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THE STANDARD ELECTROWEAK THEORY AND ITS EXPERIMENTAL TESTS

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1 INTRODUCTION

These lectures on electroweak interactions start with a short summary of the Glashow–Weinberg–Salam theory [1] and then cover in detail the main subjects of present interest in phenomenology: the Higgs sector and the open problem of the experimental investigation on the origin of the Fermi scale of mass $G_F^{-1/2}$; the structure of radiative corrections; the strategy for the experimental verification of the theory; and finally a discussion of the status of LEP physics. The important domain of flavour-changing processes and of CP violation is not reviewed here.

The modern electroweak theory inherits the phenomenological successes of the $(V-A)\otimes(V-A)$ four-fermion low-energy description of weak interactions [2], and provides a well-defined and consistent theoretical framework including weak interactions and quantum electrodynamics in a unified picture.

As an introduction, in the following we recall some salient physical features of the weak interactions. The weak interactions derive their name from their intensity. At low energy the strength of the effective four-fermion interaction of charged currents is determined by the Fermi coupling constant G_F. For example, the effective interaction for muon decay is given by

$$\mathcal{L}_{\rm eff} = (G_F/\sqrt{2})\,[\bar{\nu}_\mu \gamma_\alpha (1-\gamma_5)\mu]\,[\bar{e}\gamma^\alpha(1-\gamma_5)\nu_e]\ , \qquad (1)$$

with [3]

$$G_F = 1.16637(2)\times 10^{-5}\ {\rm GeV}^{-2}\ . \qquad (2)$$

In natural units $\hbar = c = 1$, G_F has dimensions of $({\rm mass})^{-2}$. As a result, the intensity of weak interactions at low energy is characterized by $G_F E^2$, where E is the energy scale for a given process ($E \approx m_\mu$ for muon decay). Since

$$G_F E^2 = G_F m_p^2 (E/m_p)^2 \simeq 10^{-5}(E/m_p)^2\ , \qquad (3)$$

where m_p is the proton mass, the weak interactions are indeed weak at low energies (of order m_p). The quadratic increase with energy cannot continue for ever, because it would lead to a violation of unitarity. In fact, at large energies the propagator effects can no longer be neglected, and the current–current interaction is resolved into current–W gauge boson vertices connected by a W propagator. The strength of the weak interactions at high energies is then measured by g_W, the $W-\mu$–ν_μ coupling, or, even better, by $\alpha_W = g_W^2/4\pi$ analogous to the fine-structure constant α of QED. In the standard electroweak theory, we have

$$\alpha_W = \sqrt{2}\,G_F\, m_W^2/\pi = \alpha/\sin^2\theta_W \cong 1/30\ . \qquad (4)$$

That is, at high energies the weak interactions are no longer so weak.

The range r_W of weak interactions is very short: it is only with the experimental discovery of the W and Z gauge bosons that it could be demonstrated that r_W is non-vanishing. Now we know that

$$r_W = \hbar/m_W c \simeq 2.5\times 10^{-16}\ {\rm cm}\ , \qquad (5)$$

corresponding to $m_W \simeq 80$ GeV. This very large value for the W (or the Z) mass makes a drastic difference, compared with the massless photon and the infinite range of the QED force. The experimental limits on the photon mass [3] are listed in the following. From a laboratory experiment, one obtains $m_\gamma < 10^{-14}$ eV by a method based on the vanishing of the electric field inside a cavity with conducting walls, predicted by Gauss' law. In fact, the exact r^{-2} behaviour of the electric field corresponds to $m_\gamma = 0$. From the observed distribution of planetary magnetic fields (the field should be damped by an extra factor $e^{-m_\gamma r}$ if $m_\gamma \neq 0$) the Pioneer probe to Jupiter obtained $m_\gamma < 6\times 10^{-16}$ eV. Finally, indirect evidence from galactic magnetic fields indicates that $m_\gamma < 3\times 10^{-27}$ eV. Thus, on the one hand, there is very good evidence that the photon is massless. On the other hand, the weak bosons are very heavy. A unified theory of electroweak interactions has to face this striking difference.

Another apparent obstacle in the way of electroweak unification is the chiral structure of weak interactions: in the massless limit for fermions, only left-handed quarks and leptons (and right-handed antiquarks and antileptons) are coupled to W's. This clearly implies parity and charge-conjugation violation in weak interactions.

The universality of weak interactions and the algebraic properties of the electromagnetic and weak currents [the conservation of vector currents (CVC), the partial conservation of axial currents (PCAC), the algebra of currents, etc.] have been crucial in pointing to a symmetric role of electromagnetism and weak interactions at a more fundamental level. The old Cabibbo universality for the weak charged current [4]:

$$\begin{aligned} J_\alpha^{\rm weak} = {} & \bar{\nu}_\mu\gamma_\alpha(1-\gamma_5)\mu + \bar{\nu}_e\gamma_\alpha(1-\gamma_5)e + \cos\theta_c\,\bar{u}\gamma_\alpha(1-\gamma_5)d + \\ & \sin\theta_c\,\bar{u}\gamma_\alpha(1-\gamma_5)s + \dots\ , \end{aligned} \qquad (6)$$

suitably extended, is naturally implied by the standard electroweak theory. In this theory the weak gauge bosons couple to all particles with couplings that are proportional to their weak charges, in the same way as the photon couples to all particles in proportion to their electric charges [in Eq. (6), $d' = \cos\theta_c\, d + \sin\theta_c\, s$ is the weak-isospin partner of u in a doublet. The (u,d') doublet has the same couplings as the (ν_e,ℓ) and (ν_μ,μ) doublets].

Another crucial feature is that the charged weak interactions are the only known interactions that can change flavour: charged leptons into neutrinos or up-type quarks into down-type quarks. On the contrary, there are no flavour-changing neutral currents at tree level. This is a remarkable property of the weak neutral current, which is explained by the introduction of the GIM mechanism [5] and has led to the successful prediction of charm.

The natural suppression of flavour-changing neutral currents, the separate conservation of e,μ and τ leptonic flavours, the mechanism of CP violation [6] through the phase in the quark-mixing matrix, are all crucial features of the Standard Model. Many examples of new physics tend to break the selection rules of the standard theory. Thus the experimental study of rare flavour-changing transitions is an important window on possible new physics.

In the following sections we shall see how these properties of weak interactions fit into the standard electroweak theory.

2 GAUGE THEORIES

In this section we summarize the definition and the structure of a gauge Yang–Mills theory [7],[8]. We will list here the general rules for constructing such a theory. Then in the next section these results will be applied to the electroweak theory.

Consider a Lagrangian density $\mathcal{L}[\phi, \partial_\mu \phi]$ which is invariant under a D dimensional continuous group of transformations:

$$\phi' = U(\theta^A)\phi \qquad (A = 1, 2, ..., D) . \tag{7}$$

For θ^A infinitesimal, $U(\theta^A) = 1 + ig\sum_A \theta^A T^A$, where T^A are the generators of the group Γ of transformations (7) in the (in general reducible) representation of the fields ϕ. Here we restrict ourselves to the case of internal symmetries, so that T^A are matrices that are independent of the space–time coordinates. The generators T^A are normalized in such a way that for the lowest dimensional non-trivial representation of the group Γ (we use t^A to denote the generators in this particular representation) we have

$$\mathrm{tr}(t^A t^B) = 1/2\delta^{AB} . \tag{8}$$

The generators satisfy the commutation relations

$$[T^A, T^B] = iC_{ABC}T^C . \tag{9}$$

In the following, for each quantity V^A we define

$$\mathbf{V} = \sum_A T^A V^A . \tag{10}$$

If we now make the parameters θ^A depend on the space–time coordinates $\theta^A = \theta^A(x_\mu)$, $\mathcal{L}[\phi, \partial_\mu \phi]$ is in general no longer invariant under the gauge transformations $U[\theta^A(x_\mu)]$, because of the derivative terms. Gauge invariance is recovered if the ordinary derivative is replaced by the covariant derivative:

$$D_\mu = \partial_\mu + ig\mathbf{V}_\mu , \tag{11}$$

where V_μ^A are a set of D gauge fields (in one-to-one correspondence with the group generators) with the transformation law

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} - (1/ig)(\partial_\mu U)U^{-1} . \tag{12}$$

For constant θ^A, $\mathbf{V}$ reduces to a tensor of the adjoint (or regular) representation of the group:

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} \simeq \mathbf{V}_\mu + ig[\theta, \mathbf{V}_\mu] , \tag{13}$$

which implies that

$$V'^C_\mu = V^C_\mu - gC_{ABC}\theta^A V^B_\mu , \tag{14}$$

where repeated indices are summed up.

As a consequence of Eqs. (11) and (12), $D_\mu\phi$ has the same transformation properties as ϕ:

$$(D_\mu\phi)' = U(D_\mu\phi) . \tag{15}$$

Thus $\mathcal{L}[\phi, D_\mu\phi]$ is indeed invariant under gauge transformations. In order to construct a gauge-invariant kinetic energy term for the gauge fields V^A, we consider

$$[D_\mu, D_\nu]\phi = ig\{\partial_\mu \mathbf{V}_\nu - \partial_\nu \mathbf{V}_\mu + ig[\mathbf{V}_\mu, \mathbf{V}_\nu]\}\phi \equiv ig\mathbf{F}_{\mu\nu}\phi , \tag{16}$$

which is equivalent to

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - gC_{ABC}V^B_\mu V^C_\nu . \tag{17}$$

From Eqs. (7), (15) and (16) it follows that the transformation properties of $F^A_{\mu\nu}$ are those of a tensor of the adjoint representation

$$\mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^{-1} . \tag{18}$$

The complete Yang–Mills Lagrangian, which is invariant under gauge transformations, can be written in the form

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}\sum_A F^A_{\mu\nu}F^{A\mu\nu} + \mathcal{L}[\phi, D_\mu\phi] . \tag{19}$$

For an Abelian theory, as for example QED, the gauge transformation reduces to $U[\theta(x)] = \exp[ieQ\theta(x)]$, where Q is the charge generator. The associated gauge field (the photon), according to Eq. (12), transforms as

$$V'_\mu = V_\mu - \partial_\mu\theta(x) . \tag{20}$$

In this case, the $F_{\mu\nu}$ tensor is linear in the gauge field V_μ so that in the absence of matter fields the theory is free. On the other hand, in the non-Abelian case the $F^A_{\mu\nu}$ tensor contains both linear and quadratic terms in V^A_μ, so that the theory is non-trivial even in the absence of matter fields.

3 THE STANDARD MODEL OF THE ELECTROWEAK INTERACTIONS

In this section, we summarize the structure of the standard electroweak Lagrangian and specify the couplings of $W^\pm$ and Z, the intermediate vector bosons (IVBs).

For this discussion we split the Lagrangian into two parts by separating the Higgs boson couplings:

$$\mathcal{L} = \mathcal{L}_{\mathrm{symm}} + \mathcal{L}_{\mathrm{Higgs}} . \tag{21}$$

We start by specifying $\mathcal{L}_{\rm symm}$, which involves only gauge bosons and fermions:

$$\mathcal{L}_{\rm symm} = -\frac{1}{4}\sum_{A=1}^{3} F^A_{\mu\nu}F^{A\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar\psi_L i\gamma^\mu D_\mu \psi_L + \bar\psi_R i\gamma^\mu D_\mu\psi_R \ . \qquad (22)$$

This is the Yang–Mills Lagrangian for the gauge group $SU(2)\otimes U(1)$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and} \quad F^A_{\mu\nu} = \partial_\mu W^A_\nu - \partial_\nu W^A_\mu - g\epsilon_{ABC}\, W^B_\mu W^C_\nu \qquad (23)$$

are the gauge antisymmetric tensors constructed out of the gauge field B_μ associated with $U(1)$, and W^A_μ corresponding to the three $SU(2)$ generators; ϵ_{ABC} are the group structure constants [see Eqs. (9)] which, for $SU(2)$, coincide with the totally antisymmetric Levi-Civita tensor (recall the familiar angular momentum commutators). The normalization of the $SU(2)$ gauge coupling g is therefore specified by Eq. (23).

The fermion fields are described through their left-hand and right-hand components:

$$\psi_{L,R} = [(1\mp\gamma_5)/2]\psi, \quad \bar\psi_{L,R} = \bar\psi[(1\pm\gamma_5)/2] \ , \qquad (24)$$

with γ_5 and other Dirac matrices defined as in the book by Bjorken–Drell [9]. In particular, $\gamma_5^2 = 1, \gamma_5^\dagger = \gamma_5$. Note that, as given in Eq. (24),

$$\bar\psi_L = \psi_L^\dagger\gamma_0 = \psi^\dagger[(1-\gamma_5)/2]\gamma_0 = \bar\psi[\gamma_0(1-\gamma_5)/2]\gamma_0 = \bar\psi[(1+\gamma_5)/2] \ .$$

The matrices $P_\pm = (1\pm\gamma_5)/2$ are projectors. They satisfy the relations $P_\pm P_\pm = P_\pm, P_\pm P_\mp = 0, P_+ + P_- = 1$.

The sixteen linearly independent Dirac matrices can be divided into γ_5-even and γ_5-odd according to whether they commute or anticommute with γ_5. For the γ_5-even, we have

$$\bar\psi\Gamma_E\psi = \bar\psi_L\Gamma_E\psi_R + \bar\psi_R\Gamma_E\psi_L \qquad (\Gamma_E \equiv 1, i\gamma_5, \sigma_{\mu\nu}) \ , \qquad (25)$$

whilst for the γ_5-odd,

$$\bar\psi\Gamma_O\psi = \bar\psi_L\Gamma_O\psi_L + \bar\psi_R\Gamma_O\psi_R \qquad (\Gamma_O \equiv \gamma_\mu, \gamma_\mu\gamma_5) \ . \qquad (26)$$

In the Standard Model the left and right fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form $\bar\psi_L\psi_R$ + h.c.) are forbidden in the symmetric limit. In particular, all ψ_R are singlets in the minimal Standard Model. But for the moment, by ψ_R we mean a column vector, including all fermions in the theory that span a generic reducible representation of $SU(2)\otimes U(1)$. The standard electroweak theory is a chiral theory, in the sense that ψ_L and ψ_R behave differently under the gauge group. In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing ψ_L and ψ_R. Fermion masses will be introduced, together with $W^\pm$ and

Z masses, by the mechanism of symmetry breaking. The covariant derivatives $D_\mu\psi_{L,R}$ are explicitly given by

$$D_\mu\psi_{L,R} = \left[\partial_\mu + ig\sum_{A=1}^{3} t^A_{L,R}W^A_\mu + ig'\frac{1}{2}Y_{L,R}B_\mu\right]\psi_{L,R} \ , \qquad (27)$$

where $t^A_{L,R}$ and $1/2Y_{L,R}$ are the $SU(2)$ and $U(1)$ generators, respectively, in the reducible representations $\psi_{L,R}$. The commutation relations of the $SU(2)$ generators are given by

$$[t^A_L, t^B_L] = i\,\epsilon_{ABC}t^C_L \quad \text{and} \quad [t^A_R, t^B_R] = i\epsilon_{ABC}t^C_R \ . \qquad (28)$$

We use the normalization (8) [in the fundamental representation of $SU(2)$]. The electric charge generator Q (in units of e, the positron charge) is given by

$$Q = t^3_L + 1/2\,Y_L = t^3_R + 1/2\,Y_R \ . \qquad (29)$$

Note that the normalization of the $U(1)$ gauge coupling g' in (27) is now specified as a consequence of (29).

All fermion couplings to the gauge bosons can be derived directly from Eqs. (22) and (27). The charged-current (CC) couplings are the simplest. From

$$g(t^1W^1_\mu + t^2W^2_\mu) = g\left\{[(t^1+it^2)/\sqrt{2}](W^1_\mu - iW^2_\mu)/\sqrt{2}] + \text{h.c.}\right\} = g\left\{[(t^+W^-_\mu)/\sqrt{2}] + \text{h.c.}\right\} \ , \qquad (30)$$

where $t^\pm = t^1 \pm it^2$ and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$, we obtain the vertex

$$V_{\bar\psi\psi W} = g\bar\psi\gamma_\mu\left[(t^+_L/\sqrt{2})(1-\gamma_5)/2 + (t^+_R/\sqrt{2})(1+\gamma_5)/2\right]\psi W^-_\mu + \text{h.c.} \qquad (31)$$

In the neutral-current (NC) sector, the photon A_μ and the mediator Z_μ of the weak NC are orthogonal and normalized linear combinations of B_μ and W^3_μ:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu \ ,$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W\, W^3_\mu \ . \qquad (32)$$

Equations (32) define the weak mixing angle θ_W. The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. Recalling Eq. (29) for the charge matrix Q, we immediately obtain

$$g\,\sin\theta_W = g'\cos\theta_W = e \ , \qquad (33)$$

or equivalently,

$$\text{tg}\,\theta_W = g'/g \qquad (34)$$

Once θ_W has been fixed by the photon couplings, it is a simple matter of algebra to derive the Z couplings, with the result

$$\Gamma_{\bar\psi\psi Z} = g/(2\,\cos\theta_W)\bar\psi\gamma_\mu[t^3_L(1-\gamma_5) + t^3_R(1+\gamma_5) - 2Q\sin^2\theta_W]\psi Z^\mu \ , \qquad (35)$$

where $\Gamma_{\bar{\psi}\psi Z}$ is a notation for the vertex. In the minimal Standard Model, $t_R^3 = 0$ and $t_L^3 = \pm 1/2$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs. (31) and (35), we anticipate that large masses, as experimentally observed, are provided for $W^\pm$ and Z by $\mathcal{L}_{\text{Higgs}}$. For left–left CC couplings, when the momentum transfer squared can be neglected with respect to m_W^2 in the propagator of Born diagrams with single W exchange, from Eq. (31) we can write

$$\mathcal{L}_{\text{eff}}^{\text{CC}} \simeq (g^2/8m_W^2)[\bar{\psi}\gamma_\mu(1-\gamma_5)t_L^+\psi][\bar{\psi}\gamma^\mu(1-\gamma_5)t_L^-\psi] \,. \tag{36}$$

By specializing further in the case of doublet fields such as $\nu_e - e^-$ or $\nu_\mu - \mu^-$, we obtain the tree-level relation of g with the Fermi coupling constant G_F measured from μ decay [see Eq. (2)]:

$$G_F/\sqrt{2} = g^2/8m_W^2 \,. \tag{37}$$

By recalling that $g\, \sin\theta_W = e$, we can also cast this relation in the form

$$m_W = \mu_{\text{Born}}/\sin\theta_W \,, \tag{38}$$

with

$$\mu_{\text{Born}} = (\pi\alpha/\sqrt{2}G_F)^{1/2} \simeq 37.2802 \text{ GeV} \,, \tag{39}$$

where α is the fine-structure constant of QED ($\alpha \equiv e^2/4\pi = 1/137.036$).

In the same way, for neutral currents we obtain in Born approximation from Eq. (35) the effective four-fermion interaction given by

$$\mathcal{L}_{\text{eff}}^{\text{NC}} \simeq \sqrt{2}\, G_F\rho_0\bar{\psi}\gamma_\mu[...]\psi\bar{\psi}\gamma^\mu[...]\psi \,, \tag{40}$$

where

$$[...] \equiv t_L^3(1-\gamma_5) + t_R^3(1+\gamma_5) - 2Q\sin^2\theta_W \tag{41}$$

and

$$\rho_0 = m_W^2/m_Z^2\ \cos^2\theta_W \,. \tag{42}$$

All couplings given in this section are obtained at tree level and are modified in higher orders of perturbation theory. In particular, the relations between m_W and $\sin\theta_W$ [Eqs. (38) and (39)] and the observed values of ρ ($\rho = \rho_0$ at tree level) in different NC processes, are altered by computable electroweak radiative corrections, as discussed in Section 7.

The gauge-boson self-interactions can be derived from the $F_{\mu\nu}$ term in $\mathcal{L}_{\text{symm}}$, by using Eq. (32) and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$. Defining the three-gauge-boson vertex as in Fig. 1, we obtain ($V \equiv \gamma, Z$)

$$\Gamma_{W^-W^+V} = ig_{W^-W^+V}[g_{\mu\nu}(q-p)_\lambda + g_{\mu\lambda}(p-r)_\nu + g_{\nu\lambda}(r-q)_\mu] \,, \tag{43}$$

with

$$g_{W^-W^+\gamma} = g\ \sin\theta_W = e \quad \text{and} \quad g_{W^-W^+Z} = g\ \cos\theta_W \,. \tag{44}$$

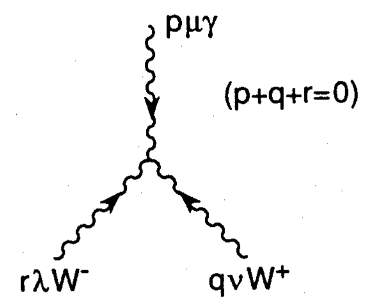

Fig. 1. The three-gauge boson vertex

We now turn to the Higgs sector [10] of the electroweak Lagrangian. Here we simply review the formalism of the Higgs mechanism applied to the electroweak theory. In the next section we shall make a more general and detailed discussion of the physics of the electroweak symmetry breaking. The Higgs Lagrangian is specified by the gauge principle and the requirement of renormalizability to be

$$\mathcal{L}_{\text{Higgs}} = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi) - \bar{\psi}_L\Gamma\psi_R\phi - \bar{\psi}_R\Gamma^\dagger\psi_L\phi^\dagger \,, \tag{45}$$

where ϕ is a column vector including all Higgs fields; it transforms as a reducible representation of the gauge group. The quantities Γ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The potential $V(\phi^\dagger\phi)$, symmetric under $SU(2) \otimes U(1)$, contains, at most, quartic terms in ϕ so that the theory is renormalizable. Spontaneous symmetry breaking is induced if the minimum of V which is the classical analogue of the quantum mechanical vacuum state (both are the states of minimum energy) is obtained for non-vanishing ϕ values. Precisely, we denote the vacuum expectation value (VEV) of ϕ, i.e. the position of the minimum, by v:

$$\langle 0|\phi(x)|0\rangle = v \neq 0 \,. \tag{46}$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by v:

$$M = \bar{\psi}_L\ \mathcal{M}\psi_R + \bar{\psi}_R\mathcal{M}^\dagger\psi_L \,, \tag{47}$$

with

$$\mathcal{M} = \Gamma \cdot v \,. \tag{48}$$

In the minimal Standard Model, where all left fermions ψ_L are doublets and all right fermions ψ_R are singlets, only Higgs doublets can contribute to fermion masses. There are enough free couplings in Γ, so that one single complex Higgs doublet is indeed sufficient to generate the most general fermion mass matrix. It is important to observe that by a suitable change of basis we can always make the matrix $\mathcal{M}$ Hermitian, γ_5-free, and diagonal. In fact, we can make separate unitary transformations on ψ_L and ψ_R according to

$$\psi'_L = U\psi_L, \quad \psi'_R = V\psi_R \tag{49}$$

and consequently

$$\mathcal{M} \to \mathcal{M}' = U^\dagger \mathcal{M} V . \tag{50}$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\text{symm}}$. For quarks, the Cabibbo–Kobayashi–Maskawa [4],[11] unitary transformation relates the mass eigenstates d, s and b to the CC eigenstates d', s', b', i.e. the states coupled by W emission to u, c, and t, respectively. The NC is then automatically diagonal in flavour at tree level (GIM mechanism [5]). In the case of leptons, if the neutrinos are massless then clearly there is no mixing.

If only one Higgs doublet is present, the change of basis that makes $\mathcal{M}$ diagonal will at the same time diagonalize also the fermion–Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. This is not true, in general, when there are several Higgs doublets. But one Higgs doublet for each electric charge sector i.e. one doublet coupled only to u-type quarks, one doublet to d-type quarks, one doublet to charged leptons would also be all right [12], because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets it is also possible to generate CP violation by complex phases in the Higgs couplings [13]. In the presence of six quark flavours, this CP-violation mechanism is not necessary. In fact, at the moment, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomena.

We recall that the Standard Model, with N fermion families with the observed quantum numbers, is automatically free of γ_5 anomalies [14] owing to cancellation of quarks with lepton loops.

We now consider the gauge-boson masses and their couplings to the Higgs. These effects are induced by the $(D_\mu\phi)^\dagger(D^\mu\phi)$ term in $\mathcal{L}_{\text{Higgs}}$ [Eq. (45)], where

$$D_\mu\phi = \left[\partial_\mu + ig\sum_{A=1}^{3} t^A W_\mu^A + ig'(Y/2)B_\mu\right]\phi . \tag{51}$$

Here t^A and $1/2Y$ are the $SU(2)\otimes U(1)$ generators in the reducible representation spanned by ϕ. Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that the photon remains massless is equivalent to the condition that the vacuum is electrically neutral:

$$Q|v\rangle = (t^3 + 1/2Y)|v\rangle = 0 . \tag{52}$$

The charged W mass is given by the quadratic terms in the W field arising from $\mathcal{L}_{\text{Higgs}}$, when $\phi(x)$ is replaced by v. We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2 |(t^+ v/\sqrt{2})|^2 W_\mu^+ W^{-\mu} , \tag{53}$$

whilst for the Z mass we get [recalling Eq. (32)]

$$1/2m_Z^2 Z_\mu Z^\mu = |[g\cos\theta_W t^3 - g'\sin\theta_W (Y/2)]v|^2 Z_\mu Z^\mu , \tag{54}$$

where the factor of 1/2 on the left-hand side is the correct normalization for the definition of the mass of a neutral field. By using Eq. (52), relating the action of t^3 and $1/2Y$ on the vacuum v, and Eqs. (34), we obtain

$$1/2m_Z^2 = (g\cos\theta_W + g'\sin\theta_W)^2 |t^3 v|^2 = (g^2/\cos^2\theta_W)|t^3 v|^2 . \tag{55}$$

For Higgs doublets

$$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}, \quad v = \begin{pmatrix}0\\ v\end{pmatrix} , \tag{56}$$

we have

$$|t^+ v|^2 = v^2, \quad |t^3 v|^2 = 1/4v^2 , \tag{57}$$

so that

$$m_W^2 = 1/2g^2v^2, \quad m_Z^2 = 1/2g^2v^2/\cos^2\theta_W . \tag{58}$$

Note that by using Eq. (37) we obtain

$$v = 2^{-3/4} G_F^{-1/2} = 174.1 \text{ GeV} . \tag{59}$$

It is also evident that for Higgs doublets

$$\rho_0 = m_W^2/m_Z^2 \cos^2\theta_W = 1 . \tag{60}$$

This relation is typical of one or more Higgs doublets and would be spoiled by the existence of Higgs triplets etc. In general,

$$\rho_0 = \sum_i \left[(t_i)^2 - (t_i^3)^2 + t_i\right] v_i^2 / \sum_i 2(t_i^3)^2 v_i^2 \tag{61}$$

for several Higgses with VEVs v_i, weak isospin t_i, and z-component t_i^3. These results are valid at the tree level and are modified by calculable electroweak radiative corrections, as discussed in Sections 7 and 8.

If only one Higgs doublet is present, then the fermion–Higgs couplings are in proportion to the fermion masses. In fact, from the Yukawa couplings $g_{\phi f f}(\bar{f}_L \phi f_R + h.c.)$, the mass m_f is obtained by replacing ϕ by v, so that $m_f = g_{\phi f f} v$.

With only one complex Higgs doublet, three out of the four Hermitian fields are removed from the physical spectrum by the Higgs mechanism and become the longitudinal modes of W^+, W^-, and Z. The fourth neutral Higgs is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be around for each additional doublet.

Finally, the couplings of the physical Higgs H to the gauge bosons can be simply obtained from $\mathcal{L}_{\text{Higgs}}$, by the replacement

$$\phi(x) = \begin{pmatrix}\phi^+(x)\\ \phi^0(x)\end{pmatrix} \to \begin{pmatrix}0\\ v + (H/\sqrt{2})\end{pmatrix} , \tag{62}$$

[so that $(D_\mu\phi)^\dagger(D^\mu\phi) = 1/2(\partial_\mu H)^2 + ...$], with the result

$$\begin{aligned}\mathcal{L}[H,W,Z] = g^2(v/\sqrt{2})W_\mu^+W^{-\mu}H + (g^2/4)W_\mu^+W^{-\mu}H^2 \\ +[(g^2vZ_\mu Z^\mu)/(2\sqrt{2}\cos^2\theta_W)]H \\ +[g^2/(8\cos^2\theta_W)]Z_\mu Z^\mu H^2 \ . \end{aligned} \tag{63}$$

We have thus completed our summary of the standard electroweak theory and of the $W^\pm, Z$ couplings.

4 THE HIGGS SECTOR

The gauge symmetry of the Standard Model was difficult to discover because it is well hidden in nature. The only observed gauge boson that is massless is the photon. The graviton is still unobserved, even at the classical level of gravitational waves; the gluons are presumed massless but unobservable because of confinement, and the W and Z weak bosons carry a heavy mass. Actually the main difficulty in unifying weak and electromagnetic interactions was the fact that e.m. interactions have infinite range ($m_\gamma = 0$), whilst the weak forces have a very short range, owing to $m_{W,Z} \neq 0$.

The solution of this problem is in the concept of spontaneous symmetry breaking, which was borrowed from statistical mechanics.

Consider a ferromagnet at zero magnetic field in the Landau–Ginzburg approximation. The free energy in terms of the temperature T and the magnetization $\mathbf{M}$ can be written as

$$F(\mathbf{M},T) \simeq F_0(T) + 1/2\,\mu^2(T)\mathbf{M}^2 + 1/4\,\lambda(T)(\mathbf{M}^2)^2 + \dots \ . \tag{64}$$

This is an expansion which is valid at small magnetization. The neglect of terms of higher order in $\mathbf{M}^2$ is the analogue in this context of the renormalizability criterion. Also, $\lambda(T) > 0$ is assumed for stability; F is invariant under rotations, i.e. all directions of $\mathbf{M}$ in space are equivalent. The minimum condition for F reads

$$\partial F/\partial M = 0, \quad [\mu^2(T) + \lambda(T)\mathbf{M}^2]\mathbf{M} = 0 \ . \tag{65}$$

There are two cases. If $\mu^2 > 0$, then the only solution is $\mathbf{M} = 0$, there is no magnetization, and the rotation symmetry is respected. If $\mu^2 < 0$, then another solution appears, which is

$$|\mathbf{M}_0|^2 = -\mu^2/\lambda \ . \tag{66}$$

The direction chosen by the vector $\mathbf{M}_0$ is a breaking of the rotation symmetry. The critical temperature T_{crit} is where $\mu^2(T)$ changes sign:

$$\mu^2(T_{\text{crit}}) = 0 \ . \tag{67}$$

It is simple to realize that the Goldstone theorem holds. It states that when spontaneous symmetry breaking takes place, there is always a zero-mass mode in the spectrum. In a classical context this can be proven as follows. Consider a Lagrangian

$$\mathcal{L} = |\partial_\mu\phi|^2 - V(\phi) \tag{68}$$

symmetric under the infinitesimal transformations

$$\phi \to \phi' = \phi + \delta\phi, \quad \delta\phi_i = i\delta\theta t_{ij}\phi_j \ . \tag{69}$$

The minimum condition on V that identifies the equilibrium position (or the ground state in quantum language) is

$$(\partial V/\partial\phi_i)(\phi_i = \phi_i^0) = 0 \ . \tag{70}$$

The symmetry of V implies that

$$\delta V = (\partial V/\partial\phi_i)\delta\phi_i = i\delta\theta(\partial V/\partial\phi_i)t_{ij}\phi_j = 0 \ . \tag{71}$$

By taking a second derivative at the minimum $\phi_i = \phi_i^0$ of the previous equation, we obtain

$$\partial^2 V/\partial\phi_k\partial\phi_i(\phi_i = \phi_i^0)t_{ij}\phi_i^0 + \frac{\partial V}{\partial\phi_i}(\phi_i = \phi_i^0)t_{ik} = 0 \ . \tag{72}$$

The second term vanishes owing to the minimum condition, Eq. (70). We then find

$$\partial^2 V/\partial\phi_k\partial\phi_i\ (\phi_i = \phi_i^0)t_{ij}\phi_j^0 = 0 \ . \tag{73}$$

The second derivatives $M_{ki}^2 = (\partial^2 V/\partial\phi_k\partial\phi_i)(\phi_i = \phi_i^0)$ define the squared mass matrix. Thus the above equation in matrix notation can be read as

$$M^2 t\phi^0 = 0 \ , \tag{74}$$

which shows that if the vector $(t\phi^0)$ is non-vanishing, i.e. there is some generator that shifts the ground state into some other state with the same energy, then $t\phi^0$ is an eigenstate of the squared mass matrix with zero eigenvalue. Therefore, a massless mode is associated with each broken generator.

When spontaneous symmetry breaking takes place in a gauge theory, the massless Goldstone mode exists, but it is unphysical and disappears from the spectrum. It becomes, in fact, the third helicity state of a gauge boson that takes mass. This is the Higgs mechanism. Consider, for example, the simplest Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}\,F_{\mu\nu}^2 + |(\partial_\mu - ieA_\mu)\phi|^2 + \frac{1}{2}\mu^2\phi^*\phi - (\lambda/4)(\phi^*\phi)^2 \ . \tag{75}$$

Note the 'wrong' sign in front of the mass term for the scalar field ϕ, which is necessary for the spontaneous symmetry breaking to take place. The above Lagrangian is invariant under the $U(1)$ gauge symmetry

$$A_\mu \to A'_\mu = A_\mu - (1/e)\partial_\mu\theta(x), \quad \phi \to \phi' = \phi\,\exp[i\theta(x)] \ . \tag{76}$$

Let $\phi^0 = v \neq 0$, with v real, be the ground state that minimizes the potential and induces the spontaneous symmetry breaking. Making use of gauge invariance, we can make the change of variables

$$\phi(x) \to (1/\sqrt{2})[\rho(x) + v] \exp[i\zeta(x)/v] ,$$
$$A_\mu(x) \to A_\mu - (1/ev)\partial_\mu \zeta(x). \tag{77}$$

Then $\rho = 0$ is the position of the minimum, and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\rho^2A_\mu^2 + e^2\rho v A_\mu^2 + \mathcal{L}(\rho) . \tag{78}$$

The field $\zeta(x)$, which corresponds to the would-be Goldstone boson, disappears, whilst the mass term $\frac{1}{2}e^2v^2A_\mu^2$ for A_μ is now present; ρ is the massive Higgs particle.

The Higgs mechanism is realized in well-known physical situations. For a superconductor in the Landau–Ginzburg approximation the free energy can be written as

$$F = F_0 + \frac{1}{2}\mathbf{B}^2 + |(\nabla - 2ie\mathbf{A})\phi|^2/4m - \alpha|\phi|^2 + \beta|\phi|^4 . \tag{79}$$

Here $\mathbf{B}$ is the magnetic field, $|\phi|^2$ is the Cooper pair (e^-e^-) density, $2e$ and $2m$ are the charge and mass of the Cooper pair. The 'wrong' sign of α leads to $\phi \neq 0$ at the minimum. This is precisely the non-relativistic analogue of the Higgs model of the previous example. The Higgs mechanism implies the absence of propagation of massless phonons (states with dispersion relation $\omega = kv$ with constant v). Also the mass term for $\mathbf{A}$ is manifested by the exponential decrease of $\mathbf{B}$ inside the superconductor (Meissner effect).

Thus the Higgs effect is not an abstract device, but it is endowed with a precise physical reality. In the electroweak theory it is absolutely necessary in order to give masses to the W's and Z^0 and to the fermions, as well as to ensure the correct high-energy behaviour required by renormalizability. However a more profound physical reality could be hidden behind or accompany the Higgs formalism. In fact, the Higgs mechanism is at present without experimental support. Actually the clarification of the physical origin of the electroweak symmetry breaking is one of the most important problems for experimental particle physics in the next decade. There are arguments indicating that the minimal Standard Model with fundamental Higgs fields cannot be the whole story and that some kind of new physics must necessarily appear near the Fermi scale. The most famous argument of this type is based on the so-called 'hierarchy problem', which we now describe. There is no unification of the fundamental forces in the Standard Model, because a separate gauge group and coupling is introduced for each interaction. On the other hand, the structural unity implied by the common, restrictive property of gauge invariance strongly suggests the possibility that all the observed interactions actually stem from a unified theory at some more fundamental level. The idea of unification at energies of order $m_{\rm GUT} \approx 10^{15}$ GeV, below the energy scale where quantum gravity becomes effective at masses of the order of the Planck mass, $m_P \approx 10^{19}$ GeV, has been much studied in recent years. However, the question remains whether unification without the inclusion of quantum gravity is

really plausible. It is clear that the inclusion of gravity must induce major changes in the physics of the Standard Model at energies of order m_P and possibly even below.

Thus, at least because of the fact that gravity is not included in the Standard Model, new physics must necessarily emerge at some large energy scale Λ (equivalent to some small distance scale). Then the problem is to understand what order of magnitude can reasonably be expected for Λ. In particular, we can ask whether it is natural to expect that Λ may be as large as m_P or $m_{\rm GUT}$. In other words, is it possible that the Standard Model holds without any new physical input up to the energy scale of quantum gravity? The answer is probably negative, because then we could not naturally explain the enormous value of m_P/m_W, i.e. the ratio between the Planck and the Fermi scales of masses.

To develop this point further, we recall that in the Standard Model the fermion and vector-boson masses are all specified in terms of the VEV of the Higgs field v, according to Eqs. (48) and (58). The value of v is determined by the curvature scale of the Higgs potential V:

$$V(\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + (\lambda/4)(\phi^\dagger\phi)^2 , \tag{80}$$

according to

$$v = \mu/\sqrt{\lambda} . \tag{81}$$

The observed values of the masses require for v (and, therefore, roughly for μ as well) that $v \approx 10^2$ GeV [see Eq. (59)].

If $\Lambda \approx (10^{15}$–$10^{19})$ GeV, then we face the problem of justifying the presence of two so largely different mass scales in a single theory (the so-called hierarchy problem). In general, if Λ is very large in comparison with μ, then, even if we set by hand a small value for μ at the tree level, the radiative corrections would make μ increase up to nearly the order of Λ. This problem is particularly acute in theories with scalars, as in the Standard Model, because the degree of divergence of mass corrections is quadratic, whilst the same divergences are only logarithmic for spin-1/2 fermions.

One general way out would be that the limit $\mu \to 0$ corresponds to an increase of the symmetry of the theory. In fact, the observed value of μ^2 can be decomposed as

$$\mu^2 = \mu_0^2 + \delta\mu^2 , \tag{82}$$

where μ_0 is the tree-level value and $\delta\mu^2$ arises from the loop quantum corrections. If no new symmetry is induced when $\mu \to 0$ and no other non-renormalization theorem is operative, then a small value for μ^2/Λ^2 can only arise from an unbelievably precise cancellation between μ_0^2/Λ^2 and $\delta\mu^2/\Lambda^2$. If, however, $\mu = 0$ leads to an additional symmetry, then $\delta\mu^2$ must be proportional to μ_0^2, because for $\mu_0 = 0$ both the tree diagrams and the loop corrections must respect the symmetry. Then, if one starts from a small value of μ_0^2/Λ^2, the radiative corrections would preserve the smallness of μ^2/Λ^2.

For fermions, chiral symmetry is added when $m_f \to 0$, because the axial currents are also conserved in this limit, as their divergence is proportional to the fermion mass. Chiral symmetry and the logarithmic degree of divergence for fermion masses considerably alleviate the hierarchy problem in theories with no fundamental scalars.

In the Standard Model no additional symmetry is gained for $\mu_0 = 0$. This is also seen from the explicit formula for $\delta\mu^2$ at the one-loop level, which shows that $\delta\mu^2$ is not proportional to μ_0^2:

$$\mu^2/\Lambda^2 = (\mu_0^2/\Lambda^2) + (1/128\pi^2)(d/dv)^2 \times \times \sum_J (2J+1)(-1)^{2J} m_J^2(v) + \dots , \quad (83)$$

where terms which vanish with $\Lambda \to \infty$ are indicated by the dots. The sum over J includes both particles and antiparticles (counted separately) of spin J and mass m_J (expressed as a function of v). Thus we are forced to the conclusion that in the Standard Model the natural value for $\delta\mu^2/\Lambda^2$ is of order one or so. Therefore, as Λ can be interpreted as the energy scale where some essentially new physical ingredient becomes important, we are led to expect that the validity of the present framework cannot be extended beyond $\Lambda \approx$ (1–10) TeV. Note that here the discussion is on the relation between bare and renormalized masses, where the cut-off dependence is hidden. In the renormalization procedure a physical value is simply assigned to m_H and it is left to the bare mass and the cut-off to adjust to each other. The naturalness problem arises if the divergences are seen as a low-energy effect, to be eventually removed by some new physics at the scale Λ (e.g. by gravity at $M_{\rm Pl}$). Then the large momentum cut-off and the scale of new physics can be physically identified.

The problem of explaining the Fermi scale is seen to be closely connected to the Higgs mechanism and to the consequent presence of scalars, which makes the problem of testing the Higgs sector particularly crucial.

One possible solution is that the Higgses are really scalar fundamental fields, but naturality is restored by supersymmetry.

Supersymmetry (SUSY for short) [15] relates bosons and fermions, so that in a multiplet that forms one representation of supersymmetry there is an equal number of bosonic and fermionic degrees of freedom. This implies that SUSY generators are spin-1/2 charges Q_α. SUSY leads to an extension of the Poincaré algebra. Besides the obvious algebraic relations between Q_α and the Poincaré generators, which specify the spinorial transformations of Q_α under Lorentz transformations and its invariance under translations, the essentially new relation is the anticommutator

$$\{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma_\mu)_{\alpha\beta} P^\mu , \quad (84)$$

where P^μ is the energy–momentum four-vector, which generates space–time translations. If all fundamental symmetries are gauge symmetries, then also SUSY is presumably a local symmetry. This immediately leads to the realm of gravity. In fact, the product of two local SUSY transformations is a translation with space–time-dependent parameters, as follows from Eq. (84). But a translation with space–time-dependent parameters is a general coordinate transformation. As ordinary gravity can be seen to arise from gauging the Poincaré group, a similar gauging of the Poincaré algebra

enlarged by SUSY generators leads to an extended version of gravity, called supergravity. In fact, supersymmetry and supergravity play a crucial role in most present attempts at constructing a sensible theory of quantum gravity, including superstring theories [16] that at present represent the most advanced and promising project for a theory of gravity (and of all particle interactions).

Theorists like SUSY for several reasons. SUSY is the maximum symmetry compatible with a non-trivial S-matrix in a local relativistic field theory. The powerful constraints between couplings and masses implied by SUSY drastically reduce the degree of singularity of field theory as deduced from power counting. In some cases a finite field theory is even obtained. For example, $N = 4$ extended SUSY Yang–Mills theories are finite in four dimensions. In general, more powerful non-renormalization theorems are deduced for SUSY theories. This property may solve many naturalness problems of the Standard Model. In particular, the hierarchy problem can be solved in SUSY theories because the mass divergences of bosons and fermions become the same, and the quadratic divergences of scalars are reduced to logarithmic divergences as for spin-1/2 fermions. Thus in SUSY theories, we automatically obtain

$$(d/dv)^2 \sum_J [(2J+1)(-1)^{2J} m_J^2(v)] = 0 , \quad (85)$$

because of a cancellation between bosons and fermions. Of course the cancellation is only exact in the limit of unbroken SUSY. But we know that SUSY must be broken because the SUSY partners of ordinary particles have not been observed. In broken SUSY, the Λ appearing in Eq. (83) can be identified with the mass scale that is typical of SUSY partners of ordinary states. Thus if Λ is of order $G_F^{-1/2}$ or at most $O(1\ \text{TeV})$, then the hierarchy problem would be solved in a natural way. This is the only known way out of the hierarchy problem compatible with fundamental scalar Higgs fields.

We have seen that most theorists working on quantum gravity and superstrings tend to consider SUSY as 'established' at m_P and beyond. For economy, we are naturally led to also try to use SUSY at low energy to solve some of the problems of the Standard Model, including the hierarchy problem. It is therefore very important that it was indeed shown [17] that models where SUSY is softly broken by gravity do offer a viable alternative. We stress again that the supersymmetric way is very appealing to theorists. In fact, it would represent the ultimate triumph of a continuous line of progress obtained by constructing field theories with an increasing degree of exact and/or broken symmetry and applying them to fundamental interactions. Also the value of the ratio of knowledge versus ignorance would be remarkably large in the case of SUSY: the correct degrees of freedom for a description of physics up to gravity would have been identified, the Hamiltonian would be essentially known, and the theory would, to a large extent, be computable up to m_P. In the limit of exact supersymmetry and exact gauge $SU(2)\otimes U(1)$, all particles are massless. When supersymmetry is broken while $SU(2) \otimes U(1)$ is still preserved, ordinary particles remain massless while sparticles become massive. It is important to note that observed particles are precisely those whose mass terms are forbidden in the $SU(2) \otimes U(1)$ limit, while sparticle masses are allowed. For example, quark and lepton masses are forbidden while squark or slepton masses are allowed, the gauge boson masses are forbidden but the gaugino masses are allowed. Thus the fact that all ordinary particles were observed but no sparticles is not unnatural. When finally the $SU(2) \otimes U(1)$ symmetry breaking is switched on, the mass μ^2 in the Higgs potential naturally takes a value of the order of the scale of sparticle masses and all ordinary particles acquire a mass.

The alternative main avenue to physics beyond the Standard Model is compositeness or, more generally, the existence of new strong forces. For example, the electroweak symmetry could be broken by condensates of new fermions attracted by a new force with $\Lambda_{\rm new} \approx m_F \approx G_F^{-1/2}$($\Lambda_{\rm new}$ being the analogous quantity of $\Lambda_{\rm QCD}$), as in technicolour theories [18]. Recently it has been proposed [19] that a very heavy top mass ($m_t \geq 230$ GeV) could induce the electroweak symmetry breaking. The Higgs would be a sort of $t\bar{t}$ bound state with mass $m_H \simeq O(m_t)$. This model (of the Nambu–Jona–Lasinio type) is non-renormalizable and involves many *ad hod* four-fermion interactions to fix the fermion masses. Arguments were given indicating that, if m_H is $O(m_t)$, then this view of the Higgs is just a matter of language without great physical significance [20] when compared with the standard picture. Or the Higgs scalar can be a composite of new fermions bound by a new force [21]. In the last two cases there are unsolved problems related to the fermion masses. Or the $SU(2)\otimes U(1)$ gauge symmetry can be a low-energy fake [22]. At high energies $\gtrsim m_F$, the W and Z^0 would be resolved into their constituents. Another possibility is that the Higgs mass becomes large [23] [$O(1$ TeV)]. Then, as we shall see, the weak interactions become strong and a spectroscopy of tightly bound weak- interaction resonances appears (e.g. WW, WZ, or ZZ states).

However, it is fair to say that the compositeness alternative is not at all so neat and clearly formulated as the supersymmetric option. On the contrary, in many respects the compositeness way is not well defined at all and leads to many unsolved problems.

Of course, the two avenues are not necessarily mutually exclusive, and theoretical frameworks where both appear have been considered [24].

Searching for the standard Higgs particle appears to be a good way to organize the experimental solution of the symmetry-breaking problem.

The search for the Higgs is being pursued at LEP 1 and will continue at LEP 200. Indeed all previous limits on the Higgs mass m_H have been dwarfed by only a few months of LEP operation. For the standard Higgs we have at present the following combined limits from the four LEP experiments [25]:

$$m_H \gtrsim 62.5 \text{ GeV}. \tag{86}$$

This limit is obtained from negative searches of the process $e^+e^- \rightarrow HZ^* (\rightarrow f\bar{f})$.

As is well known, the value of the Higgs mass is not predictable even in the minimal Standard Model with a single Higgs doublet. What is certainly true is that the Higgs boson cannot be too heavy or the perturbative theory becomes sick and breaks down [26]. If $m_H \geq O(1$ TeV) the perturbative rates for $VV \rightarrow VV$ scattering ($V \equiv W, Z$) violate the unitarity limit [23] for $\sqrt{s} \gg m_W$. More important than this, in non-

asymptotically free gauge theories there are Landau poles where the coupling constant blows up according to the renormalization group improved perturbation theory (unless the renormalized coupling is not vanishing so that the theory is a free theory, i.e. trivial). This phenomenon is also present in QED, but it would only occur beyond the Planck scale of mass, so that the problem can be solved at such large energies by embedding the theory in a larger context (e.g. grand unification). The coupling of the quartic term $\lambda(\phi^+\phi)^2$ in the Higgs potential increases with m_H^2 ($m_H^2 \sim \lambda/G_F$); see Eqs. (59) and (81). In addition, for a given m_H, λ increases logarithmically with energy because the theory is not asymptotically free in the Higgs sector. Thus the position of the Landau pole depends on m_H. Imposing that the Landau pole be far enough for the theory to make sense up to a scale Λ, gives a bound [27] on the standard Higgs mass which is plotted in Fig. 2, taken from Ref. [28]. We see that for a light Higgs, i.e. $m_H \leq$ 180–200 GeV, the perturbative regime is valid up to $M_{\rm GUT}$ or M_P. For a heavier Higgs the value of Λ decreases until eventually $\Lambda \sim m_H$: for $m_H \sim 1$ TeV, the theory is valid up to $\Lambda \sim 1$ TeV.

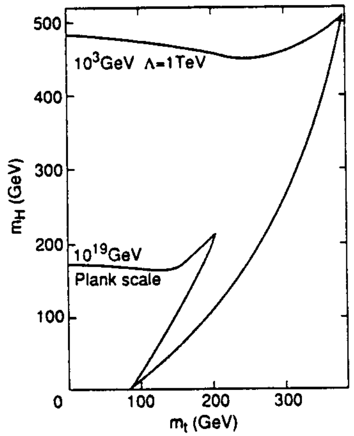

Fig. 2. Combined limits (from Ref. [28] on m_H and m_t from vacuum stability and avoiding the Landau pole up to a scale Λ

We can understand these results by the following crude simplification [26]. The renormalization group equation for the quartic coupling λ, in the limit of neglecting gauge and Yukawa couplings, becomes:

$$\frac{d\lambda(t)}{dt} = \frac{3}{4\pi^2}\,\lambda^2(t)\,, \tag{87}$$

with $t = \ln \Lambda/v$, where v is the Higgs vacuum expectation value and Λ is the scale where λ is evaluated. The coefficient $\beta_0 = 3/4\pi^2$ is obtained from one-loop corrections to the quartic coupling in the $\lambda(\phi^+\phi)^2$ theory. The normalization of v and λ, in physical terms, is here chosen such that

$$\lambda \equiv \lambda(v) = \sqrt{2}\, G_F\, m_H^2 \tag{88}$$

$$v = (2\sqrt{2}\, G_F)^{-1/2} \simeq 174 \text{ GeV} . \tag{89}$$

When $\lambda(t)$ is large the gauge and Yukawa couplings can be neglected with the exception of the top Yukawa coupling, which can become large if $m_t \geq v : g_{\text{top}} = m_t/v$. By solving (87) one obtains

$$\lambda(t) = \frac{\lambda}{1 - 3/4\pi^2 \lambda t} . \tag{90}$$

The minus sign in the denominator, typical of non-asymptotically free theories, implies the increase of $\lambda(t)$ with the scale Λ up to an infinite value which is obtained for $3/4\pi^2\ \lambda t = 1$. In order to avoid the Landau pole the condition

$$\frac{3}{4\pi^2}\ \lambda t = \frac{3}{4\pi^2}\ \sqrt{2}\, G_F\, m_H^2\ \ln\ \Lambda/v < 1 \tag{91}$$

must be imposed. This condition is equivalent to:

$$m_H \leq \frac{893 \text{ GeV}}{\sqrt{\ln\ \Lambda/174 \text{ GeV}}} \tag{92}$$

or $m_H < 144,\ 165,\ 675$ GeV for $\Lambda = 10^{19}, 10^{15}, 10^3$ GeV respectively.

We see that this simple model reproduces the quantitative features of the bounds on $m_{\rm H}$ in Fig. 2 fairly well. The curves in Fig. 2 are obtained by a more refined renormalization group treatment of the problem, with inclusion of gauge and top effects. The obvious criticism to the above approach is that a perturbative evaluation of the β function is not justified in the vicinity of the Landau pole. Thus it is very interesting that the validity of the bound has been confirmed by recent computer simulations of the electroweak theory on the lattice [29]. The precise value of the upper limit on m_H depends on the exact definition of Λ and on where one fixes the line between acceptable and not acceptable. In fact the lattice results nicely extrapolate the perturbative evaluation and impose limits on m_H such that:

$$m_H \leq (8\text{–}10) m_W \simeq 0.6\text{–}0.8 \text{ TeV} . \tag{93}$$

It is thus fair to conclude that, to the best of our knowledge, the internal consistency of the Standard Model demands that the Higgs mass is below 1 TeV.

In Fig. 2 there is also a forbidden region at large m_t and small m_H. This boundary is determined by the requirement of vacuum stability [26],[30].

At tree level the scalar potential can be written in the form:

$$V(\varphi) = -\mu^2|\varphi|^2 + \frac{\mu^2}{2v^2}|\varphi|^4 . \tag{94}$$

The quantum corrections can be computed by expanding in the number of loops. At one loop one obtains:

$$V(\varphi) = -\mu^2|\varphi|^2 + \frac{\mu^2}{2v^2}|\varphi|^4 + \gamma|\varphi|^4 \left(\ln \frac{|\varphi|^2}{v^2} - \frac{1}{2}\right) \tag{95}$$

with

$$\gamma = \frac{3 \sum\limits_{\text{vectors}} m_v^4 + \sum\limits_{\text{scalars}} m_s^4 - 4 \sum\limits_{\text{fermions}} m_f^4}{64\pi^2 v^4} \tag{96}$$

It is simple to check that also in the corrected form v is an extremum of $V(\phi)$. In the minimal Standard Model with one Higgs doublet and three fermion families one obtains

$$\gamma = \frac{3m_Z^4 + 6m_W^4 + m_H^4 - 12m_t^4}{64\pi^2 v^4} . \tag{97}$$

The extra factor of three in front of m_t^4 is of course due to colour.

For the realization of spontaneous symmetry breaking and stability of the theory one requires that i) the extremum at $\varphi = v$ is a minimum, i.e. $V(v) < 0$, and ii) $V(\varphi) \to +\infty$ for $|\varphi| \to \infty$, so that the Hamiltonian is bound from below.

At small m_t, $m_t < 80$ GeV, the first requirement leads to the Linde–Weinberg limit $m_H^2 > 2\gamma v^2$, or $m_H \geq 7$ GeV. This limit is by now void, because of the experimental lower bounds on both m_t and m_H. For the second requirement to be fulfilled, $m_{\rm H}$ must increase with m_t in order to prevent γ from becoming too negative [30]. At large $|\varphi|$ the one-loop evaluation of $V(\varphi)$ is not sufficient, and one needs a resummation of the large logarithms $\log \varphi^2/v^2$. The results are shown in Fig. 2 [28]. The above limits are only valid in the minimal Standard Model with one Higgs doublet. Note that in case that there are two or more Higgs doublets the limits refer to some average mass. Thus for the lightest Higgs the lower limit can be easily evaded but the upper limit is *a fortiori* valid. The bottom line is that either the Higgs is found below $\simeq 1$ TeV or new physics beyond the Standard Model should appear. At least one should see the onset of a new non-perturbative regime where the weak interactions become strong.

In conclusion there are solid arguments for new physics near the Fermi scale of mass $G_F^{-1/2}$. Either a fundamental scalar Higgs exists and naturalness is restored by supersymmetry, or new strong forces will manifest themselves, drastically changing the framework of the Standard Model beyond O(1 TeV). A new non-perturbative regime will set up, with new resonances, and the physics will become less predictable above that energy. An important point is that all conceivable possibilities are very complex. Each of them implies a rich new spectrum of states and phenomena: the whole spectrum of superpartners in SUSY; new hadrons, excited vector bosons, etc., in the composite alternative. The new physics is in all cases distributed over a large interval of energies. The low-lying fringes of the new spectroscopy, or at least their virtual effects, should already be accessible to LEP 1 and LEP 200. A lot of discoveries are expected at the LHC, to be followed by more at the SSC.

5 BASIC RELATIONS FOR PRECISION TESTS

In the standard electroweak theory, there are a number of basic relations that one wants to verify experimentally with the best possible precision. The same quantity $\sin^2\theta_W$ appears in all these relations.

First, $\sin^2\theta_W$ can be measured from the value of m_W. Starting from the tree-level relations [Eqs. (33) and (37), (38)], $\sin^2\theta_W = e^2/g^2$ and $G_F/\sqrt{2} = g^2/8m_W^2$, we obtain

$$\begin{aligned}\sin^2\theta_W &= (\pi\alpha/\sqrt{2}\,G_F)/[m_W^2(1-\Delta r)] \\ &= (37.2802\text{ GeV})^2/[m_W^2(1-\Delta r)]\,, \end{aligned} \tag{98}$$

where $\Delta r \neq 0$ owing to the effect of radiative corrections.

Then $\sin^2\theta_W$ is also related to the ratio of the vector boson masses. At tree level [see Eq. (42)],

$$\sin^2\theta_W = 1 - m_W^2/\rho_0 m_Z^2\,, \tag{99}$$

where $\rho_0 = 1$ in the Standard Model with only doublets of Higgs bosons. In general, this relation is also modified by radiative corrections:

$$\sin^2\theta_W = 1 - m_W^2/\rho_{\text{mass}} m_Z^2\,, \tag{100}$$

with $\rho_{\text{mass}} = \rho_0(1+\delta\rho_{\text{mass}})$. However, as we shall discuss in detail later, Eq. (99) with $\rho_0 = 1$ is often adopted as a definition of $\sin^2\theta_W$ at all orders. Clearly in this case, $\rho_{\text{mass}} = 1$ by definition.

Finally, $\sin^2\theta_W$ can be obtained for neutral-current couplings. At tree level, the four-fermion interaction from Z exchange is given [see Eqs. (41) and (42)] by

$$\begin{aligned}M_{if} &= [\sqrt{2}\,G_F m_Z^2/D(s)] \\ &\quad \rho_0(J_3^i - 2\sin^2\theta_W J_{em}^i)(J_3^f - 2\sin^2\theta_W J_{em}^f)\,, \end{aligned} \tag{101}$$

where $D(s)$ is the Z propagator, and J_3^f and J_{em}^f are the weak isospin and electromagnetic currents for the fermion f. Excluding pure QED corrections, electroweak radiative corrections modify M_{if} according to

$$\begin{aligned}M_{if} &= [\sqrt{2}\,G_F m_Z^2/D(s)] \\ &\quad \rho_{if}(J_3^i - 2k_i\,\sin^2\theta_W J_{em}^i)(J_3^f - 2k_f\,\sin^2\theta_W J_{em}^f) + \dots\,, \end{aligned} \tag{102}$$

where $\rho_{if} = \rho_0(1+\delta\rho_{if})$; $k_a = 1 + \delta k_a (a = i, f)$ are in general different for different fermions and depend on the scheme adopted (for example, δk_a depend on the definition of $\sin^2\theta_W$). The ellipsis indicates possible additional small non-factorizable terms.

6 INPUT PARAMETERS

For LEP physics [31], a self-imposing set of input parameters is given by $\alpha_s, \alpha, G_F, m_Z$, m_f and m_H. Clearly the Fermi coupling $G_F = 1.16637(22) \times 10^{-5}\text{GeV}^{-2}$ is conceptually more complicated than $\alpha_{weak} = \frac{g^2}{4\pi}$ (which would more naturally accompany $\alpha = 1/137.036$ and α_s) or $\sin^2\theta_W$ or m_W, but is preferred for practical reasons because it is known with all the desirable accuracy. Similarly, m_Z has now been measured at LEP with remarkable precision. This preliminary task of LEP in view of precision tests of the Standard Model has been accomplished to a nearly final degree of accuracy (see Section 10 for details on the line-shape measurement).

The LEP results on m_Z, as summarized at the La Thuile and Moriond '93 conferences [32],[33] are reported in Table 1 [1]. The resulting relative precision is impressive: $\delta m_Z/m_Z = 8 \times 10^{-5}$.

Table 1: Results on m_Z from LEP

Experiment	m_Z GeV
ALEPH	91.187 ± 0.007
DELPHI	91.186 ± 0.007
L3	91.196 ± 0.007
OPAL	91.181 ± 0.007
Average	91.187 ± 0.0033(stat.) ± 0.0063 (LEP) ≃ 91.187 ± 0.007

Among the quark and lepton masses, m_f, the main unknown is the top quark mass. Our ignorance of m_t is at present a serious limitation for precise tests of the electroweak theory because the radiative corrections are relatively large for large m_t and depend quadratically on m_t [31]. This fact can be used to put stringent constraints on m_t from the existing electroweak measurements, in particular an upper bound on m_t, to be discussed in detail later. As for lower bounds on m_t the best results arise from the failure to observe the t quark at e^+e^- and hadron colliders. LEP and SLC lead to a model-independent bound $m_t \gtrsim 45$ GeV. From CDF [34] one learns that $m_t \gtrsim 108$ GeV, provided that the t quark production rate and semi-leptonic branching ratio are as predicted by the Standard Model.

The mass of light quarks is in general an ambiguous concept. Fortunately, in the present context what matters is the contribution of light quarks (including c and b) to the vacuum polarization loop. The relevant polarization function can be related via a dispersion relation to the $e^+e^- \to$ hadrons cross-section and computed from the actual data. The value described in the next section is obtained as a result. The error is a non-negligible source of ignorance and could only be improved by more precise data, especially in the region between the J/ψ and the Υ.

[1] When writing this talk I decided to update the experimental data by also including results that were not yet available when it was given.

The Higgs mass m_H is largely unknown. One of the most impressive performances of LEP up to now has been the dwarfing of all previous lower bounds on m_H. The LEP bound is summarized in Eq. (86). The basis for the theoretical upper bound on m_H was reviewed in Section 4. As a result the calculation of radiative corrections is now done for 60 GeV $< m_H <$ 1 TeV. The sensitivity of the radiative corrections to variations of m_H in the range 60 GeV $< m_H <$ 1 TeV is not large. In a sense, this level of accuracy fixes the goal for precision tests of the Standard Model because the clarification of the symmetry-breaking sector of the theory is the main target of present-day experiments.

Finally, for electroweak calculations involving hadrons, the value of the QCD coupling α_s must also be specified. The best value of α_s at the Z mass, obtained from all existing experiments is given by [35] $\alpha_s(m_Z) = 0.118 \pm 0.007$. (Note that I am conservative on errors, which are dominated by theoretical uncertainties.) The QCD corrections to processes involving quarks are typically of order α_s/π. As a consequence the stated error on α_s leads to a few per mille relative uncertainty on the corresponding predictions.

7 LARGE CONTRIBUTIONS TO RADIATIVE CORRECTIONS

A set of important quantitative contributions to the radiative corrections arise from large logarithms [e.g. terms of the form $(\alpha/\pi \ln(m_Z/m_{f_\ell}))^n$ where f_ℓ is a light fermion], and from quadratic terms in m_t, i.e. terms proportional to $G_F m_t^2$.

The sequences of leading and close-to-leading logarithms are fixed [36] by well-known and consolidated techniques (β functions, anomalous dimensions, penguin-like diagrams, etc.). For example, large logarithms dominate the running of α from m_e, the electron mass, up to m_Z, with the result that [37]

$$\alpha(m_Z)/\alpha = 1/(1-\delta\alpha)\ . \tag{103}$$

At present, the best value of $\delta\alpha$, obtained by extracting the relevant effective light-quark masses from the data on $e^+e^- \to$ hadrons, is given by [37]

$$\delta\alpha = 0.0596 \pm 0.0009 \text{ or } \alpha(m_Z)^{-1} = 128.87 \pm 0.12. \tag{104}$$

Large logarithms of the form $[\alpha/\pi \ln(m_Z/\mu)]^n$ also enter, for example, in the relation between $\sin^2\theta_W$ at the scales m_Z (LEP, SLC) and μ (e.g. the scale of low-energy neutral-current experiments).

The quadratic dependence on m_t [38] (and on other possible widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD, the running of α and α_s at a scale Q is not affected by heavy quarks with mass $M \gg Q$. According to an intuitive decoupling theorem [39], diagrams with heavy virtual particles of mass M can be ignored at $Q \ll M$ provided that the couplings do not grow with M and that

the theory with no heavy particles is still renormalizable. In spontaneously broken gauge theories, one important difference is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). The upper limit on m_t from radiative corrections arises from this phenomenon. Other subtler sources of non-decoupling are related, for example, to the presence of chiral anomalies [14] (which may not be completely cancelled if heavy particles are removed). Another very important consequence is that precision tests of the electroweak theory may be sensitive to new physics even if the new particles are too heavy for their direct production. With the value of m_t being continuously pushed up by experiment, the quantitative importance of the terms of order $G_F m_t^2$ is increasingly large. Both the large logarithms and the $G_F m_t^2$ terms have a simple structure and are to a large extent universal, i.e. common to a wide class of processes. Their study is important for an understanding of the pattern of radiative corrections. One can also derive approximate formulae (e.g. improved Born approximations, see Section 9), which can be useful in cases where a limited precision may be adequate.

8 DETERMINATION OF $\sin^2\theta_W$ FROM m_Z

Once $\alpha, \alpha_s, G_F, m_Z, m_f$ and m_H have been chosen as input parameters, $\sin^2\theta_W$ is a derived quantity. We now consider the calculation of $\sin^2\theta_W$ beyond the tree approximation. A precise definition of $\sin^2\theta_W$ must be specified before its value can be computed.

At tree level the relation between $\sin^2\theta_W$ and m_Z is obtained from Eqs.(98) and (99) (with $\Delta r = 0$):

$$\sin^2\theta_W\ \cos^2\theta_W = (\pi\alpha/\sqrt{2}\,G_F)/(\rho_0 m_Z^2)\ . \tag{105}$$

Beyond the tree level, the quantity Δr [40] is introduced by radiative corrections:

$$\sin^2\theta_W\ \cos^2\theta_W = (\pi\alpha/\sqrt{2}\,G_F)/[\rho_0 m_Z^2(1-\Delta r)]\ , \tag{106}$$

where $\Delta r \equiv \Delta r(\alpha, \alpha_s, G_F, m_Z, m_f, m_H)$ is of course different for different definitions of $\sin^2\theta_W$. In the following, some particularly interesting definitions of $\sin^2\theta_W$ will be discussed. We now set $\rho_0 = 1$. Let us first consider the usual definition [41]:

$$\sin^2\theta_W \equiv s_W^2 = 1 - (m_W^2/m_Z^2)\ . \tag{107}$$

From now on, the symbol s_W^2 will always refer to this specific definition of $\sin^2\theta_W$. Clearly, given m_Z from LEP, s_W^2 is directly equivalent to m_W. In this case Δr specifies the relation between m_Z and m_W:

$$(1 - m_W^2/m_Z^2)m_W^2 = (\pi\alpha/\sqrt{2}\,G_F)/(1-\Delta r)\ . \tag{108}$$

The value of Δr as a function of the input parameters has been studied in great detail. In particular, the following results are obtained [37], [40], [42]:

$$\begin{aligned} 1/(1-\Delta r) &= \alpha(m_Z)/\alpha \cdot 1/[1 + (c_W^2/s_W^2)\delta\rho] + \text{'small'} \\ &= 1/(1-\delta\alpha) \cdot 1/[1 + (c_W^2/s_W^2)\delta\rho] + \text{'small'}\ , \end{aligned} \tag{109}$$

where $c_W^2 = 1 - s_W^2$; $\delta\alpha$ was defined in Eqs. (103) and (104), and $\delta\rho \to \delta\rho_t$, for large m_t with $\delta\rho_t$ given by [37], [43]

$$\delta\rho_t = (3G_F m_t^2/8\pi^2\sqrt{2}) + (3G_F m_t^2/8\pi^2\sqrt{2})^2(19-2\pi^2)/3 + O[(G_F m_t^2)^3] . \tag{110}$$

By 'small' in Eq. (109), we mean terms (which at one-loop accuracy are known) without large logs and/or leading powers of $G_F m_t^2$; $\delta\rho_t$ is the dominant term, for large m_t, of the famous ρ-parameter first studied in Refs. [38]. The two-loop term was obtained in Ref. [43] and the geometric series resummation was advocated in Ref. [42].

Going back to the Z-exchange amplitude near the resonance and the parameters ρ_{if}, k_i, and k_f defined in Eq. (102) with the definition of $\sin^2\theta_W \equiv s_W^2$, we obtain [37]

$$\rho_{if} = 1 + \delta\rho + \text{'small'} \tag{111}$$

and

$$k_f = 1 + (c_W^2/s_W^2)\delta\rho + \text{'small'} \tag{112}$$

(here $f \neq b$; the b-quark will be reconsidered in the following), where the 'small' terms are non-universal (i.e. process-dependent). Note that k_f contains additional 'large' terms with respect to those included in Δr and ρ_{if}. As these 'large' terms are also universal, this suggests that ks_W^2 could be a better effective $\sin^2\theta_W$ than s_W^2 for physics at the Z pole. Before going into this matter, we will add some comments on $\delta\rho$. For any weak isospin fermion doublet (e.g. new heavy quarks or leptons), the quantity '$3m_t^2$' in $\delta\rho_t$ (at one loop) becomes [37],[40]

$$\text{'}3m_t^2\text{'} = N_c[m_u^2 + m_d^2 - 2m_u^2 m_d^2/(m_u^2 - m_d^2)\ln\, m_u^2/m_d^2] , \tag{113}$$

where N_c is the number of colours. For negligible $m_{d,u}$, '$3m_t^2$' $\to N_c m_{u,d}^2$, whilst for $m_u = m_d + \epsilon$: '$3m_t^2$' $\to N_c \frac{4}{3}\epsilon^2$ to leading order in ϵ (the result vanishes for unsplit doublets). Similarly, many more kinds of broken weak isospin multiplets can contribute to '$3m_t^2$' [44] (squarks and sleptons [45], charged Higgses [46], etc.). The upper limit on 'm_t' $\lesssim$ 200 GeV (see Section 11) together with the direct lower limit on the t-quark mass, $m_t \gtrsim 108$ GeV, leave little space for heavy multiplets, except for nearly degenerate ones.

The contribution of the neutral Higgs mass to $\delta\rho$ is only logarithmic at one loop, (see, e.g., Eq. (158) in Section 12). There are no m_H^2 terms but only logs, because the 'custodial' $SU(2)$ symmetry is not broken in the Higgs sector [23]. Power terms appear only at two loops [47], but their effect is sizeable only for $m_H \gtrsim 1$ TeV.

We now consider a different class of definitions of $\sin^2\theta_W$. Assume that we fix $k_f = 1$ [defined by Eq. (102)] for one given Z vertex (e.g. $k_f = 1$ in $Z \to e^+e^-$ for on-shell Z). Then it follows from the previous discussion that δk is 'small' for all neutral-current processes (for $f \neq b$) near the Z pole. Let us introduce the notation

$$\sin^2\theta_W(\text{from } Z \to e^+e^-) \equiv \bar{s}_W^2 . \tag{114}$$

In this case, Eq. (102) becomes

$$M_{fi} = [\sqrt{2}\, G_F m_Z^2/D(s)] \\ \rho_{if}(J_3^i - 2\bar{s}_W^2 J_{em}^i)(J_3^f - 2\bar{s}_W^2 J_{em}^f) + \text{'small'} , \tag{115}$$

where ρ_{if} is given by Eq. (110) in terms of $\delta\rho$, which is specified in Eqs. (110). In the present case,

$$\bar{s}_W^2\bar{c}_W^2 = (\pi\alpha/\sqrt{2}G_F)/[m_Z^2(1-\Delta\bar{r})] , \tag{116}$$

and we find [37]

$$\bar{s}_W^2 \simeq s_W^2 + c_W^2\delta\rho + \text{'small'} , \tag{117}$$

In other words, apart from 'small' terms, we have

$$\bar{s}_W^2 \simeq 1 - (m_W^2/\rho m_Z^2) \tag{118}$$

with $\rho \simeq 1 + \delta\rho$. It is interesting to note that $\bar{s}_W^2$ is less dependent than s_W^2 on m_t^2. In fact [37],[40]

$$\Delta r \simeq \delta\alpha - (c_W^2/s_W^2)\delta\rho , \tag{119}$$

$$\Delta\bar{r} \simeq \delta\alpha - \delta\rho , \tag{120}$$

so that the amplifying factor c_W^2/s_W^2 in front of $\delta\rho$ is missing in $\Delta\bar{r}$. Note that given m_Z, $\bar{s}_W^2$ is known with an accuracy of about ± 0.002 when m_t varies between 60 and 210 GeV (see Fig. 3). There is a class of definitions of $\sin^2\theta_W$ in the literature that correspond to $\bar{s}_W^2$ if 'small' terms are neglected: $\bar{s}_W^2$ of Hollik [48]; $s^{*2}(m_Z^2)$ of Lynn and Kennedy [49]; $(\sin^2\theta_W)_{\overline{MS}}(m_Z^2)$ [50],[51], etc. Note that because of Eqs. (103) and (120) one can also write:

$$\bar{s}_W^2\bar{c}_W^2 = [\pi\alpha(m_Z)/\sqrt{2}G_F]/\rho m_Z^2 . \tag{121}$$

9 IMPROVED BORN APPROXIMATION

For precision tests of the electroweak theory, the complete one-loop radiative corrections (plus higher-order and possibly exponentiated purely photonic corrections) are mandatory and are indeed available for all processes of practical relevance. However, in many cases a less accurate estimate can be enough. For this purpose, it is useful to know [37] that formulae as simple as those of the Born approximation can be written in a way that takes all 'large' corrections into account. As a result, for $e^+e^- \to f\bar{f}$, with $f \neq e, b$, the amplitude for γ and Z exchange near the resonance can be written in the form

$$M_{f\bar{f}} = Q_e Q_f[4\pi\alpha(m_Z)/s]J_{em}^e J_{em}^f \\ +\sqrt{2}\, G_F\rho m_Z^2/(s - m_Z^2 + is\, \Gamma_Z/m_Z)\bar{J}^e\bar{J}^f , \tag{122}$$

where $\rho = 1 + \delta\rho, \delta\rho$ being given by Eq. (110), and

$$J_{em}^f = \gamma_\mu , \quad \bar{J}^f = \gamma_\mu[I_3^f(1-\gamma_5) - 2Q_f\bar{s}_W^2] , \tag{123}$$

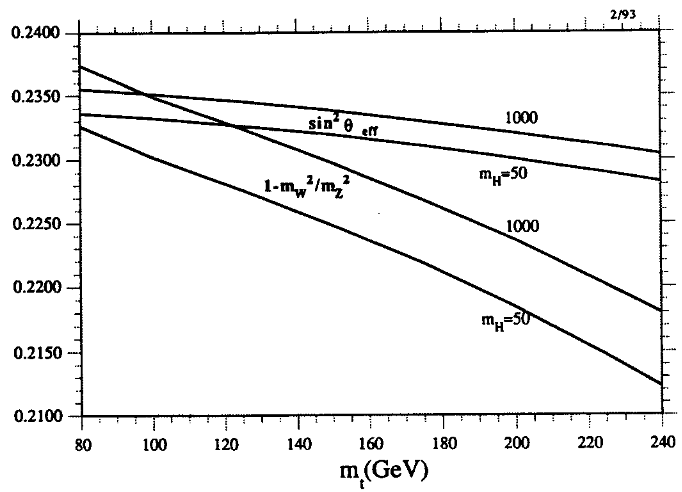

Fig. 3. The behaviour of s_W^2 and $\bar{s}_W^2$ [defined by Eqs. (107) and (114), respectively] as functions of m_t, for $m_Z = 91.187$ GeV, $m_H = 50$–1000 GeV.

with Q_f and I_3^f being respectively the electric charge and the third component of the weak isospin (e.g. for $f = \mu^-, Q = -1, I_3 = -1/2$). The important features are the replacement of α-fixed with α-running, the inclusion of the s-dependence for the total width Γ_Z in the resonant denominator (see Section 10), the presence of the factor of ρ multiplying G_F, and the use in the Z couplings of the effective $\sin^2\theta_W = \bar{s}_W^2$, introduced in the previous section. Clearly for $f = e$, i.e. for Bhabha scattering, the t-channel exchange is also to be included. The improved Born approximations include the real parts of self-energies (sometimes [49] called 'oblique' corrections). All large logs and all $G_F m_t^2$ terms are included. What are left out are 'small' corrections from imaginary parts of self-energies, vertices, and boxes.

For $f = b$ there are additional large terms from the vertex corrections [52] of the type shown in Fig. 4. The longitudinal W modes are, also in this case, responsible for the presence of quadratic mass terms. We can simply modify the improved Born approximation in order to include these terms as well. The recipe [37],[53] is as follows: in Eqs. (122) and (123), replace ρ by

$$\sqrt{\rho\rho_b}, \quad \rho_b \equiv \rho\left(1 - \frac{4}{3}\delta\rho\right) , \tag{124}$$

and $\bar{s}_W^2$ by

$$\bar{s}_W^2\left(1 + \frac{2}{3}\delta\rho\right) \equiv \bar{s}_W^2\ k_b \ . \tag{125}$$

Note that whilst many sorts of heavy particles can contribute to $\delta\rho$, only the t-quark (or a new t') contributes to $\delta\rho_b$, because the W nearly always turns a b-quark into a t (or a t') quark.

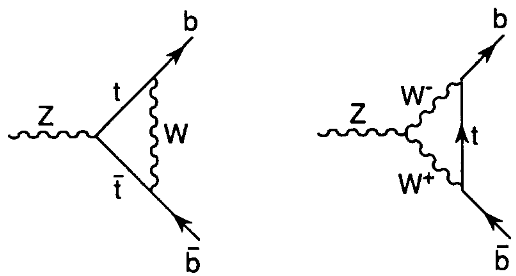

Fig. 4. Vertex corrections that contain large $G_F m_t^2$ terms

As a final example, the improved Born approximation for the inclusive widths $\Gamma[Z \to f\bar{f}(\gamma, g)]$ (including photons and gluons in the final state), is given by

$$\Gamma[Z \to f\bar{f}(\gamma, g)] = N_c(G_F\rho m_Z^3/24\pi\sqrt{2})[1 + (1 - 4|Q_F|\bar{s}_W^2)^2] , \tag{126}$$

where

$$N_c = \begin{cases} 1 \quad [1 + (3\alpha/4\pi)Q_f^2] & \text{(leptons)} \\ 3 \quad [1 + (3\alpha/4\pi)Q_f^2]\ [1 + \alpha_s(m_Z)/\pi + \ldots] & \text{(quarks)} \end{cases} . \tag{127}$$

In the same approximation the total width is given simply by

$$\Gamma_Z = \sum_f \Gamma\ [Z \to f\bar{f}(\gamma, g)] + \text{rare decays} , \tag{128}$$

where the rare decays [54] (e.g. $Z \to H\mu^+\mu^-$ for a relatively light Higgs) are numerically unimportant in the Standard Model. For $\Gamma(Z \to b\bar{b})$, replace ρ by ρ_b and $\bar{s}_W^2$ by $\bar{s}_W^2 k_b$, where ρ_b and k_b are given in Eqs. (124) and (125). Note that in the Standard Model for $f \neq b$, Eq. (126) can also be written in the form [by using Eq. (121)]:

$$\Gamma[Z \to f\bar{f}(\gamma, g)] = N_c[\alpha(m_Z)m_Z/(48\ \bar{s}_W^2\bar{c}_W^2)][1 + (1 - 4|Q_F|\bar{s}_W^2)^2] \ . \tag{129}$$

Actually, the 'large' $G_F m_t^2$ corrections are empirically not so large (i.e. there is a strong upper limit on m_t). As a consequence, as will be discussed in Section 12, an improved Born approximation based on the Standard Model at tree level plus pure QED corrections is well supported by the data.

10 THE Z LINE-SHAPE

A considerable amount of work has deservedly been devoted to the theoretical study of the Z line-shape [55]. The present experimental accuracy on m_Z obtained at LEP is $\delta m_Z = \pm 7$ MeV (recall Table 1, Section 6). This small error was obtained by a precise calibration of the LEP energy scale achieved in '91 by taking advantage of the transverse polarization of the beams and implementing a sophisticated resonant spin depolarization method [56]. Similarly, a measurement of the total width to an accuracy $\delta\Gamma = \pm 7$ MeV has by now been achieved. The prediction of the Z line-shape in the Standard Model to such an accuracy has posed a formidable challenge to theory, which has been successfully met. For the inclusive process $e^+e^- \to f\bar{f}X$, with $f \neq e$ (for simplicity, we leave Bhabha scattering aside) and X including γ's and gluons, the physical cross-section can be written in the form of a convolution [55]:

$$\sigma(s) = \int_{z_0}^{1} dz\, \hat{\sigma}(zs) G(z,s) \,, \tag{130}$$

where $\hat{\sigma}$ is the reduced cross-section, and $G(z,s)$ is the radiator function that describes the effect of initial-state radiation; $\hat{\sigma}$ includes the purely weak corrections, the effect of final-state radiation (of both γ's and gluons), and also non-factorizable terms (initial- and final-state radiation interferences, boxes, etc.) which, being small, can be treated in lowest order and effectively absorbed in a modified $\hat{\sigma}$. The radiator $G(z,s)$ has an expansion of the form [55]

$$\begin{aligned} G(z,s) &= \delta(1-z) + \alpha/\pi(a_{11}L + a_{10}) + (\alpha/\pi)^2(a_{22}L^2 + a_{11}L + a_{20}) \\ &\quad + \ldots + (\alpha/\pi)^n \sum_{i=0}^{n} a_{ni}L^i \,, \end{aligned} \tag{131}$$

where $L = \ln s/m_e^2 \simeq 24.2$ for $\sqrt{s} \simeq m_Z$. All first- and second-order terms are known exactly. The sequence of leading and next-to-leading logs can be exponentiated (closely following [57] the formalism of structure functions in QCD). For $m_Z \approx 91$ GeV, the convolution displaces the peak by +110 MeV, and reduces it by a factor of about 0.74. The exponentiation is important in that it amounts to a shift of about 14 MeV in the peak position.

A model-independent analysis [58] of the reduced cross-section $\hat{\sigma}$ leads to the following general expression (here $m \equiv m_Z$):

$$\begin{aligned} \hat{\sigma}(s) &= [12\pi\Gamma_e\Gamma_f/|D(s)|^2][s/m^2 + R_f(s-m^2)/m^2 + \Gamma/m\, I_f + \ldots] \\ &\quad + [4\pi\alpha^2(m^2)Q_f^2 N_c/3s] \,, \end{aligned} \tag{132}$$

with N_c given by Eq. (127) and

$$D(s) = s - m^2 + im\Gamma[s/m^2 + \epsilon(s-m^2)/m^2] + \ldots \,. \tag{133}$$

This form of the resonant term was obtained by starting from a general renormalizable field theory (the Standard Model being a particular case). Near the resonance,

$(s-m^2)/m^2 \approx \Gamma/m$ is of order α (or α_W). As only a perturbative calculation of $\hat{\sigma}$ is possible in α or α_W, at the same level of accuracy we can expand vertices, propagators, etc., in $(s-m^2)/m^2$ near the resonance. For example, for the inverse propagator, we can write the expansion

$$\begin{aligned} D(s) &= s - m^2 + \Pi(s) \\ &= s - m^2 + \text{Re}\,\Pi(m^2) + (s-m^2)\text{Re}\,\Pi'(m^2) + i\,\text{Im}\,\Pi(m^2) + \\ &\quad i(s-m^2)\text{Im}\,\Pi'(m^2) + \ldots \\ &= [1 + \text{Re}\,\Pi'(m^2)]\{s - m^2 + im\,\Gamma[s/m^2 + \epsilon(s-m^2)/m^2 + \ldots]\} \,, \end{aligned} \tag{134}$$

with

$$m\Gamma = \text{Im}\,\Pi(m^2)/[1 + \text{Re}\,\Pi'(m^2)] \,, \tag{135}$$

and $\text{Re}\,\Pi(m^2) = 0$ because of the specific definition of $m \equiv m_Z$ that is adopted. (Recently a different definition of m_Z was discussed [59]. The present prescription would lead to problems related to gauge invariance at the two-loop level. Within the one-loop framework it is perfectly all right and is generally adopted because of its calculational simplicity.) The overall factor $1+\text{Re}\,\Pi'(m^2)$ is reabsorbed in the numerator. The parameter ϵ measures the deviation from the scaling behaviour of the s-dependent width; ϵ is noticeably different from zero only if, in the final state of Z decays, there are important channels with massive particles (by now we know that this is ruled out). For example, for $Z \to A\bar{A}, \epsilon_A \approx (4m_A^2/m^2)B(Z \to A\bar{A})$. The following identity is valid:

$$\begin{aligned} D(s) &= s - m^2 + im\,\Gamma[s/m^2 + \epsilon(s-m^2)/m^2 + \ldots] \\ &= [1 + i\gamma(1+\epsilon)](s - \bar{m}^2 + i\bar{m}\bar{\Gamma}) \,, \end{aligned} \tag{136}$$

with $\gamma = \Gamma/m$ and

$$\bar{m} = m[1 - (\gamma^2/2)(1+\epsilon) + \ldots] \,, \tag{137}$$

$$\bar{\Gamma} = \Gamma[1 - (\gamma^2/2)(1+3\epsilon) + \ldots] \,. \tag{138}$$

As the factor $1+i\gamma(1+\epsilon) \simeq \exp i\gamma(1+\epsilon)$ cannot be observed from the absolute square of $D(s)$, we see that, on the one hand, at $\epsilon = 0$ the replacement of the constant Γ by the s-dependent width $s\Gamma/m^2$ leads to a variation of the apparent mass by

$$\delta m = -\frac{1}{2}\gamma^2 m \simeq -34 \text{ MeV} \,, \tag{139}$$

which is clearly an important effect [60],[61]. On the other hand, the additional effect from ϵ,

$$\delta m_\epsilon = -\frac{1}{2}\gamma^2\epsilon m \simeq -34\epsilon \text{ MeV} \,, \tag{140}$$

is certainly small because ϵ cannot exceed several per mille or so at most. Note that the effect of ϵ cannot be disentangled from m in the fit, so that it leads to an ambiguity in the determination of m. In conclusion, ϵ can be safely neglected.

Of the two parameters R_f and I_f which appear in Eq. (132) for $\hat{\sigma}$ (in addition to the inclusive partial widths Γ_e and Γ_f), I_f, like ϵ, depends only on the spectrum of particles below the Z. In fact, I_f is determined by absorptive parts of vertices and boxes. Thus large deviations from the Standard Model value cannot occur for I_f. In the Standard Model, I_f is very small [for $\mu^+\mu^-, I_\mu \simeq -(1\text{–}2) \times 10^{-2}$], and as it is multiplied by Γ/m, it can be neglected. The main contribution to R_f is already present at the Born level and arises from γ–Z interference. Higher-order corrections are relatively important, especially for muons. New physics, for example a heavy Z', can modify R_f because the non-resonating background is changed. In the Standard Model, R_f is small [for muons, $R_\mu \simeq (4.5\text{–}6) \times 10^{-2}$, for hadrons $R_h \simeq (0.75\text{–}1) \times 10^{-1}$]. Determining R_f from a fit is difficult. In first approximation, it can be fixed at its Standard Model value. Realistic deviations from the Standard Model would not appreciably affect the determination of m and Γ.

In conclusion, a model-independent analysis of the reduced cross-section proves that a modified Breit–Wigner with s-dependent width, plus photon exchange and interference is a perfectly adequate basis for the experimental study of the line-shape. Whilst in the Minimal Standard Model all the parameters $\Gamma, \Gamma_e, \Gamma_f, R_f$ and I_f can be computed from m_Z, given m_t and m_H, the general expression of $\hat{\sigma}$ will, in principle, allow a model-independent measurement of the Z parameters. Starting from Eq. (132) for $\hat{\sigma}$, approximate analytic solutions of the convolution integral can be found [58], [62]–[64]. The resulting compact analytic expressions are sufficiently precise for most applications.

11 PRECISION TESTS OF THE STANDARD ELECTROWEAK THEORY

It is clear that the set of input parameters specified in Section 6 can be separated into two parts. On the one hand, $\alpha, G_F, m_Z, m_{f_{light}}$ are well known and the ambiguities associated with these quantities on the radiative corrections are quite small ($m_{f_{light}}$ is actually replaced by $\alpha(m_Z)$ computed via the experimental data on $e^+e^- \to$ hadrons, as discussed in Section 7). We can add α_s to this class, in that, if it is true that the experimental error on α_s is relatively large, it only enters as a small correction to electroweak processes involving hadrons and is practically irrelevant for purely leptonic processes. Also, by fitting the electroweak data one cannot obtain a better value for $\alpha_s(m_Z)$ than that derived from QCD tests. On the other hand m_t and m_H are largely unknown. Thus, for each relevant observable, one can only express the prediction of the Standard Model as a function of m_t and m_H, obtained by using the best available calculations of radiative corrections, with $\alpha, \alpha_s, G_F, m_Z$ and $m_{f_{light}}$ fixed at their experimental values with the corresponding errors. By comparing these predictions with experiment, one can check their mutual consistency and derive constraints on m_t and m_H.

Actually the sensitivity on m_H is so small that for all the measured quantities the ambiguity due to varying m_H in the range 60 GeV $< m_H <$ 1 TeV is below the

present experimental error, so that for practical purposes, at the present stage of accuracy, the relevant predictions can be plotted as functions of m_t in the form of a band of values determined by $\delta m_H, \delta m_Z(\delta\alpha_s)$ (see Figs. 5 to 14).

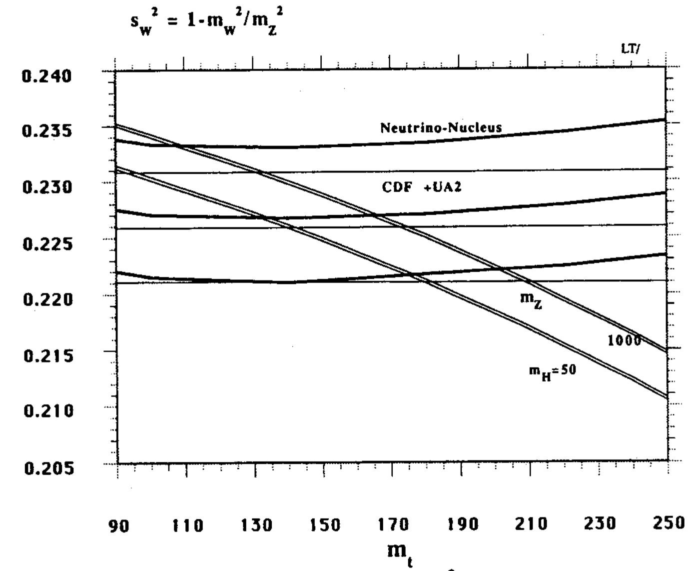

Fig. 5. Labelled by m_Z is the theoretical prediction for s_W^2 obtained from m_Z as a function of m_t for $m_H = 50$–1000 GeV. The error bands implied by the CDF and UA2 measurements of m_W/m_Z and by the data on R_ν are also shown.

Note that from this point of view $\sin^2\theta_W$ is not a primary quantity. It is not part of the set of input parameters. It is a derived quantity that one could even decide not to introduce at all. I stress this point in order to make it clear that all disputes over which is the better definition of $\sin^2\theta_W$ beyond the tree level are completely secondary. First of all it is always true that physical results are independent of definitions. Differences in physical results obtained from a different definition of input parameters (scheme dependence) can at most occur by terms of higher order, due to the truncation of the perturbative series at a given order. But, for $\sin^2\theta_W$, its precise definition is only necessary to compute it from the input parameters, but cannot matter for the prediction of observables because, with the choice specified above, $\sin^2\theta_W$ is not taken as an input parameter of the theory. And in fact, the emphasis on $\sin^2\theta_w$ is fading away with the improved refinement of precision tests of the Standard Model.

The widespread use of expressing the experimental result for each given observable in terms of the corresponding value of $\sin^2\theta_W$ (within a specified definition for it) is no longer adequate at the present level of sophistication. For comparing the constraining power of different experiments it would be more appropriate to quote the range of

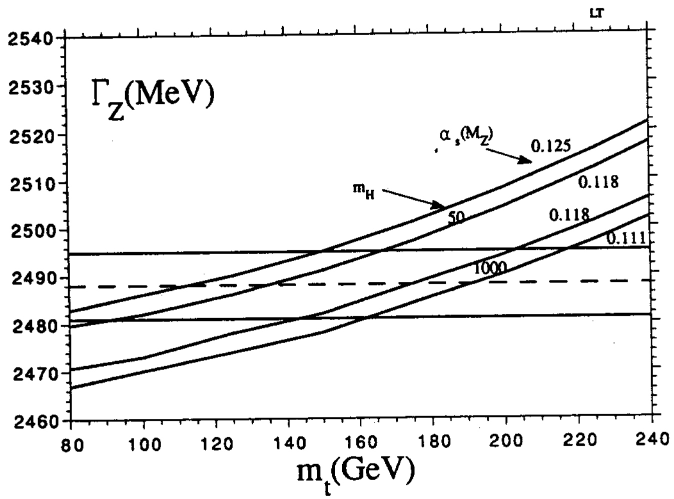

Fig. 6. Γ_Z vs m_t as predicted by the Standard Model for m_H = 50–1000 GeV and $\alpha_s(m_Z)$ = 0.111–0.125 compared with the LEP result

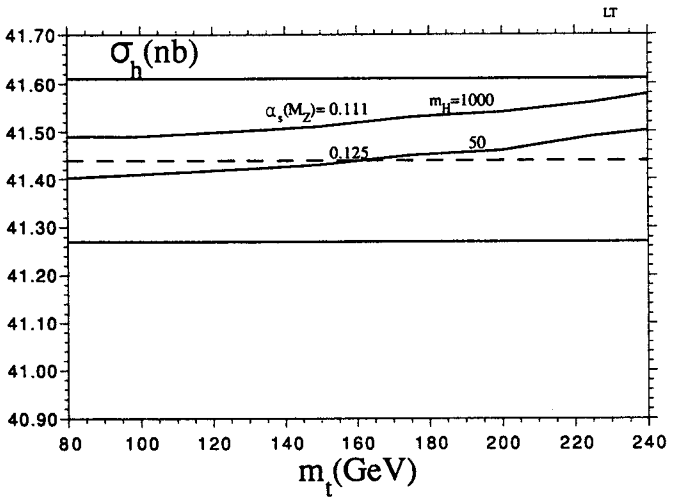

Fig. 7. σ_h vs. m_t as predicted by the Standard Model for m_H = 50–1000 GeV and $\alpha_s(m_Z)$ = 0.111–0.125 compared with the LEP result

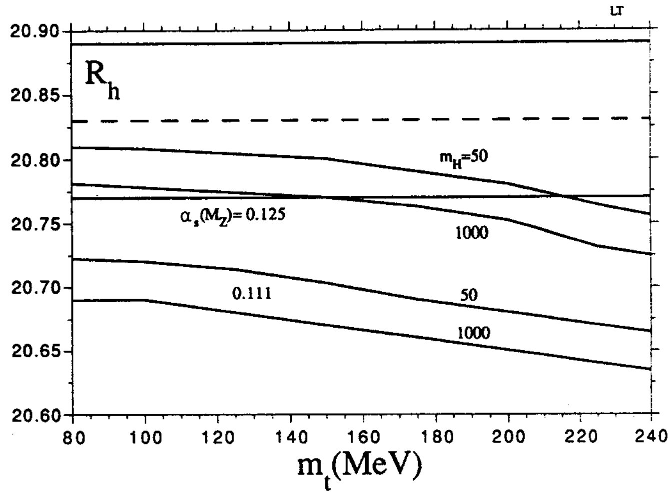

Fig. 8. R_h vs. m_t as predicted by the Standard Model for m_H = 50–1000 GeV and $\alpha_s(m_Z)$ = 0.111–0.125 compared with the LEP result

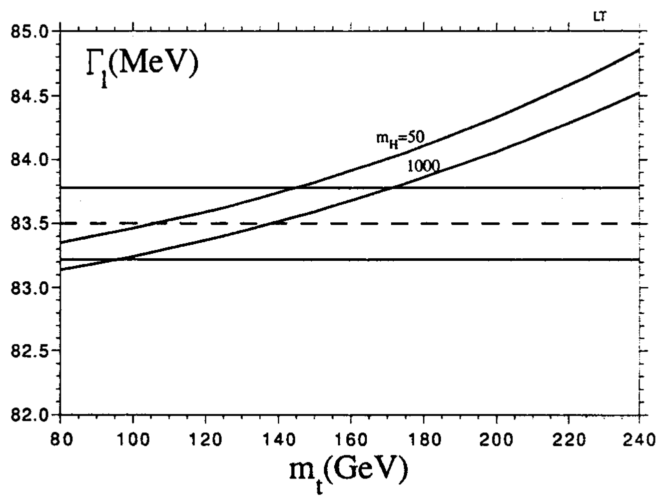

Fig. 9. Γ_ℓ vs. m_t as predicted by the Standard Model for m_H = 50–1000 GeV compared with the LEP result

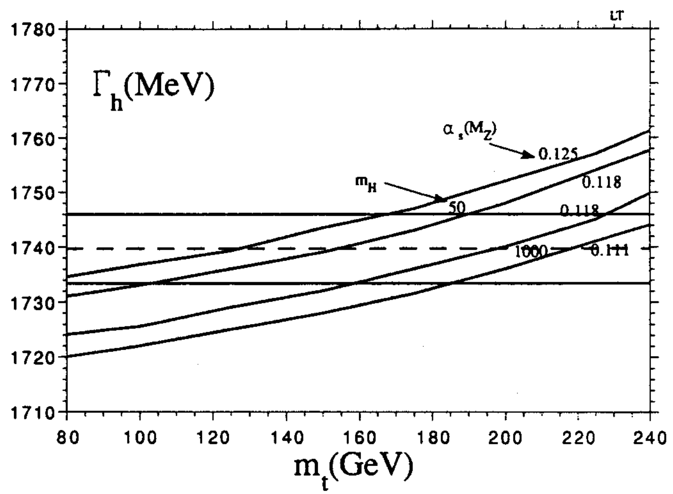

Fig. 10. Γ_h vs. m_t as predicted by the Standard Model for m_H = 50–1000 GeV and $\alpha_s(m_Z)$ = 0.111–0.125 compared with the LEP result

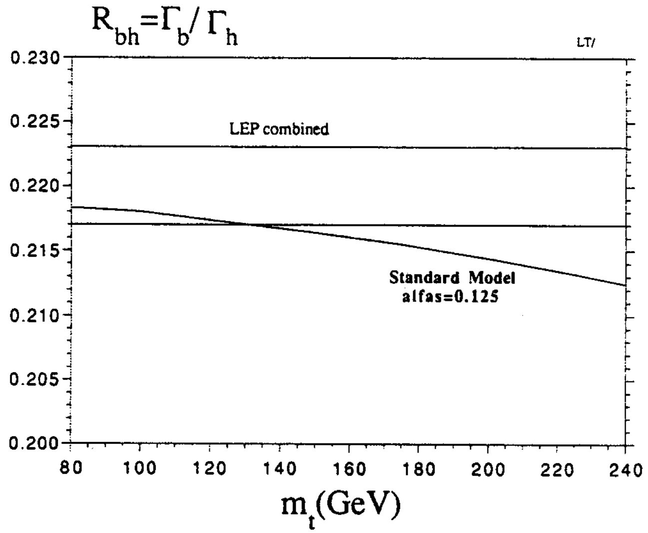

Fig. 11. R_{bh} vs. m_t as predicted by the Standard Model with $\alpha_s(m_Z) = 0.125$. (The dependence on α_s is very mild; $R_{bh} \simeq \left(1 + 0.15\frac{\alpha_s}{\pi}\right)$.)

m_t implied by each of them [65]. In fact, $\sin^2\theta_W$ is just one particular observable of the theory. Outside the domain of precision tests, with appropriate definitions of $\sin^2\theta_W$, as was seen in Section 9, one can write simple improved Born approximations that include the main contributions of radiative corrections (e.g. large logarithms and terms of order $G_F m_t^2$). While, for precision tests, the use of as complete as possible radiative corrections is mandatory, these approximate formulae are very useful for our understanding of the pattern of radiative corrections and for everyday-life estimates of rates and experimental sensitivities.

We go back to the definition of s_W^2 given in Eq. (107). Clearly in this case the observables s_W^2 and m_W are directly equivalent given that m_Z is among the input parameters. In the Standard Model, s_W^2 can be computed from the input parameters by the relation [see Eq. (108)]:

$$s_W^2 c_W^2 \equiv \left(1 - \frac{m_W^2}{m_Z^2}\right)\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F}\frac{1}{m_Z^2}\frac{1}{1-\Delta r} \tag{141}$$

where $c_W^2 = 1 - s_W^2$ and $\Delta r \equiv \Delta r(\alpha_s, G_F, m_Z, m_f, m_H)$ is the effect of radiative corrections. The quantity Δr as a function of the input parameters has been studied in great detail [40]. The result for s_W^2, obtained starting from the average LEP value for m_Z (see Table 1), as a function of m_t, is plotted in Fig. 5, where the uncertainty for 50 GeV $< m_H <$ 1 TeV is also visible. We see that m_t is the main unknown in the calculation of m_W/m_Z from m_Z, followed in importance by the ambiguities from varying the Higgs mass in the above range and the value of $\alpha(m_Z)$, while the remaining uncertainty from the experimental error on m_Z is very small.

When the available direct experimental information on m_W/m_Z is added, the sensitivity of s_W^2 to m_t provides a very strong constraint on m_t. The ratio m_W/m_Z is directly measured at hadron colliders but can also be indirectly obtained (assuming the validity of the Standard Model) from the ratio $R_\nu = \sigma^{NC}/\sigma^{CC}$ of neutral current (NC) to charged current (CC) cross-sections in neutrino–nucleus deep inelastic scattering. The value of m_W/m_Z has been measured at hadron colliders [66]. From CDF and UA2 we have the results reported in Table 2, where the PDG'92 value is also shown (which includes in addition UA1 data on $m_W - m_Z$). We use the latter value in the following.

Table 2

Experiment	m_W/m_Z	$s_W^2 = 1 - m_W^2/m_Z^2$
CDF	0.8763±0.0043	0.2320±0.0075
UA2	0.8813±0.0041	0.2234 ± 0.0072
Average	0.8789±0.0030	0.2275±0.0052
PDG'92	0.8798±0.0028	0.2259±0.0049

By combining m_W/m_Z from PDG'92 [3] with the LEP value for m_Z one obtains

$m_W = 80.23 \pm 0.26$ GeV. The corresponding average value of s_W^2 is also shown in Fig. 5 as a horizontal band, obviously independent of m_t, in the s_W^2–m_t plane.

As is well known, the value of s_W^2 extracted from R_ν is also nearly independent of m_t in the interesting range of values for the top mass. This fact arises from a largely accidental cancellation [67], specific to this process and to the Standard Model, between two different sources of m_t dependence, as discussed in the following.

The ratio $R_\nu = \sigma_{NC}/\sigma_{CC}$ for ν–N scattering is given in terms of s_W^2 by [see Eqs. (101) and (102)]:

$$R_\nu = \rho_{\nu N}^2 \left(\frac{1}{2} - k_{\nu N} s_W^2 + \frac{5}{9}(k_{\nu N} s_W^2)^2 (1+r)\right) + \dots , \qquad (142)$$

where $r = (\sigma^{\bar{\nu}}/\sigma^{\nu})_{CC} \simeq 0.4$ is also measured. The tree approximation (with $\rho_0 = 1$) is recovered for $\rho_{\nu N} = k_{\nu N} = 1$. Some large logarithms from the radiative corrections to σ^{CC} are also included in $\rho_{\nu N}$. But for the sake of this argument we are only considering the $G_F m_t^2$ terms. For fixed $R_\nu =$ (the experimental value) and $s_W^2 \sim 0.23$ there is a strong cancellation in the Standard Model between the m_t dependence of $\rho_{\nu N} \simeq 1 + \delta\rho$ and of $k_{\nu N} \simeq 1 + c_W^2/s_W^2 \delta\rho$ [Eqs. (111),(112)], so that as a result $\delta s_W^2 \simeq 0.2\delta\rho$, where $\delta\rho$ is given in Eq. (110). For realistic values of m_t the resulting contribution of the quadratic m_t terms is no longer dominant.

The most precise experimental results on R_ν were obtained by the CHARM [68] and CDHS [69] collaborations at CERN. Recently, a new result from the CCFR Collaboration at FNAL has been published. The original results on $\sin^2\theta_W$ were given for fixed m_t and m_H. CHARM obtained $s_W^2 = 0.236 \pm 0.005$ (exp) $\pm$ 0.005 (th) for $m_t = 45$ GeV and $m_H = 100$ GeV, while the CDHS result was $s_W^2 = 0.2275 \pm 0.005$ (exp) $\pm$ 0.005 (th) for $m_t = 60$ GeV and $m_H = 100$ GeV. The theoretical error arises from hadronic uncertainties and the effect of the charm threshold. An average at $m_t = 60$ GeV and $m_H = 100$ GeV gives $s_W^2 = 0.232 \pm 0.006$ (where the error 6×10^{-3} is obtained as $6 \times 10^{-3} = \sqrt{(5/\sqrt{2})^2 + 5^2} \times 10^{-3}$). The new result from CCFR [70] quoted for $m_t = 150$ GeV and $m_H = 100$ GeV, $s_W^2 = 0.2222 \pm 0.057$ lowers the average at $m_t = 60$ GeV to something like 0.229±0.006. The corresponding combined result at different values of m_t and m_H can also be obtained from the known form of the radiative corrections. The result is shown [71] in Fig. 5.

There are many more, less precise experimental results on s_W^2 from low- energy neutral current data, most of them being well known since a long time [72],[73]. These additional data are all consistent among them and with the results in Fig. 5.

We now consider the implications for the standard electroweak theory of the LEP results on the Z partial widths and asymmetries. About 2 million Z events were collected by the four LEP experiments in the '89–'91 runs and 3 million more were stored on tape in '92. The final results from the analysis of the '89–'91 data plus some preliminary results from '92 run are now available [32],[33] and will be discussed in the following. From the shape of the resonance, measured by energy scanning, one obtains m_Z and Γ_Z. Since in '92 there has been no scanning, i.e. all data were

collected at the Z peak, there is essentially no progress on m_Z and Γ_Z with respect to the '91 results. At the peak the basic measurements are those of the hadronic peak cross-section σ_h (deconvoluted from all QED effects: $\sigma_h = \frac{12\pi}{m_Z^2}\frac{\Gamma_e\Gamma_h}{\Gamma_Z^2}$), the ratio R_h of hadrons to leptons: $R_h = \Gamma_h/\Gamma_\ell$ (either using e-μ-τ universality, which is supported by the data within the present accuracy, or separately for each charged lepton type) and the ratio $R_{bh} = \Gamma_b/\Gamma_h$ of the $Z \to b\bar{b}$ to the hadronic widths. From the above primary measurements one can also derive the experimental values of $\Gamma_h, \Gamma_\ell, \Gamma_b$ and $\Gamma_{\rm inv}$ (or equivalently N_ν the number of light neutrinos, which is now $N_\nu = 2.99 \pm 0.03$) given in Table 3. In Figs. 6 to 11, we compare the data on the Z widths (collected in Table 3) with the predictions of the Standard Model, obtained with the programme ZFITTER [74] which includes a state-of-the-art set of electroweak radiative corrections.

Table 3: Summary of combined data used in the text

M_Z	91.187 ± 0.007
m_W/m_Z	0.8798±0.0028
Γ_T (MeV)	2488 ± 7
$R = \Gamma_h/\Gamma_\ell$	20.83±0.06
$\sigma_h = \frac{12\pi\Gamma_e\Gamma_h}{m_Z^2\Gamma_T^2}$ (nb)	41.44 ± 0.17
Γ_ℓ(MeV)	83.52 ± 0.28
Γ_h (MeV)	1739.7 ± 6.3
Γ_b (MeV)	383 ± 6
$R_{bh} = \Gamma_b/\Gamma_h$	0.220 ± 0.003
A_{FB}^ℓ	0.0164 ± 0.0021
$A_{\rm pol}^\tau$	0.142 ± 0.017
A^e	0.130 ± 0.025
A_{FB}^b	0.098 ± 0.09
g_v/g_a (all asymmetries)	0.0725 ± 0.0033

Additional important information is provided by the measurement of a number of asymmetries. In particular we refer to the forward–backward asymmetries for charged leptons $\left(A_{FB}^\ell\right)$ and for the b-quark $\left(A_{FB}^b\right)$ and the τ polarization asymmetry $\left(A_{pol}^\tau\right)$. The value of those asymmetries, combined over the LEP experiments, are given in the following [32],[33]. For A_{FB}^ℓ deconvoluted for all QED effects, one has:

$$A_{FB}^\ell(\sqrt{s} = m_Z) = 0.0164 \pm 0.0021 \qquad (143)$$

which corresponds to $x = g_V/g_A = 0.0743 \pm 0.0048$ via the relation:

$$A_{FB}^\ell = \frac{3x^2}{(1+x)^2}. \qquad (144)$$

For A_{FB}^b the result, after correction for the $B - \bar{B}$ mixing effect and for QCD effects [75],[76], is given by

$$A_{FB}^b(\sqrt{s} = m_Z) = 0.098 \pm 0.09 \,. \qquad (145)$$

For A^{τ}_{pol} one has:

$$A^{\tau}_{pol}(\sqrt{s} = m_Z) = \frac{2x}{1+x^2} = 0.142 \pm 0.017 \tag{146}$$

Actually, from the angular dependence of the τ polarization, the analogue of A_τ for the electron, A_e, can be measured. LEP finds:

$$A_\ell = 0.130 \pm 0.025 \tag{147}$$

Assuming universality the values in Eqs.(146), (147) can be combined:

$$A_\tau = 0.138 \pm 0.014\ . \tag{148}$$

which is the value shown in Fig. 14. It is important to mention that recently the first results on A_{LR} with polarized e^- beams were published by the SLD collaboration at SLAC [77]. The measured value is $A_{LR} = 0.100 \pm 0.044$. The combined value of $x = g_V/g_A$ from all asymmetries measured at LEP is shown in Table 3. The experimental results on the asymmetries are compared with the Standard Model predictions in Figs. 12–14.

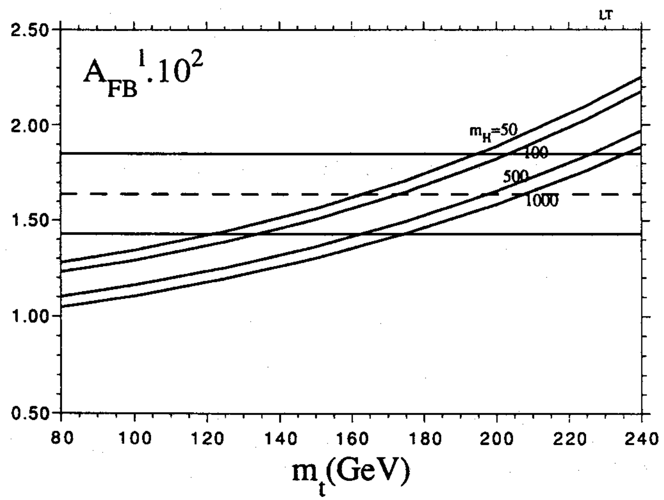

Fig. 12. $A^{\ell}_{FB}\times10^2$ vs. m_t as predicted the Standard Model for $m_H = 50$–1000 GeV compared with the LEP result

All measurements are in good agreement with the Standard Model and a consistent range of m_t values is indicated. When the data are fitted in the Standard Model

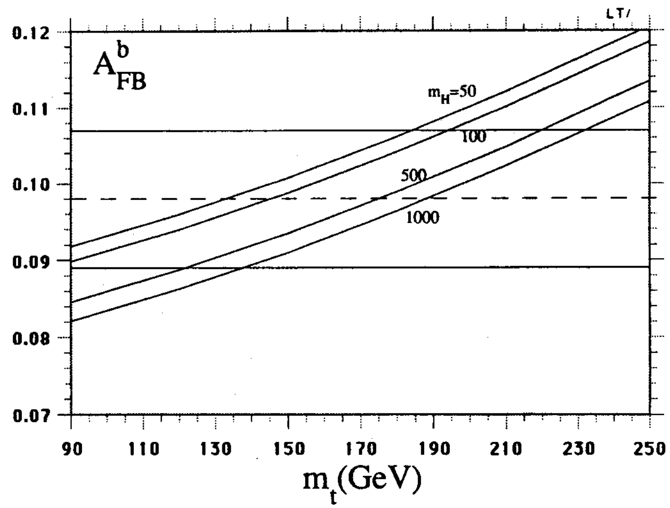

Fig. 13. A^b_{FB} vs. m_t as predicted by the Standard Model for $m_H = 50$–1000 GeV compared with the LEP result

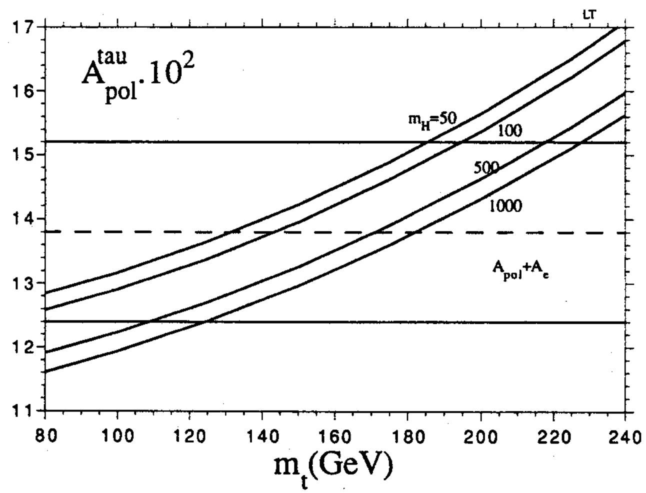

Fig. 14. A^{τ}_{pol} vs. m_t as predicted by the Standard Model for $m_H = 50$–1000 GeV compared with the LEP result

(taking the experimental correlations also into account, see Ref. [32] with $\alpha_s(m_Z) = 0.118 \pm 0.007$ fixed, one obtains

$$\begin{aligned} m_t &= 132 \pm 22 \text{ GeV for } m_H \sim 60 \text{ GeV} \\ &= 174 \pm 17 \text{ GeV for } m_H \sim 1 \text{ TeV} \end{aligned}$$

When also the data on m_W/m_Z [66], R_ν [69],[70] and atomic parity violation on Cs [78] are added, the corresponding results are

$$\begin{aligned} m_t &= 135 \pm 18 \text{ GeV for } m_H \sim 60 \text{ GeV} \\ &= 174 \pm 15 \text{ GeV for } m_H \sim 1 \text{ TeV} \end{aligned}$$

With the progress of LEP the direct impact of other electroweak tests on m_t is becoming marginal. The χ^2 value prefers small m_H, but within 2σ value of m_H as large as 1 TeV or more are allowed. If $\alpha_s(m_z)$ is left free, the fit leads to the value [2]

$$\alpha_s(m_Z) = 0.128 \pm 0.008$$

while the central values of m_t are shifted down by ~ 2 GeV.

Finally, it is interesting to note that the observed magnitude of the radiative corrections implies a lower bound on m_t as well, i.e., $m_t > 80$ GeV at 95% c.l.

12 TOWARD A MODEL-INDEPENDENT ANALYSIS OF THE DATA

Recently much attention has been devoted to develop a strategy for a model independent analysis of precision electroweak data [79]–[84]. The general framework consists in defining a set of variables describing possible deviations from the Standard Model. These variables are chosen in such a way as to minimise the negative impact of our ignorance of the top quark mass m_t and to be as much as possible sensitive to new physics in particularly problematic sectors of the theory, such as the Higgs sector. A very common approach [79]–[81], in terms of the variables S, T and U, is based on the assumption that the dominant effects of new physics only appear through vacuum polarization diagrams [85]. Alternative assumptions, formulated in the language of effective Lagrangians [82] or of dominance of lowest dimension terms in an operator expansion [83] or in physical terms [86], lead to the same or to similar sets of variables.

In a previous work [84], with S. Jadach, we have advocated a more pragmatic approach where no specific assumptions are *a priori* formulated. Rather, in the first paper of Ref. [84] the variables ϵ_1, ϵ_2 and ϵ_3 were defined in general from the basic observables $m_W/m_Z, \Gamma_\ell$ and A^ℓ_{FB} (the forward-backward asymmetry at the Z-peak for charged leptons). We always assume charged lepton universality, which is supported by the data at the present level of accuracy, so that Γ_ℓ and A^ℓ_{FB} refer to

[2] Note that from the experimental value of R_h alone, shown in Table 3, which is often used by QCD experts, one would obtain $\alpha_s(m_Z) = 0.131 \pm 0.010$ by using $(R_h)_0 = 19.945 \pm 0.03$, the updated value of the QCD-uncorrected prediction [32].

the corresponding average data. These observables were chosen because they are particularly simple, are all defined at the intermediate gauge boson scale of mass, are precisely measured, do not involve important hadronic effects (like $\alpha_s(m_Z)$ or the $Z \to b\bar{b}$ vertex [87]) and each of them carries a qualitatively different information. In terms of these observables ϵ_1, ϵ_2 and ϵ_3 were defined in Ref. [84] in a way that, if the assumption of vacuum polarization dominance is made, then ϵ_1 would be linearly related to T, ϵ_2 to U and ϵ_3 to S. But one virtue of this approach is that the most interesting physical results are already obtained at a completely model independent level, without assumptions like the dominance of vacuum polarization diagrams. One example is the result that present data favour models of new physics that lead to negative corrections to ϵ_3 with respect to the Standard Model, like suitable versions of extended gauge models [88] or the Minimal Supersymmetric Standard Model [89],[90], while simple technicolour models [91],[92] are disfavoured.

Of course some assumptions are needed when additional observables other than $m_W/m_Z, \Gamma_\ell$ and A^ℓ_{FB} are introduced and one wants to relate them to the same set of epsilons in order to make their determination from the data more precise. One can formulate a hierarchy of simple and rather general assumptions valid in large classes of models which are needed in order to relate the epsilons to a progressively larger set of observables. First, very mild assumptions are required in order that all other observables connected to charged leptons at the Z pole, e.g. the τ-polarization asymmetry $A^\tau_{\rm pol}$ and the left-right asymmetry A_{LR}, are uniquely determined by the ϵ_i. This is true in all models where the contributions of new physics only occur either through vacuum polarization terms and/or in $Z \to \ell^+\ell^-$ vertex corrections of the form $\Delta V_\mu(Z \to \ell^+\ell^-) = \bar{u}(\Delta g_A \gamma_5 + \Delta g_V)\gamma_\mu u$ with Δg_V and Δg_A real form factors universal for all charged lepton flavours. These two kinds of contributions cannot be disentangled if we only consider on-shell Z properties in the charged lepton sector. Stronger hypotheses about possible forms of new physics are needed if one wants to relate the epsilons to observables involving quarks, like the hadronic widths or asymmetries, or to observables measured at low q^2, far from the Z pole (e.g. deep inelastic neutrino-nucleus scattering [93],[94] , ν-e scattering [95] and atomic parity violation in atomic physics [96],[97]). For example, we could restrict ourselves to the case of new physics in oblique corrections with the assumption that the q^2-dependence of the relevant vacuum polarization amplitudes is weak enough for all second and higher order derivatives in q^2 to be safely neglected [79]–[81]]. However, because m_t is unknown, the observables related to the b-quark, e.g. the b partial width and the forward-backward b asymmetry A^b_{FB}, in principle cannot be obtained from the ϵ_i's even if the new physics only occurs in oblique and/or in universal Z-vertex corrections. This is because of the large m_t-dependent Standard Model corrections to the $Z \to b\bar{b}$ vertex [87]. As a consequence the relation between the previous ϵ_i's and Γ_b or A^b_{FB} can only be specified for a given value of m_t. In practice, the sensitivity to the new quadratic terms in m_t^2 from the $Z \to b\bar{b}$ vertex is only present in Γ_b, and consequently in the hadronic and total widths Γ_h and Γ_t, while it is almost non-existent, at realistic values of m_t, for A^b_{FB}. For this reason and because of the dependence on $\alpha_s(m_Z)$, the precise measurements of Γ_T, Γ_h and Γ_b were not included in the previous analysis [84] (while A^b_{FB} was included).

In the second paper of Ref. [84] we have overcome the above limitation by adding a new parameter, ϵ_b, which describes the m_t-dependent part of the $Z \to b\bar{b}$ vertex, and by correspondingly enlarging the set of basic observables with the inclusion of Γ_b. From the quantities $m_W/m_Z, \Gamma_\ell, A^\ell_{FB}$ and Γ_b the four epsilons can be defined without need of specifying m_t. The important point is that, in the Standard Model, for all observables at the Z pole, the whole dependence on m_t arising from one-loop diagrams only enters through the epsilons. Furthermore, the same is obviously true for any extension of the Standard Model such that all possible deviations only occur through vacuum polarization diagrams and/or the $Z \to b\bar{b}$ vertex. At the same time we have improved the previous analysis (first paper of Ref. [84]) by slightly modifying the original definition of the epsilons in order to achieve a number of advantages: the modified variables are exactly zero in the Standard Model in the limit of neglecting all pure electroweak loop corrections (i.e. when only the predictions from the tree level Standard Model plus pure QED corrections are taken into account), are more directly related to the forward-backward asymmetry at the peak as by now precisely specified by the LEP experiments [98] and some small top quark mass dependences are better taken into account. In the present article we have updated the analysis described in the second paper of Ref. [84] by including the new results presented at the La Thuile and Moriond '93 Conferences [98],[99].

12.1 Basic Definitions

In the Standard Model all the observables can be computed starting from the input parameters, a self-imposing set of which is given by $\alpha, \alpha_s, G_F, m_Z, m_f, m_H$. In the following we use the values $\alpha^{-1} = 137.036, G_F = 1.166372 \times 10^{-5}\ \mathrm{GeV}^2, m_Z = 91.187$ GeV. Among the fermion masses m_f, the main unknown quantity is the top quark mass m_t. The CDF lower limit on $m_t, m_t > 108$ GeV, is valid in the Standard Model [100]. The failure of direct searches at LEP imposes the bound $m_H > 62.5$ GeV on the Higgs mass [101]. Also, the Standard Model is affected with serious pathologies for $m_H > 0.6\text{-}1\ TeV$, when the Landau pole associated to the $\lambda(\phi^+\phi)^2$ coupling comes too close to the physical region in parameter space [102].Thus in evaluating the Standard Model predictions we vary m_H in the range 60 GeV-1 TeV. Finally direct measurements of $\alpha_s(m_Z)$ from many different experiments lead to a relatively precise value of $\alpha_s(m_Z)$ [103]. In the present discussion we use $\alpha_s = 0.118 \pm 0.007$ [103].

We start from the basic observables $m_W/m_Z, \Gamma_\ell$ and A^ℓ_{FB} and Γ_b. From these four quantities one can isolate the corresponding dynamically significant corrections $\Delta r_W, \Delta\rho_N, \Delta k_N$ and ϵ_b, which contain the small effects one is trying to disentangle, and are defined in the following. First we introduce Δr_W as obtained from m_W/m_Z by the relation:

$$\left(1 - \frac{m_W^2}{m_Z^2}\right)\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2}\, G_F m_Z^2 (1 - \Delta r_W)}\ . \qquad (149)$$

Here $\alpha(m_Z) = \alpha/(1 - \Delta\alpha)$ is fixed to the conventional value 1/128.87 so that the effect of the running of α due to known physics is extracted from $(1 - \Delta r) =$

$(1 - \Delta\alpha)(1 - \Delta r_W)$. In fact, in the Standard Model the value of $1/\alpha(m_Z)$ should be 128.87 ± 0.12 [104]. A possible departure from this value would then be included in Δr_W. The present definition of Δr_W is unchanged with respect to both papers of Ref. [84]. In order to define $\Delta\rho_N$ and Δk_N we consider the effective vector and axial-vector couplings g_V and g_A of the on-shell Z to charged leptons, given by the formulae:

$$\Gamma_\ell = \frac{G_F m_Z^3}{6\pi\sqrt{2}}(g_V^2 + g_A^2)\left(1 + \frac{3\alpha}{4\pi}\right) \qquad (150)$$

$$A^\ell_{FB}(\sqrt{s} = m_Z) = \frac{3 g_V^2 g_A^2}{(g_V^2 + g_A^2)^2} = \frac{3x^2}{(1+x^2)^2}\ . \qquad (151)$$

Note that Γ_ℓ stands for the inclusive partial width $\Gamma(Z \to \ell\ell + \text{photons})$. With respect to the definition of g_V and g_A in the first Ref. [84] we stress the following differences, which have been introduced in the second. First, we have extracted from $(g_V^2 + g_A^2)$ the factor $(1 + 3\alpha/4\pi)$ which is induced in Γ_ℓ from final state radiation. Second, by the asymmetry at the peak in Eq. (151) we mean the quantity which is now commonly referred to by the LEP experiments (denoted as A^0_{FB} in Ref. [98]), which is corrected for all QED effects, including initial and final state radiation and also for the effect of the imaginary part of the γ vacuum polarization diagram. The last effect was instead not corrected for in the first [84]. In terms of g_A and $x\ g_V/g_A, \Delta\rho_N$ and Δk_N are given by [84]:

$$g_A = -\frac{\sqrt{\rho_N}}{2} = -\frac{1}{2}\left(1 + \frac{\Delta\rho_N}{2}\right)$$
$$x = \frac{g_V}{g_A} = 1 - 4\bar{s}_W^2 = 1 - 4(1 + \Delta k_N)s_0^2\ . \qquad (152)$$

The index N for 'new' is there to recall that the corresponding definition was changed in the second Ref. [84] with respect to the first. In fact the formal relations with the couplings are identical but the definition of g_V and g_A has been modified (it is only for simplicity that we do not append the index N to them too). In Eq. (152) $\bar{s}_W^2$ is an effective $\sin^2\theta_W$ for on-shell Z, defined by this very equation, while s_0^2 is the corresponding quantity before non pure-QED corrections, given by:

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2} \qquad (153)$$

with $c_0^2 = 1 - s_0^2$ ($s_0^2 = 0.231184$ for $m_Z = 91.187$ GeV).

We now define ϵ_b from Γ_b, the inclusive partial width for $Z \to b\bar{b}$ according to the relation

$$\Gamma_b = \frac{G_F m_Z^3}{6\pi\sqrt{2}}\,\beta\left(\frac{3-\beta^2}{2}\, g_{bV}^2 + \beta^2 g_{bA}^2\right) N_C R_{QCD}\left(1 + \frac{\alpha}{12\pi}\right)\ . \qquad (154)$$

where $N_C = 3$ is the number of colours, $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$, with $m_b = 4.8$ GeV, R_{QCD} is the QCD correction factor given by

$$R_{QCD} = 1 + 1.2a - 1.1a^2 - 13a^3\ ;\quad a = \frac{\alpha_S(m_Z)}{\pi} \qquad (155)$$

(for $\alpha_s(m_Z) = 0.118, R_{QCD} = 1.0428$) and g_{bV}, g_{bA} are specified as follows

$$g_{bA} = -\frac{1}{2}\left(1+\frac{\Delta\rho_N}{2}\right)(1+\epsilon_b)$$
$$\frac{g_{bV}}{g_{bA}} = \frac{1-\frac{4}{3}\bar{s}_W^2+\epsilon_b}{1+\epsilon_b} = \frac{1-\frac{4}{3}(1+\Delta k_N)s_0^2+\epsilon_b}{1+\epsilon_b} \quad (156)$$

with $\Delta\rho_N, \Delta k_N, \bar{s}_W^2$ and s_0^2 as defined in Eqs. (150)–(153). The parameter ϵ_b in the Standard Model is closely related to the quantity $-\mathrm{Re}\{\delta_{b-\mathrm{vertex}}\}$ defined in the last of Refs. [87].

As is well known [105], in the Standard Model, $\Delta r_W, \Delta\rho_N, \Delta k_N$ and ϵ_b,for sufficiently large m_t, are all dominated by quadratic terms in m_t of order $G_F m_t^2$. As new physics can more easily be disentangled if not masked by large conventional m_t effects, it is convenient to keep $\Delta\rho_N$ and ϵ_b while trading Δr_W and Δk_N for two quantities with no contributions of order $G_f m_t^2$. We thus introduce the following linear combinations [80],[84]:

$$\epsilon_{N1} = \Delta\rho_N$$
$$\epsilon_{N2} = c_0^2\Delta\rho_N + \frac{s_0^2\Delta r_W}{(c_0^2-s_0^2)} - 2s_0^2\Delta k_N$$
$$\epsilon_{N3} = c_0^2\Delta\rho_N + (c_0^2-s_0^2)\Delta k_N\ . \quad (157)$$

In analogy with $\Delta\rho_N$ and Δk_N the new epsilons are labelled ϵ_N. Clearly ϵ_{N2} and ϵ_{N3} no longer contain terms of order $G_F m_t^2$ but only logarithmic terms in m_t. The leading terms for large Higgs mass, which are logarithmic, are mainly contained in ϵ_{N1} but are also present in ϵ_{N3}. In the Standard Model one has the following 'large' asymptotic contributions [87],[106],[107]:

$$\epsilon_{N1} = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} - \frac{3G_F m_W^2}{4\pi^2\sqrt{2}}\mathrm{tg}^2\theta_W \ln\left(\frac{m_H}{m_Z}\right) + \ldots$$
$$\epsilon_{N2} = -\frac{G_F m_W^2}{2\pi^2\sqrt{2}}\ln\left(\frac{m_t}{m_Z}\right) + \ldots$$
$$\epsilon_{N3} = \frac{G_F m_W^2}{12\pi^2\sqrt{2}}\ln\left(\frac{m_H}{m_Z}\right) - \frac{G_F m_W^2}{6\pi^2\sqrt{2}}\ln\left(\frac{m_t}{m_Z}\right) + \ldots$$
$$\epsilon_b = -\frac{G_F m_t^2}{4\pi^2\sqrt{2}} + \ldots \quad (158)$$

The relations between the basic observables and the epsilons can be linearised and inverted, leading to the formulae

$$\epsilon_{N1} = -0.9882 + 0.011963\Gamma_\ell\,(\mathrm{MeV}) - 0.1511x$$
$$\epsilon_{N3} = -0.7146 + 0.009181\Gamma_\ell\,(\mathrm{MeV}) - 0.69735x$$
$$\epsilon_b = -0.62\epsilon_{N1} + 0.24\epsilon_{N3} + 0.436\left[\frac{\Gamma_b}{\Gamma_{b0}} - 1\right]$$
$$\epsilon_{N2} = \frac{1}{c_0^2-s_0^2}\left[c_0^2\epsilon_{N1} - 2s_0^2\epsilon_{N3} + s_0^2\Delta r_W\right] = 1.43\epsilon_{N1} - 0.86\epsilon_{N3} + 0.43\Delta r_W \quad (159)$$

where $x = g_V/g_A$ is defined in Eq. (151) from A_{FB}^ℓ. The quantity Γ_{b0} is the value of Γ_b in the limit when all epsilons are neglected. With $\alpha_s(m_Z) = 0.118, \Gamma_{b0} = 379.4$ MeV [more in general its value is given by Eqs. (154)–(156) and (175)].

Equations (159) are also used to obtain, by definition, the behaviour of the epsilons in the Standard Model as functions of m_t and m_H: by means of the latest version of the programme ZFITTER [108] we compute Γ_ℓ, Γ_b, x and Δr_W as functions of m_t and m_H; we obtain the results for $\epsilon_{N1}, \epsilon_{N2}$ and ϵ_{N3} shown in Fig. 15, while the result for ϵ_b, which is practically independent of m_H, is plotted in Fig. 16 for $\alpha_s(m_Z) = 0.118$. The above results were also checked by using the new code TOPAZ0 [109]. As already mentioned, in the Standard Model the dependence on m_t and m_H of all observables at the Z pole only enters through the epsilons (see Section 12.3). This is also true in any extension which deviates from the Standard Model in vacuum polarizations and/or in the $Z \to b\bar{b}$ vertex. This is an exact statement if one loop corrections are only considered. At two loops, some residual dependence on m_t is induced in the various observables from terms of order α_s^2 [110], which can be reabsorbed in the epsilons only in an approximate but sufficiently accurate way.

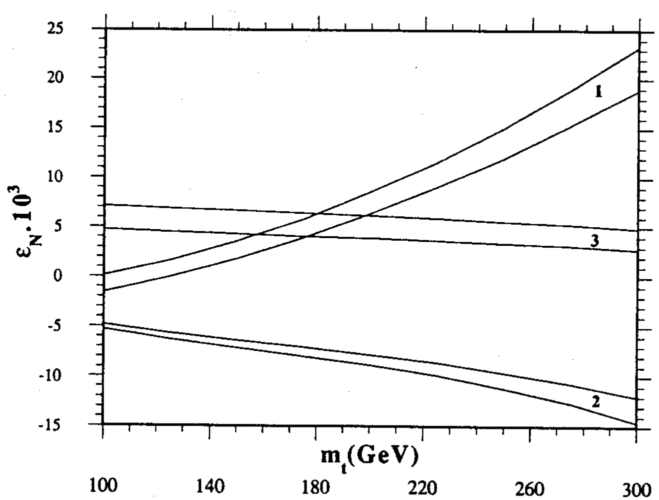

Fig. 15. $\epsilon_{N1}, \epsilon_{N2}$ and ϵ_{N3} vs. m_t as predicted in the Standard Model for m_H = 50–1000 GeV

12.2 Determination of the Epsilons from the Data

The experimental results that will be used in the present analysis are collected in Table 3 (Section 11). We start by deriving from the data on the input observables

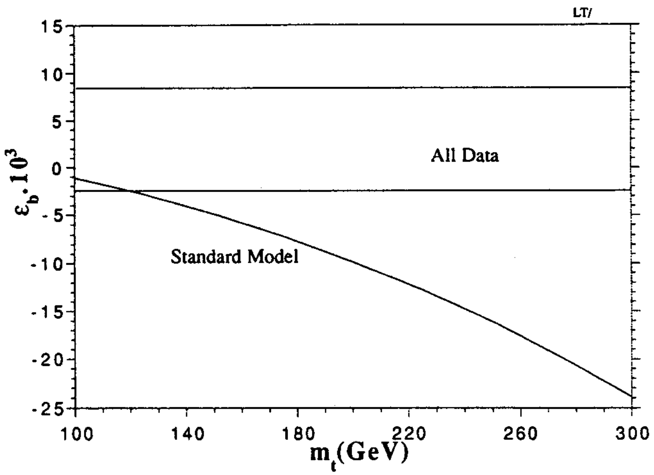

Fig. 16. ϵ_b vs. m_t as predicted by the Standard Model (there is no dependence on m_H). The band is the value fitted in Eqs. (180) from all the data analyzed with $\alpha_s(m_Z) = 0.118 \pm 0.007$.

the values of the epsilons that follow by directly applying their definition, with no dynamical assumptions.

By combining the value of m_W/m_Z [111] with the LEP value for the Z mass [98], both given in Table 3, and using Eq. (149) one obtains:

$$\Delta r_W = (-16 \pm 16) \times 10^{-3} \,. \qquad (160)$$

From the LEP results on the charged lepton partial width and the forward-backward asymmetry, assuming e-μ-τ universality, given in Table 3, one finds:

$$g_A^2 = 0.2499 \pm 0.0009 \qquad (161)$$

$$x = g_V/g_A = 0.0743 \pm 0.0048 \quad \text{or} \quad \bar{s}_W^2 = 0.2314 \pm 0.0012 \,. \qquad (162)$$

One can now use Eqs. (152) to derive the results for $\Delta\rho_N$ and Δk_N:

$$\epsilon_{N1} = \Delta\rho_N = (-0.3 \pm 3.4) \times 10^{-3} \qquad (163)$$

$$\Delta k_N = (0.9 \pm 5.0) \times 10^{-3} \,. \qquad (164)$$

The corresponding results for ϵ_{N2} and ϵ_{N3} are obtained from Eqs. (157) or (159):

$$\epsilon_{N2} = (-7.6 \pm 7.6) \times 10^{-3} \qquad (165)$$

$$\epsilon_{N3} = (0.4 \pm 4.2) \times 10^{-3} \qquad (166)$$

Finally from the value of Γ_b listed in Table 3 one finds, by using Eqs. (154)–(156)) or (159):

$$\epsilon_b = (4.4 \pm 7.0) \times 10^{-3} \qquad (167)$$

To proceed further, and include other measured observables in the analysis we need to make some dynamical assumptions. The minimum amount of model dependence is introduced by including other purely leptonic quantities at the Z pole, such as $A^\tau_{\rm pol}, A_e$ (measured [98] from the angular dependence of the τ polarization) and A_{LR}. At this stage, one is simply relying on lepton universality. Indeed, with the new definition of $x = g_V/g_A$ in Eq. (151) in terms of the peak asymmetry, defined with all pure QED effects, including the imaginary part of the photon vacuum polarization, subtracted away, the data on $A^\tau_{\rm pol}, A_e$ and A_{LR} can be taken into account by using the simple formula

$$A^\tau_{\rm pol}, A_e, A_{LR} = \frac{2 g_V g_A}{g_V^2 + g_A^2} = \frac{2x}{1+x^2} \qquad (168)$$

with the same x as defined from A^ℓ_{FB}, apart from still negligible differences. With essentially the same assumptions one can also include the data on the b-quark forward–backward asymmetry A^b_{FB}. In fact it turns out that A^b_{FB} is almost unaffected by the $Z \to b\bar{b}$ vertex correction. Schematically the reason is that $A^b_{FB} = 3\eta_e\eta_b$ with $\eta = \frac{g_V g_A}{(g_V^2+g_A^2)}$, so that $\delta A^b_{FB} = 3(\eta_e\delta\eta_b + \eta_b\delta\eta_e)$. We see that the sensitivity on η_b, which contains the $Z \to b\bar{b}$ vertex correction, is strongly suppressed by the small factor η_e. This is confirmed by an accurate numerical calculation of A^b_{FB}. If we define a new quantity $(\bar{s}^2_W)_b$ by the identity:

$$A^b_{FB} = 3\frac{1-4(\bar{s}^2_W)_b}{1+(1-4(\bar{s}^2_W)_b)^2}\,\frac{\beta(1-\frac{4}{3}(\bar{s}^2_W)_b)}{\beta^2+\frac{3-\beta^2}{2}(1-\frac{4}{3}(\bar{s}^2_W)_b)^2}\,, \qquad (169)$$

where $\beta = \sqrt{1-4m_b^2/m_Z^2}$, we can explicitly evaluate the relation, as a function of m_t and m_H, between $(\bar{s}^2_W)_b$ and the similar quantity $(\bar{s}^2_W)_{FB}$ previously defined from the charged lepton asymmetry A^ℓ_{FB} and check that, with the new definition of A^ℓ_{FB}, they indeed practically coincide.

As a result of the previous discussion, we can combine the values of $x = g_V/g_A$ obtained from the whole set of asymmetries measured at LEP and we get the value:

$$x = g_V/g_A = 0.0725 \pm 0.0033 \quad \text{or} \quad \bar{s}^2_W = 0.2319 \pm 0.0008 \,, \qquad (170)$$

which coincides with the result quoted in Ref. [98]. At this stage the best values of $\epsilon_{N1}, \epsilon_{N2}$ and ϵ_{N3} are modified according to

$$\begin{aligned} \epsilon_{N1} &= \Delta\rho_N = (0.0 \pm 3.4) \times 10^{-3} \\ \epsilon_{N2} &= (-8.3 \pm 7.7) \times 10^{-3} \\ \epsilon_{N3} &= (1.6 \pm 3.5) \times 10^{-3} \\ \epsilon_b &= (-4.5 \pm 7.0) \times 10^{-3} \,. \end{aligned} \qquad (171)$$

All observables measured on the Z peak at LEP can be included in the analysis provided that we assume that all deviations from the Standard Model are only contained in vacuum polarization diagrams (without demanding a truncation of the q^2 dependence of the corresponding funtions) and/or the $Z \to b\bar{b}$ vertex. Note that this is true for whatever partition of the new effect between g_{bV} and g_{bA}, because only one combination of them is measured in Γ_b, while, as already mentioned, A^b_{FB} is nearly independent of the $Z \to b\bar{b}$ vertex. A set of primary measurements at LEP contains $\Gamma_Z, R = \Gamma_h/\Gamma_l$, the hadronic peak cross-section σ_h (with a 3×3 correlation matrix given in Ref. [99], $R_{bh} = \Gamma_b/\Gamma_h$ plus the asymmetries that lead to the value of x in Eq. (170). The relations between these quantities and the epsilons can be written down in the following linearised form:

$$\begin{aligned} \Gamma_Z &= \Gamma_{Z0}[1 + 1.35\epsilon_{N1} - 0.46\epsilon_{N3} + 0.35\epsilon_b] \\ R &= R_0[1 + 0.28\epsilon_{N1} - 0.36\epsilon_{N3} + 0.50\epsilon_b] \\ \sigma_h &= \sigma_{h0}[1 - 0.03\epsilon_{N1} + 0.04\epsilon_{N3} - 0.20\epsilon_b] \\ R_{bh} &= R_{bh0}[1 - 0.06\epsilon_{N1} + 0.07\epsilon_{N3} + 1.79\epsilon_b] \\ x &= x_0[1 + 17.6\epsilon_{N1} - 22.9\epsilon_{N3}] \end{aligned} \tag{172}$$

We stress that, at the one-loop level, all the dependence on m_t and m_H of the quantities in Eqs. (172) enters through the epsilons and not in the various numerical coefficients. This is no longer true for those two-loop effects, unlike those computed in Ref. [112], which are not vacuum polarization or $Z \to b\bar{b}$ vertex corrections. For example, the dependence on m_t in R_{QCD} introduced by the α_s^2 terms of Ref. [109], was not included in Eq. (155). As a consequence, their effect on Γ_b is, by definition, found in ϵ_b. Their remaining effect on all other observables we consider is numerically irrelevant.

We have made a global fit of the data on $m_W/m_Z, \Gamma_Z, R, \sigma_h, R_{bh}$, given in Table 3 and the results on the asymmetries, summarised by the value of x in Eq. (170). For LEP data, we have taken the correlation matrix for Γ_Z, R and σ_h given in Ref. [99], while we have considered the additional information on R_{bh} and x as independent. The results of our fit are:

$$\begin{aligned} \epsilon_{N1} &= \Delta\rho_N = (-0.3 \pm 3.2) \times 10^{-3} - 0.1\,\delta a_s \\ \epsilon_{N2} &= (-8.5 \pm 7.6) \times 10^{-3} - 0.11\delta a_s + 0.23\delta a \\ \epsilon_{N3} &= (1.5 \pm 3.3) \times 10^{-3} - 0.04\delta a_s - 0.77\delta a \\ \epsilon_b &= (3.1 \pm 4.5) \times 10^{-3} - 1.42\delta a_s \end{aligned} \tag{173}$$

where the dependence on the values of $\alpha_s(m_Z)$ and $\alpha(m_Z)$ has also been indicated. If we neglect possible contributions from δa and add in quadrature the effect of a ±0.007 error on $\alpha_s(m_Z)$ around the central value 0.118, we obtain:

$$\begin{aligned} \epsilon_{N1} &= \Delta\rho_N = (-0.3 \pm 3.2) \times 10^{-3} \\ \epsilon_{N2} &= (-8.5 \pm 7.6) \times 10^{-3} \\ \epsilon_{N3} &= (1.5 \pm 3.3) \times 10^{-3} \\ \epsilon_b &= (3.1 \pm 6.5) \times 10^{-3} \end{aligned} \tag{174}$$

which shows that the errors induced from $\alpha_s(m_Z)$ are entirely negligible with the exception of the error on ϵ_b.

Provided that all deviations from the Standard Model are only contained in vacuum polarizations and/or in the $Z \to b\bar{b}$ vertex, Eqs. (172),(173) apply. For that to be true, it is important that we focus on observables at the intermediate gauge boson scale of mass. To include in the analysis lower energy observables as well, a stronger hypothesis needs to be made: only vacuum polarization diagrams are allowed to vary from the Standard Model ones and only in their constant and first derivative terms in a q^2-expansion [79]–[81], a likely picture, e.g. in technicolour theories [91],[92]. In such a case, to Eqs. (159) one can add the linearized expressions, in terms of the epsilons, of the ratio R_ν of neutral to charged current processes in deep inelastic neutrino scattering on nuclei [93],[94]

$$R_\nu = 0.3115 \times 0.83\epsilon_{N1} - 0.27\epsilon_{N3} \tag{175}$$

or of the 'weak charge' Q_W measured in atomic parity violation experiments on Cs [96],[97]

$$Q_W = -72.72 \pm 0.13 - 102\epsilon_{N3} \tag{176}$$

(with the error due to the theoretical uncertainties in the atomic wave functions).

It is now possible to include these low energy observables in an overall fit. Using, together with the data in Table 3, the present average value from νN scattering

$$\langle R_\nu \rangle = 0.312 \pm 0.003 \tag{177}$$

(which takes the recent CCFR result [94] into account) and the result from caesium data [96],[97]

$$Q_W = -71.04 \pm 1.81\,, \tag{178}$$

one obtains the global fit

$$\begin{aligned} \epsilon_{N1} &= \Delta\rho_N = (-0.1 \pm 2.7) \times 10^{-3} \\ \epsilon_{N2} &= (-8.2 \pm 7.5) \times 10^{-3} \\ \epsilon_{N3} &= (1.3 \pm 3.1) \times 10^{-3} \\ \epsilon_b &= (2.0 \pm 5.4) \times 10^{-3}\,. \end{aligned} \tag{179}$$

Here the error induced from $\alpha_s(m_Z)$ was taken into account in the same way as in Eqs. (177).

12.3 Discussion and Conclusion

The results of the various fits are collected in Table 4. The results for ϵ_b and ϵ_{N2} are compared with the Standard Model predictions in Figs. 16 and 17. We also show in Figs. 18–20 the projected ellipses in the planes $\epsilon_{N1} - \epsilon_{N3}, \epsilon_{N1} - \epsilon_b, \epsilon_{N3} - \epsilon_b$ for different fits together with the corresponding Standard Model predictions.

We note that, with the progress of LEP, the effect of low energy data on the epsilons is decreasing in importance. We also observe that all the epsilons are compatible with zero within 1σ (or slightly more for ϵ_{N2}, as a consequence of the fact that Δr_W in

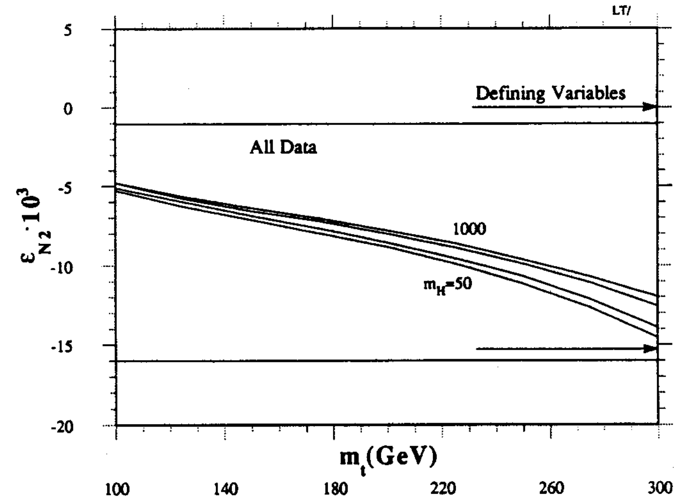

Fig. 17. ϵ_{N2} vs. m_t and m_H as predicted in the Standard Model compared with the value fitted from all the data in Eqs. (179). The arrows indicate the limits of the band if only the defining variables of Eq. (165) are used.

Eq. (160) is displaced from zero by 1σ). This means that the tree level Standard Model plus pure QED corrections is quite a good approximation and that the possibility of measuring the pure weak radiative corrections is marginal within the present accuracy. However the apriori size of the purely weak corrections could well be larger than the measured values. As a consequence strong contraints are obtained both in the Standard Model (upper and lower limits on the top mass) and beyond.

Table 4: Summary of the different determinations of the epsilon parameters

	Only defining variables	Defining variables plus all asymmetries	All LEP data	'All data'
$\epsilon_{N1} \cdot 10^3$	-0.3± 3.4.	0.0 ± 3.4	-0.3± 3.2	-0.1± 2.7
$\epsilon_{N2} \cdot 10^3$	-7.6± 7.6	-8.3 ± 7.5	-8.5 ± 7.6	-8.2 ± 7.5
$\epsilon_{N3} \cdot 10^3$	0.4± 4.2	1.6± 3.5	1.5±3.3	1.3± 3.1
$\epsilon_b \cdot 10^3$	4.4 ±7.0	4.5± 7.0	3.1 ± 5.5	2.9±5.4

By comparing the results from the global fit given in Table 4, column 3, with those in column 2, obtained from fitting $m_W/m_Z, \Gamma_\ell, \Gamma_b$ and the value of x derived from all the asymmetries, we observe that the effect on $\epsilon_{N1}, \epsilon_{N2}$, and ϵ_{N3} of the additional information from Γ_Z, R and σ_h is quite modest. Their central values are displaced by less than 1σ and the gain on the errors is very limited. This *a posteriori* supports

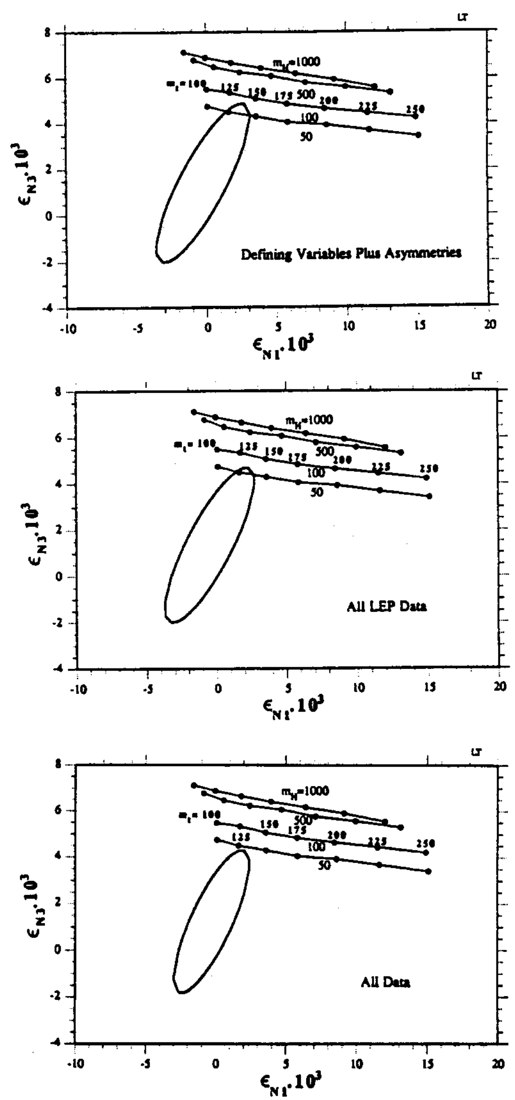

Fig. 18. Plots of ϵ_{N3} vs. ϵ_{N1} for different fits presented in Table 4: a) column 2; b) column 3; c) column 4. The projections on the axis of the ellipses correspond to the 1σ errors displayed in Table 4. The curves refer to the Standard Model predictions for different values of m_t and m_H.

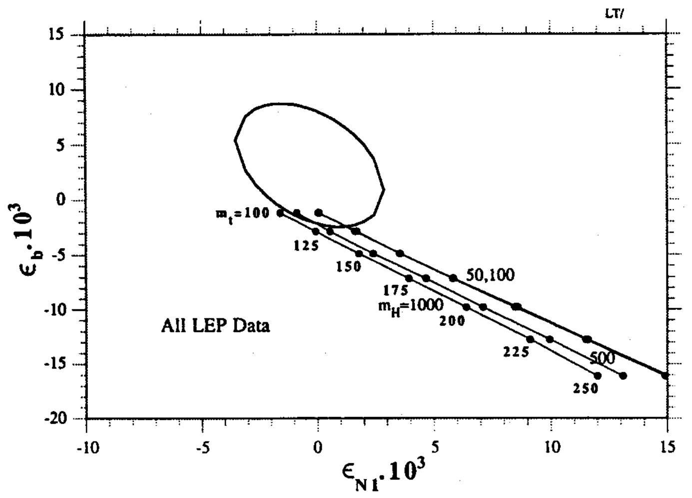

Fig. 19. Plots of ϵ_b vs. ϵ_{N1} for the fit presented in Table 4, column 3 (all LEP data). The projections on the axis of the ellipses correspond to the 1σ errors displayed in Table 4. The curves refer to the Standard Model predictions for different values of m_t and m_H with $\alpha_s(m_Z) = 0.118 \pm 0.007$.

the procedure proposed in the first paper in Ref. [84], where, for the determination of $\epsilon_{N1}, \epsilon_{N2}$, and ϵ_{N3}, the hadronic quantities were discarded because of the top dependence introduced by the $Z \to b\bar{b}$ vertex and of the need of specifying $\alpha_s(m_Z)$. It is possible to make contact with the approach in terms of the variables S, T and U [79],[81] only under the assumption of dominance of new physics effects through vacuum polarization corrections. In such a case, the precise relation with the epsilons is given by:

$$\begin{aligned} \epsilon_{N1} &= \epsilon_{N10} + \alpha T \\ \epsilon_{N2} &= \epsilon_{N20} - \frac{\alpha}{4s_0^2} U \\ \epsilon_{N3} &= \epsilon_{N30} + \frac{\alpha}{4s_0^2} S \,. \end{aligned} \tag{180}$$

Note that S, T and U have only meaning in terms of a reference set of values for m_t and m_H, while our epsilons are defined from measurable quantities without need of specifying given values of m_t and m_H (because our reference point is the improved Born approximation given by the Standard Model at tree level plus pure QED corrections which is independent of m_t and quite well supported by all present data). The quantities $\epsilon_{N10}, \epsilon_{N20}$ and ϵ_{N30} are the Standard Model values at the reference point, where also ϵ_b would have to be fixed.

It is interesting to observe that the central value of ϵ_b is always positive, while

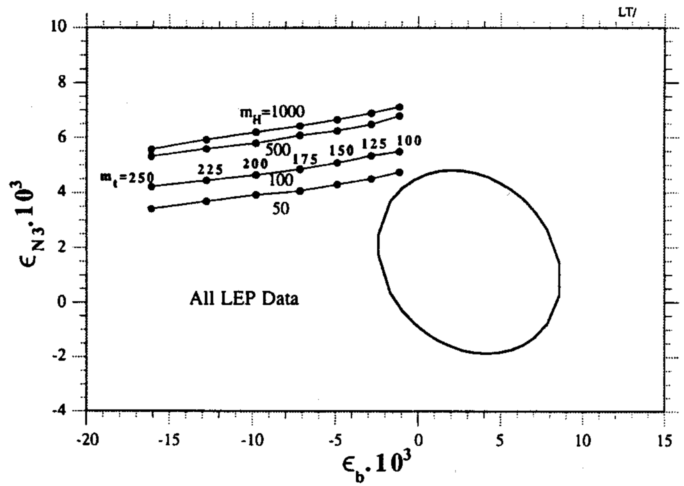

Fig. 20. Plots of ϵ_{N3} versus ϵ_b for the fit presented in Table 4, column 3 (all LEP data). The projections on the axis of the ellipses correspond to the 1σ errors displayed in Table 4. The curves refer to the Standard Model predictions for different values of m_t and m_H with $\alpha_s(m_Z) = 0.118 \pm 0.007$.

the predictions of the Standard Model are negative for $m_t > 100$ GeV (Fig. 16). Thus, besides ϵ_{N3} we have identified another interesting variable for constraining models of new physics: those extensions of the Standard Model that push the prediction of ϵ_b in the positive direction are favoured. Another interesting consequence of this result is that one can obtain a rather strong limit on m_t, independent of $\Delta\rho_N$ (i.e. ϵ_{N1}). Suppose in fact that one assumes that new physics leads to arbitrary effects in vacuum polarization functions but that all vertices are instead negligibly affected. Then our analysis would apply and, moreover, ϵ_b would be described by the Standard Model prediction as a function of m_t. For $\alpha_s(m_Z) = 0.118 \pm 0.007$, we obtain from ϵ_b the limit

$$m_t < 195 \text{ GeV} \qquad (95\% \text{ c.l.}) \tag{181}$$

By a similar approach a weaker limit was obtained in Ref. [113] (where only the data on Γ_b and R were taken into account, while also the results on Γ_Z and σ_h have a sizeable impact). There may be some question on the meaning of the confidence level assigned to this bound as the data on ϵ_b at 1σ intersect the Standard Model theoretical curve in a region of m_t, which is mostly excluded by the CDF limit : we have taken the ratio of the Gaussian tail above the limit and the area from 108 GeV to infinity to be 5%. If we did not know that the region below 100 GeV is forbidden, the corresponding limit would be $m_t < 160$ GeV. Note that if we perform a complete fit in the Standard Model of the same set of data, with $\alpha_s(m_Z) = 0.118 \pm 0.007$, we obtain the results $m_t = 132 \pm 22$ GeV for $m_H = 50$ GeV and $m_t = 175 \pm 18$ GeV for

$m_H = 1$ TeV, with a better χ^2 for the light Higgs case (3.0 vs. 7.1).

It is important to stress that the fitted value of ϵ_b and its excess with respect to the Standard Model prediction for realistic values of m_t depend on the input range of $\alpha_s(m_Z) = 0.118 \pm 0.007$, which has been derived from all existing experiments, assuming the validity of QCD [103]. It is interesting to repeat the fit without constraining $\alpha_s(m_Z)$. By considering all LEP data (as for the results in Eqs. (174) one obtains in this case:

$$\begin{aligned}\epsilon_{N1} &= \Delta\rho_N = (0.03 \pm 3.3) \times 10^{-3} \\ \epsilon_{N3} &= (1.5 \pm 3.4) \times 10^{-3} \\ \epsilon_b &= (4.7 \pm 8.1) \times 10^{-3} \\ \alpha_s(m_Z) &= 0.115 \pm 0.015 \ .\end{aligned} \tag{182}$$

(Note that a fit of the same data in the Standard Model with $\alpha_s(m_Z)$ unconstrained gives $\alpha_s(m_Z) = 0.128 \pm 0.009$ [98]). By comparison with Eqs. (174) we see that, first, ϵ_{N1} and ϵ_{N3} change by only a small fraction of their errors. Second, the central value of ϵ_b becomes even more positive, but the error increases. Third, the value of $\alpha_s(m_Z)$ goes down to a central value, in perfect agreement with the world average. As a matter of fact, when $\alpha_s(m_Z)$ is a free parameter, ϵ_b is essentially determined from Eqs. (172) by R_{bh} alone, which is almost α_s-independent, with ϵ_{N1} and ϵ_{N3} fixed by Γ_ℓ and $x = g_V/g_A$. Thus, in principle, one can experimentally decide if the observed excess of ϵ_b is due to $\alpha_s(m_Z)$ or to the $Z \to b\bar{b}$ vertex, because the pattern of deviations for the various observables is different in the two cases. At present the precision of the data, especially those on R_{bh}, is not sufficient to really decide this issue. The data change rapidly, so the picture could be soon modified, but the present data, taken at face value, suggest that the long-standing result that the central value of $\alpha_s(m_Z)$ comes out a bit too large from the line-shape (or the value of R in Table 3) could be due to an anomaly in the $Z \to b\bar{b}$ vertex.

The decrease of ϵ_{N3} with respect to the Standard Model prediction is possible in many models, for example extended gauge models [88] or the Minimal Supersymmetric Standard Model (MSSM) [89],[90]. In particular, it has been shown in Ref. [91], that in the MSSM a sizeable reduction of ϵ_{N3}, of the order of the measured shift, is induced by a Z wave function renormalisation effect if electroweak gaugino-higgsinos ('charginos') are close to their experimental limits. This effect, by itself, would reduce by the same percentage all widths. Then it would not at all affect R_{bh}, so that it would not cure the apparently high value of ϵ_b. In the MSSM there are corrections to the $Z \to b\bar{b}$ vertex from charged Higgs exchange and from chargino and s-top exchange [114],[115]. Charged Higgs exchange goes in the direction of suppressing $\Gamma(Z \to b\bar{b})$ [114]. Chargino and s-top exchange can overcompensate this effect for light charginos and s-top [115]. However, even for charginos near their experimental limit and for a s-top mass $m_{st} = 100$ GeV the resulting effect is small, especially for a not too heavy top quark, and of marginal importance with respect to the experimental error. We also note that, the vertex corrections (other than the $Z \to b\bar{b}$ vertex) being negligible in this case, the determination of the epsilons from all LEP data given in Table 4 applies. On the contrary, due to the strong q^2 dependence of the Z wave function

renormalisation in the case of gauginos and higgsinos close to threshold, the low energy data cannot be included.

In conclusion, especially in view of the envisaged accuracy that will be obtained by the electroweak tests at the end of the LEP1 phase, the study of the epsilons can provide important constraints and possibly also positive clues on new physics beyond the Standard Model. The isolation of different sectors of the theory, as allowed by the epsilons, may help to clarify the physical origin of any deviation from the Standard Model that will possibly be found or to discard theories that require some specific pattern of radiative effects on the epsilons which turns out to be incompatible with the data.

Finally, I most warmly thank the Organizing Committee of the Schladming School for their kind invitation and for the very nice hospitality received in this magnificient mountain resort.

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