one-dimensional compressible flow in a duct, Eqs. (6), (8) and (10) can be rewritten in matrix notation as

$$\mathbf{A}^u\mathbf{u}+\mathbf{C}_p^u\mathbf{p}=\mathbf{b}^u \tag{11}$$

$$\rho=\mathbf{C}^\rho\mathbf{p}+\mathbf{b}^\rho \tag{12}$$

$$\mathbf{M}^\rho\rho+\mathbf{M}^u\mathbf{u}=\mathbf{b}^c \tag{13}$$

Extending the method introduced by Watson [14] and further developed by Zedan and Schneider [15] for incompressible flows, Eq. (11) is rewritten as

$$\mathbf{u}=(\mathbf{A}^u)^{-1}\mathbf{b}^u-\mathbf{D}_p^u\mathbf{p} \tag{14}$$

where $\mathbf{D}_p^u=(\mathbf{A}^u)^{-1}\mathbf{C}_p^u$, and substituting for $\mathbf{u}$ from Eq. (14) and for ρ from Eq. (12) into Eq. (13) the resulting equation for pressure is given by

$$\mathbf{A}^p\mathbf{p}=\mathbf{b}^p \tag{15}$$

where

$$\mathbf{A}^p=\mathbf{M}^\rho\mathbf{C}^\rho-\mathbf{M}^u\mathbf{D}_p^u \tag{16}$$

$$\mathbf{b}^p=-\mathbf{M}^\rho\mathbf{C}^\rho-\mathbf{M}^u(\mathbf{A}^u)^{-1}\mathbf{b}^u \tag{17}$$

The exact solution to Eqs (11)-(13) can therefore be achieved by solving Eq. (15) for $\mathbf{p}$, followed by a direct substitution into Eqs. (14) and (12) to obtain $\mathbf{u}$ and ρ.

This method for solving the linear equations clearly illustrates how the pressure influences velocity through Eq. (14) and density through Eq. (12) such that together velocity and density satisfy the algebraic representation of mass conservation. Also, this method for solving the linear equations for pressure, velocity and density is relatively straightforward and readily implemented numerically. The computational requirements, in particular, for the evaluation of $\mathbf{D}_p^u$ and the storage of $\mathbf{A}^p$ are considerable. To reduce these computational requirements the iterative segregated approach is adopted where approximations to $\mathbf{D}_p^u$ are introduced.

Generalized Segregated Approach

Given an estimate of pressure, denoted by $\mathbf{p}^*$, the corresponding velocity, $\mathbf{u}^*$, which satisfies the algebraic representation of momentum conservation and the corresponding density, ρ^*, from the algebraic representation of the equation of state, are given by

$$\mathbf{u}^*=(\mathbf{A}^u)^{-1}(\mathbf{b}^u-\mathbf{C}_p^u\mathbf{p}^*) \tag{18a}$$

or,

$$\mathbf{u}^*=(\mathbf{A}^u)^{-1}\mathbf{b}^u-\mathbf{D}_p^u\mathbf{p}^* \tag{18b}$$

and

$$\rho^*=\mathbf{C}^\rho\mathbf{p}^*+\mathbf{b}^\rho \tag{19}$$

Since the $\mathbf{p}^*$ is not in general correct, the $\mathbf{u}^*$ velocity and ρ^* density will not together satisfy mass conservation. To improve the estimate of the $\mathbf{u}^*$ velocity and ρ^* density it is necessary to subtract out the effect of $\mathbf{p}^*$ on $\mathbf{u}$ and ρ and add in the effect of an improved pressure estimate. Approximating the influence of $\mathbf{p}^*$ on $\mathbf{u}^*$ by $-\overline{\mathbf{D}}_p^u\mathbf{p}^*$ and the influence of the improved pressure estimate, $\overline{\mathbf{p}}$, by $-\overline{\mathbf{D}}_p^u\mathbf{p}^*$, the improved estimate of velocity, $\overline{u}$, is given by

$$\overline{\mathbf{u}}=\mathbf{u}^*+\overline{\mathbf{D}}_p^u\mathbf{p}^*-\overline{\mathbf{D}}_p^u\overline{\mathbf{p}} \tag{20a}$$

or,

$$\overline{\mathbf{u}}=\mathbf{u}^*-\overline{\mathbf{D}}_p^u(\overline{\mathbf{p}}-\mathbf{p}^*) \tag{20b}$$

Without further approximation, the improved estimate of density, $\overline{\rho}$, is given by

$$\overline{\rho}=\rho^*-\mathbf{C}^\rho\mathbf{p}^*+\mathbf{C}^\rho\overline{\mathbf{p}} \tag{21a}$$

or,

$$\overline{\rho}=\rho^*+\mathbf{C}^\rho(\overline{\mathbf{p}}-\mathbf{p}^*) \tag{21b}$$

where $\mathbf{C}^\rho\mathbf{p}^*$ and $\mathbf{C}^\rho\overline{\mathbf{p}}$ represent the influence of $\mathbf{p}^*$ and $\overline{\mathbf{p}}$, respectively, on the density.

By requiring that the $\overline{\mathbf{u}}$ velocity and $\overline{\rho}$ density satisfy mass conservation, given by Eq. (13), the following equation for $\overline{\mathbf{p}}$ results.

$$\overline{\mathbf{A}}^p\overline{\mathbf{p}}=\overline{\mathbf{b}}^p \tag{22}$$

where

$$\overline{\mathbf{A}}^p=\mathbf{M}^\rho\mathbf{C}^\rho-\mathbf{M}^u\overline{\mathbf{D}}_p^u \tag{23}$$

$$\overline{\mathbf{b}}^p=-\mathbf{M}^\rho(\rho^*-\mathbf{C}^\rho\mathbf{p}^*)-\mathbf{M}^u(\mathbf{u}^*+\overline{\mathbf{D}}_p^u\mathbf{p}^*) \tag{24}$$

Solving Eq. (22) for $\overline{\mathbf{p}}$ the solution for $\overline{\mathbf{u}}$ and $\overline{\rho}$ are readily determined from Eqs. (20) and (21), respectively.

It is important to note that if $\overline{\mathbf{D}}_p^u$ is chosen to be $\mathbf{D}_p^u$ exactly, then $\overline{\mathbf{p}}$, $\overline{\mathbf{u}}$ and $\overline{\rho}$ will satisfy Eqs. (11), (12) and (13) exactly. However, because an approximate evaluation of D_p^u is used to obtain $\overline{\mathbf{p}}$,$\overline{\mathbf{u}}$ and $\overline{\rho}$, these solutions will not satisfy Eqs. (11)-(13) exactly unless the choice for $\mathbf{p}^*$ happened to be correct.

Using the approach described above the solutions of Eqs. (11)-(13) can be determined from repeated application of Eqs. (18)-(24) with $\mathbf{p}^*$ to be taken from the previous value of $\overline{\mathbf{p}}$. With an appropriate choice for $\overline{\mathbf{D}}_p^u$ this iterative method will converge but it is not clear that the computational requirements associated with this iterative method would be any less than the requirements of a direct method. However, if $\overline{\mathbf{D}}_p^u$ is a good approximation of $\mathbf{D}_p^u$, then the values of $\overline{\mathbf{p}}$, $\overline{\mathbf{u}}$ and $\overline{\rho}$ after one iteration will adequately represent $\mathbf{u}$, $\mathbf{p}$ and ρ. As a result, the computational requirements of the iterative method may be considerably less than that of a direct method.

The resulting segregated approach is implemented by executing the following sequence of steps:

(1) Guess a pressure field, $\mathbf{p}^*$.

(2) Evaluate the coefficients of the momentum conservation equation and the equation of state, Eqs. (11) and (12) and solve for $\mathbf{u}^*$ and ρ^* using $\mathbf{p}^*$.

(3) Evaluate the coefficients of the mass conservation equation, Eq. (13), and the pressure equation, Eq. (15) and solve for $\overline{\mathbf{p}}$.

(4) Evaluate the improved estimate for velocity, $\overline{\mathbf{u}}$, and density, $\overline{\rho}$, from Eqs. (20) and (21), respectively.

(5) Evaluate the coefficients of the energy conservation equation, Eq. (9), and solve for the temperature, T.

(6) Using the $\overline{\mathbf{p}}$ found in step 3 as the new $\mathbf{p}^*$, return to step 2. Repeat this cycle, until the desired convergence is achieved, to obtain the solution for $\mathbf{p}$, $\mathbf{u}$, ρ and T at the end of the time step.

(7) Repeat steps 1 to 6 for each time step until the solution at the prescribed time, or steady state conditions are obtained.

In the approach described above the solutions of the tentative velocity, $\mathbf{u}^*$ the tentative density ρ^*, the approximate pressure, $\overline{\mathbf{p}}$, and the temperature, T, are determined separately, in an uncoupled manner. This approach to solving the coupled equations can appropriately described as segregated. In fact, without prescribing how $\overline{\mathbf{D}}_p^u$ is to be evaluated, the method is the compressible flow extension of the generalized segregated approach to solving incompressible fluid flows described previously [11].

It is evident that the performance of a segregated method is dependent on the approximations introduced to determine $\mathbf{D}_p^u$. With a detailed discussion of this consideration for incompressible flows already presented by Van Doormaal and Raithby [11], only a summary of the discussion is presented here.

On the Structure of $\mathbf{D}_p^u$

In an effort to ensure that the computational requirements of the segregated method are minimized a desirable structure for $\mathbf{D}_p^u$ arises from relating each nodal velocity, through $\mathbf{D}_p^u$, to only the two nodal pressures that stagger the velocity node. In this case the $\mathbf{D}_p^u$ matrix would have zero entries everywhere except along two diagonals.

There are at least two advantages to this form of $\mathbf{D}_p^u$. The first is that the diagonal entries of $\mathbf{D}_p^u$ may be readily evaluated, thereby keeping the computational requirements low. Secondly, the computational storage requirements of $\mathbf{A}^p$ from Eq.(23) are minimal. It is because of these advantages that most segregated methods adopt this simple form of $\mathbf{D}_p^u$. However, there are at least two major shortcomings of this practice.

The first shortcoming is that for high Reynolds number flows and with relatively high values of the time step it has been shown [11] that nodal pressures which are physically distant from a nodal velocity can have a significant influence on the velocity. For such cases the simple form of $\mathbf{D}_p^u$ is not appropriate. Only for smaller time steps does this simple form become appropriate [11]. The second shortcoming of this simple form of $\mathbf{D}_p^u$ is that, without taking special care, the convergence of a segregated method often degrades significantly with grid refinement.

With a knowledge, then, of these potential limitations of segregated methods, these methods can be used in an appropriate fashion. In the next section various approximations to $\mathbf{D}_p^u$ leading to the compressible flow extension of SIMPLE, SIMPLER, SIMPLEC and SIMPLEX are reviewed. Subsequently, the results of numerical experiments designed to evaluate the relative performance of these methods are presented.

EXAMPLES OF THE SEGREGATED APPROACH

The preceeding description of the generalized segregated approach is given in terms of the dependent variables, pressure, velocity and density using matrix notation, with the solution of the temperature, determined from the conservation of energy, being completely decoupled from the solution of the remaining dependent variables. In what follows the segregated approach is reformulated and cast in terms of corrections to the nodal values of pressure, velocity and density. Although the two descriptions are algebraically equivalent, the latter is employed because it is the form most commonly implemented in a computer code. Also, in the interest of minimizing the effects of computer round-off, use of the correction form is recommended [10].

Segregated Approach in Terms of Nodal Values

Given an estimate for pressure, p^*, the nodal values for the u^* velocities and ρ^* densities are determined from

$$a_p^u u^*_P = \sum a_{nb}^u u^*_{nb} - c^u(p^*_E - p^*_P) + b^u \tag{25}$$

and

$$\rho^*_P = c^\rho p^*_P + b^\rho \tag{26}$$

Introducing corrections to the p^* pressure and u^* velocity denoted by p' and u' such that

$$p' = \bar{p} - p^* \tag{27}$$

$$u' = \bar{u} - u^* \tag{28}$$

approximating that

$$u'_P = -\bar{d}^u(p'_E - p'_P) \tag{29}$$

and combining with the definition of u', the $\bar{u}$ velocity is given by

$$\bar{u}_P = u^*_P - \bar{d}^u(p'_E - p'_P) \tag{30}$$

Similarly, introducing a correction to the ρ^* density denoted by ρ' such that

$$\rho' = \bar{\rho} - \rho^* \tag{31}$$

the $\bar{\rho}$ density is given by

$$\bar{\rho}_P = \rho^*_P + c^\rho p'_P \tag{32}$$

By requiring that the $\bar{u}$ velocities and $\bar{\rho}$ densities satisfy mass conservation, that is

$$m_p^\rho \bar{\rho}_P + m_e^\rho \bar{\rho}_E m_w^\rho \bar{\rho}_W + m_e^u \bar{u}_P + m_w^u \bar{u}_W = b^c \tag{33}$$

the following equation for p' results:

$$a_p^p p'_P = a_e^p p'_E + a_w^p p'_E + b^{p'} \tag{34}$$

where

$$a_p^p = m_p^\rho c^\rho + m_e^u \bar{d}^u + m_w^u [\bar{d}^u]_W \tag{35}$$

$$a_e^p = -m_e^\rho [c^\rho]_E + m_e^u \bar{d}^u \tag{36}$$

$$a_w^p = -m_w^\rho [c^\rho]_W - m_w^u [\bar{d}^u]_W \tag{37}$$

$$b^{p'} = b^c - m_p^\rho \rho^*_P - m_e^\rho \rho^*_E - m_w^\rho \rho^*_W - m_e^u u^*_P - m_w^u u^*_W \tag{38}$$

and where the subscripts on the coefficients enclosed by the [ ] brackets refer to coefficients written for the equation of the corresponding subscript. For terms not enclosed by the [ ] brackets the P-control volume is implied. Upon solving Eq. (34) for p', $\bar{p}$ is obtained from

$$\bar{p} = p^* + p' \ , \tag{39}$$

$\bar{u}$ is obtained from Eq. (30) and $\bar{\rho}$ from Eq. (32).

At this point it is instructive to note the algebraic equivalence of Eqs.(18) and (25), Eqs. (19) and (26), Eqs. (20b) and (30), Eqs. (21b) and (32) and Eqs. (22) and (34). In the case of Eqs. (30), (32) and (34) which are expressed in terms of p', Eq. (39) provides the simple linear transformation to $\bar{p}$ which appears in Eqs. (20b), (21b) and (22). In noting the equivalence of Eqs. (22) and (34) it is worthy to note that the approximate form of $\mathbf{D}_p^u$ results in a tridiagonal matrix for A^p thereby minimizing the computational storage requirements.

Four Segregated Methods

Using the generalized segregated approach described above, a number of segregated methods can be generated by introducing different approximations to evaluate the $\bar{d}^u$ coefficient of Eqs. (29) and (30). The evaluation of $\bar{d}^u$ and the approximation introduced for several segregated methods including SIMPLE, SIMPLER, SIMPLEC and SIMPLEX, all proposed originally for incompressible flows, are listed in Table 1.

It is important to note here that the evaluations of $\bar{d}^u$ distinguish the segregated methods from one another, and that the same evaluations of $\bar{d}^u$ used for incompressible flows are applicable to compressible flows. The major difference between segregated methods for incompressible flows and segregated methods for compressible flows is the additional consideration in compressible flows of the variations in density in the mass conservation equation.

SIMPLE In the original form of SIMPLE [5], the approximations which are introduced to evaluate $\bar{d}^u$ often result in an overestimation of the magnitude of p' which in turn leads to slow convergence or to cause divergence of the

TABLE 1

EVALUATIONS OF $\bar{d}^u$

Method	Approximation	$\bar{d}^u$
SIMPLE	$\sum a^u_{nb} u'_{nb} = 0$	c^u / a^u_p
SIMPLER	$\sum a^u_{nb} u'_{nb} = 0$	c^u / a^u_p
SIMPLEC	$\sum a^u_{nb}(u'_{nb} - u'_P) = 0$	$c^u / (a^u_p - \sum a^u_{nb})$
SIMPLEX	$\Delta p_{ALL} = \Delta p_P$ for u_P	$a^u_p \bar{d}^u_P = \sum a^u_{nb} \bar{d}^u_{nb} + c^u$

method. To remedy this, for incompressible flows, the correction of pressure is under-relaxed by

$$\bar{p} = p^* + \alpha_p p' \quad (40)$$

where Patankar [17] recommends $\alpha_p \approx 0.8$. However, for compressible flows Eq. (40) is not necessarily appropriate since the pressure now influences both velocity and density. Therefore, for compressible flows a value of unity for α_p is used.

SIMPLER In place of under-relaxing p' Patankar [9,17] in his SIMPLER method for incompressible flows introduces the "Pressure Update of Patankar" (PUP) as a second stage to SIMPLE. For compressible flows a similar update of pressure can be adopted. By introducing a second set of improved pressure, velocity and density estimates, $\bar{\bar{p}}$, $\bar{\bar{u}}$ and $\bar{\bar{\rho}}$ and substituting into momentum conservation and the equation of state

$$a^u_p \bar{\bar{u}}_P = \sum a^u_{nb} \bar{\bar{u}}_{nb} - c^u (\bar{\bar{p}}_E - \bar{\bar{p}}_P) + b^u \quad (41)$$

$$\bar{\bar{\rho}}_P = c^\rho \bar{\bar{p}}_P + b^\rho \quad (42)$$

With a second set of pressure and velocity corrections, $p'' = \bar{\bar{p}} - \bar{p}$ and $u'' = \bar{\bar{u}} - \bar{u}$, and introducing the approximation that $\sum a^u_{nb} u_{nb}'' \approx 0$, which is similar to that of SIMPLE, Eq. (41) becomes

$$\bar{\bar{u}}_P = u^{**}_P - \bar{d}^u (p''_E - p''_P) \quad (43)$$

where

$$u^{**}_P = (\sum a^u_{nb} \bar{u}_{nb} - c^u (\bar{p}_E - \bar{p}_P) + b^u) / a^u_p \quad (44)$$

Similarly by introducing a second density correction, $\rho'' = \bar{\bar{\rho}} - \bar{\rho}$, Eq. (42) can be expressed as

$$\bar{\bar{\rho}}_P = \rho^{**}_P + c^\rho p''_P \quad (45)$$

where

$$\rho^{**}_P = c^\rho \bar{p}_P + b^\rho = \bar{\rho}_P \quad (46)$$

By requiring that $\bar{\bar{u}}$ and $\bar{\bar{\rho}}$ satisfy mass conservation, an equation for p'' similar to Eq. (34) results

$$a^p_p p''_P = a^p_e p''_E + a^p_w p''_W + b^{p''} \quad (47)$$

where

$$b^{p''} = b^c - m^\rho_p \bar{\rho}_P - m^\rho_e \bar{\rho}_E - m^\rho_w \bar{\rho}_W - m^u_e \bar{u}_P - m^u_w \bar{u}_W \quad (48)$$

Upon solving Eq. (47) for p'' the improved pressure estimate is determined from $\bar{\bar{p}} = \bar{p} + p''$ and the $\bar{\bar{u}}$ velocities and $\bar{\bar{\rho}}$ densities from Eqs. (43) and (45), respectively.

The application of PUP differs in two ways from the description of SIMPLER provided by Patankar [9,17]. The first difference arises from the use of a second pressure correction, p'', not used by Patankar. However, PUP can be implemented in terms of the $\bar{\bar{p}}$ pressure by rearranging Eqs. (43) and (45) into the form given by

$$\bar{\bar{u}}_P = \hat{u}_P - \bar{d}^u (\bar{\bar{p}}_E - \bar{\bar{p}}_P) \quad (49)$$

$$\bar{\bar{\rho}}_P = \hat{\rho}_P + c^\rho \bar{\bar{p}}_P \quad (50)$$

where $\hat{u}_P = (\sum a^u_{nb} \bar{u}_{nb} + b^u) / a^u_p$ and $\hat{\rho}_P = b^\rho$. Now by requiring that $\bar{\bar{u}}$ and $\bar{\bar{\rho}}$ conserve mass an equation for $\bar{\bar{p}}$ results that is consistent with Patankar's proposal.

The second difference is the order of the SIMPLE and PUP stages. In the preceeding description PUP follows SIMPLE where in the SIMPLER described by Patankar, PUP precedes SIMPLE. The order of the PUP and SIMPLE stages described in this paper is identical to the order described by Raithby and Schneider [18] in their study of methods for solving incompressible flows and is in many ways similar to the PISO method [19,20] developed for solving incompressible and compressible flows. In fact, for steady incompressible flows the PISO method is identical to the method described here. For transient incompressible flows as well as compressible flows further study is required to determine the significance of any difference between the two methods.

SIMPLEC The introduction of PUP as a second stage is an attempt to correct errors in the pressure which result from making a poor approximation to $\mathbf{D}_p{}^u$. In the SIMPLEC method of Van Doormaal and Raithby [10] a more 'consistent' approximation is introduced. The results of numerical experiments indicate that for incompressible flows SIMPLEC is substantially more economic than SIMPLE and that SIMPLEC is usually less expensive than SIMPLER.

SIMPLEX In the three segregated methods described above, no care is taken to ensure that the rate of convergence will not decrease with grid refinement. An attempt to address this concern is made in SIMPLEX [11] where the influence of nodal values of pressure further from a nodal velocity is accounted for. This is accomplished in SIMPLEX by using extrapolation to express all pressure differences in the domain in terms of the pressure difference local to the velocity. Again, for incompressible flows, the computational advantage of SIMPLEX over the previous methods reviewed here, particularly for fine grids, has been demonstrated [11].

ONE-DIMENSIONAL DEMONSTRATION PROBLEMS

To illustrate the applicability of the segregated methods and to evaluate the relative convergence behaviour of these methods numerical experiments were performed on two laminar one-dimensional compressible duct flows.

Subsonic Demonstration Problem

The first demonstration problem is that of a subsonic laminar flow through the 3:2 converging duct shown in Fig. 2 with an inlet Mach number of 0.3. Using a Reynolds number of 10^7 steady state numerical solutions were determined to within the round-off limit of the computer using 20, 40, 80, 160, and 320 nodes. Using these solutions, numerical experiments were performed to evaluate the sensitivity of the convergence of the segregated methods to the number of nodes, N, and the time step, Δt. For each grid and for a prescribed time step the coefficients of the linearized algebraic representations of mass conservation, momentum and energy conservation and the equation of state were determined using the steady state solutions for p, u, ρ and T. Subsequently, the pressure was set throughout the domain to the value of the outlet and the convergence of each segregated method was monitored as the solution of the linearized algebraic equations with fixed coefficients was obtained. It is important to emphasize that these tests reveal how well the solution methods used treat the coupling between p, u and ρ in the linear equation set. Good performance at this level is a prerequisite to the satisfactory solution of the non-linear set.

In Fig. 3 the number of cycle, K required by each of the methods on each of the grids, to determine the pressure to within 0.5 percent of its correct value is presented as a function of the size of the time step. The results indicate

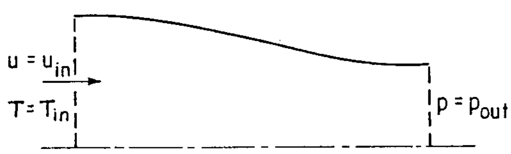

Fig. 2 Problem of one-dimensional subsonic flow through a 3:2 converging duct with Mach 0.3 inlet.

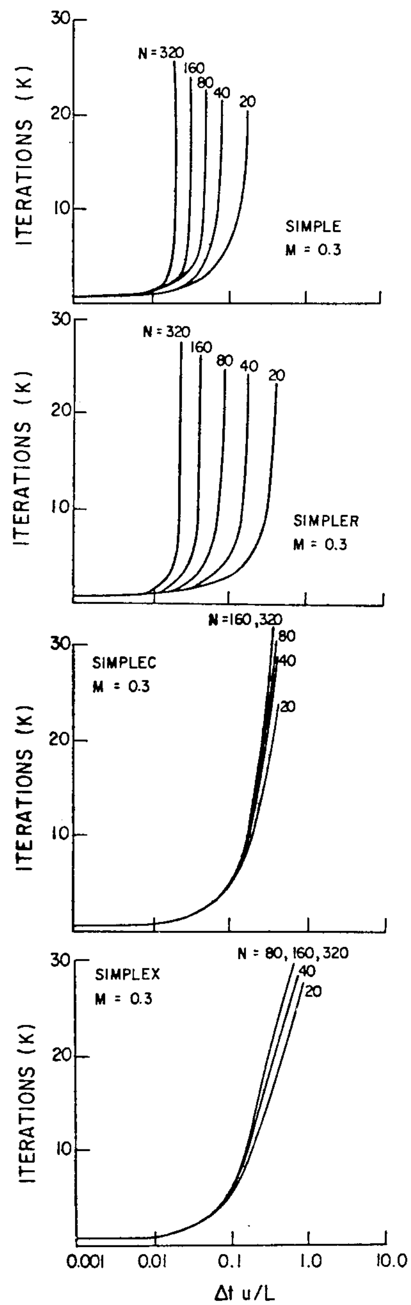

Fig. 3 Iterative convergence behaviour of a) SIMPLE, b) SIMPLER, c) SIMPLEC and d) SIMPLEX for one-dimensional subsonic test problem.

that for sufficiently small time steps all methods require only one cycle. This is due to the fact that the coefficients of the algebraic equations which are based on the steady state solution are held fixed. However, as the size of the time step increases the iterative requirements of all methods increase monotonically. This behaviour is due to the limitations imposed on all methods by using a very simple structure for $\mathbf{D}_p^u$ which does not appropriately account for the significant influence of nodal pressures which are far from a nodal velocity. The results also show that the behaviour of both SIMPLEC and SIMPLEX is relatively independent of the number of nodes while the behaviour of both SIMPLE and SIMPLER suffers with grid refinement. In summary, these results indicate that for subsonic flow the behaviour of segregated methods using large time steps is determined by the approximations to $\mathbf{D}^u$.

Supersonic Demonstration Problem

The second demonstration problem is that of a supersonic laminar flow through the 3:2 converging duct, shown in Fig. 4, with an inlet Mach number of 2.0. Using the same Reynolds number, the same grids and a similar numerical experiment as in the subsonic case, the sensitivity of the convergence of the segregated methods to the number of nodes and the time step were determined.

The results shown in Fig. 5 again indicate that for small Δt only one cycle was required by all methods to achieve an accuracy of 0.5 percent and that the behaviour of SIMPLEC and SIMPLEX is similar to that found in the subsonic case. However, in contrast to the poor behaviour of SIMPLE and SIMPLER on the subsonic problem, the behaviour of these methods for the supersonic problem are quite favourable. As shown in Fig. 5c, both SIMPLE and SIMPLER exhibit only a marginal sensitivity to the time step and no sensitivity to spatial discretization. These results, at first, seem surprising especially in light of the poor approximations used to evaluate d^u. In fact, in both SIMPLE and SIMPLER the value of d^u is underestimated resulting in an overestimation of pressure corrections for incompressible and subsonic flows. However, for supersonic flows, it appears that the underestimation of d^u is not detrimental to the convergence behaviour of SIMPLE and SIMPLER. This is due to the fact that for supersonic flows the primary role of pressure is to influence the density through the equation of state so that mass is conserved. As a result the influence of pressure on velocity can be ignored. To illustrate this point the numerical experiments were repeated for the supersonic problem using SIMPLE0 where 0 is appended to SIMPLE to denote that d^u is everywhere set to zero. The results shown in Fig. 5c indicate that the iterative convergence behaviour is identical to that of SIMPLE and SIMPLER. Therefore, the superior performance of SIMPLE and SIMPLER for supersonic flows is not due to a better approximate evaluation of d^u but rather that the underestimation of d^u is not detrimental to the performance of these methods. Unfortunately, the same is not true for multi-dimensional flows.

SEGREGATED APPROACH FOR TWO-DIMENSIONAL PROBLEMS

Up to this point the segregated approach and methods have been considered in a one-dimensional context. In what follows the extension of the segregated approach and methods to two-dimensions is reviewed. Subsequently, the applicability of these methods is demonstrated on a two-dimensional compressible flow problem.

Differential Equations

The differential equations expressing the conservation of mass, momentum and energy for a laminar two-

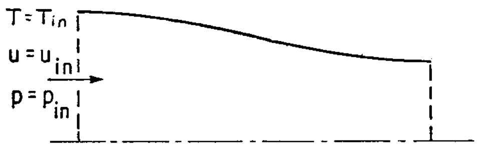

Fig. 4 Problem of one-dimensional supersonic flow through a 3:2 converging duct with Mach 2.0 inlet.

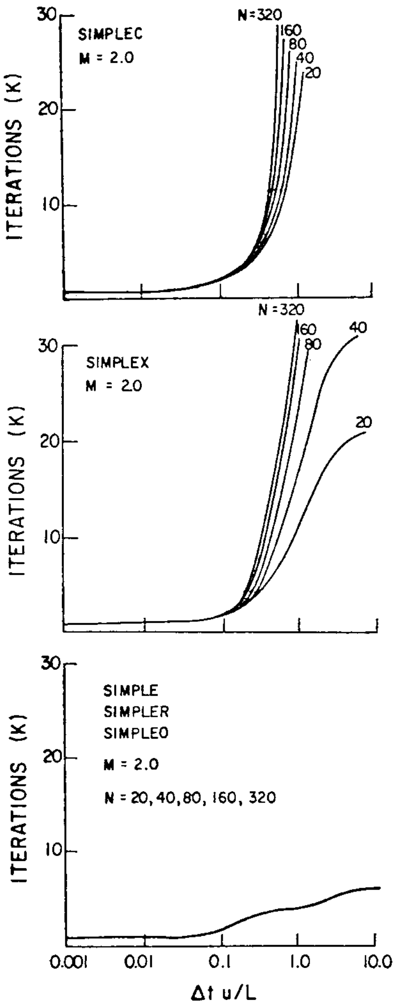

Fig. 5 Iterative convergence behaviour of a) SIMPLEC, b) SIMPLEX, and c) SIMPLE, SIMPLER and SIMPLEO for one-dimensional supersonic test problem.

dimensional viscous compressible flow of a perfect gas can be expressed in Cartesian Coordinates as

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0 \qquad (51)$$

$$\frac{\partial}{\partial t}(\rho u)+\frac{\partial}{\partial x}(\rho u u)+\frac{\partial}{\partial y}(\rho v u)=-\frac{\partial p}{\partial x} \qquad (52)$$

$$+\frac{\partial}{\partial x}\left[\mu\frac{\partial u}{\partial x}\right]+\frac{\partial}{\partial y}\left[\mu\frac{\partial u}{\partial y}\right]+\frac{\mu}{3}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\frac{\partial}{\partial t}(\rho v)+\frac{\partial}{\partial x}(\rho u v)+\frac{\partial}{\partial y}(\rho v v)=-\frac{\partial p}{\partial y} \qquad (53)$$

$$+\frac{\partial}{\partial x}\left[\mu\frac{\partial v}{\partial x}\right]+\frac{\partial}{\partial y}\left[\mu\frac{\partial v}{\partial y}\right]+\frac{\mu}{3}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

$$\frac{\partial}{\partial t}(\rho T)+\frac{\partial}{\partial x}(\rho u T)+\frac{\partial}{\partial y}(\rho v T)=\frac{k}{C_p}\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right) \qquad (54)$$

$$+\frac{1}{C_p}\left(\frac{\partial p}{\partial t}+\frac{\partial}{\partial x}(up)+\frac{\partial}{\partial y}(vp)-p\frac{\partial u}{\partial x}-p\frac{\partial v}{\partial y}\right)$$

where the additional dependent variable, v, is the component of velocity in the y coordinate direction.

Algebraic Equations

Employing the staggered grid shown in Fig. 6, the same discretization and linearization techniques described in the one-dimensional context, the linearized algebraic representations of the conservation of mass, momentum and energy can be expressed as

$$m_p^\rho \rho_P+m_e^\rho \rho_E+m_w^\rho \rho_W+m_n^\rho \rho_N+m_s^\rho \rho_S \qquad (55)$$

$$+m_e^u u_P+m_w^u u_W+m_n^v v_P+m_s^v v_S=b^c$$

$$a_p^u u_P=\sum a_{nb}^u u_{nb}-c^u(p_E-p_P)+b^u \qquad (56)$$

$$a_p^v v_P=\sum a_{nb}^v v_{nb}-c^v(p_N-p_P)+b^v \qquad (57)$$

$$a_p^T T_P=\sum a_{nb}^T T_{nb}+b^T \qquad (58)$$

where

$$\sum a_{nb}^u u_{nb}=a_e^u u_E+a_w^u u_W+a_n^u u_N+a_s^u u_S$$

$$\sum a_{nb}^v v_{nb}=a_e^v v_E+a_w^v v_W+a_n^v v_N+a_s^v v_S$$

$$\sum a_{nb}^T T_{nb}=a_e^T T_E+a_w^T T_W+a_n^T T_N+a_s^T T_S$$

and, where the equation of state is again represented by

$$\rho_P=c^\rho p_P+b^p \qquad (59)$$

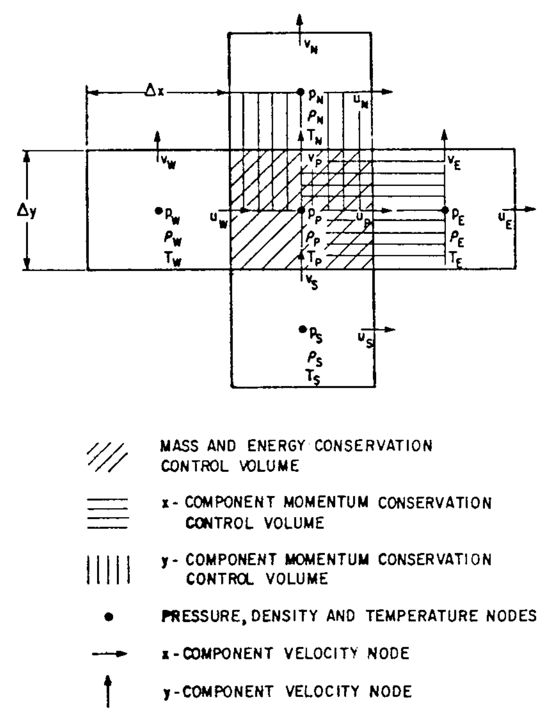

Fig. 6 Staggered arrangement of mesh, nodes and control volumes for two-dimensional flow.

Generalized Segregated Approach

The generalized segregated approach presented previously within the context of one-dimensional flow in a duct is readily extended to multi-dimensions. In terms of nodal values and considering only the influence of nodal pressures which are adjacent to the nodal components of velocity, the improved estimates of the components of velocity, $\bar{u}$ and $\bar{v}$, can be related to the pressure correction by

$$\bar{u}_P = u^*_P - \bar{d}^u (p'_E - p'_P) \tag{60}$$

$$\bar{v}_P = v^*_P - \bar{d}^v (p'_N - p'_P) \tag{61}$$

where, given p^*, an estimate for pressure, u^* and v^* are determined from

$$a^u_p u^*_P = \sum a^u_{nb} u^*_{nb} - c^u (p^*_E - p^*_P) + b^u \tag{62}$$

$$a^v_p v^*_P = \sum a^v_{nb} v^*_{nb} - c^v (p^*_N - p^*_P) + b^v \tag{63}$$

By requiring that the improved estimates of velocity $\bar{u}$ and $\bar{v}$ given by Eqs. (60) and (61) and the improved estimate of density given by Eq. (32) satisfy mass conservation, Eq. (55), the following equation of pressure correction is obtained:

$$a^p_p p'_P = a^p_e p'_E + a^p_w p'_W + a^p_n p'_N + a^p_s p'_S + b^{p'} \tag{64}$$

where

$$a^p_p = m^\rho_e + m^u_e \bar{d}^u - m^u_w [\bar{d}^u]_W + m^v_n \bar{d}^v - m^v_s [\bar{d}^v]_S \tag{65}$$

$$a^p_e = -m^\rho_e [c^\rho]_E + m^u_e \bar{d}^u \tag{66}$$

$$a^p_w = -m^\rho_w [c^\rho]_W - m^u_w [\bar{d}^u]_W \tag{67}$$

$$a^p_n = -m^\rho_n [c^\rho]_N + m^v_n \bar{d}^v \tag{68}$$

$$a^p_s = -m^\rho_s [c^\rho]_S - m^v_s [\bar{d}^v]_S \tag{69}$$

$$b^{p'} = b^c - m^\rho_p \rho^*_P - m^\rho_e \rho^*_E - m^\rho_w \rho^*_W - m^\rho_n \rho^*_N - m^\rho_s \rho^*_S \tag{70}$$
$$- m^u_e u^*_P - m^u_w u^*_W - m^v_n v^*_P - m^v_s v^*_S$$

and where the evaluations of $\bar{d}^u$ are given in Table 1 and, for SIMPLE and SIMPLER

$$\bar{d}^v = \frac{c^v}{a^v_p} \tag{71}$$

for SIMPLEC

$$\bar{d}^v = \frac{c^v}{a^v_p - \sum a^v_{nb}} \tag{72}$$

and for SIMPLEX

$$a^v_p [\bar{d}^v]_P = \sum a^v_{nb} [\bar{d}^v]_{nb} + c^v \tag{73}$$

Solving Eq. (64) for p', the $\bar{p}$ pressures determined from $\bar{p} = p^* + p'$, and the $\bar{u}$ velocities, $\bar{v}$ velocities and $\bar{\rho}$ densities are determined from Eqs. (60), (61) and (32), respectively.

TWO-DIMENSIONAL DEMONSTRATION PROBLEM

For the purpose of demonstrating the applicability of the segregated methods and of demonstrating their relative performance, the steady state supersonic (Mach 2) flow around a flat plate oriented normal to the flow, as shown in Fig. 7a, was considered. The solution of the discrete equations of motion using a uniform coarse 22x18 grid was determined to the round-off limits of single precision FORTRAN-H on an IBM 4341 Group II computer. The isobars and streamlines from this solution are plotted respectively in Figs. 7b and 7c. Figure 7b shows that a bow shock is predicted, while Fig. 7c shows the flow deflection around the the plate with a subsonic region surrounding the plate. The shock smearing that is evident results from the coarse grid and from the numerical diffusion that is inherent in the discretization used. Because the present study focuses on

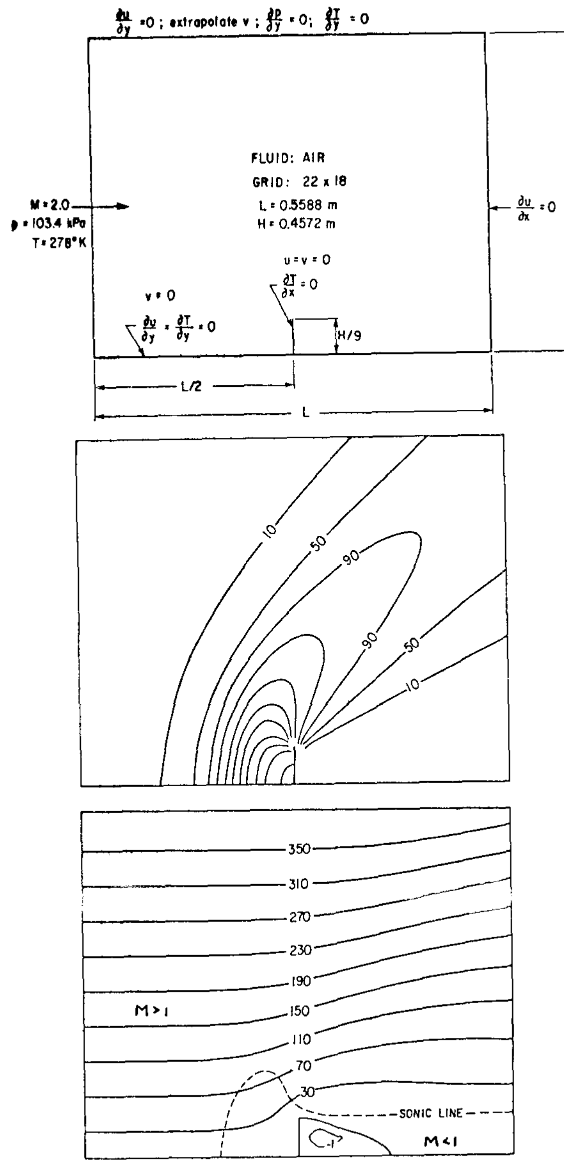

Fig. 7 Flow around a flat plate, a) geometry and boundary conditions, b) isobars (kPa) of pressure in excess of inlet pressure and c) streamlines.

the applicability of solution methods, the improvement of accuracy was not considered.

A number of tests were carried out to establish the sensitivity of solution time of the segregated methods to the number of cycles used for each time step and to the accuracy to which each linear set of equations was solved. Details of these tests have been reported by Van Doormaal [22]. It was found that, for each time step, it was best to permit only one cycle (ie. to solve only one set of linear equations) except for the continuity equation where a second coefficient evaluation and solution were performed. The MSIP solver of Schneider and Zedan [21] was used to solve the segregated linear equation set for each variable.

The performance of the various solution methods was tested by initializing all dependent variables to inlet conditions, and advancing the solution through time steps Δt

until the computed variables were all within 0.5 percent of the previously obtained steady state solution. The computational effort is plotted against Δt in Fig. 8 for each of the four segregated methods. At small Δt the behaviour of all methods is similar; the slightly higher effort for SIMPLER and SIMPLEX reflects the need to solve an extra linear equation. For lager Δt SIMPLEC and SIMPLEX become distinctly superior to SIMPLE and SIMPLER. This is due to the relatively poor approximations inherent in d^u and d^v in SIMPLE and SIMPLER. For multi-dimensional flows this detrimental effect of the underestimation of d^u and d^v is to be expected, even in supersonic cases, because there is likely to be at least one coordinate direction along which the component of the flow is subsonic. An attractive feature of SIMPLEX for this particular test problem is its relative insensitivity to the size of Δt when large time steps are taken.

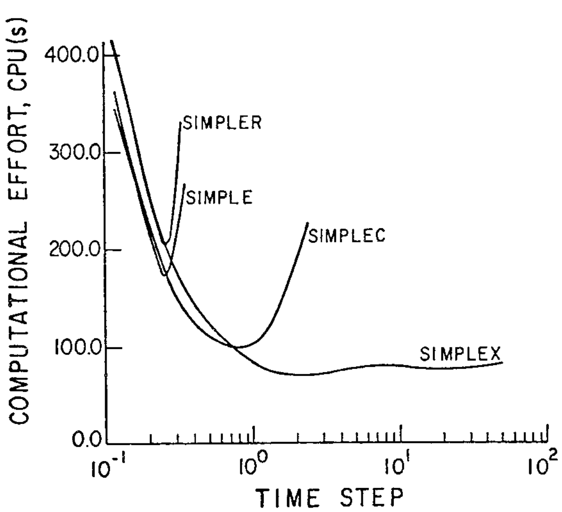

Fig. 8 Computational requirements versus time step of SIMPLE, SIMPLER, SIMPLEC and SIMPLEX for flow around a flat plate.

CONCLUDING REMARKS

A number of aspects of the segregated approach to solving viscous compressible fluid flows have been presented in this paper. Included in these are the extension to the solution of compressible flows of an interpretation of the role of pressure in the segregated approach, of the generalized view of the segregated approach, and of the segregated methods SIMPLE, SIMPLER, SIMPLEC and SIMPLEX developed originally for the solution of incompressible flows. The present paper also develops an understanding of the strengths and shortcomings of the segregated methods. Many of these aspects are demonstrated on one-dimensional and two-dimensional compressible flow problems. In particular, with the appropriate linearization of mass conservation and with the appropriate approximation of the influence of pressure on velocity, the segregated approach has been demonstrated to be equally applicable to supersonic and subsonic compressible flows as well as compressible flows.

ACKNOWLEDGEMENTS

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