The Size-Grading of Sand by Wind

By R. A. Bagnold

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In the course of experiments on the changes which take place in the size-grading of the grains when sands are picked up, transported and redeposited by a wind, it was found that by plotting the proportional weights of the constituent sieve-separated grades on a log scale, important changes can be observed in the character of the grading which are imperceptible when the usual grading diagram is used. Let the abscissa of the usual diagram be $R = \log$ diameter, so that δR is the log ratio between successive sieve apertures. If δp is the weight of a grade (total weight of sample unity), then the ordinate $\phi = \dfrac{\delta p}{\delta R}$. With an infinite number of sieves $\phi = \dfrac{dp}{dR}$. The whole area included under the diagram is $\int \phi \, dR = 1$.

1—The Grading Diagram

Fig. 1*a* shows the grading curve of a sample of dune sand, together with a suitably chosen "normal probability curve" $\phi' = \dfrac{a}{\sqrt{\pi}} e^{-a^2(R-R_0)^2}$, where R_0 is log (most frequent diameter). Though comparison is difficult owing to the low values of the ordinates representing the extreme grades, it is sometimes assumed that the grain-size distribution may be a random effect.

In fig. 1*b* the same two curves are plotted with $y = \log_{10} \phi$ as ordinate. The probability curve becomes a parabola

$$y' = \log_e \phi' = -a^2(R-R_0)^2 + \log B,$$

while in the case of the actual sand sample the curve is seen to depart very considerably from the parabolic probability form. The curves given by the analyses of a number of sand samples, when plotted in the manner of fig. 1*b*, suggested to me that the extreme grades tended to die away in proportional weight exponentially on each side of the mean diameter. Though I can find no theoretical reason why this should be so, as an approximate assumption it provides a useful method of specifying the grading of a sand by three quantities, the mean diameter, and two coefficients defining the slopes of the straight lines of fig. 1*b*. Sands in which the grading can be specified in this way I will call "regular sands".

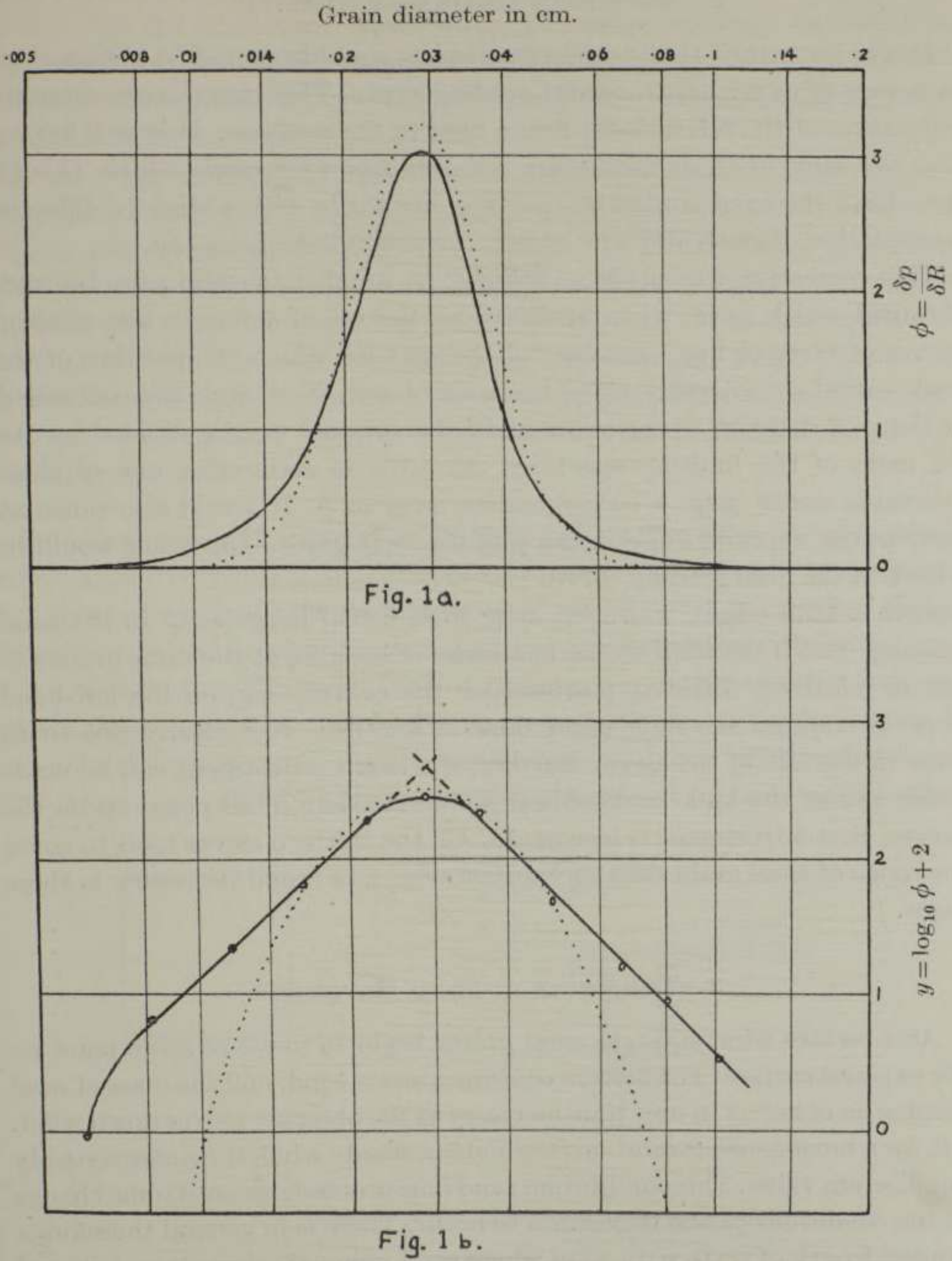

FIG. 1—Comparison of sand-grading curve (heavy line) with probability curve (dotted). *a*, ordinates on usual linear scale; *b*, ordinates on log scale.

2—Method of Grain-size Analysis

It was important that as many points as possible should be obtained on each side of every experimental grading curve. This meant very accurate estimation of the relative size limits used in the analyses. It is well known that the same analysis results are not obtained with sieves which, though they have the same apparent aperture, are made with a slightly different combination of mesh and wire gauge. A correction is necessary.

This correction was made as follows: A number of sand samples were obtained which gave, when analysed by the set of sieves in use, grading curves of more or less "regular" shape, but for which the position of the peak varied considerably along the scale of size. Now since the ordinate ϕ of the ϕ-R diagram is the quotient of the original weight divided by the log ratio of the limiting apertures, an error in estimating one of these apertures would cause a corresponding error in ϕ. It would also cause an error in the opposite sense in the ϕ of the next grade. The result would be a kink in the final grading curve.

Such a kink might, however, arise from a real irregularity in the sand grading. But if the kink occurs in a series of samples at the same grain size but in relatively different positions on the curves—e.g. on the left-hand slope of one and the right-hand slope of another—it is clearly due to an error in the size of the sieve. Further, if a slight adjustment can be made which causes the kink to disappear simultaneously in all positions on the curves, that adjustment is legitimate. Of the thirteen sieves used to cover the range of sand grain sizes such adjustment was found necessary in three cases.

3—The Cycle of Sand Movement

At a certain wind strength sand grains begin to move at some point on the exposed surface. The motion continues down wind, and the mass of sand passing an observer in unit time increases as the observer moves down wind, till, for a homogeneous sand surface under a steady wind, it reaches a steady equilibrium value. This equilibrium sand flow proceeds on until some change in the conditions causes deposition to occur. There is in general therefore a limited length of surface up wind where sand removal takes place, followed by a length in which sand is neither removed nor deposited, followed again by a length where deposition occurs.

Sand Removal—The threshold wind required to start the removal depends on the diameter of the grains which cover and protect the surface. If the wind is feeble a removal takes place of the finer grades exposed on the surface, but the movement ceases when the surface becomes stabilized by the protection afforded by exposure of sufficient large grains on the surface. This limited removal clearly continues locally for a longer time if the large grains are relatively few.

If the wind is strong enough to move the largest grains, then removal proceeds indefinitely. In this case sand is ultimately removed in the same grading proportions as those of the underlying bed. Since large grains can be rolled along the surface by the bombardment of far smaller ones even by a gentle wind, removal also takes place indefinitely if the oncoming wind is carrying a little fine sand removed from elsewhere up wind. There is a distinction, affecting the grading of the removed sand, between sand removal by direct pick-up and removal stimulated by the bombardment of on-coming grains.

Transportation—The mass flow of removed sand consists (*a*) of a small proportion of very fine grains in true suspension; (*b*) of grains in saltation over the surface; (*c*) of the surface creep of grains rolled or impelled along the surface by the forward impact of grains descending from the saltation. Owing to a continuous interchange of grains between the saltation and the surface creep a rigorous distinction is impossible. In my experiments I define the surface creep as consisting of grains of such low velocity that they tumble over into a narrow transverse slot (fig. 2 inset) if such is cut in the

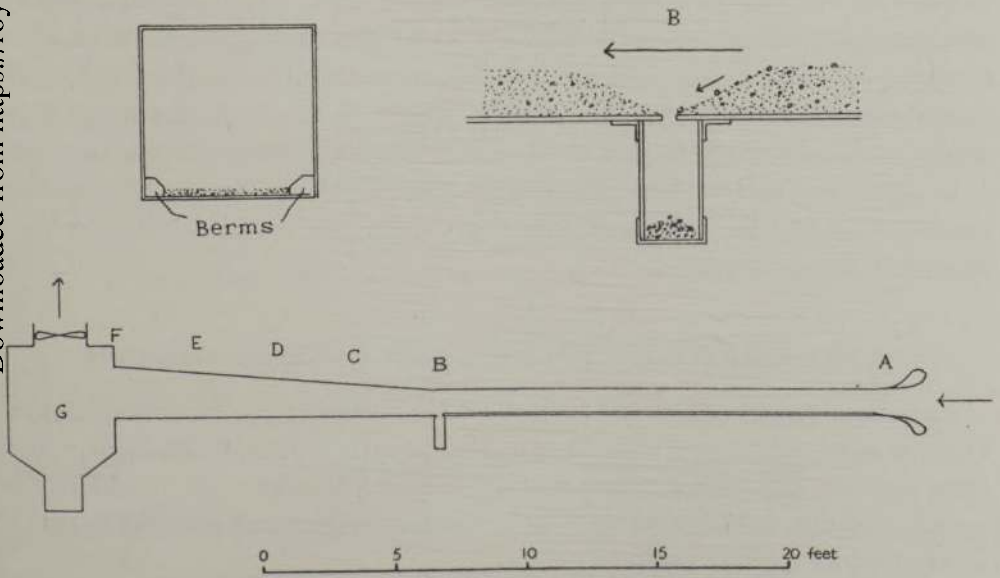

Fig. 2—General arrangement of apparatus.

floor beneath the sand bed. The rate of flow of the surface creep is of the order of 25 % of the total flow.

The surface creep consists of a large mass of sand (per unit area of surface) moving very slowly, whereas the saltation, carrying the bulk of the flow, consists of a relatively small mass moving at a speed of the order of half that of the wind. Hence in their journey down wind the grains in surface creep lag far behind those in saltation. It is doubtful, therefore, whether even over a continuous homogeneous sand bed a true homogeneous condition of sand flow can ever ensue over a considerable length of path. Wherever saltation occurs there is a complementary surface creep of relatively larger grains, and so the surface sand, except where deposition is taking place, tends to become coarser than that immediately beneath.

Deposition—There appear to be three ways by which grains reach their final resting places in a sand deposit. (*a*) True sedimentation, in which grains falling through slowly moving air strike the surface with insufficient forward velocity either to be carried on again or to jerk other surface grains forward. In the case of the very fine grains carried in true suspension this is probably the only method of deposition possible. (*b*) Accretion. The grains in saltation in a diminished wind may strike the surface with sufficient velocity to jerk other grains forward and so to contribute to a diminished surface creep, but some of the impacting grains may themselves come to rest in the process. Here the deposit is built up from the surface creep and from the saltation simultaneously. (*c*) Encroachment. If the surface is not continuous but contains an obstruction of any sort, e.g. an abrupt step up or down, the surface creep is held up while the saltation passes on. The resulting deposit, which need not require any diminution in the wind for its occurrence, consists only of the surface creep. In the case of a steep slope down, such as the slip-face of a dune, the surface grains roll over the crest of it and come to rest because they become sheltered from the impelling bombardment of the saltation. The slope encroaches down wind forming a relatively coarse deposit.

4—Experiments and Notation used to Express Results

Experiments were designed to find out by imitating the foregoing natural cycle of sand movement under controlled conditions (*a*) the changes which take place in the slopes of the asymptotes of "regular" sands and in the position of the peaks; and (*b*) how a sand of haphazard grading is transformed into a regular sand.

The wind tunnel used has already been described (Bagnold 1936). Deposition was brought about by inserting an expansion section at the down wind end to check the velocity of the air stream. The arrangement is shown in fig. 2. The sand bed was laid in the parallel tunnel AB. A transverse slot 2 cm. long and 0·5 cm. wide was cut in the centre of the floor at B, through which a sample of the surface creep could fall into a container. The sand removed by the wind from AB was deposited between B and F. Removal close to A could be stimulated by introducing a trickle of sand at A.

Difficulty was encountered owing to a tendency for the finer grains to separate out sideways and to collect in the zones of slow-moving air along the tunnel walls. This was reduced partially by adding smooth wood berms at each side as shown inset in fig. 2. The surface of the berm being free from the drag of the sand movement, the wind at the edge of the sand layer could, by a suitable adjustment of the width of the berms, be made as stong as that over the central sand portion of the floor.

Observations were made on (*a*) composition of the original bed sand; (*b*) composition of surface creep trapped at B; (*c*) relative weights and compositions of the deposits collected from BC, CD, DE, EF and of any carry-over found in the box G.

Results will be shown in diagrams of the form of fig. 1*b*, and the following terms will be used.

The "Carrier Grade" is the sand of grain diameters lying between the size limits bounding the rounded portion of the curve. It forms the bulk of the saltation and is the driving agent which impels the larger grains along the surface.

The "Peak Size" R_0 is defined by the intersection of the two asymptotes. The "Peak Height" $y_0 = \log_{10}\phi_0$ is the height of the intersection.

The "Small Grade Coefficient" s is the slope of the left-hand asymptote representing the grading of grains finer than the carrier grade. The "Coarse Grade Coefficient" c similarly refers to the right-hand side:

$$\frac{dy}{dR} = s, \; .$$

$$y = \log_{10}\phi = \int s\,.\,dR + y_0 = s(R - R_0) + y_0$$

$$\phi = \phi_0 e^{2\cdot 3s(R-R_0)}, \quad \text{left-hand side,}$$

similarly

$$\phi = \phi_0 e^{2\cdot 3c(R-R_0)}, \quad \text{right-hand side.}$$

Some sand samples are composed of a number of constituent sands each having a different grading. If the area $\int \phi\, dR$ of the whole curve plotted as in fig. 1*a* is unity for a sample of unit weight, the grading of any constituent is shown comparatively by reducing all its ordinates ϕ by a constant f representing the ratio of the weight of the constituent to that of the whole. On the log scale of fig. 1*b* the constituent is represented by a curve lowered bodily through a distance $\log f$.

The "width of a sand" is defined by the expression $1/s+1/c$ which is the length of the base of the triangle cut off by any abscissa divided by the height.

5—RESULTS

(*a*) *Regular Sands*

In fig. 3*a* the heavy curve shows the grading of a wide sand compiled artificially from a stock range of separate grades. The grading coefficients s and c are 1·9 and 3·0 respectively. The peak size is 0·034 cm. The irregularity in the two finest grades was made accidentally during the mixing. It was purposely left uncorrected.

The thin continuous curves represent the grading of the constituent deposits collected in the sections CD, DE, EF of the expansion tunnel and that carried over into the box G. Each has been reduced in scale according to the relative weight of the deposit to the total weight collected. The deposit in BC was inseparable from the surface creep which accumulated there. It is omitted from fig. 3*a* for clearness.

It will be seen that the slopes of the deposit curves become steeper and that the peak size falls as the wind slackens, but that in spite of this the grading coefficients, though larger than those of the original sand, are both nearly constant throughout the deposition ($s_{\text{dep}} = 2{\cdot}5$, $c_{\text{dep}} = 9$). Since, outside the limits of the carrier grade, any ordinate $\phi_R = \phi_0 e^{2\cdot3s(R-R_0)}$, the sum of all ordinates ϕ_R of a set of deposits 1, 2, 3, etc. themselves lie on an exponential curve

$$(\phi_1+\phi_2+\phi_3+\text{ etc.}) = e^{2\cdot3sR}(\phi_{0_1}e^{-sR_{0_1}}+\phi_{0_2}e^{-sR_{0_2}}+\phi_{0_3}e^{-sR_{0_3}}+\text{ etc.}).$$

If, therefore, ${}_s\Phi_c$ represents the composition of unit mass of a regular sand with grading coefficients s and c, any group of regular sands of varying peak size but with constant grading coefficients can be mixed together in proportions A, B, C, etc., to form another regular sand

$$\Sigma\, {}_s\Phi_c = {}_s\Phi_c(A+B+C+\text{ etc.}).$$

The line MN joining the peaks of the four fractional deposits is approximately straight. Its slope $T = \dfrac{dY_0}{dR_0}$ is a measure of the rate of change of relative mass deposited at any place with the peak grain size at that place. $1/T$ may be called the "separation" caused by the process. Its value is here 0·11. It is remarkably small; only 1·55 % of the deposit had so far separated out that its peak size had shifted from 0·034 cm. for the original mixture to 0·029 cm.

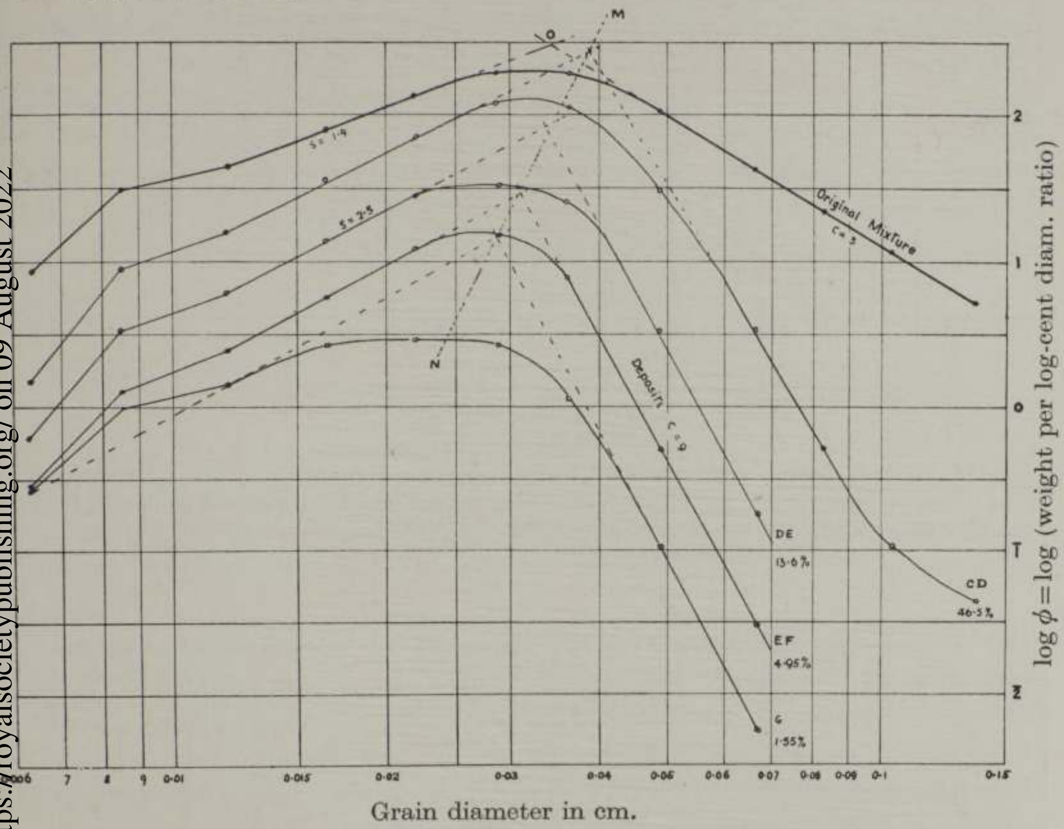

FIG. 3*a*—Last stage of removal by a moderate wind. Bed becoming coarser with consequent shift of the line MN to the right. Original bed mixture shown by heavy line $\left(\int \phi\, dR = 1\right)$. Fractional deposits in various places in the tunnel CD, DE, etc. shown by thin curves for which the ordinates have been reduced according to the proportion of sand by weight found at each place; e.g. for deposit in CD $\int \phi\, dR = 0{\cdot}465$. Total deposit for which $\int \phi\, dR = 0{\cdot}334 + 0{\cdot}465 + 0{\cdot}136 +$ etc. $= 1$ is shown by dotted line in fig. 3*b*.

Fig. 3*b* shows the composition of the surface creep (cross dotted), the first deposit BC omitted from fig. 3*a*, and the total sand movement (broken line) which is the sum of all the fractional deposits. On the left-hand side it will be noticed that the irregularity in the extreme grades is faithfully

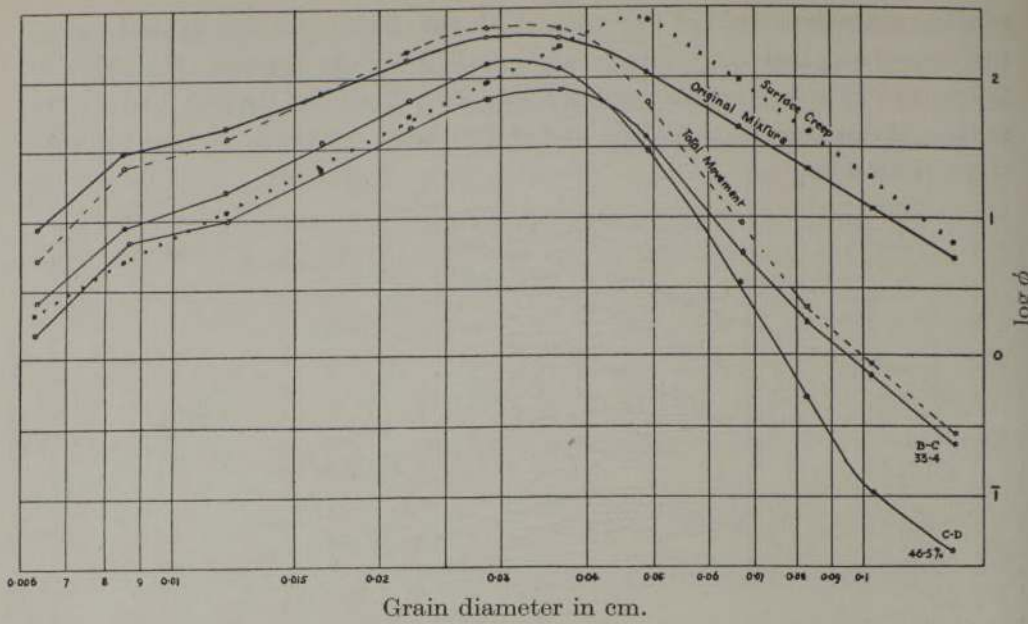

FIG 3*b*—Other details omitted from fig. 3*a*. Extreme up-wind deposit between *B* and *C* shown by continuous curve. Total sand removed and deposited shown dotted. Surface creep collected by trap in tunnel floor shown cross dotted.

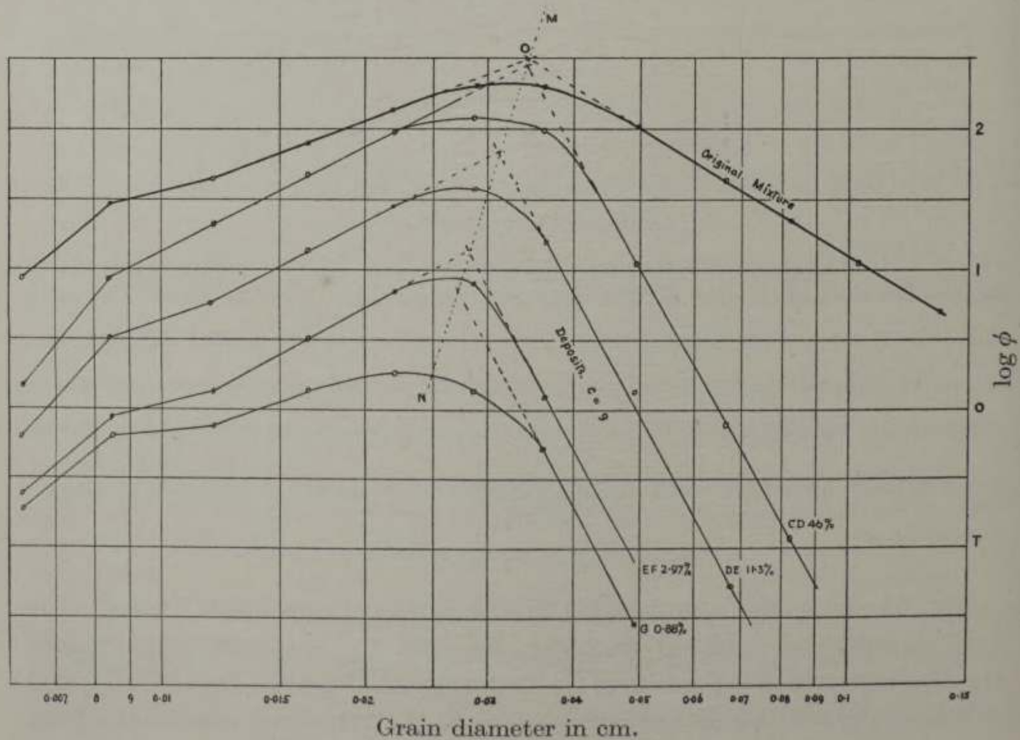

FIG. 4—Early stage of removal by moderate wind.

reproduced in all the deposits, but that it has been almost flattened out in the case of the surface creep. On the right-hand side, the surface creep has taken up a new slope, steeper than the original but considerably flatter than that of the deposits. The peak size of the surface creep has shifted to the right. The advance of the surface creep and its mingling with the up-wind deposits is well marked by the extreme deposit grades bending up to become parallel with it. This is an example of the sum of two mixtures differing in grading coefficient.

The above was the result of a run under a moderate wind maintained till the surface became stable and motion ceased. During the later part of the run the failing supply of fine grains caused the carrier grade to become coarser. Consequently the line MN joining the peaks of the deposits passes to the right of the peak O of the original bed mixture. Fig. 4 resulted from a run of the same period—3 min.—when a fresh surface of the same sand was kept disturbed by a small incoming fall of sand at the mouth. The grading coefficients of the deposits are unchanged, but the line MN has shifted to the left so that it now passes through O. Since, however, the coarse grading coefficient c for the total sand removal is greater than that of the bed mixture, the consequent concentration of the coarsest grades on the removal bed must in time shift the whole deposit pattern to the right again.

In fig. 5 the same sand was exposed to a considerably stronger wind, above the threshold required to move the largest grains. On the left, the increased wind has made the slope of the deposits approach very closely to that of the bed mixture; but the slope of the surface creep has remained unchanged. The peak size of the surface creep has shifted still farther to the right, and the long line of eight analysis points illustrates well the persistence of the exponential relationship. Again the irregularity has straightened out. On the right the slope of the deposits remains unchanged at $c = 9$. The duration of the run was only 2 min. but the rapid removal, with its consequent coarsening of the removal bed, has already shifted the line MN to the right, and has steepened the right-hand slope of the surface creep. In the final state of steady movement the peak of the surface creep shifts so far to the right that its right-hand coefficient rises to the common value of 9 as shown by the dotted line in the figure.

Fig. 6 shows the behaviour of a natural "fine silver sand" as obtained from a builder's merchant. The peak size is finer than that of the artificial mixture; and the composition curve is typical of the sand of desert dunes. The wind in this case was lower than that in fig. 5 but was of corresponding strength having regard to the threshold wind required to move the coarsest grains. On the left the deposit slope is again nearly parallel to that of the original mixture. On the right it has taken up the same value as before, $c = 9$. The coarse grades of the surface creep have again taken up a slope intermediate between those of the deposits and of the original mixture.

Owing to the relative lack of coarse grains in this sand, the coarsening of the removal bed, as indicated by the shift of the line MN to the right, is considerably less than in the case of the "wide" sand.

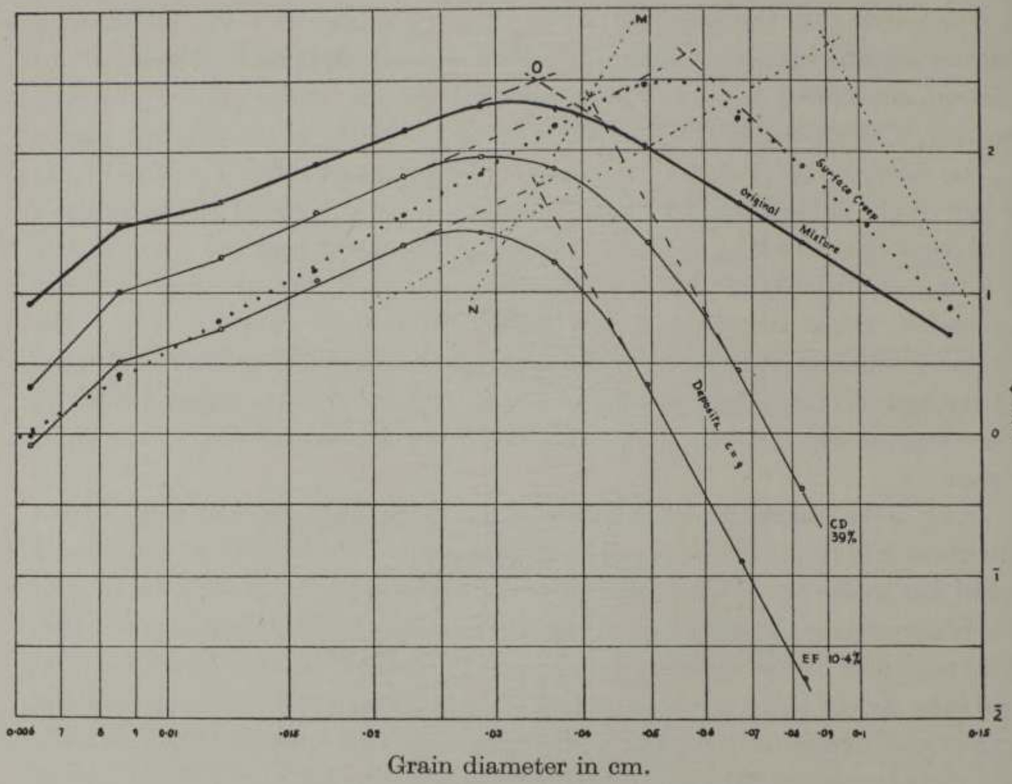

FIG. 5—Grading effects of strong wind.

(*b*) *Irregular Sands*

The foregoing experiments show that regular sands tend to retain their exponential grading no matter how their coefficients may change. Further experiments were made to ascertain whether, and if so in what part of the cycle, a non-regular sand is transformed into a regular sand. The natural sand of fig. 6 was used; and by the addition of more sand (*a*) the extreme fine grade, and (*b*) the extreme coarse grade were separately increased about tenfold. Since changes in the grading on one side of a curve appear to have no appreciable effect on the other side, the irrelevant halves of the curves are omitted from fig. 7.

If the distortion n is defined as the ratio of the content of a grade actually present in a mixture to that of the same grade if it conformed to the general pattern of the regular grading curve, then the difference in the height y of the ordinate in the figure between the straight line and the actual curve is $\log n$.

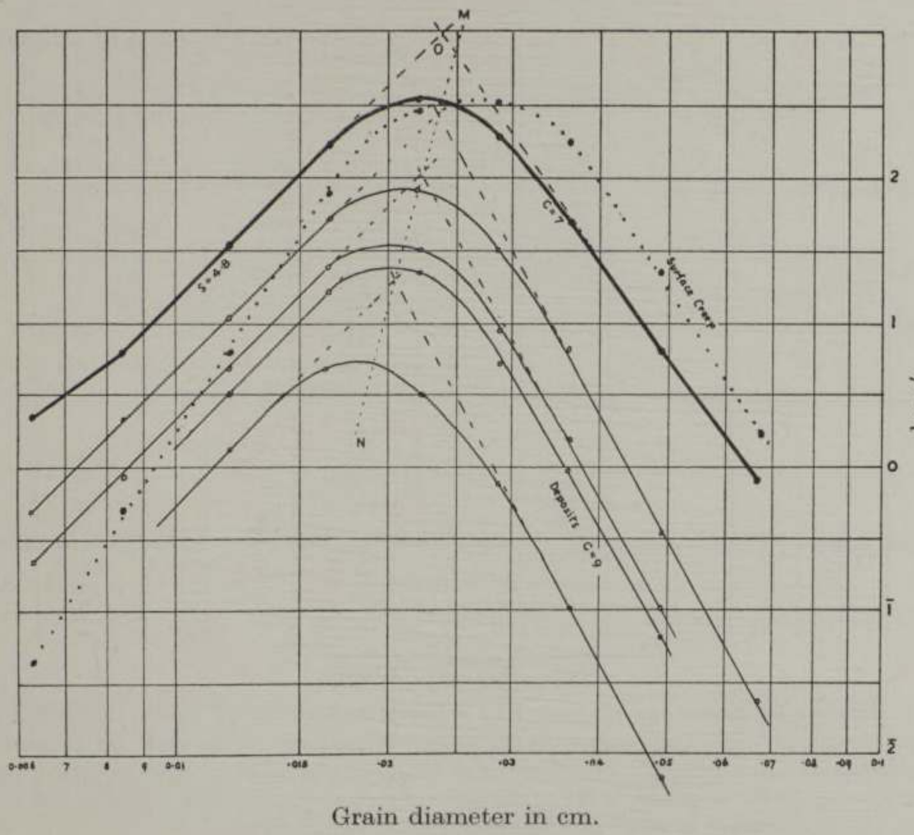

FIG. 6—A "narrow" natural sand. Original bed mixture shown by heavy curve. Fractional deposits shown by thin curves. Surface creep shown cross dotted.

In fig. 7*a* the distortion artificially caused was about sevenfold for the extreme fine grade. In the deposits it will be seen that n has dropped, most pronouncedly in the early up-wind deposits, and to a lesser extent as the deposit dies off down wind. Most of the excess of the finest grade was left behind on the removal bed, where it formed smooth sheets over which the wind passed without causing any movement on them. Confirmation of this rather surprising rejection of fine sand in the pick-up process is given by fig. 3*b* where the slope of the total sand removed is steeper than that of the removal bed. In the case of the surface creep as trapped at B the distortion is much reduced.

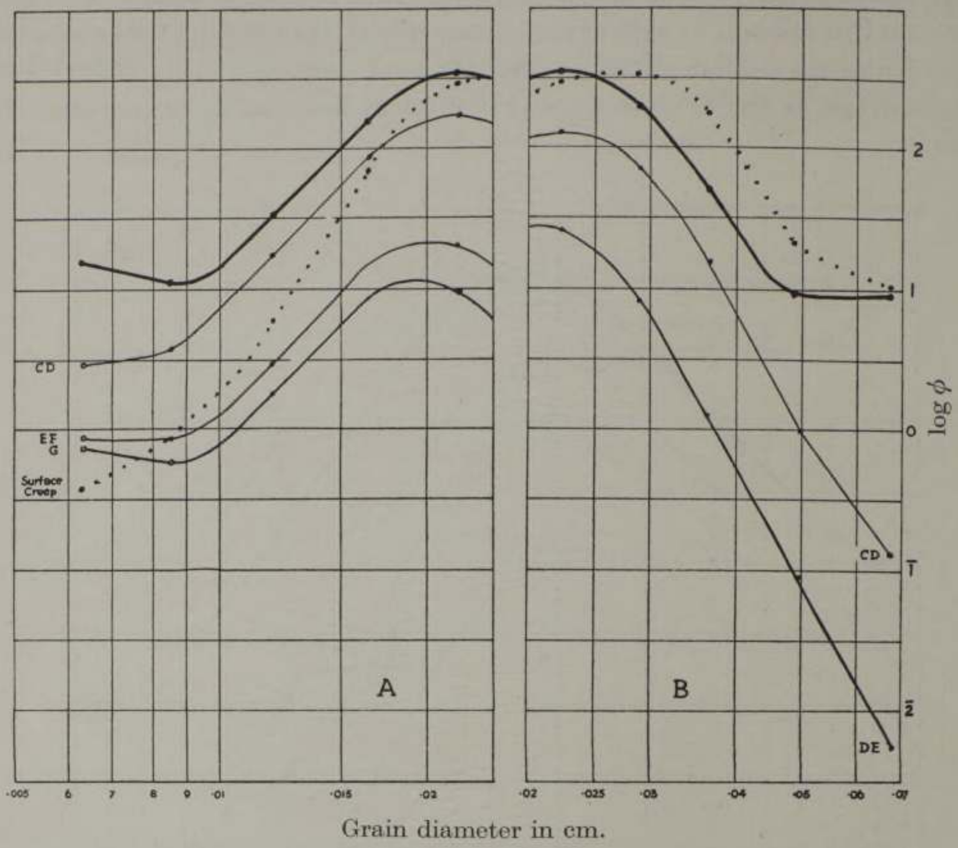

FIG. 7—Effects of distortion of pattern of original bed mixture. Bed mixture shown by heavy curves—extreme grades on right and left increased, in turn, in weight by the addition of extra sand. On the right (coarse grains) the regular exponential grading tends to be restored in the successive down-wind deposits. On the left (fine grains) the restoration is most pronounced in the surface creep.

Fig. 7*b* shows the changes which took place in a distortion on the right of the curve, n was 10 for the original mixture. It was reduced least, to 6·3, in the case of the surface creep, and was progressively more reduced in the deposits till it had disappeared altogether as the deposit died away towards the down-wind end of the expansion tunnel. Again most of the excess of the distorted grade was left behind on the removal bed, where in this case it formed typical "residue ridges" (Bagnold 1937).

The tendency for sand grains to grade themselves according to the exponential relation $\log_{10}\phi = -c(R-R_0)$ is not confined to natural sands composed of a complete continuity of grain size. Fig. 8 shows the result of two experiments with a mixture consisting of a narrow carrier grade of almost uniform size and two coarser grades separated by gaps. As before, the grading tends to become linear, and c increases towards the apparent limiting value of 9 under the action of the wind.

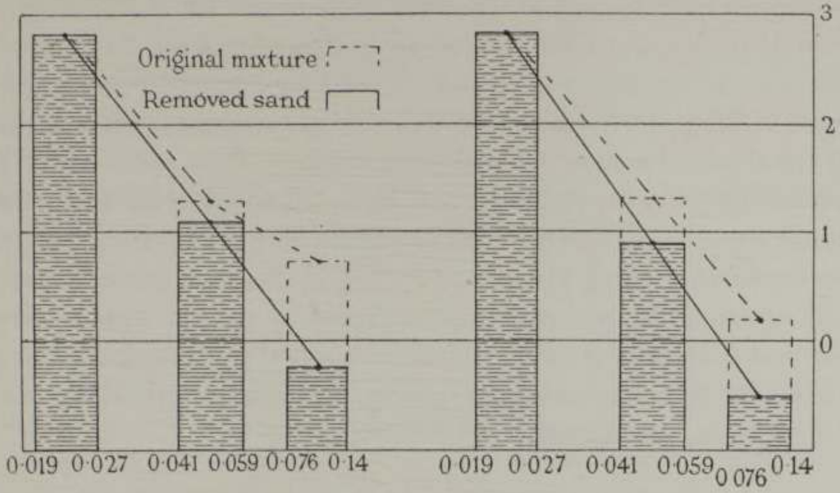

FIG. 8—Discontinuous grading. Tendency of thin sands each of nearly uniform grain size to assume and maintain the general exponential grading relation.

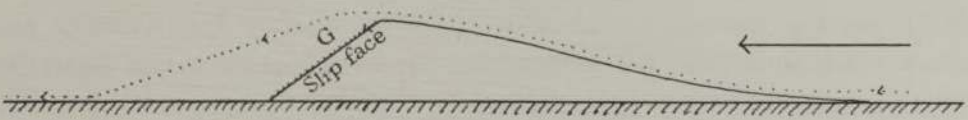

FIG. 9—Separation of saltation from surface creep at dune crest.

(c) *Grading of an Encroachment Deposit*

Observation of the movement of sand over a typical dune such as that shown in fig. 9 indicates that the dune advances down wind by the trapping of the surface creep, which is driven by the saltation over the crest into the shelter of the leeward slip-face. It seems, therefore, that most of the mass of the dune, above the level of the foot of the slip-face, is composed of sand which has been graded by a process similar to that of the slot method used to collect the surface creep; and it is this process which appears to be most effective in producing the exponential form of the left-hand side of the grading curve. Thus, while the steepness and the exponential grading of the right-hand side may be regarded as arising from the removal process, those of the left-hand side appear to be due to the separation of the surface creep at the dune crest. This is born out by the fact that the grading curves of samples I have analysed of sands taken from the great undulating desert sand accumulations, where no crest or slip-face exist, all have a very flat left-hand side with but little approach to regularity of form.

Since the analysis of samples of true dune sands also showed that in many cases the left-hand side of the curve was steeper than that of the surface creep in any of the preceding experiments, some further grading process was looked for. Such was found in the elutriating or winnowing action of the upward air current due to the wind eddy in the lee of the crest. For in imitation of this upward air current an in-draught of air was made to rise through the collecting slot at B (fig. 2); and as a result, though there was no change in the right-hand side of the curve or in the position of the peak, the left-hand side became considerably steeper. It is noteworthy that in this process, too, the altered left-hand side still retained its linear form.

It would seem from the above results that there is experimental evidence that the assumed grading law of the form $\phi \propto d^n$ is something more than a practical approximation, and that deposits of grains which have been removed and transported by the wind from a bed composed of sand graded in any irregular way tend to become graded according to the above law, most of the superfluous grains being left behind on the bed.

SUMMARY

The size-grading of samples of eolian sand deposits when plotted as weight per log-cent change in grain diameter against log diameter give curves which show that the extreme grades on each side of the mean diameter die away in percentage weight according to an exponential law. The exponential coefficients on each side differ in value. These and other coefficients defined in the paper numerically describe the grading of sand in a manner independent of the usual diagrammatic representation and of the sieve sizes used in its analysis.

By imitating experimentally a simple cycle of sand movement consisting of removal from a sand bed of known composition, transportation over the bed, and deposition under various conditions, the factors controlling the value of each coefficient separately have been investigated.

The tendency of sand to grade itself according to the above bi-exponential pattern was confirmed by experiments with sands whose grading pattern had been artificially distorted by the addition of extra sand of certain grades. The process by which the extra sand is rejected by the mixture is described.

The results are applied to the size-grading of a typical sand dune.

REFERENCES

Bagnold, R. A. 1936 *Proc. Roy. Soc.* A, **157**.
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