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The spin evolution of nascent neutron stars

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ABSTRACT

The loss of angular momentum owing to unstable r-modes in hot young neutron stars has been proposed as a mechanism for achieving the spin rates inferred for young pulsars. One factor that could have a significant effect on the action of the r-mode instability is fallback of supernova remnant material. The associated accretion torque could potentially counteract any gravitational-wave-induced spin-down, and accretion heating could affect the viscous damping rates and hence the instability. We discuss the effects of various external agents on the r-mode instability scenario within a simple model of supernova fallback on to a hot young magnetized neutron star. We find that the outcome depends strongly on the strength of the magnetic field of the star. Our model is capable of generating spin rates for young neutron stars that accord well with initial spin rates inferred from pulsar observations. The combined action of r-mode instability and fallback appears to cause the spin rates of neutron stars born with very different spin rates to converge, on a time-scale of approximately 1 year. The results suggest that stars with magnetic fields $\leq 10^{13}$ G could emit a detectable gravitational wave signal for perhaps several years after the supernova event. Stars with higher fields (magnetars) are unlikely to emit a detectable gravitational wave signal via the r-mode instability. The model also suggests that the r-mode instability could be extremely effective in preventing young neutron stars from going dynamically unstable to the bar-mode.

Key words: accretion, accretion discs – gravitation – magnetic fields – waves – stars: neutron.

1 MOTIVATION

Neutron stars may, by virtue of their oscillations, act as sources of detectable gravitational radiation. Of particular interest are the r-mode oscillations, quasi-toroidal currents that arise in rotating stars owing to the action of the Coriolis force. Andersson (1998) and Friedman & Morsink (1998) first showed that the r-modes of a perfect fluid star succumb to the gravitational radiation-driven Chandrasekhar–Friedman–Schutz instability for all rates of stellar rotation. Subsequent results suggest that the instability might grow on short time-scales, with positive feedback increasing the gravitational wave signal to potentially detectable levels (Owen et al. 1998). The r-mode instability is likely to be most important in hot young rapidly rotating neutron stars. For such stars there appears to be a window of temperatures and spin rates in which the r-modes can grow faster because of the emission of gravitational waves than they are damped by viscosity. The consequent loss of angular momentum provides a plausible mechanism for achieving the spin rates inferred for young pulsars (Andersson, Kokkotas & Schutz 1999). In fact, the excitement regarding the r-mode instability stems to a large extent from the fact that the first studies suggested that it would spin a newly born neutron star to roughly 20 ms, in good agreement with the inferred initial spin period of the Crab pulsar.

Recent efforts to improve our understanding of the unstable r-modes have focused on the interior physics, aiming to shed light on issues such as the role of general relativity (Lockitch, Andersson & Friedman 2001), superfluidity (Lindblom & Mendell 2000), the crust–core interface (Bildsten & Ushomirsky 2000; Lindblom, Owen & Ushomirsky 2000), and possible exotic states of matter such as hyperons (Jones 2001; Lindblom & Owen 2002). As we will show in this paper, another factor that may significantly affect the r-mode instability is accretion. Neutron stars are born in a messy environment, surrounded by debris from the supernova progenitor. In the first few months some of this material will fall back on to the young star. Intuitively, one would expect this to affect the action of the r-mode in two ways. First, accretion torques could counteract the gravitational wave induced spin-down, and perhaps cause the star to spin-up despite the presence of a large-amplitude r-mode. One can readily estimate that accretion of a total mass ΔM can spin a neutron star up to a rotation period

$$P \approx 3\left(\frac{\Delta M}{0.1\,\mathrm{M}_\odot}\right)^{-1}\ \mathrm{ms}.$$

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This estimate indicates that rapid supernova fallback may play a key role in determining the spin evolution of a nascent neutron star. Secondly, rapid accretion would be associated with strong heating, which would affect the strongly temperature-dependent viscosities, and hence the longevity and strength of the r-mode instability and any associated gravitational wave signal. Accretion is also expected to delay the formation of a crust until the star has cooled to a bulk temperature $\sim 10^9$ K (Haensel 1997). This should be compared with the case of isolated neutron stars in which the crust is expected to form at $\sim 10^{10}$ K. This delayed crust formation could be significant since the presence of a crust would suppress any unstable r-mode considerably (Bildsten & Ushomirsky 2000; Andersson & Kokkotas 2001). A delay in crust formation could therefore increase the longevity of any gravitational wave signal.

Our aim with this paper is to address two key questions. First, can the combined action of supernova fallback and r-mode instability give rise to spin rates that match those inferred for young pulsars? Secondly, what are the implications for the gravitational wave signal from the r-mode instability? In order to address these questions we develop a simple model of supernova fallback on to a hot young magnetized neutron star, modelling both spin evolution and gravitational wave emission.

2 THE MODEL

2.1 Spin evolution

In order to be able to compare our results with previous work and assess directly the role of the induced accretion torque, we employ a simple phenomenological spin evolution model analogous to that devised by Owen et al. (1998) (see also Ho & Lai 2000; Andersson & Kokkotas 2001).

We assume that we can model the angular momentum of the star as the sum of bulk angular momentum J and the canonical angular momentum of the r-mode J_c. The total torque on the star $\dot{J}$ can therefore be written as

$$\dot{J} = \dot{I}\Omega + I\dot{\Omega} + \dot{J}_c, \tag{1}$$

where the dots indicate time derivatives. Ω is the angular velocity of the star. For simplicity we will assume that the star does not rotate differentially despite the presence of a large-amplitude r-mode and a strong accretion torque. $I = \tilde{I}MR^2$ represents the moment of inertia of the star. We will model the star as an $n = 1$ polytrope, which means that $\tilde{I} = 0.261$. For this particular stellar model, accretion will affect the mass M of the star but not the radius R.

In our analysis we will only account for radiation from the (dominant) $l = m = 2$ current multipole of the r-mode. The canonical angular momentum of the mode is

$$J_c = -\frac{3\Omega\alpha^2\tilde{J}MR^2}{2}, \tag{2}$$

where α is the mode amplitude (as defined by Owen et al. 1998) and $\tilde{J} = 1.635 \times 10^{-2}$ for an $n = 1$ polytrope.

The r-mode is driven by gravitational radiation and damped by viscosity. We thus assume that the canonical angular momentum, which is proportional to the square of the perturbation, evolves according to

$$\frac{\dot{J}_c}{2J_c} = -\frac{1}{\tau}, \tag{3}$$

where

$$\tau^{-1} = -|\tau_g|^{-1} + \tau_b^{-1} + \tau_s^{-1}, \tag{4}$$

and $\tau_{g,b,s}$ are the time-scales associated with growth of the linear perturbation caused by gravitational radiation back reaction and dissipation of energy caused by bulk and shear viscosity, respectively. From equation (3) it can be seen that the mode is unstable (in an isolated star) if $\tau < 0$. In this case a small perturbation will lead to an unbounded growth of the mode.

Equations (1)–(3) can be combined to give equations for the evolution of α and Ω:

$$\dot{\alpha} = -\alpha\left(\frac{1}{\tau} + \frac{\dot{\Omega}}{2\Omega} + \frac{\dot{M}}{2M}\right) \tag{5}$$

and

$$\dot{\Omega} = \left(\frac{\dot{J}}{I} - \frac{\dot{M}\Omega}{M} - \frac{3\tilde{J}\alpha^2\Omega}{\tilde{I}\tau}\right). \tag{6}$$

We initiate our evolutions by assuming that a small-amplitude r-mode (with $\alpha = 10^{-6}$) is present. Variation of the initial amplitude has no significant effect on the subsequent evolution.

Equations (5) and (6) only apply when α is small. To model the behaviour once the r-mode reaches the non-linear regime we assume that it saturates at some value α_s. The larger the value of α_s, the greater the fraction of the angular momentum of the star the r-mode can carry (we consider $0.01 \le \alpha_s \le 1$ in this paper). When the mode is saturated the spin evolution is described by

$$\dot{\Omega} = \frac{\dot{J}}{I}\left(1 - \frac{3\tilde{J}\alpha_s^2}{2\tilde{I}}\right)^{-1} - \frac{\dot{M}\Omega}{M}. \tag{7}$$

As discussed by Owen et al. (1998), the bulk of gravitational wave emission and associated spin-down will occur during the saturated phase.

Eventually, the mode will become stable again, de-saturate and die away; when this happens we switch back to using equations (5) and (6). Note that equation (5) indicates that the r-mode is unstable, in the sense that $\dot{\alpha} > 0$, whenever

$$\frac{1}{\tau} + \frac{\dot{\Omega}}{2\Omega} + \frac{\dot{M}}{2M} < 0. \tag{8}$$

This is the appropriate criterion for the onset of instability in a star that is spun up by accretion (Ho & Lai 2000). Compare this with the instability criterion for an isolated star, $\tau < 0$.

We use the estimates given by Andersson & Kokkotas (2001) for the gravitational radiation reaction time-scale caused by the $l = m = 2$ current multipole:

$$\tau_g \approx -47 M_{1.4}^{-1} R_{10}^{-4} P_{-3}^{6}\ \mathrm{s}, \tag{9}$$

and the bulk and shear viscosity damping time-scales:

$$\tau_b \approx 2.7 \times 10^{11} M_{1.4} R_{10}^{-1} P_{-3}^{2} T_9^{-6}\ \mathrm{s}, \tag{10}$$

$$\tau_s = 6.7 \times 10^{7} M_{1.4}^{-5/4} R_{10}^{23/4} T_9^{2}\ \mathrm{s}, \tag{11}$$

where we have used the notation $M_{1.4} = M/1.4\,\mathrm{M}_\odot$, $R_{10} = R/10\,\mathrm{km}$, $P_{-3} = P/1\,\mathrm{ms}$ and $T_9 = T/10^9\,\mathrm{K}$. We have used these time-scales because it allows us to make a direct comparison with the earlier work of Owen et al. (1998). However, it should be noted that this is probably the most optimistic scenario with regard to the lifetime of unstable r-modes. We do not take into account any damping effects that may arise from the interaction of mode and magnetic field (Rezzolla, Lamb & Shapiro 2000), or from the presence of exotic particles such as hyperons (Jones 2001; Lindblom & Owen 2002).

2.2 Torques

The total torque on the star $\dot{J}$ is taken to be the sum of torques caused by gravitational radiation, accretion and magnetic dipole radiation. The torque arising from gravitational wave emission from the $l = m = 2$ current multipole is

$$\dot{J}_{\rm g} = 3\tilde{J}\Omega\alpha^2 MR^2\tau_{\rm g}^{-1}. \tag{12}$$

In order to model the accretion torque we need to know the quantity and rate of fallback on to the young neutron star. Several authors have examined this problem and concluded that a significant quantity of matter (up to $\sim 0.1\,{\rm M}_\odot$) could fall back on to the neutron star during the first few months of its life (Colgate 1971; Chevalier 1989; Houck & Chevalier 1991). Accretion of such a large amount of matter on to a neutron star is hypercritical; that is, orders of magnitude above the Eddington limit (cf. Bethe, Brown & Lee 2000). Such rapid accretion would be advection dominated, with energy loss occurring via neutrino emission as matter is processed at the surface of the neutron star. Advection-dominated accretion flows have been examined in detail by Narayan & Yi (1994, 1995). They conclude that advection could dominate in cases where the mass accretion rate is very high; precisely the conditions likely to prevail around a newly formed neutron star.

Advection-dominated accretion on to a magnetized neutron star has been examined by Menou et al. (1999). Building on earlier work (for example, Illarionov & Sunyaev 1975; Ghosh & Lamb 1978), they model a rotating magnetized neutron star surrounded by a magnetically threaded accretion disc. Accreting matter follows magnetic field lines and gives up angular momentum on reaching the surface, exerting a spin-up torque. The contribution of the magnetically threaded disc is, however, more complex. Interaction between the stellar field and the disc results in a positive torque for small radii, where the field lines rotate more slowly than the local Keplerian speed of the gas. For larger radii, the field lines rotate more quickly than the local Keplerian speed, resulting in a negative torque. If the spin period becomes very short, or the rate of fallback of material on to the magnetosphere drops, the star can enter what is known as the propeller regime. Then the rapidly rotating magnetosphere exerts a centrifugal barrier that inhibits further accretion. Accreting matter is flung away from the star and the star experiences a spin-down torque. An accreting magnetized star can therefore experience either a spin-up or a spin-down torque.

To proceed we need to determine the likely rate and quantity of supernova fallback. Mineshige et al. (1997) model the supernova fallback problem assuming that the fallback material possesses some angular momentum [cf. Chevalier (1989) and Brown & Weingartner (1994), who model the problem under the assumption of Bondi spherical accretion]. Mineshige et al. find a fallback rate $\dot{M}_{\rm f} \propto t^{-n}$, with $1 < n < 2$. We therefore use

$$\dot{M}_{\rm f} = Ct^{-3/2}, \tag{13}$$

where C is set by choosing the total mass to be accreted $M_{\rm acc}$ between some initial time $t_{\rm i}$ after the supernova explosion and $t = \infty$. The accretion rate is related to the fallback rate by

$$\dot{M} = \begin{cases} \dot{M}_{\rm f} & \text{propeller off,} \\ 0 & \text{propeller on.} \end{cases} \tag{14}$$

Following Chevalier (1989) and Mineshige et al. (1997) we choose $M_{\rm acc}$ to be $\leq 0.1\,{\rm M}_\odot$. We choose $t_{\rm i} = 100\,{\rm s}$ as this is the time after the supernova at which we expect there to be a recognizable compact object (Burrows & Lattimer 1986). If the propeller effect is not active, we assume that hypercritical accretion continues until the accretion rate falls to $\dot{M} \leq 10^{-4}\,{\rm M}_\odot\,{\rm yr}^{-1}$. This corresponds to the cut-off point below which hypercritical accretion is no longer possible. We follow Chevalier (1989) and Bethe et al. (2000) in assuming that below this rate photons begin to diffuse out of the shocked region and further accretion proceeds at (or below) the Eddington rate ($\sim 10^{-8}\,{\rm M}_\odot\,{\rm yr}^{-1}$).

If, on the other hand, the propeller effect becomes active, the situation is less clear. The propeller effect prevents accretion and trapped photons may escape as soon as the propeller becomes active. However, would the pressure of these photons reduce the rate of fallback of material on to the magnetosphere? We model two possibilities: (i) continued exponentially decaying fallback and (ii) constant accretion at the Eddington rate. For stars with magnetic fields of $\leq 10^{13}\,{\rm G}$ there is little quantitative difference between the two models, however, there are noticeable differences for higher fields.

A typical young neutron star is expected to have a magnetic field $B \sim 10^{12}\,{\rm G}$ (see Table 1), although a magnetar could have a field strength as high as $10^{15}\,{\rm G}$ (see, for example, Thompson & Duncan 1996). We assume that the magnetic field strength is not affected by accretion or the action of the mode, even though it is by no means clear that this would be the case in practice. Several authors have suggested that the r-mode might lead to differential rotation and hence to changes in the magnetic field (see, for example, Spruit 1999; Rezzolla et al. 2000). In the earliest stages after the supernova the accretion rate is likely to be so high that the magnetic field of the neutron star is completely confined. The pressure of accreting material will be so much greater than the magnetic pressure that the magnetospheric radius, defined by

$$r_{\rm m} = \left[\frac{B^2R^6}{2\dot{M}(2GM)^{1/2}}\right]^{2/7}, \tag{15}$$

with B taken to be the polar magnetic field of the star, is smaller than the radius of the neutron star. The magnetic field remains confined to the neutron star until the accretion rate falls sufficiently for the field to begin to exert an influence outside the star.

The accretion torque $\dot{J}_{\rm a}$ will vary depending on whether the magnetic field is confined and whether the propeller effect is operational. When the field is completely confined we use the results of Narayan & Yi (1994, 1995) for the accretion torque. Their models of advection-dominated accretion flows suggest orbital velocities with $v^2 \approx \frac{2}{7}v_{\rm k}^2$, where $v_{\rm k}$ is the Kepler velocity of a particle orbiting the star. Hence

$$\dot{J}_{\rm a} = \dot{M}R\left(\frac{2GM}{7R}\right)^{1/2}. \tag{16}$$

As the accretion rate falls, $r_{\rm m}$ grows and eventually exceeds the neutron star radius R. At this stage we switch to the equations of Menou et al. (1999), as the torque will now be affected by the interaction of the magnetic field and the fallback material:

$$\dot{J}_{\rm a} = 2r_{\rm m}^2\Omega_{\rm k}(r_{\rm m})\left(1 - \frac{\Omega}{\Omega_{\rm k}(r_{\rm m})}\right)\begin{cases} \dot{M} & \text{propeller off,} \\ \dot{M}_{\rm f} & \text{propeller on,} \end{cases} \tag{17}$$

where $\Omega_{\rm k}(r_{\rm m})$ is the Keplerian angular velocity evaluated at the magnetospheric radius. This magnetized accretion torque falls as the star spins up, and for $\Omega > \Omega_{\rm k}(r_{\rm m})$ the torque will, in fact, be negative. The star is then in the propeller regime. During this phase

Table 1. Young pulsars (PSR) that have been identified with known supernova remnants (SNR). Where the braking index is not provided by observation, it is assumed to be similar to that of the Crab ($n = 2.5$). The lower part of the table illustrates the difficulty of estimating t_{SNR}.

PSR	P (ms)	n	SNR	SNR age (kyr)	t_c (kyr)	P_0 (ms)	B (10^{12} G)
B0531+21	33	2.52	Crab	0.95 (SN1054)	1.2	18	3.8
B0833−45	89	1.4 (a)	Vela	9–30	11	13–57	3.4
B0540−69	50	1.81	N158A	0.8−1.1	1.7	34–39	5.0
PSR 1951+32	39.5		CTB 80	38−74 (b)	107	21–33	0.49
J1811−1926	65		G11.2−0.3	1.7 (SN386)	24	63	1.7
J1846−0258	324		Kes 75	0.9−4.3	0.7	315 (c)	49
J205+6449	66		3C58	0.82 (SN1181)	5.4	60	3.6
J1124−5916	135		G292.0+1.8	1.7	2.9	90	10
B1853+01	267		W44	20 (d)	20	$\ll P$	7.6
B1509−58	151	2.83	MSH15−52	(e)	1.6	(e)	15
J1119−6127	408	2.91	G292−05	(e)	1.6	(e)	41
J1016−5857	107		G284.3−1.8	~10 (e)	21	(e)	3
B1757−24	125		G5.4−1.2	39−170 (f)	15.5	(f)	4

Notes: (a) The measurement technique for n is somewhat unorthodox. (b) Age estimate made using the proper motion of the pulsar. (c) Using $t_{\mathrm{SNR}} = 900$ yr – for larger t_{SNR} it is not possible to estimate P_0 using equation (29). (d) As $t_{\mathrm{SNR}} = t_c$ it is not possible to estimate a value of P_0 using equation (29) – we can only say that P_0 is small. (e) Age of supernova remnant too uncertain to make a reliable estimate of P_0. (f) Estimated age of remnant much larger than t_c making estimate of P_0 infeasible.
Most references from Kaspi (2000), additional data from: B0833−45 (Lyne et al. 1996), B0540−69 (Zhang et al. 2001; Reynolds 1985), PSR 1951+32 (Foster et al. 1994; Migliazzo et al. 2002), J1811−1926 (Torii et al. 1999), J1846−0258 (Gotthelf et al. 2000), J205+6449 (Murray et al. 2002), J1124−5916 (Camilo et al. 2002), B1853+01 (Cox et al. 1999), B1509−58 (Kaspi et al. 1994), J1119−6127 (Crawford et al. 2001), J1016−5857 (Camilo et al. 2001), B1757−24 (Gaensler & Frail 2000).

matter is prevented from accreting on to the star and fallback material is instead expelled from the system by the rotating magnetosphere. Menou et al. (1999) review the assumption that accretion ceases and conclude that accretion is indeed minimal when the propeller is active. We do not attempt to model the complicated transition regime between the two phases in detail.

Once magnetic field confinement ends we could also include a torque caused by emission of magnetic dipole radiation, $\dot{J}_m$, where

$$\dot{J}_m = -\frac{2B^2R^6\Omega^3}{3c^3}. \tag{18}$$

This term was, however, found to be negligible compared with the other torques acting on the young neutron star. Its effects will not be discussed further in this paper.

2.3 Temperature evolution

Viscosity, the main agent damping the r-mode in our model, depends critically on temperature. We include three factors in modelling the temperature evolution: modified URCA cooling, shear viscosity reheating, and accretion heating. We assume the star to be isothermal even though this may not be appropriate for a newly born accreting star (Burrows & Lattimer 1986). Still, this assumption simplifies the analysis considerably and all previous studies of the r-mode instability have assumed a uniform temperature distribution.

The primary cooling mechanism for a young neutron star will be the modified URCA reaction, in which neutrons decay via the weak interaction, emitting neutrinos. We do not model any rapid cooling effects arising from the direct URCA reactions in the core of the star (cf. Lattimer et al. 1994; Page et al. 2000). The cooling rate owing to the modified URCA reaction, $\dot{\epsilon}_u$, is given by Shapiro & Teukolsky (1983):

$$\dot{\epsilon}_u = 7.5\times10^{39} M_{1.4}^{2/3} T_9^8 \,\mathrm{erg\,s^{-1}}. \tag{19}$$

The neutron star will be heated by the action of shear viscosity on the r-mode oscillations. The heating rate caused by shear viscosity, $\dot{\epsilon}_s$, is given by Andersson & Kokkotas (2001):

$$\begin{aligned}\dot{\epsilon}_s &= \frac{2\alpha^2\Omega^2 MR^2\tilde{J}}{\tau_s}\\ &= 8.3\times10^{37}\alpha^2\Omega^2\tilde{J}M_{1.4}^{9/4}R_{10}^{-15/4}T_9^{-2}\,\mathrm{erg\,s^{-1}}.\end{aligned} \tag{20}$$

Accretion heating will have two components. The first contribution arises when accreting matter undergoes nuclear burning at the surface of the star. We assume that accreting matter will release approximately 1.5 MeV of energy per nucleon (Brown & Bildsten 1998). If we assume that every nucleon reaching the surface is burnt, then energy is liberated at a rate of

$$\dot{\epsilon}_n = \frac{\dot{M}}{m_B}\times 1.5\,\mathrm{MeV} = 4\times10^{51}\dot{M}_{1.4}\,\mathrm{erg\,s^{-1}}, \tag{21}$$

where m_B is the mass of a baryon.

A second contribution arises because the flow is assumed to be advection dominated. Matter falling in towards a star will liberate $\sim GM/R$ of potential energy per unit mass. In a non-advection-dominated flow most of this energy would be dissipated as heat in the accretion disc. In an advection-dominated flow, in contrast, this energy is carried in with the flow of matter. We assume that the advected potential energy is used to generate neutrinos near the surface of the star, and that these neutrinos are radiated isotropically from the point of generation. We therefore assume that half of the neutrinos escape without interacting with the star and that the remaining half are radiated into the star, where they scatter and interact with the stellar material. The mean free path λ of inelastic scattering of neutrinos with electrons in the neutron star matter is

$$\lambda \sim 2\times10^{10}\left(\frac{\rho_{\mathrm{nuc}}}{\bar{\rho}}\right)^{7/6}\left(\frac{0.1\,\mathrm{MeV}}{E_\nu}\right)^{5/2}\mathrm{cm}, \tag{22}$$

where $\rho_{\rm nuc} \sim 2.8 \times 10^{14}\,{\rm g\,cm^{-3}}$ and $\bar{\rho}$ is the average density of the star. For an $n = 1$ polytrope we have

$$\frac{\bar{\rho}}{\rho_{\rm nuc}} = 0.7 \frac{M}{{\rm M}_\odot}. \tag{23}$$

We assume that the neutrino energy is $E_\nu = 1\,{\rm MeV}$ (Shapiro & Teukolsky 1983). Assuming fully efficient scattering, and noting that $\lambda > R$, the heating rate caused by advected potential energy may be estimated as

$$\dot{\varepsilon}_{\rm h} \sim \frac{R}{\lambda} \frac{GM\dot{M}}{R} = 8 \times 10^{51} M_{1.4}^{13/6} \dot{M}_{1.4}\,{\rm erg\,s^{-1}}. \tag{24}$$

We estimate the heat capacity C_ν by assuming the thermal energy of the neutron star to reside almost exclusively in degenerate neutrons (Shapiro & Teukolsky 1983). Neglecting interactions, the heat capacity of such a system is given by

$$C_\nu = 1.6 \times 10^{39} M_{1.4}^{1/3} T_9\,{\rm erg\,K^{-1}}. \tag{25}$$

Equation (25) shows that $C_\nu \propto T$, hence the equation of thermal balance of the star is

$$\frac{\rm d}{{\rm d}t}\left(\frac{1}{2} C_\nu T\right) = \dot{\varepsilon}_{\rm s} - \dot{\varepsilon}_{\rm u} + \dot{\varepsilon}_{\rm n} + \dot{\varepsilon}_{\rm h}. \tag{26}$$

Manipulation of equation (26) yields an expression for the temperature evolution of the star. We initiate our evolutions at $T = 10^{11}$ K and terminate them at $T = 10^9$ K, the temperature at which we expect a crust to form (Haensel 1997) and suppress the r-mode instability in an accreting star.

3 OBSERVATIONAL CONSTRAINTS

Before we proceed to the discussion of the results obtained from our simple model it is useful to consider the constraints provided by observations of young neutron stars. It is standard practice to assume that a pulsar spins down purely as a result of magnetic dipole radiation. Given the observed period and the spin-down rate we can then estimate the magnetic field strength from

$$B = 3.2 \times 10^{19} (P\dot{P})^{1/2}\,{\rm G}. \tag{27}$$

Similarly, one can infer the characteristic age $t_{\rm c}$ of the pulsar from the observed data:

$$t_{\rm c} = \frac{P}{2\dot{P}}. \tag{28}$$

Here it is assumed that the star was born spinning much faster that its current rate. The use of this estimate is supported by the fact that, in the case of the Crab pulsar, the age calculated in this way accords well with the known age.

If observations also provide the braking index, n, and an estimate of pulsar age, $t_{\rm SNR}$, the initial spin period of the pulsar P_0 is inferred from

$$P_0 = P\left[1 - (n-1) t_{\rm SNR} \frac{\dot{P}}{P}\right]^{1/(n-1)}, \tag{29}$$

where n is given by

$$n = \frac{P\ddot{P}}{\dot{P}^2} - 2. \tag{30}$$

In most cases the largest uncertainty in estimating the initial spin period is associated with the age of the pulsar, $t_{\rm SNR}$. Ideally, one would like to make an association with a known historical supernova. This would then, as in the case of the Crab, provide a precise age estimate and a reasonably reliable initial spin period. Unfortunately, this kind of data are the exception rather than the norm. Typically, the independent age is estimated by measuring the size of the supernova remnant and comparing it with theoretical models for the expansion rate. This procedure is complicated by many factors and the results are associated with large error bars.

A sample of data for young pulsars that are claimed to be associated with known supernova remnants is given in Table 1. The reliability of the inferred initial spin periods can be assessed by comparing the characteristic age of the pulsar to the estimated age of the supernova remnant. For most of the cases listed in Table 1, these estimates are consistent. In these cases the association between supernova remnant and pulsar seems to be correct and the spin of these pulsars appears to have evolved primarily because of the action of magnetic dipole radiation.

The data for young pulsars clearly support the notion that neutron stars are not formed spinning near the Kepler limit. This would be consistent with both the r-mode instability scenario (where a rapidly spinning, newly born neutron star is spun down to a period of say 10 ms in a few months) and the possibility that magnetic core–envelope coupling in the supernova progenitor leads to most neutron stars being born rotating slowly (Spruit & Phinney 1998).

There is ongoing discussion in the literature as to whether the (obviously simplistic) magnetic dipole model adequately describes the spin evolution of young pulsars. Concerns stem from discrepancies between various age estimates for some young pulsars. Calculations would also be complicated by magnetic field decay (Colpi, Geppert & Page 2000) or growth (Gaensler & Frail 2000). Another problem is that all measurements of the braking index n are lower than the value of 3 predicted by the magnetic dipole model. Several authors have suggested that the propeller effect, driven by ongoing low-rate accretion of supernova remnant material, could explain the discrepancies (Marsden, Lingenfelter & Rothschild 2001; Menou, Perna & Hernquist 2001). The same mechanism might also explain the properties of the anomalous X-ray pulsars (AXPs) (Chatterjee, Hernquist & Narayan 2000; Perna, Hernquist & Narayan 2000; Alpar 2001; Menou et al. 2001), without having to invoke large magnetic field strengths (Thompson & Duncan 1996). See Rothschild, Lingenfelter & Marsden (2002) for a review of AXP properties and the competing accretion-driven and magnetar models.

The relation of these propeller–fallback models to the one used in this paper is simple. Because we are interested in gravitational wave emission, we model only the very earliest stages of fallback and terminate our evolutions at crust formation. At crust formation ($t \sim 1$ yr) there is still material in the fallback disc, although the fallback rate has dropped significantly. The papers listed above focus more on the subsequent long period of sub-Eddington accretion. Although they do discuss the early super-Eddington accretion phase (see, for example, Chatterjee et al. 2000), they do not model this phase in detail and neglect the effects of gravitational wave emission completely.

4 RESULTS FROM THE FALLBACK MODEL

We are now prepared to discuss the results obtained from our phenomenological r-mode instability/supernova fallback model, and compare the predicted spin rates to the data in Table 1.

If we consider the data in Table 1, and the fact that the r-modes are certainly stable for periods longer than (say) 20 ms (see

Andersson & Kokkotas 2001), it would seem as if the instability may only be relevant for a small sample of nascent neutron stars. This would be in agreement with the model of Spruit & Phinney (1998), which suggests that most neutron stars are born rotating slowly. Our study will show, however, that supernova fallback may change this picture considerably, with accretion-induced spin-up driving the star into the window where the r-mode is likely to be unstable. In other words, significant supernova fallback enhances the probability that a young neutron star evolves through a phase where the r-mode instability is active.

Let us consider the effect of supernova fallback on a neutron star born spinning at a reasonably small fraction of the Kepler rate. Taking the birth spin period as 10 ms, we evolve the star according to our phenomenological model. Typical results for the resultant spin evolution are shown in Fig. 1. The data shown in the figure suggest the following picture. After an initial spin-up phase, the r-mode becomes unstable and begins to grow. For a typical young neutron star with magnetic field $\sim 10^{12}$ G the r-mode instability provides the main spin-down mechanism. For larger magnetic fields the propeller effect becomes active at an earlier stage of the evolution and is the main spin-down mechanism for highly magnetized stars. Nevertheless, the r-mode is important in even the most highly magnetized stars. Switching off the r-mode leads to a marked reduction in spin-down, despite the fact that the mode never actually saturates in the star with the largest magnetic field. Note also that our model predicts that supernova fallback will delay the appearance of the r-mode signal compared with the case of an isolated star, for which the r-mode is predicted to saturate after ~ 10 min (Owen et al. 1998). This can be seen from equation (8): the high initial accretion rate overwhelms all of the other terms in the expression and α will not grow until $\dot{M}$ has fallen.

In our model we have assumed that the magnetic field does not influence $\dot{M}_{\mathrm{f}}$ (although it does affect $\dot{M}$ via the propeller effect). Recent work by Igumenshchev & Narayan (2002) on spherical fallback, however, suggests that the presence of a magnetic field may reduce $\dot{M}_{\mathrm{f}}$ by perhaps a factor of 10. If we assume such a reduced $\dot{M}_{\mathrm{f}}$ from the time at which magnetic field suppression ends, we find little impact on spin rate or gravitational wave emission for a 10^{12}-G star. The effect on a 10^{14}-G star is, however, more pronounced. Initial spin-up and subsequent propeller-driven spin-down are much reduced, as is the emitted gravitational wave amplitude. Note, however, that we only examine the first ~ 1-year post-supernova and that the effects of slower fallback may be more important at later times.

We have terminated the evolutions once the star reaches a temperature of 10^9 K. At this point one would expect the crust to form, which is predicted to increase viscous damping of the r-mode dramatically (Bildsten & Ushomirsky 2000; Lindblom et al. 2000). In Fig. 2 we compare the spin period at the end of the our evolutions with the values of P_0 inferred for young pulsars using the magnetic dipole model (as listed in Table 1). This figure should, of course, be considered with caution since the inferred spin periods are associated with significant error bars. Still, our model clearly captures the main trend of the data, namely that the initial spin period inferred from the magnetic dipole model increases with magnetic field. Given the many uncertainties in both the theoretical model and the observational data the agreement between the results is rather good.

The fallback model suggests that the outcome is insensitive to the spin period at the beginning of the evolution (the true 'birth' spin rate). The combined action of supernova fallback and the r-mode instability causes the spin rates of stars with very different birth spins to converge within ~ 1 yr. This is shown in Fig. 3.

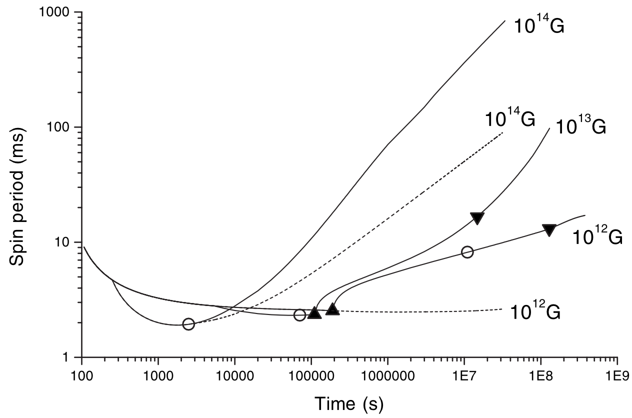

Figure 1. The solid lines show the evolution of the spin period for newly born accreting neutron stars with an unstable r-mode and different magnetic field strengths (key: open circles, propeller switches on; upwards triangles, r-mode saturates; downwards triangles, r-mode becomes stable). Note that the r-mode never reaches the saturation amplitude for the star with the highest magnetic field. The dashed lines, given for comparison for the two extreme magnetic field strengths, show the spin evolution if an unstable r-mode is not present.

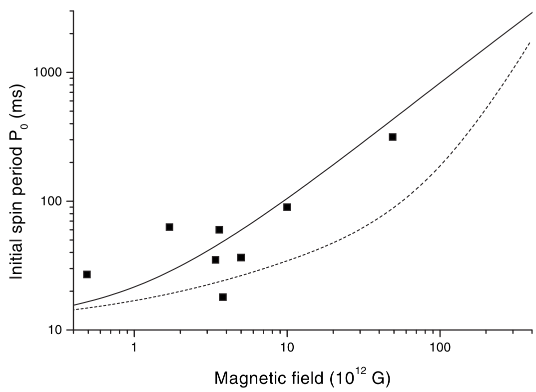

Figure 2. Spin rates predicted by the fallback model at crust formation (within 1–10 yr of the supernova) compared with values inferred for young pulsars using the standard magnetic dipole spin-down model (black squares). The solid line is for a model where fallback rate continues to decay exponentially when the propeller switches on; the dashed line for one where the fallback rate then drops to the Eddington limit. (Note that the observational data are associated with large error bars that are not included in the figure.)

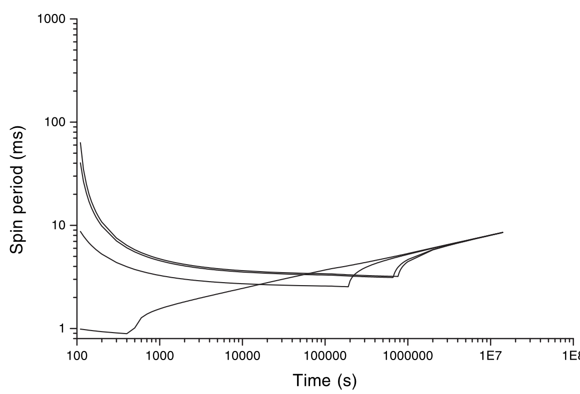

Figure 3. The combined action of the r-mode instability and fallback accretion causes the spin rates of stars with birth spin rates in the range 1 ms to 1 s to converge to ~ 10 ms within ~ 1 yr. The data shown are for a star with $B = 10^{12}$ G.

This result is notable given the uncertainty over the spin rates at which neutron stars are formed. It may, in fact, suggest that the true 'birth' spin is not the key factor that determines the spin period at (say) 1 year post-supernova. This observation has implications for determining whether or not a young neutron star undergoes a phase where the r-modes are unstable. This will, in turn, affect the relevance of the r-modes as a gravitational wave source. Of particular interest is the fact that our results suggest that fallback accretion may lead to even a very slowly spinning neutron star being spun up sufficiently for the r-modes to become unstable after ~ 10 min. However, we conclude that the magnetic field is far more important in determining the spin rate at 1 year post-supernova than both the birth spin rate and the possible r-mode spin-down phase.

The key parameter in determining the role of the r-mode instability is the saturation amplitude α_s. As has been demonstrated by recent hydrodynamical simulations (Stergioulas & Font 2001; Lindblom, Tohline & Vallisneri 2001), the r-modes may not saturate until the amplitude has reached values of the order of unity. The effect of varying α_s is shown in Fig. 4. These results are exactly what one would expect: the r-mode needs to be able to grow to a large amplitude if it is to counteract the strong accretion torque during the fallback phase.

One might expect fallback accretion to lead to strong heating, and that this would have a significant effect on the duration of the r-mode gravitational wave signal. We analyse the role of accretion heating in Fig. 5. We find that even though rapid accretion increases the temperature by roughly 10 per cent throughout the initial evolution, it does not lengthen the r-mode instability phase. Far more important in this respect is reheating due to the action of shear viscosity on the r-mode, which prolongs the r-mode instability phase for both an accreting and a non-accreting star. The amount of viscous heating is essentially the same in our current model as it would be in the case of an isolated young neutron star.

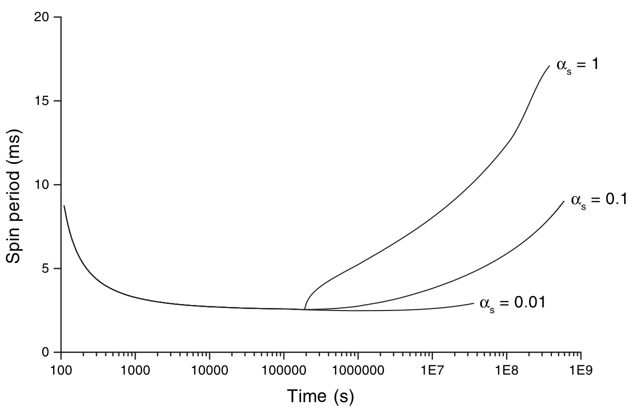

Figure 4. The effect on spin evolution of changing α_s for a star with $B = 10^{12}$ G.

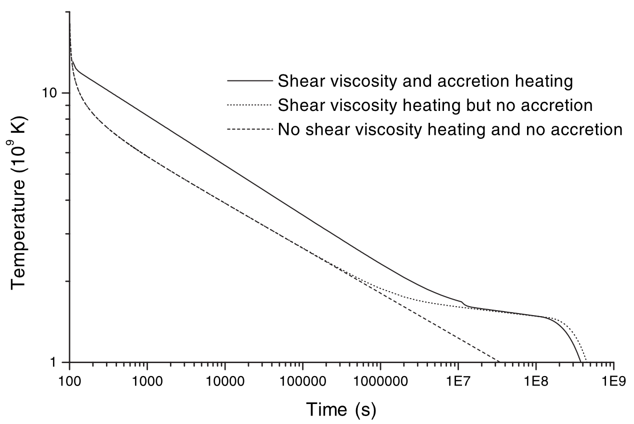

Figure 5. The effect of shear viscosity reheating and accretion heating on the temperature evolution of a young neutron star.

In order to assess the detectability of the gravitational waves due to the r-modes, we use the prescription of Owen et al. (1998). The true strain of the gravitational waves, h, is given by

$$h(t) = 7.54 \times 10^{-23} \alpha \tilde{J} \Omega^3 M_{1.4} R_{10}^3 \frac{15\,\mathrm{Mpc}}{D}. \tag{31}$$

We consider sources in the Virgo Cluster, for which the distance to the source, $D \approx 15$ Mpc. Results from our model are similar to results from previous studies (such as Owen et al. 1998). They show that the unstable r-modes may lead to a gravitational wave signal that could be detectable by a second-generation detector such as LIGO II. The main difference between our results and those of Owen et al. (1998) is the duration of the signal. We predict that, if continued accretion were to take place and prevent crust formation until the core temperature has fallen to 10^9 K, the signal may last for several years. We find that the peak gravitational wave signal amplitude remains almost unchanged as we vary the saturation amplitude by two orders of magnitude, even though this has a noticeable effect on the final spin period. Note, however, that these conclusions are only relevant for neutron stars with fields $< 10^{13}$ G. It is unlikely that we would observe a signal from the r-modes of neutron stars with larger magnetic fields. This conclusion agrees well with studies of Rezzolla et al. (2000) and Ho & Lai (2000). In our case the reduction in signal with increasing magnetic field is a result mainly of increased propeller-effect spin-down. This is because the gravitational wave amplitude depends so strongly on spin rate (see equation 31). It should, of course, be pointed out that we have not taken account of any possible interaction between the r-mode and the magnetic field, which may further decrease the relevance of the r-modes in magnetized stars.

Finally, we have considered the possibility that the fallback accretion torque might be able to spin the star up to (and beyond) the mass-shedding limit (for the canonical $n = 1$ polytropic neutron star used in our evolutions this corresponds to $P \approx 0.8$ ms). Depending on the overall stiffness of the supranuclear equation of state, neutron stars spinning near the mass-shedding limit may be subject to the dynamical bar-mode instability (Shapiro & Teukolsky 1983). This would lead to the formation of a highly non-axisymmetric bar-like configuration which is expected to be a strong source of gravitational waves with frequencies of the order of kHz (Houser, Centrella & Smith 1994). Whether a young neutron star becomes bar-mode unstable is therefore of great interest to gravitational wave theorists. Interestingly, our evolutions indicate that the r-mode instability is remarkably efficient in preventing the star from reaching the mass-shedding limit during the fallback phase. Even stars born with periods as short as 0.85 ms were unable to reach the Kepler threshold. The r-mode grows rapidly and is able to spin-down the star, even in the initial stages where the accretion torque is the highest. This suggests that r-mode-induced spin-down might counteract accretion spin-up to prevent all but the most rapidly rotating young neutron stars from becoming bar-mode unstable. For comparison we also investigated the case where there was no unstable r-mode. In this case the propeller effect switches on far earlier and can also act to prevent the star from going bar-mode unstable. Consider the $B = 10^{12}$ G star shown in Fig. 1, which is born spinning at $P = 10$ ms. If the unstable r-mode is present, the propeller effect will not switch on until ~ 200 d. If the r-mode is absent, however, the propeller will come on after only ~ 20 d. For a $B = 10^{12}$ G star born spinning at $P = 1$ ms, the difference is even more pronounced. In this case the propeller will switch on after ≈ 3 d when there is no r-mode, as compared with ~ 200 d when the unstable r-mode is present.

5 CONCLUDING REMARKS

We commenced this work with two prime objectives: to understand the impact of supernova fallback accretion on the unstable r-modes of neutron stars and any associated gravitational wave signal, and to investigate the combined influence of accretion and r-mode instability on the spin rates of young neutron stars. Our model, simple as it is, has offered insight into both questions. Our results suggest that newly formed neutron stars accreting fallback material will indeed experience a phase of perhaps several years during which the r-mode is unstable. This would have positive ramifications for detection of gravitational waves from the unstable r-modes of neutron stars. Moreover, the model suggests that this is true even for stars that are born with low spin rates. The effect of magnetic field is, however, crucial. While stars with canonical ($\leq 10^{13}$ G) field strengths are predicted to go r-mode unstable, the strong propeller effect associated with magnetars appears to preclude the r-mode from growing to the saturation level. The external torque simply spins the star down fast enough that the r-mode is not given time to grow. It is thus unlikely that we will see a strong r-mode gravitational wave signal from a highly magnetized star. The likelihood of obtaining a gravitational wave signal from an unstable bar-mode is also low in our model. Spin-down owing to emission of radiation from the r-modes is able to counteract even the strongest accretion torque and prevent all but the stars with the most rapid initial spins (and obviously those born spinning faster than the Kepler limit) from going dynamically unstable to the bar-mode. The propeller effect has a similar preventive effect for the stars with the highest magnetic fields.

With regard to our second question, the predictions of the model are consistent with the spin rates inferred from observations of young pulsars. A canonical neutron star is spun down within a few years by the combined action of fallback and r-mode to a period similar to that inferred for the Crab pulsar at its birth. Neutron stars with stronger magnetic fields, for which the propeller effect is the dominant spin-down mechanism, achieve far longer periods in the first year post-supernova. Even for such highly magnetized stars, however, the effect of an unstable r-mode on spin rate is significant.

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