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Research Article

THE STATIONARY ANALYSIS OF A RETRIAL QUEUE WITH MULTISERVER IN n-LIMITED CAPACITY

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ABSTRACT

In this paper, we study the stationary analysis of the model M/M/3/n+1 with linear retrial rates and with state dependent parameters by introducing the bivariate process $\{(C(t), Q(t)), t \geq 0\}$. Some numerical results are also presented.

INTRODUCTION

A retrial queueing system is described by an arriving customer, finds the server busy, joins the retrial group to try again for service after a random amount of time. Retrial queueing systems have been widely used to model many problems in modern telephone switching systems, computer and communication systems. For detailed survey one can see yang and Templeton (1987). Most papers assume that each orbiting customer seeks service independently of each other after a random time exponentially distributed with a fixed rate. Nevertheless, there are other queueing situations in which the retrial rate does not depend on the number of customers in the orbit. Some notable works in this directions are Fayolle (1986) and Martin and Artalejo (1995). Artalejo and Gomez-Corral (1997), in their paper incorporate both possibilities by assuming that time intervals between successive repeated attempts are exponentially distributed with parameter $\alpha(1-\delta_{0j})+j\mu$, when the orbit size is j.

Gomez-Correl and Ramalhoto (1999) assumed the time intervals between successive repeated attempts to be exponentially distributed with parameter $\alpha_i(1-\delta_{0j})+j\mu_i$, and they find the stationary distributions of the bivariate Markov processes associated with *M/M*/2/2+1 and *M/M*/3/3 queues.

The purpose of this paper is to analyse the retrial queueing model *M/M*/3/n+1 using the technique of Gomez-Correl and Ramalhoto (1999).The rest of the article is organized as follows: We describe the Mathematical model in section 2. In section 3, we carry out the stationary analysis *M/M*/3/n+1 retrial queueing model. Section 4 contains some numerical results corresponding to the model in section 3.

MATHEMATICAL MODEL

We consider a retrial queueing system with c servers and d waiting positions. When the c servers are busy, an arriving customer (called primary customer) occupies a waiting position and, when one server becomes free, one of the waiting customers immediately enter the servers. Otherwise, When the c servers are busy and the d waiting positions are occupied, the customer immediately enter the orbit (called orbit customer). The state of the system at time t is described by the bivariate process $\{(C(t), Q(t)), t \geq 0\}$, where C(t) is the total number of servers and waiting position occupied and Q(t) denotes the number of orbiting customers. The model is denoted by *M/M*/r/r+d. The arrival rates of the primary customer is λ_i if C(t) = i and the rate of orbit customer equals β_{ij} where C(t) = i and Q(t) = j. The service rate equals ν_i when C(t) = i. The state space S = {0, 1, 2, ... c} x Z_+ noted in Asmussen (1987), of the Markov process $\{X(t), t \geq 0\}$ is ergodic if and only if there exists a probability solution $P = \{(P_{0j}, P_{1j}, \ldots P_{cj}), j \geq 0\}$ to equality PQ = 0, where Q is the infinitesimal matrix of the process $\{X(t), t \geq 0\}$. In this case the vector P is the stationary distribution of $\{X(t), t \geq 0\}$. In section 3, we take C(t) = i, $i \in \{0, 1, 2, 3, 4\}$ called model 1.

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The Analysis

Let $\{X(t), t \geq 0\}$ be a time homogeneous Markov process, where X(t) the bivariate process (C(t), Q(t)), C(t) is the number of customers in the system and Q(t) is the number in the orbit. Here the bivariate limit process X takes values on the lattice semi-strip S = {0, 1, 2, 3, 4} x Z_+. The infinitesimal matrix is $Q = (q_{ij})$, where

$$Q = \begin{pmatrix} A_0 & C & 0 & 0 & 0 & \ldots \\ B_1 & A_1 & C & 0 & 0 & \ldots \\ 0 & B_2 & A_2 & C & 0 & \ldots \\ 0 & 0 & B_3 & A_3 & C & \ldots \\ 0 & 0 & 0 & B_4 & A_4 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{pmatrix}$$

with $(A_0+C)e = o$, $(B_i + A_i + C)e = 0$, $e = (1,1,1, \ldots)'$,

$$A_i = \begin{pmatrix} -(\lambda_0+\beta_{0i}) & \lambda_0 & 0 & 0 & 0 \\ \beta_1 & -(\lambda_1+\nu_1+\beta_{1i}) & \lambda_1 & 0 & 0 \\ 0 & \nu_2 & -(\lambda_2+\nu_2+\beta_{2i}) & \lambda_2 & 0 \\ 0 & 0 & \nu_3 & -(\lambda_3+\nu_3+\beta_{3i}) & \lambda_3 \\ 0 & 0 & 0 & \nu_4 & -(\lambda_4+\nu_4) \end{pmatrix}$$

$$B_i = \begin{pmatrix} 0 & \beta_{0i} & 0 & 0 & 0 \\ 0 & 0 & \beta_{1i} & 0 & 0 \\ 0 & 0 & 0 & \beta_{2i} & 0 \\ 0 & 0 & 0 & 0 & \beta_{3i} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 \end{pmatrix}$$

$i = 0, 1, 2, 3, \ldots$, $\beta_{10} = 0$, $\beta_{20} = 0$, and $\beta_{30} = 0$, where $\beta_{ij} = \alpha_i(1-\delta_{oj})+j\mu_i$.

The Markov process $\{X(t), t \geq 0\}$ is Ergodic if and only if there exist a solution $P = (P_0, P_1, P_2, \ldots P_i, \ldots)$, where $P_i = (P_{io}, P_{i1}, P_{i2}, P_{i3}, P_{i4})$, the matrix equation.

$PQ = 0$ ... (3.1)

This is equation to

$(\lambda_0+\beta_{oj}) P_{0j} = \nu_1 P_{1j}, j \geq 0$ ... (3.2)

$(\lambda_1+\nu_1+\beta_{1j}) P_{1j} = \lambda_0 P_{0j} + \beta_{0j+1} P_{0j+1} + \nu_2 P_{2j}, j \geq 0$ ... (3.3)

$(\lambda_2+\nu_2+\beta_{2j}) P_{2j} = \lambda_1 P_{1j} + \beta_{1j+1} P_{1j+1} + \nu_3 P_{3j}, j \geq 0$ ... (3.4)

$(\lambda_3+\nu_3+\beta_{3j}) P_{3j} = \lambda_2 P_{2j} + \beta_{2j+1} P_{2j+1} + \nu_4 P_{4j}, j \geq 0$ ... (3.5)

$(\lambda_4+\nu_4) P_{4j} = \lambda_3 P_{3j} + \beta_{3j+1} P_{3j+1} + \lambda_4 P_{4j-1}, j \geq 0$ ... (3.6)

We define generating functions

$P_i(z) = \sum_{j=0}^{\infty} P_{ij} z^j$, i = 0, 1, 2, 3, 4 ... (3.7)

Applying (3.9) on both sides of (3.2), (3.3), (3.4), (3.5), (3.6), we get

$(\lambda_0+\alpha_0)\, P_0(z) + \mu_0 z P_0^{'}(z) = \nu_1 P_1(z) + \alpha_0 P_{00}$ ... (3.8)

$\mu_1 z^2 P_1^{'}(z)+(\lambda_1+\nu_1+\alpha_1)zP_1(z)+\alpha_0P_{00}=\mu_0 zP_0^{'}(z)+(\lambda_0 z+\alpha_0)P_0(z)+\nu_2 zP_2(z)+\alpha_1 zP_{10}$ ... (3.9)

$\mu_2 z^2 P_2^{'}(z)+(\lambda_2+\nu_2+\alpha_2)zP_2(z)+\alpha_1P_{10}=\mu_1 zP_1^{'}(z)+(\lambda_1 z+\alpha_1)P_1(z)+\nu_3 zP_3(z) + \alpha_2 zP_{20}.$ .. (3.10)

$\mu_3 z^2 P_3^{'}(z)+(\lambda_3+\nu_3+\alpha_3)zP_3(z)+\alpha_2P_{20}=\mu_2 zP_2^{'}(z)+(\lambda_2 z+\alpha_2)P_2(z)+\nu_4 zP_4(z)+ \alpha_3 zP_{30}$ ... (3.11)

$(\lambda_4+\nu_4 - \lambda_4 z)zP_4(z)+\alpha_3P_{30}=\mu_3 zP_3^{'}(z)+(\lambda_3 z+\alpha_3)P_3(z)$ ... (3.12)

Multiplying (3.9) to (3.12) by z^{-1} and adding the resulting equalities and (3.8), we get.

$\lambda_4 zP_4(z) = \sum_{i=0}^{3} \mu_i zP_i^{'}(z)+\alpha_i(P_i(z)-P_{i0})$ ... (3.13)

Differentiating the equation (3.8) with respect to z

$\nu_i P_i^{'}(z)=(\lambda_0+\alpha_0+\mu_0)P_0^{'}(z) +\mu_0 z\, P_0^{''}(z)$ ... (3.14)

From (3.2), (3.8) and (3.14) we can write (3.9) as

$\nu_1\nu_2 zP_2(z) = \mu_0\mu_1 z^3 P_0^{''}(z) + \{((\lambda_0 + \alpha_0 + \mu_0)\, \mu_1+\mu_0(\lambda_1+\alpha_1+\mu_1))z^2$ ... (3.15)

$-\mu_0\nu_1 z\}P_0^{'}(z)+(((\lambda_0+\alpha_0)(\lambda_1+\alpha_1)+\alpha_0\nu_1)z-\alpha_0\nu_1)P_0(z)$ $+(-(\alpha_0(\lambda_1+\nu_1)+(\lambda_0+\alpha_0)\alpha_1)z+\alpha_0\nu_1)P_{00}$

Differenting (3.15 with respect to z and multiply the resulting relation by z and after some algebraic manipulation we get,

$\nu_1\nu_2 z^2 P_2^{'}(z) = \mu_0\mu_1 z^4 P_0^{'''}(z) + \{((\lambda_0 + \alpha_0 +3\mu_0)\, \mu_1+\mu_0(\lambda_1+\alpha_1+\mu_1))z^3$... (3.16)

$\mu_0\nu_1 z^2\}P_0^{''}(z)+(((\lambda_0+\alpha_0+\mu_0)(\lambda_1+\alpha_1+\mu_1)+(\alpha_0+\mu_0)\nu_1)z^2- \alpha_0\nu_1 z)P_0^{'}(z) +$ $\alpha_0\nu_1P_0(z)-\alpha_0\nu_1P_{00}$

Substituting (3.2), (3.3), (3.8) and (3.14) to (3.16) into (3.10) and rearranging leads to the following equality.

$\nu_1\nu_2\nu_3 zP_3(z) = Az^4 P_0^{'''}(z) + (Bz^3+Cz^2)P_0^{''}(z)+(Dz^2+Ez)P_0^{'}(z) + (Fz+G)P_0(z)$ $+ (Hz+I)P_{00} + (\alpha_0+\mu_0)\nu_1\alpha_2 zP_{01}$ ... (3.17)

Where

A	=	$\mu_0\mu_1\mu_2$
B	=	$\mu_0\mu_1(\lambda_2+\nu_2+\alpha_2) + ((\lambda_0 + \alpha_0 +3\mu_0)\, \mu_1+\mu_0(\lambda_1+\nu_1+\alpha_1))\mu_2$
C	=	$- \mu_0(\nu_1\mu_2+\mu_1\nu_2)$,
D	=	$\mu_0((\lambda_1+\nu_1+\alpha_1)\, (\lambda_2 + \alpha_2) + (\nu_1+\alpha_1)\nu_2) + (\lambda_0+\alpha_0+ \mu_0)\, \mu_1(\lambda_2+\alpha_2+\nu_2) + ((\lambda_0 + \alpha_0 + \mu_0)\, (\lambda_1+\alpha_1+\mu_1) + (\alpha_0 +\mu_0)\nu_1)\mu_2$,
E	=	$- (\nu_1(\alpha_0\mu_2+ \mu_0(\lambda_2+\nu_2+\alpha_2))+ ((\lambda_0+\alpha_0)\mu_1+ \mu_0(\alpha_1+\mu_1))\nu_2)$,
F	=	$((\lambda_0+\alpha_0)\, (\lambda_1 + \alpha_1) + \alpha_0\nu_1)\, (\lambda_2+\alpha_2)+((\lambda_0+\alpha_0)\, \alpha_1 + \alpha_0\nu_1)\nu_2$,
G	=	$-(\alpha_0\nu_1\, (\lambda_2+\nu_2+\alpha_2-\mu_2) + (\lambda_0 + \alpha_0)\, \alpha_1\nu_2)$,
H	=	$-(\lambda_0\lambda_1\alpha_2+(\alpha_0(\lambda_0+\nu_1)+ (\lambda_0+\alpha_0)\, \alpha_1)(\alpha_2 + \lambda_2) + (\alpha_0\nu_1 +(\alpha_0+\lambda_0)\alpha_1)\nu_2)$,
I	=	$\lambda_0\alpha 1\nu_2 + (\alpha_0((\nu_2+\lambda_2 + \alpha_2 - \mu_2)+\alpha_1\nu_2)$,

Differenting (3.17) with respect to z and multiply by z, we get

$\nu_1\nu_2\nu_3 z^2\, P'_3(z) = \mu_0\mu_1\mu_2 z^5 P_0^{IV}(z) + ((3\, \mu_0\mu_1\mu_2 + A)\; z^4+ Bz^3)\, P_0^{III}(z)$
$+ ((2A + C)\, z^3 + (B + D)\, z^2)\, P_o^{II}(z)$
$+ ((C + E)\, z^2 + Fz)\, P'_O(z) - GP_o(z) - H\, P_{00}$ ...(3.18)

Substituting (3.2), (3.3), (3.4), (3.9), and (3.15) to (3.18) into (3.11), we get

$\nu_1\nu_2\nu_3\nu_4 z^2\, P_4(z) = G_1 z^6 P_0^{IV}(z) + (G_2 z^5+ G_3 z^4)\, P_0^{III}(z)+(G_4 z^4+G_5 z^3+G_6 z^2)P_0^{''}(z)$

$+(G_7 z^3+G_8 z^2+G_9 z)P_0^{'}(z)+(G_{10} z^2+G_{11} z+G_{12})P_0(z)$

$+ (G_{713} z^2 + G_{14}\, z +G_{15})\, P_{00} + (G_{16} z^2+G_{17} z)P_{01}$ ... (3.19)

where $G_1 = \mu_0\mu_1\mu_2\mu_3,$

$G_2 = ((3\mu_0\mu_1\mu_2 + A)\, \mu_3 + (\lambda_3 +\nu_3 + \alpha_3)\, \mu_0\mu_1\mu_2),$
$G_3 = (B\mu_3 - \nu_3 - \mu_0\mu_1\mu_2),$
$G_4 = ((A + C)\, \mu_3 + A(\lambda_3 +\nu_3 + \alpha_3\mu_3) - \lambda_2\mu_0\mu_1\, \nu_3),$
$G_5 = (B+D)\mu_3+B(\lambda_3+\nu_3+\alpha_3)-\mu_2\nu_3)((\lambda_0+3\mu_0+\alpha_0)\mu_1+\mu_0(\lambda_1+\nu_1+\alpha_1)-\alpha_2\mu_0\mu_1\nu_3,$
$G_6 = \mu_0\mu_2\nu_1\nu_3,$
$G_7 = (C(\lambda_3 +\nu_3 + \alpha_3 + \mu_3)+E\mu_3 - \lambda_2\, \nu_3\, ((\lambda_0 +\mu_0 +\alpha_0)\, \mu_1 + \mu_0\, (\lambda_1 +\nu_1 + \alpha_1)))$
$G_8 = (F\mu_3+(\lambda_3+\nu_3+\alpha_3)D-\nu_3\mu_2((\lambda_0+\mu_0+\alpha_0)(\lambda_1+\mu_1+\alpha_1)+\nu_1(\mu_0+\alpha_0)))$ $+ \lambda_2\nu_3\mu_0\nu_1 - \alpha_2\, \nu_3((\lambda_0 +\mu_0 +\alpha_0)\, \mu_1 +\mu_0(\lambda_1 +\nu_1 + \alpha_1))$
$G_9 = (\mu_2\nu_3\nu_1\alpha_0 + \mu_0\nu_3\nu_1\alpha_2)$
$G_{10} = ((\lambda_3+\nu_3 + \alpha_3)\, E - \lambda_2\nu_3\, ((\lambda_0 + \alpha_0)\, (\lambda_1 + \alpha_1) + \nu_1\alpha_0)),$
$G_{11} = (- F\mu_3 + F\, (\lambda_3+\nu_3 + \alpha_3) + \lambda_2\nu_3\alpha_0\nu_1 - \alpha_2\nu_3\, (\lambda_0 + \alpha_0)\, (\lambda_1 + \alpha_1) + \nu_1\alpha_0)),$
$G_{12} = (-\alpha_0\nu_1\nu_3\, (\mu_2 - \alpha_2)),$

$G_{13} =$ $\{\lambda_2\nu_3 (\alpha_0(\lambda_1+\nu_1) + \alpha_1 (\lambda_1+\alpha_0)) - \alpha_3\alpha_2 \lambda_0 \lambda_1 (\lambda_2 + \nu_2) - \alpha_3 \lambda_0 \lambda_1\nu_2 + (\lambda_3 + \nu_3 + \alpha_3)\}$

$G_{14} =$ $H (\lambda_3 + \nu_3 + \alpha_3 - \mu_3) + \alpha_2\nu_3\lambda_2\lambda_1 - \alpha_0\nu_3\lambda_0\nu_1 + \alpha_2\nu_3 (\alpha_0 (\lambda_1 + \alpha_1) + \alpha_1 (\lambda_0 + \alpha_0))$

$G_{15} =$ $(\alpha_0\nu_1\nu_3 (\mu_2 - \alpha_2))$,

$G_{16} =$ $(\nu_1\alpha_2 (\lambda_3 + \nu_3 + \alpha_3) (\alpha_0+ \mu_0) + \alpha_3\nu_1 (\lambda_2 + \nu_2)(\alpha_0 +\mu_0)) + (\alpha_3\nu_3(\alpha_1+\mu_1) (\alpha_0+ \mu_0 + \lambda_0))$,

$G_{17} =$ $(-\alpha_2\nu_1\nu_3 (\mu_0 + \alpha_0))$,

For convenience of notation, we re-express some previous equations. First, from (3.9) we consider the relation

$$\nu_1\nu_2\nu_3\nu_4 P_1(z) = a_1zP'_0(z)+ a_2P_0(z) + a_3 P_{00}. \qquad ...(3.20)$$

where $a_1 = \mu_0\nu_2\nu_3\nu_4$; $a_2 = (\lambda_0 + \alpha_0) \nu_2\nu_3\nu_4$ and $a_3 = - \alpha_0\nu_2\nu_3\nu_4$,

From (3.14), we have that

$$\nu_1\nu_2\nu_3\nu_4 P'_1(z) = b_1 zP''_0(z) + b_2 P'_0(z) \qquad ...(3.21)$$

where $b_1 = \mu_0\nu_2\nu_3\nu_4$; $b_2 = (\lambda_0 + \alpha_0 + \mu_0) \nu_2\nu_3\nu_4$

we can write the equations (3.15) as

$$\nu_1\nu_2\nu_3\nu_4 zP_2 (z) = c_1z^3P''_0(z) +(c_2z^2 + c_3z) P'_0(z) + (c_4z + c_5) P_0(z) + (c_6z + c_7) P_{00} \qquad ... (3.22)$$

where $c_1 = \mu_0\mu_1\nu_3\nu_4$; $c_2 = ((\lambda_0 + \alpha_0 + \mu_0) \mu_1 + \mu_0 (\lambda_1 + \alpha_1 + \nu_1))\nu_3\nu_4$,

$c_3 = - \mu_0\nu_1\nu_3\nu_4$; $c_4 = ((\lambda_0 + \alpha_0) (\lambda_1 + \alpha_1) + \alpha_0 \nu_1)\nu_3\nu_4$,

$c_5 = - \alpha_0\nu_1\nu_3\nu_4$; $c_6 =-(\alpha_0(\lambda_1+\nu_1)+(\lambda_0+\alpha_0)\alpha_1)\nu_3\nu_4$; $c_7 = \alpha_0\nu_1\nu_3\nu_4$

From (3.16) we deduce that

$$\nu_1\nu_2\nu_3\nu_4z^2P'_2 (z)=d_1z^4P'''_0 (z)+(d_2z^3+d_3z^2)P''_0(z)+(d_4z^2+d_5z)P'_0(z) +d_6P_0(z)+d_7P_{00}, \qquad ...(3.23)$$

where $d_1 = \mu_0\mu_1\nu_3\nu_4$, $d_2 = ((\lambda_0 + \alpha_0 + 3\mu_0) \mu_1 + \mu_0 (\lambda_1 +\nu_1 + \alpha_1)) \nu_3\nu_4$,

$d_3 = \mu_0\nu_1\nu_3\nu_4$, $d_4= ((\lambda_0 +\alpha_0 + \mu_0) (\lambda_1 + \alpha_1 + \mu_1) + (\alpha_0 + \mu_0)\nu_1) \nu_3\nu_4$,

$d_5 = -\alpha_0\nu_1\nu_3\nu_4$, $d_6 = \alpha_0\nu_1\nu_3\nu_4$, $d_7 = -\alpha_0\nu_1\nu_3\nu_4$

From (3.17) we obtain

$$\nu_1\nu_2\nu_3\nu_4 zP_3 (z) = e_1z^4P'''_0(z) + (e_2z^3 + e_3z^2) P''_0(z) + (e_4z^2 + e_5(z) P'_0(z) + (e_6z + e_7) P_0 (z) + (e_8z + e_9) P_{00} + e_{10}zP_{o1} \qquad ...(3.24)$$

where $e_1 = \mu_0\mu_1\mu_2\nu_4$, $e_2 = \nu_4A$. $e_3 = \nu_4 B$, $e_4 = \nu_4C$, $e_5 = \nu_4D$, $e_6 = \nu_4E$, $e_7 = \nu_4F$,

$e_8 = \nu_4G$, $e_9 = \nu_4H$, $e_{10} = (\alpha_0+ \mu_0) \alpha_2 \nu_1\nu_4$

From (3.18),

$$\nu_1\nu_2\nu_3\nu_4 z^2P'_3 (z) = f_1z^5P^{IV}_0 (z) + (f_2z^4 + f_3z^3)P'''_0(z) +(f_4z^3 + f_5z^2) P''_0(z) + (f_6z^2 + f_7z) P'_0(z) + f_8P_0(z) + f_9 P_{00} \qquad ...(3.25)$$

where $f_1 = \mu_0\mu_1\mu_2\nu_4$, $f_2 = (3\mu_0\mu_1\mu_2 +A)\nu_4$, $f_3 = \nu_4 B$, $f_4 = (2A+C)\nu_4$, $f_5 = (B+D)\nu_4$,

$f_6 = (C+E)\nu_4$, $f_7 = F\nu_4$, $f_8 = -\nu_4G$; $f_9 = -\nu_4H$;

From (3.19) we obtain

$$\lambda_4\nu_1\nu_2\nu_3\nu_4 z^2P_4 (z) = g_1z^6P^{IV}_0(z) + (g_2z^5 + g_3z^4) P'''_0(z) + (g_4z^4 + g_5z^3 + g_6z^2) P''_0(z) + (g_7z^3 + g_8z^2 + g_9z) P'_0(z) + (g_{10}z^2 + g_{11}z + g_{12}) P_0(z) + (g_{13}z^2 + g_{14}z + g_{15}) P_{00} +(g_{16}z^2 + g_{17}) P_{o1} \qquad ...(3.26)$$

where $g_1=\lambda_4G_1$, $g_2= \lambda_4G_2$, $g_3=\lambda_4G_3$, $g_4=\lambda_4G_4$; $g_5=\lambda_4G_5$, $g_6= \lambda_4G_6$,

$g_7=\lambda_4G_7$, $g_8=\lambda_4G_8$; $g_9 = \lambda_4G_9$ $g_{10}= \lambda_4G_{10}$;$g_{11} = \lambda_4 G_{11}$ $g_{12}=\lambda_4G_{12}$;

$g_{13} = \lambda_4G_{13}$; $g_{14} = \lambda_4 G_{14}$; $g_{15} = \lambda_4 G_{15}$ $g_{16} = \lambda_4 G_{16}$; $g_{17} = \lambda_4 G_{17}$

Now using the set of equations (3.20) to (3.25) we have that, after some tedious algebra the equality (3.13)can be expressed as follows

$$\lambda_4\nu_1\nu_2\nu_3\nu_4 z^2P_4 (z) = 1_1z^5P^{IV}_0(z) + (l_2z^4 + l_3z^3)P'''_0(z) + (l_4z^3 + l_5z^2) P''_0(z) + (l_6z^2+l_7z)P'_0(z)+(l_8z+l_9)P_0(z)+(l_{10}z+l_{11})P_{00}+l_{12}z P_{o1} \qquad ...(3.27)$$

where

$l_1=f_1\mu_3$; $l_2 = (\mu_2d_1+f_2\mu_3+\alpha_3\mu_0\mu_1\mu_2)$, $l_3=f_3\mu_3$; $l_4 = (\mu_2d_2+f_4\mu_3+\alpha_2c_1+A\alpha_3+\mu_1b_1)$;

$l_5=(d_3\mu_2+f_5\mu_3+B\alpha_3)$, $l_6 = (\nu_1\nu_2\nu_3\mu_0+\mu_1b_2+\mu_2d_4+f_6\mu_3+c_2\alpha_2+a_1\alpha_1+C\alpha_3)$;

$l_7=(d_5\mu_2+f_7\mu_3+D\alpha_3+c_3\alpha_2)$; $l_8 = (\nu_1\nu_2\nu_3\alpha_0+\alpha_1a_2+\alpha_2c_4+E\alpha_3)$;

$l_9=(d_6\mu_2-f_8\mu_3+F\alpha_3+c_5\alpha_2)$;

$l_{10}=\{-\alpha_2\nu_3\lambda_0\lambda_1-\alpha_3\lambda_0\lambda_1(\lambda_2+\nu_2)+\alpha_1a_3+c_6\alpha_2+G\alpha_3-\alpha_2\lambda_0\lambda_1(\nu_2-\nu_3))$

$l_{11}=(d_7\mu_2-f_9\mu_3+c_7\alpha_2+H\alpha_3)$;

$l_{12}=(\alpha_2\nu_1(\alpha_0+\mu_0) (\alpha_3+\nu_3) + \alpha_3(\lambda_0+\mu_0) (\lambda_2+\nu_2)- \alpha_3\nu_2(\lambda_0+\alpha_0+\nu_0) (\alpha_1+\mu_1))$

Then we deduce from (3.26) and (3.27) that the generating function $P_0(z)$ satisfies the following fourth order differential equations.

$$(A_1z^6 + A_2z^5) P_0^{IV}(z) + (A_3z^5 + A_4z^4+A_5z^3)P_0'''(z) +(A_6z^4 + A_7z^3+A_8z^2)P_0''(z) +(A_9z^3 + A_{10}z^2+A_{11}z)P_0'(z) +(A_{12}z^2+A_{13}z+A_{14})P_0(z)+(A_{15}z^2+A_{16}z+A_{17})P_{00}+(A_{18}z^2+A_{19}z)P_{01}= 0 \qquad ...(3.28)$$

where $A_1 = g_1$, $A_2 = l_1$, $A_3 = g_2$, $A_4 = g_3-l_2$, $A_5 = l_3$, $A_6 = g_4$, $A_7 = g_5-l_4$, $A_8 = g_6 - l_5$,

$A_9 = g_7$, $A_{10} = g_8 - l_6$;$A_{11}= g_9-l_7$, $A_{12} = g_{10}$, $A_{13} = g_{11} - l_8$, $A_{14} = g_{12} - l_9$,

$A_{15} = g_{13}$, $A_{16} = g_{14} - l_{10}$, $A_{17} = g_{15}-l_{11}$, $A_{18} = g_{16}$, $A_{19} = g_{17} - l_{12}$

Replacing the generating function $P_0(z)$ and its derivatives in the above differential equation and rearranging its terms, we conclude that the sequence $\{P_{0j}, j\geq 0\}$ satisfies

$$P_{0j} = \chi_{j-1} P_{0j-1} - \bar{\chi}_{j-2} P_{0j-2}, j \geq 3 \quad \text{... (3.29)}$$

where

$$\chi_{j-1} = -\frac{(A_{13} + (j-1)A_{10} + (j-1)(j-2)A_7 + (j-1)(j-2)(j-3)A_4 + (j-1)(j-2)(j-3)(j-4)A_2}{A_{14} + jA_{11} + j(j-1)A_8 + j(j-1)(j-2)A_5}, j \geq 4$$

$$\bar{\chi}_{j-2} = -\frac{(A_{12} + (j-2)A_9 + (j-2)(j-3)A_6 + (j-2)(j-3)(j-4)A_3 + (j-2)(j-3)(j-4)(j-5)A_1}{A_{14} + jA_{11} + j(j-1)A_8 + j(j-1)(j-2)A_5}, j \geq 4$$

$$P_{03} = \chi_2 P_{02-1} - \bar{\chi}_1 P_{01};\ P_{02} = \chi_1 P_{01} - \bar{\chi}_0 P_{00}$$

where

$$\chi_1 = \frac{-(A_{18} + A_{13} + A_{10})}{A_{14} + 2A_{11} + 2A_8};\ \chi_2 = \frac{-(A_{13} + 2A_{10} + 2A_7)}{A_{14} + 3A_{11} + 6A_8 + A_5};$$

$$\bar{\chi}_1 = \frac{-(A_{12} + A_9)}{A_{14} + 3A_{11} + 6A_8 + A_5};\quad \bar{\chi}_2 = \frac{-(A_{15} + A_{12})}{A_{14} + 2A_{11} + 2A_5}$$

It follows by induction form (3.28) that

$P_{01} = \frac{B_0}{C_0} P_{00}$, where $B_0 = -(A_{13} + A_6)$; $C_0 = A_{11} + A_{14} + A_{19}$

$$P_{0j} = \eta_{j-1} P_{00} \quad \text{... (3.30)}$$

where $\eta_{j-1} = \eta_{j-2}\chi_{j-1} + \eta_{j-3}\chi_{j-2}\,,\ j \geq 3$

Theorem 3.1. If $|\lim_{j\to\infty}\eta_j| = +\infty$, then the stationary distribution of $\{X(t), t \geq 0\}$ is given by

$$P_{00} = \left(\sum_{i=0}^{4}\sum_{j=0}^{\infty} M_{ij}\right)^{-1},$$

$$P_{ij} = M_{ij}P_{00}, (i,j) \in E - \{(0,0)\}$$

where

$$M_{0j} = \eta_{j-1}\,, j \geq 1$$
$$M_{1j} = \left(\frac{\lambda_0+\beta_{0j}}{\nu_1}\right)\eta_{j-1}\,,\ j \geq 0$$

$$M_{2j} = \nu_1^{-1}\nu_2^{-1}\left(\left((\lambda_1+\nu_1+\beta_{1j})\beta_{0j} + (\lambda_1+\beta_{1j})\lambda_0\right)\eta_{j-1} - \nu_1\beta_{0j+1}\eta_j\right)$$

$$\begin{aligned}M_{3j} = (\nu_1\nu_2\nu_3)^{-1}\{&\{((\lambda_2+\nu_2+\beta_{2j})(\nu_1+\beta_{1j}) + (\nu_2+\beta_{2j})\nu_1)\beta_{0j}\\&+ (\lambda_2+\beta_{2j})(\lambda_1+\beta_{1j})\lambda_0 + \nu_2\beta_{1j}\lambda_0\}\eta_{j-1}\\&- ((\lambda_2+\nu_2+\beta_{2j})\beta_{0j+1}\nu_1 + \beta_{1j+1}\nu_2(\lambda_0+\beta_{0j+1}))\eta_j\}\end{aligned}$$

$$\begin{aligned}M_{4j} = (\nu_1\nu_2\nu_3\nu_4)^{-1}\{&\{((\lambda_3+\nu_3+\beta_{3j})(\lambda_2+\nu_2+\beta_{2j})(\lambda_1+\beta_{1j}) + (\nu_2+\beta_{2j})\nu_1)\beta_{0j}\\&+ ((\lambda_2+\beta_{2j})(\lambda_1+\beta_{1j}) + \nu_2\beta_{1j})\lambda_0 - \lambda_2\nu_3(\lambda_1+\nu_1+\beta_{1j})\beta_{0j} - \lambda_2\nu_3(\lambda_1+\beta_{1j})\lambda_0\}\eta_{j-1}\\&-\{(\lambda_3+\nu_3+\beta_{3j})(\lambda_2+\nu_2+\beta_{2j})\beta_{0j+1}\nu_1 - \beta_{1j+1}\nu_2(\lambda_0+\beta_{0j+1}) - \beta_{2j+1}\nu_3(\lambda_1+\nu_1+\beta_{1j+1})\\&-\beta_{2j+1}\nu_3(\lambda_1+\beta_{1j+1})\lambda_0 + \beta_{0j+1}\lambda_2\nu_1\nu_3\}\eta_j + \beta_{2j+1}\nu_1\nu_3\beta_{0j+2}\eta_{j+1}\}\end{aligned}$$

Notice, that the stationary probabilities $P_{ij},(i,j)\in E$, have been written in terms of P_{00}. Hence, the computation of the stationary distribution of $\{X(t), t \geq 0\}$ is reduced to find P_{00} to any desired accuracy by using the equation (3.30).

Numerical study

Numerical calculations were performed to obtain the values of the probabilities, for fixed values of parameters $\lambda_i = 1/ i+1$, $\nu_i = 1 / i+2$ and $\beta_{ij} = \alpha_i(1-\delta_{0j}) + j\mu_i$, where $\mu_i = 1/2i$ and $\alpha_i = 1/i+3$, $0\leq i \leq 4$, $j \geq 0$. Some selective results are exhibited in table 4.1

Table 1 The steady state probabilities

λ_0	0.2	0.4	0.6	0.8	1.0
p00	0.0052	0.0053	0.0055	0.0057	0.0059
p01	0.0006	0.0003	0.0002	0.0001	0
p02	0.0007	0.0005	0.0004	0.0003	0.0002
p03	0.0003	0.0002	0.0001	0.0001	0.0001
p04	0	0	0	0	0
p05	0	0	0	0	0
:	:	:	:	:	:
p10	0.0034	0.0070	0.0109	0.0152	0.0198
p11	0.0010	0.0008	0.0006	0.0004	0.0002
p12	0.0012	0.0011	0.0011	0.0010	0.0009
p13	0.0004	0.0004	0.0004	0.0003	0.0003
p14	0	0	0.0001	0.0001	0.0001
p15	0	0	0	0	0
:	:	:	:	:	:
p20	0.0062	0.0137	0.0216	0.0302	0.0395
p21	0.0047	0.0039	0.0029	0.0019	0.0008
p22	0.0089	0.0082	0.0076	0.0070	0.0064
p23	0.0041	0.0038	0.0035	0.0032	0.0029
p24	0.0002	0.0003	0.0006	0.0007	0.0008
p25	0	0	0	0	0
:	:	:	:	:	:
p30	0.0048	0.0170	0.0297	0.0435	0.0584
p31	0.0117	0.0091	0.0059	0.0022	0.0020
p32	0.0460	0.0432	0.0404	0.0373	0.0339
p33	0.0288	0.0260	0.0239	0.0220	0.0200
p34	0.0019	0.0028	0.0051	0.0062	0.0067
p35	0	0	0	0	0
:	:	:	:	:	:
p40	0.1032	0.1399	0.1837	0.2333	0.2887
p41	0.2422	0.2022	0.1669	0.1318	0.0951
p42	0.3682	0.3329	0.3047	0.2780	0.2510
p43	0.1771	0.1693	0.1595	0.1487	0.1369
p44	0.0144	0.0168	0.0306	0.0370	0.0394
p45	0	0	0	0	0
:	:	:	:	:	:

We can extend it to n-limited capacity in similar mannar.

CONCLUSION

If we use n-limited capacity model then we can get service in quickly.

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