THE START-UP RESPONSE OF PIPE FLOW TO A STEP CHANGE IN FLOW RATE

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The start-up response of pipe flow to a step input of constant flow rate given by an automatic solenoid valve has been studied experimentally by the use of electrochemical technique. The variation of velocity distribution and velocity gradient at the pipe wall with the passage of time has been measured far downstream from the inlet section, where flow becomes fully developed in the steady state. The velocity profiles of the start-up flow development show a trend essentially different from those of steady-state flow development at different distances in the entrance region of a circular pipe: they show a minimum at the axis and a maximum in the intermediate region between the axis and the wall as the result of non-uniformity of acceleration in the central core (annular jet effect). The development of the laminar boundary layer with time could be regarded as that of the constant-stress layer near the wall. Still, the velocity profiles in the laminar boundary layer at different times are similar to each other.

Introduction

A fluid is contained in a very long horizontal pipe. Initially the fluid is at rest. At time $t=0$ the step input of a constant flow rate is introduced into the pipe. Owing to viscous friction, an initially very thin laminar boundary layer is formed on the pipe wall and its thickness increases with time. The fully-developed velocity profile is established after the edge of the boundary layer coincides with the axis of the pipe.

Correspondingly, the pressure gradient in the period of flow development must differ from that of a fully-developed flow.

The start-up problem of this kind is of considerable industrial significance, particularly in the analysis of the performance of hydraulic systems. It will also be of importance in re-examining models of processes that involve both time and a spatial coordinate. Unfortunately, no general relationship is available to predict the transient start-up response and to develop a theoretical model.

Previous theoretical investigations[1,2,6], most of which have been done for the transient start-up following the sudden imposition of a constant pressure gradient to a fluid at rest, remain unproved by measurements.

This type of acceleration should be carefully distinguished from the acceleration of a fluid in the entrance region of a pipe in steady state.

The present experimental research is concerned with the transient start-up response following the step change in flow rate from zero to a fixed quantity. Naturally, in this case the question arises whether there exists an analogy between unsteady start-up flow development and steady flow development in the entrance region of a circular pipe.

The experiments have been performed by the use of electrochemical technique[3,5].

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Experimental Apparatus and Instrumentation

The experimental apparatus is shown schematically in **Fig. 1**. The flow system has been constructed from materials which are inert in the presence of the electrolyte. A centrifugal feed pump and an automatic electromagnetic valve (solenoid valve) are used to introduce a step change in flow rate into the pipe. An acrylic resin convergent nozzle with a contraction ratio of 49: 1 precedes the approximately 4.6 m-long entry section and the test section consisting of 28 mm I.D. stainless steel pipes. The inner surface of the nozzle exit has been smoothed flush with the pipe wall to provide an inlet which is made free from disturbances. A 450 mm long section of stainless pipe downstream of the test section serves as the anode. Discontinuities at joints have been carefully eliminated so that flow through the pipe line could be considered to become fully developed. The remainder of the flow lines are PVC plastic piping.

Velocity electrode

The variation of velocity distribution with time is measured by traversing a blunt-nose cathode probe at the test section ($x/D=165.5$). The tip of the velocity electrode has a glass-fused, 0.10 mm dia. platinum wire, as shown in **Fig. 2**.

The following calibration relationship is necessary to obtain the fluid velocity (α, β: experimental constants).

$$i=\alpha+\beta u^{1/2} \quad (1)$$

Velocity-gradient electrode

The variation of velocity gradients at the wall with time is measured with very small rectangular platinum cathodes—as shown in Fig. 2—which have been embedded in the inert, stainless steel pipe wall. 0.106 mm thick platinum sheets have been inserted and cemented into approximately 0.5 mm wide and 32 mm long circular arc slits hollowed normally to flow direction. The protruding sheets have been sanded down with progressively finer grades of emery paper and buffed to be flush with the wall surface of the pipe. This operation produces a velocity-gradient electrode which is a small rectangular platinum surface having the contour of the pipe wall.

The following equation is employed to calculate the local velocity gradient at the wall:

$$s=1.90(i/FAC)^3(L/\boldsymbol{D}^2) \quad (2)$$

The entry section and the test section have twelve velocity-gradient electrodes along the axis altogether.

Electrolyte fluid

The working fluid is an aqueous solution having equimolar concentrations (about 0.01 M) of potassium ferri- and ferrocyanide and a 1 M concentration of sodium hydroxide. It has a kinematic viscosity of approximately 1.10×10^{-2} cm^2/sec in the temperature range of these experiments.

An electrolytic reaction is carried out between a velocity electrode or a velocity-gradient electrode and a much larger anode pipe downstream. The electric circuit being used is shown in **Fig. 3**.

The problems encountered in measuring the velocity field by the use of electrochemical technique are similar to those encountered in using the hot-wire anemometer: the time response, the uniformity of the flow over the probe, and the effect of nonlinearities. These problems have been discussed in the literature[3,5]. However, there are almost no studies of the response of mass transfer to step-change flow except for the velocity electrode experiments of Ito *et al.*[4]. Judging from the start-up response curves obtained for the smallest velocity change $u=5$ cm/sec, the time constant could be overestimated to be 0.1 sec for the velocity electrode and 0.3 sec for the velocity-gradient electrode in the experimental range. It can be seen from Fig. 5 in their paper[4] that the time constant of the smallest electrode is about 0.03 sec for the velocity change $u=5$ cm/sec. The

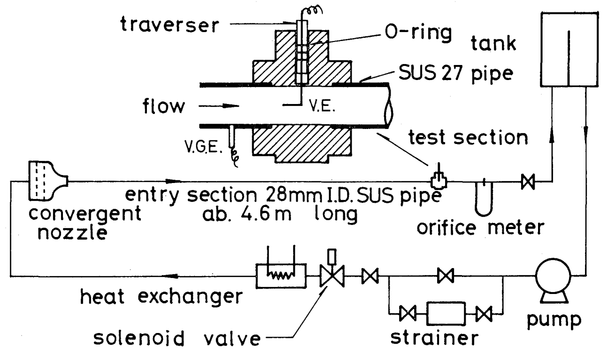

Fig. 1 Experimental apparatus

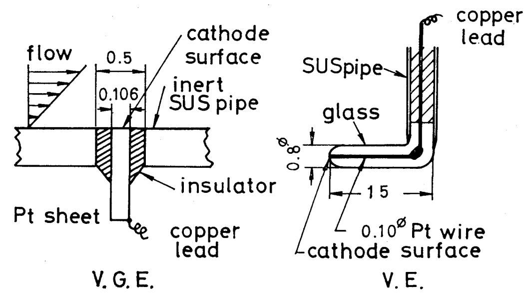

Fig. 2 Velocity electrode and velocity-gradient electrode

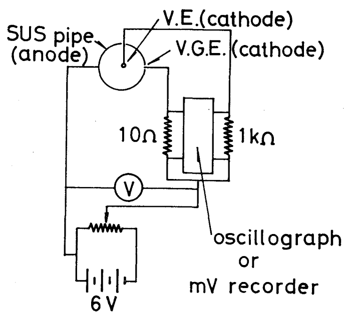

Fig. 3 Electrode circuits

blunt-nose velocity electrode used in the present work is different in shape from the cylindrical electrode of Ito *et al.*, but the former is much smaller than the latter. The above overestimation of the time constant is valid.

Experimentally it is impossible to produce an exact step function, but an input with a fast rise time compared to process response time can be produced, so that the step function can be reasonably approximated. The step change in flow rate used has been successfully confirmed by performing a preliminary measurement of the velocity just a bit below the exit of the convergent nozzle, except for the oscillations at the start at low Reynolds numbers. The result is shown in **Fig. 4**.

Experimental Results

First of all, the velocity distribution in steady

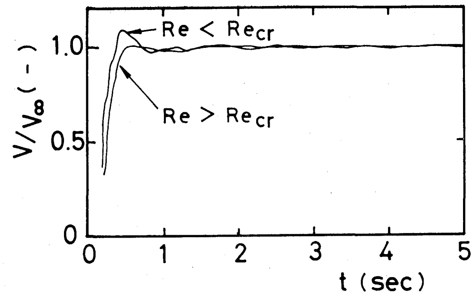

Fig. 4 Step change in flow rate

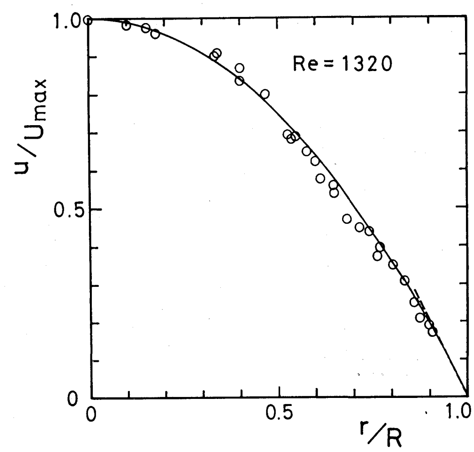

Fig. 5 Parabolic velocity distribution of steady-state laminar flow

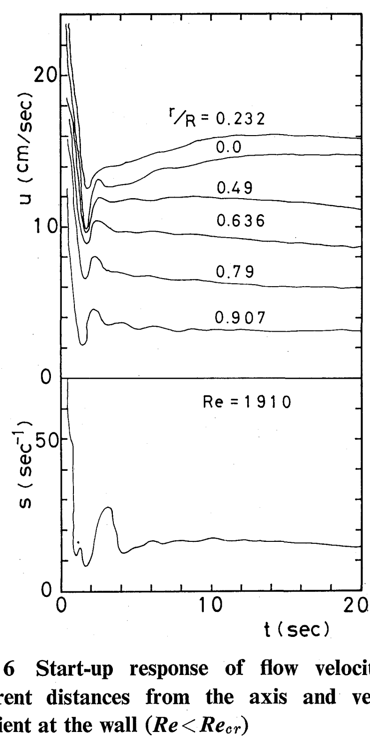

Fig. 6 Start-up response of flow velocity at different distances from the axis and velocity gradient at the wall ($Re < Re_{cr}$)

state was measured only in the downstream section (test section) of the pipe (at $x/D=165.5$), where the flow would become fully developed. The experimental result, as shown in **Fig. 5**, is reduced to the parabolic velocity distribution that characterizes fully-developed laminar flow.

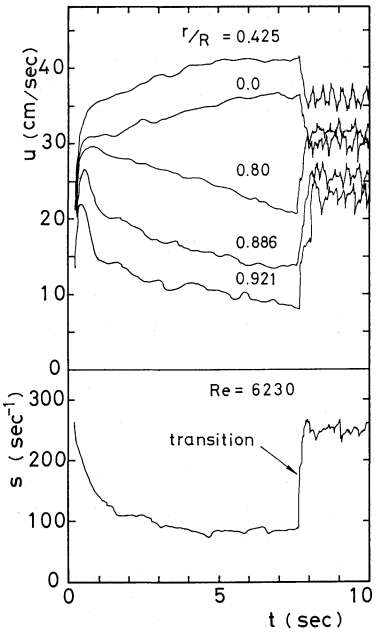

Fig. 7 Start-up response of flow velocity at different distances from the axis and velocity gradient at the wall ($Re > Re_{cr}$)

Start-up response was measured in the test section to two types of step input of flow rate: the first type was for such a small flow rate that the fully-developed flow would remain laminar ($Re < Re_{cr}$), and the second was for such a large flow rate that the transition from laminar to turbulent flow would occur at an intermediate instant ($Re > Re_{cr}$).

Figures 6 and **7** show the variation of flow velocity at different distances r/R from pipe axis and velocity gradient at the pipe wall. At lower Reynolds numbers ($Re < Re_{cr}$), there appear remarkable oscillations at the start, which die out in a very short period of several seconds. Auxiliary experiments have been performed to explore what such initial oscillations indicate. With the aid of a miniature pressure transducer using Piezo resistance effect (TOYODA PMS5-1H), the static pressure has been measured through the pressure tap (1 mm dia. hole) at $x/D=145$. As shown in **Fig. 8**, the variation of static pressure also shows similar oscillations only at lower Reynolds numbers. In addition, it has been found by visual observation using immiscible droplets that such oscillations come from the oscillations in flow rate, and fluid velocities vary in the same phase over the pipe cross-section.

The time which elapses before the oscillation disappears shortens with increasing Reynolds number: about 4 sec at $Re=1910$ and about 2.5 sec at $Re=2300$. Still, the time is quite small compared with the process response time in which the entire change takes place. After the decay of the initial oscillations, the laminar boundary layer increases in thickness, while the central core beyond the boundary layer is accelerated to compensate the decrease in the rate of flow near the wall.

On the other hand, it should be noted from Fig. 7 that there do not appear any initial oscillations from the start at higher Reynolds numbers ($Re > Re_{cr}$). At the start a laminar boundary layer is formed on the wall, and it grows with time. At an intermediate instant there suddenly appear random oscillations that characterize the generation of turbulence. However, it should be noted in Fig. 8 that even at $Re > Re_{cr}$ there is no transition effect in the variation of the static pressure. In general, the time required for the transition from laminar to turbulent flow shortens with increasing Reynolds number, just as does the transition length in the entrance region of steady-state pipe flow. Nevertheless, the transition time ranged very widely even at the same Reynolds number: transition time $t_{tr} = 7.5 \sim 14$ sec (avg. value $=$ 7.6 sec) at $Re = 6230$ and $t_{tr} = 2.5 \sim 9$ sec (avg. value $=$ 3.3 sec) at $Re = 11900$. After the transition to turbulent boundary layer, the boundary layers gradually merge into each other at the center line, and finally the flow reaches its fully-developed turbulent form. It actually took about thirty seconds to attain fully-developed turbulent flow after a step change in the input.

As shown in **Figs. 9** and **10**, the development of the velocity distribution can be illustrated by a sequence of velocity profiles at various times after the disappearance of initial oscillations. It was very difficult to measure the initial velocity distribution. However, as can be seen from the velocity profile at $\theta = 2.81 \times 10^{-3}$ (i.e., $t = 0.5$ sec) in Fig. 10, the initial velocity profile may be approximated by a constant uniform velocity.

The velocity profiles are seen to show a trend essentially different from those in the entrance region of steady laminar pipe flow: the peculiar velocity distribution is due to non-uniformity of acceleration in the central core. The velocity profiles of the present results show a minimum at the axis and a maximum in the intermediate region ($0 < r/R < 1$), i.e., at the edge of boundary layer, while those in the entrance region of steady pipe flow always show a maximum at the axis ($r/R = 0$) decreasing monotonically to the wall ($r/R = 1$). This phenomenon can be called the "annular jet effect."

The flow rate in question could be kept approximately constant at all times from the start of the motion until the transition occurred. But after the transition to turbulent flow, it gradually decreased, asymptotically approaching another steady flow rate.

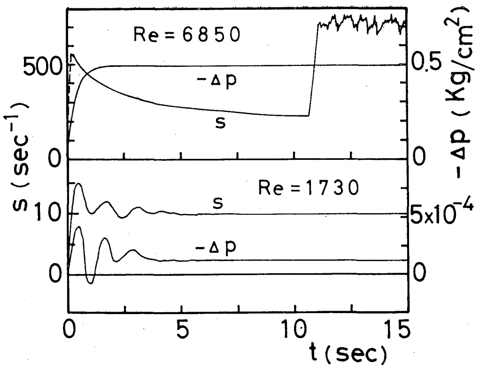

Fig. 8 Static pressure change

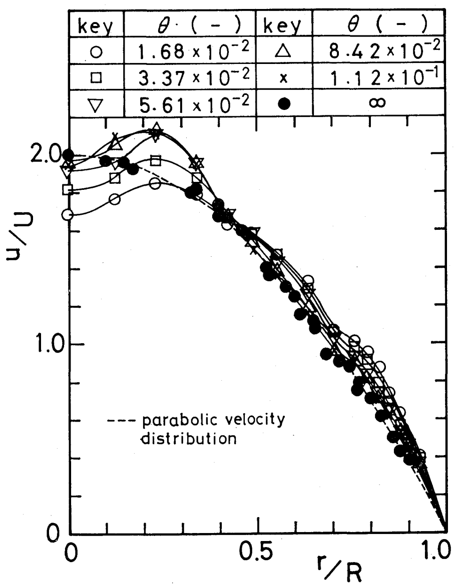

Fig. 9 Development of the velocity distribution (*Re*=1910)

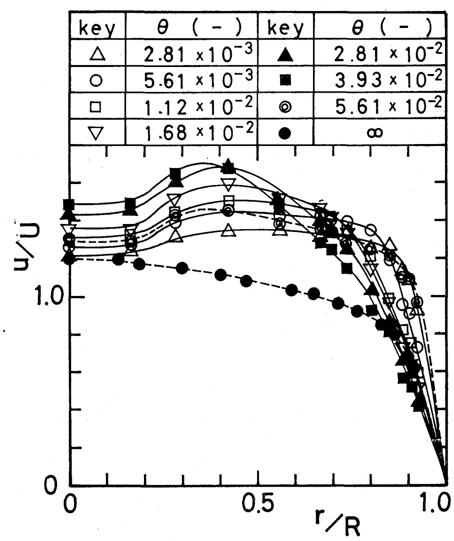

Fig. 10 Development of the velocity distribution (*Re*=6230)

As the result, the fully-developed turbulent velocity distribution has gone down a bit in Fig. 10.

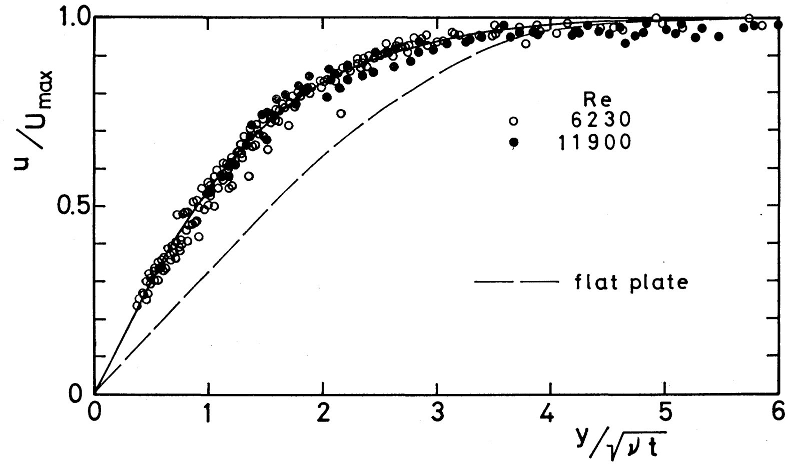

Fig. 12 Velocity distribution in the boundary layer developing with time

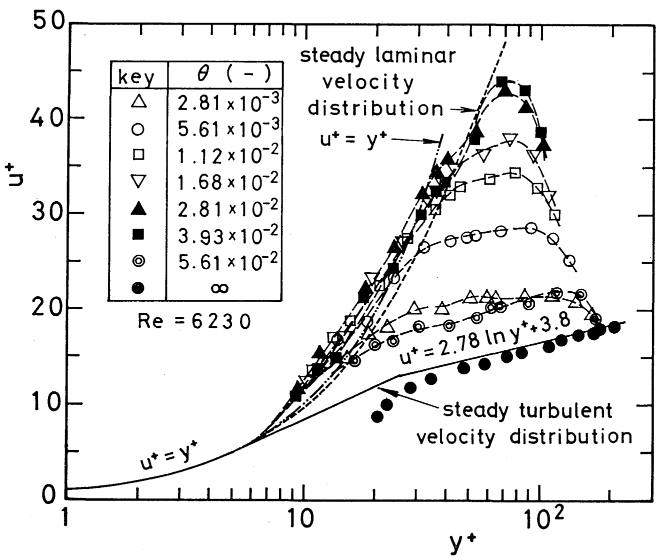

Fig. 11 u^+ vs. y^+ plot of the developing velocity distribution

Figure 11 shows a kind of universal velocity profiles plotted with the aid of the velocity-gradients measured at every instant. Each experimental curve on the wall side coincides with the extension of the velocity curve $u^+=y^+$, which can be obtained by integrating Newton's law of viscosity with the assumption that $\tau \doteqdot \tau_0$ over the laminar sublayer. The development of the laminar boundary layer could be regarded as that of the constant-stress layer near the wall. As aforementioned, the fully-developed velocity distribution should have been reduced to the logarithmic velocity distribution if the flow rate could have been kept constant also after the transition.

By using $\sqrt{\nu t}$ as a measure of momentum penetration thickness, the velocity distribution in the laminar boundary layer was re-plotted in **Fig. 12**. It can be seen that there still exists similarity among the velocity profiles at different times. It is also worth noting here that this curve representing similar velocity profiles shows a trend of the boundary layer with accelerated external flow: the velocity distribution is flatter near the edge of the boundary layer and steeper near the wall than that for a flat plate in tangential flow.

Still, considerable work remains to be done in studying systematically the non-uniformity of acceleration in the central core.

Conclusion

A new kind of acceleration (annular jet effect) has been found to occur in start-up pipe flow to a step input of constant flow rate.

The velocity profiles of this kind of start-up flow development show a trend different from those of steady-state flow development at different distances in the entrance region: they show a minimum at the axis and a maximum in the intermediate region between the axis and the wall.

The development of the laminar boundary layer with time can be regarded as that of the constant-stress layer near the wall.

Still, the velocity profiles in the boundary layer at different times are similar to each other.

Nomenclature

A	= surface area of test electrode	[cm²]
C	= bulk concentration of ferricyanide ion	[g-equiv./cm³]
D	= pipe diameter	[cm]
D	= diffusivity of ferricyanide ion	[cm²/sec]
F	= Faraday constant	[coul/g-equiv.]
i	= electric current	[amp]
L	= length of test electrode	[cm]
p	= static pressure	[Kg/cm²]
R	= pipe radius	[cm]
Re	= DU/ν; Reynolds number	[—]
r	= radial coordinate	[cm]
s	= $(\partial u/\partial y)_{y=0}$; velocity gradient at pipe wall	[1/sec]
t	= time	[sec]
U	= average velocity	[cm/sec]
U_{max}	= maximum velocity	[cm/sec]
u	= flow velocity	[cm/sec]
u^*	= $\sqrt{\tau_0/\rho}$; friction velocity	[cm/sec]
u^+	= u/u^*; dimensionless velocity	[—]

V	=	volumetric flow rate	[cm³/sec]
V_∞	=	steady-state volumetric flow rate	[cm³/sec]
x	=	axial coordinate	[cm]
y	=	distance from pipe wall	[cm]
y^+	=	yu^*/ν; dimensionless distance from pipe wall	[—]
θ	=	$t/(R^2/\nu)$; dimensionless time	[—]
ν	=	kinematic viscosity	[cm²/sec]
ρ	=	density	[g/cm³]
τ_0	=	shear stress at pipe wall	dyne/cm²]

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HEAT-TRANSFER IN A TAYLOR VORTEX FLOW

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The characteristics of heat transfer affected by Taylor vortex motion were investigated experimentally by the use of electrochemical method and theoretically with the aid of the non-linear theory suggested by J. T. Stuart. The present non-linear theory, which considers the effect of the fundamentals of disturbances on heat transfer, especially in the wide gap problem, is also applicable to the heat-transfer problem in the range of $1 \leqq Re/Re_{cr} \leqq 2$ for fluid with $Pr=1$. The local Nusselt numbers (Sherwood numbers) measured by means of electrochemical technique show a remarkable sinusoidal periodicity in the axial direction due to Taylor vortex motion, except for their cycloidal periodicity in a very short supercritical range of Reynolds number.

The values of skin friction and heat-transfer coefficient calculated by the semiempirical modification of the equilibrium amplitude of supercritical disturbances agree quite well with measurements over an unexpectedly wide range of Reynolds number ($2 \leqq Re/Re_{cr} \leqq 20$) where higher-order disturbances should be taken into account.

Introduction

The present paper is part of the second phase of an investigation of the transport phenomena which control the rate of heat transfer in an annulus formed by two concentric cylinders with rotation of the inner cylinder. Consideration is restricted to the case of no axial flow of fluid in the annulus. Taylor[9,10], in a very early paper, investigated both theoretically and experimentally the instability of viscous isothermal flow in the narrow annulus between two concentric rotating cylinders and found that, when the Reynolds number exceeds a critical value, the disturbance takes the form of pairs of counter-rotating toroidal vortices (Taylor vortices) spaced regularly along the axis of the cylinders. A sketch of this flow pattern is shown in **Fig. 1**.

A number of experiments have tried to correlate the friction factor and over-all heat-transfer coefficient of such a system with the so-called Taylor number. However, there are almost no experimental investigations available of the local variation of heat- or mass-transfer due to Taylor vortex motion. One of the objectives of the present paper is to measure the local rate of heat-transfer enhanced by such a vortex motion. The experiments are performed by the electrochemical method under an assumption of analogy between heat and mass transfer. Measurements can be regarded as those of heat transfer to a constant-temperature wall from a fluid of large Prandtl number.

On the other hand, Stuart[11,12] introduced an

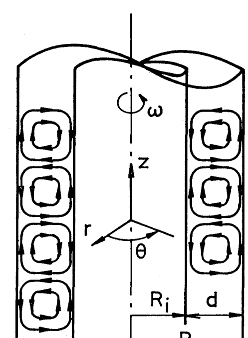

Fig. 1 Flow configuration of Taylor system of vortices and the coordinate system

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