LETTERS TO THE EDITOR.

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Non-Euclidean Geometry.

MR. FRANKLAND (NATURE, September 7) has raised the old problem of Bertrand's proof of the parallel-axiom by a consideration of infinite areas. This is perhaps the most subtle and the most specious of all the attempted proofs, and this character it owes to the fact that a process of reasoning which is sound for finite magnitudes is extended to a field which is beyond our powers of comprehension—the field of infinity. The fallacy which underlies Bertrand's proof becomes more apparent in Legendre's simpler device ("Éléments de Géométrie," 12 éd., Note ii.). A straight line divides a plane in which it lies into two congruent parts—this, of course, has no real meaning, since we are dealing with infinite areas, but such is the argument—and two rays from a point enclose an (infinite) area which is less than half the whole plane. Hence, if two intersecting lines are both parallel to the same straight line, the area of half the plane can be enclosed within an area which is less than half the plane.

This is the same sort of paradox as the well-known one by which the part is made to appear equal to, or even greater than, the whole. The even numbers 2, 4, 6, . . . form a part of the aggregate of integral numbers 1, 2, 3, . . ., but a (1, 1) correspondence can be established between them, viz. to $2n$ in the part corresponds n in the whole aggregate, and to n in the whole corresponds $2n$ in the part. Hence the part is equal to the whole. And, again, a (2, 1) correspondence can be established between the part and the whole, viz. to $4n$ in the part corresponds n in the whole, while the numbers of the form $4n+2$ have no correspondent. Thus the part is greater than the whole.

Mr. Frankland's comparison of the areas of a circle and a regular inscribed polygon is not quite fair to the polygon. The area of a regular N-gon, as its radius tends to infinity, tends to a finite limit, $\pi k^2(N-2)$, which, of course, tends to infinity as N is increased. The area of a circle is $4\pi k^2 \sinh^2 r/2k$, which also tends to infinity as r is increased. The first he calls a linear infinity, and the second an exponential infinity, and certainly e^x/x^n tends to infinity with x, if n is any finite number. But what is the relation between r and N? If we take the expression for the area of a regular N-gon inscribed in a circle of radius r, and then let N increase, we get a limit $4\pi k^2 \sinh^2 r/2k$, which is the expression for the area of the circle. Again, if in the regular N-gon with infinite radius we inscribe a circle, its area is $2\pi k^2(\operatorname{cosec} \frac{\pi}{N} - 1)$ and this always bears a finite ratio to the area of the N-gon; it is thus an infinity of the same order, if N is increased indefinitely, and the N-gon, the inscribed circle, and the circumscribed circle all tend to the same geometrical limit—the absolute.

The fact that the cuspidal edge of the surface of rotation of the tractrix forms a line of discontinuity in this representation, and that none of the types of surfaces of constant negative curvature exactly images the hyperbolic plane in the properties belonging to analysis situs, appears to be no objection to hyperbolic geometry. An exactly similar difficulty occurs in the representation of elliptic geometry, since there is no continuous surface of constant positive curvature on which two geodesics have but one point of intersection. Geometry has become entirely a matter of postulation; but, at the same time, it is of interest to observe that the non-Euclidean geometries are capable of being truly represented, even within a restricted field, in Euclidean space.

D. M. Y. SOMMERVILLE.

The University, St. Andrews, September 30.

Elements of Comet 1911*f*.

FROM M. Quénisset's observation of September 23, and my own of September 26 and 30, I obtain the following approximate elements:—

$$T = 1911 \text{ November } 12{\cdot}67$$
$$\omega = 123^\circ\ 13{\cdot}4'$$
$$\Omega = 35^\circ\ 36{\cdot}6'$$
$$i = 102^\circ\ 19{\cdot}8'$$
$$\log q = \bar{1}{\cdot}89116.$$

The comet is now receding from the earth and approaching the sun, and there is no reason to expect much increase in brilliancy. The only point of interest is that when at the descending node on December 16 it will be about half a million miles outside the earth's orbit. The difference of the heliocentric longitudes of the earth and comet will, however, be 132°, so that no near approach is possible.

J. B. DALE.

Craigness, New Malden, Surrey, October 3.

Rainfall in the Summer of 1911 and of 1912.

HAS Mr. MacDowall the courage to apply his own experience, to which he refers in NATURE of September 28, to "supply long-range forecasts of months, seasons, &c."? Will he publish in advance a forecast for the winter 1911–12 or for the spring and summer of 1912, such as he considers could have been done for the summer of 1911? Or is it only after the event that he can discover what points in the past have to be considered and in what grouping they have to be compared in order to yield an *a posteriori* "forecast"?

HUGH ROBERT MILL.

62 Camden Square, London, N.W., October 2.

Miniature Rainbows.

WHEN returning one day in August of last year from the Farne Islands to Berwick in a pleasure steamer, I was standing in the bow of the boat, and was much struck by the display of a permanent rainbow in the spray that was thrown up. The rainbow was inverted, the result, presumably, of my position above it. The sea was very rough, and thus the spray was constant.

EDWARD A. MARTIN.

285 Holmesdale Road, South Norwood, S.E.

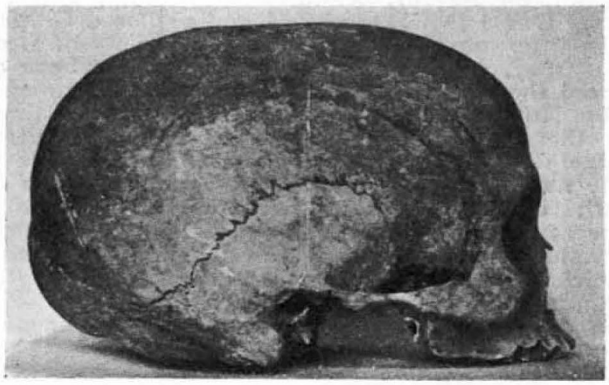
THE STONE AGES OF SOUTH AFRICA.[1]

THE papers in this volume are a very full and important addition to the work already published by Mr. J. P. Johnson; but it is doubtful whether they bring us any nearer to a solution of one of the most interesting questions connected with archæological or palæontological discoveries in South or Central Africa—namely, the approximate age to which the existence of man can be traced back in South Africa, East Africa, the Congo basin, West Africa, and the Sudan. Though Dr. Péringuey would seem, from one or two phrases, to lean to the theory of a very ancient date for the human colonisation of tropical Africa, he has to admit repeatedly that so far no cogent evidence has been produced in the shape of geological features associated with the finds of human remains or implements to indicate, as positively as is the case in Europe and Asia, the period in the earth's history with which such remains are to be associated.

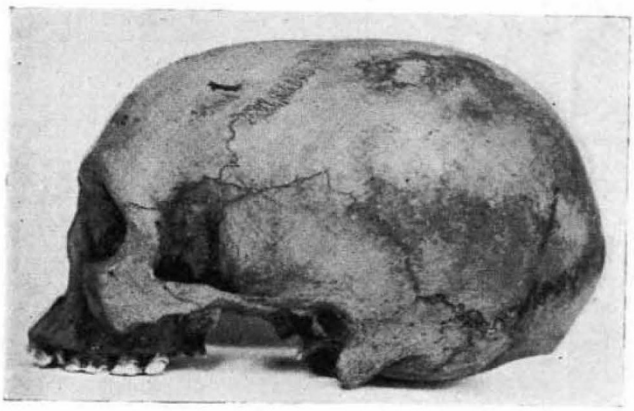
As our knowledge advances towards perfection, as we become better and better able to read that new Bible, the book of the Earth itself, we may have to revise our estimate of the ages of the hitherto discovered prehistoric, palæolithic, and eolithic human remains in Europe and Asia. Still, there can be little

[1] Annals of the South African Museum: vol. viii., part 1, containing the Stone Ages of South Africa, &c. By Dr. L. Péringuey, with further contributions by Mr. A. L. Du Toit and Dr. F. C. Shrubsall. (London: Printed for the Trustees of the South African Museum by West, Newman and Co.) Price 40*s*.

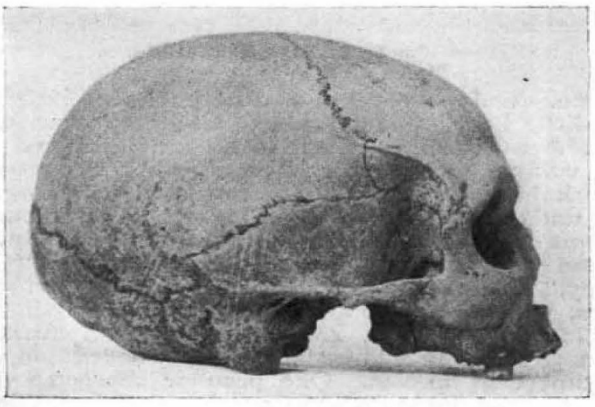
question within a few thousand years, more or less, of the relative ancientness of the calvarium, the molar teeth, and thigh-bones of *Pithecanthropus erectus* in Java; of the *Homo primigenius* type of lower jaw found near Heidelberg; and of the Grimaldi skeletons,



(i) Strandlooper.



(ii) Bushman.



(iii) Hottentot.
Crania from South Africa.

in any part of Europe or temperate Asia argues some degree of antiquity in the specimens, because we know more or less historically the period at which they were abandoned for implements or weapons of metal. But in tropical Africa no such argument can apply, for a few small portions of the continent are still in the Palæolithic, or even Eolithic, Stone age. The south-western extremity of Africa was certainly in the palæolithic age 400 years ago, when Europeans first arrived there; though the Bantu tribes bordering on the Hottentot and Bushman domain had for many centuries before been mining, smelting, and using copper and iron.

So far as the reviewer is concerned, he continues to adhere obstinately to the belief, not as yet shaken by any registered fact, that man is a relatively recent immigrant into Southern Africa, and that Africa in general south of the Sahara Desert and of Somaliland has been much more newly populated by man (almost entirely of the negro subspecies) than has been the case with Asia—the original birthplace of the human species and genus—and with Europe.

Much interesting palæontological and geological evidence is collected and laid before the reader in chapter vii. (p. 74 and onwards). In this, Dr. Péringuey mentions that a portion of a molar tooth of a mastodon has been found in close proximity to a deposit of palæoliths and fragments of stone, evidently used as human implements, at Barkly West, in Cape Colony. Human implements have also been found in association with the lower jaw of an extinct horse of large size and of the gigantic long-horned buffalo—*Bos* or *Bubalus baini* (a close ally of the *Bos antiquus* of Algeria). But from the rock drawings in Algeria we know that this gigantic North African buffalo not only was contemporaneous with man, but even with neolithic man, and only seems to have become extinct a few thousand years ago. Similarly, in the extremity of South Africa a mastodon, a large horse, and the South African type of gigantic buffalo may have lingered down to quite a late period, since this same portion of the continent contains in a living state at the present day creatures which became extinct in Europe a hundred thousand years ago. (In this portion of his narrative Dr. Péringuey persists in confusing *Hyena brunnea* of South Africa with *Hyena striata* of East, West, and North Africa and Western Asia. I write under correction, but had always believed that *Hyena brunnea* was a very distinct species which hitherto had not been found north of the Zambezi and South Angola, and, though allied to the various types of *striata* far more than to *crocuta,* was nevertheless a very distinct species.)

Some of Dr. Péringuey's deductions are very interesting, especially as combined with the observations of Dr. Shrubsall. One of these would seem to be that the earliest human invaders of South Africa were of somewhat higher culture, of different head-form, and better brain development than the modern Bushman. These people are the now celebrated Strandloopers. At one time it was assumed, on the strength of some very prognathous skulls found in the coast regions of South Africa, that the Strandlooper was more "simian" (if that word may be applied to a very slight approximation towards the basal human type) than the Bushman. This deduction would seem to be wrong. The sub-nasal prognathism in the skulls of the earliest cave-men of Strandlooper types is less than in the Kalahari Bushmen and the Nama Hottentots of to-day, or in the general mass of negroes. One of the Strandlooper skulls, according to Dr. Shrubsall, has a more prominent nose and face than the typical negro, and in some respects recalls the river-bed type of early Europeans. The cranial capacity of these primitive Strandloopers was distinctly greater than either Bushmen or Hottentots, and this feature is present in the oldest skulls. One of these has a cranial capacity of 1600 c.c., while in a female skull of the Bush race from the Kalahari Desert there is a capacity of only 950 c.c.

Moreover, it would seem to be as though there had been a marked degeneration not only in the physical conformation of the pre-Bantu inhabitants of South Africa from the earliest Strandlooper type downwards, but also in the character and size of the stone implements manufactured by these primitive South African peoples.

Dr. Péringuey in some of his remarks (p. 168) would seem to regard the Bushman as not being a primitive race, but an example of retrogression in some directions and a singular advance in others. He puts forward the interesting hypothesis that the ancestors of the Bushman having discovered the potency of vegetable and animal poisons, gave all their attention to the manufacture and shooting of poisoned arrows, and therefore no longer cared to fabricate large stone weapons.

Dr. Péringuey writes as a South African, and South Africans are apt to hold heretical notions regarding the Bantu. One is that there is a Bantu physical type of negro, which is not the case; and the other is that the Bantu languages in their present form are of immense antiquity, and came to Africa from India. The Bantu-speaking peoples of South Africa vary in physical type, just as they do in the rest of Bantu Africa, and do not present any collective difference from millions of other negroes not speaking a Bantu language. As to this language family, I have given at different times reasons which appear to me conclusive for supposing that it cannot have originated in North Central Africa more than some 3,000 years ago. It was brought into existence in the heart of Africa, just like its neighbour Hausa, by the intrusion of some half-white race similar to the Hamite or the Fula.

This book gives interesting illustrations of the steatopygia and peculiarities of the external genitalia of the Bushman race. H. H. JOHNSTON.

CALIFORNIAN TREES.[1]

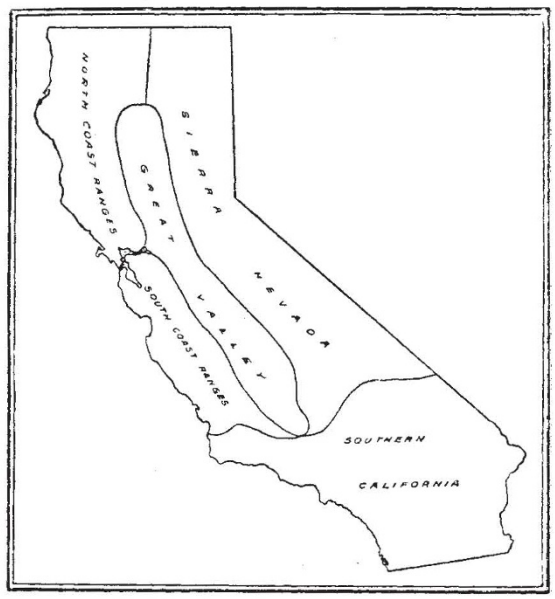
PERHAPS the noblest and most fascinating of all subjects for the writer and student of trees is the sylva of California. The arboreal vegetation of no other area of similar dimensions rivals it in interest or in the size of its individual types. Three trees alone —the two Sequoias ("Big Tree" and "Redwood") and the Douglas Fir—give to it a unique distinction, and they are supplemented by a group of scarcely less wonderful pines, firs, and spruces. Whilst it is the immense coniferous trees that give to the Californian sylva its remarkable fascination, many of the "broad-leaved," or non-coniferous, species are scarcely inferior in interest and distinction. There are, for instance, the magnificent Madroña—*Arbutus Menziesii* —a close ally of the Killarney arbutus, but reaching 125 feet in height, with a trunk 5 feet in diameter; the golden chestnut (*Castanopsis chrysophylla*), its leaves a tawny gold beneath, also over 100 feet high; the Mountain dogwood (*Cornus Nuttallii*), an ally of our Cornelian cherry, but often 50 to 60, sometimes 100, feet high, with its beautiful white involucres 6 inches across. Mr. Jepson, therefore, may well be congratulated on his subject.

To us in the British Isles it possesses an exceptional interest, because most of the Californian trees can be cultivated in the open air in many parts of our country. Nowhere else, indeed, out of California itself, can its coniferous trees be seen to such perfection as in the Perthshire valleys and in various places in the south and west of England and Ireland.

[1] Memoirs of the University of California. Vol. ii., "The Silva of California." By W. L. Jepson. Pp. 480+85 plates+3 maps. (London: T. Fisher Unwin; Berkeley: University Press, 1910.) Price 2*l.* 2*s.* net.

The history of the Californian sylva as known to Europeans strikes one as curiously recent. Botanical knowledge began with the visits of the Malaspina and the Vancouver expeditions. The latter, a voyage of survey organised by the British Government, touched California about 1793. Most of the botanical work accomplished on this journey was done by Archibald Menzies, but he only penetrated a few miles from the coast. David Douglas visited and explored California in the interests of the Horticultural Society of London about 1827, and with him may be said to have commenced the real revelation of its sylva. It was carried on by Nuttall, Fremont, Kellogg, Brewer, Bolander, and others. But even the existence of the "big trees" (*Sequoia gigantea*) was not definitely and authoritatively made known till nearly the middle of the nineteenth century, although hunters and wandering pioneers had previously brought home accounts of marvellous trees—mostly received, however, with the scepticism the stories of such folk obtain.

Among the later investigators of the Californian sylva a foremost place is held by the author of this



The Five Forest Provinces of California.

volume. Only two years ago he published an admirable little book, "The Trees of California," of which the present elaborate work is an amplification. The new volume opens with an interesting essay on the remarkable topography of California, its climate, rainfall, and tree distribution. The two great mountain systems (the Coast Ranges and the Sierra Nevada) enclose a great oval plain known as the "Great Valley," drained by the San Joaquin and Sacramento rivers, which meet about midway, and empty into the Pacific. This region, 400 miles long and averaging about 50 miles in width, is sparsely wooded and weak in number of species. One peculiar characteristic of its scenery is the park-like grouping and disposition of a few species of oak, chiefly the Valley oak (*Quercus lobata*) and the Live oak (*Q. Wislizenii*). They never form forests as the coniferous trees of the foot hills and mountain slopes do, and scarcely anywhere on this central plain does an aggregation of individuals amount to more than what may be termed a grove.

Besides the great valley of the Sacramento and San Joaquin, the author discusses his subject from the point of view of four other great geographical areas: