THE *SM/M/N* QUEUEING SYSTEM WITH BROADCASTING SERVICE

MOON HO LEE, ALEXANDER N. DUDIN, AND VALENTINA I. KLIMENOK

Received 12 November 2005; Accepted 28 April 2006

We consider a multiserver queueing model with the semi-Markovian arrival process and exponential service time distribution. Novel customers admission discipline is under study. The customer, which sees several free servers upon arrival, is served simultaneously by all these servers. Such situation occurs, for example, in modeling wireless communication network with broadcasting. Systems with infinite buffer and with losses are investigated. Stationary distributions of a queue, waiting and sojourn times, and the main performance measures are calculated.

1. Introduction

Multiserver queueing systems model many real-life objects and processes and have got a lot of attention in the literature since the pioneering works by the Danish mathematician and engineer A.K. Erlang in the early 19th. In this paper we consider the model that differs from all previous works by the customer admission discipline. The standard assumption is that the customer is served by one server. In our paper, we investigate the case when the customer gets a service from all servers that were idle at the customer arrival epoch. Such a discipline is realistic in modeling, for example, the wireless communication network with broadcasting. If the system has many antennas, it makes sense to employ all of the free antennas to transmit the arriving information unit. It creates some redundancy, but it can help to decrease the average information delivering time if the system is not overloaded while the transmission time has high variation. The diversification of the ways of transmission helps to decrease the average time of delivering of a first copy of the broadcasted information.

We assume here that the input flow is described in terms of the *SM* (*semi-Markovian*) arrival process. It means that the successive interarrival times are defined by the sojourn times of some semi-Markovian stochastic process in its states. The *SM* arrival process is

Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2006, Article ID 98171, Pages 1–18
DOI 10.1155/MPE/2006/98171

maximally general among known-in-literature descriptors of the arrival process which still allows to get analytically tractable results for characteristics of the queueing model. The *SM* arrival process is, generally speaking, the correlated process and so it well suits for modeling the real-life flows in modern telecommunication networks. As a particular case, the *SM* arrival process includes the set of all *GI* (general independent) recurrent service processes in which the interarrival times are independent identically arbitrary distributed random variables. Service time distribution in this model is assumed to be exponential.

Multiserver models with the *SM* arrival process and standard service discipline were recently considered in [1–4]. In [3, 4], batch arrivals are allowed. Analysis is based on the theory of piecewise Markov process and transform approach. Applications to investigation of the transmission of MPEG (motion picture experts group) frame sequence on ATM network are presented. In [1, 2], the models are analyzed where the buffer capacity is finite and the service process is of MSP (Markovian service process) type. *MSP* is a direct analogue of the well-known MAP (Markovian arrival processes).

The main distinction of the model considered in the present paper from the models analyzed in the mentioned above papers consists of another service discipline which we call the broadcasting service discipline. Such a discipline suggests that if the arriving into the system customer meets several free servers upon arrival, all these servers start, independently of others, the service of this customer. Such a discipline is realistic, for example, in multiantenna communication networks The multiserver model of the *MAP/PH/N* type with the broadcasting service discipline was recently considered in [5]. In our present paper, we impose more strong assumptions about the service process (the class of *PH* phase-type distributions is much more rich than the set of exponential distributions considered in the present paper), but the arrival process of *SM* type, considered in this paper, is an essentially more wide class than the set of the Markovian arrival processes. So, the model considered in [5] could be exploited for performance evaluation and capacity planning in real-life systems in situations when the service time distribution cannot be well approximated by the exponential distribution. While the present model allows to consider more complicated arrival processes, it is worth to mention that, except our paper [5], the model studied in this paper was not previously investigated in the literature even in the much simpler case of the stationary Poisson arrival process and exponential service time distribution.

Due to the different assumptions about the input flow and service process, analysis of the stochastic processes in the model in this paper and the model in [5] is quite different. In [5], the multidimensional continuous time Markov chain is under study. Here we have to analyze first the two-dimensional discrete time Markov chain embedded at the epochs of the customers arrival into the system. In both cases, the investigated stochastic process is not directly immersed into some well-known class of random processes and its investigation is not quite straightforward.

The rest of the paper is organized as follows. In Section 2, the model is described. In Section 3, it is analyzed for the case of an infinite buffer. Numerical procedures for calculating the stationary-state distributions of the number of customers in the system at the customer arrival and arbitrary epochs are described. Distribution functions of the waiting and sojourn time are derived. The case of unreliable customers service is touched.

The system with losses is investigated in brief in Section 4. Section 5 contains some concluding remarks.

2. Mathematical model

We consider an N-server queueing system. The flow of customers arriving into the system is of *SM* type. It means that the arrivals are directed by some semi-Markovian process. Let us denote this process by ν_t, $t \geq 0$. The process ν_t, $t \geq 0$, has a finite-state space $\{1,\dots,K\}$. Behavior of the process is described by the semi-Markovian kernel $A(t)$. This kernel is the square matrix of size K with entries $(A(t))_{k,k'}$, k, $k' = \overline{1,K}$. Function $(A(t))_{k,k'}$ has the meaning of the probability that sojourn time of the process in the state k will be no longer than t and after that the process jumps into the state k', not necessary different from the state k, k, $k' = \overline{1,K}$. The matrix $A(\infty)$ has a meaning of the one-step probability matrix of the Markov chain embedded at epochs of all jumps of the process ν_t, $t \geq 0$. It is assumed that the embedded Markov chain is irreducible, and sojourn times of the process $\nu_t, t \geq 0$, in its states are positive and finite. Then the stationary distribution of the embedded Markov chain exists. Denote by $\boldsymbol{\theta}$ the row vector of the stationary distribution of the embedded Markov chain. It is well known that this vector is the unique solution to the following system of equations:

$$\boldsymbol{\theta} = \boldsymbol{\theta} A(\infty), \qquad \boldsymbol{\theta}\mathbf{e} = 1. \tag{2.1}$$

Here and in the sequel $\mathbf{e}$ is the column vector consisting of 1's.

The customers in the *SM* arrival process arrive at the epochs of jumps of the semi-Markovian process ν_t, $t \geq 0$. The value λ defined by formula

$$\lambda^{-1} = \boldsymbol{\theta} \int_0^\infty t dA(t)\mathbf{e} \tag{2.2}$$

is called the average intensity or fundamental rate of the *SM* arrival process.

The servers of the system are assumed to be independent of each other and identical. Service time distribution is assumed to be exponential with the positive finite parameter μ.

If the arriving into the system customer meets several free servers upon arrival, all these servers start, independently of others, the service of this customer. Since this epoch, all multiple copies of this customer are considered as the different customers serving in the system. Further, the system does not distinguish the customers having unique or multiple copies in the system. So, multiple copies cannot be deleted from the system before they get a service even if new customers requiring a service arrive or if some copies of an original customer already finish the service in the system.

If all the servers are busy upon arrival, we will distinguish the following two variants of the system behavior: (i) the customer is placed into the buffer of an infinite capacity and then it will be picked up from the queue according to the *FIFO* (first in, first out) discipline; (ii) the customer leaves the system forever, that is, it is lost by a system.

These two variants are coded in the literature in Kendall's denotations as the *SM/M/N* and *SM/M/N/N* systems, respectively. We consider these variants in this paper sequentially.

3. The *SM/M/N* system

3.1. Distribution of the number of customers in the system at arrival epochs. Let i_t, $i_t \geq 0$, be the number of customers in the system at epoch t, $t \geq 0$. Our aim is to study the stationary behavior of the process i_t, $i_t \geq 0$. However, this process is a non-Markovian one and its direct investigation is not possible. So, we apply the method of embedded Markov chains for its investigation. To this end, we first consider the states of the process i_t, $i_t \geq 0$, only at the epochs $t_n - 0$ immediately before the nth customer arrival into the system, $n \geq 1$. Let us denote $i_n = i_{t_n - 0}$, $n \geq 1$, and $\nu_n = \nu_{t_n - 0}$, $n \geq 1$.

It is easy to see that the two-dimensional process

$$\zeta_n = (i_n, \nu_n), \quad n \geq 1,\ i_n \geq 0,\ \nu_n = \overline{1,K}, \tag{3.1}$$

is an irreducible discrete time Markov chain.

Denote the stationary probabilities of this process as

$$\pi(i,k) = \lim_{n \to \infty} P\{i_n = i,\ \nu_n = k\}, \quad i \geq 0,\ k = \overline{1,K}. \tag{3.2}$$

The problem of the establishing conditions for existence of the limits (3.2) will be discussed a bit later.

Let us enumerate the states of the Markov chain ζ_n, $n \geq 1$, in the lexicographic order and form the row-vectors $\boldsymbol{\pi}_i$ of the stationary-state probabilities $\pi(i,k)$, corresponding to the state i of the first component of the chain:

$$\boldsymbol{\pi}_i = (\pi(i,1), \pi(i,2), \ldots, \pi(i,K)), \quad i \geq 0. \tag{3.3}$$

Analogously, we form the matrices $P_{i,l}$ of one-step transition probabilities of the Markov chain ζ_n, $n \geq 1$, as

$$P_{i,l} = \left(P\{i_{n+1} = l, \nu_{n+1} = k' \mid i_n = i, \nu_n = k\}\right)_{k,k' = \overline{1,K}}, \quad i,l \geq 0. \tag{3.4}$$

Lemma 3.1. *The nonzero matrices $P_{i,l}$, $i,l \geq 0$, of one-step transition probabilities are calculated by*

$$P_{i,l} = \Omega_{i+1-l}, \quad N \leq l \leq i+1, \quad i \geq N-1, \tag{3.5}$$

$$P_{i,l} = \int_0^\infty C_N^l e^{-l\mu t} \int_0^t N\mu \frac{(N\mu t)^{i-N}}{(i-N)!} \left(e^{-\mu y} - e^{-\mu t}\right)^{N-l} dy\, dA(t), \quad 0 < l < N,\ i \geq N, \tag{3.6}$$

$$P_{i,l} = P_{N-1,l} = \mathcal{A}_l, \quad i = \overline{0, N-1},\ l = \overline{1,N}, \tag{3.7}$$

$$P_{i,0} = \mathcal{A}_0, \quad i = \overline{0, N-1}, \tag{3.8}$$

$$P_{i,0} = A(\infty) - \sum_{l=1}^{N-1} P_{i,l} - \sum_{l=0}^{i-N+1} \Omega_l, \quad i \geq N, \tag{3.9}$$

where

$$\Omega_l = \int_0^\infty \frac{(N\mu t)^l}{l!} e^{-N\mu t} dA(t), \quad l \geq 0, \tag{3.10}$$

$$\mathcal{A}_l = \int_0^\infty C_N^l e^{-l\mu t} (1 - e^{-\mu t})^{N-l} dA(t), \quad l = \overline{0,N}. \tag{3.11}$$

The proof of the lemma consists of analysis of one-step transitions. It is straightforward and so it is omitted.

Lemma 3.2. *The matrix $\mathcal{P} = (P_{i,l})_{i\geq 0,\, l\geq 0}$ of one-step transition probabilities of the Markov chain ζ_n, $n \geq 1$, has the following structure:*

$$\mathcal{P} = \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \cdots & \mathcal{A}_{N-1} & \mathcal{A}_N & O & O & O & \cdots \\ \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \cdots & \mathcal{A}_{N-1} & \mathcal{A}_N & O & O & O & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \cdots & \mathcal{A}_{N-1} & \mathcal{A}_N & O & O & O & \cdots \\ P_{N,0} & P_{N,1} & P_{N,2} & \cdots & P_{N,N-1} & \Omega_1 & \Omega_0 & O & O & \cdots \\ P_{N+1,0} & P_{N+1,1} & P_{N+1,2} & \cdots & P_{N+1,N-1} & \Omega_2 & \Omega_1 & \Omega_0 & O & \cdots \\ P_{N+2,0} & P_{N+2,1} & P_{N+2,2} & \cdots & P_{N+2,N-1} & \Omega_3 & \Omega_2 & \Omega_1 & \Omega_0 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}. \tag{3.12}$$

The statement of the lemma directly stems from Lemma 3.1.

Two essential distinctions of the matrix $\mathcal{P} = (P_{i,l})_{i\geq 0,\, l\geq 0}$ of one-step transition probabilities of its prototype in the case of the classical service discipline have to be mentioned. The first one is that the matrix $\mathcal{P}$ in the form (3.12) is not completely low Hessenbergian as its prototype. This distinction makes the investigation of the Markov chain more complicated. The second distinction consists of the fact that the first N block rows of the matrix $\mathcal{P}$ coincide. It appears that this distinction simplifies investigation and, eventually, results, which will be derived below, have more nice analytic form comparing the system with the classical service discipline.

Theorem 3.3. *Stationary distribution $\boldsymbol{\pi}_i$, $i \geq 0$, of the Markov chain ζ_n, $n \geq 1$, exists if and only if the inequality*

$$\rho = \frac{\lambda}{N\mu} < 1 \tag{3.13}$$

is fulfilled, where λ is the fundamental rate of the arrival process defined by formula (2.2).

Vectors $\boldsymbol{\pi}_i$, $i \geq 0$, are computed as follows:

$$\boldsymbol{\pi}_0 = \boldsymbol{\theta}(I - \mathcal{R})\left(I + \mathcal{R} - \sum_{k=1}^{N} \mathcal{A}_k - \sum_{k=1}^{N} \mathcal{B}_k\right), \tag{3.14}$$

$$\boldsymbol{\pi}_k = \boldsymbol{\theta}(I - \mathcal{R})(\mathcal{A}_k + \mathcal{B}_k), \quad k = \overline{1, N-1}, \tag{3.15}$$

$$\boldsymbol{\pi}_i = \boldsymbol{\theta}(I - \mathcal{R})\mathcal{R}^{i-N+1}, \quad i \geq N, \tag{3.16}$$

where the matrices $\mathcal{A}_k$, $k = \overline{0,N}$, *are defined by formula (3.11), the matrices* $\mathcal{B}_k$, $k = \overline{1,N}$, *are computed by*

$$\mathcal{B}_k = N\mathcal{R}C_N^k \sum_{m=0}^{N-k} C_{N-k}^m (-1)^{N-k-m}(mI - N\mathcal{R})^{-1}\left(\alpha\left(\mu(N-m)\right) - \mathcal{R}\right), \tag{3.17}$$

the matrix $\mathcal{R}$ *is the minimal nonnegative solution to the equation*

$$\mathcal{R} = \alpha\left(N\mu(I - \mathcal{R})\right), \tag{3.18}$$

I *is identity matrix,* $\boldsymbol{\theta}$ *is the vector defined as solution to the system (2.1), and* $\alpha(s)$ *is the matrix Laplace-Stieltjes transform of the semi-Markovian kernel:*

$$\alpha(s) = \int_0^\infty e^{-st} dA(t), \quad \operatorname{Re} s > 0. \tag{3.19}$$

Proof. It is well known that the vectors $\boldsymbol{\pi}_i$, $i \geq 0$, defining the stationary distribution of the Markov chain ζ_n, $n \geq 1$, satisfy Chapman-Kolmogorov's equations (or equilibrium equations),

$$(\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \ldots)\mathcal{P} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \ldots). \tag{3.20}$$

It can be seen from the structure (3.12) of the transition probability matrix $\mathcal{P}$ that, starting from the $(N+1)$th block row, the matrix becomes below Heisenbergian. So, results by Neuts [6] concerning the so called *GI/M/*1 type Markov chains (or Markov chains possessing skip-free to the right property) can be applied in some extent. In particular, because the limiting behavior of the Markov chain (and stability condition) does not depend on the transitions of the Markov chain in the boundary states, stability condition of form (3.13) directly follows from [6].

To prove relations (3.14)–(3.16) we will solve, step by step, equilibrium equations (3.20) with the transition probability matrix $\mathcal{P}$ of form (3.12).

The kth, $k \geq N+2$, equation of the system (3.20) can be rewritten as

$$\boldsymbol{\pi}_k = \sum_{l=k-1}^{\infty} \boldsymbol{\pi}_l \Omega_{l+1-k}, \quad k \geq N+1. \tag{3.21}$$

By the direct substitution, we can make sure that the probability vectors $\boldsymbol{\pi}_k$, which satisfy system (3.21), have the following form:

$$\boldsymbol{\pi}_k = \mathbf{c}\mathcal{R}^{k-N+1}, \quad k \geq N, \tag{3.22}$$

where $\mathbf{c}$ is some constant vector and the matrix $\mathcal{R}$ is solution to (3.18). See Neuts' book [6] for more reasonings and explanations.

The kth, $k = \overline{2,N}$, equation of the system (3.20) can be rewritten as

$$\boldsymbol{\pi}_k = \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \mathcal{A}_k + \sum_{m=N}^{\infty} \boldsymbol{\pi}_m P_{m,k}, \quad k = \overline{1,N-1}, \tag{3.23}$$

where the matrices $\mathcal{A}_k$, $k = \overline{1, N-1}$, are defined by formula (3.11) and the matrices $P_{m,k}$ are defined by formula (3.6).

By substituting expressions (3.6) and (3.22) into the infinite sum in (3.23) and calculating this sum, we get the relation

$$\boldsymbol{\pi}_k = \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \mathcal{A}_k + \mathbf{c}\mathcal{B}_k, \quad k = \overline{1, N-1}, \tag{3.24}$$

where the matrices $\mathcal{B}_k$ are given by formula (3.17).

The $(N+1)$th equation of the system (3.20) can be rewritten as

$$\boldsymbol{\pi}_N = \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \Omega_0 + \sum_{m=N}^{\infty} \boldsymbol{\pi}_m \Omega_{k+1-N}. \tag{3.25}$$

By substituting expressions (3.10) and (3.22) into the infinite sum in (3.25), calculating this sum, and taking into account that $\Omega_0 = \mathcal{A}_N$, we conclude that relation (3.24) holds good for $k = N$ as well.

The first equation of the system (3.20) can be written as

$$\boldsymbol{\pi}_0 = \sum_{l=0}^{N-1} \boldsymbol{\pi}_l P_{N-1,0} + \sum_{m=N}^{\infty} \boldsymbol{\pi}_m P_{m,0}. \tag{3.26}$$

Taking into account (3.8) and (3.9), we rewrite this equation as

$$\boldsymbol{\pi}_0 = \sum_{l=0}^{\infty} \boldsymbol{\pi}_l A(\infty) - \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \sum_{i=1}^{N} \mathcal{A}_i - \sum_{l=N}^{\infty} \boldsymbol{\pi}_l \sum_{m=1}^{N-1} P_{l,m} - \sum_{l=N}^{\infty} \boldsymbol{\pi}_l \sum_{i=0}^{L-N+1} \Omega_i. \tag{3.27}$$

It is evident that $\sum_{l=0}^{\infty} \boldsymbol{\pi}_l = \boldsymbol{\theta}$. Taking into account formula (2.1), we conclude that the first term in the right-hand side of (3.27) is equal to $\boldsymbol{\theta}$. After the routine calculation of the infinite sums in the right-hand side of (3.27) including the change of order of summation and the use of formulae (3.6), (3.10), and (3.22), we yield expression

$$\boldsymbol{\pi}_0 = \boldsymbol{\theta} - \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \sum_{i=1}^{N} \mathcal{A}_i - \mathbf{c} \sum_{k=1}^{N-1} \mathcal{B}_k - \mathbf{c}\mathcal{R}(I - \mathcal{R})^{-1} + \mathbf{c}\Omega_0. \tag{3.28}$$

Mention that the inverse matrix in (3.28) exists due to Hadamard's theorem because it is known (see [6]) that the minimal nonnegative solution to (3.18) is sub stochastic. Taking into account the explicit form of the matrix $\mathcal{B}_N$, we modify the expression (3.28) as

$$\boldsymbol{\pi}_0 = \boldsymbol{\theta} - \sum_{l=0}^{N-1} \boldsymbol{\pi}_l \sum_{i=1}^{N} \mathcal{A}_i - \mathbf{c} \sum_{k=1}^{N} \mathcal{B}_k - \mathbf{c}\mathcal{R}^2(I - \mathcal{R})^{-1}. \tag{3.29}$$

By summing (3.29) and (3.24) for $k=\overline{1,N}$, we get the following expression for still an unknown vector:

$$\boldsymbol{\delta}=\sum_{l=0}^{N-1}\boldsymbol{\pi}_l:\boldsymbol{\delta}=\left[\boldsymbol{\theta}-\mathbf{c}\left(\mathcal{B}_N+\mathcal{R}^2(I-\mathcal{R})^{-1}\right)\right](I+\mathcal{A}_N)^{-1}. \tag{3.30}$$

Mention that the inverse matrix in (3.30) exists due to Hadamard's theorem because the matrix $\mathcal{A}_N$ is substochastic.

Recall that we have derived two different expressions, (3.22) and (3.24), for the vector $\boldsymbol{\pi}_N$. By equating them, we get

$$\boldsymbol{\pi}_N=\mathbf{c}\mathcal{R}=\boldsymbol{\delta}\mathcal{A}_N+\mathbf{c}\mathcal{B}_N. \tag{3.31}$$

Taking into account formulae (3.30) and (3.31), we get the explicit expressions for unknown up to this moment vectors $\boldsymbol{\delta}$ and $\mathbf{c}$:

$$\mathbf{c}=\boldsymbol{\delta}=\boldsymbol{\theta}(I-\mathcal{R}). \tag{3.32}$$

By substituting the obtained expressions for vectors $\boldsymbol{\delta}$ and $\mathbf{c}$ into the formulae (3.22), (3.24), and (3.29), we prove formulae (3.16), (3.15), and (3.14) correspondingly. This completes the proof of the theorem. □

Corollary 3.4. *Average number L of customers in the system at the customers arrival epochs is computed by*

$$L=\sum_{k=1}^{\infty}k\boldsymbol{\pi}_k\mathbf{e}=\boldsymbol{\theta}(I-\mathcal{R})\left[\sum_{k=1}^{N}k(\mathcal{A}_k+\mathcal{B}_k)+\mathcal{R}^2\left(N(I-\mathcal{R})+I\right)(I-\mathcal{R})^{-2}\right]\mathbf{e}. \tag{3.33}$$

Average number N_s of servers, which process an arbitrary customer in the system, is computed by

$$N_s=\left[\sum_{k=0}^{N-1}(N-k)\boldsymbol{\pi}_k+\sum_{k=N}^{\infty}\boldsymbol{\pi}_k\right]\mathbf{e}=\boldsymbol{\theta}(I-\mathcal{R})\left[N(I+\mathcal{R})-\sum_{k=1}^{N}k(\mathcal{A}_k+\mathcal{B}_k)\right]\mathbf{e}+\boldsymbol{\theta}\mathcal{R}\mathbf{e}. \tag{3.34}$$

3.2. Distribution of the number of customers in the system at arbitrary epochs. Having the stationary distribution of the embedded Markov chain ζ_n, $n\geq 1$, been computed, now we can calculate the stationary distribution of the non-Markovian process i_t, $i_t\geq 0$, of the number of customers in the system at epoch t, $t\geq 0$.

Let us denote the stationary probabilities of the two-dimensional process (i_t,ν_t), $t\geq 0$, by

$$p(i,k)=\lim_{t\to\infty}P\{i_t=i,\ \nu_t=k\},\quad i\geq 0,\ k=\overline{1,K}, \tag{3.35}$$

and form the row-vectors $\mathbf{p}_i$ of the stationary-state probabilities $p(i,k)$, corresponding to the state i of the first component of the chain:

$$\mathbf{p}_i=\left(p(i,1),p(i,2),\dots,p(i,K)\right),\quad i\geq 0. \tag{3.36}$$

Theorem 3.5. *The stationary-state probability vectors* $\mathbf{p}_i$, $i \geq 0$, *are computed by*

$$\mathbf{p}_i = \rho\boldsymbol{\theta}\mathcal{R}^{i-N}(A(\infty) - \mathcal{R}), \quad i \geq N,$$

$$\begin{aligned}
\mathbf{p}_l = {} & \lambda \sum_{m=l-1}^{N-1} \boldsymbol{\pi}_m C_{m+1}^{l} \sum_{k=0}^{m+1-l} C_{m+1-l}^{k} \frac{1}{\mu(l+k)} (A(\infty) - \alpha(\mu(l+k))) \\
& + \lambda\boldsymbol{\theta}(I - \mathcal{R})\mathcal{R}NC_N^l \sum_{m=0}^{N-l} C_{N-l}^{m}(-1)^{N-l-m}(mI - N\mathcal{R})^{-1} \\
& \times \left[\frac{A(\infty) - \alpha(\mu(N-m))}{\mu(N-m)} - (N\mu(I-\mathcal{R}))^{-1}(A(\infty) - \mathcal{R})\right], \quad l = \overline{1, N-1},
\end{aligned} \tag{3.37}$$

$$\mathbf{p}_0 = \boldsymbol{\theta} - \sum_{l=1}^{N-1} \mathbf{p}_l - \rho\boldsymbol{\theta}(I - \mathcal{R})(A(\infty) - \mathcal{R}).$$

Proof exploits in [7, Theorem 6.12] and is straightforward. So, it is omitted.

3.3. Distribution of the waiting and sojourn times. Let $V(x)$ and $W(x)$ be distribution functions of sojourn time and waiting time of an arbitrary customer in the system under study.

Theorem 3.6. *Distribution function* $W(x)$ *of the waiting time is calculated by*

$$W(x) = 1 - \boldsymbol{\theta}\mathcal{R}e^{-N\mu(I-\mathcal{R})x}\mathbf{e}. \tag{3.38}$$

Proof. It is clear that waiting time of the arbitrary (tagged) customer is equal zero if the customer meets free servers in the system. Probability of this event is equal to $\sum_{k=0}^{N-1} \boldsymbol{\pi}_k \mathbf{e} = \boldsymbol{\delta}\mathbf{e} = \boldsymbol{\theta}(I - \mathcal{R})\mathbf{e}$. When the tagged customer meets all servers busy and i, $i \geq 0$, customers waiting in a queue (probability of this event is equal to $\boldsymbol{\pi}_{N+i}\mathbf{e}$), its conditional waiting time distribution is Erlangian of order $i+1$ with intensity of the phase equal to $N\mu$.

By the direct applying the formula of total probability and using Theorem 3.3, we get formula

$$\begin{aligned}
W(x) &= \sum_{k=0}^{N-1} \boldsymbol{\pi}_k \mathbf{e} + \sum_{i=N}^{\infty} \boldsymbol{\pi}_i \int_0^x N\mu \frac{(N\mu u)^{i-N}}{(i-N)!} e^{-N\mu u} du\mathbf{e} \\
&= \boldsymbol{\theta}(I - \mathcal{R})\left[I + N\mu\mathcal{R}\int_0^x e^{-N\mu(I-\mathcal{R})u} du\right]\mathbf{e} \\
&= \boldsymbol{\theta}(I - \mathcal{R})\mathbf{e} + \boldsymbol{\theta}\mathcal{R}(I - e^{-N\mu(I-\mathcal{R})x})\mathbf{e} = 1 - \boldsymbol{\theta}\mathcal{R}e^{-N\mu(I-\mathcal{R})x}\mathbf{e}.
\end{aligned} \tag{3.39}$$

The theorem is proved. □

Corollary 3.7. *Average waiting time* W_1 *of customers in the system is computed by*

$$W_1 = \frac{\boldsymbol{\theta}\mathcal{R}(I - \mathcal{R})^{-1}\mathbf{e}}{N\mu}. \tag{3.40}$$

Variance D_W of the waiting time of customers in the system is computed by

$$D_W = \frac{2\boldsymbol{\theta}\mathcal{R}(I-\mathcal{R})^{-2}\mathbf{e}}{(N\mu)^2} - L^2. \tag{3.41}$$

Because the considered service discipline supposes that the customer, which sees i free servers upon arrival, is served by all these servers, independently of each other, we have to clarify what you mean when speaking about the sojourn time of a customer in the system. Here we mean that the sojourn time of a customer in the system is the time since the customer arrival to the system till the earliest epoch of this customer service completion (epoch of the delivering of the first copy of a customer).

It is easy to calculate that the time since the service beginning of this customer till the finish of the service of the first among i copies of this customer has exponential distribution with the parameter $i\mu$, $i = \overline{1,N}$.

Theorem 3.8. *Distribution function $V(x)$ of the sojourn time is calculated by*

$$\begin{aligned} V(x) = \boldsymbol{\theta}(I-\mathcal{R})\Bigg\{ &\Bigg(I+\mathcal{R}-\sum_{k=1}^{N}(\mathcal{A}_k+\mathcal{B}_k)\Bigg)(1-e^{-N\mu x}) + \sum_{k=1}^{N-1}(\mathcal{A}_k+\mathcal{B}_k)(1-e^{-(N-k)\mu x}) \\ &+N\mathcal{R}\Big[\big(N(I-\mathcal{R})\big)^{-1}(I-e^{-N\mu(I-\mathcal{R})x}) \\ &\quad - e^{-\mu x}\big(N(I-\mathcal{R})-I\big)^{-1}(I-e^{-(N\mu(I-\mathcal{R})-I\mu)x})\Big]\Bigg\}\mathbf{e}. \end{aligned} \tag{3.42}$$

The proof is straightforward. It is based on the formula of total probability. Reasonings, which are presented before the theorem formulation, about the sojourn time in the case of a customer arrival when not all servers are busy are taken into account. Also, we took into account that, for a customer who sees all servers busy upon arrival and i, $i \geq 0$, customers in a queue, conditional distribution of the sojourn time is convolution of the Erlangian distribution of order $i+1$ with intensity of the phase equal to $N\mu$ (conditional waiting time distribution) and the exponential distribution with intensity μ (conditional service time distribution).

Corollary 3.9. *The mean sojourn time V_1 in the system is computed by*

$$V_1 = \sum_{i=0}^{N-1}\frac{\boldsymbol{\pi}_i\mathbf{e}}{\mu(N-i)} + W_1 + \frac{1}{\mu}, \tag{3.43}$$

where the mean waiting time W_1 is defined in Corollary 3.7.

The second-order initial moment V_2 of the sojourn time distribution is computed by

$$V_2 = 2\left\{\sum_{i=0}^{N-1}\frac{\boldsymbol{\pi}_i\mathbf{e}}{\big(\mu(N-i)\big)^2} + \boldsymbol{\theta}\mathcal{R}\big(N\mu(I-\mathcal{R})\big)^{-2}\mathbf{e} + \frac{\boldsymbol{\theta}\mathcal{R}\big(N\mu(I-\mathcal{R})\big)^{-1}\mathbf{e}}{\mu} + \frac{1}{\mu^2}\right\}. \tag{3.44}$$

Jitter J (variance of the sojourn time) is calculated by

$$J = V_2 - V_1^2. \tag{3.45}$$

3.4. The case of unreliable servers. It is intuitively clear that the broadcasting service discipline has the following advantage. Because the customer can get the service simultaneously and independently in several servers, and the failure in one or even several servers, does not mandatorily cause failure of the customer service in the system, this discipline can improve, comparing to the classical discipline, the quality of the customers service in the cases when the servers are not absolutely reliable. The service of a customer is not absolutely reliable, for example, in a channel of a telecommunication network due to the possibility of the errors' occurrence on physical layer of information transmission. So, these cases should be analyzed aiming to model the real-life systems.

Thus, let us consider in brief the system with nonreliable servers. We assume here that the service of a customer is not interrupted when the error occurs, but just this customer will be considered as not served properly. In terms of telecommunications it means that the information unit, for example, message, will not be delivered in a good shape. So, we suppose that the errors process does not impact the process of service, but it impacts the result of the service. Situation when the error's occurrence impacts the service process in general, for example, it causes the break of the service, deserves a separate treatment, and is not considered in this paper.

We consider here two types of the error mechanisms. The first type assumes the independent errors' occurrence in the servers. The service of an arbitrary customer in an arbitrary server can fail with some known probability q, $0 \le q < 1$. The second type assumes that there exists a stationary Poisson arrival process of errors which has a known intensity φ. Arrival of such an error causes the failure of the service in all servers currently providing a service to customers.

Let us denote $P_+^{(k)}$ probability that an arbitrary customer will be delivered successfully through the considered queueing system under the kth type of the error mechanism, $k = 1,2$, and $\widetilde{P}_+^{(k)}$ is the value of the corresponding probability in the system with the classical service discipline.

Theorem 3.10. *Probabilities $P_+^{(k)}$, $k = 1,2$, that an arbitrary customer will be successfully delivered through the queueing system, are computed by*

$$P_+^{(k)} = 1 - \psi_1^{(k)} \boldsymbol{\theta}\mathcal{R}\mathbf{e} - \boldsymbol{\theta}(I - \mathcal{R})\left[\left(I + \mathcal{R} - \sum_{l=1}^{N}(\mathcal{A}_l + \mathcal{B}_l)\right)\psi_N^{(k)} + \sum_{l=1}^{N-1}(\mathcal{A}_l + \mathcal{B}_l)\psi_{N-l}^{(k)}\right]\mathbf{e}, \tag{3.46}$$

where

$$\psi_i^{(1)} = q^i, \quad \psi_i^{(2)} = \frac{\varphi}{\varphi + i\mu}, \quad i = \overline{1,N}. \tag{3.47}$$

The proof directly follows from the evident formula

$$P_{+}^{(k)} = \left[\sum_{l=0}^{N-1} \boldsymbol{\pi}_l\left(1-\psi_{N-l}^{(k)}\right) + \left(1-\psi_1^{(k)}\right)\sum_{l=N}^{\infty}\boldsymbol{\pi}_l\right]\mathbf{e} \tag{3.48}$$

and Theorem 3.3.

Note that it is easy to show that probabilities $P_{+}^{(k)}$ of successful delivering are greater than the corresponding probabilities $\widetilde{P}_{+}^{(1)} = 1-q$ and $\widetilde{P}_{+}^{(2)} = \mu/(\mu+\varphi)$ in the system with the classical service discipline.

3.5. The *GI/M/N*, *D/M/N*, and *M/M/N* systems. In the case of the *GI/M/N* system, the results are a bit simplified. In this case the semi-Markovian kernel $A(t)$ is a scalar interarrival distribution function. Correspondingly, all vectors and matrices, which present in the results of the previous subsections, are scalars. Scalar equation (3.18) has a unique solution in the interval $(0,1)$ if stability condition (3.13) is fulfilled.

In the special case of the *D/M/N* system where the arrival flow is deterministic:

$$A(t) = \begin{cases} 0, & t \le T, \\ 1, & t > T, \end{cases} \tag{3.49}$$

the values $\mathcal{A}_k, \mathcal{B}_k$, $k = \overline{0,N}$, in Theorem 3.3, are calculated by

$$\begin{gathered} \mathcal{A}_k = C_N^k e^{-k\mu T}\left(1-e^{-\mu T}\right)^{N-k}, \\ \mathcal{B}_k = N\mathcal{R}C_N^k \sum_{m=0}^{N-k} C_{N-k}^m (-1)^{N-k-m} \frac{\mathcal{R}-e^{-\mu(N-m)T}}{N\mathcal{R}-m}, \quad k = \overline{0,N}, \end{gathered} \tag{3.50}$$

the number $\mathcal{R}$ is the single root of equation $\mathcal{R} = e^{-\mu N(1-\mathcal{R})T}$ in the interval $(0,1)$.

In the case of the *M/M/N* system, the root $\mathcal{R}$ of (3.18) is calculated in explicit form and is equal to the traffic intensity (load) ρ. So, the results are essentially simplified.

Corollary 3.11. *In the case of the M/M/N system with broadcasting, the stationary distribution p_i, $i \ge 0$, is computed by*

$$\begin{gathered} p_i = p_0\sigma_i, \quad i = \overline{0,N}, \\ p_i = p_0\sigma_N\rho^{i-N}, \quad i > N, \\ p_0 = \rho(1-\rho)\sigma_N^{-1}, \end{gathered} \tag{3.51}$$

where

$$\sigma_i = \prod_{k=1}^{i} \frac{\lambda+(k-1)\mu}{k\mu}, \quad i = \overline{0,N}. \tag{3.52}$$

Average number of customers L in the system is calculated by

$$L = N\rho + \frac{\rho^2}{1-\rho} + \rho(1-\rho)\sigma_N^{-1}\sum_{i=0}^{N-1} i\sigma_i. \tag{3.53}$$

Average sojourn time V_1 of a customer in the system is calculated by

$$V_1 = \frac{\rho}{N\mu}\left(N + \frac{1}{1-\rho}\right) + \rho(1-\rho)\sigma_N^{-1}\sum_{i=0}^{N-1}\frac{\sigma_i}{(N-i)\mu}. \tag{3.54}$$

Probability $P_+^{(k)}$ that an arbitrary customer will be successfully delivered through the queueing system (if the service is not reliable) is computed by

$$P_+^{(k)} = 1 - \psi_1^{(k)}\rho - \rho(1-\rho)\sigma_N^{-1}\sum_{i=0}^{N-1}\sigma_i\psi_{N-i}^{(k)}, \tag{3.55}$$

where the values $\psi_i^{(k)}$, $i = \overline{1,N}$, $k = 1,2$, are given by formula (3.47).

Detailed numerical comparison of the main performance characteristics of the system with broadcasting service with the corresponding measures of the classical $M/M/N$ system, $N > 1$, is not intended to be presented in this paper. We mention only several observations based on such a comparison.

(i) Little's formula $L = \lambda V_1$ is valid for the classical $M/M/N$ system and it does not hold good for the $M/M/N$ broadcasting system.

(ii) The broadcasting system has smaller value of the probability to have an empty system.

(iii) Average number L of customers in the system is always larger for the broadcasting system.

(iv) Relation of the average sojourn time V_1 of a customer in the systems depends on the system parameters. For example, if we fix $\lambda = 1$ (it can be always done without the loss of generality) and fix the traffic intensity $\rho = 0.5$ and then increase the number of servers N with the corresponding fitting service intensity μ, we get the following result. For $1 < N \leq 3$, the classical service discipline gives smaller value of the average sojourn time V_1. For $N > 3$, the broadcasting service discipline is better. For example, for $N = 50$ we have $V_1 = 25.00$ for the classical service discipline and $V_1 = 19.95$ for the broadcasting service discipline.

So, the broadcasting discipline provides 20 percent lesser average sojourn time. When the number of servers grows, the advantage of this discipline continues to increase.

If we assume now that the traffic intensity ρ is equal to 0.1, then for $N = 50$ we have $V_1 = 5.00$ for the classical service discipline and $V_1 = 1.6326$ for the broadcasting service discipline. This means that the delivering time of a customer is three times less in the case of the broadcasting service discipline. If we further decrease the traffic intensity ρ, the benefit of broadcasting service discipline can become very big.

The positive effect of the broadcasting in situations when errors in a transmission can occur is confirmed by the numerical experiment. In Table 3.1, we present the value κ of the relative improvement (in percent) of the probability of a successful customer delivering by means of broadcasting discipline compared to the classical discipline for three different values of the system load ρ and four values of the probability $\bar{q} = 1 - q$ of

Table 3.1. Improvement κ of the probability of a successful customer delivering.

ρ	$\bar{q} = 0.9$	$\bar{q} = 0.8$	$\bar{q} = 0.7$	$\bar{q} = 0.5$	$\bar{q} = 0.4$
0.5	3.5%	7.3%	12.0%	23.0%	57.0%
0.25	6.4%	15.0%	23.0%	48.0%	65.0%
0.1	8.9%	19.0%	33.3%	73.0%	104.0%

a customer service without an error. The value κ is computed by

$$\kappa = \frac{P_+^{(1)} - \widetilde{P}_+^{(1)}}{\widetilde{P}_+^{(1)}} \times 100\%, \tag{3.56}$$

where $P_+^{(1)}$ is the value of the probability of a successful customer delivering by means of broadcasting discipline and $\widetilde{P}_+^{(1)}$ is the value of this probability under the classical discipline.

One can see that in case when the load of the system is small and so the sending of a customer to several parallel servers is not rare, the profit from broadcasting in respect of more reliable customers delivering can be essential. This profit increases if the quality of the service in a server becomes worse. The number of the servers in this experiment was assumed to be $N = 15$. The profit increases also with the grow of the number of servers in a system.

In the case of the second mechanism of error's occurrence, the profit from using the broadcasting discipline behaves analogously.

These observations as well as the possible reduction of the delivering time, what was mentioned above, motivate importance of investigation of the broadcasting service discipline.

4. The *SM/M/N/N* system

Assume now that the system has no buffer, and the arriving customer that meets all servers busy is lost.

The behavior of the system here is described by the two-dimensional irreducible discrete time Markov chain

$$\zeta_n = (i_n, \nu_n), \quad n \geq 1,\ i_n = \overline{0,N},\ \nu_n = \overline{1,K}, \tag{4.1}$$

where components (i_n, ν_n) have the same meaning as in the previous section.

Denote the stationary probabilities of this Markov chain as

$$\pi(i,k) = \lim_{n\to\infty} P\{i_n = i, \nu_n = k\}, \quad i = \overline{0,N},\ k = \overline{1,K}, \tag{4.2}$$

and the row-vectors

$$\boldsymbol{\pi}_i = (\pi(i,1), \pi(i,2), \ldots, \pi(i,K)), \quad i = \overline{0,N}. \tag{4.3}$$

Stationary distribution (4.2) of the finite-state irreducible Markov chain under study exists for all values of the system parameters.

As in the previous section, we form the matrices $P_{i,l}$ of one-step transition probabilities of the Markov chain ζ_n, $n \geq 1$, by

$$P_{i,l} = \left(P\{i_{n+1} = l, \nu_{n+1} = k' \mid i_n = i, \nu_n = k\}\right)_{k,k'=\overline{1,K}}, \quad i,l = \overline{0,N}. \tag{4.4}$$

Lemma 4.1. *The matrix $\mathcal{P} = (P_{i,l})_{i=\overline{0,N},\ l=\overline{0,N}}$ of one-step transition probabilities of the Markov chain ζ_n, $n \geq 1$, has the following structure:*

$$\mathcal{P} = \begin{pmatrix} \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \dots & \mathcal{A}_{N-1} & \mathcal{A}_N \\ \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \dots & \mathcal{A}_{N-1} & \mathcal{A}_N \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathcal{A}_0 & \mathcal{A}_1 & \mathcal{A}_2 & \dots & \mathcal{A}_{N-1} & \mathcal{A}_N \end{pmatrix}. \tag{4.5}$$

Theorem 4.2. *Stationary distribution $\boldsymbol{\pi}_i$, $i \geq 0$, of the Markov chain ζ_n, $n \geq 1$, is computed as follows:*

$$\boldsymbol{\pi}_i = \boldsymbol{\theta}\mathcal{A}_i, \quad i = \overline{0,N}, \tag{4.6}$$

where the matrices $\mathcal{A}_i$, $i = \overline{0,N}$, are defined by formula (3.11).

Proof of this theorem is straightforward and is omitted.

Theorem 4.3. *Probability P_{loss} of a loss of an arbitrary customer is computed by*

$$P_{\text{loss}} = \boldsymbol{\theta}\alpha(N\mu)\mathbf{e}. \tag{4.7}$$

Theorem 4.4. *Distribution function $V(x)$ of the sojourn time is calculated by*

$$V(x) = 1 - \boldsymbol{\theta}\sum_{k=0}^{N-1}\mathcal{A}_k e^{-\mu(N-k)x}\mathbf{e}. \tag{4.8}$$

The proof easy follows from the formula of total probability:

$$V(x) = P_{\text{loss}} + \boldsymbol{\theta}\sum_{k=0}^{N-1}\mathcal{A}_k\left(1 - e^{-\mu(N-k)x}\right)\mathbf{e}. \tag{4.9}$$

Theorem 4.5. *Distribution function $V(x)$ of the sojourn time for customers, which are not lost, is calculated by*

$$V(x) = \frac{\boldsymbol{\theta}}{1 - P_{\text{loss}}}\sum_{k=0}^{N-1}\mathcal{A}_k\left(1 - e^{-\mu(N-k)x}\right)\mathbf{e}. \tag{4.10}$$

Corollary 4.6. *The mean sojourn time V_1 in the system for customers, which are not lost, is computed by*

$$V_1 = \frac{\boldsymbol{\theta}}{1 - P_{\text{loss}}}\sum_{k=0}^{N-1}\frac{\mathcal{A}_k}{\mu(N-k)}\mathbf{e}. \tag{4.11}$$

The second initial moment V_2 of the sojourn time in the system for customers, which are not lost, is computed by

$$V_2 = \frac{\boldsymbol{\theta}}{1 - P_{\text{loss}}} \sum_{k=0}^{N-1} \frac{2\mathcal{A}_k}{(\mu(N-k))^2}\mathbf{e}. \tag{4.12}$$

Jitter J of delivering time is computed by $J = V_2 - V_1^2$.

Corollary 4.7. *In the case of the M/M/N/N system with broadcasting, the stationary distribution π_i, $i \geq 0$, is computed by*

$$\pi_i = \sigma_i \left(\sum_{k=0}^{N} \sigma_k \right)^{-1} = \frac{\sigma_i \rho}{\sigma_N(1+\rho)}, \quad i = \overline{0,N}, \tag{4.13}$$

where the values σ_i, $i = \overline{0,N}$, are defined by formula (3.52).

Probability P_{loss} of a loss of an arbitrary customer is computed by

$$P_{\text{loss}} = \frac{\rho}{1+\rho}. \tag{4.14}$$

The mean sojourn time V_1 in the system for customers, which are not lost, is computed by

$$V_1 = \frac{\rho}{\mu\sigma_N} \sum_{i=0}^{N-1} \frac{\sigma_i}{N-i}. \tag{4.15}$$

Comparing the value of the loss probability P_{loss} for the broadcasting discipline with the corresponding probability $\widetilde{P}_{\text{loss}}$ for the classical discipline, which is given by the famous B-formula by A. K. Erlang:

$$\widetilde{P}_{\text{loss}} = \frac{(N\rho)^N/N!}{\sum_{l=0}^{N}((N\rho)^l/l!)}, \tag{4.16}$$

we can see that, for $N > 1$, loss probability in the the case of the broadcasting discipline is higher.

However, comparing the value of the mean sojourn time V_1 for customers, which are not lost, for the broadcasting discipline with the corresponding mean sojourn time $\widetilde{V}_1$ for the classical discipline, which is given by $\widetilde{V}_1 = \mu^{-1}$, we can see that, even for $N = 2$, the mean sojourn time V_1 in the the case of the broadcasting discipline can be smaller. For example, if the intensity λ of the arrival process satisfies inequality $\lambda < 3\mu$, where μ is the service intensity, then the mean sojourn time V_1 is less than 75 percent of the mean sojourn time $\widetilde{V}_1$. With the decrease of the traffic intensity $\rho = \lambda/N\mu$ and the increase of the number of the servers N, advantage of the broadcasting discipline with respect to the mean sojourn time can become huge.

So, conclusion about the preference of the classical or broadcasting disciplines in any concrete system modeled by the multiserver queueing model without a buffer should be made in each concrete situation depending on the relation of the system requirements to

Table 4.1. Probabilities of a successful customer delivering by means of the broadcasting and classical disciplines.

ρ	$\bar{q} = 0.9$	$\bar{q} = 0.8$	$\bar{q} = 0.7$	$\bar{q} = 0.5$
0.5	0.8949; 0.6417	0.7955; 0.6128	0.6060; 0.5791	0.4972; 0.4912
0.25	0.9; 0.7816	0.8; 0.7594	0.7; 0.7324	0.5; 0.6558
0.1	0.9; 0.8994	0.8; 0.8873	0.7; 0.8720	0.5; 0.8243
0.05	0.9; 0.9470	0.8; 0.9402	0.7; 0.9315	0.5; 0.9030

the loss probability and response time. If the loss probability is more important, the classical discipline is better. However, if the response time is more essential, the broadcasting discipline can be more preferable.

If the service in a server can be implemented with error, as it was described in Section 3.4, the following statement holds good.

Corollary 4.8. *Probability $P_+^{(k)}$ of successful delivering of a customer in the case of kth type of the error occurrence mechanism is computed by*

$$P_+^{(k)} = \sum_{i=0}^{N-1} \frac{\sigma_i \rho}{\sigma_N (1+\rho)} \left(1 - \psi_{N-i}^{(k)}\right), \quad k = 1,2, \tag{4.17}$$

where the values $\psi_i^{(k)}$, $i = \overline{1,N}$, $k = 1,2$, are given by formula (3.47).

Numerical experiments show that if the error in servers can occur, then the broadcasting discipline can be more preferable even with respect to loss probability if the service of a customer with error is considered to be equivalent to the customer loss. Table 4.1 shows the value of the probabilities $P_+^{(1)}$ and $\widetilde{P}_+^{(1)}$ of successful customer delivering by means of the broadcasting and classical disciplines correspondingly for several values of a probability $\bar{q} = 1 - q$ of the service in a server without an error and several values of the traffic intensity ρ. The first number among two numbers for each system corresponds to $\widetilde{P}_+^{(1)}$. The second one, separated by the character ";", corresponds to $P_+^{(1)}$.

The number of servers in this experiment is $N = 15$. Because the traffic intensity ρ is chosen here to be small, there is practically no losses of customer in the case of the classical discipline. So, probability of successful delivering is completely defined by a probability $\bar{q}$ for $\rho \geq 0.25$. It is evidently seen that with the increase of the error probability and the decrease of the traffic intensity the broadcasting discipline becomes more preferable.

5. Conclusion

We have analyzed the stationary distribution of the queue and waiting and sojourn times in the *SM/M/N* type queueing systems with infinite buffer or losses when the customer arriving into the system is served simultaneously by all free servers. The diversity of services increases the load of the servers. But, as follows from numerical results, it helps to get more quick delivering of the customers. Also, if the service in the channel can be provided with an error, the considered discipline allows to increase, sometimes essentially, the probability of successful service in the system.

Acknowledgments

This research was supported by the MIC (Ministry of Information and Communication), Korea, under the ITSIP (IT Foreign Specialist Inviting Program), supervised by the IITA (Institute of Information Technology Assessment), KOTEF, and the International Cooperation Research Program of the Ministry of Science and Technology in Korea.

References

[1] A. V. Pechinkin and V. V. Chaplygin, *Stationary characteristics of the SM/MSP/n/r queueing system*, Automation and Remote Control **65** (2004), no. 9, 1429–1443.

[2] A. V. Pechinkin and S. I. Trishechkin, *The SM_2/MSP/n/r system with a random-server discipline and common storage*, Automation and Remote Control **64** (2003), no. 11, 1742–1755.

[3] H. Takagi and D.-A. Wu, *Multiserver queue with semi-Markovian batch arrivals*, Computer Communications **27** (2004), no. 6, 549–556.

[4] H. Takagi and D.-A. Wu, *Multiserver queue with semi-Markovian batch arrivals with application to the MPEG frame sequence*, Internet Performance and Control of Network Systems III, Proceedings of SPIE, vol. 4865, 2002, pp. 178–189.

[5] M. H. Lee, A. N. Dudin, and V. I. Klimenok, *Multi-server queueing model with broadcasting service*, submitted for publication.

[6] M. F. Neuts, *Matrix-Geometric Solutions in Stochastic Models. An Algorithmic Approach*, Johns Hopkins Series in the Mathematical Sciences, vol. 2, Johns Hopkins University Press, Maryland, 1981.

[7] E. Çinlar, *Introduction to Stochastic Processes*, Prentice-Hall, New Jersey, 1975.

Moon Ho Lee: Institute of Information and Communication, Chonbuk National University, Chonju 561-765, South Korea
E-mail address: moonho@chonbuk.ac.kr

Alexander N. Dudin: Institute of Information and Communication, Chonbuk National University, Chonju 561-765, South Korea
Current address: Department of Applied Mathematics and Computer Science, Belarusian State University, Minsk 220030, Belarus
E-mail addresses: dudin_alex@yahoo.com ; dudin@bsu.by

Valentina I. Klimenok: Department of Applied Mathematics and Computer Science, Belarusian State University, Minsk 220030, Belarus
E-mail address: klimenok@bsu.by