

**The Seiberg-Witten Equations and  
Applications to the Topology of  
Smooth Four-Manifolds**

by

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