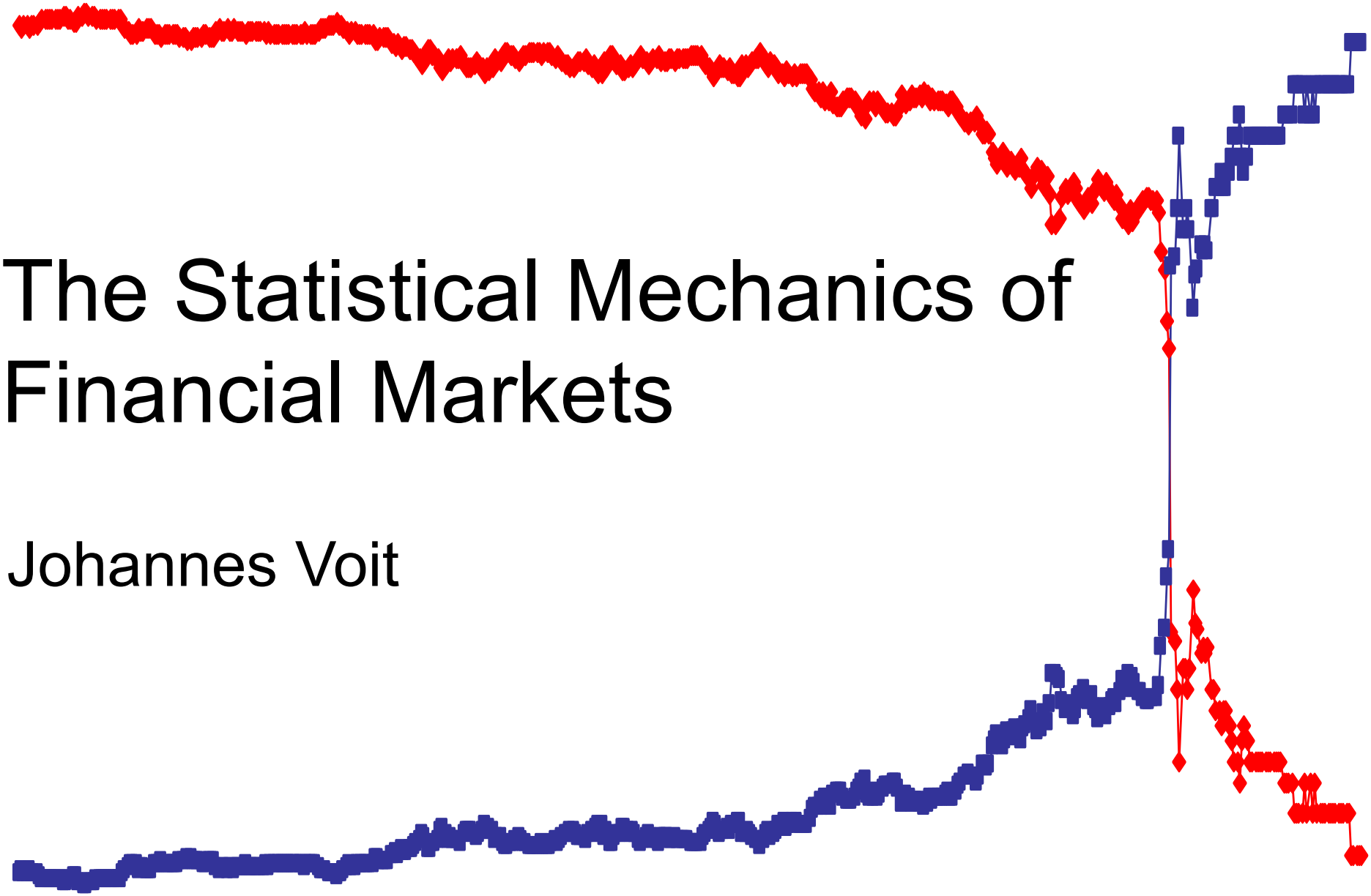


The Statistical Mechanics of Financial Markets

Johannes Voit

Overview

1. Why statistical physicists care about financial markets
2. The standard model - its achievements and failures
3. Option pricing
4. Crashes
5. Introduction to risk measurement

On August 12, 2011 at Academia Sinica

“The Financial Crisis 2007-2009:
How Did It Come? Will It Happen Again?”

What does business management achieve?

- Why do banks exist? (Definitions and theorems, cooperation and competition, etc.)
- The loan
- The deposit
- Regulation
- Financial reporting
- Bank management
- Accounting
- Organization of banks
- Open questions

What Is Risk?

- resecum (lat.), ριζικον (gr.) = cliff
- Something unpleasant happening?
- Hazard, a chance of bad consequence, loss or exposure to mischance (Oxford dictionary)
- Any event or action that may adversely affect an organization's ability to achieve its objectives and execute its strategies (McNeil, Frey, Embrechts)
- Deviation of a specified quantile of a (profit-and-) loss distribution from its expectation value (Ali Samad-Khan)
  - NB: this statement apparently defines risk through its measurement process!

Why Risk Measurement?

- "You only can manage what you measure"
- Determination of risk capital and capital adequacy
  → banking regulation
  → economic capital
  – The amount of capital shareholders should invest in a company in order to limit its probability of default to a certain confidence level
- Management tool
  → Basis for limit setting
- Insurance premiums
  → compensate insurance for bearing the risk of claims

The Role of Capital As a Buffer against Risk

- Regulatory capital (→ banking regulation)
  - Is the amount of capital the supervisors require a bank to hold
- Economic capital (applies to both banks and non-bank corporates)
  - (theoretically) Is the amount of capital a bank‘s shareholders would choose to cover its risk / ensure continuous operation in the absence of external regulation
  - BIS: capital which a bank holds based on its own assessment of risk
  - Gives full benefit of risk diversification
- What is capital?
  - Capital needed → risk measurement
  - Capital available
    - Regulatory: recognized capital constituents
    - Economic: value of all assets

What Risks Faces a Bank? (And Any Other Corporation, And Any Other Individual)

- Market risk
- Counterparty default risk
- Liquidity risk

Defining the essential risks

- Market risk is the risk of loss of a position in a security, portfolio, etc. due to changes in market conditions.
- Credit risk is the risk of loss due to a counterparty in a financial contract not satisfying her contractual obligations.
- Operational risk is the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. Includes legal risk. Excludes strategic and reputational risk.
- Liquidity risk is the risk that an asset cannot be traded fast enough to prevent a loss, resp. remain solvent (insolvency risk). Refunding risk … increased cost of refunding due to market illiquidity.

How To Measure Risk?

- Notional amount
  - Sum of notional values of securities
  - When modified by “risk weights”, often used by regulators, e.g. standardized approach in Basel framework for market and credit risk
  - Neglects effects of hedging
  - Neglects the effects of diversification

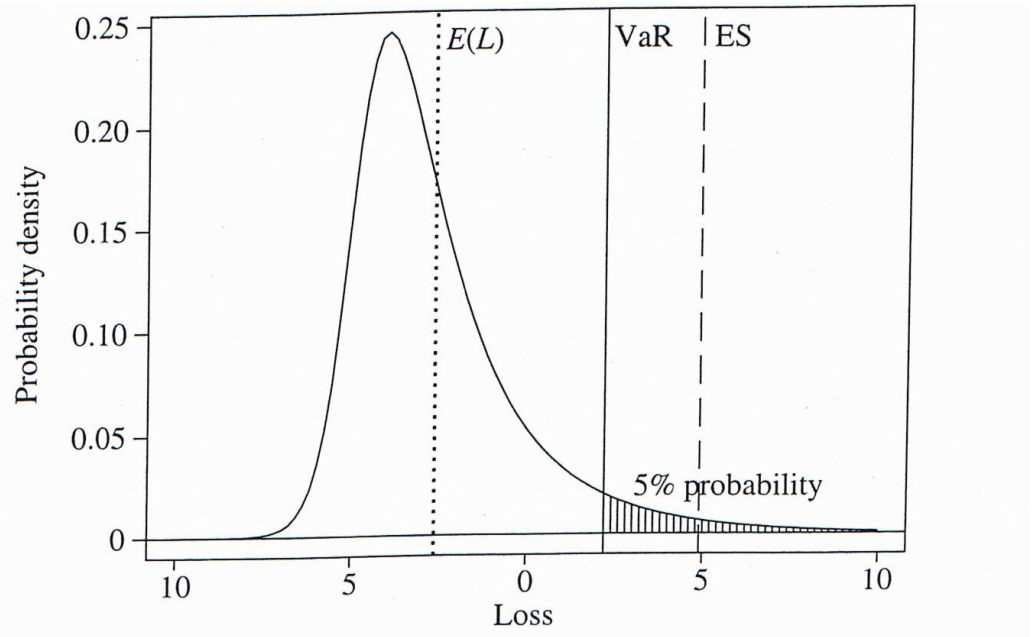
- Variance / standard deviation = volatility
  - Second moment must exist
  - Only symmetric distributions
  - Convergence properties under fat-tailed distributions

Value at risk is the most popular risk measure (I)

- P(L≤l) … probability that a loss L is below a certain value l
- Idea: maximum loss not exceeded with a given (high) probability

$$VaR_\alpha = \inf_l \{P(L > l) \le 1 - \alpha\} = \inf_l \{P(L \le l) \ge \alpha\}$$

- Quantile of loss distribution
- Typically, $\alpha = 0.95$ or 0.99



Source: McNeil, Frey, Embrechts

On notation

- "The Statistical Mechanics of Financial Markets" uses log-returns over specified time scale τ
- (Spot) price of an asset at time t: S(t)
- Continuous compounding with rate μ: $S(t+\tau) = S(t) \exp(\mu \tau)$
- Log-return $\delta S_\tau(t) = \ln \frac{S(t)}{S(t-\tau)}$
- Frequent proxy for forward-looking risk measures

$$\delta S_\tau(t+\tau) = \ln \frac{S(t+\tau)}{S(t)} \approx \ln \frac{S(t)}{S(t-\tau)} = \delta S_\tau(t)$$

- Translational invariance in time assumed, "future ≈ past"

Value at risk is the most popular risk measure (II)

- $L = -\delta S_\tau(t+\tau) = -\delta S_\tau(t)$ [when $\delta S_\tau(t+\tau)$ negative, stationary] contains a time scale τ, dependent on scale of main business
  - τ = 1 day for trading limits
  - τ = 10 day for market risk management
  - τ = 1 year for credit and operational risk management
- Definition of VaR is a practical working definition, accurate mathematical definition can be given
- When VaR is calculated on bank level, α is related to default probability of bank, and therefore to its rating
- Conversely, a given target rating determines α
- $VaR_\alpha - \langle L \rangle$ is a related risk measure
  - Sometimes called value at risk, mean-VaR, unexpected losses

Default probabilities determine rating scores

- Default probabilities (PD) translate into confidence levels α for risk measurement: PD = 1 - α

Investment grade

Junk bonds

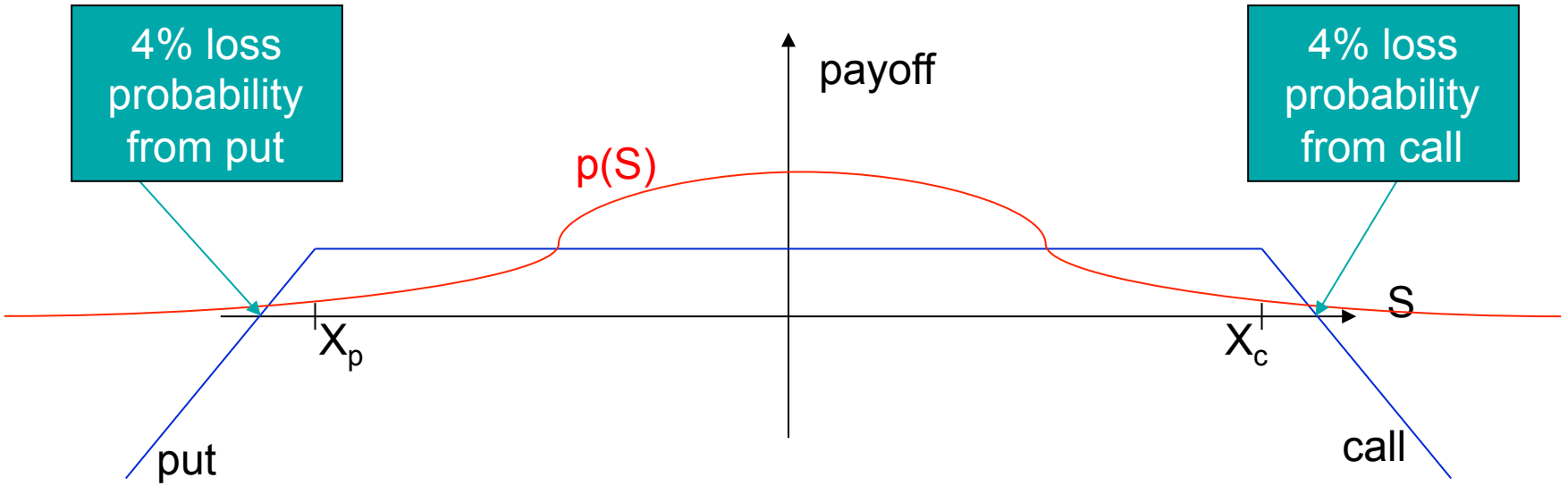
S&P	Moody ‘s	Implied PD
AAA	Aaa	≤ 0.01%
AA+	Aa1	0.02%
AA	Aa2	0.03%
A	A2	0.07%
BBB+	Baa1	0.12%
BBB	Baa2/Baa3	0.3%
BB+	(Ba1)	0.6% (0.9%)
BB	Ba2	1.3%
B	B2	6.7%
CCC		20%
D		defaulted

Pros and cons of VaR as a risk measure

+ Implements managerial view: clear-cut separation of what can be managed (events below confidence level) and of what cannot be managed (events above confidence level)
+ Recognized by regulators (cf. below, Basel framework for market risk)
- VaR is not subadditive:
  - Assume a bank with two portfolios with loss variables L_1 and L_2, and VaR_1 and VaR_2 at the same confidence level α
  - Loss of bank is $L = L_1 + L_2$
  - Then $VaR_{1+2} \leq VaR_1 + VaR_2$ (subadditivity property) IS NOT NECESSARILY SATISFIED

Examples for the failure of VaR

- Short position in far-out-of-the-money call and put options



- No risk at 95% confidence level in each separate position
- However, significant risk to combined position
- Failure of VaR observed in many other instances

Coherent Risk Measures

- A coherent risk measure $\rho(X)$ satisfies the following four properties
  (X,Y … values of positions, i.e. “risk” comes from negative X,Y)

- Subadditivity: $\rho(X+Y) \leq \rho(X) + \rho(Y)$
  [$\rho(X+Y) = \rho(X) + \rho(Y) \Leftrightarrow$ X and Y perfectly correlated]
- Translation invariance (risk-free condition): $\rho(X+rn) = \rho(X) - n$
  r … risk-free interest rate
- Positive homogeneity of degree 1: $\rho(\lambda X) = \lambda\, \rho(X)$
- Monotonicity: $\rho(X) \leq \rho(Y)$ if $X \geq Y$

Expected shortfall is a coherent risk measure

- Expected shortfall $ES_\alpha = \frac{1}{1-\alpha} \int_{\mathrm{VaR}_\alpha}^{\infty} L\, p(L)\, dL \quad \left(\geq VaR_\alpha\right)$
- Simplified definition for integrable loss variables with continuous distributions, accurate definition can be given
- VaR just controls probability of bad event, not its consequences

There Is a Big Gap in Risk Measurement

- Aggregation of individual securities to portfolio risk measurement mainly by Monte Carlo simulation
  - Alternative 1: historical simulation
  - Alternative 2: variance-covariance model
    (Gaussian world, VaR $\propto\sigma$, prefactor dependent on confidence level)

$$VaR_{tot} = \sqrt{VaR_1^2 + 2\,\rho_{12}\,VaR_1\,VaR_2 + VaR_2^2}$$

- Systematic aggregation from portfolio level to bank level almost not feasible (copulas)

References

- Johannes Voit, *The Statistical Mechanics of Financial Markets,* 3rd ed., Springer Verlag, 2005 and World Publishing Corporation, Beijing 2010
- Alexander J. McNeil, Rüdiger Frey, Paul Embrechts, *Quantitative Risk Management,* Princeton University Press, 2005