The Solar Neutrino Problem

Martin Lahrz
martin.lahrz@gmail.com
University Umeå

Abstract

Since neutrinos are the only known particles being able to leave the core of the Sun directly, they were supposed to approve the Standard Solar Model. However, the number of detected neutrinos was only one third of the predicted number.[10] After theoretical computations and other experiments had corroborated our understanding of the Sun, the question of the missing neutrinos remained as the solar neutrino problem. Not until 2001 neutrino oscillations could explain this discrepancy[13] and gave in turn new insights about the neutrinos themselves.

1 Neutrinos and their classification

1.1 From Pauli to the Standard Model

A two-body decay produces a discret energy spectrum since the conservation of energy and momentum ensure an unique solution. From the point of view in the early 1900s in the beta decay one electron is leaving the core which is changing its atomic number by one. However, in 1911 L. Meitner and O. Hahn verified a continuous spectrum of the electrons coming from the beta decay which was a contradiction. Additionally, the conservation of angular momentum was violated, too.

In 1930 W. Pauli postulated a neutral particle with spin 1/2 which he called neutron[24]. Because of J. Chadwick's discovery of the todays neutron in 1932, E. Fermi, who worked out the theory about this particle, renamed it later to "little neutron" – neutrino.

In a three-body problem the energy spectrum of each participant would be continuous. According to Pauli the particle should be similar to light quanta but not travelling with the speed of light and the mass should be at the same magnitude as the electron mass. Simultaneously this particle could permeate matter as good as x-rays or even better since its cross section is very small. In fact, Pauli himself thought it would not be possible to see it for a long time. Indeed, this succeeded for the first time by C. Cowan and F. Reines[18] in 1956.

So far, only the electron neutrino was known but J. Steinberger, M. Schwartz and L.M. Ledermann discovered with the muon neutrino a second generation in 1962[19]. When M.L. Perl found the tauon in 1975[25], physicists expected also a corresponding neutrino generation. In fact, it was verified at the DONUT experiment in 2000[12] as the latest particle of the Standard Model being directly observed.

1.2 Properties

Neutrinos are neutral charged elementary particles. As leptons they are fermions and have a spin of 1/2 which was already postulated by Pauli. As mentioned previously, three different flavors are known: electron neutrinos, muon neutrinos and tau neutrinos while each type has basically an antiparticle. It is for sure that there are exactly three generation with a mass less than $45.6\,GeV$[20].

Neutrinos interact through the weak force, primarily, and throught the gravitational force which is in most cases neglectable, however. Since they do not interact neither through the electromagnetic force nor through the strong force, their cross section is extremly small. On the one hand, this allows them to cross matter almost unaffected. Especially they are able to leave the Sun's core directly which make them to the only known information source about this region. On the other hand, it is very hard to verify these neutrinos here on Earth.

The Standard Model of particle physics describes neutrinos as massless. According to the special theory of relativity this leads to the assumption that neutrinos must travel at the speed of light c. In fact, experiments with the MINOS detector measured their speed to $(v - c)/c = 5.1 \pm 2.9 \cdot 10^{-5}$ (at 68% C.L.)[1] which at least means that it is very close to c. However, observed neutrino oscillations, playing an important part in the solution of the solar neutrino problem, require a nonzero mass.

2 The solar neutrino problem

2.1 The Standard Solar Model

The Standard Solar Model (SSM) desribes the structure and the thermodynamical properties of the Sun. According to the SSM the Sun generally consists of hydrogen plasma which is designed by the equilibrium of the gravitational force and the energy density. At the core temperatures of about $15.7 \cdot 10^6\,K$[21] and pressures of $150\,g/cm^3$[8] enable nuclear fusion where mainly a proton is transformed into a neutron.

To verify the SSM different radiation or fluxes might be observed: photons in the sense of light, plasma namely the solar wind, and neutrinos. In contrast to all other particles including photons, neutrinos are coming right from the core and therefore can give particular insights into the Sun.

2.2 Neutrinos from the Sun

Whenever a proton is converted in a neutron a quark has to change its flavor. The only interaction being responsible for this is the weak interaction and therefore the nuclear beta decay. Thus, each time a neutron is formed, an electron neutrino is produced, too.[9]

$$p + X \rightarrow n + e^+ + \nu_e + X \qquad (1)$$

These neutrinos have a continuous energy spectrum with a maximal energy depending on the reaction, see Tab. 2.1 as well as Fig. 2.1. However, the inverse beta decay of ^7Be produces a discret energy spectrum with lines at $0.9\,MeV$ and $0.4\,MeV$.

The flux of the ^8B neutrinos is of particular importance since it is $\Phi(^8B) \propto T^{25}$, so highly sensitive.[5] This is only one example how neutrinos might give applicable information about a property of a system.

2.3 Homestake

The first investigation of solar neutrions were made by R. Davis in the mid-1960's.[4] The so-called Homestake experiment was based on the neutrino reaction on chlorine making an isotope of argon

$$\nu_e +^{37} Cl \rightarrow e^- +^{37} Ar \qquad (2)$$

Table 2.1: *Neutrino production from the proton-proton chain reaction (upper) and the CNO cycle (bottom) based on the SSM. The total solar flux at the Earth is $6.5 \cdot 10^{10}\,cm^{-2}s^{-1}$. The majority of the solar neutrinos come from the pp chain (more than 91%); while the ^7Be, pep, and ^8B chains correspond to about 7%, 0.2%, and 0.008% of the total flux, respectively. The hep contribution is minuscule and mostly neglected.[9] This is table 7 of [6].*

Reaction	Label	Flux $cm^{-1}s^{-1}$
$p + p \rightarrow {}^2H + e^+ + \nu_e$	pp	$5.95 \cdot 10^{10}$
$p + e^- + p \rightarrow {}^2H + \nu_e$	pep	$1.40 \cdot 10^8$
$^3He + p \rightarrow {}^4He + e^+ + \nu_e$	hep	$9.3 \cdot 10^3$
$^7Be + e^- \rightarrow {}^7Li + \nu_e$	^7Be	$4.77 \cdot 10^9$
$^8B \rightarrow {}^8Be^* + e^+ + \nu_e$	^8B	$5.05 \cdot 10^6$
$^{13}N \rightarrow {}^{13}C + e^+ + \nu_e$	^{13}N	$5.48 \cdot 10^8$
$^{15}O \rightarrow {}^{15}N + e^+ + \nu_e$	^{15}O	$4.80 \cdot 10^8$
$^{17}F \rightarrow {}^{17}O + e^+ + \nu_e$	^{17}F	$5.63 \cdot 10^6$

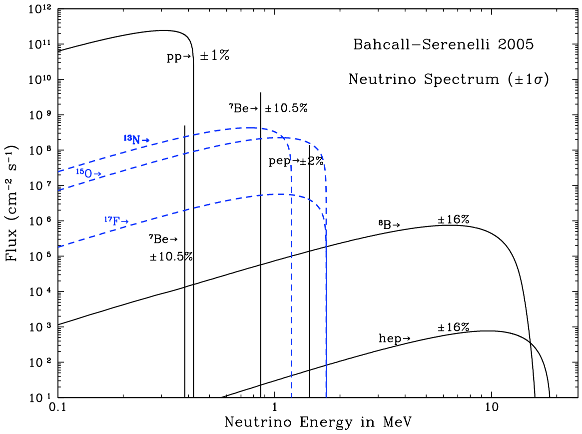

Figure 2.1: *The solar neutrino spectra predicted by the SSM. Note, that the continuum neutrino fluxes at one astronomical unit are given in units of $cm^2s^{-1}MeV^{-1}$, while the line ones are given in $cm^{-2}s^{-1}$. This is figure 2 of [7].*

where the minimum energy of the neutrinos have to be $0.814\,MeV$. This reaction is very rare and according to Fig. 2.1 a large percentage of incoming neutrinos just drop out. In fact, about one atom of argon was produced each week in a tank containing about $380\,m^3$ of the dry-cleaning fluid, perchlorethylene.[9] However, neutrinos coming from Earth were precluded easily since there are no sources connected with these energies.

Already the first results from 1968 showed two aspects. First, argon atoms were produced, i.e. the experiment was working. But second, the number of measured events was only about a quarter of the expected number. Finally, the experiment ended in 1995 and led to the following result, which still was unsatisfying:[10]

$$\Phi_{Cl}\,(Homestake) = 2.56 \pm 0.16 \pm 0.16\,SNU\,,$$

while the SSM predicted

$$\Phi_{Cl}\,(SSM) = 7.6^{+1.3}_{-1.1}\,SNU\,.$$

Note: A SNU (Solar Neutrino Unit) is the product of the solar neutrino fluxes, measured or calculated, and the calculated cross sections. Hence one SNU equals one capture per second and per 10^{36} target atoms.[9]

In consideration of this result, either the Standard Solar Model or the Standard Model of particle physics had to be revised. On the one hand, other observation confirmed the SSM. On the other hand, a change of the neutrino flavor contradict the Standard Model of particle physics since neutrinos have no mass according to this model. In fact, neutrino oscillations were postulated in the case of a nonzero mass by B. PONTECORVO in 1957[26].

2.4 SAGE and GALLEX/GNO

Since Homestake was only able to detect high energy neutrinos, two other experiments working with gallium were conceived. The Soviet-American Gallium Experiment (SAGE) was located in the Baksan laboratory in Russia, the Gallium Experiment (GALLEX) and later Gallium Neutrino Observatory (GNO) were an European project situated in Italy. The underlying reaction

$$\nu_e +^{71} Ga \rightarrow e^- +^{71} Ge \tag{3}$$

is very similar to Eq. (2) but already sensitive for energies of at least $0.233\,MeV$ including *pp*, ^7Be, ^8B, and *pep* neutrinos. The expected flux was more than 15 times higher than with chlorine. Indeed, both independent experiments came to a similar results[9]

$$\Phi_{Cl}\,(SAGA) = 69.9^{+4.4+3.7}_{-4.3-3.2}\,SNU$$
$$\Phi_{Cl}\,(GALLAX/GNO) = 70.8 \pm 4.5 \pm 3.8\,SNU$$

but the flux predicted by the SSM[9]

$$\Phi_{Cl}\,(SSM) = 129^{+9}_{-7}\,SNU$$

was still about twice as large.

2.5 Kamiokande and SuperKamiokande

In 1987 the Kamiokande experiment changed the reference method fundamentally using a water Čerenkov detector instead of a radiochemical one like the previous experiments did. The neutrino-electron scattering process[1]

$$\nu_e + e^- \rightarrow \nu_e + e^- \tag{4}$$

accelerated the electrons to a velocity above the speed of light in water which cause characteristic Čerenkov radiation. This radiation can be evaluated by photomultiplier tubes in intensity and direction. The primal experiment was supposed to determine the direction of incoming neutrinos in general. It succeeded to show that the neutrinos are actually coming from the direction of the Sun.

The directly following project was the SuperKamiokande experiment in 1996 where basically the main principle was retained. Although the threshold of $5\,MeV$ was about six times higher than at Homestake, hence first of all ^8B neutrinos could be observed, about 15 events per day were detected. And this was again distinctly larger than at the radiochemical experiments.

The SuperKamiokande yielded two insights. First, the measured neutrino flux of energies in the range of $6.5 - 20\,MeV$ was

$$\begin{aligned}\Phi_{ES}\,(SK) &= 4017 \pm 105^{+116}_{-116}\,SNU\\ &= \left(2.42 \pm 0.06^{0.10}_{-0.07}\right) \cdot 10^6\,cm^{-2}s^{-1}\end{aligned}$$

and therewith conform with the Kamiokande flux of $2.80 \pm 0.19 \pm 0.33 \cdot 10^6\,cm^{-2}s^{-1}$[11]. The ration of the SSM predicted rate was still only 35.8%. These data include only the first 300 days but are already representative enough to got published in [11]. Later the last value was corrected to 46.5%[9] which is not much better. Second, a possible day/night rate asymmetry was proved but the effective discrepancy of[9]

$$A_{DN} = 2\frac{\Phi_D - \Phi_N}{\Phi_D + \Phi_N} = -0.021 \pm 0.020^{+0.013}_{-0.012}$$

wherein no significant difference could be seen.[11] As it later transpired, contrary to the fact the Earth shields

---

[1] This reaction is also sensitive to muon and tau neutrinos which was used in the SNO experiment later.

solar neutrinos, the number of electron neutrinos at the night side is larger than on the day side which is why the Sun occurs brighter at night.

Finally, the SuperKamiokande was looking for anti-neutrinos and the conversion probability from a solar electron neutrino to an electron anti-neutrino, respectively. They found an upper limit of 0.8% of the total flux between $8\,MeV$ and $20\,MeV$ predicted by the SSM[17].

2.6 Sudbury Neutrino Observatory

The Sudbury Neutrino Observatory (SNO) was the first detector explicitly searching for electron neutrinos as well as myon or tau neutrinos from the ^8B reaction. The detector was quite similar to Kamiokande but worked with heavy water D_2O instead of light water. This allowed analysing three reaction at once.[16] First, the elastic scattering (ES) of electrons by neutrinos similar to Eq. (4) but this time explicit sensitive for all flavors of neutrinos,

$$\nu_x + e^- \;\rightarrow\; \nu_x + e^- \qquad (x = e,\, \mu,\, \tau)\,. \tag{5}$$

This reaction guarantees the Sun as source of the neutrinos. Second, a charged current (CC) reaction

$$d + \nu_e \;\rightarrow\; p + p + e^-\,, \tag{6}$$

which is sensitive for electron neutrinos only. Third, a neutral-current (NC) reaction

$$d + \nu_x \;\rightarrow\; \nu_x + p + n \qquad (x = e,\, \mu,\, \tau)\,, \tag{7}$$

where a deuteron is fragmented into a proton and a neutron. This is sensitive for all types equally. This leads to the following relations between specific neutrino flux and to be measured rates, which are

$$\begin{aligned} \Phi_{CC} &= \Phi_e \\ \Phi_{ES} &= \Phi_e + 0.15\Phi_{\mu\tau} \\ \Phi_{NC} &= \Phi_e + \Phi_{\mu\tau} \end{aligned} \tag{8}$$

Finally, the resulting neutrino fluxes determined by SNO were[13] [14]

$$\begin{aligned} \Phi_{total} &= \left(5.54^{+0.33+0.36}_{-0.31-0.34}\right) \cdot 10^6\, cm^{-2}s^{-1} \\ \Phi_e &= \left(1.76 \pm 0.05 \pm 0.09\right) \cdot 10^6\, cm^{-2}s^{-1} \\ \Phi_{\mu\tau} &= \left(3.41 \pm 0.45^{+0.48}_{-0.45}\right) \cdot 10^6\, cm^{-2}s^{-1}\,. \end{aligned}$$

Not only the predicted total flux of ^8B neutrinos by the SSM is

$$\Phi_{SSM} = \left(5.05^{1.01}_{-0.81}\right) \cdot 10^6\, cm^{-2}s^{-1}\,,$$

and therewith very close, but also the ascertained muon and tau neutrion rate is 5.3σ above zero[13], i.e. it has to be assumed that flavor transformation exists with a probability bordering on certainty. In addition, the amount of electron neutrinos of the total flux coincides with the measurements of the former experiments, with respect to influences of the other neutrino flavors, very well. The solar neutrino problem seems to be solved.

3 Neutrino oscillations

The SNO experiment has shown that neutrinos can change their flavor. One possible explanation might be neutrino oscillations.

Like every particle neutrinos can be described as a wave function or a superposition of wavefunctions which is summarized by

$$\left|\nu_\alpha\right\rangle = \sum_i U_{\alpha i} \left|\nu_i\right\rangle\,. \tag{9}$$

We can only measure the resulting eigenstate on the left-hand side of Eq. (9), which we call flavor. It is generally accepted that there are three different flavors, namently electron, muon and tau neutrinos corresponding to the three charged leptons, i.e. $\alpha = e,\, \mu,\, \tau$. On the right-hand side we find the mass eigenstates $\left|\nu_i\right\rangle$ with $i = 1,\, 2,\, 3$ and the mixing matrix $U_{\alpha i}$ which is called the Pontecorvo-Maki-Nakagawa-Sakata (MNSP) matrix and is basically a rotation matrix with three Euler angles. It describes a charged-current interaction, i.e. a weak interaction mediated by the W boson, in the leptonic sector.[9]

The assumption that only electron neutrinos are produced in the Sun is right. Now, we describe the propagation of these neutrinos in vacuum, which is given in interstellar space in good approximation, as a plane wave. Since these neutrinos are ultrahigh-energetic, i.e. $p \gg m$ and $t \approx L$, we can approximate $E = \sqrt{p^2 + m^2} \approx p + \frac{m^2}{2p} \approx E + \frac{m^2}{2E}$ and it is

$$\left|\nu_i\left(t\right)\right\rangle = e^{-im_i^2 L/2E} \left|\nu_i\left(0\right)\right\rangle\,. \tag{10}$$

The eigenstates $\left|\nu_i\right\rangle$ are of course the mass eigenstates. But this means, that they are travel with different velocities depending on m_i. The electron neutrino, as a superposition of different $\left|\nu_i\right\rangle$, flows apart. Constructive interference then leads to the transition probability from flavor α to β

$$\begin{aligned} P_{\alpha\beta} &= \left|\left\langle\nu_\beta\left(0\right)|\nu_\alpha\left(t\right)\right\rangle\right|^2 \\ &= \left|\sum_i U_{\alpha i} U^*_{\beta i} e^{-im_i^2 L/2E}\right|^2\,. \end{aligned} \tag{11}$$

Table 3.1: *Fit values for the three-flavor neutrino oscillation parameters based on global data including solar, atmospheric, reactor and accelerator experiments. Last update Sept 2007. This is table D1 of [23].*

parameter	best fit	2σ	3σ
$\Delta m^2_{21}\,[10^{-5}\,eV^2]$	7.6	7.3 – 8.1	7.1 – 8.3
$\Delta m^2_{31}\,[10^{-3}\,eV^2]$	2.4	2.1 – 2.7	2.0 – 2.8
$\sin^2\theta_{12}$	0.32	0.28 – 0.37	0.26 – 0.40
$\sin^2\theta_{23}$	0.50	0.38 – 0.63	0.34 – 0.67
$\sin^2\theta_{13}$	0.007	≤ 0.033	≤ 0.050

Assuming two types of neutrinos the mixing matrix U only depends on one mixing angle θ. Now, the transition probability can be written as

$$P_{\alpha\beta} = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right) , \qquad (12)$$

where $\Delta m^2 = m^2_\alpha - m^2_\beta$. According to the Standard Model of particles physics this difference would always give zero since all masses are zero. However, we can observe neutrino oscillations, so the model has to be extended.

In the case of three neutrino types there are not only two parameters but at least five. Depending on possible symmetry violations there could be more. Current measurements, not only of solar neutrinos but also in particle accelerators and of atmospheric as well as reactor neutrinos, yield the values given in Tab. 3.1 by [23].

In matter scattering becomes important. As used in the SNO experiment, the cross section for elastic scattering and charged current reactions are different for electron and non-electron neutrinos. The Hamiltonian depends on the electron density and resonances occurs now. This is called the MIKHEEV-SMIRNOV-WOLFENSTEIN (MSW) effect and is responsible for the day/night asymmetry and the energy dependence of the transistion probability.[3]

4 Current research and future questions

The Solar Neutrino Problem seems to be solved. However, some questions are still of interest such as the total mass of the neutrinos or the energy dependence of neutrino oscillations.

Yet, ultra-high energetic neutrinos above $5\,MeV$ were investigated. Nevertheless, low energy neutrinos up to $2\,MeV$ include many information. Unfortunately, a new generation of detectors is needed since these energies are below threshold for production of Čerenkov light and radiochemical reactions have too small cross sections. Also background radiation becomes important again. The ^7Be neutrinos are monoenergetic so that a time dependence can be measured well. The KamLAND[2] and Borexino[3] are supposed to research at this energy scale. And finally, *pp* neutrinos represent about 91% of the total neutrino flux but have a maximum energy of $0.42\,MeV$.

Since the observation of neutrino oscillations only leads to the mass difference between two types, just a lower limit can be estimated assuming the lightest neutrino is indeed massless. An upper limit can be estimated by measuring the electron energy spectrum at the beta decay while this ends at one neutrino mass before the total energy of the system is reached. The mass of an electron antineutrino could be indicated to $< 2\,eV$[2]. The KATRIN experiment[4] was planed to be started in 2009 and is designed to measure the mass of the electron antineutrino using the tritium beta decay to an upper limit of $0.2\,eV$[15].

If a neutrinoless double beta decay is possible, neutrinos would be their own antiparticles. The evidence for this decay mode is 97% (2.2σ)[22] but still in discussion since an evidence of 5σ is necessary to prove a theory.

References

[1] MINOS Collaboration: P. Adamson. Measurement of neutrino velocity with the minos detectors and numi neutrino beam. *Physical Review D*, 76:072005, 2007.

[2] C. Amsler and Particle Data Group. Review of particle physics, 2008.

[3] J. N. Bahcall, P. I. Krastev, and A. Yu Smirnov. Is large mixing angle msw the solution of the solar neutrino problems? *Physical Review D*, 60:093001, 1999.

[4] John N. Bahcall. Solar neutrinos. *Phys. Rev. Lett.*, 17(7):398–401, 1966.

[5] John N. Bahcall. How many σs is the solar neutrino effect? *Phys. Rev. C*, 65(1):015802, 2001.

[6] John N. Bahcall, M. H. Pinsonneault, and Sarbani Basu. Solar models: current epoch and time dependences, neutrinos, and helioseismological properties, 2001.

---

[2] Kamioka Liquid Scintillator Antineutrino Detector

[3] Boron Experiment

[4] Karlsruhe Tritium Neutrino Experiment

[7] John N. Bahcall, Aldo M. Serenelli, and Sarbani Basu. New solar opacities, abundances, helioseismology, and neutrino fluxes, 2005.

[8] Sarbani Basu, William J. Chaplin, Yvonne Elsworth, Roger New, and Aldo M. Serenelli. Fresh insights on the structure of the solar core, 2009.

[9] Alain Bellerive. Review of solar neutrino experiments. *International Journal of Modern Physics A*, 19:1167, 2004.

[10] B. T. Cleveland et al. Measurement of the solar electron neutrino flux with the Homestake chlorine detector. *Astrophys. J.*, 496:505–526, 1998.

[11] The Super-Kamiokande Collabora. Measurements of the solar neutrino flux from super-kamiokande's first 300 days. *ERRATUM-IBID.*, 81:4279, 1998.

[12] DONUT Collaboration. Observation of tau neutrino interactions. *Physics Letters B*, 504:218, 2001.

[13] SNO Collaboration. Direct evidence for neutrino flavor transformation from neutral-current interactions in the sudbury neutrino observatory. *Physical Review Letters*, 89:011301, 2002.

[14] SNO Collaboration. Independent Measurement of the Total Active ^8B Solar Neutrino Flux Using an Array of ^3He Proportional Counters at the Sudbury Neutrino Observatory. *Physical Review Letters*, 101:111301, 2008.

[15] The KATRIN Collaboration. Katrin design report 2004, 2004.

[16] The SNO Collaboration. The sudbury neutrino observatory. *NUCL.INSTRUM.METH.A*, 449:172, 2000.

[17] Super-Kamiokande collaboration Gando. Search for anti-electron-neutrinos from the sun at super-kamiokande-i. *Physical Review Letters*, 90:171302, 2003.

[18] C. L. Cowan, F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire. Detection of the free neutrino: A Confirmation. *Science*, 124:103–104, 1956.

[19] G. Danby. Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos. *Phys. Rev. Lett.*, 9:36–44, 1962.

[20] S. Eidelman. Review of Particle Physics. *Physics Letters B*, 592:1+, 2004.

[21] G. Fiorentini and B. Ricci. What have we learnt about the Sun from the measurement of the ^8B neutrino flux? *Physics Letters B*, 526:186, 2002.

[22] H. V. Klapdor-Kleingrothaus, A. Dietz, H. L. Harney, and I. V. Krivosheina. Evidence for neutrinoless double beta decay. *MOD.PHYS.LETT.A*, 16:2409, 2001.

[23] M. Maltoni, T. Schwetz, M. A. Tortola, and J. W. F. Valle. Status of global fits to neutrino oscillations, 2004.

[24] W. Pauli. Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen, 1930.

[25] M. L. Perl, G. S. Abrams, A. M. Boyarski, M. Breidenbach, D. D. Briggs, F. Bulos, W. Chinowsky, J. T. Dakin, G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, G. Hanson, F. B. Heile, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke, B. A. Lulu, V. Lüth, D. Lyon, C. C. Morehouse, J. M. Paterson, F. M. Pierre, T. P. Pun, and P. A. Rapidis. Evidence for Anomalous Lepton Production in $e^+ - e^-$ Annihilation. *Phys. Rev. Lett.*, 35(22):1489–1492, 1975.

[26] B. Pontecorvo. Lepton mixing and neutrino oscillations. *J.Exptl. Theoret. Phys.*, 33:549, 1957.