

THE SHORT PERIOD LIGHT AND VELOCITY VARIATIONS IN ALPHA VIRGINIS

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SUMMARY

The light variation of α Virginis (Spica) has been followed over a total of 26 nights in 1968, 1969 and 1970. On seven nights in 1970, three-colour observations were made. Both the 4-day variation with the orbital period and the 4-hr variation due to Beta Canis Majoris type pulsation (already reported in a previous paper) were evident; the amplitude of the short period appears to be randomly variable suggesting that the mode of pulsation is unstable.

New spectra have been obtained and the radial velocities discussed together with others published from 1890 to 1956. The 4-hr period is also found in part of this data but some of the earlier velocities indicate the presence of at least three more periodicities, including one of about 6 hr. It is suggested that the 4-hr period currently observed is only a first overtone pulsation and that the 6-hr period is the fundamental. The period of the 4-hr pulsation is variable although the form of the variation cannot be uniquely determined.

The phase relationships between the light, colour and velocity variations during the 4-hr period are investigated and the temperature variation estimated. The phase relationships are the same as for other β CMa stars for which these parameters have been observed; that is, maximum light and temperature occur at, or very close to, minimum radius.

1. INTRODUCTION

In an earlier paper on the light variations in Spica during 1968 (to be referred to as Paper I (1)) it was shown that the primary of the system is a β Canis Majoris (or β Cephei) variable with a period of about 4 hr. In addition, a light variation with the 4-day orbital period of this well known spectroscopic binary was found and shown to be due to the ellipsoidal distortion of the primary by the secondary.

Observations on five more nights in 1969 and 11 nights in 1970 have since been obtained, giving a total of 990 observations in a natural 'v' system (in 1968, 1969 and 1970) and also 240 observations in 'b' and 'u' (in 1970 only).

In addition, 41 spectra have been obtained at 6.7 \AA mm^{-1} on the 74-inch coude spectrograph at Mount Stromlo Observatory. These have been used, together with the velocities published by Baker (2), Struve & Ebbighausen (3) and Struve *et al.* (4), to redetermine the orbital parameters. The velocities were also analysed for short periods, as has been done by Smak (5). In some cases, the periods found differ significantly from those given by Smak.

2. PHOTOMETRIC OBSERVATIONS

The observations were made on the 16-inch reflector at Siding Spring Observatory, Coonabarabran, Australia. For the 1968 observations (Paper I) and on the

first two nights of the 1969 series, a refrigerated RCA 1P21 photomultiplier was used. Subsequently, it was replaced by an uncooled EMI 6094S. The filters are (nominally) the same as those specified for the *UBV* system by Johnson & Morgan (6); the auxiliary equipment was detailed in Paper I—a General Radio d.c. amplifier, a voltage to frequency converter and an electronic counter.

For the three-colour observations a 30 s integration time was used but when only *v* observations were made, the integration time was varied between 50 and 100 s according to the airmass (rarely above 2.0) and the quality of the night. The colour system will be referred to as '*ubv*'; conversion was not made to the *UBV* system since the light and colour variations were so small as to be virtually unaffected by the transformation coefficients.

The comparison stars θ Vir and 73 Vir have been discussed in Paper I, as also has the method of reduction. We remark again here, that because of the large angular distance of these stars from Spica and since they are respectively 3.4 and 5.0 mag fainter than Spica (and therefore measured less accurately), an extinction coefficient was determined for each night, together with a 'gain drift' polynomial in time. The minimum order of the polynomial was chosen (generally about 3 to 5) that gave a constant magnitude throughout the night for each comparison star, with an r.m.s. deviation close to that expected for an individual integration. The term 'gain drift' is used because in general the brightness of all stars increased during the night by about 0.5 per cent per hour. The effect is not due to the G.R. amplifier, but may be at least partly due to temperature effects on the filter and photocell. It seems too large to be explained by varying extinction and since the Fabry lens and photomultiplier are rigidly mounted together (and the lens well focused), the effect is not due to flexure of the system. This effect is well known to photometrists and we mention it since it has a very significant effect on our necessarily non-differential method of variable star photometry of bright stars with a small telescope.

As in Paper I, we shall use the term '1 m.mag' to refer to 0.001 mag.

For the colour reductions on the seven nights in 1970, we used the following formulae:

$$(b-v)_o = (b-v)_{\text{obs}} - \text{Air Mass} \{k_{b-v} - 0.038 (B-V)\} + A_1 t + A_2 t^2 + \dots$$

$$(u-b)_o = (u-b)_{\text{obs}} - \text{Air Mass} \{k_{u-b} - 0.041 (U-B)\} + B_1 t + B_2 t^2 + \dots$$

where $(b-v)$ and $(u-b)$ are the colours on the natural system outside the atmosphere (subscript 'o') and observed ('obs'), k is the extinction coefficient, the A 's and B 's are constants and t the time with respect to midnight. The 'gain drift' terms in the colours were considerably smaller than those in the *v* magnitudes. The colour terms in the extinction were found both from two nights of measuring blue stars at high air mass and from α , θ and 73 Vir, once it was established that α Vir varied very little in colour. It should be noted that the $(U-B)$ term in the $(u-b)$ extinction applies only to B and A0 stars. The term is very important; its omission would cause an apparent change in the value of $(u-b)$ for (α Vir— θ Vir) by 4.1 m.mag over a range of one air mass.

3. PHOTOMETRIC ERRORS

Table I shows the r.m.s. errors, in m.mag, of individual 60 s integrations for *v* and 30 s integrations for the colours of α Vir, θ Vir and 73 Vir.

TABLE I

r.m.s. errors (in m.mag) of individual integrations

	<i>V</i>	<i>v</i> r.m.s. (60 s)	(<i>b-v</i>) r.m.s. (30 s)	(<i>u-b</i>) r.m.s. (30 s)
α Vir	0.97	2.0	3.2	3.4
θ Vir	4.38	2.7	3.4	4.1
73 Vir	5.93	5.8	4.2	5.8

Accurate measurements were made of the ratio of the input resistances of the G.R. amplifier. They show clearly a change in the ratio of the 10^7 and 10^8 ohm steps by 5 m.mag over a 20°C change in dome temperature. The voltage range steps were found to be constant to about 0.4 m.mag. Appropriate gain corrections have been applied to each night, so that the magnitudes to be discussed refer to real differences in the *v* magnitudes.

There appeared to be real changes in the magnitude difference $v(\theta \text{ Vir} - 73 \text{ Vir})$; in particular, on 1968 June 17, the value was 16 m.mag brighter than the mean nightly value for 1968. This is over five times the standard deviation for the other nights. Since the value of $v(\alpha \text{ Vir} - \theta \text{ Vir})$ lies well *below* the 4-day curve for this night, we suspect that it is θ Vir which varies. Its variations, however, were never detected *during* any one night's observations, and are not expected to have significantly affected the 4-hr light curve in α Vir.

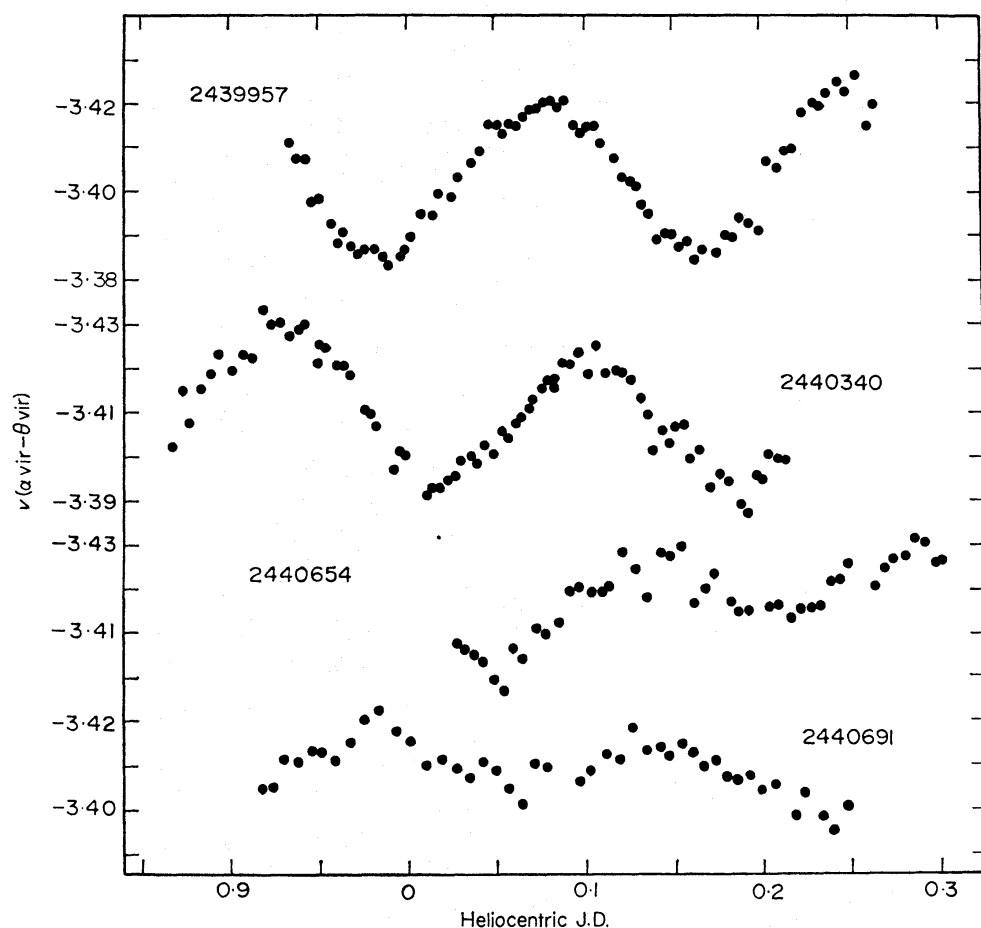


FIG. 1. *Four light curves for α Vir on the dates shown. The top one is from 1968, the second from 1969 and the lower two from 1970.*

4. PHOTOMETRIC RESULTS

4.1 The 4-hr period

Fig. 1 illustrates four light curves for α Vir, showing clearly the change in the mean amplitude of the variations from 1968 to 1970. From the 1968 data alone, we obtained a period for the variation of $0^d.173765$, using the Fourier analysis computer program described in Paper I. This period analysis was affected by the light variations with the 4-day period and Smak (5) determined a different period of $0^d.173795$ from the times of maxima and minima in our published 1968 data.

We have treated all our 990 observations to remove as much as possible of the

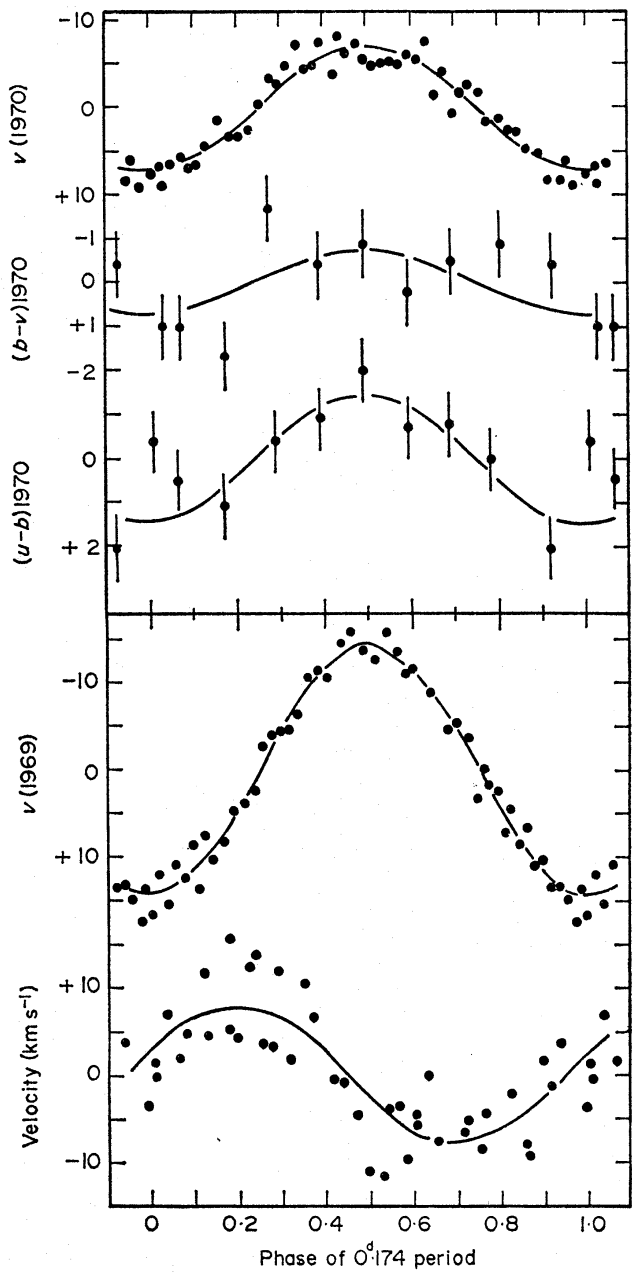


FIG. 2. Upper section: mean light and colour curves from seven nights in 1970. Lower section: mean light and velocity curves for 1969. The v curves have been normalized to phase 0.5. The ordinate units for the v , $(b-v)$ and $(u-b)$ curves are $m.mag$ ($=0.001 mag$). The errors bars shown for the colours are $\pm 0.7 m.mag$.

orbital light variation, by optimizing the parameters involved in the formula derived in Paper I. We have been able to remove only 85 to 90 pc of this variation, assuming it is due only to ellipsoidal distortion of the primary. The residual orbital variation (3 m.mag range) will have an insignificant effect on the determination of the 4-hr period.

The short period has been re-determined by applying the Fourier analysis program to these 'treated' data. The value is $0^d.1737853 \pm 0^d.0000004$. When this period was subtracted from the data and a search made of the frequency spectrum from 10^6 days to $0^d.05$, no other periods were found with a significant amplitude. It is conceivable, in view of the progressive, if erratic, decrease in the amplitude of the 4-hr period that there is a beat period of the order of several years. There is no correlation of the amplitude variations with the orbital period and they are so random that we are inclined to treat them as 'noise' fluctuations in the pulsation of the star and not to assume undiscovered periodicities.

The ephemeris for the 4-hr period over the interval 1968 to 1970 is:

$$t_{\text{maximum light}} = \text{J.D.} - 2439927.0128 + 0.1737853 E \\ \pm 0.0010 \pm 0.0000004$$

4.2 *Colour variations and phase relations*

Since the colour variation was not detected on any one night, all 240 30 s integrations in (*b-v*) and in (*u-b*) from the seven nights were combined; a least squares fit for each colour was made to a sine curve with the above 4-hr period.

The top half of Fig. 2 shows the mean *v* magnitude variation, the mean (*b-v*) and the mean (*u-b*) variations during seven nights in 1970. The data are equal r.m.s. error points (with the error bars of 0.7 m.mag shown for the colours) and are averages of four integrations for *v* and of 24 integrations for the colours.

The lower half of Fig. 2 shows the mean *v* magnitude variation and the mean 4-hr velocity variation (discussed later) for 1969. The mean ranges of all the variations in Fig. 2, the phases (with maximum light at 0.5) and the standard deviations of the parameters are as follows.

1970	Magnitude range	14.3 ± 1.0 m.mag
	Phase of maximum light	0.500 ± 0.011
	(<i>b-v</i>) range	1.5 ± 0.8 m.mag
	Phase of max. negative (<i>b-v</i>)	0.48 ± 0.10
	(<i>u-b</i>) range	3.0 ± 0.8 m.mag
	Phase of max. negative (<i>u-b</i>)	0.47 ± 0.05
1969	Magnitude range	29.0 ± 1.2 m.mag
	Phase of maximum light	0.500 ± 0.007
	Velocity range	15.8 ± 2.0 km s ⁻¹
	Phase of mean velocity at minimum star radius	0.445 ± 0.025

It is evident that maximum temperature occurs at maximum light, as with other βCMa stars. Although maximum light lags behind minimum radius by 0.055 ± 0.032 of the phase (or 14 ± 8 min) this difference is of limited significance; other βCMa stars do, however, show similar phase differences.

4.3 Temperature variation

If we assume that the primary is 6.4 times as bright as the secondary (7), and that the colours of the two stars are the same, then in 1970 the v flux ($\mathcal{F}(v)$) of the primary changed by 1.55 ± 0.10 pc from maximum to minimum in the 4-hr cycle. $\mathcal{F}(u)/\mathcal{F}(b)$ and $\mathcal{F}(b)/\mathcal{F}(v)$ changed by 0.32 ± 0.10 and 0.16 ± 0.10 pc, respectively.

It seems reasonable to suppose that since the light variation in 1970 was half that observed in 1969, the velocity variation in 1970 (had it been observed) would have been approximately halved also. With this assumption, using the value $8.1 R_{\odot}$ for the radius of the primary (7), we find that the contribution of an assumed *radial* pulsation to the change in v brightness is 1.0 ± 0.3 pc. This is due to the change in surface area only. Since the surface area is *smallest* at *maximum* temperature, the total change in v due to the temperature variation is 2.6 ± 0.4 pc.

To find the temperature variation from the ($b-v$) changes, we plotted the ratio of the flux at the effective wavelength of the b filter to the flux at the effective wavelength of the v filter against T_e ; the model atmospheres of van Citters & Morton (8) and Bradley & Morton (9) were used. We then interpolated to find the slope at the temperature of α Vir. The temperature changes indicated by the ($u-b$) and v variations were found similarly. The results are

$$\text{from } \Delta v: \quad \Delta T = 300 \pm 45^{\circ}\text{K},$$

$$\text{from } \Delta(b-v): \quad \Delta T = 320 \pm 200^{\circ}\text{K},$$

$$\text{from } \Delta(u-b): \quad \Delta T = 90 \pm 30^{\circ}\text{K}.$$

These results show that the observed ($u-b$) variation is far smaller than would be expected from the change in v . Watson, in his recent study (10), has found that generally the ratio $\Delta v : \Delta(u-b)$ (on a narrow-band photometric system) lies between 1 : 1 and 2 : 1. However, he discovered that the ratio for β Cru was more than 5 : 1, which is evidently, in spite of the different colour systems, comparable with that observed for α Vir.

5. RADIAL VELOCITIES OF α VIRGINIS

5.1 New radial velocities

The data used in this analysis are from 41 spectra on which the lines of both the primary and the secondary are visible separately. They are all Eastman Kodak IIaO plates taken on the coude spectrograph of the 74-inch Mount Stromlo reflector at 6.7 \AA mm^{-1} reciprocal dispersion. Thirty-eight were taken by R.R.S., two by Dr J. Norris and one by Dr R. Watson.

The plates were measured on the oscilloscope setting device at Mount Stromlo (11) by N.R.L. As the primary lines are broad, shallow and often asymmetric, it was attempted to set consistently on the region near the peak of the profile. The lines of the secondary appeared to be much sharper but were often difficult to distinguish from grain noise. During the measurement an integer weight was assigned to each line; this weight ranged from zero up to eight.

The reductions were carried out by a computer program, the positions of the comparison lines being fitted by the method of least squares to a sixth order polynomial. The r.m.s. deviation from this curve was about 0.012 \AA . The star lines used and their rest wavelengths (Table II) were the same as those used by

Struve *et al.* (4), except that two additional lines, H8 and H ϵ , were measured. The lines He I 4120 Å and Mg II 4481 Å were found to have systematic deviations of between -15 and -30 km s $^{-1}$ from the mean and were not used in its determination. In spite of the high dispersion, He I 4120 Å is probably blended with O II 4119 Å and Mg II 4481 Å with Al III 4480 Å.

TABLE II
List of line wavelengths

Wavelength (Å)	Element	Wavelength (Å)	Element
3889.051	H8	4143.759	He I
3970.074	H ϵ	4267.15	C II
4009.270	He I	4340.468	H γ
4026.218	He I	4387.928	He I
4101.737	H δ	4471.507	He I
4120.857*	He I	4481.228*	Mg II

* The velocities from these lines were not used (see text).

The remaining star lines nearly all showed systematic deviations although they were smaller than those of the rejected lines. In order to take this into account, as well as the fact that some line positions were measured more accurately than others, the following method of weighting was adopted.

- Step 1. Using the integer weights assigned during measurement, a mean velocity was calculated together with the standard deviation of a measurement of unit weight.
- Step 2. The weight of each line velocity was then reduced, if necessary, so that the deviation of the line velocity from the mean value was less than the standard deviation of a measurement of that weight, i.e.

$$|v_L - \bar{v}| < \frac{\sigma_1}{\sqrt{k}}$$

where σ_1 is the s.d. of an observation of a measurement of unit weight,

k is the new weight of the line,

v_L is the line velocity,

and \bar{v} is the mean velocity.

- Step 3. A new weighted mean was then calculated.

The final product of the program was the heliocentric mean velocity together with the heliocentric Julian date of the observation. The internal error of each measurement of primary velocity was about 2.7 km s $^{-1}$, while for the secondary it was about 5 km s $^{-1}$.

Nearly all the lines measured show some systematic deviation from the mean velocity. These deviations are different for primary and secondary lines and also vary with the phase of the orbit. For example, H γ shows deviations of $+8 \pm 2$ km s $^{-1}$ and -6 ± 5 km s $^{-1}$ for the primary and secondary respectively near periastron and $+1 \pm 1$ km s $^{-1}$ and $+24 \pm 7$ km s $^{-1}$ just after apastron. These deviations can probably be explained by blending with unresolved lines and blending between

primary and secondary components. A further effect probably caused only by blending is that the mean difference between the velocities of the hydrogen and helium lines of the primary near periastron is $+15 \pm 5 \text{ km s}^{-1}$, whereas just after apastron the mean difference is $-2 \pm 1 \text{ km s}^{-1}$.

The list of primary and secondary velocities measured is given in Table III.

TABLE III
Radial velocities of alpha Virginis in 1969

Helio. J.D.	V_1 km s ⁻¹	V_2 km s ⁻¹	Helio. J.D.	V_1 km s ⁻¹	V_2 km s ⁻¹
2440 +			2440 +		
283.0008	+92.9	-145.8	283.2126	+104.9	-167.9
283.0175	+96.0	-167.1	283.2231	+105.0	-153.8
283.0279	+96.8	-131.6	283.2335	+104.3	-157.8
283.0376	+107.1	-159.9	283.2453	+102.9	-169.2
283.0467	+101.1	-152.1	283.2543	+107.8	-132.8
283.0557	+109.2	-161.6	283.2627	+100.7	-162.5
283.0675	+108.8	-159.2	283.2717	+96.6	-163.9
283.0779	+107.7	-150.2	283.2821	+89.7	-159.5
283.0925	+96.8	-170.3	283.2883	+97.8	-166.4
283.1029	+87.0	-166.1	317.1800	-148.4	+191.7
283.1113	+94.4	-142.3	344.9312	-103.6	+169.1
283.1217	+94.1	-170.7	345.0589	-126.8	+144.0*
283.1300	+91.3	-162.8	364.9004	-70.0	+127.3
283.1404	+92.5	-165.3	364.9101	-74.0	+127.1
283.1495	+95.0	-160.5	365.8434	-91.0	+136.6
283.1592	+97.5	-163.3	365.8475	-84.7	+128.6
283.1668	+90.4	-155.7	365.8635	-87.3	+136.3
283.1786	+103.7	-164.2	265.8691	-89.6	+126.1
283.1890	+96.6	-165.1	365.8871	-86.3	+121.1
283.1960	+107.1	-167.0	365.8934	-75.3	+132.6
283.2036	+105.2	-162.6			

* Indicates rejected velocity.

5.2 The orbit of α Vir

(a) *The computer program.* From the new radial velocities it is evident that, as expected, there is a 4-hr period in the velocities of the primary. In order to obtain the best possible values of the orbital parameters this short period must also be taken into account. The orbital reduction program employed fits the equation

$$V_1 = \gamma_1 + K_1\{e \cos \omega + \cos (v + \omega)\} + A \sin \{2\pi/p(t - \tau)\}$$

to the primary velocities and simultaneously fitted

$$V_2 = \gamma_2 + K_2\{e \cos (\pi + \omega) + \cos (\pi + v + \omega)\}$$

to the secondary velocities. The program also allows for the rotation of the line of apsides. The parameters fitted by the program are

- γ_1, γ_2 the mean velocity of the primary and secondary, respectively.
- K_1, K_2 the amplitude of the orbital variation of the primary and secondary, respectively.
- T the time of periastron passage.
- ω the longitude of periastron at T .

- e* the eccentricity of the orbit.
- P* the period of orbital motion taken from *periastron to periastron*.
- Q* the period of rotation of the line of apsides.
- A* the amplitude (half the *range*) of the short period variation.
- p* the period of the short period variation.
- τ* zero phase for the short period variation.

Any of these parameters could be treated either as an unknown or as a constant, in which case a value would be inserted for that parameter. The optimization was performed using a simplex method (12). This program is described in detail in a paper by Herbison-Evans & Lomb (13).

TABLE IV
Orbital parameters of α Virginis

Epoch	1908	1934	1956	1969	Adopted
γ_1 (km s ⁻¹)	3 ± 2	-2 ± 2	-1 ± 2	0 ± 1	0 ± 2
K_1 (km s ⁻¹)	129 ± 2	128 ± 1	120 ± 1	120 ± 2	124 ± 4
γ_2 (km s ⁻¹)	1 ± 3	2 ± 3	9 ± 3	-2 ± 2	2 ± 3
K_2 (km s ⁻¹)	209 ± 2	201 ± 2	192 ± 1	189 ± 3	197 ± 8
<i>e</i>	0.11 ± 0.02	0.11 ± 0.01	0.15 ± 0.01	0.18 ± 0.02	0.14 ± 0.03
<i>P</i> (days)	4.0149 ± 0.0003	4.01447 ± 0.00005	4.0145 ± 0.0002	(4.01450)	4.01454 ± 0.00003
ω (°)	328 ± 9	39 ± 9	97 ± 8	145 ± 7	142 ± 8
<i>T</i> (j.d.)	2417955.84 ± 0.09	2426041.26 ± 0.10	2435563.57 ± 0.09	2440284.78 ± 0.07	2440284.76 ± 0.08
<i>Q</i> (years)	(130)	(130)	(130)	(130)	128 ± 12
<i>a sin i</i> (km)	1.85 × 10 ⁷ ± 0.03	1.80 × 10 ⁷ ± 0.02	1.70 × 10 ⁷ ± 0.02	1.68 × 10 ⁷ ± 0.03	1.76 × 10 ⁷ ± 0.08
$m_1 \sin^3 i$ (<i>M</i> _☉)	9.7 ± 0.4	8.8 ± 0.3	7.5 ± 0.2	7.1 ± 0.4	8.2 ± 1.0
$m_2 \sin^3 i$ (<i>M</i> _☉)	6.0 ± 0.3	5.6 ± 0.2	4.7 ± 0.1	4.5 ± 0.3	5.2 ± 0.7
m_1/m_2	1.62 ± 0.04	1.58 ± 0.03	1.60 ± 0.03	1.58 ± 0.05	1.59 ± 0.03
s.d. prim. (km s ⁻¹)	10.7	7.5	7.0	4.5	
s.d. second (km s ⁻¹)	15.0	13.0	4.3	10.1	

Values in brackets are assumed (see text).

(b) *The orbital elements.* For the 1969 velocities *Q* was fixed at 130 years and because of the short baseline, *P* was also fixed. For the first run of the program all points were assigned weight unity; this run indicated the r.m.s. residuals about the primary and secondary velocity curves. In subsequent program runs, each primary

and secondary velocity was given a weight inversely proportional to the r.m.s. error about its respective curve.

There are three earlier lists of velocities of Spica from which we have re-determined the orbital and short period parameters using the above program. These are the lists of Baker (2) in 1909, Struve & Ebbinghausen (3) in 1934 and Struve *et al.* (4) in 1958. We used only the velocities which were determined from spectra in which primary and secondary lines were evidently fairly clearly resolved; that is, either both primary and secondary velocities were published, or the primary velocity was so large that blending was unlikely to be serious. In each case Q was fixed at 130 years and weights were assigned in the same manner as for the 1969 data. The results are given in Table IV, which does not include any of the parameters of the short period as these will be dealt with in the next section. It suffices to say here that for each epoch the period, amplitude and zero point found for the short period was in close agreement with the values found by the Fourier analysis methods discussed in the next section.

It is apparent from Table IV that there is a variation in the K 's which is outside the errors. However, each set of velocities was obtained using different lines, dispersions and measuring apparatus, so one must expect the blending effects to be different for each series. Consequently, we believe the variations in the K 's can be ascribed to instrumental and measuring effects. Similarly, the variation in γ_2 is probably not real.

The final γ 's, K 's and e are derived after an inspection of the elements at the different epochs. P and T come from a least squares fit to the times of periastron determined at the four epochs; Q and ω also come from a least squares fit. The errors quoted for the final parameters are estimated from the scatter between the four values; note that they are often larger than the individual internal errors of each series.

5.3 *Fourier analysis of the radial velocity residuals*

The data used in this section were residuals from the primary orbital velocity curves that were calculated for each series of velocities in the previous section. In addition, Vogel's velocities of 1889 to 1891 (14) were also investigated after removal of the orbital variation. The ratio of the amplitude of the β CMa sine waves in these orbital curve residuals to the final r.m.s. deviation (i.e. the signal to noise ratio) is very low—about 1 : 1. Also, the distribution of observing times is unfavourable to the determination of short periods. Consequently, we discuss below the analysis of the data in some detail.

(a) *Method of analysis.* The computer program employed fits by least squares a series of sine curves of different frequencies to the data. The reduction in the sum of squares of the residuals due to each sine wave is then plotted against frequency. This is then the frequency spectrum of the observations and the peaks correspond to real periods in the data and to their aliases. The alias structure due to a single period is similar to that in periodogram analysis (15). Basically, there is a set of aliases surrounding the main peak at constant intervals of about $1/T$ cycles per day (c/d), where T is the total time span of the observations. The relative heights of these aliases depend largely on the lengths of the intervals between observations. As groups of observations are necessarily separated by one day or multiples of one day, there is a similar set of aliases at separations of about 1 c/d

from the true frequency. The heights of these aliases depend on the range of the sidereal times of each group of observations, in the sense that the greater the range, the more rapid is the decrease in the heights of the aliases.

Some of the data to be analysed extended over several years, consequently it was necessary to have a method of broadening the peaks. The method used, which was similar to that suggested by Fitch (16), consisted of subdividing the observations into groups extending over equal times (Δt), fitting a sine curve to each group and then summing the reductions in the residuals. The widths of the peaks are then proportional to $1/\Delta t$. If the spacing of the observations did not allow this then the analysis was carried out not on all the points but on the largest consecutive subset of the observations. Once a main peak was found a 'high-resolution' spectrum of its frequency region was calculated using all the points and the full baseline of the observations.

Ambiguities could arise due to either set of aliases. Where it was difficult to distinguish the true peak from its $1/T$ c/d aliases a 'synthetic' curve with the same frequency and amplitude as the most likely period was analysed. This synthetic curve was of course sampled at the same Julian dates as the observations. Comparison of the synthetic spectrum with the observed spectrum usually revealed the correct peak. To differentiate between a period and its 1 c/d aliases the heights of the aliases were plotted against frequency. This gave, at least approximately, the centre of the envelope of the aliases which should be the correct period. A comparison with the relative heights of the 1 c/d aliases of a synthetic curve was usually also made.

(b) *1969 residuals.* The spectrum was searched from $0^{\text{d}}.18$ to $0^{\text{d}}.16$. A whole series of nearly equal height peaks was obtained. This is not surprising in view of the distribution of observing times—30 on one night and the remaining 11 spread over 82 days. However, one peak was slightly larger than the others and as it gave a period which was close to the period of the light variations it was accepted as the main period. The data were then 'pre-whitened' by subtracting this period and the spectrum searched from $0^{\text{d}}.3$ to $0^{\text{d}}.085$. No other periods were found.

(c) *1956 residuals.* The 1956 observations consisted of 30 observations spread over 96 days and six observations more than a year later. In consequence, in the preliminary search of the spectrum only the first 30 points were used. With these points the spectrum was searched from $0^{\text{d}}.3$ to $0^{\text{d}}.15$. Four main peaks were found: $0^{\text{d}}.17381$, $0^{\text{d}}.17279$, $0^{\text{d}}.15986$, $0^{\text{d}}.15905$. The frequency difference between the first two and the last two peaks is 0.5 c/d—a result of the spacing of the observations. When these peaks were searched with all the observations the first period gave the best fit, the second slightly worse and the last two substantially worse. In the absence of further information differentiation between the first two periods would be difficult but as the first agrees with the period of the light variations that was taken as the true period. In order to check whether the highest peak of the 'high resolution' spectrum was the correct one a synthetic curve was analysed. This synthetic spectrum was in close agreement with the observed spectrum with regard to shape, relative heights of the aliases and the amplitudes of the aliases.

The period thus found was then subtracted from the data and the spectrum searched from $14^{\text{d}}.0$ to $0^{\text{d}}.12$. No clear period was obtained although the spectrum was extremely noisy.

(d) 1934 *residuals*. The data were grouped in lots of 130 days and searched from $0^d.15$ to $0^d.2$. One peak ($0^d.1738$) was clearly the largest—this was searched with the full baseline of the observations. Three nearly equal peaks were found. To differentiate between them a synthetic curve with the frequency of the central peak was analysed. Again a very good fit was obtained showing that the central peak is the correct one. In Table V the amplitudes and percentage heights of the four nearest $1/T$ c/d aliases of the main period are given, both for the observed spectrum and the synthetic spectrum. The period chosen by Smak (5) is the first alias of the correct period.

TABLE V

The fit of a synthetic curve of period $0^d.173790$ to the 1934 R.V. residuals

Period (days)	$0^d.173831$		$0^d.173811$		$0^d.173790$		$0^d.173770$		$0^d.173750$	
	Obs	Syn	Obs	Syn	Obs	Syn	Obs	Syn	Obs	Syn
Amplitude (km s^{-1})	6.1	6.3	8.2	8.2	9.1	9.1	8.2	8.3	6.9	7.2
Percentage height	57	60	85	87	100	100	89	90	65	71

The data were then prewhitened with this period and searched from $7^d.0$ to $0^d.1$ but this time in groups of 71 days. Three possible periods were found: $0^d.496$, $0^d.180$, $0^d.122$. All three were characterized by the fact that they seemed to exist in only two of the four groups into which the data had been divided. Of the three periods $0^d.180$ was the only one to appear in consecutive groups and also, in contrast to the other periods, it had a similar amplitude in both groups. Thus this is the period which is most likely to be real. The groups of data in which this period was found were the first two which consisted altogether of 25 points spread over 492 days with a 300-day break in the middle. These 25 points were analysed separately with the $0^d.173790$ period removed. The best period seems to be $0^d.1796$ which has a range of 21.0 km s^{-1} , however, due to the small number of points there is considerable ambiguity both with the $1/T$ c/d aliases and the 1 c/d aliases.

(e) 1908 *residuals*. The data were searched from $0^d.17$ to $0^d.18$. A small peak was found at $0^d.17382$ but there were many other peaks of comparable height in the region. Thus the peak at $0^d.17382$, which Smak (5) takes as the period in 1908, can probably be attributed to noise. To see if there is any other period present, the data was grouped in 90 day lots and searched from $0^d.4$ to $0^d.14$. Three possible periods were found: $0^d.201$, $0^d.252$ and $0^d.338$ but only one of these can be real as their frequencies are separated by 1 c/d. On searching through these peaks with the full baseline of the observations the $0^d.252$ period gave the best fit. As the original orbital fit to the velocities had assumed a period near $0^d.1738$ a new orbital reduction was carried out using the $0^d.252$ period and a new set of residuals was calculated. With these new data a plot of the height of the 1 c/d aliases was made against frequency. From this it was seen that while the $0^d.252$ period was the most probable the $0^d.338$ period was also a possibility. However, a synthetic curve of period $0^d.252$ gave a much better fit to the observed aliases than a synthetic curve of $0^d.338$ period. Fig. 3 shows the fit of the $0^d.252$ period to the 1908 velocities. The reduction in the sum of squares for the $0^d.252$ period is 48 per cent while for Smak's assumed period of $0^d.17382$ it is only 23 per cent.

The new residuals were prewhitened with the $0^d.252$ period and the spectrum searched from $10^d.0$ to $0^d.1$. No other significant period was found.

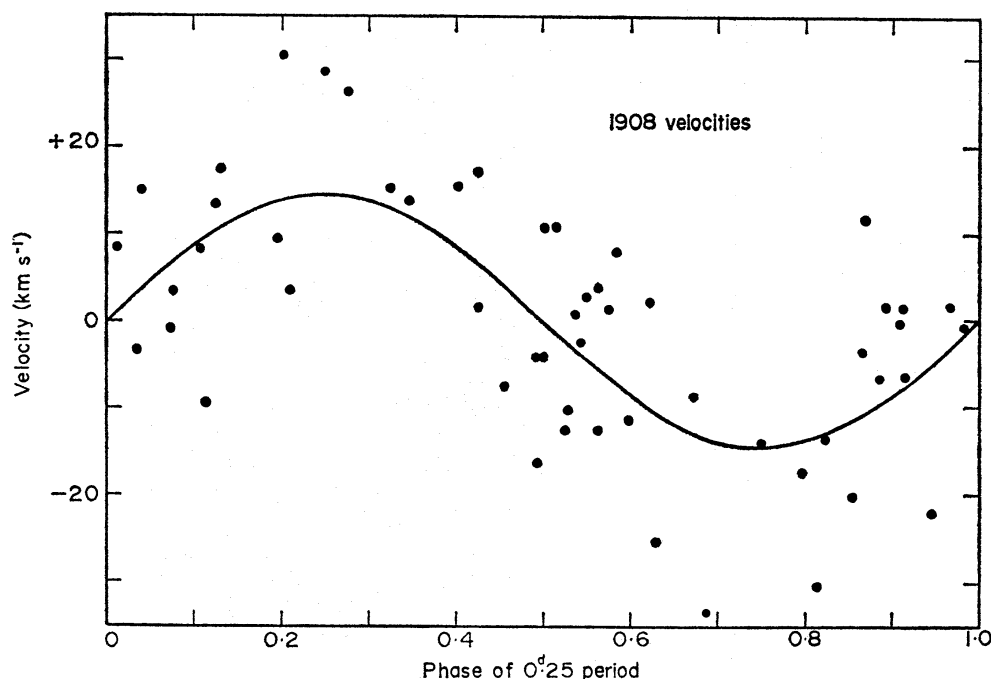


FIG. 3. This graph shows the fit of the 0.25219 period to the 1908 velocities of Baker (2).

(f) 1890 residuals. These data are of low accuracy as the velocities are based only on the measures of one blended line. They consist of three observations in 1889, 21 in 1890 and three in 1891. As a first step only the 1890 observations were used. A set of possible periods with their frequencies separated by steps of 1 c/d were found. All eight of these periods in the region 0^d.11 to 0^d.48 were then searched with the full baseline of the observations. The heights of the peaks were then plotted and compared with the aliases of synthetic curves. No unique identification could be made—the two which gave the best fit are given in Table VI.

TABLE VI

Short Periods: results of the Fourier analyses of the orbital velocity residuals

Epoch	Heliocentric J.D. (max. vel.)	Period (days)	Range (2 A) (km s ⁻¹)	Percentage reduction in Σ(residuals) ²
1969	2440283.047 ±0.004	0.17380 ±0.00002	16 ±2	66
1956	2435561.068 ±0.007	0.173796 ±0.000005	17 ±3	51
1934	2425977.123 ±0.009	0.173790 ±0.000001	18 ±3	38
1908	2417686.656 ±0.013	0.25219 ±0.00001	31 ±5	48
1890	—	0.1961 or 0.2439	22 22	37 37

6. DISCUSSION

6.1 Variation of the 4-hr period

As we have already mentioned, the period determined by Smak (5) from the 1934 velocities is an alias of the correct period and the period of 0^d.17382 which he

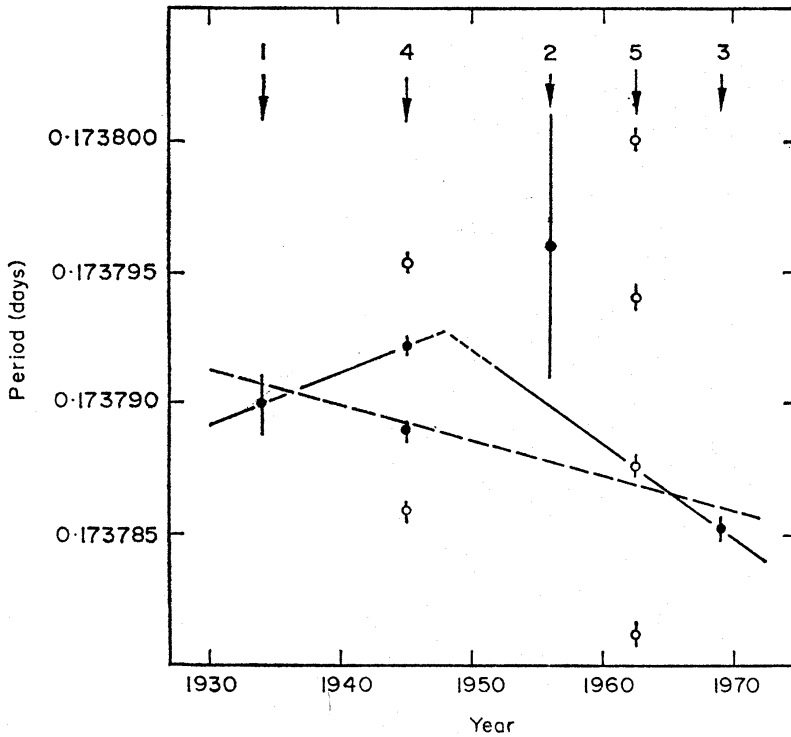


FIG. 4. Possible variations of the 4-hr period. The plot is fully described in the text (Section 6.1).

found in the 1908 velocities seems to be a noise peak in the frequency spectrum. Consequently, we are unable to accept his value of a 5 s per century decrease in the 4-hr period.

However, there does appear to be a change in the 4-hr period from the time of its appearance in published data in 1934 up to 1970. Fig. 4 shows, with standard deviation error bars, the following data.

1. The period of variation in the 1934 velocities (from Table VI).
2. The period of variation in the 1956 velocities (from Table VI).
3. The period of variation in the 1968, 1969 and 1970 magnitudes (from Section 4.1).
4. Four possible values of the mean period between 1934 and 1956 corresponding to four consecutive integer numbers of oscillations over this time baseline.
5. Four possible values of the mean period between 1956 and 1969 (from velocities alone) corresponding to four consecutive integer numbers of oscillations over this time baseline.

The frequency spectrum for the 1934 and the 1956 velocities together shows two equal height peaks (the filled circles at date 1945 in Fig. 4) and two lower peaks (the open circles). The two filled circles thus represent the most probable values of the mean period over this interval. Unfortunately, the four highest peaks in the frequency spectrum of the combined 1956 and 1969 velocities were of closely similar heights; thus the analysis does not indicate which of these four values of the period is most probable.

The dashed line shown on Fig. 4 represents a possible value of the rate of decrease of the 4-hr period if the rate is assumed to be linear; this line corresponds

to a decrease at the rate of 1.0 s per century. However, it is evident that the data might be fitted just as well by an increase in the period until about 1950, followed by a decrease over the last 20 years. It is clear that the period is variable but the large time intervals between the sets of published data do not permit an unambiguous determination of the form of the change.

6.2 *The other short periods*

The other period present in the 1934 velocities, the most probable value of which is 0.1796, seems to indicate that α Vir may have exhibited another typical β CMa star phenomenon—that of two similar periods. The period of the beat between the 0.1738 and the 0.1796 periods is a little over 5 days.

The existence of the 0.25219 (6-hr) period in 1908 and the absence of the 4-hr period presents an interesting problem. If the 6-hr period is the fundamental period of pulsation of the star, then the period observed at the present time (4 hr) is presumably the first overtone. This pulsation mode might be expected to be less stable than the fundamental, which may be the explanation for the apparently random variations in the amplitude over the three years of photometric observations. The $\log P$ of the fundamental period predicted for α Vir by Watson from his $\log P/\theta_e/\log g$ relation (10) is -0.71 . This is in better agreement with the 4-hr period ($\log P = -0.76$) than with the 6-hr period ($\log P = -0.60$). However, we note that Watson's relation is defined only by the slow rotating 'classical' β CMa variables and in fact for two other fast rotating β CMa stars, λ and κ Sco, the observed values of $\log P$ (17) are 0.10 greater than those predicted by Watson. Consequently, the poor fit of the 6-hr period to the $\log P/\theta_e/\log g$ relation may not preclude the possibility of its being the fundamental period.

In the 1890 velocities there is no evidence for any of the three subsequently observed periods of 0.2522, 0.1796 or 0.1738. Although there is some uncertainty in the period at this time, it is not one of the above three values.

In summary we may state that in 1908 the star was pulsating only in the fundamental mode; in 1934, the first overtone and another close period were evident, although this latter period had vanished by the end of the interval over which these velocities were obtained. Finally, in the 1956 and 1969 velocities and in the 1968 to 1970 magnitudes only the first overtone period is evident.

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NOTE ADDED IN PROOF

On four nights of observation in 1971, the light of α Vir varied during each night by no more than 5 m.mag; no periods can be determined.