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The Spillover Effect of Compulsory Insurance

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Abstract

The assumption usually made in the insurance literature that risks are always insurable at the desired level does not hold in the real world: some risks are not—or are only partially—insurable, while others, such as civil liability or health and workers' injuries, must be fully insured or at least covered for a specific amount. We examine in this paper conditions under which a reduction in the constrained level of insurance for one risk increases the demand of insurance for another independent risk. We show that it is necessary to sign the fourth derivative of the utility function to obtain an unambiguous spillover effect. Three different sufficient conditions are derived if the expected value of the exogenous risk is zero. The first condition is that risk aversion be standard—that is, that absolute risk aversion and absolute prudence be decreasing. The second condition is that absolute risk aversion be decreasing and convex. The third condition is that both the third and the fourth derivatives of the utility function be negative. If the expected value of the exogenous risk is positive, a wealth effect is added to the picture, which goes in the opposite direction if absolute risk aversion is decreasing.

Key words: Insurance demand, multiple sources of risk, compulsory insurance, standard risk aversion, prudence.

1. Introduction

The interdependence of different sources of risk affecting final wealth has been recently examined within the framework of expected utility. This interdependence has long been pointed out in finance, but assumptions were made, such as the normality of the distribution of returns, to simplify research. Typically, if risks are normally distributed and independent, the demand for each specific risk depends only on the characteristics of this risk. However, the separability of decisions relative to independent risks does not hold in general. Under more general distributions, such a separability property holds only if the utility function is quadratic or exponential. For example, without any other assumption than risk aversion, two independent risks that are undesirable when considered in isolation can be jointly desirable. Samuelson [1963] pointed out that a large proportion of the population could view this possibility as a consequence of the law of large numbers, but this is clearly a misinterpretation of it. Pratt and Zeckhauser [1987] described the necessary and sufficient condition on the utility function—called properness—that guarantees that two independent undesirable risks can never be jointly desirable. A sufficient condition for properness is that all odd derivatives of the utility function u be positive and all even derivatives negative.

A weakness of the concept of properness is that it appeared to be useless to analyze the effect of a change in one risk on the demand for another risk. Kimball [1993] examined a stronger restriction on utility functions that yields unambiguous comparative statics properties for this problem. Namely, risk aversion is "standard" if any loss-aggravating risk[1] always lowers the demand for any other independent risk. Kimball shows that the necessary and sufficient condition for standardness is that both absolute risk aversion $-u''(w)/u'(w)$—and absolute prudence $-u'''(w)/u''(w)$—be decreasing in wealth.[2]

In this paper, we consider a specific class of changes in risk. Namely, we examine the case of a change in the scale of the risk—that is, a change from risk $k\tilde{x}$ to risk $k'\tilde{x}$. Changes in scale arise in many real-world problems. Elmendorf and Kimball [1992] examine the problem of the change in the labor income tax rate $(1 - k)$ on the demand for risky assets. Since gross labor incomes $\tilde{x}$ are risky, this is a typical application of the problem addressed above. Alternatively, consider a mutual fund that can invest in different financial instruments under some regulatory constraints as the existence of upper limits of investment in some specific classes of assets. The question is, what is the effect of a change in the upper limit for one asset on the demand for other assets? We will illustrate this analysis by considering the problem of a change in the compulsory rate of insurance in one risk on the demand of insurance for other independent risks.[3]

The introduction or change in the compulsory rate of insurance raises several interesting issues about welfare effects of insurance programs. Indeed, compulsory insurance is usually required to ensure adequate judgements to third parties or to reduce negative externalities that would arise in the absence of social security programs. If compulsory insurance produces compensating changes on the freely chosen insurance coverages, this in turn affects welfare. For instance, if compulsory insurance requirements reduce the demand for insurance for other risks, it creates the externality one was trying to remove by introducing compulsory insurance, and it can undermine policy objectives. The question of complementarity or substitution between compulsory insurance and other insurance lines is thus a potentially important one.

The paper is organized as follows. In Section 2, we present the general model. We then consider in Section 3 the effect of imposing full insurance on one risk either in the small and in the large. Section 4 is allocated to the more general problem of analyzing the spillover effect of a marginal change in the compulsory rate of insurance, and some concluding remarks are provided in the last section.

2. The model

We consider the problem of a risk-averse individual who faces two sources of risk. In the absence of insurance, the agent's final wealth is $w_{00} - \tilde{x}_1 - \tilde{x}_2$, where $\tilde{x}_1$

and $\tilde{x}_2$ are two independent random variables representing potential losses on specific assets. An insurance policy on risk i is characterized by a couple $(\alpha_i, P_i(\alpha_i))$ where α_i is the rate of retention and P_i is the premium paid prior to the realization of the loss. We assuume that the insurance pricing is linear with $\lambda_i \geq 0$ denoting the loading factor. Namely, the insurance tariff on risk i takes the following form:

$$P_i(\alpha_i) = (1 + \lambda_i)(1 - \alpha_i)\mu_i, \tag{1}$$

where μ_i is the expected loss for risk i. The net wealth when retaining a share of α_1 of risk 1 and a share α_2 of risk 2 is a random variable that is defined as

$$\tilde{w}(\alpha_1, \alpha_2) = w_{00} - \sum_{i=1}^{2}[\alpha_i\tilde{x}_i + P_i(\alpha_i)] = w_0 + \sum_{i=1}^{2}\alpha_i\tilde{y}_i, \tag{2}$$

where $w_0 = w_{00} - \sum_{i=1}^{2} (1 + \lambda_i)\mu_i$ is the net wealth in case of full insurance, and $\tilde{y}_i = (1 + \lambda_i)\mu_i - \tilde{x}_i$ is the net gain of not buying insurance.[4] Because λ_i is assumed to be nonnegative, so is the expected value of $\tilde{y}_i$.

We assume that there is a compulsory rate of retention on risk $i = 2$—that is, $\alpha_2 = k$ is imposed to the consumer exogenously. Depending on the expected-utility-maximizing level of retention on risk 2, this constraint is either an uninsurability constraint (if the optimal rate of retention is smaller than k) or a compulsory insurance constraint (if the optimal rate of retention is larger than k). The problem of the expected-utility-maximizing consumer is to select the optimal rate of retention $\alpha_1^*(k)$ on risk 1, given the exogenous rate of retention k on risk 2. This problem is written as

$$\alpha_1^*(k) \in \arg\max_{\alpha_1} E_{12}u(w_0 + \alpha_1\tilde{y}_1 + k\tilde{y}_2), \tag{3}$$

where the utility function u is increasing and strictly concave. The strict concavity of u together with the nonnegativity of the loading factor implies that $\alpha_1^*(k)$ is nonnegative. For the sake of simplicity, we do not take into account the feasibility constraint $\alpha_1 \leq 1$. The first-order condition for program (3) is

$$E_{12}u'(w_0 + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)\tilde{y}_1 = 0. \tag{4}$$

The second-order condition is always satisfied.

We consider the effect of a change in the compulsory rate of retention on risk 2 on the optimal rate of retention on risk 1. The usual method for dealing with a change in background risk is to define an indirect utility function v defined as follows (see Kihlstrom, Romer, and Williams [1981]; Nachman [1982]; Eeckhoudt and Kimball [1992]):

$$v(w; k) = E_2u(w + k\tilde{y}_2). \tag{5}$$

An individual with direct utility function $u(.)$ who faces background risk $k\tilde{y}_2$ would behave as an individual free of background risk with utility $v(.; k)$. Therefore, by defining such an indirect utility function, problem (3) simplifies to the standard problem of insurance demand as formulated by Mossin [1968], since concavity is preserved under transformation (5). The index of absolute risk aversion of the indirect utility function is

$$A(w; k) = -\frac{\frac{\partial^2 v}{\partial w^2}(w; k)}{\frac{\partial v}{\partial w}(w; k)} = -\frac{E_2 u''(w + k\tilde{y}_2)}{E_2 u'(w + k\tilde{y}_2)}. \tag{6}$$

The problem of the comparative statics effect of a change in the compulsory rate of retention is equivalent to the problem of the effect of a change in the shape of the utility function.[5] As shown by Arrow [1971] and Pratt [1964], a sufficient condition for a change in the shape of the utility function to have an unambiguous effect on the optimal insurance demand at any wealth level is that this change takes the form of a concave (or convex) transformation:[6]

$$\frac{\partial \alpha_I^*}{\partial k}(k) \leq 0 \text{ at any initial wealth level} \Leftrightarrow \frac{\partial A}{\partial k}(w; k) \geq 0 \ \forall w. \tag{7}$$

In the remainder of the paper, we determine the condition under which an increase in the compulsory rate of retention on one risk does increase indirect risk aversion, therefore increasing the demand for insurance for any other independent risk. Using (7), the problem simplifies to the search for sufficient conditions on u to guarantee that A is increasing in k. The partial derivative of A with respect to k is written as

$$\frac{\partial A}{\partial k}(w; k) = \frac{E_2[u''(w + k\tilde{y}_2)\tilde{y}_2]E_2 u''(w + k\tilde{y}_2) - E_2[u'''(w + k\tilde{y}_2)\tilde{y}_2]E_2 u'(w + k\tilde{y}_2)}{(E_2[u'(w + k\tilde{y}_2)])^2}. \tag{8}$$

Clearly, the concavity of u is not sufficient to guarantee that the right side of this equality be positive.

3. The effect of compulsory full insurance

In this section, we consider the simpler case of departing from compulsory full insurance.

3.1. The effect of partial insurance

We first consider the effect of introducing partial insurance "in the small"—that is, when an *infinitesimal* retention rate is imposed to the policyholder. Observe that

$$\frac{\partial A}{\partial k}(w; 0) = \frac{(u''(w))^2 - u'''(w)u'(w)}{(u'(w))^2} E\tilde{y}_2 = A_0'(w)E\tilde{y}_2, \tag{9}$$

where $A_0(w) = A(w; 0) = -\frac{u''(w)}{u'(w)}$ is the absolute risk aversion of the original utility function. Since $E\tilde{y}_2 = \lambda_2\mu_2 \geq 0$, it appears that increasing absolute risk aversion is necessary and sufficient to guarantee that departing from compulsory full insurance increases the demand of insurance for the other risk, at the margin. More specifically, equation (9) yields the following property:

Proposition 1: *Assume that the loading factor on the compulsory insurance line is positive ($\lambda_2 > 0$). Departing from full insurance on risk 2 reduces (increases) the optimal rate of retention on risk 1 at the margin if absolute risk aversion is increasing (decreasing):*

$$\frac{\partial \alpha_1^*}{\partial k}(k = 0) \begin{cases} \leq 0 & \text{if } A_0' \geq 0; \\ \geq 0 & \text{if } A_0' \leq 0. \end{cases} \tag{10}$$

If $\lambda_2 = 0$, departing from full insurance on risk 2 has no effect at the margin on the optimal insurance coverage of risk 1.

Common wisdom indicates that independent risks should be substitutes in the sense that an increase in one risk reduces the demand for the other. However, Proposition 1 states that independent risks with a positive expectation are rather complementary "in the small," since absolute risk aversion is generally assumed to be decreasing. To get an intuition of this property, it is useful to decompose the right side of equation (8) as follows:

$$\begin{aligned} \frac{\partial A}{\partial k}(w; k) = {} & \frac{E_2[u''(w + k\tilde{y}_2)(\tilde{y}_2 - E\tilde{y}_2)]E_2[u''(w + k\tilde{y}_2)]}{(E_2[u'(w + k\tilde{y}_2)])^2} \\ & - \frac{E_2[u'''(w + k\tilde{y}_2)(\tilde{y}_2 - E\tilde{y}_2)]E_2[u'(w + k\tilde{y}_2)]}{(E_2[u'(w + k\tilde{y}_2)])^2} \\ & + \frac{\partial A}{\partial w}(w; k)\, E\tilde{y}_2. \end{aligned} \tag{11}$$

The second term in the RHS of (11) clearly corresponds to a wealth effect. Under decreasing absolute risk aversion (DARA), it is always negative since, as shown by Kihlstrom, Romer and Williams [1981], DARA is preserved under transformation (5). The point is that, under a positive loading, increasing the rate of retention increases expected wealth, implying in turn less risk aversion. The first term in (11) corresponds to a pure zero-mean risk effect. Its sign is ambiguous at this level of generality. However, when evaluated at $k = 0$, the risk effect vanishes and the total effect reduces to the wealth effect. Typically, the risk effect is a second-order effect with respect to the wealth effect, which is a first-order effect. This is apparent by using the Arrow-Pratt approximation.[7]

3.2. *The effect of a global change*

We now consider the effect of a global change "in the large" that is, when a "large" retention rate is imposed to the policyholder on risk $\tilde{y}_2$. As shown above, the unique relevant information to sign the spillover effect of departing from full insurance in the small is the sign of the derivative of absolute risk aversion. We show in the next proposition that more information is required to perform comparative statics in the large, even in the absence of a wealth effect.

Proposition 2: *Assume that $\lambda_2 = 0$. If absolute risk aversion is decreasing and convex, then departing from full insurance on risk 2 reduces the optimal rate of retention on risk 1: $\alpha_1^*(k) \leq \alpha_1^*(0)$. If absolute risk aversion is increasing and concave, then departing from full insurance on risk 2 increases the optimal rate of retention on risk 1: $\alpha_1^*(k) \geq \alpha_1^*(0)$.*

Proof: We consider the case of decreasing and convex absolute risk aversion. Observe that

$$\begin{aligned} A(w;k) &= \frac{E_2u''(w + k\tilde{y}_2)}{E_2u'(w + k\tilde{y}_2)} \\ &= E_2[\gamma(w, \tilde{y}_2)\, A_0(w + k\tilde{y}_2)] \qquad (12) \\ &= E_2[A_0(w + \tilde{y}_2)] + E_2[(\gamma(w, \tilde{y}_2) - 1)\, A_0(w + k\tilde{y}_2)], \end{aligned}$$

where

$$\gamma(w, y_2) = u'(w + ky_2)/E_2u'(w + k\tilde{y}_2).$$

This is a decreasing function of y_{22}, by risk aversion and $k > 0$. Because A_0 is convex, the Jensen inequality yields

$$E_2[A_0(w, \tilde{y}_2)] \geq A(w + kE\tilde{y}_2) = A_0(w),$$

—that is, the first term in (12) is larger than $A(w)$. We would be done if the second term in (12) is positive. Define z such that $\gamma(w, z) = 1$. For any $ky_2 < z$, $(\gamma(w, y_2) - 1)$ is positive and $A_0(w + ky_2)$ is larger than $A_0(w + z)$, since A_0 is decreasing. Therefore, the integrand $(\gamma(w, y_2) - 1)A_0(w + ky_2)$ is larger than $(\gamma(w,_2) - 1)A_0(w + z)$. Similarly, for any $ky_2 > z$, $(\gamma(w,_2) - 1)$ is negative and $A_0(w + ky_2)$ is smaller than $A_0(w + z)$. Therefore, the integrand $(\gamma(w, y_2) - 1)A_0(w + ky_2)$ is also larger than $(\gamma(w, y_2) - 1)A_0(w + z)$. It yields

$$E_2[(\gamma(w, \tilde{y}_2) - 1)A_0(w + k\tilde{y}_2)] \geq A_0(w + z)E_2[(\gamma(w, \tilde{y}_2) - 1)] = 0,$$

since the expectation of γ is 1 by definition. It implies that $A(w; k) \geq A_0(w)$, i.e. indirect risk aversion is reduced by allowing full insurance at fair price. The result follows from applying the Arrow-Pratt theorem. The case with increasing and concave absolute risk aversion can be treated symmetrically. ■

Let us assume that absolute risk aversion is decreasing and convex.[8] In the large, departing from full insurance at fair price increases the willingness to pay for insurance in other lines of insurance. In other words, under the same circumstances, introducing an upper limit of insurance in a specific insurance line with fair price increases the optimal coverage in other lines of insurance. The risk effect, which is zero at the margin around full insurance, is consistent with intuition in the large. Notice that the risk effect goes in the opposite direction with respect to the wealth effect, since absolute risk aversion is here assumed to be decreasing. It implies that the convexity of absolute risk aversion is not sufficient to sign the effect of compulsory insurance with unfair prices. The same contradictory effects also apply for the case of increasing and concave absolute risk aversion.

The convexity of decreasing absolute risk aversion is a natural assumption. Indeed, absolute risk aversion cannot be concave everywhere, otherwise risk aversion would eventually become negative. Moreover, all familiar DARA utility functions exhibit convex absolute risk aversion. It is noteworthy that

$$A_0'(w) = A_0(w)[A_0(w) - P_0(w)] \tag{13}$$

and

$$A_0''(w) = A_0(w)[2A_0'(w) + P_0(w)(T_0(w) - A_0(w))], \tag{14}$$

with $P_0(w) = - u'''(w)/u''(w)$ is the coefficient of absolute prudence (see Kimball [1990]) and $T_0(w) = -u''''(w)/u'''(w)$ is the coefficient of temperance (Kimball [1991a]; Eeckhoudt, Gollier, and Schlesinger [1993]). Decreasing absolute risk aversion requires that P_0 be larger than A_0, implying in turn that $u''' > 0$. According to equation (14), decreasing and convex absolute risk aversion requires an addition that T_0 be also larger than A_0, yielding $u'''' < 0$. A necessary condition for absolute risk aversion to be decreasing and convex is that the derivatives of u alternate in sign at least up to the fourth derivative.

4. The effect of a marginal change in the compulsory rate of insurance

In the previous section, we addressed the problem of the effect on the optimal rate of retention on one risk when partial insurance is introduced on another risk. In this section, we consider the more general problem of the effect of a marginal change in the compulsory rate of retention, i.e. when the dnitial retention rate on risk 2 is already positive and is further increased. Proposition 3 is close to the result obtained by Elmendorf and Kimball [1992] who used a two-period two-variable model.

Proposition 3: *If u exhibits decreasing absolute risk aversion and decreasing absolute prudence, then an increase in the compulsory rate of retention decreases the optimal rate of retention on another independent risk whenever this increase is not welcomed by the insured person:*

$$A_0'(w) \leq 0 \text{ and } P_0'(w) \leq 0 \ \forall w$$

$$\Rightarrow \frac{\partial \alpha_1^*}{\partial k}(k) \leq 0 \text{ whenever } \frac{\partial}{\partial k} E_{12} u(w + \alpha_1^*(k)\tilde{y}_1 + k\tilde{y}_2) \leq 0.$$

Proof: By the concavity of the objective function of problem (3), the problem simplifies to the proof that $E_{12}[\tilde{y}_1\tilde{y}_2 u''(w + \alpha_1^*(k)\tilde{y}_1 + k\tilde{y}_2)]$ is negative. Assuming decreasing absolute prudence, Elmendorf and Kimball [1992, Lemma 5] proved that this term satisfies the following condition:

$$E_{12}[\tilde{y}_1\tilde{y}_2 u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)] \leq \frac{E_{12}[\tilde{y}_1 u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)]\, E_{12}[\tilde{y}_2 u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)]}{E_{12}[u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)]}$$

In the remainder of this proof, we show that both terms in the numerator of the above inequality are positive, therefore concluding the proof. The fact that $E_{12}[\tilde{y}_1 u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)]$ is positive comes from the fact that u''' is assumed to be positive (see the proof of Proposition 3). The proof that $E_{12}[\tilde{y}_2 u''(w + \alpha_1^*\tilde{y}_1 + k\tilde{y}_2)]$ is positive as in Elmendorf and Kimball (Lemma 4). ■

If insurance for $\tilde{y}_2$ is fair, then an increase in the compulsory rate of retention is never welcomed by risk averse individuals. Therefore, under standard risk aversion, imposing an increase in the rate of retention at fair price has the intuitive appealing spillover effect for other lines of insurance.

If λ_2 is positive, Proposition 3 says nothing about small values of k at which an increase of compulsory retention is beneficial. Indeed, under unfair insurance pricing, increasing k from 0 is first beneficial up to the optimal retention rate. Observe however that $\frac{\partial \alpha_1^*}{\partial k}(k = 0)$ is positive under decreasing absolute risk aversion, as shown in Proposition 1. It follows that under standard risk aversion and a positive insurance loading, one can expect that increasing the rate of retention on one risk will first increase the optimal rate of retention on the other risk. It will then reduce it once increasing further the compulsory rate of retention becomes detrimental to expected utility.

As already noticed, two effects take place when the loading factor is positive: a risk effect and a wealth effect, as expressed in equation (11). The wealth effect is clearly identified and signed when absolute risk aversion is either decreasing or increasing, since $A(w; \alpha)$ is decreasing (increasing) whenever $A_0(w)$ is decreasing (increasing). In the remaining of this section, we focus on the risk effect.

Proposition 4: *Assume that λ_2 is zero. Increasing the compulsory rate of retention on one risk reduces (increases) the optimal rate of retention on any other independent risk if both u''' and u'''' are negative (positive):*

$$\frac{\partial \alpha_1^*}{\partial k}(k) \begin{cases} \leq 0 & \text{if } u''' < 0 \text{ and } u'''' < 0; \\ \geq 0 & \text{if } u''' > 0 \text{ and } u'''' > 0. \end{cases}$$

Proof: Suppose that u''' and u'''' are uniformly negative. Remember that we are interested in signing

$$\frac{\partial A}{\partial k}(w; k) = \frac{E_2[u''(w + k\tilde{y}_2)\tilde{y}_2]E_2[u''(w + k\tilde{y}_2)] - E_2[u'''(w + k\tilde{y}_2)\tilde{y}_2]E_2[u'(w + k\tilde{y}_2]}{(E_2[u'(w + k\tilde{y}_2)])^2}.$$

Since u''' is negative, $u''(w + ky_2) < (>)\ u''(w)$ whenever $y_2 > (<)\ 0$. Therefore, $E_2[u''(w + k\tilde{y}_2)\tilde{y}_2]$ is less than $u''(w)E[\tilde{y}_2] = 0$. Similarly, since u'''' is also negative, $E_2[u'''(w + k\tilde{y}_2)\tilde{y}_2]$ is less than $u'''(w)E[\tilde{y}_2] = 0$. It implies that the right side of the above equality is positive. The other property can be proved symmetrically. ■

If absolute risk aversion is decreasing (implying $u''' > 0$) and if u'''' is positive, the wealth effect (Proposition 1) and the risk effect (Proposition 4) reinforce each other (in a counter-intuitive manner). It yields the following Corollary.

Corollary: *If absolute risk aversion is decreasing and if the fourth derivative of u is positive, then increasing the compulsory rate of retention on one risk increases the optimal rate of retention on any other independent risk.*

The weakness of Proposition 4 and its corollary is that no familiar utility function exhibits one of the three conditions on u, since successive derivatives of u typically alternate in sign. The fact that, under the conditions of the Corollary, increasing the retention on one risk increases the optimal retention on the other is clearly a paradox that indicates that the fourth derivative of the utility function should actually be negative.

Since standardness requires $u''' > 0$ and $u'''' < 0$, Figure 1 summarizes our findings for the case of fair insurance prices. Figure 1 suggests that the sign of the fourth derivative of u plays a more important role than the sign of the third derivative to determine the sign of the spillover effect. Notice that the case with $u''' < 0$ and $u'''' > 0$—which has not been examined in this paper—is the least natural one since it implies increasing absolute risk aversion and increasing absolute prudence.[9]

5. Conclusion

Modifying the rate of insurance coverage on one risk entails in general ambiguous spillover effects on the optimal rate of coverage for other sources of independent risk. In this paper, we presented some restrictions on the utility function that yield unambiguous spillover effects. If we assume that compulsory insurance is provided at fair price, we found that reducing the coverage on one risk has a positive spillover effect on the demand of insurance for other sources of risk if one of the two following conditions holds. The first condition is that absolute risk aversion and absolute prudence are decreasing. The second condition is that both the third and the fourth derivatives of the utility function are negative.

If compulsory insurance is provided at unfair price, then a wealth effect has to be considered in addition to the risk effect whose sign is unambiguous under one

		u''''	
		+	-
u'''	+	≥0	≤0 (if standard)
	-	?	≤0

Figure 1. Sign of $\frac{\partial \alpha_1^*}{\partial k}$ when $\lambda_2 = 0$.

of the two conditions stated above. Under decreasing absolute risk aversion, this effect goes the opposite direction since a reduction in compulsory insurance makes insured persons wealthier on average, therefore reducing their demand for insurance for other risks. We showed that if absolute risk aversion is decreasing and if the fourth derivative of the utility function is positive, then reducing the coverage on one risk reduces the optimal coverage for the other risk. This result indicates no unambiguous comparative statics property can be obtained without some restrictions on the shape of u''''. The only exception to this claim is when considering a small departure from full insurance, in which case only the wealth effect plays a role.

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Notes

1. A risk is said to be loss-aggravating if it increases expected marginal utility.
2. Eeckhoudt, Gollier and Schlesinger [1993] obtained a much stronger necessary and sufficient condition to guarantee that any Rothschild-Stiglitz increase in one risk reduces the demand for any other independent risk. Few utility functions satisfy this condition.
3. Doherty and Schlesinger [1983] discussed how the demand for insurance is influenced by the presence of another uninsurable background risk. They showed that if the two risks are positively correlated, overinsurance may be optimal.
4. Observe that this formulation is formally equivalent to the problem of selecting a portfolio (α_1, α_2) of two independent assets with excess return $\tilde{y}_1$ and $\tilde{y}_2$. Our analysis is thus useful to study the effect of a change in the institutional constraints on the portfolio composition of financial institutions.
5. Notice that not only the comparative statics effect—but also the welfare effect—of a change in compulsory insurance are in general ambiguous. As noticed by a referee, an increase in k generates a combination of a FSD improvement and an increase in risk. These two changes are in opposite direction as far as the effect on expected utility is concerned.
6. The necessary and sufficient condition on the change on $v(., k)$ that yields an unambiguous comparative statics property for any $\tilde{y}_2$ is that $v(., k + \Delta k)$ be "centrally more risk-averse around w_0" than $v(., k)$, a concept proposed by Kimball [1991b]. It means that $v'(w, k + \Delta k)/v'(w_0, k + \Delta k)$ is larger (resp. smaller) than $v'(w, k)/v'(w_0, k)$ for any $w \leq w_0$ (resp. $w \geq w_0$). If we want the property to hold for any w_0, condition (7) becomes necessary.
7. The argument relies on the classical Taylor approximation of $Eu(w + k\tilde{y}_2) \cong u(w) + kE\tilde{y}_2u'(w) + k^2E\tilde{y}_2^2u''(w)$.
8. Decreasing and convex absolute risk aversion is a sufficient condition for "weak properness," a concept defined in Gollier and Pratt [1993]. Risk aversion is weak proper if adding any unfair risk to wealth increases risk aversion.
9. Kimball [1990] provides justifications for decreasing absolute prudence.

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