



The social brain of social media – a physiological boundary to the number of online relations

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Received: 1 December 2020 / Revised: 22 May 2021 / Accepted: 24 May 2021 / Published online: 8 June 2021
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Abstract

Based on the research done by Dunbar and the resulting Social Brain Hypothesis, the present study introduced a mathematical model for the development of follower numbers and the number of followed accounts regarding users/influencers of Social Media platforms. Under very simple assumptions the mathematical model suggests that an universal upper bound to follower and followed numbers exists. The theoretical upper bound is then empirically validated by using a representative data set of 255 influencers on Instagram from the field of women's fashion. The follower numbers show convergence to a common boundary for the years 2018 to 2019 and stagnation for 2019 to 2020, while the number of followed accounts show stagnation for 2018 to 2019 and convergence for 2019 and 2020. The model in conjunction with its empirical validation therefore provides the mathematical background to establish the socio-biological Social Brain Hypothesis in the field of influencer marketing in regards to Social Media platforms.

Keywords Social media · Growth · Theoretic model · Convergence · Dunbar · Social brain hypothesis

JEL classifications C51 · M31 · O49

1 Introduction

Considering Social Media stars whose followers number into the tens of millions, it seems almost antiquated that initially Facebook introduced an upper bound on the friends lists of 5,000. Comparing it with the original mission of Facebook to connect real-world friends in the digital world it becomes, however, more comprehensible in particular referring to Dunbar's number or the Social Brain Hypothesis.

Dunbar (1992) argues that for primates the size of the circle of what might be called friends depends on the size of the neocortex of the brain. Using average values for the size of a human brain, he conjectures that the upper bounds for real-world friends among humans lies at about 150. Since then this number has been found in a number of human social groups; from hunter-gatherer tribes, to the Roman army to modern distributions of city sizes, Hernando et al. (2010). The theoretical proposition is empirically studied in more

detail by Hill and Dunbar (2003) where they find empirical values for the maximum size of social networks of 153.5 with 124.9 being the average of people one is in contact with. The results are backed up by the findings of Zhou et al. (2005), Goncalves et al. (2011) and Powell et al. (2012).

In addition to motivating the upper boundary for the number of friends in general, the findings by Dunbar can be scaled in regard to the intensity of the acquaintance. The most pronounced circle being about 5 intimate or best friends and then with each step that the intensity of the acquaintance decreases the number of people in the circle roughly triples with an absolute maximum of 1,500 people to whom one can connect a face and a name.

In light of this physiological boundary and the stated goal of Facebook, capping friend lists at 5,000 seems more than reasonable.

Considering Perret and Edler (2020) even with a sample of 255 top influencers of the field of women's fashion, the average number of followed accounts by other users lies at roughly 890 with a maximum of about 5,700. Thus, an upper bound to Social Media relations seems to exist as well, even though connecting via Social Media might be even weaker

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ties than the one's required for the absolute maximum of 1,500 real-world connections.

On the other hand, the number of followers in directed Social Media networks seems to be without boundaries with the same sample of influencer quoted above the maximum is more than 51 million followers (Gigi Hadid). Nevertheless, it is not assumed that there exists any connection at all from the direction of an influencer to any of her followers – the sample of influencers considered in this study coincides with the one used by Edler and Perret (2020) and Perret and Edler (2020) and contains only women thus female pronouns are used hereafter. The only technical upper bound is the number of users of the corresponding Social Media network.

This study provides to the literature not only by empirically establishing an upper bound on Social Media for following as well as follow relations exists but as well provides a theoretical model to describe the dynamics of the process underlying the development of these numbers. While simplistic in nature, the model also proposes that the numbers converge to a common upper bound. The empirical evaluation based on a representative sample of influencers from the field of women's fashion also hints at the fact that this upper bound functions as a universal upper bound for all actors on Social Media, independently of their relevance at the moment. The data set and its representativeness are discussed in more detail in Sect. 3.

Following a brief introduction on Social Media and influencers the theoretical model underlying this study is deduced and implications resulting from the model are discussed.

The model is then empirically validated allowing in the final concluding section to also derive practical recommendations from the results and discuss limitations of the present study.

2 Social media, friends and influencers

Social Media has been present for about five decades since the early 1970s beginning with platforms like Talkomatic—Bodgoli (2004). Social networking sites in the modern sense have been around since 1997 and the introduction of Six Degrees, Boyd and Ellison (2007). Marketing's interest in Social Media in particular in influencers – actors in a Social Media platform that have a significant followership – developed from affiliate marketing in a Social Media context over the last ten years, Kozinets et al. (2010).

Influencer-based marketing began in the early 2010s with the increased use of Social Media in marketing linking it as well to word-of-mouth marketing; a link motivated by Kozinets et al. (2010), Dost et al. (2018) and Bakker (2018). The studies by Berger (2014) and Bakker (2018) motivate influencer marketing as the digital analogy of word-of-mouth based marketing.

This link closes the loop between size of social networks among real world people and the digital world and as such networks in the context of Social Media.

As this study focuses on Instagram influencers in particular (Veirman et al. 2017); (Arora et al. 2019) and (Casalo et al. 2020) provide the background for influencer marketing on Instagram; due to severe restrictions in regard to the Instagram API current quantitative studies on Instagram are limited in scope. Concerning other platforms Bakshy et al. (2011), Aswani et al. (2017), Nebot et al. (2018) and Arora et al. (2019) focus on Twitter or Cavalli et al. (2011) and Arora et al. (2019) on Facebook. Since the empirical part of this study is focused on Instagram Haenlein et al. (2020) provides a background on the managerial aspects of influencer marketing in particular on Instagram, while studies like Riedl and Luckwald (2019) and Lee and Kim (2020) put the focus more on the effects and effectiveness of influencers' advertising activities.

No general and quantifiable definition of the term influencer exists, and all relevant studies define the term differently. A number of studies Lim et al. (2017), Lou and Yuan (2018) or Audrezet et al. (2018) put the focus of their definitions on authenticity and the position of the influencers in their networks or their large audience. On the other hand, Veirman et al. (2017) and Veirman and Hudders (2020) stress aspects like brand attitude. While terms like authenticity or brand attitude are hard to quantify, characteristics like followers, reach, posting frequency, engagement rate or growth rates are quoted—Bendoni (2017), Hall (2017) or Aggrawal et al. (2018)—as tangible indicators of an influencer's position. However, no consensus exists on relevant threshold values for these characteristics.

In the course of this study a simple definition of being an influencer is applied and agents are considered influencers if they are listed by a public ranking as relevant figures in their field and additionally exceed a lower bound of Instagram followers of 100,000 signifying that this study focuses mainly on macro-influencers with a broad and general audience. This study adopts the threshold of 100,000 followers in particular for macro-influencers even though most studies like Munawar and K. (2018) just differentiate influencer types by their target audience or like Conde (2019) and Kay et al. (2020) simply argue that micro-influencers have few followers while macro-influencers have many followers. Even in the original literature on so called micro-celebrity by Senft (2008) and the study (Khamis et al. 2017) who relate it more strongly to Social Media influencers no hard thresholds for having reached are defined. In consequence and considering (Coelho 2019) who points out the link between the number of followers and the likability (relevance) of influencers, the 100,000 follower threshold is taken from popular literature where it established itself as a convenient cut-off value.

Considering the results of Dunbar’s initial research and the conception of the Social Brain Hypothesis in Dunbar (1998) the question can be raised in how far the patterns observed in the real-world apply similarly to an internet context (Dunbar 2012) or in particular, the context of Social Media platforms. A practical example for this assumption stems from Facebook’s self-perception as a social network and its initial use of a limit of 5000 to the number of possible virtual friends (Ching et al. 2015).

Social Media in this regard is of even greater interest as it aims to replicate human social interactions in a digital or virtual environment. In this context oftentimes the difference between strong and weak ties as introduced by Granovetter (1973) is recalled with the underlying argument that Social Media particularly represents networks of weak ties, Perret and Edler (2020).

In this light, while Dunbar et al. (2015) and Dunbar (2016) argue the applicability of the Social Brain Hypothesis to a Social Media context the hypothesis itself is not unanimously accepted (Acedo-Carmona & Gomila 2016) and in particular its applicability in a Social Media context where only the weakest of ties exist can be questioned as well. Roberts et al. (2009) argue that the size of the social network is dependent on the emotional intensity of the relation between influencers and their followers, Dunbar (2016) argues that follower numbers in the millions do not necessarily have to contradict earlier results as the strength of the emotional bonds between influencers and the majority of their followers is zero or very close to it. Kanai et al. (2012) provide evidence that the density of the amygdala is related to the size of participants’ Social Media contacts, thus additionally strengthening the transfer of Dunbar’s results to the Social Media context.

Goncalves et al. (2011) provide results in this direction and studies the application of Dunbar’s results to Twitter networks and as such is the first study to provide broad scale evidence for it focused on a particular platform.

3 A growth model for social media followers

To provide this study with a sound theoretical underpinning, a mathematical interpretation of the problem at hand as well as a basis for empirical analysis, a simple mathematical model for influencer development on Social Media in general has been presented. Independent of any concrete platforms, this model has been applied to real data for a set of influencers on Instagram. This approach concretizes the research question aside from the purely biological aspects put forth by Dunbar.

Considering the development of follower numbers, a simple initial relation can be assumed

$$F_{t+1} = F_t + I_t - D_t \tag{1}$$

Here t indicates period t , F_t is the number of followers in period t , I_t the newly won followers in period t and D_t the followers lost in period t . With this notation the equation states that the number of followers in the next period depends on the current number of followers and is increased by new followers and decreased by followers lost.

The change in followers ΔF can thus be written as

$$\Delta F = F_{t+1} - F_t = I_t - D_t \tag{2}$$

Interpreting the first difference ΔF as the first discrete derivative of F_t with regard to the time t Eq. (2) can be rewritten as

$$\frac{\Delta F_t}{\Delta t} = I_t - D_t \tag{3}$$

which can be approximated by the continuous derivative of F_t for t as in (4)

$$\frac{dF_t}{dt} = I_t - D_t \tag{4}$$

This equation states that the growth rate of followers is determined as the difference between new followers gained and old followers lost. Since both the growth rate and the two parts I_t and D_t are dependent on time t this equation is a differential equation of order 1. The concrete forms of I_t and D_t determine the solution to this differential equation.

In a first step it is assumed that an influencer does not lose followers and thus D_t becomes zero and Eq. (4) reduces to the following expression (assuming that the number of new followers depends on the number of current followers)

$$\frac{dF_t}{dt} = I_t(F_t) \tag{5}$$

This equation still depends on the concrete form of I_t . Ideally, it can be assumed that I_t has one of three distinct forms. Either it is negatively quadratic over time – in the sense that for small t it is increasing while for large t it is decreasing— or it is monotonously decreasing. The possibility that the number of new followers is monotonously increasing can be excluded as it would be incompatible with the real-world environment since it would imply an unbounded exponential follower growth. Using the following form both realistic assumptions for I_t are covered

$$I_t = \alpha F_t^{-\beta} \text{ with } \alpha > 0 \tag{6}$$

If β is negative then the second case is realized and it is monotonously decreasing. If β is positive for small t and negative for large t then the first case is realized and it has a negative quadratic shape. In this case β would be

time-dependent as well and corresponding coefficients in a regression research design would become in-constant; a critical aspect that is considered in detail later on.

While parameter β is related to the speed by which followers are accrued the parameter α can be interpreted as a general accelerator. Considering two actors that report the same speed β the actor with the higher α will grow much faster and accrue high follower numbers than an influencer with a smaller α . Since the loss of followers is captured by D_t negative values for α can be ruled out leading to a strictly positive α . One possibility to imagine α could in the resources that are available to the actor to increase the number of followers, whereas β is the potential to use existing followers to generate new followers (e.g., via word-of-mouth or other channels).

Inserting (6) for I_t into (5) results in

$$\frac{dF_t}{dt} = \alpha F_t^{-\beta} \tag{7}$$

This is a simplified Bernoulli differential equation which can be solved by using the substitution $V = F^{1+\beta}$, resulting in the simple differential equation (Perret 2018)

$$\frac{dV_t}{dt} = \alpha(1 + \beta) \tag{8}$$

of which the solution is given as

$$V_t = \alpha(1 + \beta)t + C_0 \tag{9}$$

Re-substitution yields as a solution for F_t

$$F_t^{1+\beta} = \alpha(1 + \beta)t + C_0 \tag{10}$$

and thus by taking the corresponding root

$$F_t = \sqrt[1+\beta]{\alpha(1 + \beta)t + C_0} = (\alpha(1 + \beta)t + C_0)^{\frac{1}{1+\beta}} \tag{11}$$

Assuming an initial condition of $F_0=0$ —each influencer starts with zero followers—leads to $C_0=0$

$$F_t = \sqrt[1+\beta]{\alpha(1 + \beta)t} = (\alpha(1 + \beta)t)^{\frac{1}{1+\beta}} \tag{12}$$

Calculating the first derivative of function (13) gives

$$\frac{dF_t}{dt} = \alpha(1 + \beta)(\alpha(1 + \beta)t)^{\frac{-\beta}{1+\beta}} = \alpha(1 + \beta)F_t^{-\beta} \tag{13}$$

This equation shows two things. First, for a β larger than -1 – with an increasing t —the change in followers is strictly positive and the number of followers will potentially increase indefinitely. However, Eq. (13) additionally shows that in this first case a larger number of followers goes hand in hand with an absolutely speaking smaller growth rate. Making convergence to a steady state level of followers a possibility.

Returning to Eq. (2), the fact that D_t up to this point is considered to be zero still needs to be remedied. Regarding D_t a simplifying assumption would be that it is a constant share of the current number of followers – each year a fixed share of followers is lost (e.g., every year an influencer with 100,000 followers on average loses 100 of them γ will be 0.001) – and therefore

$$D_t = \gamma F_t \tag{14}$$

Inserting Eq. (9) and Eq. (14) in Eq. (2) yields

$$\frac{dF_t}{dt} = \alpha F_t^{-\beta} - \gamma F_t \tag{15}$$

whereby it again becomes a Bernoulli differential equation which has the following solution (Perret 2018)

$$F_t = \left(C_0 \exp(-\gamma(1 + \beta)t) + \frac{\alpha}{\gamma} \right)^{\frac{1}{1+\beta}} \tag{16}$$

For an increasing time t the function in this case converges toward a limit of

$$F = \left(\frac{\alpha}{\gamma} \right)^{\frac{1}{1+\beta}} \tag{17}$$

and the number of possible followers would be limited by this upper bound.

Assuming again the initial condition of $F_0=0$ —each influencer starts with zero followers—leads to

$$C_0 = -\frac{\alpha}{\gamma} \tag{18}$$

Calculating the first derivative of F_t and inserting Eq. (18) leads to

$$\frac{dF_t}{dt} = \alpha \exp(-\gamma(1 + \beta)t) F_t^{-\beta} \tag{19}$$

Equation (19) shows in analogy to Eq. (13) that larger follower numbers go hand in hand with a lower absolute growth rate of the followers. Here, however, it will lead to a convergence of the follower numbers with the long-run steady state of Eq. (17).

By log-linearizing Eqs. (16) the general estimation model (20) can be deduced if a log-linearized version of a Taylor expansion of Eq. (16) is used; a detailed deduction of Eq. (20) can be found in the appendix. Thus, the approach considering herein is similar to the approaches implemented in the context of economic growth theory when testing for β -convergence; a concept introduced by Barro and Sala-I-Martin (1990); Barro and Sala-I-Martin (1992) and used in a broad array of studies on economic convergence. The resulting estimation model is

$$\ln\left(\frac{F_{t+1}}{F_t}\right) = a + b \cdot \ln(F_t) \quad (20)$$

The structure of this model allows for estimating it by applying linear regression.

The parameter b can be written as $b = -(1 - \exp(-\gamma(1 + \beta))) > -1$ for all β . However, only for a β that is negative and comparatively large ($\beta < -1$) can b become larger than zero. This case, however, would make the model itself unstable.

Additionally, $\gamma(1 + \beta)$ is the speed by which the follower numbers of influencers converge toward the long-run steady state – Eq. (17).

Note that this type of model would also work for some cases of an Eq. (1) of the form $F_{t+1} = F_t \cdot I_t / D_t$. This, however, is not considered in this study.

An additional advantage of model (14) is that the sign of parameter b might indicate whether convergence ($b < 0$) or divergence ($b > 0$) occurs. While convergence would be the assumption underlying the modeling approach in (10) and would signify that a general upper bound on the number of followers exists, divergence would imply that modeling approach (7) would be better suited in describing follower developments and that no general upper bound exists.

While the preceding discussion focused solely on Social Media followers' similar arguments can be raised in favor of followings. An estimation model equivalent in nature to Eq. (20) could thus be used for analyzing the development of followings as well.

4 Empirical validation

4.1 Data collection

255 influencers have been selected that share the qualitative characteristic of being listed in one or more rankings as one of the most significant influencers in women's fashion and as well fulfill the requirement of having at least 100,000 followers by the time of conception of the data set in early 2018. The rankings considered in the context of this study include Block—Block (2016), Collsen—Collsen (2016), Editorial Stuff—Editorial Staff (2016), Ferrari—Ferrari (2018), Forbes—Forbes (2018), Gushcloud—Gushcloud Pte Ltd (2017), Klein—Klein (2016)—and West—West (2017).

Thus, in the selection process, mostly rankings from European and US sources have been considered providing a slightly biased perspective. However, while some Asians (foremost Chinese and Koreans) and Middle Eastern influencers have significant follower numbers their outreach is usually limited to their respective locale, and they intentionally do not target a global audience, often posting in their mother tongue alone.

As discussed above, being considered an influencer is a highly subjective assignation and borders between definitions are very fuzzy. This study does not try to conduct a census of all influencers in women's fashion but draw a representative sample. Due to a lack of information on the population in this context representativeness could only partially be assured if the sample size is sufficiently large and a workable definition of influencers per se exists.

The definition of influencers introduced in the second section, with a cutoff value of 100,000 followers fulfills the second condition. Considering thus, that the number of influencers with a followership of 100,000 and above is rather limited considering an international sample of 255 influencers can be assumed to fulfill the criterion of representativeness from a purely size oriented point of view.

Not every influencer that became part of the sample has been tested in detail for the presence of bought followers. However, them being part of an established ranking, combined with anecdotal testing, hints that all influencers in the sample have a significant followership, validating their presence.

For each of the influencers, based on their Instagram profile, for the years 2018, 2019 and 2020 (The data are collected in February) data have been collected on the number of their followers and how many other accounts in total they follow. Additional core metrics like the number of posts and the related posting frequency as well as the number of topics they cover and whether they post solely in English or other languages besides – usually their mother tongue – are noted. For each influencer the average number of likes and comments their posts received are collected. Together with the time the influencer has been already active on Instagram, their engagement rate in both regards can be calculated.

External data have been collected on the origin of the influencers, their age, the number of children they have (as all influencers in the sample that have children also post pictures of them) and whether they are active as a model. This data have been compiled from multiple sources combining popular magazines, celebrity databases and the influencers' blogs, internet pages and other Social Media presences.

All 255 influencers are women that also are active at other Social Media platform and own either a blog or their own website, thus these aspects are not controlled.

(The implemented data set is available on Figshare: <https://figshare.com/s/7e1ffcc3fba4bf272cda>).

4.2 Estimating the parameters of the growth model

The empirical model deduced in Sect. 2 as Eq. (20) has been used in a linear regression approach. Four versions of the model have been estimated, both for the number of followers and the number of followed accounts each.

As data have been available for three years first the model has been estimated for the change from 2018 to 2019 and again for the change from 2019 to 2020 resulting in what in the tables below is referred to as Model I and Model III. Considering the heterogeneity of the influencers in the sample as evidenced by the description of the data set in Perret and Edler (2020) it seemed only reasonable to keep as much of the heterogeneity constant and control for the corresponding aspects. In light of this argument for each pair of years a conditional approach is realized using all the additional data listed in the previous section.

Models I and III will be referred to as the unconditional version of the basic model as no additional control are introduced. In models II and IV corresponding controls are introduced, making them the conditional versions of the basic model. Note that models I and III represent the application of the underlying data set on Instagram influencer to the model motivated in Eq. (20).

The conditional version of Models I and III however reported significant problems with multicollinearity. The main issues with multicollinearity resulted from the origin and posting language dummies. In consequence all the posting language dummies are omitted as well as all but four origin dummies. In particular, those variables that consistently

remained insignificant and reported VIFs larger than 2 have been omitted from further analysis. The results of the conditional version of Model I summarized in Table 1 as Model II, while the results of the conditional version of Model III are summarized as Model IV. In both Tables 1 and 2 coefficients marked *** are significant at the 1% level, coefficients marked ** are significant at the 5% level and coefficients marked * are significant at the 10% level. Coefficients not marked by asterisks are insignificant with p-values greater than 0.1.

The size of the coefficients of the logarithmized followers – in this study the natural logarithm has been used to logarithmize the follower numbers and calculate the growth rate – remains more or less constant, independent whether the conditional or the unconditional version of the model is considered. For the years 2018 and 2019 the coefficient is negative and significantly different from zero while for the years 2019 and 2020 it is highly insignificant.

In the conditional form of the first model—model II—the coefficients of the first model have only been slightly changed not impacting the significance of the coefficients. When switching from model III to model IV, the coefficient takes the expected negative sign, signifying that convergence

Table 1 Regression Results – Followers *Source:* Own presentation

Variable	Model I	Model II	Model III	Model IV
Log Followers	−0.026*** (0.010)	−0.036*** (0.010)	0.005 (0.008)	−0.006 (0.008)
Followed		0.037* (0.021)		0.009 (0.013)
Frequency		0.001 (0.003)		0.004 (0.011)
Engagement Rate Likes		0.034*** (0.005)		0.020*** (0.004)
Engagement Rate Comments		0.024 (0.124)		0.022 (0.102)
Age		0.007*** (0.003)		0.003 (0.002)
Origin US		0.049** (0.025)		0.057*** (0.020)
Origin DE		0.053* (0.028)		0.040* (0.023)
Origin SW		−0.059 (0.041)		−0.060* (0.034)
Origin IT		0.065 (0.045)		0.047 (0.037)
Constant	0.509*** (0.135)	0.275 (0.156)	0.050 (0.108)	0.015 (0.130)
R ²	0.026	0.268	0.001	0.178
F-Statistic	6.781***	8.754***	0.351	5.169***
Convergence Rate	0.0263	0.0367	−0.005	0.006

takes place if all controls are held constant, but the coefficient still remains insignificant.

A t-test against a test value of -1 reveals the coefficients in any of the four models is significantly larger than -1. This shows that for the number of followers the bounded model as described by Eq. (16) applies.

Considering that Eq. (16) describes the behavior of the followers and also implies convergence of the follower numbers. An interesting question can be found in the speed by which follower number converge. For the years 2018 and 2019 the speed lies at 2.63% or 3.67% respectively signifying that the gap between the different influencers closes by 2.63% to 3.67% per year or that half of the distance is covered in 26 to 18.5 years. These time horizons are clearly beyond the expected usage time of a Social Media platform like Instagram, therefore a distinction into better or worse performing influencers or as referred to by Edler and Perret (2020) as α -, β - and γ -influencers will persist throughout its use.

This argument is strengthened by the nature of models III and IV as the coefficient no longer is significantly different from zero signifying that near stagnation will set in keeping the status quo more or less as is.

Applying the same procedure for the number of followed accounts results in Table 2. Here Models V and VII describe the unconditional version for the years 2018 and 2019 or 2019 and 2020, respectively. Models VI and VIII are the corresponding conditional versions of Models V and VII.

Even though they are not significantly different from zero, the coefficients for Models V and VI are approximately of the same size and negative. The coefficients for Models VII and VIII are also similarly sized and both negative, however, compared to Models V and VI they are more pronounced and turn out significantly different from zero.

In all four cases the coefficients are negative, but they are also significantly different from and in particular larger than -1. This shows that the model in Eq. (16) applies as well for the number of followed accounts.

Switching from the unconditional to the conditional version of the model does not impact the coefficients in any serious manner. The speeds of convergence here are larger than in the case for the followers. The halftime lies at 10.5 to 4.8 years. While this also lies beyond the normal life expectancies of most Social Media platforms (considering that Google Plus shut down after about 8 years and even MySpace can be considered almost obsolete after 17 years) to evidence near to full convergence it should suffice to

Table 2 Regression Results – Followed *Source:* Own presentation

Variable	Model V	Model VI	Model VII	Model VIII
Log Followers	-0.075 (0.048)	-0.062 (0.043)	-0.115*** (0.019)	-0.125*** (0.021)
Followed		0.000 (0.000)		0.000 (0.000)
Frequency		-0.110*** (0.009)		0.000 (0.021)
Engagement Rate Likes		-0.027** (0.013)		-0.004 (0.008)
Engagement Rate Comments		0.160 (0.332)		-0.360* (0.204)
Age		0.015** (0.007)		0.002 (0.005)
Origin US		0.005 (0.066)		0.002 (0.041)
Origin DE		0.002 (0.074)		0.015 (0.046)
Origin SW		0.007 (0.110)		-0.126* (0.067)
Origin IT		0.125 (0.119)		-0.029 (0.074)
Constant	0.595* (0.303)	0.354 (0.339)	0.868*** (0.126)	0.916*** (0.189)
R ²	0.009	0.404	0.120	0.165
F-Statistic	2.422	16.216***	34.653***	4.734***
Convergence Rate	0.078	0.064	0.122	0.134

evidence at least significant changes. However, it needs to be mentioned that the insignificance of the coefficients for the years 2018 and 2019 again might hint at a stagnating development which also would not surprise as the sample comprises mostly well-established influencers that are already active on Instagram for many years and already have an established circle of friends. The explanation for the increase in convergence in the years 2019 and 2020 could then be found in the anecdotal evidence that from 2019 to 2020 a number of influencers in the sample started to ‘declutter’ their followed lists resulting in part in severe drops in the number of followed accounts.

While the results for models I to VIII indicate that a general upper bound to the number of followers and followed accounts is very likely the very low model fit as evidenced by the low coefficients of determination make it nigh impossible to produce a decent estimate of the parameters α , β , γ required for the calculation of the upper bounds. Considering that severe discrepancies even in the selected sample of influencers still persist with speeds of convergence that are restrained at best the upper bounds are supposedly rather high and might not be covered by any number in the sequence of Dunbar’s numbers in particular the follower numbers. Assuming for the number of followers the maximum of Dunbar’s numbers of 1,500 or even thrice that number as 4,500 gets very close to the actual observed maximum number of followed accounts.

5 Conclusions

5.1 Summary

The present study has introduced a basic model for the development of follower numbers and the number of followed accounts regarding users of directed Social Media platforms. As the model not necessarily resulted in an upper bound for the numbers, a testing model was deduced and applied to data for a sample of 255 influencers from the field of women’s fashion on Instagram. The empirical estimation of the model showed that only the bounded version of the model can be accepted based on the data set.

As neither the theoretical model nor the empirical estimation required the number of total Instagram users a bound supposedly exists independent of the number of total users and the platform and Social Media seemingly has its own rules apart from any boundaries set by the hosting platform.

In all except one of the estimations the sign of the regression coefficients b is negative implying that either—if the coefficient is significantly different from zero—convergence of the numbers to a common bounding value for all influencers in the sample or—if the coefficients are not significantly

different from zero stagnation of the numbers sets in and the status quo is kept.

While the results imply stagnation for the years 2019 to 2020 the results by Perret and Edler (2020) point to significant changes at least regarding the top influencers in the implemented sample when importance is considered, which is strongly correlated with the degree / the number of followers. While this stands in contrast to the assumption of convergence, it could be the result of particular leap-frogging.

5.2 Limitations and Outlook

The theoretical model, though build on reasonable and simplistic assumptions, cannot exclude that a different form might suit the data better. While the model, as introduced in Sect. 3, theoretically allows for a sigmoid (S) shape of the development path of followers empirically such is highly unlikely considering the deduced testing model. Thus, even though indirectly, it is assumed that the growth rate of the follower and followed numbers continuously decreases over time. Considering, however, real development paths of influencers the path usually starts with a phase of relatively low but increasing growth rates followed by a phase of high but decreasing growth rates, leading to a sigmoid shape. For a broader representation of Social Media users and potential development paths the model, however, might have to be adjusted. A preliminary empirical model, to test whether this adjustment is actually required, can be realized by using a quantile regression approach (Koenker & Bassett, G., Jr., 1978) to estimate the actual shape (Perret 2019).

The model itself is not restricted by its application neither to Instagram as a Social Media platform nor to profiles with very high numbers of followers and a medium to high number of followed accounts, as via the considered sample of influencers. The insights gained in this study could thus profit from replications for different Social Media platforms and a more heterogeneous sample. A cross-platform study could furthermore point out differences between the platforms and in how far these differences impact related upper bounds.

Expanding the sample beyond influencers from the field of women’s fashion seems beneficial as well for two reasons. Even for Instagram alone it is essential to consider different influencers and even more so normal users from the lower end of the spectrum of followers. This again stresses the relevance of for testing alternative model structures or at least test whether different modelling approaches are required for different types of Social Media users.

Finally, the idea of convergence clubs issued by Quah—Quah (1996a) and Quah (1996b) – in the context of economic growth theory could be tested in this context as well. A convergence implies that different sub-groups with different convergence speeds and potentially different upper

bounds exist. While a distinction between Social Media users with small follower numbers certainly develop differently from top influencers, influencers specialized on special interests (e.g., classic cars, toys, art, etc.) certainly will report different upper bounds as their potential absolute followership is already differently proportioned. Here a worthwhile question would be whether the same dynamics are driving different special interest groups, even though on different levels. A potential tool to help this type of analysis again can be found in quantile regression approaches—Koenker and Bassett, G., Jr. (1978) – as they allow to working with and in particular testing for in-constant coefficients. A second alternative in studying dynamics between different groups (e.g., normal Social Media users vs. micro- and top-influencers) can be by using Markov transition matrices—Fingleton (1997)—which however would first require a detailed deduction of the possible groups and corresponding thresholds.

5.3 Insights for practitioners

While the results of this study are primarily of theoretical interest, a number of important insights for marketing practitioners can be deduced from them.

The result, that a common upper bound in particular for the number of followers exists, gives rise to suspect that more in-depth studies are able to determine this upper bound numerically. In this case practitioners will have an additional tool to evaluate for example an influencers relevance and his worth by estimating how far away she is from a general upper bound or from her personal conditional one. Such an upper bound thus will provide practitioners with a quantifiable approach to an influencer's realized reach as compared to the potential reach and thus will quantify part of the evaluation process of an influencer's worth. It will thereby add to the tool kit of comparative and quantitative influencer marketing.

As the theoretical model provided in this study stands independent of any particular Social Media platform, upper bounds can be established – if their existence is assured – for all relevant platforms. Thus, the marketing professional can not only evaluate an influencer's reach on one particular platform alone but has a tool available to perform cross-platform evaluations. This is of particular interest since on the one hand platforms differ according to the possible reach they offer and on the other hand influencers have different involvements with different platforms. Summarizing, it offers a tool to select the best influencer for the corresponding platform. For the influencers it offers a tool to evaluate their own presence on different platforms and quantify potentials to expand.

Following the discussion of the results it has been seen that the result of this type of analysis allows for an

evaluation whether a Social Media platform and its main actors / influencers are still growing strongly or whether they are entering a stagnation phase which would signify that the platform reached its maturity phase. Thus, a convergence analysis not only allows for a more quantified detection of up-and-coming platforms as compared to well established platform. As such an analysis is founded on the actors active on the platform, convergence analysis can serve as a discriminator between up-and-coming groups of influencers with significant growth potential as compared to well-established influencers which already have exhausted most of their growth potential.

Finally, the study established that also in the context of macro-influencers the Social Brain Hypothesis does hold and as the mathematical model establishes will always hold as long as rather modest assumptions are fulfilled. All results that build on the Social Brain Hypothesis can thus without any significant doubt be transferred to marketing in general and influencer marketing in particular. Considering that the Social Brain Hypothesis, even though it is a physiological approach, has sociological implications, it motivates the implementation of more sociological oriented approaches in marketing in general but in Social Media marketing in particular; even more so where distinct grouping and social interactions take place.

An upcoming trend that might significantly be impacted by the social implications of the Social Brain Hypothesis but as well by the established upper bounds to Social Media platforms and the physiological upper bounds established by Dunbar is the study of social shopping communities (Olbrich & Holsing 2011). As these communities are strongly based on trust among their members as in trust in recommendations and testimonials (Li 2019), the existing upper bounds on group sizes might apply as well and put a limit on the maximum number of members of an efficiently working social shopping community. It would a particular interesting question to answer in how far the upper bounds for social shopping communities differ from the physiological bounds or from the Social Media bounds established herein. Since the communities function like social networks themselves, the same type of convergence analysis conducted herein could offer insights on the development trends and the current level of development of a social shopping community. This again would offer marketing professionals the possibility to evaluate these communities and rate their own platform compared to those of competitors.

Appendix

Deduction of Eq. (20)

Division of Eq. (4) by F_t yields

$$\begin{aligned} \frac{dF_t}{F_t} &= \frac{d \ln(F_t)}{dt} = \alpha F_t^{-(1+\beta)} - \gamma \\ &= \alpha \exp(-(1+\beta) \ln(F_t)) - \gamma = h(\ln(F_t)) \end{aligned} \quad (21)$$

which can be considered a function h of $\ln(F_t)$.

In the next step a Taylor development of the function h of degree 1 at the point $\ln(F)$ is considered with F being the steady state as defined in Eq. (17).

Note: The Taylor development or Taylor approximation of degree 1 for a function $f(x)$ around at a point a is given as

$$f(x) \approx f(a) + f'(a)(x - a) \quad (22)$$

Applying the rule from Eq. (22) to Eq. (21) with $a = \ln(F)$ and $f(x) = h(\ln(F_t))$ yields

$$h(\ln(F_t)) = h(\ln(F)) + \frac{dh}{d \ln(F)} (\ln(F_t) - \ln(F)) \quad (23)$$

Inserting F from Eq. (17) leads to $h(\ln(F)) = 0$ and changes Eq. (23) to

$$h(\ln(F_t)) = \frac{d \ln(F_t)}{dt} = -\gamma(1+\beta) \left(\ln(F_t) - \frac{1}{1+\beta} \ln\left(\frac{\alpha}{\gamma}\right) \right) \quad (24)$$

which is a linear differential equation of order one that has the following solution

$$\ln(F_t) = C_0 \exp(\gamma(1+\beta)t) + \ln(F) \quad (25)$$

Inserting $t=0$ yields

$$\ln(F_0) = C_0 + \ln(F) \Leftrightarrow C_0 = \ln(F_0) - \ln(F) \quad (26)$$

Inserting Eq. (26) into Eq. (25) and rearranging the result yields

$$\begin{aligned} \ln(F_t) - \ln(F_0) &= (\ln(F) - \exp(\gamma(1+\beta)t)) \\ &\quad + (1 - \exp(\gamma(1+\beta)t)) \ln(F_0) \end{aligned} \quad (27)$$

Summarizing the left side and inserting $t=1$ gives

$$\ln\left(\frac{F_1}{F_0}\right) = (\ln(F) - \exp(\gamma(1+\beta))) + (1 - \exp(\gamma(1+\beta))) \ln(F_0) \quad (28)$$

The first parentheses can be summarized into a parameter a and the second parenthesis can be summarized into a parameter b resulting in a preliminary Eq. (29)

$$\ln\left(\frac{F_1}{F_0}\right) = a + b \cdot \ln(F_0) \quad (29)$$

While Eq. (29) is defined for the changes from the initial to the first period, considering the current period t to be the

initial condition allows to replace 0 with t and 1 with $t+1$, thus yielding Eq. (30).

$$\ln\left(\frac{F_{t+1}}{F_t}\right) = a + b \cdot \ln(F_t) \quad (30)$$

Funding Open Access funding enabled and organized by Projekt DEAL. No funding was received.

Availability of Data and Materials On demand.

Declarations

Conflict of interest No conflicting interests or competing interests.

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