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,CSD-TR 459

ABSTRACT
The string-to-string correction problem is to find a minimal sequence of edit operations for changing a given string into another given string. Extant algorithms compute a Longest Common Subsequence (LCS) of the two strings and then regard the characters nol included in the LCS as the differences. However, an LCS does not necessarily include all possible malches, and therefore does not produce the shortest edit sequence.

We present an algorithm which produces the shortest edit sequence transforming one string into another. The algorithm is optimal in the sense that it generates a minimal, covering set of common substrings of one string with respect to the olher.

Two runtime improvements of the basic algorithm are also presented. Runtime and space requirements of the improved algorilhms are comparable to LCS algorithms.

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# The String-to-String Correction Problem with Block Moves 

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## Introduction

The string-to-string correction problem is to find a minimal sequence of edit operations for changing a given string into another given string. The length of the edit sequence is a measure of the differences between the two strings Programs for determining differences in this manner are useful in the following situalions.
(1) Difference programs help determine how versions of text fles differ. For instance, computing the differences between revisions of a software module helps a programmer trace the evolution of the module during maintenance[6]. or helps create test cases for exercising changed portions of the module. Another application is the automatic generation of change bars for new editions of manuais and other documents.
(2) Frequently revised documents like programs and graphics are stored most economically as a set of differences relative to a base version[10,12]. Since the changes are usually small and typically occupy less that $10 \%$ of the space needed for a complete copy[10], difference techniques can store the equivalent of about 11 revisions in less space than would be required for saving 2 revisions (one original and one backup copy) in cleartext.
(3) Changes to programs and other data are most economically distributed as "update decks" or "dicllus", which are edit sequences that transform the old version oi a data object into the new one. This approach is used in software distribution. A related application can be found in screen editors and
graphics packages. These programs update display screens efficiently by computing the difference between the old and new screen contents, and then transmitting only the changes to the display[2].
(4) In genetics, difference algorithms compare long molecules consisting of nucleotides or amino acids. The differences provide a measure of the relaLionship between types of organisms[11].

Most of the existing programs for computing differences are based on algorithms that determine a Longest Common Subsequence (LCS). An LCS has a simple and elegant definition, and algorithms for computing an LCS have received some attention in the literature $[13,4,6,7,5,9]$. An LCS of two strings is one of the longest subsequences that can be obtained by deleting zero or more symbols from cach of the two given strings. Por example, the longest common subsequence of shanghai and sakhalin is sahai. Once an LCS has been obtained, all symbols that are not included in it are considered differences. A simultaneous scan of the two strings and the LCS isolates those symbols quickly. For example, the following edit script, based on the LCS sahai, would construct the target string sakhalin from shanghai.

M 0,1
M 2.1
A 'k"
M 5,2
A " ${ }^{\prime \prime}$
M 71
A "n"

An edit-command of the form $M$ p.l, called a move, appends the substring $S[p, \ldots, p+l-1]$ of source string $S$ to the target string, and an add command of the form $A w$ appends the string $w$ to the target string. In the above example, the edit script takes up much more space than the target string, and none of the savings mentioned earlier are realized. In practical cases, however, the common subsequence is not as fragmented, and a single move command covers a long substring. In addition, if this technique is applied to text, one usually chooses full lext lines rather than single characters as the atomic symbols. Consequently, the storage space required for a move is negligible compared to the that of an add command, and it is worth minimizing the occurrence of the add commands. Note that in the above example, the last add command could be replaced with a move, since the symbol $n$ appears in both strings.

Unfortunately, the definition of an LCS is such that the $n$ cannot be included in the LCS. The algorithm presented below does not omit such matches.

## Problem Statement

Given 2 strings $S=S[0, \ldots, \pi], n \geq 0$ and $T=T[0, \ldots, m], m \geq 0$, a block move is a triple $(p, q, l)$ such that $S[p, \ldots, p+l-1]=T[q, \ldots, q+l-1]$ ( $0 \leq p \leq n-l+1,0 \leq q \leq m-l+1, l>0$ ). Thus, a block move represents a non-empty. common substring of $S$ and $T$ with length $l$, starting at position $p$ in $S$ and position $q$ in $T$. A covering set of $T$ with respect to $S$, denoted by $\delta_{\mathcal{S}}(T)$, is a set of block moves, such that every symbol $T[i]$ that also appears in $S$ is included in exactly one block move. For example, a covering set of $T=a b c a b$ with respect to $S=a b d a$ is $\{(0,0,2),(0,3,2)\}$. A trivial covering set consists of block moves of length 1 , one for each symbol $T[i]$ that appears in $S$.

The problem is to find a minimal covering set. $\Delta_{S}(T)$, such that $\left|\Delta_{S}(T)\right| \leqslant\left|\delta_{S}(T)\right|$ for all covering sets $\delta_{S}(T)$. The coverage property of $\Delta_{S}(T)$ assures that all possible matches are included, and the minimality constraint makes the set of block moves (and therefore the edit script) as small as possible.

Because of the coverage property, it is apparent that $\Delta_{S}(T)$ includes the LCS of $S$ and $T$. (Consider the concatenation of the substrings $T\left[q_{j}, \ldots q_{j}+l_{j}-1\right]$, where $\left(p_{j}, q_{j}, l_{j}\right)$ is a block move of $\Delta_{S}(T)$, and the substrings are concatenated in order of increasing $q_{j}$.) The minimality constraint assures that the LCS cannot provide a better "parcelling" of the block moves.

## F'alse Starts

Before presenting the solution, it is useful to consider several more or less obvions approaches. all of which fail. The first approach is to use the LCS. As we have seell. an LCS has the property of not necessarily generating a covering sel oi btock tnoves, for example, the following two pairs of strings have the LCS abc. which does not include the (moved) common substring de nor the (repeated)
common substring $a b c$. The LCS match is shown on the left, $\Delta_{S}(T)$ on the right.


Heckel[3] pointed out similar problems with LCS techniques and proposed a linear-time algorithm to detect block moves. The algorithm performs adequately if there are few duplicate symbols in the strings. However, the algorithm gives poor results otherwise. For example, given the two strings aabb and bbaa. Heckel's algorithm fails to discover any common substring.

An improvement of the LCS approach is to apply the LCS extraction iteratively. For instance, after finding the initial LCS in the above examples, one could remove it from the target string $T$ and recompute the LCS. This process is repeated until only an LCS of length 0 remains. The iterative LCS strategy succeeds in finding a covering set, but not necessarily the minimal one. The following example illustrates.


Assuming again that $S$ is the source string and $T$ is the target string, the left diagram shows the match obtained via an iterative LCS algorithm. The first LCS is $c: d u$. the second one is $b$. Since $c d a$ is not a substring of $S$. we obtain a total of 3 block moves. The minimal covering sel, shown to the right, consists of 2 block moves.

Another tack is lo search for the longest common substring rather than the longest common subsequence ${ }^{*}$. Computing the longest common substring iteratively results in a covering sel, but again not necessarily a minimal one. Con-

[^0]sider the following example.


The left diagram shows the block moves obtained by searching repeatedly for the longest common substring of $S$ and $T$. The result is a set of 3 block. moves, although 2 are minimal. Searching for the longest common substring is Loo "greedy" a method, since it may mask better matches.

## Basic Algorithm

A surprisingly simple algorithrn does the job. Start at the left end of the target string $T$, and try to find prefixes of $T$ in $S$. If no prefix of $T$ occurs in $S$. remove the first symbol from $T$ and start over. If there are prefixes. choose the longest one and record it as a block move. Then remove the matched prefix [rom $T$ and try to match a longest prefix of the remaining tail of $T$, again starting at the beginning of $S$. This process continues until $T$ is exhausted. The recorded block moves constitute a $\Delta_{S}(T)$, a minimal covering set of block moves of $T$ with respect to $S$, as will be shown later. The following example illustrates several steps in the execution of the algorithm. The string to the right of the vertical bar is the unprocessed tail of $T$.

Slep 1:
S = uvwuvwxy
$\mathrm{T}=\mid \mathrm{zuvwxwu} \quad$ longest block move starting with $\mathrm{T}[0]$ : none
Step 2:

longest block move starting with $\mathrm{T}[1]$ : $(3,1,4)$
Step 3:
$S=u v \underbrace{u} \underbrace{w} \underset{\sim}{w} x$
longest block move starting with $T[5]:(2,5,2)$

In step 1, we search for a prefix of $T[0, \ldots, 6]$ in $S[0, \ldots, 7]$. Since there is none, we search for a prefix of $T[1, \ldots, 6]$ in the next step. This time we find 2 matches, and choose the longer one, starting with $S[4]$. In step 3. we search for a prefix of $T[5, \ldots, 6]$ in $S[0, \ldots, 7]$, and find the longest one at $S[2]$, length 2 . Now $T$ is exhausted and the algorithm stops. Note that in each step we start at ihe teft end of $S$ in order to consider all possible matches.
'The algorithm is presented below. Let us assume that the source string is stored in an array $S[0, \ldots, m]$, and the target string in $T[0, \ldots, n] T[q]$ is the first symbol of the unmatched tail of $T ; q$ is initially zero. The firsl refinement of the algorithm is now as follows.

```
\(\mathrm{q}:=0\);
while \(q<=n\) do
begin
```

    L.: find \(p\) and 1 such that ( \(p, q, 1\) ) is a maximal block move
    if \(l>0\) then \(p r i n t(p, q, 1)\);
    \(\mathrm{q}:=\mathrm{q}+\operatorname{Max}(1, \mathrm{l})\)
    end

Implementing the statement labelled $L$ is simple. Search $S$ from left to right for a longest possible prefix of $T[q, \ldots, n]$. Note that the search can terminate as soon as there are fewer than $l+1$ symbols left in $S$, assuming that $l$ is the length of the maximal block move found in the current iteration. Similarly, there is no possibility of finding a longer block move if the last one included $T[n]$. (We use and then as the conditional logical AND operator.)

L :

```
\(1:=0 ; p:=0 ; p C u r:=0 ;\)
while \(\mathrm{pCur}+1<=\mathrm{m}\) and \(\mathrm{q}+1<=\mathrm{n}\) do
begin \{Determine length of match between \(S[p C u r, \ldots]\) and \(T[q, .\).\(] \}\)
    |Cur := 0 ;
    while ( \(p \mathrm{Cur}+\mathrm{lCur}<=\mathrm{m}\) ) and ( \(q+1 \mathrm{Cur}<=n\) )
        and then ( \(S[p \mathrm{Cur}+\mathrm{lCur}]=\mathrm{T}[\mathrm{q}+\mathrm{lCur}]\) )
        do ICur : = ICur +1 ;
    if lCur > 1 then
    begin \{ new maximum found \}
        \(\mathrm{l}:=\) lCur: \(\mathrm{p}:=\mathrm{pCur}\)
    end;
    pCur : \(:=\mathrm{pCur}+1\)
end
```

The runtime of this algorithm is bounded by $m \pi$, and the space requirements are $m+\pi$. We now show that this algoritlim finds a $\Delta_{S}(T)$. Clearly, the set of block moves printed is a covering set, because each symbol in $T$ that is not
included in some block move is (unsuccessfully) matched against each symbol in $S$. 'l'o see that the covering set is minimal, consider $T$ below, with the matching produced by our algorithm denoted as follows. Substrings included in a block move are brackeled by "(" and ")". Substrings of symbols excluded from any block move are denoted by $X$.

$$
\cdots X(\cdots) X(\cdots)(\cdots) X(\cdots)(\cdots)(\cdots) X \cdots
$$

Suppose there is a $\delta_{S}^{\prime}(T)$ with fewer block moves than the set generated by our algorithm. Clearly, the substrings denoted by $X$ cannot be part of $\delta_{s}(T)$. because our algorithm does produce a covering set. We can therefore exclude all mnmatched substrings from consideration, and concentrate on individual sequences of contiguous block moves.

Now consider block moves that are contiguous in $T$. The only way to obtain a smaller covering set is to find a sequence of $k>1$ contiguous block moves and to "reparcel" them into a covering set of fewer moves. We will show by induction on the number of contiguous block moves that the set produced by our algorithm is minimal.

Suppose we have $k \geq 1$ contiguous block moves generated by our algorithm. This means that we have $k$ triples $\left(p_{i}, q_{i}, L_{i}\right),(1 \leq i \leq k)$ satisfying the following conditions.

$$
\begin{gather*}
\Lambda i: 1 \leq i \leq k T\left[q_{i}, \ldots, q_{i}+L_{i}-1\right]=S\left[p_{i}, \ldots, p_{i}+l_{i}-1\right]  \tag{*}\\
\Lambda i: 1 \leq i \leq k, \Lambda p: 0 \leq p \leq m-L_{i}, T\left[q_{i}, \ldots q_{i}+L_{i}\right] \neq S\left[p_{1}, \ldots, p+L_{i}\right]  \tag{**}\\
A i: 1 \leq i<k T\left[q_{i}+L_{i}\right]=T\left[q_{i+1}\right] \tag{**}
\end{gather*}
$$

The first condition is just the definition of a block move. The second condition assures that each block move starting at $T\left[q_{i}\right]$ is maximal. The third condition means that the block moves are contiguous in $T$.

We need to show that for any set of of $k$ block moves satisfying (*) to (***), any equivalent set has at least $k$ block moves. Actually, it is convenient to prove somethiug slighlly more general: For any set of $k$ block moves satisfying (*) to $\left({ }^{* *}\right)$. Any set which covers the first $k-1$ bock moves and a non-empty prefix of block move $k$ has at least $k$ block moves. Firsl, assume $k=1$. Clearly, we cannot spiit any non-emply prefix of a single block move into less than 1 covering block move. Now assume that $k>1$, and that all sets covering the first $k-2$ block
moves and any non-empty prefix of block move $k-1$ consist of at least $k-1$ block moves. Consider what we can do with non-empty prefixes of the $k$ 'th block move. 'There are two cases. The first case applies to sets that cover the original block move $k-1$ with a single move $B$. In this case, let $B=\left(p_{b}, q_{b}, l_{b}\right)$, where $p_{b} \leqslant p_{k-1}$, and $p_{b}+l_{b}=p_{k-1}+l_{k-1}$. By the induction hypothesis, $B$ is at least the $k-1 s t$ move in the equivalent set. It is impossible to append a non-empty prefix of move $k$ to $B$ since that would contradict (**). Thus we need at least $k$ moves for covering the original $k-1$ moves and a non-empty prefix of original move $k$. I'he second case applies to sets that split the original block move $k-1$ into at least i2 non-emply moves (see the diagram below).

| orig. block move no. | $\mathrm{k}-2$ | $\mathrm{k}-1$ |  | k |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| orig. set | $\ldots .)$. | $(\ldots)$ | $\ldots)$ | $(\ldots)$ | $\ldots)$ |
| $\delta^{\prime}{ }_{s}(T)$ covering $\mathrm{k}-1$ | $\ldots .$. | $\ldots)$ | $(\ldots)$ |  |  |
| $\delta^{\prime \prime}{ }_{s}(T)$ covering k | $\ldots \ldots$. | $\ldots)$ | $(\ldots$ | $\ldots)$ | $(\ldots)$ |

The only choice to reduce the number of block moves below $k$ is to coalesce the suffix of the original move $k-1$ with a non-empty prefix of move $k$. This new parcelling leaves us with (a) a set covering the original $k-2$ block moves and a non-emply prefix of block move $k-1$, (b) a new coalesced move covering a suffix of move $k-1$ and a prefix of $k$, and (c) another block move if the suffix of move $k$ is nol emply. By the induction hypothesis, we know that (a) has at least $k-1$ moves. Add to that the (non-emply) coalesced move, and we end up with at least $k$ moves for covering the firsl $k-1$ block moves and any non-cmpty prefix of move $k$. 'I'hus, any set equivalent to the block moves generated by our algorithm has at least $k$ elements. QED.

## Firsl Improvement of the Basie Algorithro

Consider it situation where the souree string $S$ has few replicated symbols. That is, $\alpha$, the size of the alphabet of $S$, is approximately equal to $m$. In this case, a significant improvement of the basic algorithm is possible. During a single sean ol $S$, we prepare an index that, for each symbol $s$ in the alphabet. lists the positions of all oceurrences of $s$ in $S$. In the basic algorithm, we replace the statement labelled $\mu$ with the following. Assume $T[q]=s$ is the first symbol of the unmatehed tail of $T$. Look up the list $L$ of occurrences of symbol $s$ in $S$.
using the above index. If the list is empty. no match is possible. Otherwise, find Lhe maximal block move among those starting with the elements of $L$ in $S$.
'lhe performance of this algorithm is as follows. Assume the average length of a block move is $l$. Then the maximal block move must be selected among $m / \alpha$ alternatives, at a cost of not more than $l+1$ comparisons each. Thus, the runtime of the algorithm is $O(l *(m / a l p h a) *(n / l))=O(m n / \alpha)$. If $m \approx \alpha$, we obtain a nearly linear algorithm.

Program text and prose have the property of few repeated lines. In program text, the only repeated lines should be empty or consist of bracketing symbols like begin and end; for all other repetitions one would normally write a subprogram. In prose text, the only repeated lines should be empty or contain formatting commands. In applying our algorilhm to prose or program text, it is therefore appropriate to choose lines as the atomic symbols. To speed up comparisons, the program should use hashcodes for lines of text rather than performing character-by-character comparisons.

We implemented a program incorporating these ideas, called bdiff, and compared it with diff[6], which uses an LCS algorithm. We executed both programs on 1400 pairs of files. Each pair consisted of 2 successive revisions of text, deposited in a data bases maintained by the Revision Control System[12]. 'I'his system stores multiple revisions of text files as differences. Almost all of Lhe sample files conlained program text. We observed that diff and bdiff execule wilt smmlar speeds, but that bdiff produces deltas that are, on the average, only aboul $1 \%$ smaller. Apparently, block moves and duplicate lines in program Lext are not frequent enough to obtain significant space savings over LCS algorithms. We expect that the situation is more advantageous for block moves in the other applications mentioned in the introduction.

## Second Improvement of the Basic Algorithm

A difierent improvement speeds up our basic algorithm even if the source string contains numerous duplicaicd symbols. The improvement involves an adiptation of the: Knuth-Morris-12rath string malching algorithm[B], which allows a pallerti of tengtli $l$ to be found in a string of lenglh $m$ in $O(m+l)$ steps. Thus, if $S$ is of length $m, T$ is of length $n$, and the average block move is of length $l_{\text {, }}$ our algorithm strould operate in $O((m+l) *(n / l))=O(m n / l)$ steps. Note that the ralio $m / l$ is a measure of the "difference" of $S$ and $T$, and that the runtime
of the algorithm is proportional to that ratio. Note also that this measure is independent of the permutation of the common substrings in $T$ with respect to $S$.

An important element in the Knuth-Morris-Pratt algorithm is an auxiliary array $N$ which indicales how far to shift a partially matched pattern or block move after a mismatch. The array $N$ is as long as the pettern, and is precomputed before the match. Precomputing $N$ poses a problem for our algorithm. Since we do not know how long a block move is going to be, we would have to precompute $N$ for the entire unprocessed tail of $T$, although we would normally use only a small portion of it. Fortunately. $N$ can also be computed incrementally. The outline of the adapted pattern matching algorithm is as follows.

Assume the next unmatched symbol is $T[q]$. Start by initializing $N[q]$ and apply the Knuth-Morris-I'ratt algorithm to find the first occurrence of $T[q]$. (Note that this is a pattern of length 1.) If this pattern cannot be found, there is no block move including $T[q]$. Otherwise, expand the pattern by 1 , compute the nexl entry in $N$, and reapply the Knuth-Morris-Pratt algorithm to find the first occurrence of the expanded pattern. Start the search with the previous match. Continue this process, until the pattern reaches a length for which there is no match. At that point. the previous match is the maximal block move.

Suppose the maximal common block move starting with $T[g]$ is $l$. The last attempled patlern match is therefore of length $l+1$, and fails. The ineremental computation of the entries $N[q, \ldots, q+l+1]$ at a lotal cost proportional to $l$ assures that the cost of the average match remains $O(m+l)$.

The detailed program is given in the appendix. It is useful for applications (3) and (4) mentioned in the introduction. The idea of incrementally computing auxiliary data structures can also be applied to the Boyer-Moore pattern matching algorithm[1], resulting in a program that runs even faster on the average.

## Reconstructing the Target String

An edil scripl that reconstructs targel string $T$ from source string $S$ is a sequence of move and add commands. The commands build a string $T$ left-loripht. liach block tnove $(p, q, l)$ in $\Delta_{S}(\%)$ is represented by a command of the [onn $M p, l$, which copies the string $S[p \ldots, \ldots+t-1]$ to the end of the string $7^{\prime}$. For any substring $T[u, \ldots, v]$ consisting entirely of symbols that do not occur in $S$, the edit script contains the command $A T[u, \ldots, v]$, which simply
appends the unmatchable substring to $T$. After completion of all edit commands, $T=T$.

In general, $T$ cannot be constructed in a single pass over $S$. because block moves may cross (cl. examples in Sect. 3). If $S$ is a sequential file, one can minimize the number of rewind operations caused by crossing block moves as follows. During the generation of the edit script, it does not matter which one of 2 or more equivalent block moves is chosen. For example, suppose we have the following equivalent, maximal block moves starting with $T[q]: B 1=\left(p_{1}, q, l\right)$ and $B 2=\left(p_{2}, q, l\right)$, with $p_{1}<p_{2}$. If the previous block move emitted had its $S$ endpoint between $S\left[p_{1}\right]$ and $S\left[p_{2}\right]$, choosing the block move $B Z$ saves one rewind operation for $S$. Our algorithms are easily modified to accommodate this idea. Rather than starting at the left end of $S$ while searching for the longest possible match, they must start with the endpoint of the previous match and "wrap around" at the end of $S$.

So far, we have presented our edit scripts as constructing $T$ separately from $S$. It is also possible to transform $S$ "in place". The following paragraphs discusses the algorithm in some detail.

Suppose we have a buffer $B[0, \ldots, \operatorname{Max}(\boldsymbol{m}, n)]$ initialized to $S$, i.e., $B[i]=S[i]$ for $0 \leq i \leq n$. The goal is to transform the contents of $B$ to 7 . The key to this algorithm is an auxiliary array $A[0, \ldots, \pi]$. which keeps track of the positions of the original symbols $S[i]$ in $B$. Initially, $A[i]=i$ for $0 \leq i \leq n$. A marker $h$ moves through $A$ from left to right. giving the index of the rightmost symbol involved in a block move so far. Thus, for the $k$ 'th move command $M p_{k}, l_{k}, h=\operatorname{Max}\left(p_{j}+l_{j}, 0 \leq j \leq k\right)$. There is also a marker $t$ indicaling the index of the last symbol processed in $B$.

The first step is to remove all symbols from $B$ which are not in $T$. This step preprocesses the edit script to isolate the symbois to be deleted, and then actually removes them from $B$. It also updates the mapping array $A$ to reflect the compression, and marks those entries of $A$ as undefined whose counterparts in $B$ were deleted. The second slep processes the edit commands in sequence. An add command sinıply inserts the given string to the right of $t$, and resets $t$ to point to the last symbol so inseried. It also updates the array $A$ for the symbols sihilted right by the insertion. For each move of the form $M p, l$, compare $p$ and the current value of $h$. lf $p>h$, then the current block move is to the right of the previous one. 'The symbols between $h$ and $p$, i.e., $B[A[h+1], \ldots, A[p-1]]$,
are not included in the current move. but will be moved later. Mark them as such and set $h$ to $p+l-1$ and $t$ to $A[h]$. Thus, the characters $S[p, \ldots, p+l-1]$ will be included in the result. Otherwise, if $p \leq h$, the current block move crosses the previous one, and a substring located before $t$ must be moved or copied forward. All symbols in that string that were marked for moving by an earlier command are now moved, the others are simply copied forward. It is conceivable that the the current block move involves symbols to the left and right of $h$. In that case, first handle the string to the left of $h$ by moving or copying elements of the string $B[A[p], \ldots, A[\operatorname{Min}(p+l-1, h)]]$ after $B[t]$. The remaining (possibly empty) string $A[h+i, \ldots, p+l-1]$ is simply included by setting $h$ to $\operatorname{Max}(p+l-1, h)$. Update $A$ to reflect the moves and shifts, and set to $A[h]$.

Below is a trace of the algorithm, transforming the string sinanghai to sakhalin by applying the edit script $M 0,1 ; M 2,1 ; A^{\prime \prime} k^{\prime \prime}: M 1,2 ; A^{\prime \prime} l^{\prime \prime} ; M 7,1 ; M 3,1$. The algorithm can be applied to update display screens efficiently, provided the display offers operations for character and line insertion and deletion. as well as a eopy/move feature. The laller feature is needed for copying and moving character strings forward in the above algorithm. I'he auxiliary array $A$ is allocated in main memory.


After removing unused symbols


After applying

$$
M 0,1 ; M 2,1
$$

$A[1]$ is marked for move.

After applying
I "k"

After applying M 1, 2 ; I * $\ell^{n}$

After applying

$$
M 7,1 ; M 3,1
$$

## Conclusions

The original string-to-string correction problem as formulated in[13] permitted the edlting commands add, delete, and change. Clearly, a change command can be simulated, with a delete followed by an add. Any sequence of add and delete commands can be transformed into an equivalent sequence of add and move commands. This transformation works since delete and move commands complement each other, provided no block moves cross or overlap. Our approach of extending the editing commands by permitting crossing block moves results in shorter edit sequences. We developed efficient algorithms for compuling those sequences. Reconstructing the target string by applying the edil sequence is efficient if the source string can be accessed randomiy.

## Appendix: Using the Knuth-Morris-Pratt Pattern Matching Algorithm.

$S$ : array[0..m] of symbol:
T: array[0..n] of symbol;
N : array $[0 . \mathrm{n}]$ of symbol:
$q:=0 ;\{$ start at left end of $T\}$
while $q<=n$ do
begin | Characters le[t in T : find longest match starting with $\mathrm{T}[\mathrm{q}]$ | $\mathrm{k}:=0_{i}\{$ start match at left end of S$\}$
$j:=q$; first symbol of pattern \}
last $:=q_{i}$ \{ last symbol of pattern \}
$N[\mathrm{q}]:=\mathrm{q}-1$; \{initialize $\mathrm{N}[\mathrm{q}]\}$
$\mathrm{iN}:=q-1 ;\{$ initialize computation of $N[q+1, \ldots]\}$
loop \{loop with exit from the middle \} \{try to find a match for T[q].T[last] \} | T[q]..T[last-1] has already been matched |
$\mathrm{kOld}:=\mathrm{k}$; \{ save last point of old match, if any \}
while ( $\mathrm{j}<=$ last) and ( $\mathrm{k}<=\mathrm{m}$ ) do begin
while $(j>=q)$ and ( $S[k]<>T[j])$
do $\mathrm{j}:=\mathrm{N}[\mathrm{j}]$; $\mathbf{k}:=\mathrm{k}+\mathrm{I}_{\mathrm{i}} \mathbf{j}:=\mathrm{j}+\mathrm{l}_{\mathrm{i}}$
end
until (j<=last) || (last=n); \{ exit from the middle \}
\{ found match; now increase last and compute $N$ [last] \} while (iN>=q) and ( $\mathrm{T}[$ last $]<>\mathrm{T}[\mathrm{iN}]$ ) do iN := $\mathrm{N}[\mathrm{iN}]$;
last := last+1; $\mathrm{iN}:=\mathrm{iN}+1$; if $T[$ last $]=T[i N]$
then $N[$ last $]:=\mathrm{N}[\mathrm{iN}]$
else $\mathrm{N}[$ last $]:=\mathrm{iN}$;
end \{ end op loop \}
\{ print match \}
if $\mathrm{j}>$ last then
begin \{found match for tail of $T$ \} print $(k-(n-q+1), q, n-q+1)$; $\mathrm{q}:=\mathrm{n}+1$;
end else if $q=$ last then
begin \{no match \} $\mathrm{q}:=\mathrm{q}+1$ :
end else
begin \{last match failed: take previous one \} print(kOld-(last-q), q, last-q) q:= last;
end
end

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[^0]:    * Recull that a subsequence may have gaps, a substring may not.

