The Shortest Path Problem Revisited: Optimal Routing for Electric Vehicles

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The Shortest Path Problem Revisited: Optimal routing for Electric Vehicles

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Introduction. Electric vehicles (EV) powered by batteries will likely play a significant role in the road traffic of the future: they can be powered by regenerative energy sources such as wind and solar power, and they can recover some of their kinetic and/or potential energy during deceleration phases. However, the unique characteristics of EVs – limited cruising range, long recharge times, and the ability to regain energy during deceleration – have an impact on algorithms used in navigation systems and route planners, since it now becomes more important to determine the most economical route rather than just the fastest or shortest one. This modification might appear insignificant at first glance, as it seems enough to simply exchange time and distance values with energy consumption in the underlying routing problem. But on a second glance, several new challenges surface, which require novel algorithms that go beyond existing solutions for route search in street networks. In our work within the German electromobility model project eE-Tour Allgäu (see Fig. 1), we developed extensions to general shortest-path algorithms that address the problem of energy-optimal routing for EVs.

Figure 1: Some of the electric vehicles used in the eE-Tour Allgäu electromobility project.

Shortest path problems with constraints. We address the problem of finding the most energy-efficient path for battery-powered electric cars with recuperation in a graph-theoretical context. That is, our goal is to find an energy efficient route for a battery-powered EV in a road network with given energy values, modeled as a directed graph $G = (E, V)$ with weight function $c : E \to \mathbb{Z}$.

This problem is similar to the *shortest path problem* (SP), which consists of finding a path P in a graph from a source vertex s to a destination vertex t such that $c(P) = \min_{Q \in U}(c(Q))$, where U is the set of all paths from s to t, and $c : E \to \mathbb{Z}$ is a weight function on the edges E of the graph. SP is polynomial; the best known algorithm for the case of non-negative edges is Dijkstra (Dijkstra 1959) with time complexity $O(n^2)$, while in the general case, Bellman-Ford (Bellman 1958), (L. R. Ford 1956) has $O(n^3)$. However, most commonly used SP algorithms like contraction hierarchies (Geisberger et al. 2008), highway hierarchies (Sanders and Schultes 2005) and transit vertex routing (Bast et al. 2007) can't be applied to our problem because of the presence of negative weights that result from recuperation.

In addition, SP does not consider the constraints that result from the discharge and recharge characteristics of the EV's battery pack, namely that it neither can be discharged below zero, nor charged above its maximum capacity. These two conditions on the charge level of the battery can be viewed as *hard* and *soft constraints*, respectively, on possible routes: a route is infeasible if there is a point where the required energy exceeds the charge level, and a route is less preferred if there is a point where energy could be recuperated but the battery's maximum capacity is exceeded. An extension of the SP, the *constrained shortest path problem* (CSP) (Joksch 1966), is to find a shortest path P from s to t among all feasible paths in a graph, where a path P is called feasible if $b(P) \leq T \in \mathbb{N}$ for an additional weight function $b : E \to \mathbb{N}$. Our problem of energy-efficient routing with recuperation can be framed as such a CSP, but CSP is known to be NP-complete (M. and D. 1979).

A tractable variant of a constrained SP problem. The problem of energy-efficient routing in the presence of rechargeable batteries can then be described as follows: given a start point and battery charge level, find a route to a target point that respects the constraints and where the remaining charge is highest. We call this problem the *prefix-bounded shortest path problem*, or for short PBSP.

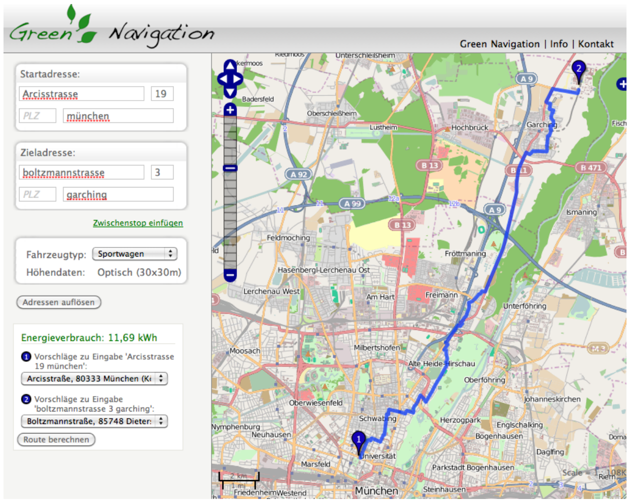

Figure 2: Screenshot of our route planner prototype, showing the energy optimal route for an EV from TUM's Munich campus to TUM's Garching campus.

If we only consider graphs with non-negative edges, the PBSP can be cast as a special case of a CSP, where the two weight functions b and c are equivalent.
However, as algorithms for CSP are exponential in the worst case, we instead propose an adaption of a general shortest path algorithm that respects the prefix-bound constraints. We could show that the additional prefix-bound constraints don't change the time complexity of the shortest path problem, and thus this algorithm solves the problem in polynomial time ($O(n^3)$ with the Bellman-Ford strategy). That is, our problem of energy-optimal routing for EVs with recuperation constitutes a tractable variant of the more general constrained shortest path problem.

Preliminary experimental results. Our proposed algorithm has been implemented and evaluated within a prototypic navigation system for energy-efficient routing, which can be accessed online at `www.greennav.org`. This system is based on OpenStreetMap (OSM) road data and freely available altitude maps of the NASA Shuttle Radar Topographic Mission (SRTM). By combining these two sources, we created a road map with elevation and cruising speed information for every point in this network. In an offline step, we then derived a graph with weights corresponding to the energy consumption of road sections. For this purpose, we used a simplistic physical model of an EV, where recuperation induces negative weights for a small percentage of edges. For a given car type, source address and destination address, our algorithm then computes a route with minimum energy costs.

Figure 2 shows the web interface to our prototype. The blue path displays the energy efficient shortest path according to the available data and vehicle model. While some of the proposed deviations from a straight (shortest) route are indeed due to energy savings, others originate from an overly simplistic vehicle model and some missing speed tags in the OSM data; future development will address these problems. Within this prototype, we evaluated our search algorithms using four different node expansion strategies. The evaluation was carried out on a section of the OSM map that covers the Allgäu region southwest of Munich, and contains 776,419 vertices and 1,713,900 edges. Preliminary results indicate that the performance of our proposed algorithm is much better than the generic CSP algorithm, and it does not suffer from its exponential worst time complexity.

Conclusion and future work. Optimal routing for electrical vehicles with rechargeable batteries will become increasingly important in the future. We began to study this problem within a graph-theoretic context. We modeled energy-optimal routing as a shortest path problem with constraints, and proposed a family of search algorithms that respect these constraints with a worst case time complexity of $O(n^3)$. Further research will study the impact of the negative/positive edge ratio in our routing graphs and the development of special tailored heuristics using the law of conservation of energy. In addition, we plan to extend our approach by modeling the energy consumption with stochastic instead of constant values for assessing the risk of running out of energy before arriving at the destination. Finally, it is interesting to extend the framework towards energy-efficient management of a fleet of EVs, for instance in car-sharing scenarios.

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