Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY
MNRASL **445,** L26–L30 (2014) doi:10.1093/mnrasl/slu129

The shape, internal structure and gravity of the fast spinner β Pictoris b

Dali Kong,[1,2] Xinhao Liao,[1] Keke Zhang[2⋆] and Gerald Schubert[3]

[1] *Key Laboratory of Planetary Sciences, Shanghai Astronomical Observatory, Shanghai 200030, China*
[2] *Center for Geophysical and Astrophysical Fluid Dynamics, University of Exeter, Exeter EX4 4QF, UK*
[3] *Department of Earth, Planetary and Space Sciences, University of California, Los Angeles, CA 90095-1567, USA*

Accepted 2014 July 29. Received 2014 July 15; in original form 2014 June 13

ABSTRACT

A young extrasolar gas giant planet, β Pictoris b, recently discovered in the β Pictoris system, spins substantially faster than the giant gas planets Jupiter and Saturn. Based on the newly measured parameters – the rotation period of the planet, its mass and radius – together with an assumption that the gas planet β Pictoris b is in hydrostatic equilibrium and made of a fully compressible barotropic gas with a polytropic index of unity, we are able to compute, via a hybrid inverse method, its non-spherical shape, internal density/pressure distribution and gravitational zonal coefficients up to degree 8. Since the mass M_β for the planet β Pictoris b is highly uncertain, various models with different values of M_β are studied in this Letter, providing the upper and lower bounds for its shape parameter as well as its gravitational zonal coefficients. If M_β is assumed to be $6M_\mathrm{J}$ with M_J being Jupiter's mass, we show that the shape of the planet β Pictoris b is approximately described by an oblate spheroid whose eccentricity at the one-bar surface is $\mathcal{E}_\beta = 0.369\,28$ with the gravitational coefficient $(J_2)_\beta = +15\,375.972 \times 10^{-6}$. It follows that our results open the possibility of constraining or inferring the mass M_β of the planet β Pictoris b if its shape can be measured or constrained. By assuming that the planet β Pictoris b will shrink to the size of Jupiter in the process of cooling down and, hence, rotate much faster, we also calculate the future shape and internal structure of the planet β Pictoris b.

Key words: planets and satellites: gaseous planets – planets and satellites: interiors.

1 INTRODUCTION

Using high-contrast near-infrared images, an extrasolar gas giant planet, β Pictoris b, located 8–15 au away from its parent star, was detected in the β Pictoris system which is about 10^6 yr old (Lagrange et al. 2009, 2010). A recent measurement (Snellen et al. 2014) reveals that the young gas giant planet β Pictoris b spins significantly faster than any planets in the Solar system. During its evolution over the next hundreds of millions of years, the size of the gas planet is expected to decrease to that of Jupiter (Baraffe et al. 2003) and, consequently, its spinning speed will become even much faster. Snellen et al. (2014) also found that the equatorial spinning velocities of giant planets Jupiter, Saturn and β Pictoris b are nearly proportional to their masses.

In the Solar system, the shape and gravity of the gas giant planet Jupiter, which provides an important constraint on the physical and chemical properties of its interior (Guillot 2007), can be accurately measured (Jacobson 2003; Seidelmann et al. 2007). As a consequence of its rapid rotation, the shape of Jupiter is non-spherical and, hence, its external gravitational potential V_J can be expanded in terms of the even Legendre functions P_n,

$$V_\mathrm{J} = -\frac{GM_\mathrm{J}}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{R_\mathrm{J}}{r}\right)^n P_n(\cos\theta)\right], \qquad (1)$$

with $r \geq R_\mathrm{J}$, where $R_\mathrm{J} = 71\,492$ km is the equatorial radius of Jupiter at the one-bar surface (Seidelmann et al. 2007), $G = 6.673\,84 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is the universal gravitational constant, $M_\mathrm{J} = 1.8986 \times 10^{27}$ kg is Jupiter's mass, n takes even integers, (r, θ, ϕ) are spherical polar coordinates with $\theta = 0$ at the axis of rotation. It is known that the shape of Jupiter is approximately described by an oblate spheroid whose eccentricity at the one-bar surface is $\mathcal{E}_\mathrm{J} = 0.3543$ and its first three zonal gravitational coefficients are given, with a high precision, by $J_2 = 14\,909.60 \times 10^{-6}$, $J_4 = -559.07 \times 10^{-6}$, $J_6 = 29.89 \times 10^{-6}$ (Jacobson 2003). By contrast, it is unlikely that the gravitational coefficients for the extrasolar giant planet β Pictoris b can be directly measured. An important question is what we can learn about β Pictoris b from the newly observed data, such as its rotation period and its radius (Snellen et al. 2014). Can we infer its shape and internal structure from the new measurements of β Pictoris b if it has similar physical properties as those of Jupiter? Can we compute the zonal gravitational

⋆ E-mail: kzhang@ex.ac.uk

coefficients for the planet β Pictoris b based on the newly observed parameters? A high uncertainty about the planet β Pictoris b is its total mass M_β (Snellen et al. 2014). Can we remove this uncertainty or provide a useful constraint by comparing the shape of theoretical models to the shape of the planet β Pictoris b which may become measurable as in the case of the exoplanet HD 189733b (Cater & Winn 2009)? This Letter attempts to answer the above questions by computing the shapes, gravitational fields and internal structures of three-dimensional non-spherical models of this rapidly rotating gas giant exoplanet.

A hybrid inverse method for calculating the non-spherical shape and internal structure of rapidly rotating gaseous bodies was recently proposed by Kong et al. (2013). The method, which employs a finite element formulation and accounts for the full effects of rotation, is carefully validated against the asymptotic solution of Chandrasekhar (1933) that is valid only for a slowly rotating gaseous body. Moreover, it is also validated with an application to Jupiter by successfully reproducing the exact shape and the zonal gravitational coefficient J_2 of Jupiter with better than 2 per cent accuracy. In comparison with the classical theory of figures (Zharkov & Trubitsyn 1978) which is based on an expansion around spherical geometry using a small rotation parameter, the new method is applicable to fast rotating planets with an arbitrary departure from spherical geometry. In this study, we shall employ this new method to construct a non-spherical model for the fast spinning giant gas planet β Pictoris b by taking full account of rotational distortion. In other words, using the newly observed parameters for the fast rotating giant planet β Pictoris b, we are able to compute the shape, three-dimensional internal structure and zonal gravitational coefficients of the planet. While information on the internal structure and gravitational field offers helpful clues to the physical state of the evolution of the planet, the shape of different theoretical models of the planet β Pictoris b may be used to remove or constraint the current high uncertainty of the mass M_β (Snellen et al. 2014).

In what follows, we begin by presenting the governing equations and the model for the planet β Pictoris b in Section 2. The result for the shape, internal structure and zonal gravitational coefficients of the planet β Pictoris b will be discussed in Section 3 while a summary and some remarks are given in Section 4.

2 DESCRIPTION OF MODEL AND GOVERNING EQUATIONS

We consider the gas giant planet β Pictoris b as being an isolated mass of gas in an equilibrium state with the total mass M_β that is rotating rapidly about the z-axis with angular velocity $\Omega_\beta \hat{z}$. The rotational effect of β Pictoris b is assumed to be sufficiently strong to flatten its body to the shape of an oblate spheroid with moderate or large eccentricity that is still within the stable limit. An equation of state (EOS) is required to construct the internal model for β Pictoris b. We shall adopt the classic polytropic EOS with index unity (Chandrasekhar 1933) for the reason that previous studies show that it provides a good qualitative approximation to the physical state of the Jovian interior (Hubbard 1999; Kong et al. 2013). With a polytrope of index unity, the pressure (p)–density (ρ) relation for the interior of the gaseous planet β Pictoris b is described by

$$p = K_\beta \rho^2, \tag{2}$$

where K_β is a constant and to be determined by the hybrid inverse method. In an inertial frame of reference, the equilibrium equations for the rotating gaseous planet β Pictoris b are

$$\boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho} \nabla p - \nabla V_\beta, \tag{3}$$

$$\nabla^2 V_\beta = 4\pi G \rho, \tag{4}$$

where $\boldsymbol{u} = \Omega_\beta \hat{z} \times \boldsymbol{r}$ with $\boldsymbol{r}$ being the position vector and V_β denotes the gravitational potential given by

$$V_\beta(\boldsymbol{r}) = -G \iiint_{\mathcal{D}} \frac{\rho\left(\boldsymbol{r}'\right) \mathrm{d}^3\boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|}, \tag{5}$$

where $\mathcal{D}$ denotes the domain of the planet which is to be determined. Equations (2)–(5) are then solved subject to the two boundary conditions

$$p = 1 \text{ bar} \quad \text{and} \quad V_\beta + V_\mathrm{c} = \text{constant} \tag{6}$$

at the bounding surface $\mathcal{S}$ of the planet, where V_c is the centrifugal potential defined as

$$V_\mathrm{c} = -\frac{\Omega_\beta}{2} \left|\hat{z} \times \boldsymbol{r}\right|^2 .$$

Here, we assume that the shape $\mathcal{S}$ of the rotationally distorted planet is an oblate spheroid with its eccentricity $\mathcal{E}_\beta$ defined as

$$\mathcal{E}_\beta = \frac{\sqrt{R_\beta^2 - R_\mathrm{p}^2}}{R_\beta},$$

where $0 < \mathcal{E}_\beta < 1$ and, R_β and R_p are the equatorial and polar radii of the planet, respectively. In our calculation, the tidal effect due to its parent star is assumed to be negligibly small. Note that the shape parameter, described by the eccentricity $\mathcal{E}_\beta$ of the planet, is a priori unknown while the total mass M_β and the equatorial radius R_β of the planet are provided by the observation. The solution of equations (2)–(5) satisfying condition (6) gives rise to the value of K_β in the EOS, the shape parameter $\mathcal{E}_\beta$ and, the density $\rho(\boldsymbol{r})$ and pressure $p(\boldsymbol{r})$ of the rapidly rotating planet β Pictoris b.

3 SHAPE, INTERNAL STRUCTURE AND GRAVITY

Although equations (2)–(5) are relatively simple, the fully three-dimensional computation of their solution, via the hybrid inverse method without assuming any spatial symmetry, is numerically expensive. This is because the solution domain $\mathcal{D}$ for solving the equations is a priori unknown and, consequently, a double iterative procedure must be adopted. First, we construct a three-dimensional finite element mesh by making a tetrahedralization of an oblate spheroid with a guessed eccentricity $\mathcal{E}_\mathrm{guess}$ and a fixed value K_fixed based on which equations (2)–(5) can be repeatedly solved by iterating over the shape parameter $\mathcal{E}_\mathrm{guess}$ such that the condition (6) is satisfied. When the internal distribution of the density $\rho(\boldsymbol{r})$ becomes available, we can then compute its total mass as

$$M_\beta = \iiint_{\mathcal{D}} \rho\left(\boldsymbol{r}'\right) \mathrm{d}^3\boldsymbol{r}',$$

which is not generally in agreement with the observed mass M_β for the planet β Pictoris b. A second iteration over K_fixed is required to determine the size of $K_\beta = K_\mathrm{fixed}$ such that the total mass M_β is exactly the same as the observed value. For a given set of the observed parameters Ω_β, R_e and M_β, we typically need about 60 numerical solutions of equations (2)–(5) in order to determine a particular set of K_β and $\mathcal{E}_\beta$ that satisfies not only the equations (2)–(5) but also the condition (6) along with the observed mass M_β. With the solution $\rho(\boldsymbol{r})$, we can compute, similar to those for Jupiter, the even zonal gravitational coefficients $(J_n)_\beta$ in the expansion

$$V_\beta(\boldsymbol{r}) = -\frac{GM_\beta}{r}\left[1 - \sum_{\text{even } n}(J_n)_\beta\left(\frac{R_\beta}{r}\right)^n P_n(\cos\theta)\right], \qquad (7)$$

which is only valid for $r \geq R_\beta$, where there is no mass and the potential $V_\beta(\boldsymbol{r})$ satisfies Laplace's equation. It should be noticed that the gravitational problem for non-spherical geometry is not straightforward because the region $R_\mathrm{p} < r < R_\beta$ has empty space in part and mass near low latitudes and, consequently, the gravitational potential $V_\beta(\boldsymbol{r})$ is, in contrast to spherical geometry, a highly complicated function of both latitude and radius.

Although the rotation period $T_\beta = 2\pi/\Omega_\beta \approx 8.1$ h and the radius $R_\beta \approx 1.65 R_\mathrm{J}$ of the planet β Pictoris b are directly or indirectly measured with a reasonable accuracy, its mass M_β is highly uncertain, $M_\beta = 11 \pm 5 M_\mathrm{J}$ (Snellen et al. 2014). In response to this uncertainty, we consider three models with different masses: (i) $M_\beta = 6M_\mathrm{J}$, $T_\beta = 8.1$ h and $R_\beta = 1.65R_\mathrm{J}$; (ii) $M_\beta = 11M_\mathrm{J}$, $T_\beta = 8.1$ h and $R_\beta = 1.65R_\mathrm{J}$; and (iii) $M_\beta = 16M_\mathrm{J}$, $T_\beta = 8.1$ h and $R_\beta = 1.65R_\mathrm{J}$. About 200 three-dimensional numerical simulations for equations (2)–(5), through the double iterative scheme, are carried out for the three models. The results are presented in Figs 1(a) and 2(a) showing the density $\rho(\boldsymbol{r})$ and pressure $p(\boldsymbol{r})$ as a function of radius r in the equatorial plane at equilibrium. The corresponding zonal gravitational coefficients $(J_n)_\beta$, up to $n = 8$ in the expansion (7), are listed in Table 1. As the mass M_β increases, the central density and pressure in the planet become larger while its shape becomes less flattened. For Model (i) with $M_\beta = 6M_\mathrm{J}$, the central density $\rho(r = 0) = 5.9538 \times 10^3$ kg m^{-3} which increases to $\rho(r = 0) = 15.032 \times 10^3$ kg m^{-3} for Model (iii) with $M_\beta = 16M_\mathrm{J}$. The shape parameter for Model (i) is $\mathcal{E}_\beta = 0.369\,28$, slightly higher than that of Jupiter, while its EOS is of the form

$$p(\boldsymbol{r}) = 568\,400 \text{ Pa m}^6 \text{ kg}^{-2} [\rho(\boldsymbol{r})]^2,$$

which not only satisfies equations (2)–(5) and the condition (6) but also its mass matches exactly the value $M_\beta = 6M_\mathrm{J}$. The corresponding two-dimensional distribution of the density ρ for Model (i) is displayed in Fig. 3(a) in a meridional plane. Since the shape parameter for Model (i) is close to that of Jupiter, its zonal gravitational coefficient $(J_2)_\beta = 0.015\,38$ is close to that of Jupiter $(J_2)_\mathrm{Jupiter} = 0.014\,91$ despite the fact that both M_β and R_β are much larger than M_J and R_J. The shape parameter $\mathcal{E}_\beta = 0.369\,28$ would be sufficiently large to be measurable (Cater & Winn 2009). When the mass M_β takes its upper bound (Snellen et al. 2014) in Model (iii), the shape parameter decreases to $\mathcal{E}_\beta = 0.233\,98$. The high uncertainty for the mass M_β of the planet β Pictoris b could be removed if its shape can be directly or indirectly measured with a reasonable accuracy.

According to the theory of planetary evolution, the size of the planet β Pictoris b will decrease to that of Jupiter and its length of day will decrease to about 3 h (Baraffe et al. 2003; Snellen et al. 2014). It is therefore of interest to look at the shape and internal structure of the future planet β Pictoris b when it shrinks to the same size as that of Jupiter but rotates much faster. Accordingly, we also consider the following three models: (iv) $M_\beta = 6M_\mathrm{J}$, $T_\beta = 3$ h and $R_\beta = R_\mathrm{J}$; (v) $M_\beta = 11M_\mathrm{J}$, $T_\beta = 3$ h and $R_\beta = R_\mathrm{J}$; and (vi) $M_\beta = 16M_\mathrm{J}$, $T_\beta = 3$ h and $R_\beta = R_\mathrm{J}$. The results of the three models are presented in Figs 1(b) and 2(b), which show the density $\rho(\boldsymbol{r})$ and pressure $p(\boldsymbol{r})$ as a function of the radius r in the equatorial plane. It can be readily compared to the possible structure of the present planet β Pictoris b displayed in Figs 1(a) and 2(a). The corresponding zonal gravitational coefficients $(J_n)_\beta$, up to $n = 8$ in the expansion (7), are listed in Table 2 for Models (iv)–(vi). We look at Model (iv) in detail which should be compared with Model (i). The shape parameter for Model (iv) increases to $\mathcal{E}_\beta = 0.455\,52$, slightly larger than that of Saturn, while its EOS is of the form

$$p(\boldsymbol{r}) = 202\,160 \text{ Pa m}^6 \text{ kg}^{-2} [\rho(\boldsymbol{r})]^2,$$

which satisfies equations (2)–(5) and the condition (6) and its mass matches the value $M_\beta = 6M_\mathrm{J}$. The corresponding two-dimensional distribution of the density ρ for Model (iv) is displayed in Fig. 3(b). For Model (iv) with $M_\beta = 6M_\mathrm{J}$, the central density is $\rho(0) = 28.198 \times 10^3$ kg m^{-3} which increases to $\rho(0) = 68.936 \times 10^3$ kg m^{-3} for Model (vi) with $M_\beta = 16M_\mathrm{J}$. There exists a close relationship between the mass M_β of the exoplanet and its shape: the shape parameter decreases to $\mathcal{E}_\beta = 0.294\,22$ for Model (vi). Even though the future planet β Pictoris b spins much faster than Saturn, its shape parameter, because of its larger mass, will be only slightly larger than that of Saturn.

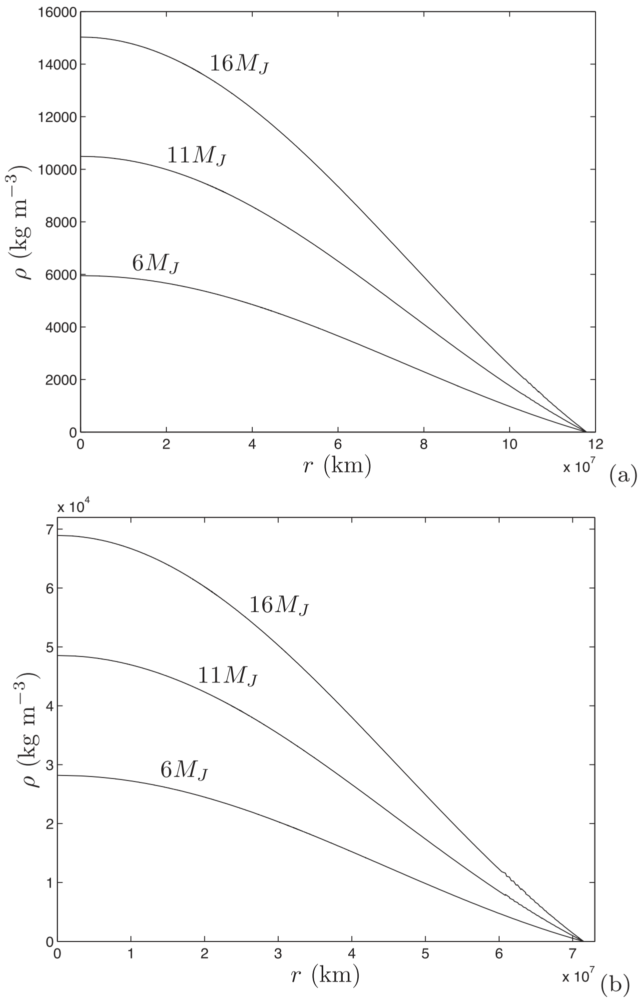

Figure 1. (a) The dependence of the density ρ on the radius r in the equatorial plane with $T_\beta = 8.1$ h and $R_\beta = 1.65R_\mathrm{J}$ for the three different models of the planet β Pictoris b with $M_\beta = 6, 11, 16M_\mathrm{J}$. (b) The dependence of the density ρ on the radius r in the equatorial plane with $T_\beta = 3$ h and $R_\beta = R_\mathrm{J}$ for the three different models with $M_\beta = 6, 11, 16M_\mathrm{J}$.

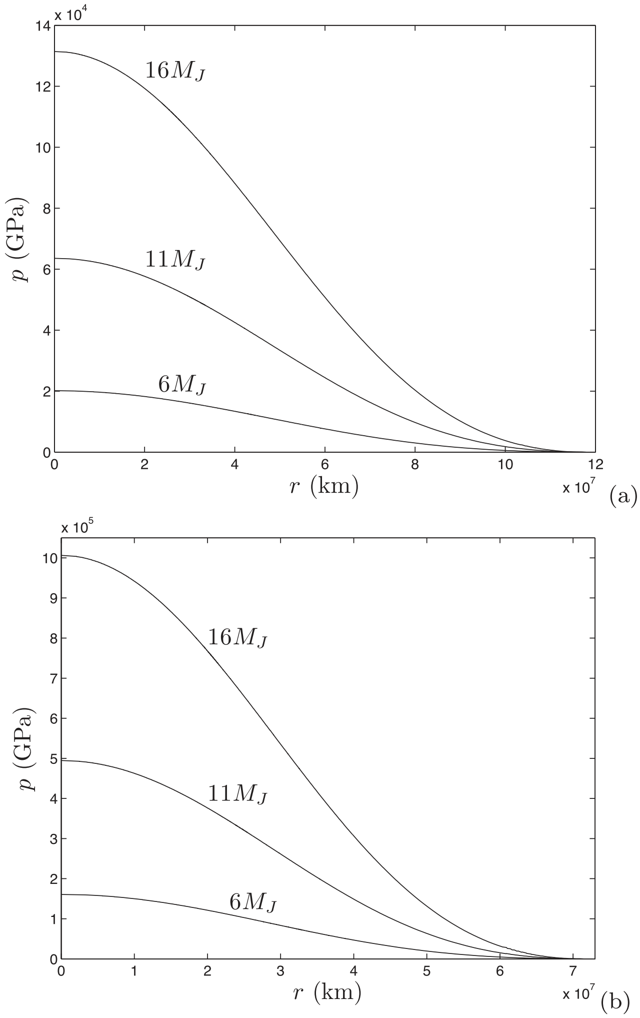

Figure 2. (a) The dependence of the pressure p on the radius r in the equatorial plane for the planet β Pictoris b when $T_\beta = 8.1$ h and $R_\beta = 1.65R_{\mathrm{J}}$ with the three difference cases $M_\beta = 5, 11, 16M_{\mathrm{J}}$. (b) The dependence of p as a function of r in the equatorial plane for the planet β Pictoris b when $T_\beta = 3$ h and $R_\beta = R_{\mathrm{J}}$ with $M_\beta = 5, 11, 16M_{\mathrm{J}}$.

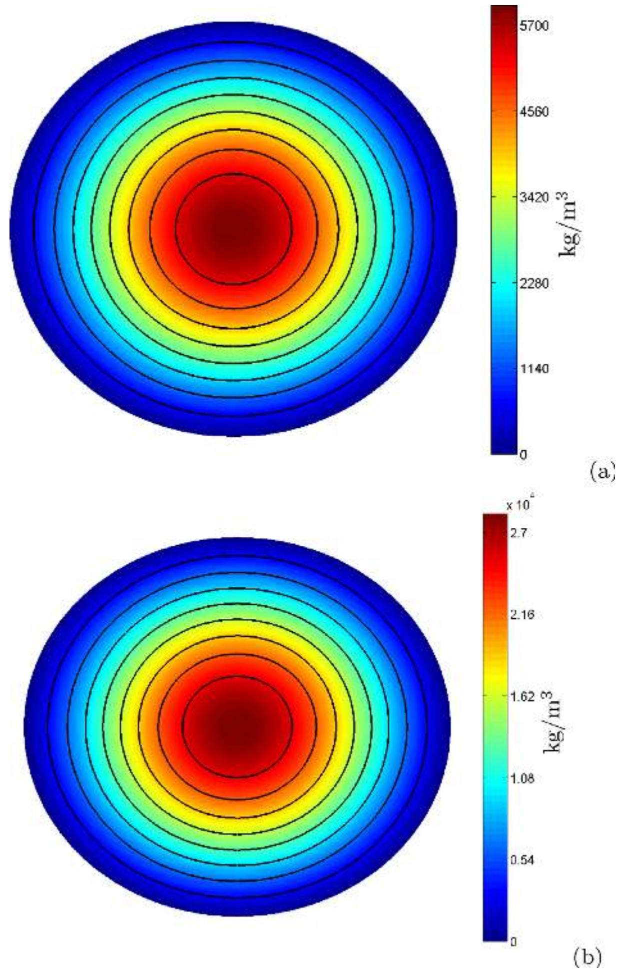

Figure 3. The interior structure from the numerical models of the planet β Pictoris b: (a) the two-dimensional density distribution ρ in a meridional plane for Model (i) with $M_\beta = 6M_{\mathrm{J}}$, $T_\beta = 8.1$ h and $R_\beta = 1.65R_{\mathrm{J}}$ at $\mathcal{E}_\beta = 0.369\,28$; (b) the two-dimensional density distribution ρ in a meridional plane for Model (iv) with $M_\beta = 6M_{\mathrm{J}}$, $T_\beta = 3.0$h and $R_\beta = R_{\mathrm{J}}$ at $\mathcal{E}_\beta = 0.455\,52$.

Table 1. Computed gravitational zonal coefficients $(J_n)_\beta$ in the expansion (7) for Models (i)–(iii) of the planet β Pictoris b.

n	Model (i) $(J_n)_\beta \times 10^6$	Model (ii) $(J_n)_\beta \times 10^6$	Model (iii) $(J_n)_\beta \times 10^6$
2	+15 375.972	+8854.802	+6216.262
4	−594.920	−197.056	−97.068
6	+32.842	+6.252	+2.160
8	−2.204	−0.237	−0.055

Table 2. Computed gravitational zonal coefficients $(J_n)_\beta$ in the expansion (7) for Models (iv)–(vi) of the future planet β Pictoris b.

n	Model (iv) $(J_n)_\beta \times 10^6$	Model (v) $(J_n)_\beta \times 10^6$	Model (vi) $(J_n)_\beta \times 10^6$
2	+23 221.461	+13 795.820	+9803.852
4	−1359.259	−478.780	−241.606
6	+113.650	+23.700	+8.488
8	−11.594	−1.428	−0.361

4 SUMMARY AND CONCLUDING REMARKS

A recent observation by Snellen et al. (2014) reveals that the young gas giant planet β Pictoris b rotates significantly faster than any of the planets in the Solar system. Based on the new observational parameters – the rotation period of the planet T_β, its mass M_β and radius R_β – together with an assumption that the gas planet β Pictoris b is in the hydrostatic equilibrium and made of a fully compressible barotropic gas with a polytropic index of unity, we have computed, via a hybrid inverse method, non-spherical shape, internal density/pressure distribution and gravitational zonal coefficients up to degree 8. Since the mass M_β for the planet β Pictoris b is highly uncertain (Snellen et al. 2014), we have considered the three different models with $M_\beta = 6, 11, 16M_{\mathrm{J}}$. They provide the upper and lower bound for the shape parameter $\mathcal{E}_\beta$ as well as the corresponding gravitational zonal coefficients of the planet. It is found that the shape parameter for β Pictoris b is in the range $0.233\,98 \le \mathcal{E}_\beta \le 0.369\,28$ while its gravitational coefficient is in the range $6216.262 \times 10^{-6} \le (J_2)_\beta \le +15\,375.972 \times 10^{-6}$ if the uncertainty of its mass is in the range $6M_J \leq M_\beta \leq 16M_J$. Since the size of the planet β Pictoris b decreases as it cools down whilst its spinning speed increases, we have also calculated the shape and gravitational field of the future planet β Pictoris b when it shrinks to the size of Jupiter. It is found that the shape parameter for the future β Pictoris b with $R_\beta = R_J$ is in the range $0.294\,22 \leq \mathcal{E}_\beta \leq 0.455\,52$ while its gravitational coefficient is in the range $+9803.852 \times 10^{-6} \leq (J_2)_\beta \leq +23\,221.461 \times 10^{-6}$ if the uncertainty of its mass remains in the range $6M_J \leq M_\beta \leq 16M_J$.

On the basis of the phenomenon that an oblate exoplanet would exhibit weak anomalies during the ingress and egress phases of the transit, Cater & Winn (2009) was able to measure the shape parameter of the giant gaseous exoplanet HD 189733b by assuming that its rotation period is tidally synchronized with its orbital period, giving upper bounds of its gravitational zonal coefficient $J_2 < 0.068$ and its eccentricity $\mathcal{E} < 0.3299$ with 95 per cent confidence. Our models suggest that the shape parameter $\mathcal{E}_\beta$ for the planet β Pictoris b may be larger than that of the exoplanet HD 189733b and, hence, would be photometrically measurable. The results presented in this Letter open the possibility of constraining or inferring the mass M_β of the planet β Pictoris b if its shape can be measured or constrained, similar to that for the exoplanet HD 189733b.

Finally, it is helpful to summarize the measured parameters and the theoretical assumptions that are used in the present calculation for the planet β Pictoris b. While the rotation period T_β, the mass M_β and the radius R_β represent the measured parameters (Snellen et al. 2014), we have made the four major assumptions in our study: (i) the fast spinning planet remains within the stable limit; (ii) the tidal effect on the planet is small and can be neglected; (iii) the planet is in hydrostatic equilibrium with the dynamic influence being negligibly weak; and (iv) the planet is made of a fully compressible barotropic gas with a polytropic index of unity. We believe that the first three assumptions are physically and dynamically reasonable. However, the fourth assumption about the EOS for the planet β Pictoris b, which is inferred from the physical properties of Jupiter (Stevenson 1982), is more uncertain and may affect the result of our calculation.

ACKNOWLEDGEMENTS

XL is supported by NSFC/11133004 and CAS under grant numbers KZZD-EW-01-3 and XDB09000000, KZ is supported by UK NERC and STFC grants and GS is supported by the National Science Foundation under grant NSF AST-0909206. The parallel computation is supported by the Shanghai Supercomputer Center.

REFERENCES

Baraffe I., Chabrier G., Barman T. S., Allard F., Hauschildt P. H., 2003, A&A, 402, 701
Cater J. A., Winn J. N., 2009, ApJ, 704, 51
Chandrasekhar S., 1933, MNRAS, 93, 390
Guillot T., 2007, Annu. Rev. Earth Planet. Sci., 33, 493
Hubbard W. B., 1999, Icarus, 137, 357
Jacobson R. A., 2003, JUP 230 Orbit Solution, http://ssd.jpl.nasa.gov/?gravity_fields_op
Kong D., Zhang K., Schubert G., Anderson J., 2013, ApJ, 763, 116
Lagrange A.-M. et al., 2009, A&A, 493, L21
Lagrange A.-M. et al., 2010, Science, 329, 57
Seidelmann P. K. et al., 2007, Celest. Mech. Dyn. Astron., 98, 155
Snellen I. A. G., Brandl B. R., de Kok R. J., Brogi M., Birkby J., Schwarz H., 2014, Nature, 509, 63
Stevenson D., 1982, Annu. Rev. Earth Planet. Sci., 10, 257
Zharkov V. N., Trubitsyn V. P., 1978, Physics of Planetary Interiors. Pachart Publishing House, Tucson, AZ