Sixth Rankine Lecture

THE STRENGTH OF SOILS AS ENGINEERING MATERIALS

by

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INTRODUCTION

Of the Rankine Lecturers so far appointed from the United Kingdom I am the first to have spent the early years of my professional life working on the design and construction of civil engineering works. Although I became deeply involved in soil testing during this period, and spent more than a year working at the Building Research Station with Dr Cooling and Professor Skempton, the tests which I performed were carried out primarily for the solution of immediate engineering problems and only secondarily as a fundamental study of soil properties.

This period no doubt left its mark, because I find that I have retained a preference for investigating naturally occurring soils, either in their undisturbed state or in the state in which they would be used for constructing the embankments of earth or rockfill dams, or other engineering works. As a consequence, I would like to direct attention to the following four aspects of the study of the strength of soils which are not only of fundamental significance, but also of immediate practical importance to the engineer:

(1) the failure criteria which are used to express the results of strength tests and which reflect the influence, if any, of the intermediate principal stress;
(2) the behaviour of soils under the high stresses implied by the greatly increased height of earth and rockfill dams now under construction;
(3) the difficulty of determining what is the in-situ undrained strength of a soil, due to the influence both of anisotropy and of unrepresentative sampling;
(4) the influence of time on the drained strength of soils.

(1) FAILURE CRITERIA

A satisfactory failure criterion should express with reasonable accuracy the relationship between the principal stresses when the soil is in limiting equilibrium. To be of practical use it should express this relationship in terms of parameters which can be used in the solution of problems of stability, bearing capacity, active and passive pressure, etc, and which can form the currency for the exchange of information about soil properties.

If we consider soil properties in terms of effective stress (Fig. 1), the most marked feature of which the failure criterion must take account is the increase in strength as the average effective stress increases. But we will wish to apply the information obtained from testing samples in axial compression in the triaxial cell (Fig. 2) (where the intermediate principal stress σ_2' is equal to the minor principal stress σ_3') to practical problems where σ_2' is greater than

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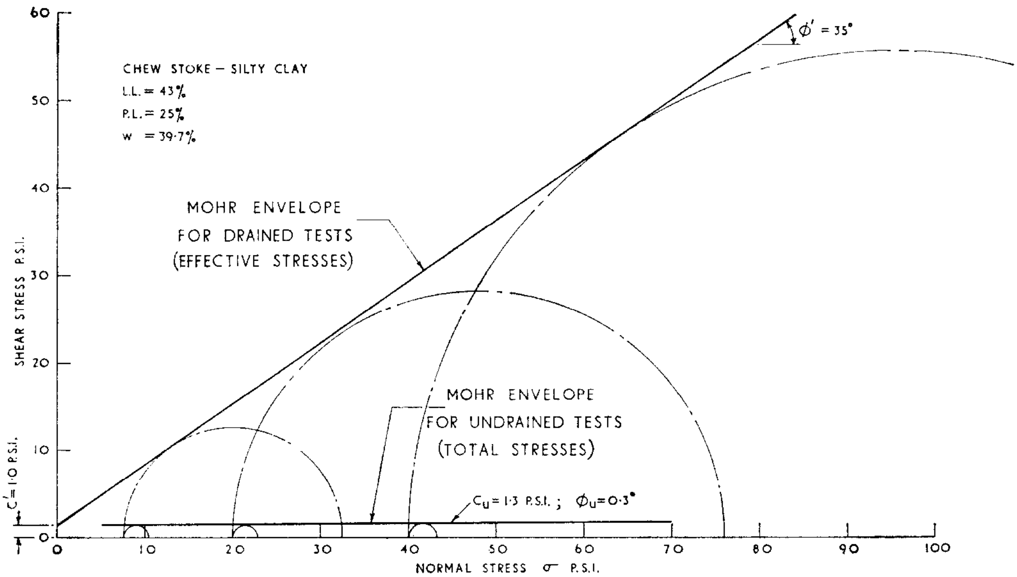

Fig. 1. Mohr envelopes for undrained and drained tests on a saturated soil, showing increase in strength with increase in effective stress

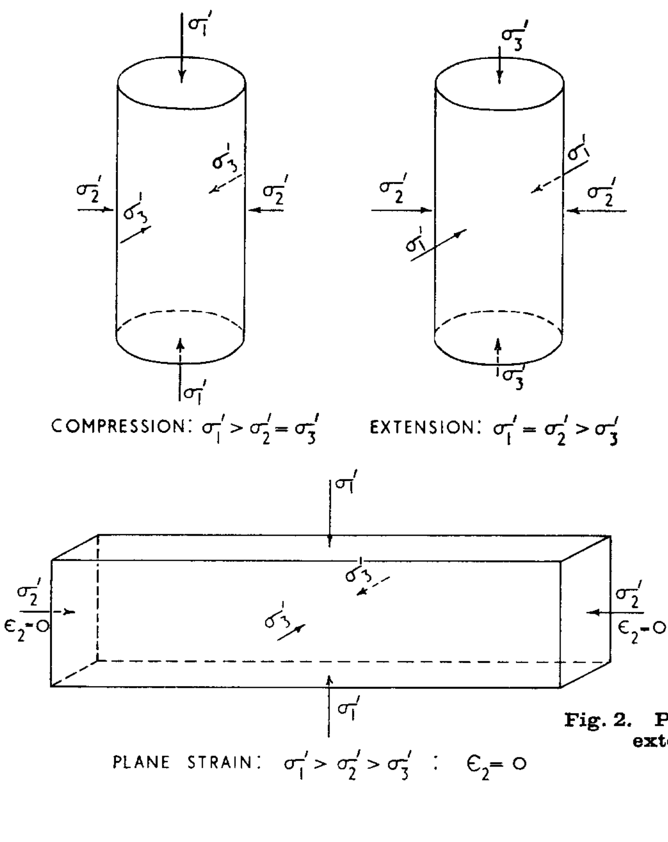

Fig. 2. Principal stresses in compression, extension and plane strain tests

σ_3' and may in the limit equal σ_1' (as in the centre of an excavation about to fail by heaving). A common special case is that of plane strain, where there is no change in length along the axis of the structure (Fig. 2). Most problems of slope stability fall into this category, and it is here that low factors of safety are most often encountered. The failure criterion should therefore also reflect the influence on the strength of the soil of the variation of σ_2' between the limiting values of σ_3' and σ_1'.

The principal failure criteria currently under discussion (see, for example, Kirkpatrick (1957), Hvorslev (1960), Scott (1963a), Roscoe *et al* (1963)) are given below. For simplicity they are given for cohesionless soils (or soils in which the cohesion intercept c' is zero). Also for simplicity σ_1', σ_2', σ_3' are chosen to denote the major, intermediate and minor effective principal stresses respectively.

The failure criteria may then be written:

Mohr–Coulomb:

$$\sigma_1'-\sigma_3' = \sin\phi' . (\sigma_1'+\sigma_3') \qquad (1)$$

Extended Tresca:[1]

$$\sigma_1'-\sigma_3' = \alpha\left(\frac{\sigma_1'+\sigma_2'+\sigma_3'}{3}\right) \qquad (2)$$

Extended von Mises:[2]

$$(\sigma_1'-\sigma_2')^2+(\sigma_2'-\sigma_3')^2+(\sigma_3'-\sigma_1')^2 = 2\alpha^2\left(\frac{\sigma_1'+\sigma_2'+\sigma_3'}{3}\right)^2 \qquad (3)$$

For the present discussion we are concerned with two points:

(1) that for a given stress system the strength should be proportional to the normal stress, and

(2) that the influence of the intermediate principal stress should be correctly indicated.

The first requirement is clearly satisfied by all three criteria. The second can be examined only on the basis of experimental evidence. It should be pointed out that whether or not yield takes place at constant volume is irrelevant to the present stage of the discussion, though it is relevant to any examination of the physical components of shear strength.

It will be noted that in the Mohr–Coulomb criterion the value of σ_2' has no influence on the strength, and the same principal stress ratio at failure would be expected for both compression and extension tests. This is in contrast to the extended Tresca and extended von Mises criteria, which both show an important difference between the stress ratios and angles of friction in compression and extension.

In *axial compression* $\sigma_2'=\sigma_3'$ and the extended Tresca (equation 2) and the extended von Mises (equation 3) both reduce to:

$$\sigma_1'-\sigma_3' = \alpha\left(\frac{\sigma_1'+2\sigma_3'}{3}\right) \qquad (4)$$

In *axial extension* $\sigma_2'=\sigma_1'$ and the extended Tresca and the extended von Mises both reduce to:

$$\sigma_1'-\sigma_3' = \alpha\left(\frac{2\sigma_1'+\sigma_3'}{3}\right) \qquad (5)$$

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[1] This is attributed by Johansen (1958) to Sandels. Roscoe *et al* (1958 and 1963) denote $\sigma_1'-\sigma_3'=q$ and $(\sigma_1'+\sigma_2'+\sigma_3')/3=p$ and take $q=\alpha p$ as the failure criterion.

[2] This is attributed to Schleicher (1925, 1926).

The influence of the intermediate principal stress on strength may be more readily appreciated in terms of the variation in ϕ' which is implied by the different failure criteria as σ_2' varies between the limits σ_3' and σ_1'.

The Mohr–Coulomb criterion (equation 1) may be written:

$$\frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'} = \sin\phi' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The relative position of σ_2' between σ_3' and σ_1' may be denoted by the parameter b, where:

$$\frac{\sigma_2'-\sigma_3'}{\sigma_1'-\sigma_3'} = b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and b varies between 0 and 1.

The extended Tresca criterion then becomes:

$$\frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'} = \frac{1}{\frac{1}{3}+\frac{2}{\alpha}-\frac{2}{3}b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the extended von Mises criterion becomes:

$$\frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'} = \frac{1}{\frac{1}{3}+\frac{2}{\alpha}.\sqrt{1-b+b^2}-\frac{2}{3}b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The variation of ϕ' (as defined by equation 6) with b (as defined by equation 7) corresponding to the two failure criteria expressed in equations 8 and 9 is illustrated in Fig. 12 and will be compared with observed values in a later section. It will be seen that the predicted ϕ' varies between $\sin^{-1}\frac{1}{\frac{2}{\alpha}+\frac{1}{3}}$ in the compression test, and $\sin^{-1}\frac{1}{\frac{2}{\alpha}-\frac{1}{3}}$ in the extension test, which is a very marked difference since the value of α at failure is typically more than 0·8.

A great many tests have been carried out, at Imperial College and elsewhere, to examine the influence of the intermediate principal stress, and some of the principal results are illustrated below.

As the accuracy of the tests is usually called in question when they fail to fit whichever theory is in vogue, it is of interest to note several points. The error in the determination of the principal stress ratio due to the distortion of the sample *at failure* has often been considerably over-exaggerated. The strain at failure in the test series to be quoted below (Cornforth, 1961) varied in compression, from $3\frac{1}{2}$% for dense sand to 6% for the middle of the range and 12% for loose sand. A sample which reached its peak stress at 6·3% axial strain is illustrated in Fig. 3. A detailed study of 4-in. diameter samples having different heights and degrees of end restraint (Fig. 4) suggests that measurement of peak strength in compression need be subject to little ambiguity (Bishop and Green, 1965).

In plane strain the failure strain ϵ_1 varied from 1·3% for dense sand to 2% in the middle of the range and 4% for loose sand. Rupture in a thin zone then occurred (Fig. 5). With these very small failure strains little uncertainty again arises in the stress calculations.

In extension the axial strain ϵ_3 at failure varied from -4% to -5% for dense and medium dense sand and rose to -9% for loose sand. In drained tests a neck begins to form at about the peak stress ratio, though it may not be very apparent to the eye (Fig. 6). If the test is stopped as soon as the peak is defined and the actual shape of the sample determined, the computed[3] value of ϕ' may be $\frac{1}{2}°$ to 1° higher at the dense end and about 2° higher at the loose end than the value based on average cross-sectional area. This correction has been made by Cornforth (1961) in the tests quoted.

[3] Based on the average cross section of a zone capable of containing a plane inclined at $45°-\phi'/2$.

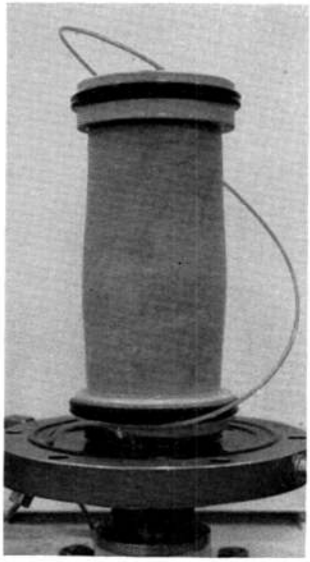

Fig. 3. 4-in. dia. × 8-in. high compression test: Ham River sand: porous disc at each end: $n_i = 40{\cdot}2\%$, $\phi'_{max} = 37{\cdot}4°$, $\epsilon_{1f} = 6{\cdot}3\%$, test stopped at $\epsilon_1 = 7{\cdot}5\%$. (Test by Green, 1966)

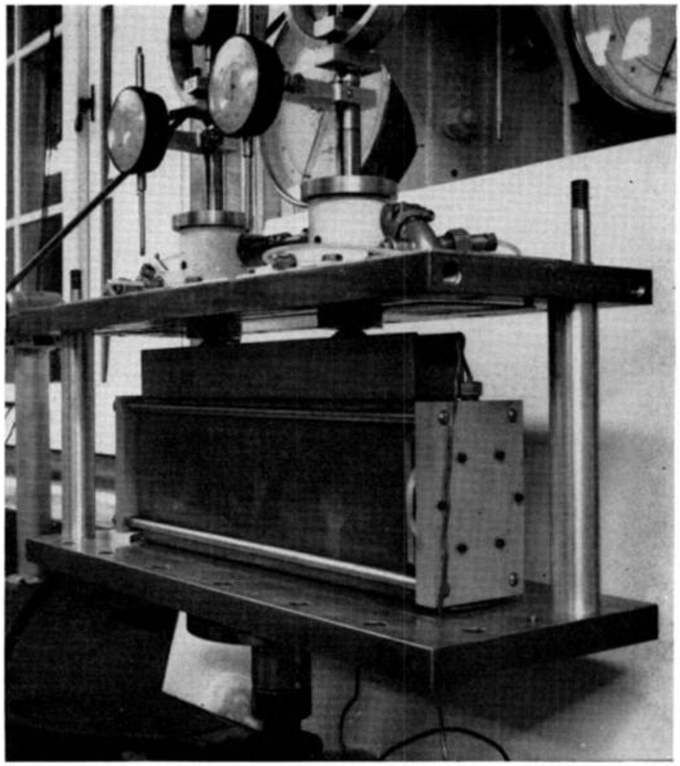

Fig. 5a. Plane strain test apparatus (with cell body removed). (Wood, 1958)

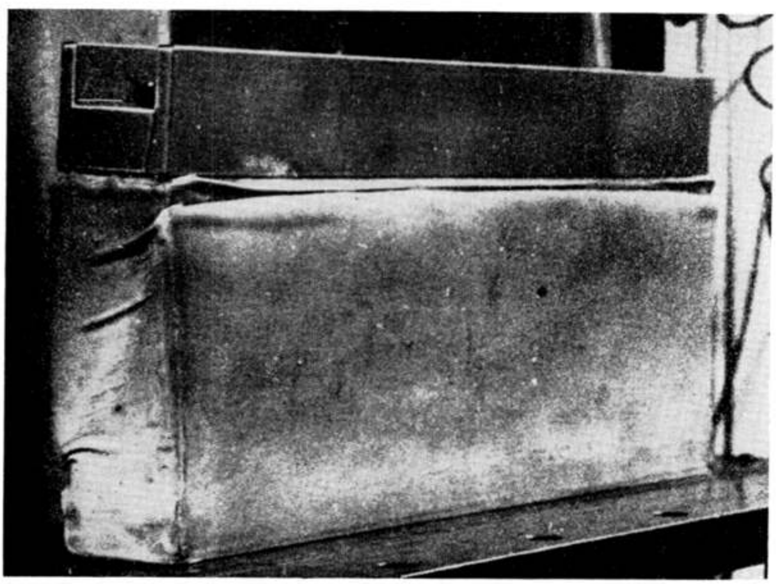

Fig. 5b. Plane strain compression test sample (4 in. × 2 in. × 16 in.) of Brasted sand after failure $n_i = 35{\cdot}6\%$, $\epsilon_{1f} = 1{\cdot}7\%$, test stopped at $\epsilon_1 = 15{\cdot}0\%$. (Test run with end plattens removed) (Cornforth, 1961)

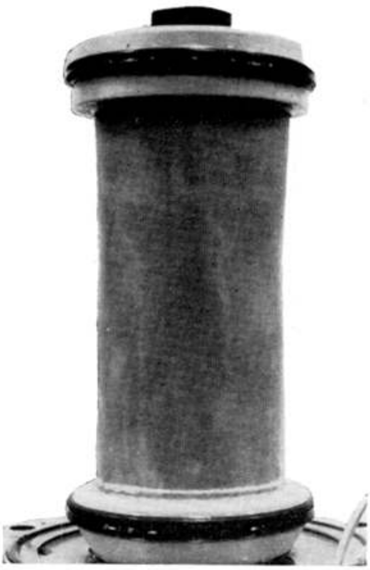

Fig. 6a. 4-in. dia. × 7-in. high extension test; Ham River sand: porous discs at each end. $n_i = 39{\cdot}2\%$, $\phi'_{max} = 40{\cdot}5°$, $\epsilon_{3f} = -7{\cdot}4\%$, test stopped at $\epsilon_3 = -8{\cdot}4\%$

Fig. 6b. 4-in. dia. × 4-in. high extension test; Ham River sand: 2/0·010-in. thick lubricated membranes at each end. $n_i = 45{\cdot}8\%$, $\phi'_{max} = 32{\cdot}6°$, $\epsilon_{3f} = -9{\cdot}5\%$, test stopped at $\epsilon_3 = -9{\cdot}8\%$

(Tests by Green, 1965)

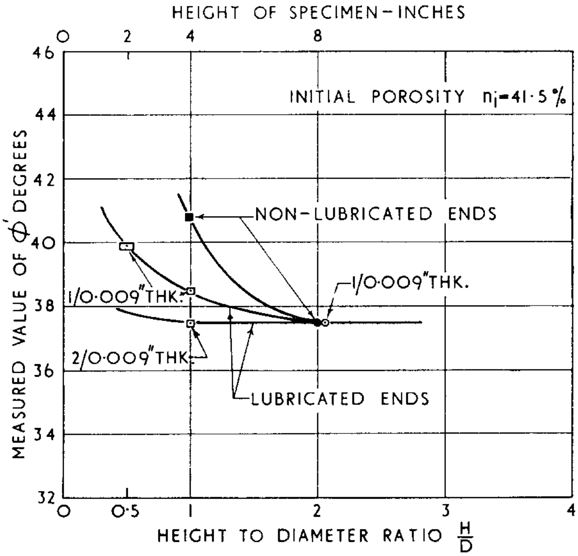

Fig. 4. Effect of sample height and degree of end restraint on measured peak ϕ' (number and thickness of lubricated membranes on end plattens indicated)

A comparison between the peak strength values, expressed in terms of ϕ' as defined above, of plane strain and axial compression tests is given in Fig. 7. This indicates values of ϕ' in plane strain higher by 4° at the dense end of the range and by $\frac{1}{2}$° in loose sand. A subsequent series of tests at Imperial College on Mol sand by Wade (1963) shows similar results. Tests on sand by Kummeneje (1957) in a vacuum triaxial apparatus and also by Leussink (1965) are in general agreement, but show less tendency to converge at higher porosities.

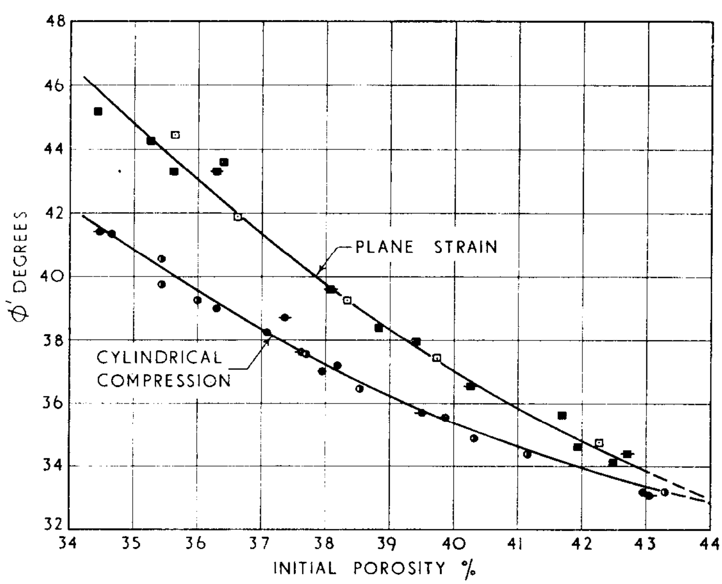

Fig. 7. Comparison of results of drained plane strain and cylindrical compression tests on Brasted sand (Cornforth, 1961)

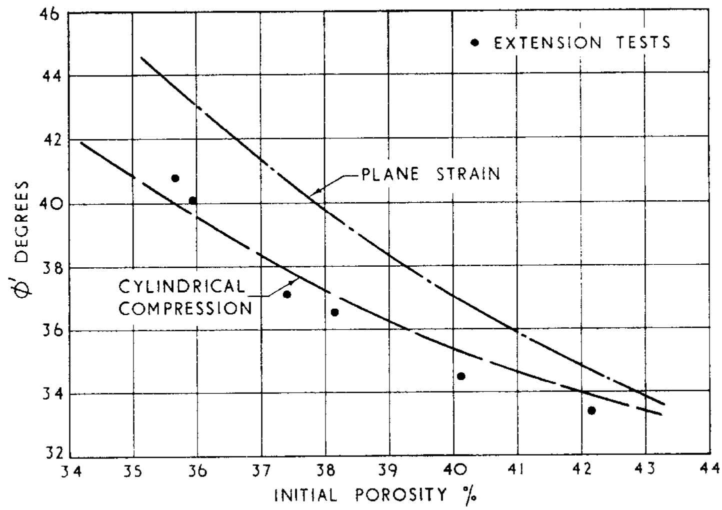

Fig. 8. Comparison of results of drained extension, compression and plane strain tests on Brasted sand (Cornforth, 1961)

The comparison between compression and extension tests is shown in Fig. 8 and it is apparent that the difference in the value of ϕ' is not significant over the range of porosities investigated. The same general conclusion is indicated by a subsequent series of tests by Green on Ham River sand,[4] using lubricated end plattens in both compression and extension tests (Fig. 9).

To examine the failure criteria a knowledge of σ_2' is required. In the compression and extension tests σ_2' is equal to the fluid pressure in the triaxial cell less the pore pressure in the sample. In the plane strain test σ_2' is determined from the load on the lubricated plattens maintaining zero strain in the σ_2' direction (Wood, 1958).

[4] Due to the variation in ϕ' with normal stress, the cell pressures in the extension tests have been selected so that the minor principal stress at failure approximates to the minor principal stress used in the compression tests.

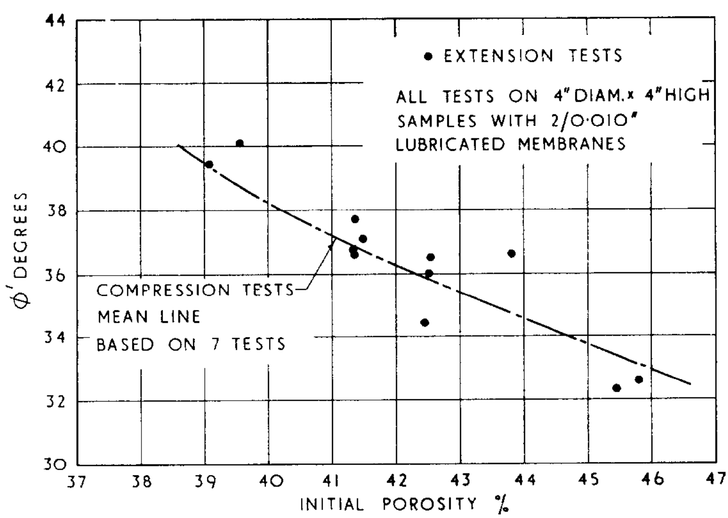

Fig. 9. Comparison of results of drained extension and compression tests on saturated Ham River sand: cylindrical samples with lubricated ends, $\frac{H}{D} = 1$ (Tests by Green, 1965)

The observed values of the ratio $\frac{\sigma_2'}{\sigma_1'+\sigma_3'}$ show a close correlation with the peak value of ϕ' (Fig. 10), over a wide range of values. The empirical expression $\frac{\sigma_2'}{\sigma_1'+\sigma_3'}=\frac{1}{2}\cos^2\phi'$ is in good agreement with the general trend, but slightly underestimates σ_2'. The expression may be derived by combining two earlier empirical relationships. Wood (1958) noted that for tests on compacted moraine carried out in the plane strain apparatus at Imperial College, the relationship between σ_2' and σ_1' at failure approximated to the expression:

$$\sigma_2' = K_0\sigma_1' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where K_0 was the coefficient of earth pressure at rest measured with zero strain in both lateral directions (i.e. when both $\epsilon_2=0$ and $\epsilon_3=0$). Tests reported by Bishop (1958) and Simons (1958) had shown that there was an empirical relationship between K_0 and ϕ' which could be represented with reasonable accuracy by an expression due to Jaky (1944 and 1948):

$$K_0 = 1-\sin\phi' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Combining these expressions and putting

$$\frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'} = \sin\phi' \quad \text{(from equation 6)}$$

we obtain

$$\frac{\sigma_2'}{\sigma_1'+\sigma_3'} = \frac{1}{2}\cos^2\phi' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

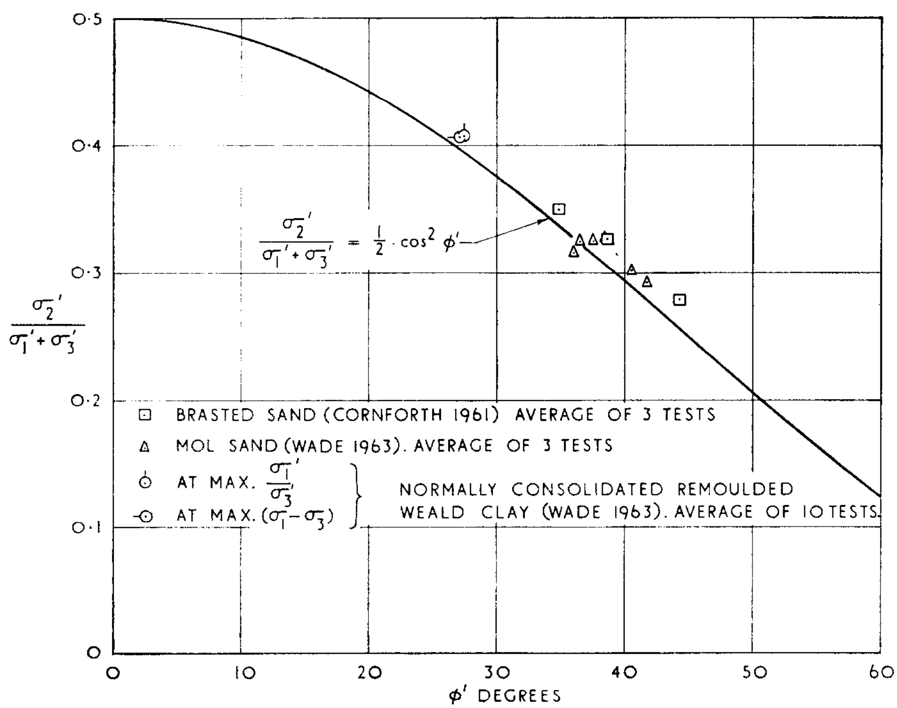

Fig. 10. Correlation with ϕ' of value of intermediate principal stress σ_2' at failure in plane strain

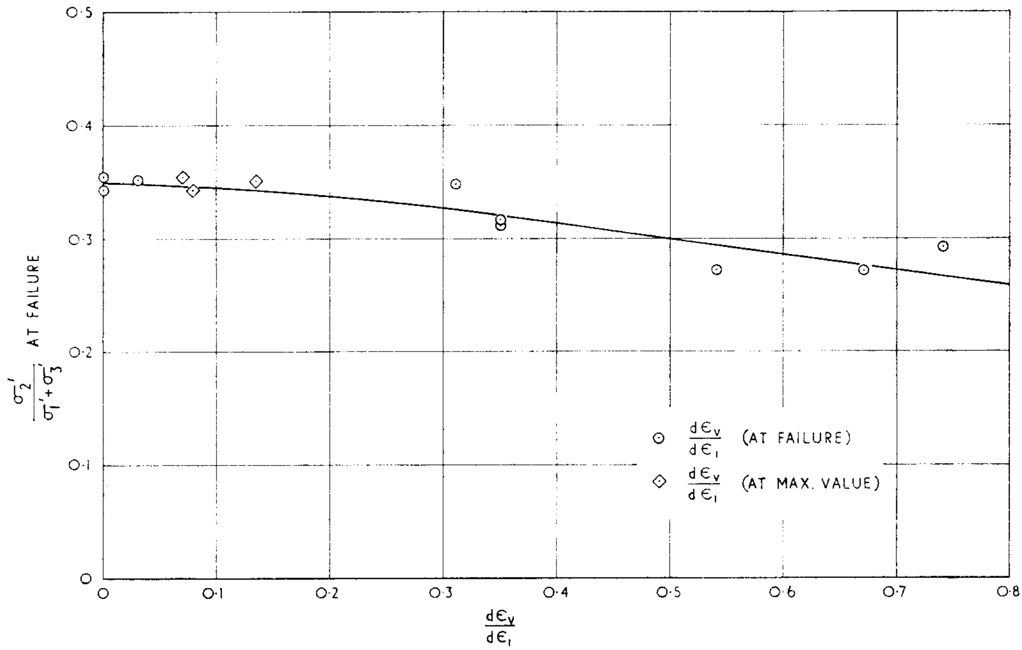

Fig. 11. Variation of $\frac{\sigma_2'}{\sigma_1'+\sigma_3'}$ with rate of volume change $\frac{d\epsilon_v}{d\epsilon_1}$ for drained tests on Brasted sand

It is of interest to note that samples which were deforming at constant volume when the peak stress ratio was reached[5] conformed to this relationship and the ratio $\frac{\sigma_2'}{\sigma_1'+\sigma_3'}$ showed no indication of being equal to $\frac{1}{2}$ as might have been expected (Fig. 11). What occurs to this ratio at strains beyond those corresponding to the maximum stress cannot be determined in the present apparatus, as zone failure always follows closely after the peak and the localized value of σ_2' cannot be measured.

With the knowledge of σ_2' we can now compare the observed values of ϕ' $\left(\text{defined by } \sin\phi' = \frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'}\right)$ with those predicted by the various failure criteria as b $\left(=\frac{\sigma_2'-\sigma_3'}{\sigma_1'+\sigma_3'}\right)$ varies between 0 and 1. Fig. 12(a) shows this comparison for loose Brasted sand, which is shearing almost at constant volume at the peak stress ratio. It will be seen that the results fit well with the Mohr–Coulomb failure criterion, whereas the extended von Mises and extended

[5] In a plane strain compression test on loose sand constant volume shear may occur firstly at the peak stress ratio, when the condition of pure shear is approximated to, and then subsequently at a lower stress ratio, when strains are largely confined to a thin slip zone in which simple shear is approximated to. In the tests on Brasted sand performed by Cornforth (1961) the peak value of ϕ' corresponding to zero rate of volume change is 34·3°. If the direction of the thin slip zone is taken to correspond to a Mohr–Coulomb slip plane a residual ϕ' of 32·3° is obtained. If, alternatively, the shear stress acting along the boundary of this zone is taken (following Hill, 1950) to be equal to the maximum shear stress within the zone, the residual value of ϕ' would be 39·2°, which is clearly unreasonable. These differences suggest that constant volume yield in pure shear may involve a failure mechanism significantly different from that associated with constant volume yield in simple shear.

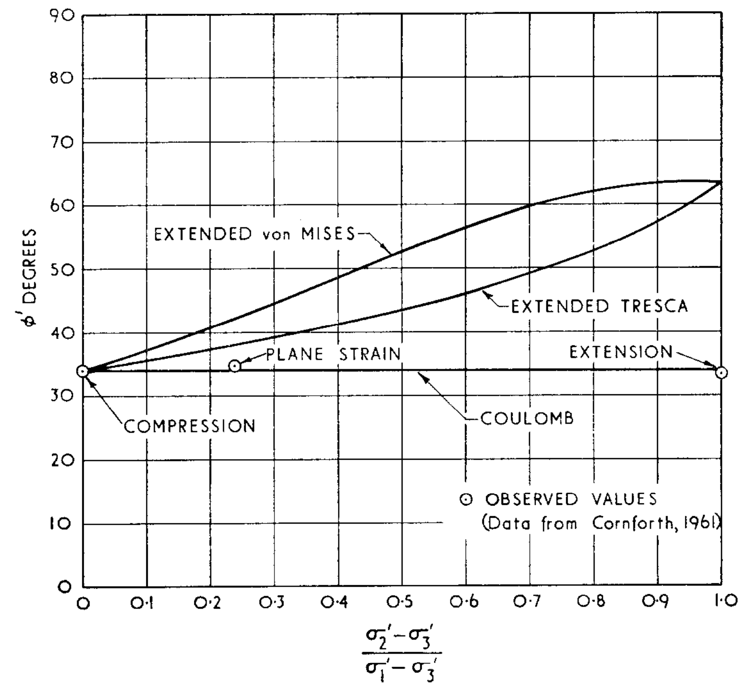

Fig. 12a. Observed and predicted values of ϕ'. Loose sand; ϕ' in compression $= 34°$ and $\alpha = 1{\cdot}375$

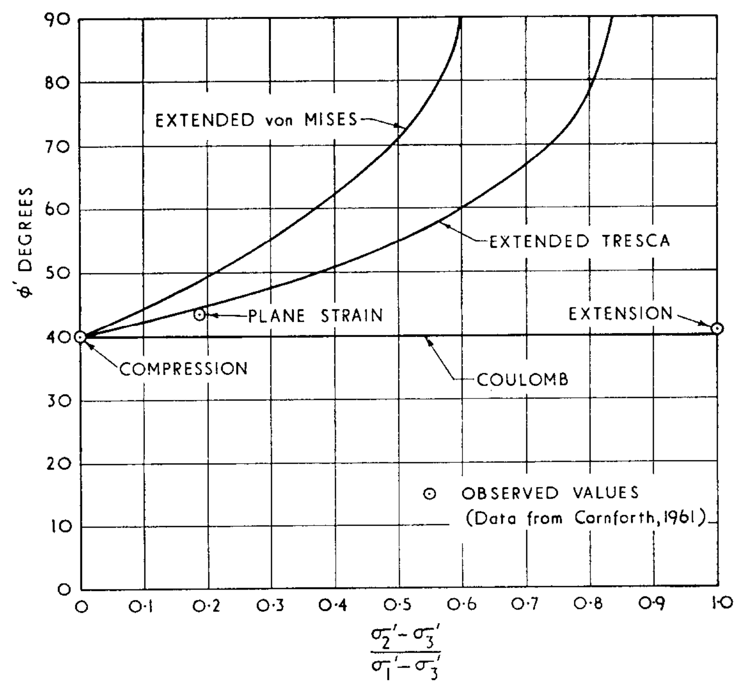

Fig. 12b. Observed and predicted values of ϕ'. Dense sand; ϕ' in compression $= 40°$ and $\alpha = 1{\cdot}636$

Tresca criteria predict values which differ from the observed values by more than the most pessimistic estimate of experimental error.

Fig. 12(b) shows that for dense sand, which is dilating at failure, the Mohr–Coulomb criterion again gives the best over-all fit, though for a comparison of the compression test and the plane strain case only the extended Tresca is in better agreement.[6] However, for the extension tests the extended Tresca and the extended von Mises both fail to predict meaningful results. The reason for this is of considerable interest.

In Fig. 13 the values of ϕ' in compression predicted by the two failure criteria are plotted against the value of the parameter α which is used in both expressions to indicate the increase in strength with normal stress. This α is the same as that used by Roscoe *et al* (1958). It will be seen that for the compression test the relationship between ϕ' and α is almost linear and in fact approximates to $\phi'=25\alpha$ over the range $\phi'=20°$ to $40°$. However, the value of ϕ' in extension predicted by both failure criteria rapidly diverges from that in compression as α increases, and becomes equal to $90°$ (i.e. $\sigma_1'/\sigma_3'=\infty$) when $\alpha=1{\cdot}5$. At this value of α the compression value of $\phi'=36{\cdot}9°$, which is well within the range of values encountered in dense sand.

The physical explanation can be seen from the representation of the failure criteria in a three-dimensional stress space (Fig. 14). If the axes σ_1', σ_2', σ_3' represent the magnitudes of the principal effective stresses in those three directions, we can select a plane on which $\sigma_1'+\sigma_2'+\sigma_3'=\text{constant}$, and a diagonal $00'$ (normal to it) for which $\sigma_1'=\sigma_2'=\sigma_3'$ (i.e., no shear

[6] If the value of $\sigma_2'/(\sigma_1'+\sigma_3')$ in the failure zone exceeds the average value recorded in the plane strain apparatus and approaches $\frac{1}{2}$ the apparent agreement with the extended Tresca criterion no longer obtains.

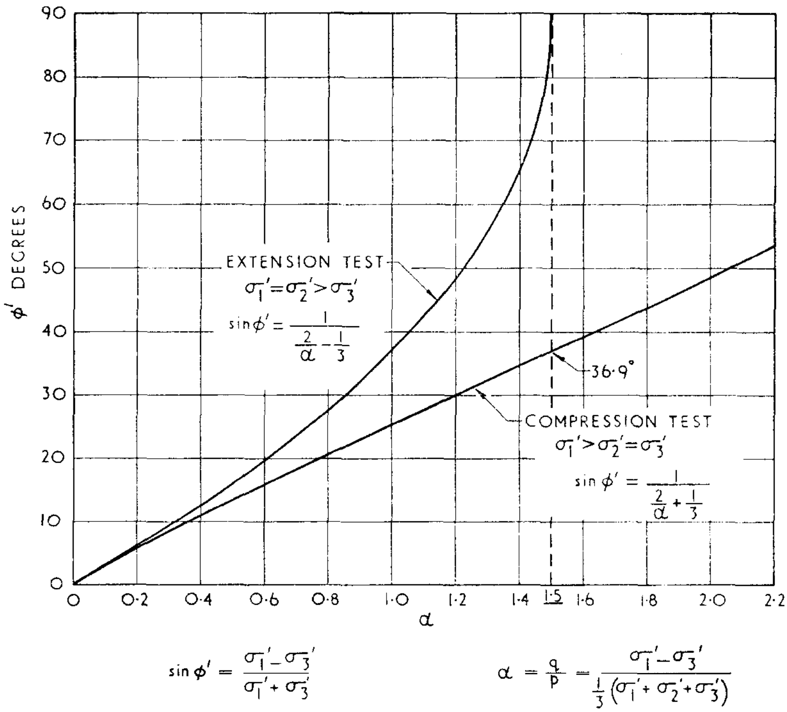

Fig. 13. Relationship between parameters used in failure criteria

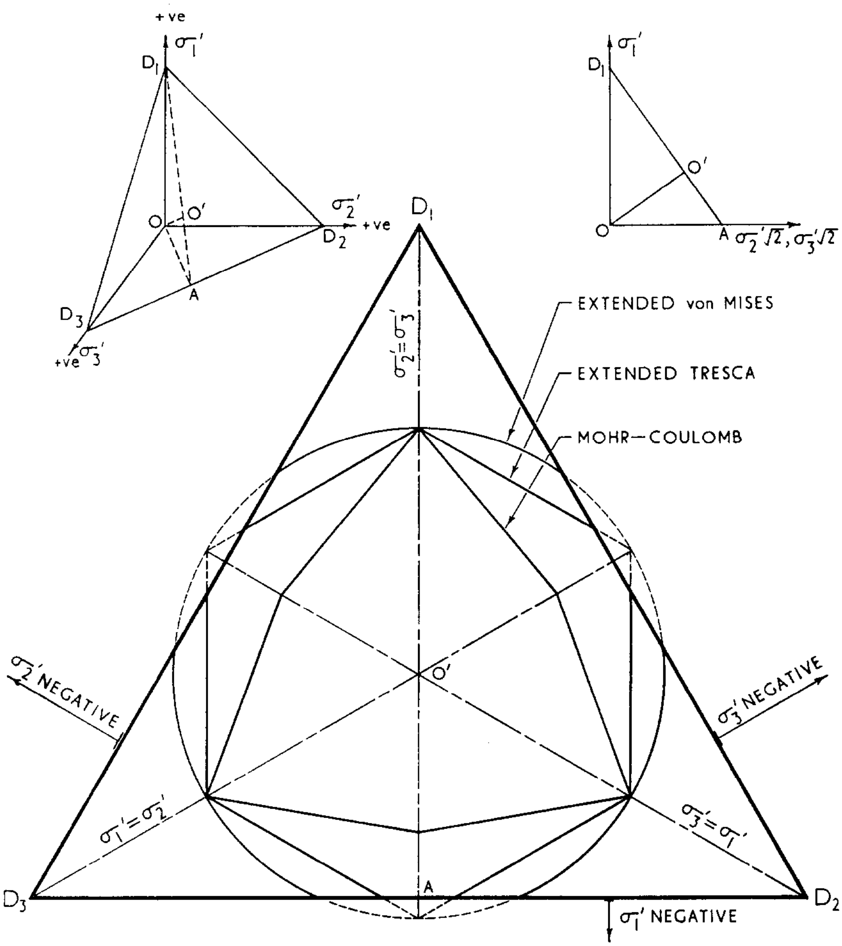

Fig. 14. Representation of failure criteria in principal effective stress space, showing boundaries of positive stress field. Sections of failure surface shown for $\phi' = 40°$ in axial compression

stress). States of limiting equilibrium expressed by the various failure criteria are then represented by pyramid shaped surfaces having their apex at 0 and showing characteristic sections on the plane $D_1D_2D_3$. These are an irregular hexagon for the Mohr–Coulomb criterion; a circle with centre 0′ for the extended von Mises criterion and the regular hexagon inscribed in this circle for the extended Tresca criterion.

In Fig. 14 these sections are plotted for $\phi' = 40°$ in axially symmetrical compression (i.e. $\alpha = 1{\cdot}636$), and it will be seen that as σ_2' moves from σ_3' to σ_1' the circle and hexagon representing the extended von Mises and Tresca criteria respectively go outside the lines D_1D_2 etc., into negative effective stress space. For a cohesionless soil (or $c' = 0$ material) this is meaningless. These failure criteria are therefore in principle unable to represent the behaviour of the denser frictional materials having a compression ϕ' of more than $36{\cdot}9°$ (i.e. $\alpha = 1{\cdot}5$) for which the extended von Mises circle is tangential to D_1D_2, D_2D_3 and D_3D_1.

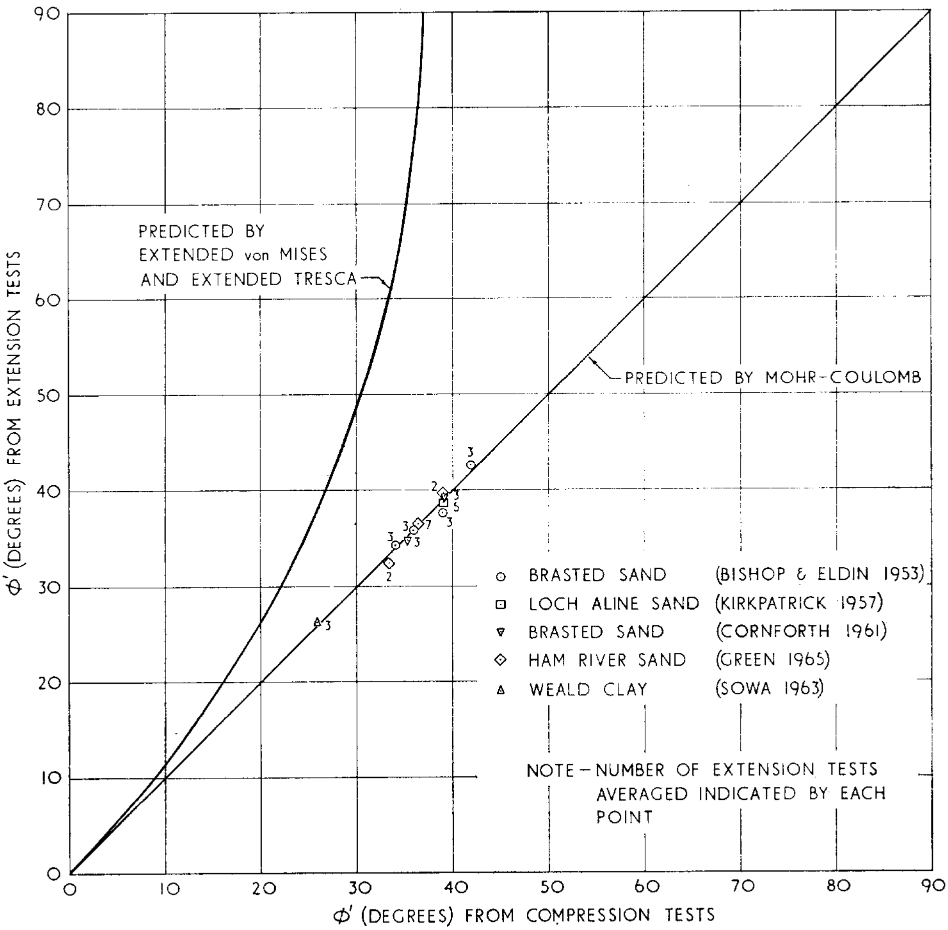

Fig. 15. Comparison of results of compression and extension tests

Even within this range the two criteria predict values of ϕ' in extension (Fig. 15), which differ from the experimental results by an amount which cannot be attributed to experimental error. The experimental results, however, strongly support the Mohr–Coulomb criterion,[7] and we must, I feel, accept the Mohr–Coulomb criterion as being the only simple criterion of reasonable generality.[8,9] It does, however, underestimate the value of ϕ' for plane strain in dense sands by up to 4° (from tests on Brasted sand by Cornforth and Mol sand by Wade).

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[7] This applies to all the data illustrated in Fig. 15, to tests reported by the Norwegian Geotechnical Institute (1958), and by Wade (1963). The tests illustrated in Fig. 15 were drained tests on saturated samples consolidated under equal all round pressure, with the exception of the series by Cornforth (1961), where consolidation with zero radial strain was used. Unpublished tests at Imperial College by Walker on dry sand consolidated under equal all round pressure agree with the tests described above. In contrast early tests by Habib (1953) and Peltier (1957) show lower values of ϕ' in extension. No obvious explanation for this difference is apparent. Tests by Haythornthwaite (1960) also show a difference, but are based on a different definition of the point of failure. On the other hand undrained tests on undisturbed clays or clays initially anisotropically consolidated from a slurry (Ladd and Bailey, 1964; Smotrych, unpublished data) show higher values of ϕ' in extension. The interpretation of the test data is, however, subject to more

Tests representing the actual state of stress or strain to be encountered in practice are obviously best for precise work in this case. The present data can be expressed in terms of an extended Mohr–Coulomb criterion involving two parameters:

$$\frac{\sigma_1'-\sigma_3'}{\sigma_1'+\sigma_3'}=\frac{K_1}{1-K_2\sqrt{b(1-b)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where $b=\dfrac{\sigma_2'-\sigma_3'}{\sigma_1'-\sigma_3'}$

and K_1 and K_2 are parameters determined from compression and plane strain tests (K_1 being equal to sin ϕ' in the compression and extension test).

Further studies of the variation of ϕ' with b for dense frictional materials now being carried out at Imperial College may, however, suggest a more elaborate expression.

It is clear, however, from Fig. 12 that from an engineering point of view the use of the extended von Mises or Tresca criteria would lead to a very substantial overestimate of strength for a wide range of b values.

(2) THE BEHAVIOUR OF SOILS UNDER HIGH STRESSES

There is one limitation, which is of considerable engineering significance, common to all the failure criteria so far discussed. The constant of proportionality between strength and normal stress, either ϕ' or α, is not really a constant when a wide range of stress is under consideration. This is of particular importance when drawing conclusions from tests on models on the one hand, where unpublished tests at Imperial College by Dr Ambraseys and Mr Sarma have shown that values of ϕ' may rise by up to 8° at stresses represented by a fraction of an inch of sand, and on the other hand in the design of high dams (Fig. 16) which are now reaching 1000 ft in height and imply considerably reduced values of ϕ'. The major principal stress in such a dam could approach 900 lb/sq. in.

Typical Mohr envelopes for various soils tested under a wide range of pressures are illustrated in Fig. 17. The marked curvature of many of the failure envelopes will be seen. It is of interest to note that the uppermost envelope[10] represents tests on one of the fill materials of the Oroville dam shown in the previous figure. The other materials include rockfill from the Infiernillo dam,[11] compacted glacial till, two dense sands, and one loose sand (Ham River), and undisturbed silt and two undisturbed clays, as well as one clay consolidated from a slurry. There is some indication that the curvature is most marked for soils which

(*a*) are initially dense or heavily compacted,
(*b*) are initially of relatively uniform grain size,
(*c*) if undisturbed, have been heavily over-consolidated.

In the coarser granular materials (the sands, gravels, and rockfills) the curvature is clearly associated with crushing of the particles, initially local crushing at interparticle contacts, and ultimately shattering of complete particles. This in turn is associated, in dense materials,

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uncertainty. The influence of undrained stress path on the value of ϕ', which is of greater importance in clays, and the anisotropy of the soil structure, are likely to play a more significant part in the behaviour of clays and more experimental data is required before general conclusions can be drawn.

[8] This conclusion was reached by Kirkpatrick (1957). However, from thick cylinder tests on dense sand he obtained an increase in ϕ' of only 2° for $b=\frac{1}{2}$.

[9] The method proposed by Johansen (1958) reduces to the Mohr–Coulomb criterion for the case when the value of ϕ' in extension is equal to that in compression.

[10] The minimum ratio of sample diameter to maximum particle size which may be used without an overestimate of the strength resulting has not been fully investigated. The ratio of only 4 was used in the tests by Hall and Gordon (1963) quoted above.

[11] Details of the special triaxial cell for samples 113 cm. diameter are given by Marsal *et al* (1965).

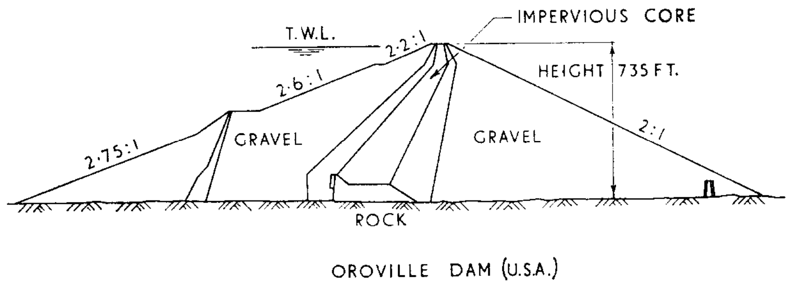
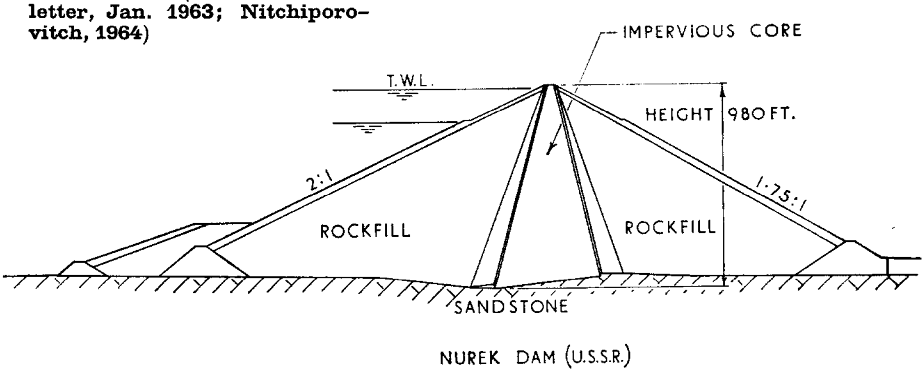


Fig. 16. Two high dams now under construction. (USCOLD Newsletter, Jan. 1963; Nitchiporovitch, 1964)



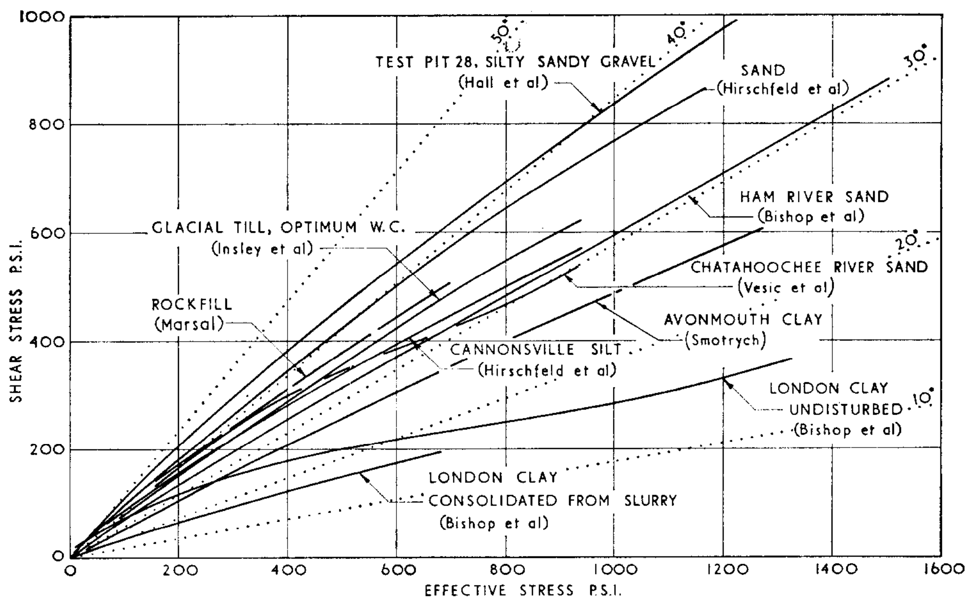


Fig. 17. Mohr envelopes for various soils under high confining pressures (all tests drained except London Clay consolidated from slurry)

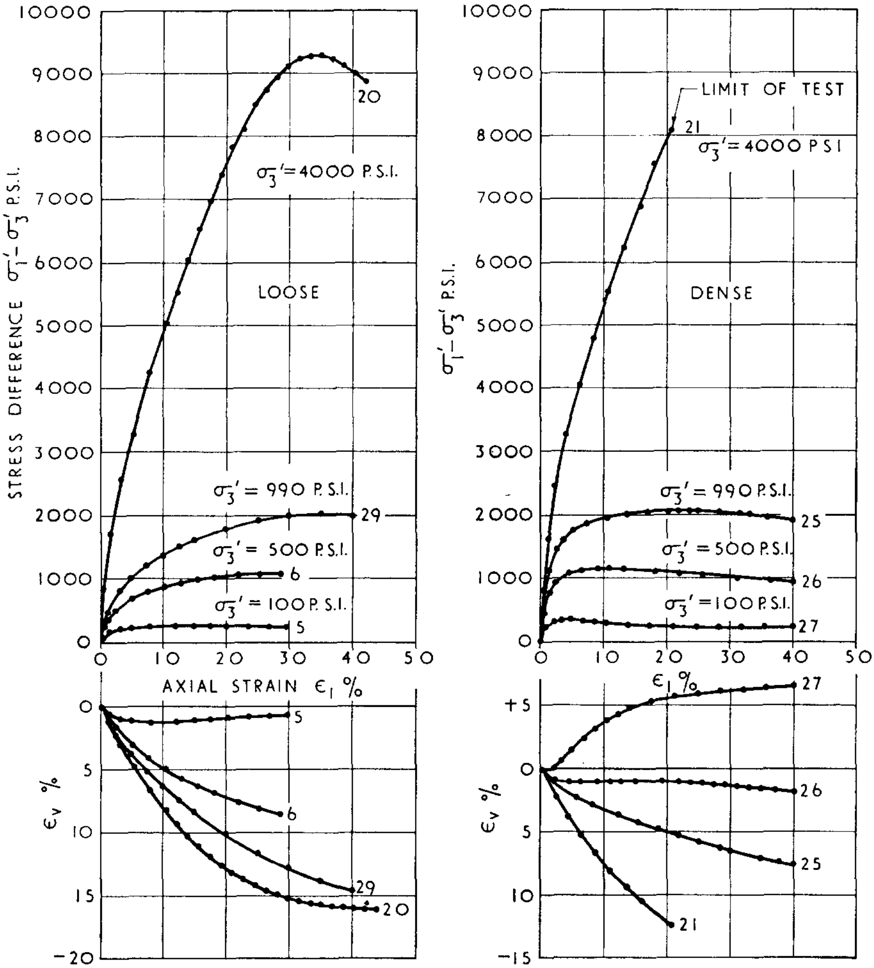


Fig. 18. Results of drained tests on saturated Ham River sand. (Tests by Skinner 1964–66)

with a greatly reduced rate of volume increase at failure, and this leads to a marked reduction in the overall value of ϕ' at failure. The dependence of ϕ' on rate of volume change was demonstrated by Taylor (1948), Bishop (1950), Hafiz (1950), and Bishop and Eldin (1953). A discussion of the various mathematical approaches to the problem is outside the scope of the present paper, particularly as the work done in crushing the particles is usually omitted from the calculations. From a practical point of view it should be noted that only London Clay and Avonmouth Clay gave ϕ' values significantly below 30°.

Typical stress, strain and volume change curves for drained tests on saturated Ham River sand are given in Fig. 18. The marked effect of pressure on the volume change during shear can be clearly seen. For loose sand the reduction in volume during shear rises rapidly with increase in σ_3' up to about 1000 lb/sq. in., and then more gradually as σ_3' is increased to 4000 lb/sq. in. Dense sand, which is strongly dilatant at low confining pressures, shows almost zero rate of volume change at failure when σ_3' reaches 500 lb/sq. in. At higher values of σ_3'

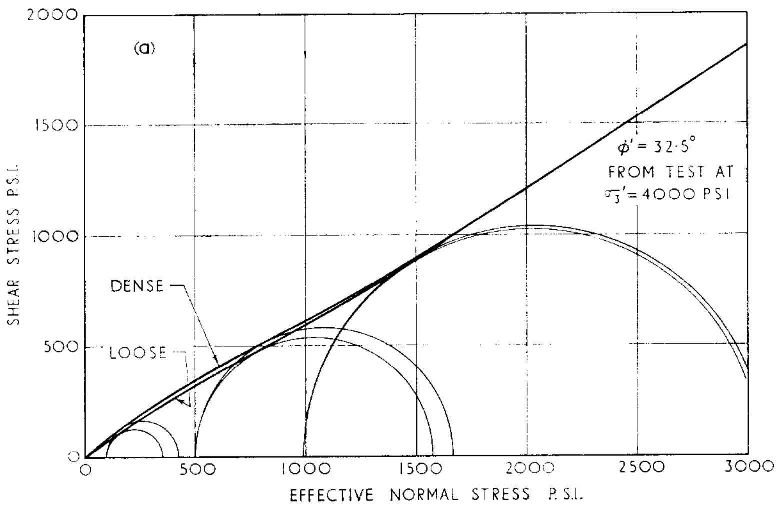


Fig. 19a. Mohr envelopes for drained tests on loose and dense Ham River sand (data from Skinner, 1964–66)

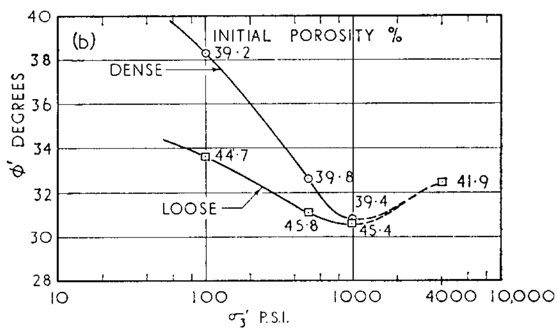


Fig. 19b. Variation of ϕ' with σ_3' (data from Skinner, 1964–66)

dense sand shows an increasingly marked reduction in volume during shear and at $\sigma_3' =$ 4000 lb/sq. in., its behaviour approximated to that of loose sand. For loose sand the stress difference at failure when $\sigma_3' = 4000$ lb/sq. in. exceeds 9000 lb/sq. in.

The Mohr envelopes for loose and dense sand are given in Fig. 19(a). The marked curvature of the envelope for the dense samples and its convergence at high stresses with that for loose samples can be seen. Fig. 19(b) shows that the difference between ϕ' for dense sand and ϕ' for loose sand drops from nearly 5° at $\sigma_3' = 100$ lb/sq. in. to only 0·2° when σ_3' is 1000 lb/sq. in.[12] It is also apparent that even for sand placed in a very loose state ϕ' is not independent of effective stress, but drops about 3° as σ_3' rises from 100 lb/sq. in. to 1000 lb/sq. in. As will be seen later, this drop is closely associated with the rate of volume change at failure.

[12] In this context ϕ' is defined by the tangent to the origin from the stress circles at the value of σ_3' under consideration.

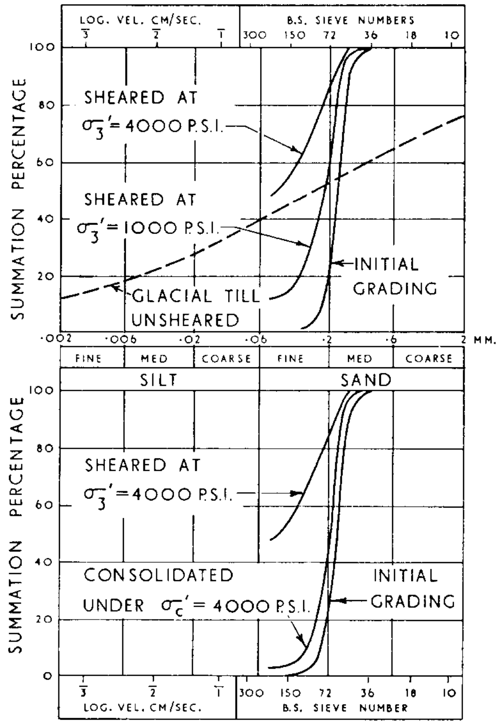


Fig. 20a. Changes in grading resulting (1) from shear at different pressures and (2) from consolidation as compared with complete shear test
Saturated Ham River sand. (Tests by Skinner, 1964–66)

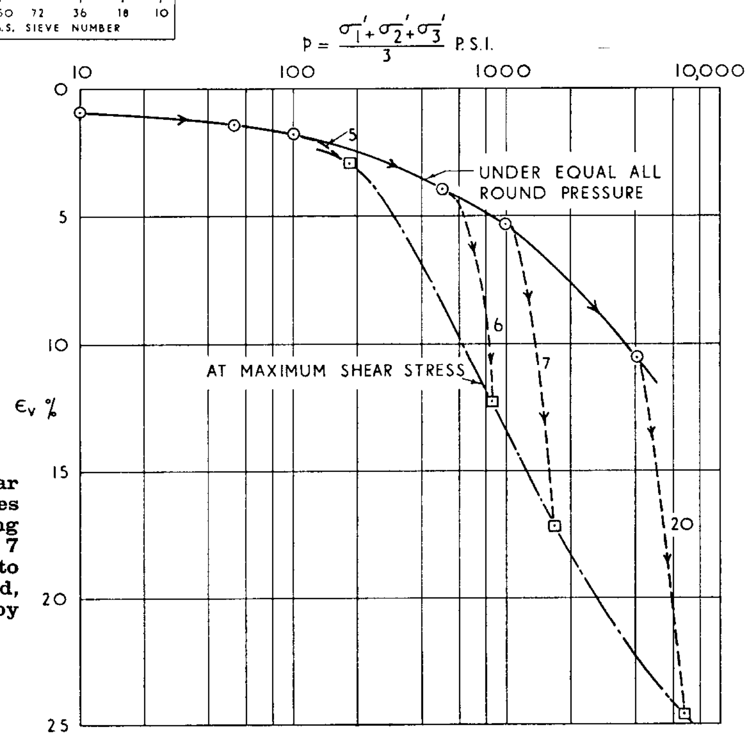


Fig. 20b. Influence of shear stress on volume changes associated with crushing of sand grains. (Test 7 terminated just prior to peak.) Ham River sand, initially loose. (Tests by Skinner, 1964–66)

In Fig. 20(a) the change in grading due to particle breakdown during compression and shear is illustrated. Several points of interest may be noted. Firstly, the combined effect of consolidation and shear leads to very marked particle breakdown even in a medium to fine sand. This effect can also be detected at much lower stresses than illustrated here. Secondly, breakdown results in a grading tending to approximate to that found in naturally occurring glacial tills, for which ϕ' has proved to be relatively insensitive to stress (Fig. 17; see also Insley *et al*, 1965).[13] This suggests the type of grading likely to be suitable for fills to withstand high stresses. Thirdly, at high stresses the particle breakdown occurs to a much greater extent during the shear stage than during the consolidation stage.

This latter point is further illustrated by plotting the associated volume changes against the average stress $\frac{\sigma_1'+\sigma_2'+\sigma_3'}{3}$ in Fig. 20(b). Where the increase in average stress is associated

[13] Further data on the breakdown of sand particles with normal and shear stress is given by Vesic and Barksdale (1963) and Borg, Friedman, Handin and Higgs (1960). The breakdown of rockfill is discussed by Marsal (1965). These points are further discussed by Bishop (1965).

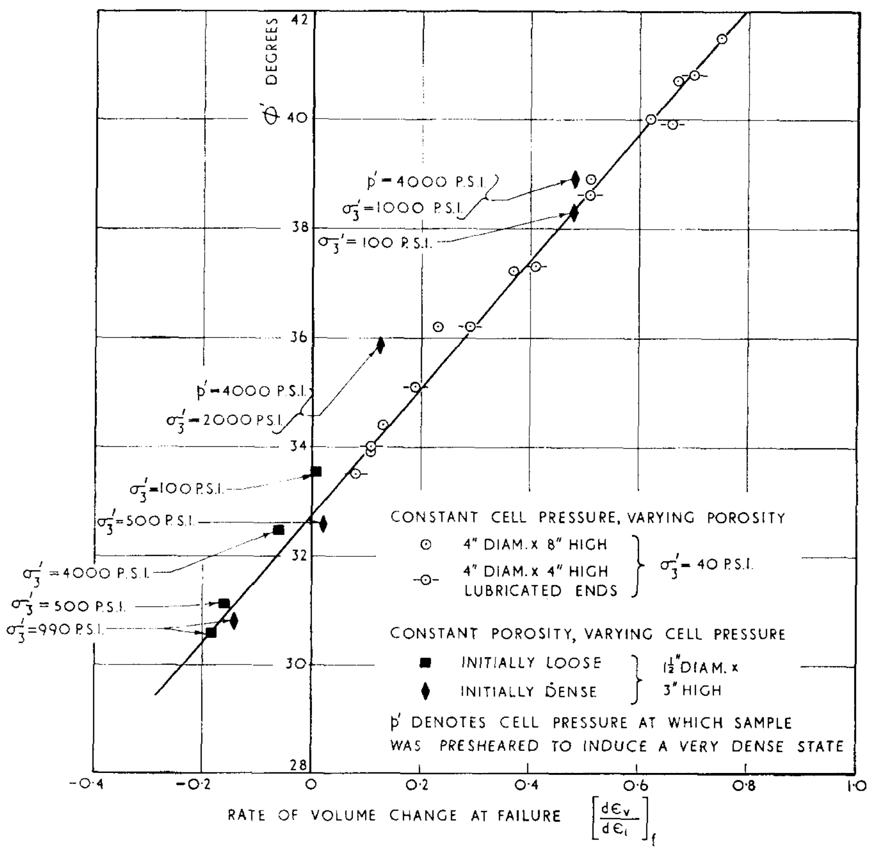


Fig. 21. The relationship between ϕ' and rate of volume change $\frac{d\epsilon_v}{d\epsilon_1}$ at failure. (Tests by Green and Skinner, 1964–66)

with a stress difference (or shear stress) the rate of volume decrease is many times greater than under all round pressure, except at low consolidation pressures.

In Fig. 21 the relationship between the value of ϕ' and the rate of volume change $\frac{d\epsilon_v}{d\epsilon_1}$ at failure (or dilatancy rate) is given. At a given pressure the dilatancy rate will vary with the initial density at which the material is placed, and a very close correlation with ϕ' will be noted. At a given initial density an increase in pressure will reduce the rate of dilatancy, and it is of interest to note how near to the previous line values obtained in this way actually lie. This is all the more remarkable when the extremely wide stress range is considered, from $\sigma_3'=40$ lb/sq. in. to $\sigma_3'=4000$ lb/sq. in. For this particular sand (Ham River), the maximum departure from the mean line is only $1\frac{1}{2}°$. The curvature of the failure envelope is thus largely accounted for by the decrease in the rate of dilatancy with increasing stress.

The variation in the rate of volume change at failure with increase in effective stress σ_3' is plotted in Fig. 22. It appears that for Ham River sand the change from positive to negative rate of dilatancy at failure occurs mainly within the stress range $\sigma_3'=0-1000$ lb/sq. in. At higher stresses the trend is reversed and failure at almost constant rate of volume change occurs after the large initial decrease in volume illustrated in Fig. 18. At failure, however, the material is no longer a fine to medium sand but a well graded silty sand with nearly 50% in the silt sizes or smaller (Fig. 20a). The greatly increased number of interparticle contacts carrying the stresses must largely counterbalance the effect of the reduction in the basic coefficient of interparticle friction with pressure (see, for example, Hafiz, 1950).

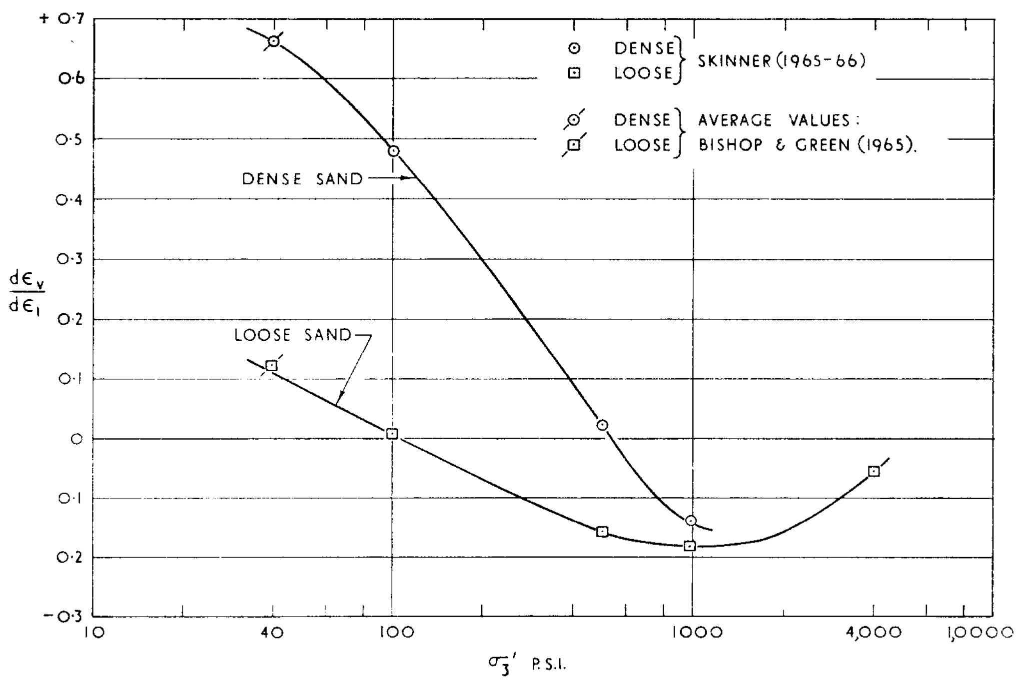


Fig. 22. Variation of rate of dilatancy at failure with increase in effective stress; saturated Ham River sand

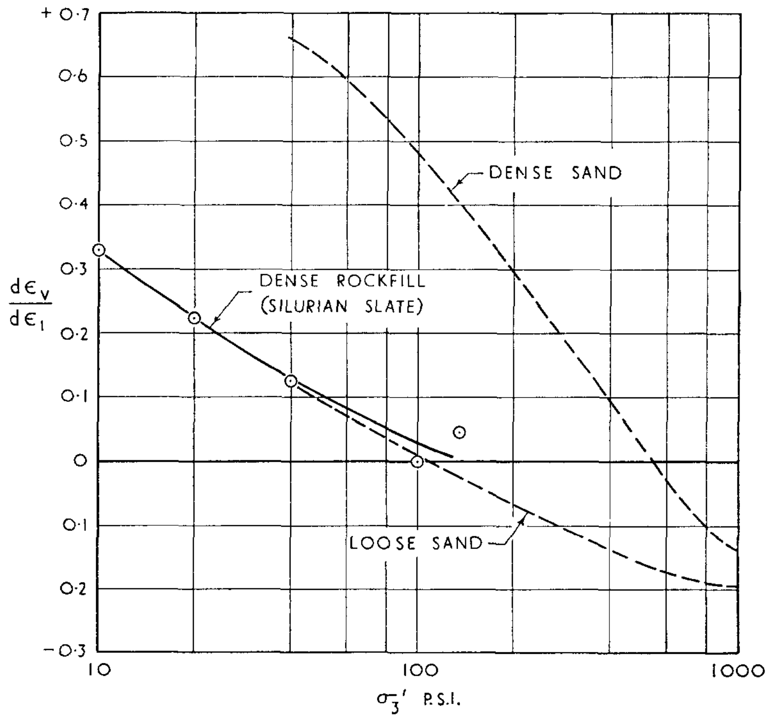


Fig. 23. Variation of rate of dilatancy at failure with increase in effective stress: dense rockfill (Silurian slate). (Tests by Tombs, 1966)

A comparison with the results of tests on 1-ft diameter samples of a compacted rockfill is made in Fig. 23. Within the working range of stress the dilatancy characteristics of the compacted rockfill (Silurian slate) approximate more closely to those of a loose sand than those of a dense sand. This is consistent with the very marked particle breakdown observed during the tests.

Provided that the influence of high pressure on ϕ' and on volume change is measured and that designs are not based on the extrapolation of low pressure tests, little difficulty is involved under drained conditions in achieving safe designs. However, fully saturated granular material under these stresses could be very dangerous under undrained or shock loading, as structural breakdown gives it an undrained stress path very similar to that of quick clay. The curves showing the relationships between stress, strain, and the build up of pore pressure are given in Fig. 24. The very small strain at failure in test 8 should be noted. The stress paths are given in Fig. 25.[14]

It will be seen that the shape of the stress path of test 8 is very similar to that observed in the low stress range for very loose sands which are susceptible to flow slides (Waterways Experiment Station, 1950; Bjerrum, 1961), and for sensitive clays (Taylor and Clough, 1951; N.G.I., unpublished data). In particular it will be noted that at the maximum value of the stress difference (represented by $\frac{1}{2}[\sigma_1'-\sigma_3']$) the value of ϕ' mobilized is 21·3° for test 8 (Fig. 25); thereafter the rise in pore pressure more than compensates for the increase in ϕ' as the residual state is approached.

[14] Further details of these tests are given by Bishop, Webb and Skinner (1965).

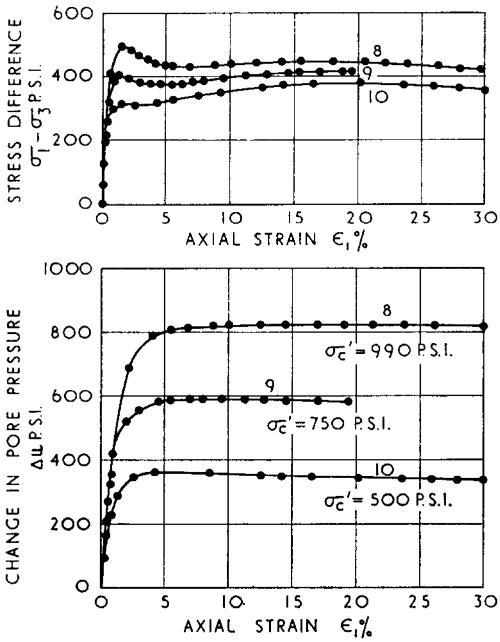


Fig. 24. Results of consolidated-undrained tests on saturated Ham River sand. (σ_c' denotes effective consolidation pressure). (Tests by Skinner, 1964)

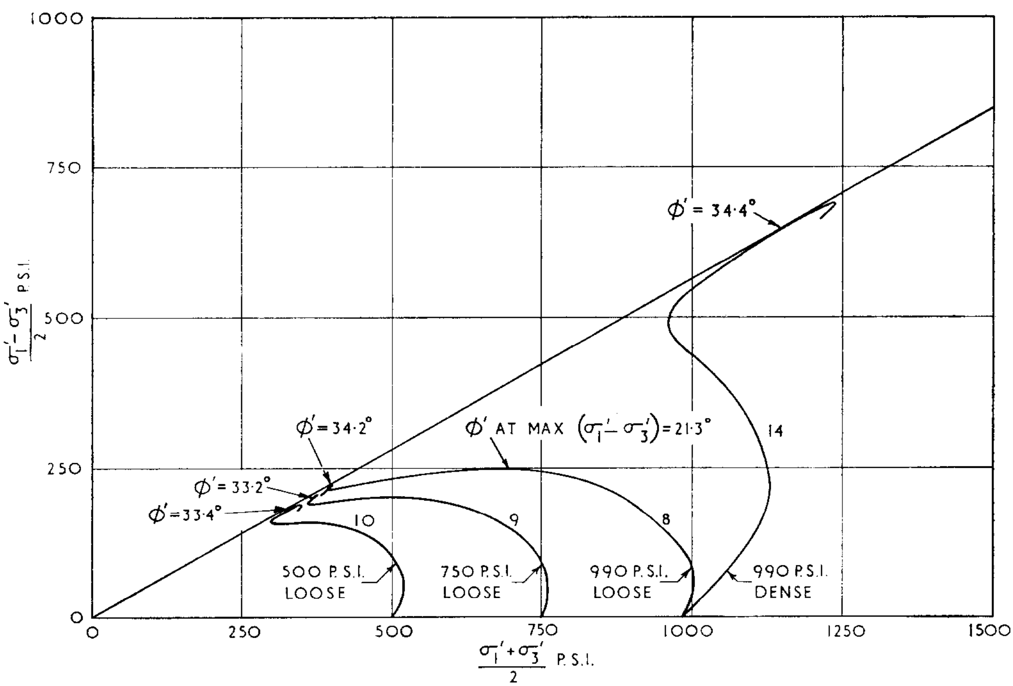


Fig. 25. Stress paths for consolidated-undrained tests on saturated Ham River sand (Tests by Skinner, 1964–66)

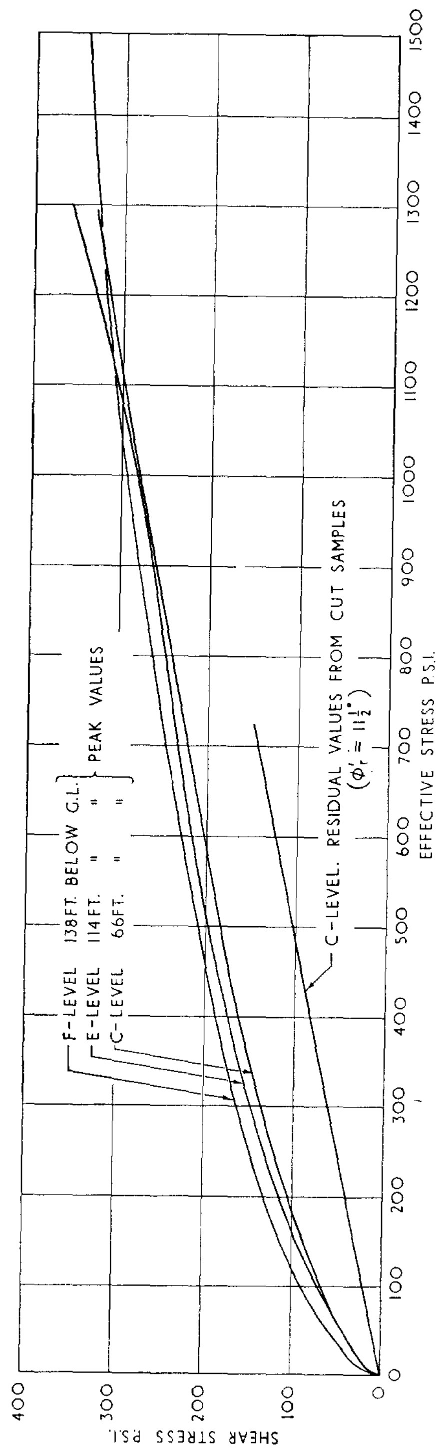


Fig. 26. Mohr envelopes for undisturbed London Clay from Ashford Common shaft (after Bishop, Webb and Lewin, 1965)

It would appear, therefore, that fill to be used fully saturated, and under high stresses, should preferably be well graded and should in any case be placed in a dense state, so that under undrained loading the pore pressure build-up is reduced and the undrained strength increased as illustrated in test 14 (Fig. 25).

There is not space to deal in detail with the behaviour under a wide range of stress of clay as an undisturbed material and as a compacted fill. One illustration must suffice. Fig. 26 shows the Mohr envelopes for undisturbed London Clay from depths of 66, 114, and 138 ft below ground level at the Ashford Common shaft.[15] The change in slope from 30° near the origin to 10° at very high stresses will be seen (the latter corresponds to an origin $\phi' = 15°$). For the same material consolidated from a slurry the value of ϕ' obtained from undrained tests with pore pressure measurement (Fig. 17) drops only from 21° near the origin to about 16° (tangent from the origin). The residual angle ϕ' from pre-cut undisturbed samples from a depth of 66 ft dropped only from $11\frac{1}{2}°$ to 11°.

(3) THE IN-SITU UNDRAINED STRENGTH OF A SOIL

In using the undrained strength of a clay in the $\phi_u = 0$ analysis for short-term loading the engineer is apt to imagine that, apart from sampling disturbance, a given sample of clay has a unique undrained strength, irrespective of the type of test (triaxial, vane or simple shear test) used to measure it and irrespective of the inclination or direction of the slip surface implied in the problem he is analysing.

In fact, as early as 1949, Professor J. Brinch Hansen and Dr R. E. Gibson showed that on theoretical grounds the laboratory undrained compression test should differ from the field vane test (Table 1) and that the in-situ strength on an inclined failure surface could differ from them both, being greatest in the case of active earth pressure and least in the case of passive earth pressure. This prediction was for a soil consolidated with zero lateral yield, i.e. the initial lateral effective stress was only K_0[16] times the vertical effective stress. The soil was thus subjected to an anisotropic stress history, but no anisotropy of the shear strength

Table 1

Predicted influence of orientation of shear plane on undrained strength (after Hansen and Gibson, 1949)

Value of $\frac{c_u}{p}$	Clay 1	Clay 2
Active earth pressure	0·331	0·282
Passive earth pressure	0·193	0·256
Failure on a horizontal plane . .	0·213	0·262
Vane test	0·191	0·252
Unconfined compression test:		
condition (a)	0·170	0·247
condition (b)	0·250	0·283

Clay 1. Normally consolidated, sensitive silty clay.
Clay 2. Normally consolidated, typical British post-glacial clay.
Condition (a), Condition (b): Probable limits of influence of stress release on sampling.

[15] The index properties at the 114 ft level, for example, are $W_L = 70$, $W_p = 27$, clay friction $= 57$, activity $= 0{\cdot}75$, initial water content $= 24{\cdot}2$. For full details see Bishop, Webb and Lewin (1965).

[16] K_0 is termed the coefficient of earth pressure at rest and is generally within the range of 0·4 to 0·7 for normally consolidated clays.

parameters in terms of effective stress was assumed in estimating the undrained strength. Their results thus indicated the influence on the excess pore pressure at failure of stress history and, in particular, of the rotation of principal stress directions in an anisotropically consolidated soil.

Their conclusions have been rather lost sight of in recent years, due to the renewed emphasis on the unique relationship between undrained strength and water content. However, test results both in the field and in the laboratory, though still rather limited, suggest that Hansen and Gibson's conclusions are substantially correct, though complicated by two additional features. These are that

(*a*) the soil structure may in fact be significantly anisotropic with respect to its shear parameters in terms of effective stress, due to orientation and/or segregation of particles during deposition, and due to orientation arising from its subsequent history;

(*b*) the degree of mobilization of these parameters (c' and ϕ') at the peak stress difference varies with the orientation of the principal stresses at failure, as does the strain at failure.

Table 2

Some examples in which c_u depends on principal stress directions during shear (tests in situ or on undisturbed samples)

1. c_u from field vane lower than c_u from piston sampler or block samples. (Vold, 1956; Coates and McRostie, 1963).
2. c_u from field vane for vertical plane lower than for horizontal plane. (Aas, 1965.)
3. c_u from block samples with axis horizontal lower than with axis vertical in lightly over-consolidated clay. (Lo, 1965)
4. c_u from block samples with axis horizontal higher than with axis vertical in heavily over-consolidated clay. (Ward, Marsland and Samuels, 1965)
5. c_u/p for simple shear (max shear stress horizontal) lower than in triaxial compression with axis vertical. (Bjerrum and Landva, 1966)
6. c_u/p for axial extension much lower than in axial compression for samples consolidated with zero lateral yield. (Ladd and Bailey, 1964; Ladd and Varallyay, 1965)

Some of the test results which bear on this problem are listed in Table 2. Of particular interest are the field tests (Fig. 27) carried out by the Norwegian Geotechnical Institute with a series of specially proportioned vanes (Aas, 1965), as they indicate the relative magnitudes of the in-situ undrained strength on the vertical and horizontal planes respectively. The ratio varies between $\frac{1}{2}$ and $\frac{2}{3}$ for the three normally consolidated clays tested. Since the conventionally proportioned vane measures mainly the strength on a vertical cylindrical surface it may underestimate the field value relevant to some engineering problems in normally or lightly over-consolidated clays. The values are for sensitive or quick clays; values for the clays usually encountered in Britain would be of great interest.

The variation in the undrained strength of lightly and heavily over-consolidated clay with the direction of the applied major principal stress is illustrated in Fig. 28. The samples were all cut from blocks taken from vertical shafts[17] and average values are given based on a large number of tests detailed in the references. In the lightly over-consolidated Welland clay the

[17] With the exception of the tests described by Bishop (1948).

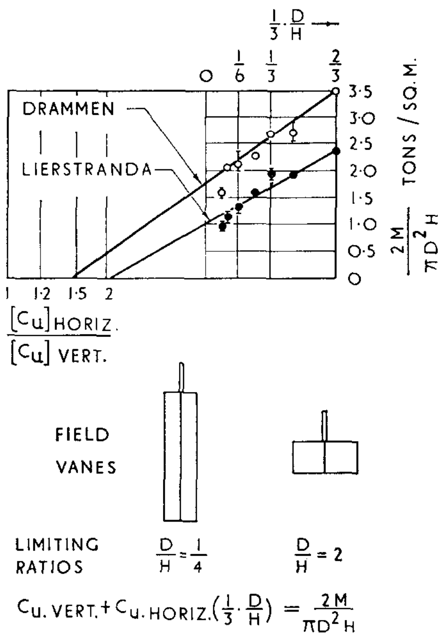


Fig. 27. Determination of anisotropy ratio $\frac{(c_u)\text{ horiz.}}{(c_u)\text{ vert.}}$ from undrained tests with vanes of different $\frac{D}{H}$ ratios. (After Aas, 1965)

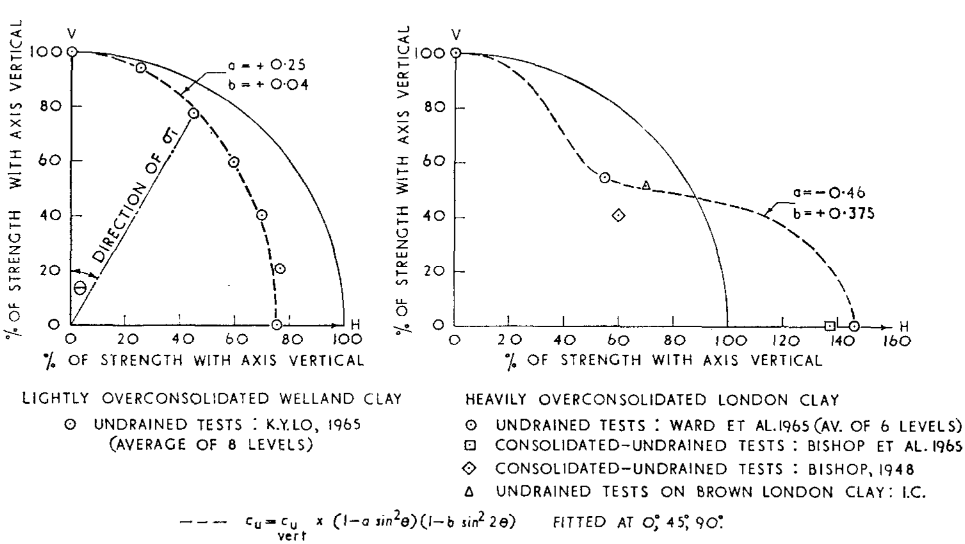


Fig. 28. Polar diagram showing variation of undrained strength with direction of applied stress: θ denotes inclination of major principal stress with respect to vertical axis

compression strength with σ_1 horizontal was about 0·75 the value with σ_1 vertical.[18] In the heavily over-consolidated London Clay the ratio of horizontal to vertical strength was 1·46 (range 1·23 to 1·63). That this difference is primarily a pore pressure phenomenon is indicated by the associated consolidated–undrained tests (Bishop, Webb and Lewin, 1965), which show ratios of the same order (e.g. 1·35), but little difference in the effective stress envelopes, the A values, however, being +0·42 for the vertical sample and +0·19 for the horizontal sample quoted.

However, inclined samples of London Clay show a great reduction in strength, presumably associated with lower shear strength parameters along the bedding planes, the undrained strength with σ_1 at 45° (i.e. the maximum shear stress parallel to the bedding planes) being 0·77 of the vertical value. A simple expression with two parameters, a and b, can be used to express these results: a reflecting the influence of pore pressures and b the directional character of c' and ϕ' as well as that of pore pressure.[19] Even in the lightly over-consolidated Welland clay the value of the second term suggests that an orientated structure is becoming manifest. Undrained tests on block samples from the brown London Clay at Maldon show a similar drop in strength in the direction of the bedding planes, as also do the earlier consolidated-undrained tests on London Clay from Walton (Bishop, 1948).

These strengths are not, of course, identical with the values of the in-situ undrained strengths in plane strain, which are of principal interest in many engineering problems, and the influence, not only of stress release on sampling,[20] but also of sample size, is of particular interest in dealing with over-consolidated clays.

Before examining these factors in more detail it should be pointed out that a drop in strength of 50% in the horizontal direction leads, in the circular arc analysis of a typical slope (Lo, 1965), to a reduction in factor of safety of only 15%–30% since the whole of the slip surface is not inclined at the least favourable angle. This, together with the reduction in measured strength which always results from sampling, has probably made the $\phi_u = 0$ analysis appear more accurate than, theoretically, it should be.

The importance of the two factors, anisotropy and the size of sample, in practical design work may be illustrated by tests carried out on the weathered London Clay foundation of a proposed embankment near Maldon in Essex. Owing to the great base width of the embankment and to the low coefficient of consolidation of the clay, undrained failure on an almost horizontal slip surface in the weathered zone represented a critical condition. I therefore asked for the normal site investigation to be augmented by a number of in-situ undrained direct shear tests on samples 2 ft × 2 ft in cross-section with their zone of maximum shear stress in the horizontal plane.

The layout of the testing equipment is shown in Fig. 29. An intact block of clay was left projecting 6 inches above the floor of the trial pit, and with the minimum of delay the shear box and loading platten were fitted over it. A load equivalent to the overburden pressure was applied through a hydraulic jack mounted beneath a strut transmitting the load to a joist

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[18] The interpretation of cylindrical compression tests on samples cut with their axis horizontal is open to some ambiguity in a soil which may have a low undrained strength when the plane of failure is vertical, but the relative motion horizontal, as in the tests by Aas (1965). Failure may occur on such a plane in the compression specimen cut with its axis horizontal, rather than on an inclined plane forming part of a conventional plane strain failure surface, as assumed by Lo (1965).

[19] The first term agrees with that used by Lo (1965) and Casagrande and Carrillo (1944). It is simpler than that proposed by Hansen and Gibson (1949), but lack of detailed information hardly justifies a more elaborate expression. The second term is assumed to represent a satisfactory working hypothesis, though, based on work by Hill (1950) on metals in plane strain, Scott (1963b) assumes a similar term to the power $-\frac{1}{2}$.

[20] Bishop and Henkel (1953), Ladd and Lambe (1963), Skempton and Sowa (1963) and Ladd and Bailey (1964) have dealt with this problem either with no rotation of the principal stresses or (Ladd and Bailey) with the special case of axial extension.

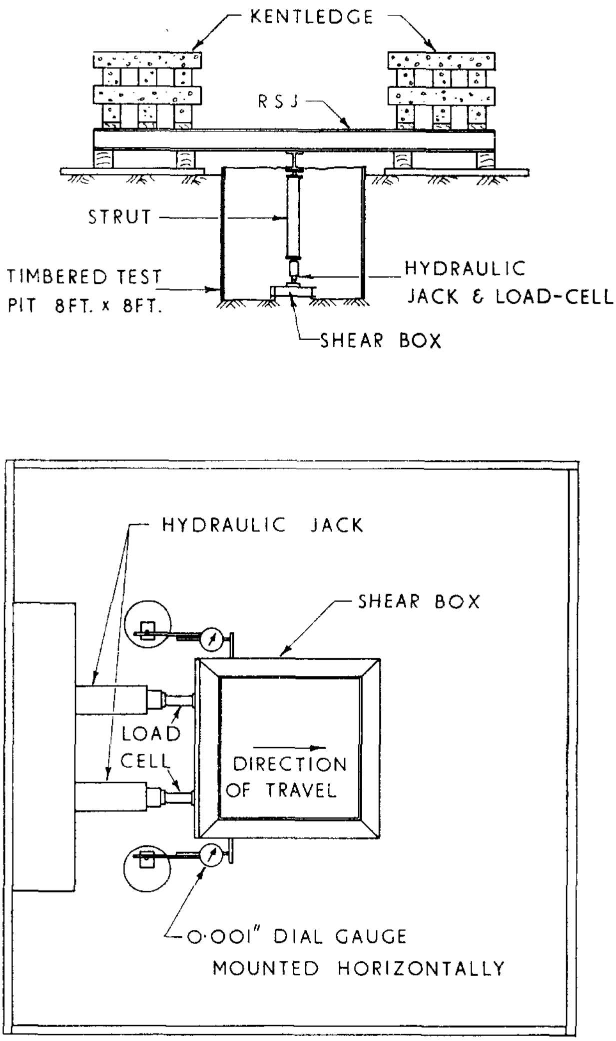


Fig. 29. Layout of direct shear test on 2 ft × 2 ft samples in the field

passing beneath kentledge on either side of the trial pit. The horizontal load was applied through two hydraulic jacks fitted with electrical load cells. Any tendency of the box to run out of the true could thus be controlled. A pair of dial micrometers recorded the horizontal displacement of the box.

The shear stress–displacement curve for the test at a depth of 11 ft in trial pit 3 is given in Fig. 30. The very small displacement (0·3 in.) at which the peak stress was reached may be

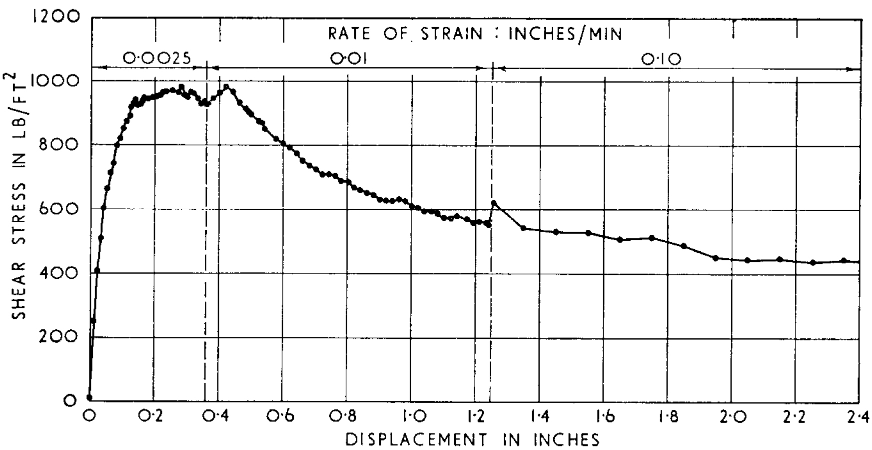


Fig. 30. Shear stress–displacement curve for undrained direct shear test on 2 ft × 2 ft sample of brown London Clay at Maldon, Essex: horizontal shear plane 11·3 ft below surface

noted. The rate of strain was controlled so that the peak stress was reached after about 1 to 2 hours. In this test, which was one of the two deep tests performed, the stress then fell off and appeared to reach an almost constant value when the limit of travel of the shear box was reached. At this stage careful sectioning of the sample revealed one or more pronounced slip surfaces running almost horizontally across the sample near the base of the shear box.

The undrained strength–depth plots are given in Fig. 31. The most notable feature is that the strengths obtained with horizontal shear on large samples are, on the average, only 55% of the strengths obtained by testing compression specimens with their axis vertical either from boreholes samples, or from tube or block samples taken in the trial pits. A reduction of this magnitude makes a conventional factor of safety of 1·5 on a conventional undrained test result on London Clay appear rather inadequate, and it is difficult to see on what grounds we can fault the in-situ tests.

Four factors may be considered in assessing the significance of the difference.

(1) *Time to failure.* The time to failure (about one hour) in the field test is greater than the duration (about 5 minutes) of the laboratory test. Other studies (La Rochelle, 1961) suggest that the effect on strength of this difference amounts to only a few per cent, but in relation to construction work the slower test in any case is the more correct.

(2) *Stress conditions in the direct shear test.* The principal stress directions in the shear box at failure are not known very precisely. An error of $(1 - \cos \phi_e) \times 100\%$ is possibly involved in estimating c_u from a shear box.[21] This is generally less than 5% for a clay of high plasticity.

(3) *Anisotropy.* Tests on block samples in the laboratory orientated so that the slip plane lay in the horizontal direction gave undrained strengths 86% of the strengths

[21] This is still controversial. Reference can be made, for example to Hill (1950), and Hansen and Gibson (1949). The interpretation of the shear box test and simple shear test will clearly be influenced by anisotropy.

with the axis vertical.[22] Similar tests on samples cut from cores taken in the trial pit with a 4-in. diameter sampler show a reduction of 87% of the strength with the axis vertical.

(4) *Sample size.* The major part (from 86%–55%) of the reduction in strength must therefore be attributed to the use in the field of a large and more representative sample.

The results of four large plate loading tests are also included on the strength–depth plot for trial pit 4. These again indicate strengths much below the values given by small laboratory samples, though rather higher than the values given by the large direct shear tests. This latter difference[23] probably reflects the fact that in the plate loading tests the slip surface is inclined to the direction of the bedding planes over much of its area, and the influence of anisotropy is reduced.

[22] Anisotropy in a larger sample may of course be more marked than in a small one due to the inclusion of a more representative structure.

[23] This difference may have been to some extent masked by the limited displacement applied in the plate loading tests.

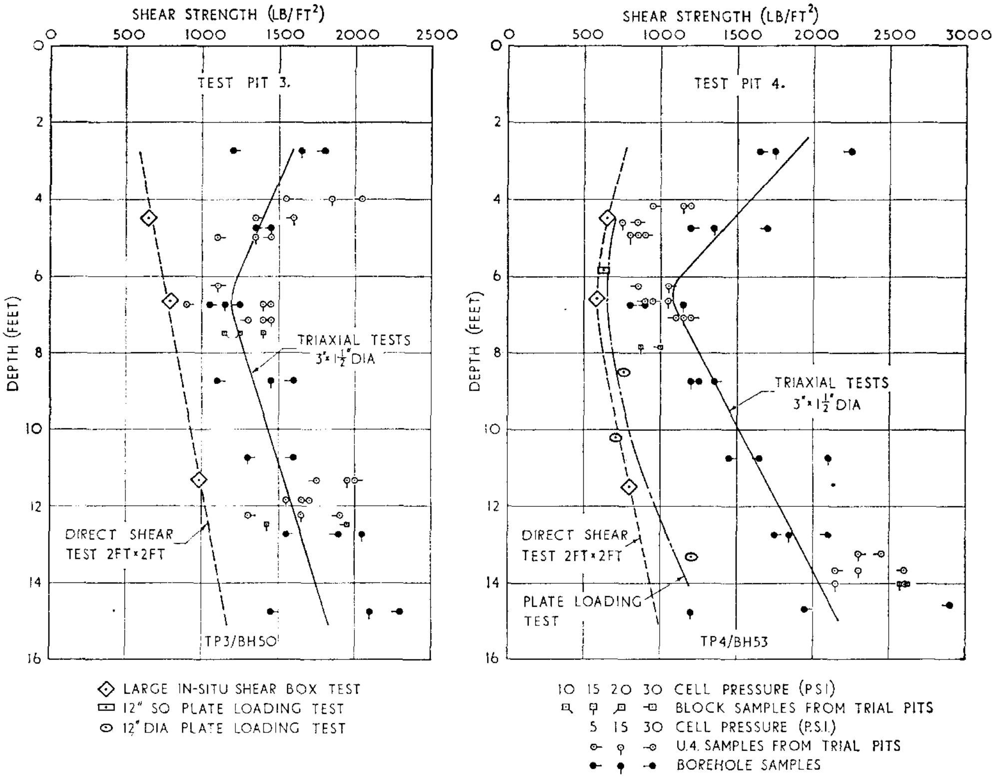


Fig. 31. Relationship between undrained strength and depth: results of 2 ft×2 ft direct shear tests and plate loading tests compared with values obtained from 3 in.×1½ in. dia. triaxial tests: brown London Clay from Maldon, Essex

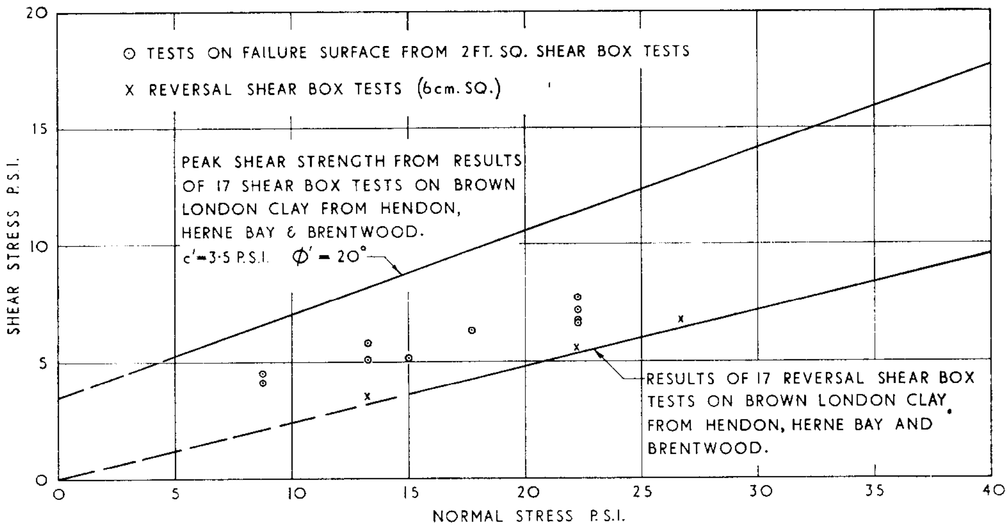


Fig. 32. Results of drained tests on samples from failure surface of 2 ft square in-situ shear box tests. (Tests by Petley, 1966)

Whitaker and Cooke (1966) have reached a similar conclusion for the deeper layers of the London Clay from the ultimate base loads measured on large bored piles. The in-situ undrained strength was, on the average, only 75% of that given by standard borehole samples tested with the axis vertical.

It is of interest to note that, in similar clay at Bradwell, Skempton and La Rochelle (1965) found that the 'undrained' strength mobilized in a slide in the side of an excavation was only about 55% of the undrained strength of 3-in. × 1½-in. diameter samples, either from boreholes or cut from blocks. In this case the slip surface was steep and not in the direction of the bedding planes, but a larger time elapsed before failure, which in itself could have accounted for a reduction to 80% of the value measured in the standard laboratory test. The further reduction of 30% due to size effect in a soil with a fissured structure at Bradwell is thus about the same as the reduction due to size effect at Maldon. This suggests that at this level in the London Clay a 2-ft square sample was adequate.

These differences overshadow refinements in our methods of analysis and suggest that we may need to rethink the means by which we determine undrained strength in engineering practice. Either samples must be large enough to include a fully representative soil structure (this could mean even larger samples in some cases than the size tested at Maldon) and they must be tested with the correct orientation, or we must apply empirical factors to the strength of our conventional laboratory specimens.

To obtain *drained* shear parameters for large samples is much more difficult. The test duration necessary for full pore pressure dissipation in the shear test described above would have been 6 months or more[24] for a clay of the type tested at Maldon. However, large scale drained tests on stiff fissured clays are clearly essential to the rational design of engineering works in or on these strata.

[24] At low stresses preferential drainage through the fissures might accelerate the consolidation of weathered samples.

It is perhaps of interest to add that drained tests run on sections cut from the Maldon in-situ tests so as to include part of the actual slip surface showed that the residual value of ϕ' had not been reached at the displacement of 2 to 3 in. applied in the tests. The test results are given in Fig. 32. The values of the residual factor R (Skempton, 1964) for 6 cm. square samples vary between 0·45 and 0·8.

(4) THE INFLUENCE OF TIME ON THE STRENGTH OF SOILS

The strength of a given soil stratum which is available to the engineer depends on time in a number of different ways.

Under a sustained load the available strength of a clay stratum changes from the undrained strength to the drained strength at a rate which depends on the coefficient of consolidation or swelling and on the length of the drainage path. The change is an increase in most foundation problems where the load increases, and a decrease in most excavation or cutting problems, where the load is decreased, and results in the first place from pore-water pressure changes in the field (for a detailed discussion see Bishop and Bjerrum, 1960).

However, as was shown by Professor Skempton in the fourth Rankine Lecture, the shear strength parameters calculated from actual slips based on a knowledge of the field pore pressure differ radically from the peak strength values measured in the laboratory in the case of over-consolidated clays of other than low plasticity.

Several factors may contribute to this discrepancy,

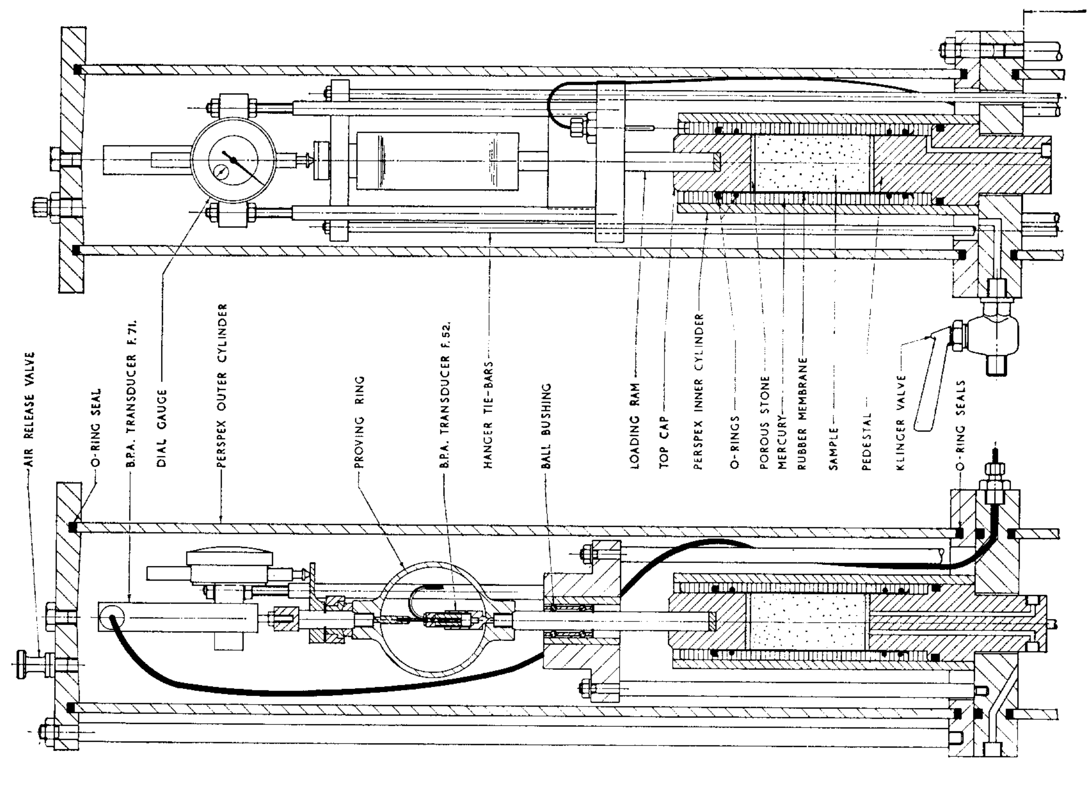
(*a*) the size and orientation of the test specimens will influence the values of the shear strength parameters measured in the laboratory;

(*b*) in soils showing brittle or work-softening stress–strain characteristics it is likely that failure in the field will be to some extent progressive, i.e. that the peak strength will not be mobilized simultaneously along the complete slip path. As pointed out by Webb, Lewin and myself in 1965, the release of stored energy on stress reduction under drained conditions in clay showing marked swelling characteristics is of special importance in this case. This approach has been developed by Dr Bjerrum in his recent Terzaghi Lecture to the American Society of Civil Engineers;

(*c*) the peak values of the drained shear strength parameters may be substantially time dependent in heavily over-consolidated clays and clay shales.

Little precise information exists on all three factors. Almost nothing really appears to be known about the time dependence of the drained peak strength of undisturbed clays. The technical difficulties of maintaining a known constant stress difference on a sample in an apparatus without leaks over periods of months and possibly years are considerable. The apparatus we are currently using at Imperial College is illustrated in Fig. 33. Several important features may be noted. The whole loading and strain measuring system is inside one continuous pressure vessel filled with oil, so that friction due to a seal on the loading ram is entirely avoided. The load is applied by two very long springs in tension, controlled by a screw adjustment at the base of the cell, and is transmitted to the cylindrical sample through a ram guided by a ball bushing. The load can be determined both from the length of the springs and from a proving ring mounted on the ram. Creep in the sample has little effect on the load in the springs due to their large extension, but in the early stages of the test such adjustment as is necessary due to the shortening of the sample and the change in its cross-sectional area can readily be made with the screw at the base of the cell. Deformation is measured by an oil filled dial micrometer reading to 10^{-4} in. and by a linear differential transformer reading to 10^{-5} in.

The sample is enclosed in a rubber membrane and is submerged in mercury to prevent loss of water through the rubber membrane, and to protect the membrane from contact with the


AIR RELEASE VALVE
O-RING SEAL
B.P.A. TRANSDUCER F.71.
DIAL GAUGE
PERSPEX OUTER CYLINDER
PROVING RING
B.P.A. TRANSDUCER F.52.
HANGER TIE-BARS
BALL BUSHING
LOADING RAM
TOP CAP
PERSPEX INNER CYLINDER
O-RINGS
POROUS STONE
MERCURY
RUBBER MEMBRANE
SAMPLE
PEDESTAL
KLINGER VALVE
O-RING SEALS

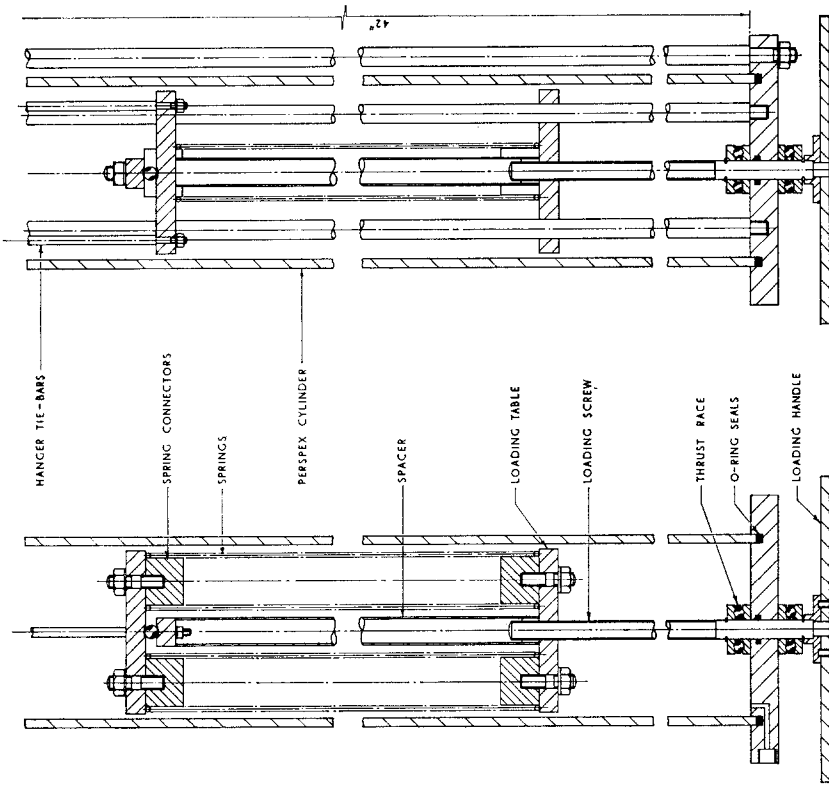


Fig. 33. Diagrammatic sections of creep cell

mineral oil. Volume change is measured with a paraffin volume gauge (Bishop and Henkel, 1962, Fig. 141) using a back pressure to ensure full saturation of the system. Volume change is also measured with a mercury-filled volume gauge, back-pressured from the cell, which is adjusted to maintain the mercury in the inner cell surrounding the sample to a constant level, determined by an electric contact.

The use of a spring-loaded system of low inertia makes the apparatus less susceptible to tremors transmitted by the structure of the building than the use of a dead load system.

The first series of tests (Fig. 34) on block samples of London Clay has been running for about seven months and has already produced some interesting information. Samples were set up and kept under sustained shear stresses, the stress levels being approximately 90%, 80%, 70%, 60%, 40%, and 16% of the peak drained strength in a test of one week's duration. These percentages are based on the results of six triaxial tests of this duration. The percentage shown first in Fig. 34 is calculated from the average of the two highest values observed. The second percentage is based on the four lower values in the series.

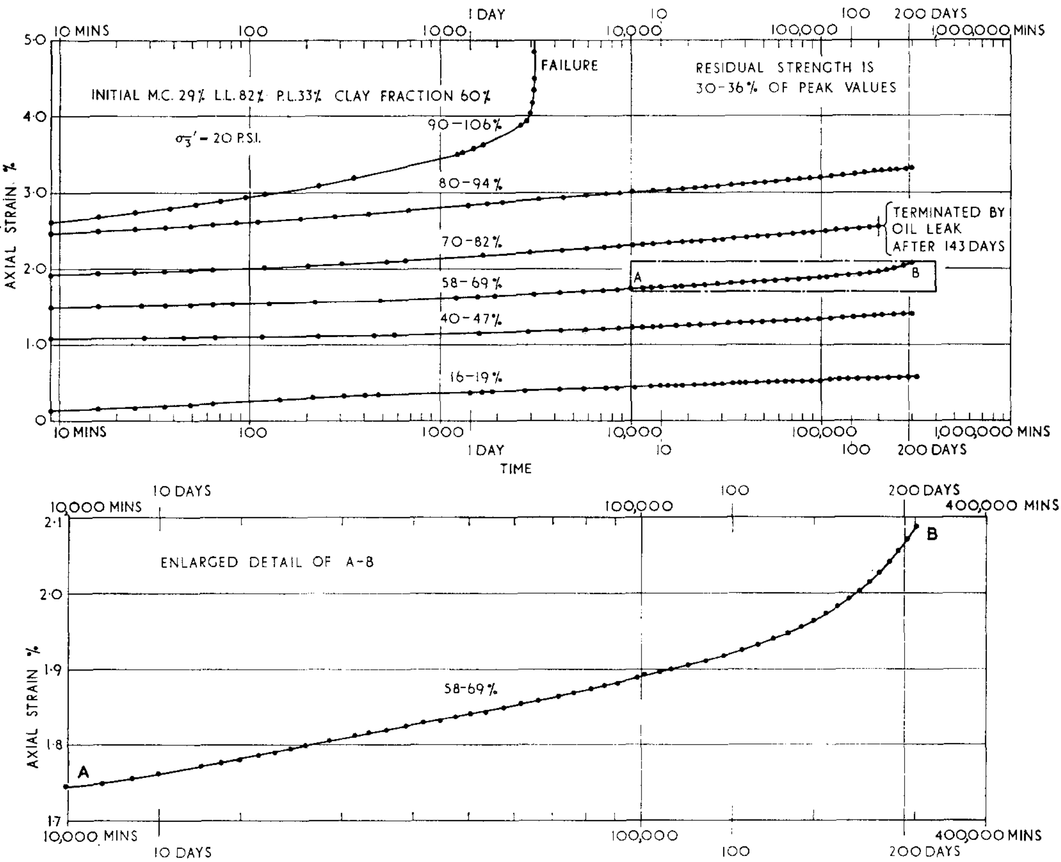


Fig. 34. Drained creep tests on undisturbed brown London Clay from Hendon. The applied principal stress differences as given above are percentages of the peak values measured in drained triaxial tests of 5 days' duration. (Tests by Lovenbury, 1965–66)

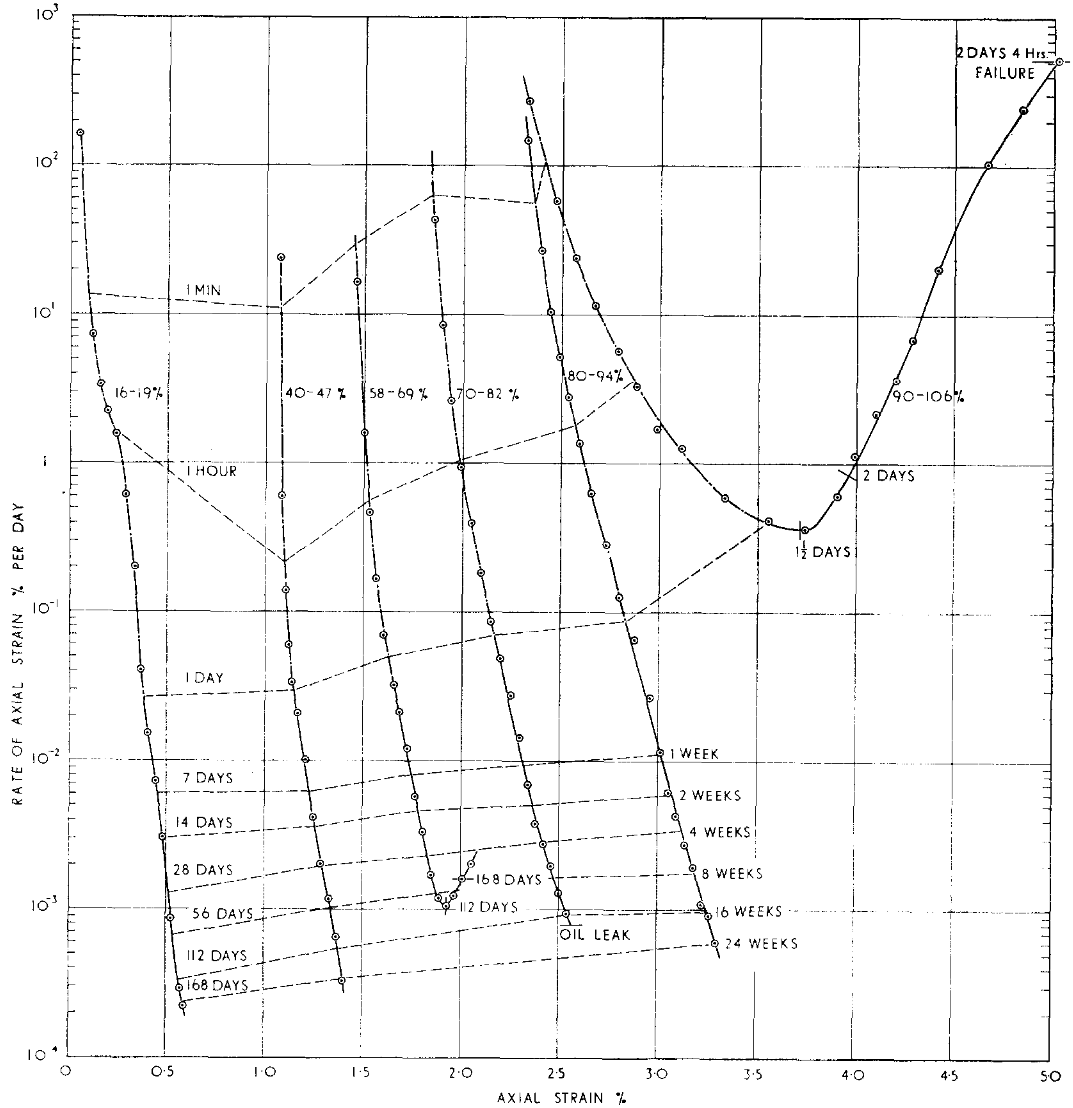


Fig. 35. Strain rate as a function of axial strain for various stress levels. (Stress levels indicated by percentages adjacent to curves). Contours represent times after application of final stress increment; drained tests on brown London Clay from Hendon. (Tests by Lovenbury 1965–66)

It will be seen that the 90% peak strength sample failed after two days, having given (at least in retrospect) sufficient warning of its intention.[25] The 80% test is still continuing with no sign of impending failure. The 70% test came to a premature end after 143 days, due, ironically, to creep in a perspex component for which we had not allowed. The 60% test showed a decreasing rate of strain for the first three and a half months (100 days), but the

[25] The period of two days is measured from the time of application of the last increment of the stress difference and full drainage under this increment would not have been achieved in the earlier part of this period.

rate has subsequently increased steadily. Whether, as the enlarged detail might suggest, this presages eventual failure is very difficult to forecast. The creep rates are given in Fig. 35, plotted against strain. Perhaps the most interesting feature of this diagram is that in undisturbed clay the creep rate does not stay constant under constant stress at any stage of the test. It either decreases fairly steadily or increases. The simpler rheological models are therefore not applicable. The 60% test, although it has speeded up, is still only proceeding at two thousandths of 1% per day and at this rate would take another 1200 days to reach the strain at which the 90% sample failed. An extrapolation of the present strain rate path suggests about 65 days more to failure.[26]

Even if the 60% sample did fail, and if we ignore the fact that the 80% sample is not yet showing a speeding-up in its strain rate, we have only accounted for part of the drop to the residual value which is approximately 30% of the higher peak drained strength. But it is a very considerable part, and may make possible a quantitative explanation in terms of the three factors listed above.

The creep tests outlined above are, of course, equally significant in relation to the performance of foundations under sustained load, where current concepts of secondary consolidation are based mainly on the results of oedometer tests with rigid lateral confinement.

CONCLUSION

In conclusion, I hope I have thrown a little light on some of the problems of soil mechanics which are both of intellectual interest and of practical importance. I hope I have also shown, in the last two sections in particular, that there is a great deal still to be found out about the actual strength of soils as engineering materials, and that not all of this can be found out in the laboratory. The investigation of full-scale failures and the carrying out of field tests of a sufficient size to be relevant and in sufficient numbers to be representative are tasks which must be shared by consulting engineers and contractors as well as by universities and research stations, and must be budgeted for.

ACKNOWLEDGEMENTS

In preparing this lecture I have been able to draw on unpublished experimental data obtained at Imperial College by G. E. Green, A. E. Skinner, H. T. Lovenbury, D. J. Petley, S. W. Smotrych, S. K. Sarma, N. N. Som, S. G. Tombs, and Dr B. P. Walker, and at the Building Research Station by P. I. Lewin. In addition I am indebted to Dr L. Bjerrum, Director of the Norwegian Geotechnical Institute for unpublished reports on field and laboratory tests.

The large shear tests at Maldon were carried out by George Wimpey and Co. Ltd under the direction of Binnie and Partners to whom I am indebted for permission to quote the results and also the results of tests on rock-fill. Acknowledgements are also due to the Science Research Council and the Civil Engineering Research Association whose grants made possible some of the tests described in this lecture.

I am particularly indebted to Gordon E. Green for assistance in assembling material for the lecture, to E. W. Harris for assistance with the illustrations, and to Dr R. E. Gibson for reading the manuscript.

[26] Since the Rankine Lecture was delivered, the speeding up of the creep rate has not increased and the time to failure cannot be predicted with any greater certainty at the present date (March 1966).

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VOTE OF THANKS

The CHAIRMAN invited Professor J. Brinch Hansen, Vice-President of the International Society for Soil Mechanics and Foundation Engineering, to propose a vote of thanks to the lecturer.

Professor Brinch Hansen said that the subject of Professor Bishop's lecture was most appropriate, as the work for which Professor Rankine became famous dealt with the behaviour of soils at failure and the application of this to practical engineering problems.

The shear strength of soils was, indeed, one of the most fundamental subjects in soil mechanics, probably the most fundamental. Different aspects of it had been studied at practically every soil mechanics laboratory in the world, but nowhere more intensively than in Great Britain. In the history of soil mechanics names, such as Skempton, Bishop, Rowe, and Roscoe would always represent milestones in the understanding of the strength properties of soils; and Imperial College came to be considered as the place where the most important work on shear strength and its practical applications had been made. In this work Professor Bishop had played a leading role.

Professor Brinch Hansen said that he did not pretend to know about all Professor Bishop's achievements but would mention a few of them. Bishop had designed and constructed a great number of different pieces of testing apparatus which were now in common use in most parts of the world. To the development and refinement of the triaxial machine, which was now generally acknowledged to be the best apparatus for measuring shear strength, Bishop had contributed more than any other person.

In the more theoretical field, mention should be made of his general method for the analysis of stability, which might well be the method most commonly used at present.

Finally, in practical engineering his new methods of designing and investigating great earth dams had proved eminently successful.

In his lecture Professor Bishop had thrown considerable light on some of the less known factors influencing shear strength, namely, intermediate principal stress, stress level, anisotropy, sampling effects, and time effects.

In the first approximation, established many years ago, the failure criterion was expressed by means of Coulomb's law, combined with the principle of effective stresses. This was simple and seemed in most cases to work well. However, in time it was found that a number of other factors, of which Coulomb's law took no account, influenced the strength to some extent. Bishop had dealt with the most important of these in his lecture.

With regard to the failure criterion, Professor Bishop had come to the conclusion that the experimental results mostly supported the Mohr–Coulomb criterion, and proposed to accept this, although it underestimated the value of ϕ' for plane strain by up to 4°.

Professor Brinch Hansen said that he was not personally very happy about this recommendation because, after all, it might mean an unnecessary extra safety factor of 2 on the bearing capacity of dense sand.

Bishop had, quite rightly, drawn attention to the effect of the stress level, which expressed itself in a curved Mohr envelope. Whatever the physical reasons for this, it was a fact which must be taken into account in their designs, especially if small scale laboratory tests were to be used as a basis for the design of full scale structures.

The often-experienced wide discrepancies between the results of bearing capacity tests in the laboratory, and theoretical values calculated on the basis of triaxial friction angles, were probably due mainly to differences in intermediate principal stress and stress level. However, the trouble was that one of these effects would usually suffice to provide an explanation. If both were admitted they might well end in the opposite ditch.

Commenting on Professor Bishop's emphasis on the effect of anisotropy, Professor Brinch Hansen said that he would, of course, be the first to agree with him. Also his point concerning the effects of sampling, or of sample size, seemed to be well established. Professor Brinch Hansen thought, therefore, that the lecturer was probably right in assuming that the apparent accuracy of the $\phi = 0$ analysis in many cases might be due to the accidental cancelling of two or more considerable errors. Incidentally, the same was probably the case in other soil mechanics calculations, for instance, in settlement analysis.

Bishop's research on the effect of time on the drained strength of London Clay was extremely interesting, as it established beyond any doubt the influence of creep, not only of the deformations but also on the shear strength itself. This seemed to be an indication that the final failure criterion would probably be expressed in strains rather than in stresses; but this belonged to the future.

They were all extremely grateful to Professor Bishop for having not only drawn their attention to some important but little-known factors influencing shear strength but having also given them the experimental background of his statements and demonstrated their significance to the practising engineer.

As usual, Professor Bishop had done all this in a very clear and scientific manner, using his considerable imagination to seek new explanations, but not giving them out until he had obtained adequate experimental proof.

On behalf of all those present he was therefore happy to propose a well-deserved vote of thanks to Professor Bishop for his important lecture.

The vote of thanks was carried with acclamation.