

The strongly-interacting light Higgs

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ABSTRACT: We develop a simple description of models where electroweak symmetry breaking is triggered by a light composite Higgs, which emerges from a strongly-interacting sector as a pseudo-Goldstone boson. Two parameters broadly characterize these models: m_ρ , the mass scale of the new resonances and g_ρ , their coupling. An effective low-energy Lagrangian approach proves to be useful for LHC and ILC phenomenology below the scale m_ρ . We identify two classes of operators: those that are genuinely sensitive to the new strong force and those that are sensitive to the spectrum of the resonances only. Phenomenological prospects for the LHC and the ILC include the study of high-energy longitudinal vector boson scattering, strong double-Higgs production and anomalous Higgs couplings. We finally discuss the possibility that the top quark could also be a composite object of the strong sector.

KEYWORDS: Beyond Standard Model, Higgs Physics, Technicolor and Composite Models.

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1. Introduction

The main goal of the LHC is to unveil the mechanism of electroweak symmetry breaking. A crucial issue that experiments should be able to settle is whether the dynamics responsible for symmetry breaking is weakly or strongly coupled. LEP1 has provided us with convincing indications in favor of weakly-coupled dynamics. Indeed, the good agreement of precision measurements with the Standard Model (SM) predictions showed that the new dynamics cannot significantly influence the properties of the Z boson, ruling out, for instance, the simplest forms of technicolor models, which were viewed as the prototypes of a strongly-interacting electroweak sector. Moreover, the best agreement between experiments and theory was obtained for a light Higgs, corresponding to a weakly-coupled Higgs self-interaction. Finally, supersymmetry, which appeared to be the most realistic realization of a light Higgs with mass stabilized under quantum corrections, received a further

boost by the LEP1 measurements of gauge coupling constants, found to be in accord with supersymmetric unification.

The situation has swayed back after the LEP2 results. The lack of discovery of a Higgs boson below 114 GeV or of any new states has forced supersymmetry into fine-tuning territory, partially undermining its original motivation. Moreover, new theoretical developments, mostly influenced by extra dimensions and by the connection between strongly-interacting gauge theories and gravity on warped geometries, have led the way to the construction of new models of electroweak symmetry breaking [1–6]. Still, the complete replacement of the Higgs sector with strongly-interacting dynamics seemed hard to implement, mostly because of constraints from electroweak data. A more promising approach is to keep the Higgs boson as an effective field arising from new dynamics [7, 8] which becomes strong at a scale not much larger than the Fermi scale. There has been various attempts to realize such scenario, including the Little Higgs [2], Holographic Higgs as Goldstone bosons [5, 6] or not [9], and other variations.

In this paper we want to study the general properties and the phenomenology of scenarios in which a light Higgs is associated with strong dynamics at a higher scale, focusing on features that are quite independent of the particular model realization. We will refer to this scenario as to the Strongly-Interacting Light Higgs (SILH). Of course, in many specific models, the best experimental signals will be provided by direct production of new states, while here we concentrate on deviations from SM properties in Higgs and longitudinal gauge boson processes. Still, we believe that our model-independent approach is useful. The tests we propose here on Higgs and gauge-boson interactions will help, in case of new discoveries, to establish if the new particles indeed belong to a strongly-interacting sector ultimately responsible for electroweak symmetry breaking. If no new states are observed, or if the resonances are too broad to be identified, then our tests can be used to investigate whether the Higgs is weakly coupled or is an effective particle emerging from a strongly-interacting sector, whose discovery has been barely missed by direct searches at the LHC.

This paper is organized as follows. In section 2, we define the SILH and construct the low-energy effective theory that describes its interactions with the SM fields. In section 3, we discuss how this effective Lagrangian is related to specific models previously proposed in the literature, like the Holographic Higgs and the Little Higgs. Then we describe in section 4, how the SILH can be tested in collider experiments. In section 5, we extend our analysis to the case of a composite top quark and finally we summarize our results and draw our conclusions in section 6.

2. The structure of SILH

2.1 Definition of SILH

The structure of the theories we want to consider is the following. In addition to the vector bosons and fermions of the SM, there exists a new sector responsible for EW symmetry breaking, which is broadly characterized by two parameters, a coupling g_ρ and a scale m_ρ describing the mass of heavy physical states. Collectively indicating by g_{SM} the SM gauge

and Yukawa couplings (basically the weak gauge coupling and the top quark Yukawa), we assume $g_{SM} \lesssim g_\rho \lesssim 4\pi$. The upper bound on g_ρ ensures that the loop expansion parameter $\sim (g_\rho/4\pi)^2$ is less than unity, while the limit $g_\rho \sim 4\pi$ corresponds to a maximally strongly-coupled theory in the spirit of naive dimensional analysis (NDA) [10]. Because of the first inequality, by a slight abuse of language, we shall refer to the new sector as “the strong sector”. The Higgs multiplet is assumed to belong to the strong sector. The SM vector bosons and fermions are weakly coupled to the strong sector by means of the $SU(3) \times SU(2) \times U(1)_Y$ gauge coupling and by means of proto-Yukawa interactions, namely interactions that in the low-energy effective field theory will give rise to the SM Yukawas.

A second crucial assumption we are going to make is that in the limit $g_{SM} = 0$, $g_\rho \neq 0$ the Higgs doublet H is an exact Goldstone boson, living in the \mathcal{G}/\mathcal{H} coset space of a spontaneously broken symmetry of the strong sector. Two minimal possibilities in which the complex Higgs doublet spans the whole coset space are $SU(3)/SU(2) \times U(1)$ and the custodially symmetric $SO(5)/SO(4)$.

The gauging of $SU(2) \times U(1)_Y$ and the non-zero Yukawas explicitly break the Goldstone symmetry of the strong sector leading to terms in the (effective) action that are not invariant under the action of \mathcal{G} on the coset space. In particular a mass term for the Higgs is generated at 1-loop. If the new dynamics is addressing the hierarchy problem, it should soften the sensitivity of the Higgs mass to short distances, that is to say below $1/m_\rho$. In interesting models, the Higgs mass parameter is thus expected to scale like $(\alpha_{SM}/4\pi)m_\rho^2$. Observation at the LHC of the new states with mass m_ρ will be the key signature of the various realizations of SILH. Here, as stated in section 1, we are interested in the model-independent effects, which could be visible in processes involving the Higgs boson and/or longitudinal gauge bosons, and which would unmistakably reveal new physics in the electroweak breaking sector.

As we shall explain below, the σ -model scale f is related to g_ρ and m_ρ by the equation

$$m_\rho = g_\rho f . \tag{2.1}$$

Fully strongly interacting theories, like QCD, correspond to $g_\rho \sim 4\pi$. In that case eq. (2.1) expresses the usual NDA relation between the pion decay constant f and the mass scale of the QCD states. On the other hand, the theories we are considering represent a “weakly coupled” deformation of this QCD-like pattern. For $g_\rho < 4\pi$, the pure low-energy effective σ -model description breaks down above a scale m_ρ , which is parametrically lower than the scale $4\pi f$ where the σ -model would become strongly coupled. The coupling g_ρ precisely measures how strong the coupling of the σ -model can become before it is replaced by a more fundamental description. The simplest example of this possibility is represented by a linear σ -model UV completion of the \mathcal{G}/\mathcal{H} non-linear theory. In this case the role of m_ρ and g_ρ^2 is played respectively by the mass of the heavy scalar modes and by their quartic coupling. For instance, in the interesting case in which the Higgs complex doublet spans $SO(5)/SO(4)$, a simple UV completion could consist of a real scalar Φ in the fundamental of $SO(5)$ and quartic potential,¹ $V = -m_\rho^2 \Phi^2 + g_\rho^2 \Phi^4$. However, once the SM couplings are

¹Along the same lines we could even describe the SM in the limit of a heavy Higgs boson, using an

turned on, such a limited UV completion fails to screen the quadratic corrections to the Higgs mass.

A more interesting possibility arises when the strong sector is composite so that the corrections to the Higgs mass are screened above the “hadron” mass scale m_ρ . Moreover if the underlying theory is a large- N gauge theory, we also expect the hadrons to interact with a coupling

$$g_\rho = \frac{4\pi}{\sqrt{N}}, \tag{2.2}$$

which becomes weaker at large N . This is also basically the picture that holds in extra-dimensional constructions where the SM is represented by a weakly-coupled boundary dynamics while the Higgs sector is part of a more strongly-coupled bulk dynamics. Examples of this type are the Holographic Goldstones [6] over a slice of AdS₅.² In these extra-dimensional realizations, the Kaluza-Klein mass and coupling play respectively the role of m_ρ and g_ρ , while the number of weakly-coupled Kaluza-Klein modes below the cut-off can be basically interpreted as N . To be explicit, consider a 5-dimensional (5D) gauge theory with 5D coupling g_5 compactified on a circle or orbifold of radius R . We have $m_\rho = 1/R$ and $g_\rho^2 \equiv g_5^2/(\pi R)$. On the other hand, according to 5D NDA, the physical cut-off of the model is $\Lambda \sim (4\pi)^2/g_5^2$. These relations then imply

$$\left(\frac{4\pi}{g_\rho}\right)^2 = \Lambda\pi R \equiv N. \tag{2.3}$$

Other models that basically fall into our class are Little Higgses [1]. There, the scale m_ρ is represented by the masses of the partners of top quark, electroweak vector bosons and Higgs, the states that soften the quadratic correction to the Higgs mass. In Little-Higgs models there is more parameter freedom, and the coupling g_ρ is more accurately described by a set of couplings that can range from weak ($\sim g_{SM}$) to strong ($\gg g_{SM}$). Nonetheless we shall still find our simplified characterization very useful.

Summarizing, in several models of interest the electroweak-breaking sector corresponds to a “deformation” of a pure σ -model, where weakly-coupled states appear below the naive cut-off $4\pi f$. It is useful to focus on the simplest possibility where just one parameter, the coupling g_ρ of the new states, characterizes this “deformation”.³

2.2 Constructing the effective action

Under the assumptions of the class of theories defined above, we now want to derive the form of the most general effective Lagrangian for the SM + Higgs fields. Since we are assuming

analogous σ -model involving only 3 (and not 4) Goldstone bosons, with $f = \langle H \rangle$, $g_\rho = \sqrt{\lambda}$ (the quartic Higgs coupling), and the physical Higgs mass playing the role of m_ρ .

²Notice that the AdS geometry only matters for the extrapolation to ultra-high scales. The same low-energy dynamics of ref. [6] could also be realized over flat space by turning on the suitable boundary terms, along the lines of ref. [11] (although ref. [11] focussed on the Higgsless limit).

³Our simplified approach based on two parameters (m_ρ, g_ρ) is close in spirit to recent studies of two (and three) site models [12]. The phenomenological goal of our paper is however complementary to that refs. [12]: while those studies focus on the physics of the new heavy states, our paper focuses on the low-energy effects in Higgs and vector boson interactions.

$g_\rho > g_{SM}$, it makes sense to focus first on the strong sector in the limit $g_{SM} = 0$, and to turn on later the couplings of this sector to the SM vectors and fermions. Indicating by T^A and T^a respectively the broken and the unbroken generators of the group \mathcal{G} , we parametrize the Goldstone field by the matrix

$$U = e^{i\Pi} \quad \Pi \equiv \Pi^A T^A. \tag{2.4}$$

In addition we assume the strong sector features a set of fields with mass of order m_ρ , which we collectively indicate by Φ . By our assumptions, the general form of the action including quantum fluctuations from scales shorter than $1/m_\rho$ must be

$$\mathcal{L} = \frac{m_\rho^4}{g_\rho^2} \left[\mathcal{L}^{(0)}(U, \Phi, \partial/m_\rho) + \frac{g_\rho^2}{(4\pi)^2} \mathcal{L}^{(1)}(U, \Phi, \partial/m_\rho) + \frac{g_\rho^4}{(4\pi)^4} \mathcal{L}^{(2)}(U, \Phi, \partial/m_\rho) + \dots \right]. \tag{2.5}$$

In the action we have kept massive degrees of freedom “integrated in” for purposes that will become momentarily more clear. One can for instance check that the structure in eq. (2.5) is obtained in the compactification of a 5D gauge theory with the identification $m_\rho \equiv 1/R$ and $g_\rho^2 = g_5^2/(\pi R)$ (provided the power divergent loops are computed by NDA, while the log-divergent and finite pieces automatically satisfy the above structure). Moreover this same structure characterizes the effective field-theory description of the string. For instance in type I compactified on a T^6 of radius $\sim 1/M_s$, we can make the identifications: $M_s = m_\rho$ and $g_s = g_\rho^2/2\pi$.

In order to get the truly low-energy effective action we should then integrate out the Φ 's and also include the quantum fluctuations at scales below m_ρ . If the structure of the terms in eq. (2.5) is the most general one, in particular if terms of all orders in derivatives appear already in the classical Lagrangian $\mathcal{L}^{(0)}$, then the presence or absence of the Φ 's has no impact on the low-energy theory. We shall first concentrate on this case. Later we shall discuss the more realistic situation where the classical Lagrangian involves at most two derivatives: in that case the structure of the higher-order terms in the low-energy action crucially depends on the quantum numbers of the Φ 's. The leading two-derivative term defines relation (2.1) for the Goldstone decay constant as well as the leading self-interactions⁴

$$\begin{aligned} \frac{m_\rho^4}{g_\rho^2} \mathcal{L}^0 &\equiv f^2 \text{Tr} (\mathcal{D}_\mu \mathcal{D}^\mu) + \dots \\ &= f^2 \text{Tr} \left[\partial_\mu \Pi \partial^\mu \Pi + \frac{1}{3} (\Pi \overleftrightarrow{\partial}_\mu \Pi) (\Pi \overleftrightarrow{\partial}^\mu \Pi) + \dots \right]. \end{aligned} \tag{2.6}$$

Here \mathcal{D}_μ is the Goldstone combination defined in eq. (A.3) of appendix A and $\Pi \overleftrightarrow{\partial}_\mu \Pi \equiv \Pi (\partial_\mu \Pi) - (\partial_\mu \Pi) \Pi$. Once we interpret Π as the Higgs doublet and include gauge covariant derivatives, we obtain that eq. (2.6) describes the following leading (dimension-6) interactions

$$\frac{c_H}{2f^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{c_T}{2f^2} \left(H^\dagger \overleftrightarrow{D}^\mu H \right) \left(H^\dagger \overleftrightarrow{D}_\mu H \right). \tag{2.7}$$

⁴When the coset generators T^A transform as a reducible representation of \mathcal{H} , in principle there will be a different f for each quadratic invariant.

Here we have made a Higgs field redefinition $H^\alpha \rightarrow H^\alpha + a(H^\dagger H)H^\alpha/f^2$ (with a an appropriate constant) to write the operator $H^\dagger H |D_\mu H|^2$ in terms of the two appearing in eq. (2.7). The coefficients c_H and c_T are fixed by the σ -model structure, up to the overall normalization which depends on the definition of f . For $\text{SO}(5)/\text{SO}(4)$ one finds $c_T/c_H = 0$, because of custodial symmetry, and for $\text{SU}(3)/\text{SU}(2) \times \text{U}(1)$ one finds $c_T/c_H = 1$.

From eqs. (2.5)–(2.6) we can deduce the rules to estimate the coefficients of the higher-dimensional operators in the low-energy effective Lagrangian

1. Each extra Goldstone leg is weighted by a factor $1/f$. For instance the addition of two Higgs doublet legs involves the factor $H^\dagger H/f^2$.
2. Each extra derivative is weighted by a factor $1/m_\rho$. When the SM subgroup is weakly gauged, the replacement $\partial_\mu \rightarrow \partial_\mu + iA_\mu \equiv D_\mu$ is in order; this same rule implies that each extra insertion of a gauge field strength $F_{\mu\nu} = -i[D_\mu, D_\nu]$ is weighted by a factor $1/m_\rho^2$.

The global symmetry \mathcal{G} is broken at tree level by the weak gauging of the SM group and by the weak interactions that underlie the origin of Yukawa terms and Higgs potential. In section 3, we shall present a more detailed analysis of all the various possibilities. For our present goal we just need to remark that if no new scale other than m_ρ is present, and simple expressions of the Goldstone field are involved, one expects these breaking terms to satisfy the same field and derivative expansions expressed by rules 1 and 2. Basically, the selection rules of \mathcal{G} and of the flavour symmetry of the SM control the overall size of the symmetry breaking terms, while rules 1 and 2 determine the counting for Higgs field and derivative insertions. We can thus formulate rule 3:

3. Higher-dimensional operators that violate the symmetry of the σ -model must be suppressed by the same (weak) coupling associated to the corresponding renormalizable interaction in the SM Lagrangian (e.g., Yukawa couplings y_f and quartic Higgs coupling λ).

For instance, the shift $H^\alpha \rightarrow H^\alpha + a(H^\dagger H)H^\alpha/f^2$ discussed before induces the operators

$$\left(\frac{c_y y_f}{f^2} H^\dagger H \bar{f}_L H f_R + \text{h.c.} \right) - \frac{c_6 \lambda}{f^2} \left(H^\dagger H \right)^3. \quad (2.8)$$

The pure σ -model contributions give $c_y/c_H = -1/3$ and $c_6/c_H = 4/3$ both in the cases of $\text{SO}(5)/\text{SO}(4)$ and $\text{SU}(3)/\text{SU}(2) \times \text{U}(1)$, with the definitions $y_f = \sqrt{2}m_f/v$ and $\lambda = m_H^2/(2v^2)$, valid up to corrections of order v^2/f^2 .

Of special phenomenological relevance are the operators involving Higgs bosons and gauge fields, and in particular those involving one or two Higgses and a pair of photons or gluons like $O_{BB} = H^\dagger H B_{\mu\nu} B^{\mu\nu}$ and $O_g = H^\dagger H G_{\mu\nu} G^{\mu\nu}$. Using the CCWZ construction [13], we derive, in appendix A, the following general structure of dimension-6 operators involving Higgs and gauge field strengths, arising by weak gauging of the SM group:

$$O_W = i \left(H^\dagger \sigma^i \overleftrightarrow{D}^\mu H \right) (D^\nu W_{\mu\nu})^i \quad O_B = i \left(H^\dagger \overleftrightarrow{D}^\mu H \right) (\partial^\nu B_{\mu\nu}) \quad (2.9)$$

$$O_{HW} = i(D^\mu H)^\dagger \sigma^i (D^\nu H) W_{\mu\nu}^i \quad O_{HB} = i(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}. \quad (2.10)$$

While, expectedly, operators involving gluons do not arise, it is also manifest that none of these operators contributes to the process $h \rightarrow \gamma\gamma$ (with real photons). On the other hand, there are contributions to $h \rightarrow Z\gamma$ from O_{HW} and O_{HB} . Notice that, using integration by parts, we could equivalently parametrize the set of operators in eqs. (2.9)–(2.10) by O_W , O_B and $O_{ZB} = H^\dagger B^{\mu\nu}(W_{\mu\nu} + B_{\mu\nu})H$ and $O_{ZW} = H^\dagger W^{\mu\nu}(W_{\mu\nu} + B_{\mu\nu})H$, where $W_{\mu\nu} \equiv W_{\mu\nu}^i \sigma^i$. Again, neither O_{ZB} nor O_{ZW} contribute to $h \rightarrow \gamma\gamma$. According to counting rule 2, all these operators have a coefficient of order $1/m_\rho^2$, as they formally involve two extra covariant derivatives with respect to a Higgs kinetic term. The absence of $O_g = H^\dagger H G_{\mu\nu} G^{\mu\nu}$ and of operators affecting the coupling between Higgs and photons like $O_{BB} = H^\dagger H B_{\mu\nu} B^{\mu\nu}$ is due to the Goldstone symmetry. Since the neutral Higgs h is both charge and color neutral, the gauging of just $SU(3)_c \times U(1)_Q$ does not break the $U(1)$ generator T_h of \mathcal{G} under which the physical Higgs boson shifts. Operators like O_g or those leading to $h \rightarrow \gamma\gamma$ (like O_{BB}) explicitly break this shift symmetry and cannot be generated upon the simple gauging of the SM group described by rule 2. In order to generate these terms, the couplings that break the symmetry generated by T_h must intervene, so that their coefficient must be suppressed by extra powers of (g_{SM}/g_ρ) . Normally one gets a g_{SM}^2/g_ρ^2 extra suppression.⁵

According to the general expression in eq. (2.5), four-derivative operators like those in eqs. (2.9)–(2.10) can arise at tree level. However in “normal” theories, the classical action including the heavy fields Φ involves at most two derivatives. Holographic Goldstone models and Little Higgs are of this type. To be more specific, these theories correspond to minimally-coupled field theories where the states have spin ≤ 1 , and all vectors are associated to (spontaneously-broken) gauge symmetries.⁶ In the case of minimally-coupled theories, higher-derivative operators like those in eqs. (2.9)–(2.10) can appear in the classical low-energy action below m_ρ only if there exists a field Φ with the appropriate quantum numbers to mediate the corresponding operator. In this respect we remark an interesting difference between O_W, O_B and O_{HW}, O_{HB} . Two linearly independent combinations of O_{HW} and O_{HB} contribute respectively to a vertex that couples an on-shell photon to two neutral states (a Higgs and a Z) and to a correction to the gyromagnetic ratio of the W . On the other hand, in minimally-coupled theories photons do not interact at tree level with neutral states and all gyromagnetic ratios are fixed to be equal to 2. In these theories O_{HW}, O_{HB} must therefore appear in $\mathcal{L}^{(1)}$ and bear an extra one-loop suppression. By the same argument O_g and O_{BB} should also arise at one-loop and moreover, because of the previous symmetry-based argument, they should be further suppressed by a Goldstone symmetry-breaking power of g_{SM}/g_ρ . The operators O_W or O_B can instead be generated in minimally-coupled theories by the tree-level exchange of heavy vector fields. We show

⁵A similar result holds in low-energy QCD. The coupling $\pi_0^2 F_{\mu\nu}^2$ vanishes at leading order in α_{EM} , and gets generated at subleading order in the quark mass m_q and also through the chiral anomaly. On the other hand, $\pi^+ \pi^- F_{\mu\nu}^2$ exists at zeroth order in both α_{EM} and m_q .

⁶The latter property sets the rule to count derivatives for massive vector fields through the requirement that the action for the eaten Goldstones be a 2-derivative one. For instance the gauge symmetry breaking term $(\partial^\mu V_\mu)^2$ counts like a four derivative object and is discarded from the classical action. Minimal coupling along with the gauge principle ensures the absence of ghosts at the scale m_ρ and a milder growth of the amplitudes at energies above the scale m_ρ .

this explicitly in appendix A, in the context of a simplified model with a heavy vector that resembles both the Little Higgs models with product of gauge groups and the Holographic Goldstone. Given that all the known examples with $g_\rho < 4\pi$ are minimally coupled, in the following of this paper, we will work under the assumption of minimal coupling.⁷ Of course this makes a difference when we consider models at $g_\rho < 4\pi$, as opposed to the genuinely strongly coupled case $g_\rho \sim 4\pi$, for which all loops are equally important in the spirit of NDA.

At the dimension-6 level, there is one last independent operator involving two Higgses and four covariant derivatives

$$\frac{1}{m_\rho^2}(D^2 H^\dagger)(D^2 H). \tag{2.11}$$

As we show in the appendix A this can be generated at tree level in a minimally coupled theory by integrating out a massive scalar transforming as an $SU(2)_L$ doublet. By the equations of motion this term can, however, be rewritten as

$$\frac{1}{m_\rho^2} \left[m_H^2 H^\alpha + \lambda H^\dagger H H^\alpha + y_f (F_L f_R)^\alpha \right]^2, \tag{2.12}$$

corresponding to effects that are all subleading to more direct corrections from the strong sector.

For completeness we should also list the dimension-6 operators involving only covariant derivatives and field strengths

$$O_{2W} = (D^\mu W_{\mu\nu})^i (D_\rho W^{\rho\nu})^i \quad O_{2B} = (\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) \quad O_{2g} = (D^\mu G_{\mu\nu})^a (D_\rho G^{\rho\nu})^a \tag{2.13}$$

$$O_{3W} = \epsilon_{ijk} W_\mu^{i\nu} W_{\nu\rho}^j W^{k\rho\mu} \quad O_{3g} = f_{abc} G_\mu^{a\nu} G_{\nu\rho}^b G^{c\rho\mu}. \tag{2.14}$$

As we show in the appendix A, see eq. (A.11), the three operators in eq. (2.13) can be generated at tree level through the exchange of massive vectors transforming respectively as a weak triplet, as a singlet and as a color octet. Their coefficients are therefore in general of order $1/(g_\rho m_\rho)^2$. The two operators in eq. (2.14) cannot arise at tree level in minimally-coupled theories. For instance O_{3W} contributes to the magnetic dipole and to the electric quadrupole of the W . They are thus generally expected with a coefficient $\sim 1/(4\pi m_\rho)^2$.

2.3 The SILH effective lagrangian

We now basically have all the ingredients to write down the low-energy dimension-6 effective Lagrangian. We will work under the assumption of a minimally coupled classical Lagrangian at the scale m_ρ .

⁷One may wonder how our results would change in a genuinely higher-spin (higher-derivative) theory like string theory. To be specific, one could consider, provided it exists, a realization of the Holographic Goldstone in weakly-coupled string theory and take the limit $M_s \sim m_{KK} \equiv m_\rho$. At first glance we would expect a drastic change. For instance, it is obvious that a photon can scatter off a dilaton, a neutral scalar, at tree level. On the other hand, a specific study of the gyromagnetic ratio g [14] of all the high-spin states of the open string remarkably gives the result $g = 2$, indicating a close similarity to a minimally-coupled theory.

Using the rules described in section 2.2, we obtain a low-energy effective action for the leading dimension-6 operators involving the Higgs field of the form

$$\begin{aligned}
 \mathcal{L}_{\text{SILH}} = & \frac{c_H}{2f^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{c_T}{2f^2} \left(H^\dagger \overleftrightarrow{D}^\mu H \right) \left(H^\dagger \overleftrightarrow{D}_\mu H \right) \\
 & - \frac{c_6 \lambda}{f^2} (H^\dagger H)^3 + \left(\frac{c_y y_f}{f^2} H^\dagger H \bar{f}_L H f_R + \text{h.c.} \right) \\
 & + \frac{i c_W g}{2m_\rho^2} \left(H^\dagger \sigma^i \overleftrightarrow{D}^\mu H \right) (D^\nu W_{\mu\nu})^i + \frac{i c_B g'}{2m_\rho^2} \left(H^\dagger \overleftrightarrow{D}^\mu H \right) (\partial^\nu B_{\mu\nu}) \\
 & + \frac{i c_{HW} g}{16\pi^2 f^2} (D^\mu H)^\dagger \sigma^i (D^\nu H) W_{\mu\nu}^i + \frac{i c_{HB} g'}{16\pi^2 f^2} (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\
 & + \frac{c_\gamma g'^2}{16\pi^2 f^2} \frac{g^2}{g_\rho^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{c_g g_S^2}{16\pi^2 f^2} \frac{y_t^2}{g_\rho^2} H^\dagger H G_{\mu\nu}^a G^{a\mu\nu}. \tag{2.15}
 \end{aligned}$$

We will later discuss the Lagrangian terms that purely involve the vector bosons. The coupling constants c_i are pure numbers of order unity. For phenomenological applications, we have switched to a notation in which gauge fields are canonically normalized, and gauge couplings explicitly appear in covariant derivatives. Also, we recall the definition $H^\dagger \overleftrightarrow{D}_\mu H \equiv H^\dagger D_\mu H - (D_\mu H^\dagger) H$.

In what follows, we will comment on the operators in eq. (2.15). Let us start with the operators involving more than two Higgs fields. As previously discussed, by using the Fierz identities for the Pauli matrices, one can write three independent operators involving four H fields and two covariant derivatives. Two are shown in our Lagrangian with coefficients c_H and c_T . The third operator $H^\dagger H |D_\mu H|^2$, can be written in terms of a combination of c_H, c_T, c_6, c_y by a Higgs field redefinition $H^\alpha \rightarrow H^\alpha + (H^\dagger H) H^\alpha / f^2$, or, which is equivalent, by using the leading order equations of motion. The operator with coefficient c_H , as we will show in section 4, plays a crucial role in testing the SILH in Higgs and vector boson scattering at high-energy colliders. The operator proportional to c_T violates custodial symmetry and gives a contribution \hat{T} to the ρ parameter

$$\Delta\rho \equiv \hat{T} = c_T \xi, \tag{2.16}$$

$$\xi \equiv \frac{v^2}{f^2}, \quad v = \left(\sqrt{2} G_F \right)^{-1/2} = 246 \text{ GeV}. \tag{2.17}$$

From the SM fit of electroweak data [16], we find $-1.1 \times 10^{-3} < c_T \xi < 1.3 \times 10^{-3}$ at 95% CL (letting also \hat{S} to vary one finds instead $-1.7 \times 10^{-3} < c_T \xi < 1.9 \times 10^{-3}$ at 95% CL). Because of this strong limit, we will neglect new effects from this operator and set c_T to zero. Indeed, the bound on c_T suggests that new physics relevant for electroweak breaking must be approximately custodial-invariant. In our Goldstone Higgs scenario this corresponds to assuming the coset $\text{SO}(5)/\text{SO}(4)$. When g_{SM} is turned on, c_T receives a model dependent contribution, which should be small enough to make the model acceptable. In the next section, we will briefly discuss the size of c_T in various models.

The coefficient c_y is universal at leading order in the Yukawa couplings, and non-universal effects will appear at order y_f^2/g_ρ^2 . This is because this term purely originates from

the non linearity in H of the σ -model matrices. Indeed, the field redefinition mentioned above precisely generates this universal c_y .

The operators proportional to c_W and c_B are generated respectively by tree-level exchange of a massive triplet and singlet vector field as explained in the previous section (see also eq. (A.11) in appendix A). Their relative importance in 2-to-2 scattering amplitudes with respect to the operator proportional to c_H is $(g^2/g_\rho^2)(c_{W,B}/c_H)$. Therefore, in weakly-coupled theories ($g_\rho \sim g$), the two contributions are comparable but, in strongly-coupled theories ($g_\rho \gg g$), the operators proportional to $c_{W,B}$ give only subleading effects. Since, as we will show in section 3, realistic models of electroweak breaking without excessive fine tuning prefer $g_\rho > g$, in most cases the contribution from $c_{W,B}$ are subleading with respect to the one from c_H .

A linear combination of the operators with coefficients c_W and c_B contributes to the \widehat{S} parameter of electroweak precision data:

$$\widehat{S} = (c_W + c_B) \frac{m_W^2}{m_\rho^2}, \tag{2.18}$$

where \widehat{S} is defined in ref. [16]. Using the SM fit of electroweak data [16], we obtain the bound $m_\rho \gtrsim (c_W + c_B)^{1/2} 2.5$ TeV at 95% CL. (this bound corresponds to assuming a light Higgs and $\Delta\rho \equiv \widehat{T} = 0$; by relaxing this request the bound becomes $m_\rho \gtrsim (c_W + c_B)^{1/2} 1.6$ TeV). In terms of the parameter ξ defined in eq. (2.17), this bound becomes

$$\xi \lesssim \frac{1.5}{c_W + c_B} \left(\frac{g_\rho}{4\pi} \right)^2. \tag{2.19}$$

As we show in section 4, new effects in Higgs physics at the LHC appear only for sizable values of ξ . Then eq. (2.19) requires a rather large value of g_ρ , unless $c_W + c_B$ happens to be accidentally small.

The operators with coefficients c_{HW} and c_{HB} originate from the 1-loop action $\mathcal{L}^{(1)}$, under our assumption of minimal coupling for the classical action. Although they are $H^2 D^4$ terms, like c_W, c_B , they cannot be enhanced above their 1-loop size by the exchange of any spin 0 or 1 massive field. In the case of a large N theory where $N \sim 16\pi^2/g_\rho^2$, these terms are down with respect to the others by $1/N$. Notice that according to this counting $\widehat{S} \sim g^2 N \xi / (16\pi^2)$, which for $\xi \sim 1$ coincides with the usual technicolor result. Recently, it has been pointed out that walking at small N might be a promising direction [15].

As discussed in section 2.2, the operators proportional to c_γ and c_g are suppressed by an extra power $(g_{SM}/g_\rho)^p$ with respect to those proportional to c_{HW} and c_{HB} . Moreover, while c_H and c_y indirectly correct the physical Higgs coupling to gluons and quarks by $O(v^2/f^2)$ with respect to the SM, the direct contribution of c_γ and c_g is of order $(v^2/f^2)(g_{SM}/g_\rho)^p$. Their effect is then important only in the weakly coupled limit $g_\rho \sim g_{SM}$. Notice that from the point of view of the Goldstone symmetry, O_{BB} and O_g are like a Higgs mass term with extra field strength insertions. According to our power counting rules we then expect their coefficient to roughly scale like m_H^2/m_ρ^4 times the trivial factors of g'^2 and g_3^2 . In the simplest models $m_H^2 \sim (g_{SM}^2/16\pi^2)m_\rho^2$. We have here assumed this simplest possibility, which accounts for the extra g_{SM}^2/g_ρ^2 appearing in eq. (2.15). More precisely, for

phenomenological purposes, we have chosen g_{SM} as the coupling of the largest contribution in the corresponding SM loop, i.e., $g_{SM}^2 = g^2 (y_t^2)$ for the operator involving photons (gluons), respectively.

Finally, for completeness, we give the dimension-6 Lagrangian for the vectors:

$$\begin{aligned} \mathcal{L}_{\text{vect}} = & -\frac{c_{2W}g^2}{2g_\rho^2m_\rho^2}(D^\mu W_{\mu\nu})^i(D_\rho W^{\rho\nu})^i - \frac{c_{2B}g'^2}{2g_\rho^2m_\rho^2}(\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}) - \frac{c_{2g}g_3^2}{2g_\rho^2m_\rho^2}(D^\mu G_{\mu\nu})^a(D_\rho G^{\rho\nu})^a \\ & + \frac{c_{3W}g^3}{16\pi^2m_\rho^2}\epsilon_{ijk}W_\mu^{i\nu}W_{\nu\rho}^jW^{k\rho\mu} + \frac{c_{3g}g_3^3}{16\pi^2m_\rho^2}f_{abc}G_\mu^{a\nu}G_{\nu\rho}^bG^{c\rho\mu}, \end{aligned} \quad (2.20)$$

where the coefficients are dictated by the arguments given in section 2.2. The operators proportional to c_{2W} , c_{2B} and c_{2g} arise from virtual tree-level exchange of massive vectors, from the second term in eq. (A.11) of appendix A. The coefficients c_{2W} and c_{2B} contribute to the electroweak precision parameters W and Y [16]

$$W = c_{2W}\frac{g^2m_W^2}{g_\rho^2m_\rho^2}, \quad Y = c_{2B}\frac{g'^2m_W^2}{g_\rho^2m_\rho^2}. \quad (2.21)$$

By using the equations of motion for the SM field strengths, these operators can be rewritten as contact interaction among the electroweak currents. Given that the experimental bound on W and Y are comparable to that on \widehat{S} , in the moderately strong coupling regime $g_\rho > g$, the constraints on c_{2W} and c_{2B} are weaker than those on $c_W + c_B$.

The Lagrangian terms in eqs. (2.15) and (2.20) include 14 dimension-6 CP-invariant operators involving only Higgs and gauge fields plus a 15th operator involving Higgs and fermions, the one associated to c_y . This result, albeit in a different basis, agrees with ref. [17] for the same class of operators.

3. Relating the SILH to explicit models

In this section, we consider a few explicit models that reduce to the effective Lagrangian (2.15) below the masses of the new states. Before reviewing these models we would like to give a synthetic but comprehensive characterization of the terms that explicitly break the Goldstone symmetry \mathcal{G} . We have already discussed in some detail the explicit breaking of \mathcal{G} induced by the weak gauging of the SM group. Here we will discuss the various possibilities for the Yukawa and Higgs couplings. We can broadly distinguish two classes of models: in the first class, the Higgs potential is fully saturated by quantum effects at the scale m_ρ , while in the second the quartic coupling is a tree-level effect (or, equivalently, it arises from quantum corrections at energies higher than m_ρ). The Georgi-Kaplan model and Holographic Goldstones are in the first class, while Little Higgses belong to the second. In what follows we shall work with canonically normalized fields, and indicate by $f_{L,R}$ the SM fermions while by $\Psi_{L,R}$ we refer to strong-sector states with mass $\sim m_\rho$.

Class 1. In this class of models, the full Higgs potential arises by one-loop effects involving SM particles. The dominant contribution is given by the top quark and its explicit form

depends on the origin of the top Yukawa coupling. We can basically distinguish two possibilities to generate the Yukawa couplings. The simplest one corresponds to models with minimal flavor violation [18] in which the only source of breaking of the $SU(3)^5$ global flavor symmetry of the SM are the Yukawa matrices themselves. This case is realized when the fermions couple to the strong sector bilinearly

$$y_f \bar{f}_L f_R O_S, \tag{3.1}$$

where O_S is some operator of the strong sector, while y_f are the SM Yukawa matrices. In the low-energy effective theory, the above term will give rise to the interaction

$$y_f \bar{f}_L f_R H P_y(H/f), \tag{3.2}$$

where P_y is a polynomial whose expansion at $\mathcal{O}(H^2)$ determines the universal coefficient c_y of the effective Lagrangian (2.15). Violations of c_y universality will come only at higher order in y_f . By simple power counting, the top-quark contribution to the Higgs potential has the form

$$V(H) \sim \frac{m_\rho^4}{g_\rho^2} \times \frac{y_t^2}{16\pi^2} \times \hat{V}(H/f). \tag{3.3}$$

Then the generic prediction of this class of models is $\langle H \rangle \sim f$ (i.e., $\xi \sim 1$) while the Higgs quartic is $\lambda \sim y_t^2 (g_\rho^2/16\pi^2)$. An upper bound around m_t is thus predicted for the Higgs mass, in some analogy with supersymmetry. Notice that, in this type of models, it is mandatory that $g_\rho \gg g_{SM}$ in order to have a Higgs mass above the experimental bound. Notice also that

$$\hat{S} \sim \frac{g_{SM}^2}{g_\rho^2} \xi, \tag{3.4}$$

which for maximal strength $g_\rho \sim 4\pi$ and $\xi \sim 1$ has the size of an electroweak loop. (By interpreting $N \sim (4\pi/g_\rho)^2$, the generic prediction for \hat{S} qualitatively corresponds to the case of N technicolors.) Then the smaller g_ρ , the more tuning on ξ is needed in order to satisfy the experimental bound on \hat{S} . These models must therefore largely satisfy our assumption $g_\rho \gg g_{SM}$. On the other hand, for g_ρ somewhat less than maximal we must tune ξ by an amount $(g_\rho/4\pi)^2 < 1$. In order to achieve that, we clearly need extra contributions to the potential, associated to other \mathcal{G} breaking couplings, possibly involving only the heavy states. Provided the extra contributions have different form than the one from the top quark, we may tune a little bit the quadratic term with respect to the quartic, thus suppressing ξ .

The second possibility to generate Yukawa couplings is to have the SM fermions couple linearly to fermionic operators of the strong sector:

$$y_L \bar{f}_L O_R + y_R \bar{f}_R O_L + \text{h.c.}, \tag{3.5}$$

where $y_{L,R}$ are matrices in flavor space. In the simplest cases, $O_{L,R}$ have definite quantum numbers under \mathcal{G} , and therefore eq. (3.5) formally determines the spurionic quantum numbers of $y_{L,R}$. The possibility of generating Yukawas from the linear couplings of eq. (3.5) was first suggested in ref. [19] for Technicolor models, and it is the one implemented in

Holographic Higgs models [6]. Writing eq. (3.5) as a function of the physical states of the strong sector Ψ , one can see that in these models the Yukawa couplings are generated through a sort of universal see-saw

$$m_\rho \left[\frac{y_L}{g_\rho} \bar{f}_L \Psi_R P_L(H/f) + \frac{y_R}{g_\rho} \bar{f}_R \Psi_L P_R(H/f) + \bar{\Psi}_L \Psi_R \right]. \quad (3.6)$$

Notice that for $y_L \sim g_\rho$ or $y_R \sim g_\rho$ respectively f_L or f_R should be considered as part of the strong sector.⁸ This remark explains the normalization of the mixing term in eq. (3.6). The effective SM Yukawa couplings after integrating out the Ψ have the form

$$y_f \sim \frac{y_L y_R}{g_\rho}. \quad (3.7)$$

For $y_L \sim y_R$ one has $y_{L,R} \sim \sqrt{y_f g_\rho}$, which is a coupling of intermediate strength. If the polynomials P_L and P_R are flavor universal so will be the c_y coefficient at leading order in the $y_{L,R}$. Nevertheless, the exchange of Ψ will give rise to non-universal (H dependent) corrections to the kinetic terms of $f_{L,R}$ that will scale like $y_{L,R}^2/g_\rho^2$. By going to canonically normalized fermions one induces then $\mathcal{O}(y_{L,R}^2/g_\rho^2)$ non-universal corrections to c_y . The top contribution to the Higgs potential receives now, in addition to terms of the form (3.3), corrections scaling like

$$V(H) \sim \frac{m_\rho^4}{g_\rho^2} \times \frac{y_{L,R}^2}{16\pi^2} \times \hat{V}(H/f). \quad (3.8)$$

For $y_L \sim y_R$ this leads to a Higgs quartic coupling $\lambda \sim (g_\rho/4\pi)^3 4\pi y_t$, and therefore a moderately heavy Higgs boson (~ 300 GeV) can in principle be obtained.⁹ For the same reason the suppression of the coefficients of O_g would be y_t/g_ρ instead of y_t^2/g_ρ^2 . Obviously, in the particular case in which the right-handed top is a singlet under the global group \mathcal{G} , its contribution to the Higgs potential eq. (3.8) will vanish. The allowed values for the couplings $y_{L,R}$ strongly depend on the quantum numbers of the mixing operators $O_{L,R}$. In the simplest case in which $O_L = (2, 1)$, $O_R = (1, 2)$ under the custodial group $\text{SO}(4) = \text{SU}(2)_L \times \text{SU}(2)_R$, the corrections to $Z\bar{b}b$ and \hat{T} are expected to be¹⁰

$$\frac{\delta g_b}{g_b} \sim \frac{y_L^2}{g_\rho^2} \xi, \quad \hat{T} = \frac{N_c y_R^4}{16\pi^2 g_\rho^2} \xi, \quad (3.9)$$

⁸For instance, for $y_R \sim g_\rho$ the linear combination of f_R and Ψ_R which is left massless by the second and third terms in eq. (3.6) has the natural interpretation of a massless composite.

⁹Whether this can be achieved in practice depends on the specific model at hand. Depending on the \mathcal{G} quantum numbers of $y_{L,R}$, the potential they generate may or may not trigger electroweak symmetry breaking. For instance in the model in ref. [6], the Higgs potential terms eq. (3.8) align to the wrong vacuum $\langle H \rangle = 0$. In that model the formally subdominant genuine top contribution eq. (3.3), which has a minimum at $\langle H \rangle \sim f$, must therefore be equally important. This is easily achieved by some little tuning of y_L and y_R thanks to an accidental numerical suppression of eq. (3.8). The result however is that the Higgs mass is bounded $\lesssim m_t$.

¹⁰Notice that \hat{T} corresponds to a contribution to the vector boson mass matrix transforming as an object with custodial isospin $I_c = 2$. Therefore, y_L , being an isospin singlet, will not contribute to \hat{T} , while y_R , being a doublet of $\text{SU}(2)_R$, will have to enter at least at fourth order.

where $N_c = 3$ is the number of colors. The experimental bounds, together with the relation $y_t \sim y_L y_R / g_\rho$ imply $\xi < 0.05$. The reason of this tight bound is that $\delta g_b / g_b$ demands a small y_L , \widehat{T} demands a small y_R , while the two couplings are constrained to have a sizeable product to reproduce y_t . A less constrained, and thus less tuned scenario, can arise in the less minimal case where $O_L = (2, 2)$, $O_R = (1, 1)$. Now y_R is a singlet under the custodial group and drops out of eq. (3.9). However y_L transforms as $(1, 2)$ under $SU(2)_L \times SU(2)_R$ and therefore

$$\frac{\delta g_b}{g_b} \sim \frac{y_L^2}{g_\rho^2} \xi, \quad \widehat{T} \sim \frac{N_c y_L^4}{16\pi^2 g_\rho^2} \xi. \quad (3.10)$$

For $y_L \sim y_t$, corresponding to a composite t_R ($y_R \sim g_\rho$), the bound from $\delta g_b / g_b$ is comparable to the one from \widehat{S} , while the one from \widehat{T} is less severe. In this case, however, the mass of the Higgs is again bounded to be parametrically $\lesssim m_t$. A larger y_L , and a possibly heavier Higgs, can remarkably be made compatible with $\delta g_b / g_b$ for the special case in which the theory possesses an additional parity P_{LR} exchanging the $SU(2)_L$ and $SU(2)_R$ group within $SO(4)$. In that case, the leading tree-level contribution to $\delta g_b / g_b$ can be naturally set to zero, and one can take $y_L \sim \sqrt{y_t g_\rho}$ without being in stark conflict with the bound on \widehat{T} . For this value of y_L flavor violating effects are of the order of the experimental bounds, as we will see in section 5. Of course our previous warnings about obtaining proper electroweak breaking still apply here.

Apart from the above exceptional case in which $O_L = (2, 2)$, $O_R = (1, 1)$, other alternatives for the quantum numbers of $O_{L,R}$ will generically have problems with $\delta g_b / g_b$ and \widehat{T} . We can take $y_L \sim y_t$ and $y_R \sim g_\rho$ in order to reduce $\delta g_b / g_b$, but in this case \widehat{T} comes always too large, $\widehat{T} \sim N_c g_\rho^2 \xi / (16\pi^2)$. A possible way to reduce \widehat{T} is to introduce into the theory custodial partners for the top whose masses m_{cust} will control the breaking of the custodial symmetry [20]. In this case the contribution to \widehat{T} will have extra suppression factors $(m_{cust}/m_\rho)^2$ that can reduce \widehat{T} below the experimental bound for $m_{cust} \sim m_t$.¹¹ This possibility implies the presence of extra light fermions that are easily accessible at the LHC or even at Tevatron.

Class 2. These are models realizing the clever Little Higgs construction by which only the quadratic term in the Higgs potential is saturated by quantum corrections at the m_ρ scale. The Higgs quartic term, in particular, is sensitive to the larger scale $\Lambda \sim 4\pi f$ and is estimated to be of order $\lambda \sim g_{SM}^2 \Lambda^2 / (16\pi^2 f^2) \sim g_{SM}^2$. We then have

$$V(H) \sim \frac{g_{SM}^2}{16\pi^2} m_\rho^2 H^2 + g_{SM}^2 H^4. \quad (3.11)$$

The minimization yields the parametric relation $v^2 / f^2 \sim g_\rho^2 / (16\pi^2)$ to be compared to $v^2 / f^2 \sim 1$ for models of Class 1. The relation $\lambda \sim g_{SM}^2$ implies that the Higgs mass can now be above the experimental bound without requiring a large g_ρ , in contrast to the models of Class 1. In a related way we have roughly $\widehat{S} \sim m_W^2 / m_\rho^2 = g^2 / 16\pi^2$, which

¹¹One must, however, check that these extra states do not have electromagnetic charge $Q = -1/3$ and mix with b_L , since this would induce large effects on $\delta g_b / g_b$. This can occur, for example, for the assignment $O_L = (3, 2)$ and $O_R = (1, 2)$.

also does not manifestly push towards a large g_ρ . Now, the result for \widehat{S} in the Little Higgs is parametrically the same of a 1-loop electroweak correction, which is typically only marginally acceptable. Better agreement with the bound on \widehat{S} can be achieved by using the flexibility the Little Higgs possesses in the strong sector parameters: g_ρ truly represents a spectrum of different couplings. In particular we can distinguish a coupling g_ρ associated to the extra gauge factors and a coupling g_T associated with the mass $m_T \sim g_T f$ of the partners of the top quark. Since \widehat{S} is controlled by the vector boson mass $m_\rho \sim g_\rho f$, while the scalar potential tends to be dominated by the top contribution, the expression for \widehat{S} is more appropriately

$$\widehat{S} \sim \frac{m_W^2}{m_\rho^2} \sim \frac{g^2}{16\pi^2} \frac{g_T^2}{g_\rho^2}, \tag{3.12}$$

showing that there is space to relax \widehat{S} with respect to its typical one-loop size, making it numerically acceptable.¹² Therefore this class of models prefers a weak coupling in the top resonance sector but still a somewhat large coupling g_ρ in the new gauge sector.

Concerning \widehat{T} , if the underlying σ -model does not possess a custodial symmetry, one will have a significant $\widehat{T} \sim \mathcal{O}(v^2/f^2) \sim g_T^2/16\pi^2$ requiring a severe tuning of parameters and disfavoring large g_T even more. But even when the model is custodially symmetric in the limit $g_{SM} = 0$, there are corrections to \widehat{T} that are potentially important at small g_ρ . This is because SM custodial breaking couplings like y_t modify at tree level the structure of the strong sector Lagrangian, in particular the potential of electroweak triplet scalars (these effects unlike the Higgs mass are not screened above the scales m_T, m_ρ , and like the Higgs quartic they formally have tree-level size). If the leading contribution to the mass of the triplets is controlled by g_ρ , then, by the isospin argument illustrated before, we generally expect $c_T \sim (y_t/g_\rho)^4$ in eq. (2.15), leading roughly to $\widehat{T} \sim y_t^4 g_T^2 / (16\pi^2 g_\rho^4)$. This also favors $g_\rho > g_T, y_t$. A more detailed scrutiny of these effects requires considering some explicit model, which is beyond the scope of our brief survey.

Notice that the general class of $\mathcal{O}(g_{SM}^2/g_\rho^2)$ (and also g_{SM}^2/g_T^2) effects including those we have just discussed, would, for $g_{SM} \sim g_\rho$, induce $\mathcal{O}(1)$ violations of the \mathcal{G}/\mathcal{H} σ -model structure below the scale m_ρ . For instance, in addition to c_T , the terms of dimension higher than 6 in the two derivative Higgs Lagrangian would not be those dictated by the \mathcal{G}/\mathcal{H} structure. In realistic LH models a weak $g_T \sim y_t$ is somewhat favored. Then in the contribution of the top partners to the Higgs-gluon coupling c_g the Goldstone suppression y_t^2/g_T^2 is $\mathcal{O}(1)$, and this operator is as important as c_H and c_y in Higgs physics. Notice however that in the weakly-coupled limit, all anomalous effects in Higgs physics are of order $v^2/f^2 \sim (g_T/4\pi)^2$, which is parametrically like a SM loop effects. Expectedly, the use of our effective Lagrangian is more motivated, the more strongly coupled the new sector is, that is the bigger v/f .

In the following, we will give three explicit examples of models with the above characteristics. We will first concentrate on the Holographic Composite Higgs model of [6, 20].

¹²Notice that if we take $g_\rho \gtrsim g_T y_t/g$ the vector boson loops dominate the mass term in the potential, in which case either the relaxation effect saturates or the mass term changes sign thus restoring electroweak symmetry. Thus we can naturally relax \widehat{S} down to $\sim (g^4/y_t^2)/16\pi^2$, which can be numerically acceptable.

Then we move to the Littlest Higgs model [2] and finally to a Little Higgs model with custodial symmetry [21].

3.1 Holographic composite Higgs model

The Holographic Higgs model [6, 20] is based on a five-dimensional theory in AdS space-time. This space-time, of constant radius of curvature $1/k$, is assumed to be compactified by two 4D boundaries. One boundary is located at $z = L_0$ (where z labels the extra dimension in conformal coordinates) and it is referred as the UV boundary, while the other one is at $z = L_1 \gg L_0$ and it is called the IR boundary. The energy scale $1/L_1$ sets the mass gap of the model (the Kaluza-Klein mass $\sim 1/L_1 \sim 1$ TeV). The bulk gauge symmetry $SO(5) \times U(1)_X \times SU(3)$ is broken down to the SM gauge group on the UV boundary and to $O(4) \times U(1)_X \times SU(3)$ on the IR. The hypercharge is defined by $Y = X + T_3^R$ where X is the $U(1)_X$ charge and T_3^R is the 3rd-component isospin of $SU(2)_R \in O(4)$.

We will follow the Holographic approach and separate the 5D gauge fields in UV-boundary fields $A(z = L_0, x)$ and bulk fields $A(z \neq L_0, x)$. This is the correct separation to make contact with the theory defined in section 2; the UV-boundary fields can be associated to the SM gauge bosons, while the bulk states correspond to the new “strong” sector. Let us analyze this new sector. Since the symmetry-breaking pattern of the bulk and IR-boundary is given by $SO(5) \rightarrow O(4)$, we expect four Goldstone bosons parametrized by the $SO(5)/SO(4)$ coset [6]:

$$\Sigma = \langle \Sigma \rangle e^{\Pi/f}, \quad \langle \Sigma \rangle = (0, 0, 0, 0, 1), \quad \Pi = \begin{pmatrix} 0_4 & H \\ -H^T & 0 \end{pmatrix}, \quad (3.13)$$

where H is a real 4-component vector, which transforms as a doublet under the weak $SU(2)$ group and can be associated with the Higgs. Apart from the Goldstones, the bulk contains a massive tower of 4D states, the gauge Kaluza-Klein modes. The mass of the lightest state is given by

$$m_\rho \simeq \frac{3\pi}{4L_1}, \quad \text{and} \quad g_\rho = \frac{m_\rho}{f} \simeq \frac{3\pi}{8} \sqrt{g_5^2 k}, \quad (3.14)$$

where g_5 is the bulk $SO(5)$ gauge coupling. In the particular case where the red-shift factor between the two boundaries L_0/L_1 is used to explain the hierarchy m_W/M_P , we obtain that $g_5^2 k \gtrsim g^2 \ln(L_1/L_0) \sim 16$ implying that the coupling among resonances is always large. As we explained before, a large g_ρ is also needed in these models to guarantee a Higgs mass above the experimental bound. The interaction between the massive Kaluza-Klein states and the fields on the UV-boundary (the SM fields) is due only to mass mixing terms. These terms only respect the SM gauge symmetry and therefore corresponds to an explicit breaking of the $SO(5)$ symmetry. For the fermion sector, we will follow ref. [20] and embed the SM fermions in the $\mathbf{5}$ representation of $SO(5)$. We can again separate each 5D fermion in a UV-boundary field, to be associated to the SM fermion, plus a bulk field. As in eq. (3.6), Yukawa couplings are generated in this model through mass mixing terms between the SM fermions and the heavy fermionic bulk modes. The size of these mixing couplings are determined by the 5D fermion masses and can be chosen to give the correct SM spectrum.

Let us calculate the contribution of this model to the coefficients of the effective operators of eq. (2.15). The coefficients c_T and c_H can be obtained from the kinetic term of the Goldstone bosons:

$$\mathcal{L}_{\text{kin}} = \frac{f^2}{2} (D_\mu \Sigma)(D^\mu \Sigma)^T. \quad (3.15)$$

In the unitary gauge where $\Sigma = (\sin h/f, 0, 0, \cos h/f)$, eq. (3.15) gives

$$\mathcal{L}_{\text{kin}} = \frac{1}{2} \partial_\mu h \partial^\mu h + m_W^2(h) \left[W_\mu W^\mu + \frac{1}{2 \cos^2 \theta_W} Z_\mu Z^\mu \right], \quad (3.16)$$

where

$$m_W(h) = \frac{gf}{2} \sin \frac{h}{f}. \quad (3.17)$$

Eq. (3.16) tells us that $\Delta\rho = 0$ or, equivalently, $c_T = 0$. This is due to the custodial $O(3)$ invariance of eq. (3.15). The value of c_H can be unambiguously computed by comparing the hWW coupling for canonical fields in eq. (3.16) (neglecting m_W^2/m_ρ^2 corrections) with the same quantity deduced from our general effective Lagrangian eq. (2.15). This second step, which requires writing eq. (2.15) in the same field basis as eq. (3.16), is performed in appendix B. From eq. (3.16) we have

$$\frac{g_{hWW}}{gm_W} = \frac{1}{gm_W(h)} \left. \frac{\partial m_W^2(h)}{\partial h} \right|_{h=\langle h \rangle} = \cos \frac{\langle h \rangle}{f} \simeq 1 - \frac{\xi}{2}, \quad (3.18)$$

and, using eq. (B.3), we find

$$c_H = 1. \quad (3.19)$$

The coefficient c_y can be similarly deduced from the calculation of the hff coupling. For the Holographic Higgs model of ref. [20] we have

$$\mathcal{L}_{\text{yuk}} = -m_f(h) \bar{f} f, \quad m_f(h) = M \sin \frac{2h}{f}, \quad (3.20)$$

where M is a constant. We then obtain

$$\frac{2m_W g_{hff}}{gm_f} = \frac{2M_W(h)}{gm_f(h)} \left. \frac{\partial m_f(h)}{\partial h} \right|_{h=\langle h \rangle} = \frac{2 \sin(\langle h \rangle / f)}{\tan(2\langle h \rangle / f)} \simeq 1 - \frac{3\xi}{2}, \quad (3.21)$$

that, comparing it with eq. (B.4), leads to

$$c_y = 1. \quad (3.22)$$

To obtain the coefficient c_6 we must match the hhh coupling obtained from the Higgs potential to eq. (B.5). In the model of ref. [20] in which the Higgs potential can approximately be written as $V(h) \simeq \sin^2 h/f [\alpha - \beta \cos^2 h/f]$ (where α and β are constants), we have

$$\frac{4m_W g_{hhh}}{gm_H^2} = \frac{4m_W(h)}{g \partial_h^2 V(h)} \left. \frac{1}{6} \frac{\partial^3 V(h)}{\partial h^3} \right|_{h=\langle h \rangle} = \frac{1 - 2 \sin^2(\langle h \rangle / f)}{\cos(\langle h \rangle / f)} \simeq 1 - \frac{3\xi}{2}. \quad (3.23)$$

From eq. (3.19), (3.23) and (B.5) we get

$$c_6 = 0. \quad (3.24)$$

In the model of ref. [6], in which the SM fermions are embedded in the spinorial representation of $SO(5)$, we find $c_y = 0$ and $c_6 = 1$.

The coefficients $c_{W,B}$, c_{2W} and c_{2B} can be obtained respectively from the parameters \widehat{S} , W and Y . The parameter \widehat{S} , at tree level, is given by [6]

$$\widehat{S} = \frac{3g^2\xi}{8g_5^2} \simeq \frac{27\pi^2}{128} \frac{m_W^2}{m_\rho^2}. \quad (3.25)$$

Using eq. (2.18) and eq. (3.25) together with the fact that the $O(4)$ symmetry of the model implies $c_W = c_B$, we obtain

$$c_W = c_B = \frac{27\pi^2}{256} \simeq 1.0. \quad (3.26)$$

For the parameters W and Y we obtain

$$W = \frac{g^2}{4g_5^2} m_W^2 L_1^2, \quad Y = \frac{g'^2}{4} \left(\frac{1}{g_5^2} + \frac{1}{g_{5X}^2} \right) m_W^2 L_1^2, \quad (3.27)$$

where g_{5X} is the bulk $U(1)_X$ gauge coupling. Using eq. (2.21), we get

$$c_{2W} \simeq \left(\frac{9\pi^2}{64} \right)^2 \simeq 1.9, \quad c_{2B} = c_{2W} (1 + r^2), \quad (3.28)$$

where $r = g_5/g_{5X}$. The coefficients $c_{HW,HB}$ and $c_{\gamma,g}$ will not be presented here. They are generated at the one-loop level and are therefore very sensitive to the details of the 5D model. Similarly, the non-universal contribution to c_y , although generated at tree-level, depends on the particular structure of the top sector and will be discussed in section 5.

Although the calculations for $c_{W,B}$ and $c_{2W,2B}$ (and therefore \widehat{S} , W and Y) are only valid for $g_\rho < 4\pi$, the predictions for c_H , c_T , c_y and c_6 at leading order can be trusted even in the non-perturbative regime. The coefficients c_H and c_T are, as we said, completely determined by the symmetry breaking pattern of the model, while c_y and c_6 depend only on the way we embed the SM fermions into the $SO(5)$ group. These coefficients are therefore independent of the five-dimensional dynamics.

3.2 Littlest Higgs model

The Littlest Higgs model [2] is based on a global $SU(5)$ symmetry and we consider the version where only a $SU(2)_L \times SU(2)_R \times U(1)_Y$ subgroup of $SU(5)$ is gauged (g_L , g_R and g' are the respective gauge couplings). As we are going to show, below the scale of the heavy new particles, this model can be described by our effective Lagrangian with a $SU(3)/SU(2)$ coset structure.

It is assumed that a UV dynamics breaks the global $SU(5)$ symmetry down to $SO(5)$. This breaking is conveniently parametrized in terms of a $SU(5)$ symmetric representation acquiring a vev of the form

$$\langle \Sigma \rangle = \begin{pmatrix} & & \mathbf{1}_2 \\ & 1 & \\ \mathbf{1}_2 & & \end{pmatrix}. \quad (3.29)$$

Among the 14 Goldstone bosons, 3 are eaten in the breaking of the gauge symmetry $SU(2)_L \times SU(2)_R$ down to the diagonal subgroup $SU(2)$ and we are left with a charged doublet $H_{1/2}$, a charged triplet ϕ_1 and a neutral singlet s_0 (the subscripts denote the $U(1)_Y$ charges of the fields). These Goldstones are parametrized by

$$\Sigma = e^{i\Pi/f} \langle \Sigma \rangle e^{i\Pi^T/f} = e^{2i\Pi/f} \langle \Sigma \rangle \quad \text{with} \quad 2\sqrt{2}i\Pi = \begin{pmatrix} is & \tilde{H} & \phi \\ -\tilde{H}^\dagger & -4is & \tilde{H}^T \\ -\phi^* & -\tilde{H}^* & is \end{pmatrix}. \quad (3.30)$$

ϕ is a 2×2 symmetric complex matrix, $\tilde{H} = i\sigma^2 H^*$ and f is the decay constant of the coset model. The interactions of the Goldstones originate from the kinetic term of the Σ field

$$\mathcal{L} = \frac{f^2}{2} \text{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma, \quad (3.31)$$

where the covariant derivative accommodates the gauging of $SU(2)_L \times SU(2)_R \times U(1)_Y$

$$D_\mu \Sigma = \partial_\mu \Sigma - ig_L (A_\mu^L \Sigma + \Sigma A_\mu^{L^T}) - ig_R (A_\mu^R \Sigma + \Sigma A_\mu^{R^T}) - ig' B_\mu (Y \Sigma + \Sigma Y), \quad (3.32)$$

with

$$A_\mu^L = A_\mu^{aL} \begin{pmatrix} \sigma^a/2 \\ \\ \\ \end{pmatrix}, \quad A_\mu^R = A_\mu^{aR} \begin{pmatrix} \\ \\ \\ -\sigma^{a^*}/2 \end{pmatrix}, \quad (3.33)$$

$$Y = \text{diag} \left(-\frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2} \right). \quad (3.34)$$

The $\langle \Sigma \rangle$ vev gives a mass to the axial part of $SU(2)_L \times SU(2)_R$, $m_{W_H}^2 = (g_L^2 + g_R^2) f^2$. While the gauge coupling of the unbroken vectorial $SU(2)$ takes its usual expression $1/g^2 = 1/g_L^2 + 1/g_R^2$. By the construction of the model, when the $SU(2)_L$ gauge coupling is turned off, the gauging of $SU(2)_R$ respects a $SU(3)$ global symmetry and the doublet and the singlet are exact Goldstone bosons while the charged triplet acquires a mass of order $g_R f$. Below the scale $g_R f$, the Littlest Higgs model has the structure described in section 2 of a $SU(3)/SU(2)$ σ -model weakly coupled to a $SU(2)_L \times U(1)_Y$ gauge sector. According to our description we should then identify $g_\rho \equiv g_R$ and the mass of the axial vector is m_ρ .

After integrating out A_R , as it is explained formally in appendix A, as well as the charged scalar triplet,¹³ the low-energy effective Lagrangian can be mapped onto the $SU(3)/SU(2)$ Lagrangian

$$2f^2 \text{Tr} [\partial_\mu \Sigma_3^\dagger \partial^\mu \Sigma_3] + \frac{1}{2} f^2 \text{Tr} |\Sigma_3^\dagger \partial_\mu \Sigma_3|^2, \quad (3.35)$$

where Σ_3 parametrizes the $SU(3)/SU(2)$ coset. This totally fixes the relative coefficient between the two independent invariants that exist in $SU(3)/SU(2)$ due to the fact that this coset decomposes into 2 irreducible representations of $SU(2)$.

¹³At the $1/f^2$ order, the integration of the scalar triplet is equivalent to the constraint $\phi^* = \frac{\tilde{H} \tilde{H}^T}{\sqrt{2}f}$.

The coefficients c_i of the corresponding SILH Lagrangian can be computed along the lines outlined in the previous subsection. The oblique parameters are found to be, in the limit $g_\rho \gg g$,

$$\widehat{S} = \frac{m_W^2}{2m_\rho^2}, \quad \widehat{T} = -\frac{m_W^2}{4m_\rho^2}, \quad W = \frac{g_L^2}{g_R^2} \frac{m_W^2}{m_\rho^2}, \quad Y = \mathcal{O}(v^6/f^6). \quad (3.36)$$

From this and eqs. (2.16), (2.18), (2.21) we deduce that

$$c_W = \frac{1}{2}, \quad c_B = 0, \quad c_T = -\frac{1}{16}, \quad c_{2W} = 1, \quad c_{2B} = 0, \quad (3.37)$$

where we have taken $c_B = 0$, since the only gauge fields integrated out form an adjoint of SU(2). The value of c_H can be easily deduced from the hWW coupling at zero momentum:

$$g_{hWW} = gm_W \left(1 - \frac{\xi}{8}\right), \quad (3.38)$$

that, together with eq. (B.3), leads to

$$c_H = \frac{1}{4}. \quad (3.39)$$

Notice that the low-energy σ -model, SU(3)/SU(2), breaks custodial symmetry and hence we obtain a non-vanishing c_T coefficient. The source of this custodial breaking is the vev of the triplet ϕ . It can always be fine-tuned away, for instance by taking $g_R \sim g_L$. In that case

$$c_T = 0, \quad c_H = \frac{5}{16}, \quad (3.40)$$

while the other coefficients are unaffected. However, the exact σ -model structure below the scale m_ρ is now lost, since the corrections of order $g_{SM}/g_\rho \sim g_L/g_R$ are important.

3.3 Little Higgs model with custodial symmetry

The littlest Higgs model with custodial symmetry [21] is based on the coset SO(9)/(SO(5) × SO(4)) with an SU(2)_L × SU(2)_R × SU(2) × U(1) subgroup gauged (g_L, g_R, g_2 and g_1 are the respective gauge couplings). This Little Higgs model will be described, below the mass of the new resonances, by a SILH Lagrangian with a SO(5)/SO(4) structure.

The global symmetry breaking of this Little Higgs model is conveniently parametrized by a symmetric representation of SO(9) taking a vev of the form

$$\langle \Sigma \rangle = \begin{pmatrix} & & & \mathbf{1}_4 \\ & & & \\ & & 1 & \\ & & & \\ \mathbf{1}_4 & & & \end{pmatrix}. \quad (3.41)$$

Among the 20 Goldstone bosons, 6 are eaten in the gauge symmetry breakings SU(2)_L × SU(2) → SU(2)_W and SU(2)_R × U(1) → U(1)_Y and we are left with a charged doublet $H_{1/2}$, a neutral triplet ϕ_0 , a charged triplet ϕ_1 and a neutral singlet s_0 (the subscripts

denote the $U(1)_Y$ charges of the fields). A convenient parametrization of these Goldstones is

$$\Sigma = e^{i\Pi/f} \langle \Sigma \rangle e^{i\Pi^T/f} = e^{2i\Pi/f} \langle \Sigma \rangle \quad \text{with} \quad 2i\Pi = \begin{pmatrix} 0_4 & H/\sqrt{2} & -\phi/2 \\ -H^T/\sqrt{2} & 0 & H^T/\sqrt{2} \\ \phi/2 & -H/\sqrt{2} & 0_4 \end{pmatrix}. \quad (3.42)$$

H is the real 4-component vector corresponding to the Goldstone doublet, while ϕ is a real 4×4 symmetric matrix and it contains the singlet and the triplets. The kinetic term of the Σ field generates the interactions among the Goldstones

$$\mathcal{L} = \frac{f^2}{4} \text{Tr} D_\mu \Sigma^\dagger D^\mu \Sigma. \quad (3.43)$$

Under the unbroken $SU(2)_W \times U(1)_Y$, the heavy vector fields transform as a neutral triplet, a neutral singlet and a charged singlet whose masses are

$$m_{W_H}^2 = (g_2^2 + g_L^2) f^2, \quad m_{B_H}^2 = (g_1^2 + g_R^2) f^2, \quad m_{A_H^\pm}^2 = g_R^2 f^2. \quad (3.44)$$

The gauge couplings of the unbroken gauge symmetries are given by the usual formulae

$$\frac{1}{g^2} = \frac{1}{g_L^2} + \frac{1}{g_2^2}, \quad \frac{1}{g'^2} = \frac{1}{g_R^2} + \frac{1}{g_1^2}. \quad (3.45)$$

By the construction of the model, when the $SU(2)$ and $U(1)$ gauge couplings are turned off, the gauging of $SU(2)_L$ and $SU(2)_R$ respects a $SO(5)$ global symmetry whose breaking to $SO(4)$ leaves the H doublet as exact Goldstone bosons while the triplets and the singlet acquire a mass of order $m_\rho \equiv g_\rho f$ (we have considered $g_L = g_R \equiv g_\rho$ for concreteness). Hence, the $SO(5)/SO(4)$ σ -model structure below the scale m_ρ , obtained after integrating out the $SU(2)_L \times SU(2)_R$ gauge fields that do not couple to fermions and integrating out as well the heavy triplet and scalar, which amounts to the constraint

$$\phi = \frac{HH^T}{2f}. \quad (3.46)$$

The oblique corrections are found to be

$$\hat{S} = \frac{m_W^2}{m_\rho^2}, \quad \hat{T} = 0, \quad W = \frac{g^2 m_W^2}{g_\rho^2 m_\rho^2}, \quad Y = \frac{g'^2 m_W^2}{g_\rho^2 m_\rho^2}, \quad (3.47)$$

which allow us to identify the coefficients of the effective Lagrangian

$$c_W + c_B = 1, \quad c_T = 0, \quad c_{2W} = 1 \quad \text{and} \quad c_{2B} = 1. \quad (3.48)$$

The value of c_H can be computed, exactly as before, by looking at the hWW coupling at $p^2 = 0$. We obtain

$$c_H = \frac{1}{2}. \quad (3.49)$$

The factor 2 of disagreement with the value, eq. (3.19), of c_H computed in the Holographic Higgs model simply fixes the relative normalization of the decay constants in the two models.

4. Phenomenology of SILH

In this section we analyze the effects of the SILH interactions and study how they can be tested at future colliders. Let us start by considering the new interaction terms involving the physical Higgs boson. For simplicity, we work in the unitary gauge and write the SILH effective Lagrangian in eq. (2.15) only for the real Higgs field h (shifted such that $\langle h \rangle = 0$). We reabsorb the contributions from c_f and c_6 to the SM input parameters (fermion masses m_f , Higgs mass m_H , and vacuum expectation value $v = 246$ GeV). Similarly, we redefine the gauge fields and the gauge coupling constants and we make the gauge kinetic terms canonical. In this way, the SILH effective Lagrangian is composed by the usual SM part (\mathcal{L}_{SM}), written in terms of the usual SM input parameters (physical masses and gauge couplings), by new Higgs interactions (\mathcal{L}_h), and new interactions involving only gauge bosons (\mathcal{L}_V) which, at leading order, are given by

$$\begin{aligned} \mathcal{L}_h = \xi \left\{ \frac{c_H}{2} \left(1 + \frac{h}{v} \right)^2 \partial^\mu h \partial_\mu h - c_6 \frac{m_H^2}{2v^2} \left(v h^3 + \frac{3h^4}{2} + \dots \right) + c_y \frac{m_f}{v} \bar{f} f \left(h + \frac{3h^2}{2v} + \dots \right) \right. \\ \left. + \left(\frac{h}{v} + \frac{h^2}{2v^2} \right) \left[\frac{g^2}{2g_\rho^2} (\hat{c}_W W_\mu^- \mathcal{D}^{\mu\nu} W_\nu^+ + \text{h.c.}) + \frac{g^2}{2g_\rho^2} Z_\mu \mathcal{D}^{\mu\nu} \left[\hat{c}_Z Z_\nu + \left(\frac{2\hat{c}_W}{\sin 2\theta_W} - \frac{\hat{c}_Z}{\tan \theta_W} \right) A_\nu \right] \right. \right. \\ \left. \left. - \frac{g^2}{(4\pi)^2} \left(\frac{c_{HW}}{2} W^{+\mu\nu} W_{\mu\nu}^- + \frac{c_{HW} + \tan^2 \theta_W c_{HB}}{4} Z^{\mu\nu} Z_{\mu\nu} - 2 \sin^2 \theta_W c_{\gamma Z} F^{\mu\nu} Z_{\mu\nu} \right) + \dots \right. \right. \\ \left. \left. + \frac{\alpha g^2 c_\gamma}{4\pi g_\rho^2} F^{\mu\nu} F_{\mu\nu} + \frac{\alpha_s y_t^2 c_g}{4\pi g_\rho^2} G^{a\mu\nu} G_{\mu\nu}^a \right] \right\} \end{aligned} \quad (4.1)$$

$$\hat{c}_W = c_W + \left(\frac{g_\rho}{4\pi} \right)^2 c_{HW} \quad (4.2)$$

$$\hat{c}_Z = \hat{c}_W + \tan^2 \theta_W \left[c_B + \left(\frac{g_\rho}{4\pi} \right)^2 c_{HB} \right] \quad (4.3)$$

$$c_{\gamma Z} = \frac{c_{HB} - c_{HW}}{4 \sin 2\theta_W} \quad (4.4)$$

$$\begin{aligned} \mathcal{L}_V = -\frac{\tan \theta_W}{2} \hat{S} W_{\mu\nu}^{(3)} B^{\mu\nu} - ig \cos \theta_W g_1^Z Z^\mu (W^{+\nu} W_{\mu\nu}^- - W^{-\nu} W_\mu^+) \\ - ig (\cos \theta_W \kappa_Z Z^{\mu\nu} + \sin \theta_W \kappa_\gamma A^{\mu\nu}) W_\mu^+ W_\nu^- \end{aligned} \quad (4.5)$$

$$\hat{S} = \frac{m_W^2}{m_\rho^2} (c_W + c_B), \quad g_1^Z = \frac{m_Z^2}{m_\rho^2} \hat{c}_W \quad (4.6)$$

$$\kappa_\gamma = \frac{m_W^2}{m_\rho^2} \left(\frac{g_\rho}{4\pi} \right)^2 (c_{HW} + c_{HB}), \quad \kappa_Z = g_1^Z - \tan^2 \theta_W \kappa_\gamma. \quad (4.7)$$

In \mathcal{L}_V we have included only trilinear terms in gauge bosons and dropped the effects of O_{2W} , O_{2B} , O_{3W} . In \mathcal{L}_h we have kept only the first powers in the Higgs field h and the gauge fields. We have defined $W_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm - \partial_\nu W_\mu^\pm$ (and similarly for the Z_μ and the photon A_μ) and $\mathcal{D}_{\mu\nu} = \partial_\mu \partial_\nu - \square g_{\mu\nu}$. Notice that for on-shell gauge bosons $\mathcal{D}_{\mu\nu} A^{\mu i} = M_i^2 A_\nu^i$. Therefore \hat{c}_W and \hat{c}_B generate a Higgs coupling to gauge bosons which is proportional to mass, as in the SM, and do not generate any Higgs coupling to photons. Notice also that the corrections to trilinear vector boson vertices satisfy the relation $g_1^Z = k_Z + \tan^2 \theta_W k_\gamma$ [23].

The new interactions in \mathcal{L}_h , see eq. (4.1), modify the SM predictions for Higgs production and decay. At quadratic order in h , the coefficient c_H generates an extra contribution to the Higgs kinetic term. This can be reabsorbed by redefining the Higgs field according to $h \rightarrow h/\sqrt{1 + \xi c_H}$ (see appendix B for an alternative redefinition of the Higgs that removes the derivative terms of the Higgs — first term of eq. (4.1)). The effect of c_H is then to renormalize by a factor $1 - \xi c_H/2$, with respect to their SM value, the couplings of the canonical field h to all other fields. Notice that the Higgs field redefinition also shifts the value of m_H (but not of v or m_f).

We can express the modified Higgs couplings in terms of the decay widths in units of the SM prediction, expressed in terms of physical pole masses (for a review of the Higgs properties in the SM, see [22]),

$$\Gamma(h \rightarrow f\bar{f})_{\text{SILH}} = \Gamma(h \rightarrow f\bar{f})_{\text{SM}} [1 - \xi(2c_y + c_H)] \quad (4.8)$$

$$\Gamma(h \rightarrow W^+W^-)_{\text{SILH}} = \Gamma(h \rightarrow W^+W^{(*)-})_{\text{SM}} \left[1 - \xi \left(c_H - \frac{g^2}{g_\rho^2} \hat{c}_W \right) \right] \quad (4.9)$$

$$\Gamma(h \rightarrow ZZ)_{\text{SILH}} = \Gamma(h \rightarrow ZZ^{(*)})_{\text{SM}} \left[1 - \xi \left(c_H - \frac{g^2}{g_\rho^2} \hat{c}_Z \right) \right] \quad (4.10)$$

$$\Gamma(h \rightarrow gg)_{\text{SILH}} = \Gamma(h \rightarrow gg)_{\text{SM}} \left[1 - \xi \text{Re} \left(2c_y + c_H + \frac{4y_t^2 c_g}{g_\rho^2 I_g} \right) \right] \quad (4.11)$$

$$\Gamma(h \rightarrow \gamma\gamma)_{\text{SILH}} = \Gamma(h \rightarrow \gamma\gamma)_{\text{SM}} \left[1 - \xi \text{Re} \left(\frac{2c_y + c_H}{1 + J_\gamma/I_\gamma} + \frac{c_H - \frac{g^2}{g_\rho^2} \hat{c}_W}{1 + I_\gamma/J_\gamma} + \frac{\frac{4g^2}{g_\rho^2} c_\gamma}{I_\gamma + J_\gamma} \right) \right] \quad (4.12)$$

$$\Gamma(h \rightarrow \gamma Z)_{\text{SILH}} = \Gamma(h \rightarrow \gamma Z)_{\text{SM}} \left[1 - \xi \text{Re} \left(\frac{2c_y + c_H}{1 + J_Z/I_Z} + \frac{c_H - \frac{g^2}{g_\rho^2} \hat{c}_W}{1 + I_Z/J_Z} + \frac{4c_{\gamma Z}}{I_Z + J_Z} \right) \right]. \quad (4.13)$$

Here we have neglected in $\Gamma(h \rightarrow W^+W^-, ZZ)_{\text{SILH}}$ the subleading effects from c_{HW} and c_{HB} , which are parametrically smaller than a SM one-loop contribution. The loop functions I and J are given in appendix C.

The leading effects on Higgs physics, relative to the SM, come from the three coefficients c_H , c_y , $c_{\gamma Z}$, although $c_{\gamma Z}$ has less phenomenological relevance since it affects only the decay $h \rightarrow \gamma Z$. The rules of SILH select the operators proportional to c_H and c_y as the most important ones for LHC studies, as opposed to totally model-independent operator analyses [24–26] which often lead to the conclusion that the dominant effects should appear in the vertices $h\gamma\gamma$ and hgg , since their SM contribution occurs only at loop level. Therefore, we believe that an important experimental task to understand the nature of the Higgs boson will be the extraction of c_H and c_y from precise measurements of the Higgs production rate (σ_h) and branching ratios (BR_h). The contribution from c_H is universal for all Higgs couplings and therefore it does not affect the Higgs branching ratios, but only the total decay width and the production cross section. The measure of the Higgs decay width at the LHC is very difficult and it can be reasonably done only for rather heavy Higgs bosons, well above the two gauge boson threshold, while the spirit of our analysis is to consider the Higgs as a pseudo-Goldstone boson, and therefore relatively light. However, for a light Higgs, LHC experiments can measure the product $\sigma_h \times BR_h$ in many different

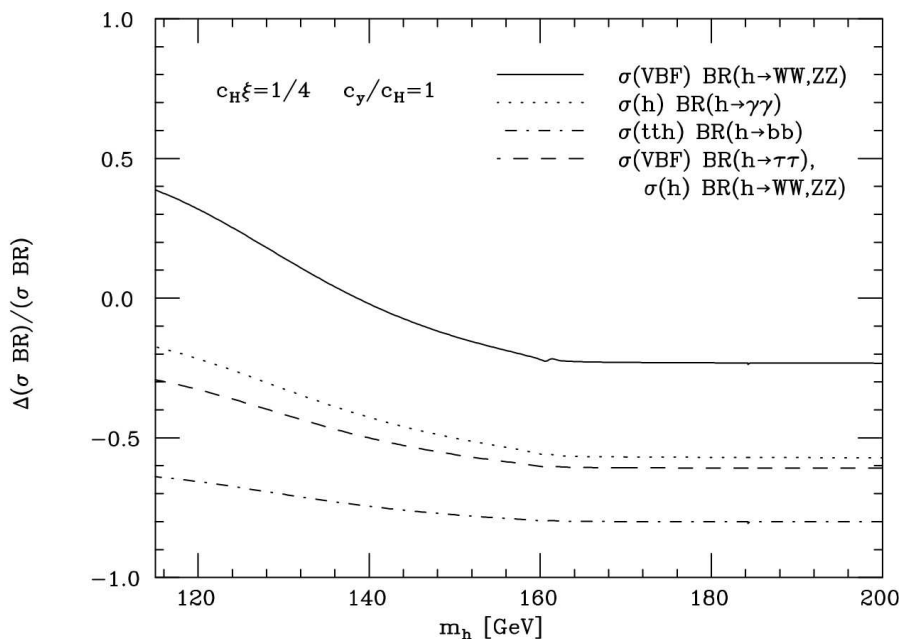


Figure 1: The deviations from the SM predictions of Higgs production cross sections (σ) and decay branching ratios (BR) defined as $\Delta(\sigma BR)/(\sigma BR) = (\sigma BR)_{\text{SILH}}/(\sigma BR)_{\text{SM}} - 1$. The predictions are shown for some of the main Higgs discovery channels at the LHC with production via vector-boson fusion (VBF), gluon fusion (h), and topstrahlung (tth). The SILH Lagrangian parameters are set by $c_H\xi = 1/4$, $c_y/c_H = 1$ and we have included also the terms quadratic in ξ , not explicitly shown in eqs. (4.8)–(4.13).

channels: production through gluon, gauge-boson fusion, and top-strahlung; decay into b , τ , γ and (virtual) weak gauge bosons. At the LHC with about 300 fb^{-1} , it is possible to measure Higgs production rate times branching ratio in the various channels with 20–40 % precision [27], although a determination of the b coupling is quite challenging [28]. This will translate into a sensitivity on $|c_H\xi|$ and $|c_y\xi|$ up to 0.2–0.4.

In figure 1, we show our prediction for the relative deviation from the SM expectation in the main channels for Higgs discovery at the LHC, in the case $c_H\xi = 1/4$ and $c_y/c_H = 1$ (as in the Holographic Higgs). For $c_y/c_H = 0$, the deviation is universal in every production channel and is given by $\Delta(\sigma BR)/(\sigma BR) = -c_H\xi$.

Cleaner experimental information can be extracted from ratios between the rates of processes with the same Higgs production mechanism, but different decay modes. In measurements of these ratios of decay rates, many systematic uncertainties drop out. Our leading-order ($g_\rho \gg g_{SM}$) prediction is that $\Delta[\Gamma(h \rightarrow ZZ)/\Gamma(h \rightarrow W^+W^-)] = 0$, $\Delta[\Gamma(h \rightarrow f\bar{f})/\Gamma(h \rightarrow W^+W^-)] = -2\xi c_y$, $\Delta[\Gamma(h \rightarrow \gamma\gamma)/\Gamma(h \rightarrow W^+W^-)] = -2\xi c_y(1 + J_\gamma/I_\gamma)^{-1}$. However, the Higgs coupling determinations at the LHC will still be limited by statistics, and therefore they can benefit from a luminosity upgrading, like the SLHC. At a linear collider, like the ILC, precisions on $\sigma_h \times BR_h$ can reach the percent level [29], providing a very sensitive probe on the new-physics scale. Moreover, a linear collider can test the

existence of c_6 , since the triple Higgs coupling can be measured with an accuracy of about 10% for $\sqrt{s} = 500$ GeV and an integrated luminosity of 1000 fb^{-1} [30].

Deviations from the SM predictions of Higgs production and decay rates, could be a hint towards models with strong dynamics, especially if no new light particles are discovered at the LHC. However, they do not unambiguously imply the existence of a new strong interaction. The most characteristic signals of a SILH have to be found in the very high-energy regime. Indeed, a peculiarity of SILH is that, in spite of the light Higgs, longitudinal gauge-boson scattering amplitudes grow with energy and the corresponding interaction becomes strong, eventually violating tree-level unitarity at the cutoff scale. Indeed, the extra Higgs kinetic term proportional to $c_H \xi$ in eq. (4.1) prevents Higgs exchange diagrams from accomplishing the exact cancellation, present in the SM, of the terms growing with energy in the amplitudes. Therefore, although the Higgs is light, we obtain strong WW scattering at high energies.

From the operator $O_H \equiv \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H)$ in eq. (2.15), using the equivalence theorem [31], it is easy to derive the following high-energy limit of the scattering amplitudes for longitudinal gauge bosons

$$\mathcal{A}(Z_L^0 Z_L^0 \rightarrow W_L^+ W_L^-) = \mathcal{A}(W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0) = -\mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) = \frac{c_H s}{f^2}, \quad (4.14)$$

$$\mathcal{A}(W^\pm Z_L^0 \rightarrow W^\pm Z_L^0) = \frac{c_H t}{f^2}, \quad \mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{c_H (s+t)}{f^2}, \quad (4.15)$$

$$\mathcal{A}(Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0) = 0. \quad (4.16)$$

This result is correct to leading order in s/f^2 , and to all orders in ξ in the limit $g_{SM} = 0$, when the σ -model is exact. The absence of corrections in ξ follows from the non-linear symmetry of the σ -model, corresponding to the action of the generator T_h , associated with the neutral Higgs, under which v shifts. Therefore we expect that corrections can arise only at $\mathcal{O}(s/m_\rho^2)$. The growth with energy of the amplitudes in eqs. (4.14)–(4.16) is strictly valid only up to the maximum energy of our effective theory, namely m_ρ . The behaviour above m_ρ depends on the specific model realization. In the case of the Little Higgs, we expect that the amplitudes continue to grow with s up to the cut-off scale Λ . In 5D models, like the Holographic Goldstone, the growth of the elastic amplitude is softened by KK exchange, but the inelastic channel dominate and strong coupling is reached at a scale $\sim 4\pi m_\rho/g_\rho$. Notice that the result in eqs. (4.14)–(4.16) is exactly proportional to the scattering amplitudes obtained in a Higgsless SM [31]. Therefore, in theories with a SILH, the cross section at the LHC for producing longitudinal gauge bosons with large invariant masses can be written as

$$\sigma(pp \rightarrow V_L V_L' X)_{c_H} = (c_H \xi)^2 \sigma(pp \rightarrow V_L V_L' X)_H, \quad (4.17)$$

where $\sigma(pp \rightarrow V_L V_L' X)_H$ is the cross section in the SM without Higgs, at the leading order in $s/(4\pi v)^2$. With about 200 fb^{-1} of integrated luminosity, it should be possible to identify the signal of a Higgsless SM with about 30–50% accuracy [32–34]. This corresponds to a sensitivity up to $c_H \xi \simeq 0.5\text{--}0.7$.

In the SILH framework, the Higgs is viewed as a pseudo-Goldstone boson and therefore its properties are directly related to those of the exact (eaten) Goldstones, corresponding to the longitudinal gauge bosons. Thus, a generic prediction of SILH is that the strong gauge boson scattering is accompanied by strong production of Higgs pairs. Indeed we find that, as a consequence of the $O(4)$ symmetry of the H multiplet, the amplitudes for Higgs pair-production grow with the center-of-mass energy as eq. (4.14),

$$\mathcal{A}(Z_L^0 Z_L^0 \rightarrow hh) = \mathcal{A}(W_L^+ W_L^- \rightarrow hh) = \frac{c_H s}{f^2}. \quad (4.18)$$

Notice that scattering amplitudes involving longitudinal gauge bosons and a single Higgs vanish. This is a consequence of the Z_2^4 parity embedded in the $O(4)$ symmetry of the operator O_H , under which each Goldstone change sign. Non-vanishing amplitudes necessarily involve an even number of each species of Goldstones.

Using eqs. (4.14), (4.15) and (4.18), we can relate the Higgs pair production rate at the LHC to the longitudinal gauge boson cross sections

$$2\sigma_{\delta,M}(pp \rightarrow hhX)_{c_H} = \sigma_{\delta,M}(pp \rightarrow W_L^+ W_L^- X)_{c_H} + \frac{1}{6} \left(9 - \tanh^2 \frac{\delta}{2} \right) \sigma_{\delta,M}(pp \rightarrow Z_L^0 Z_L^0 X)_{c_H}. \quad (4.19)$$

Here all cross sections $\sigma_{\delta,M}$ are computed with a cut on the pseudorapidity separation between the two final-state particles (a boost-invariant quantity) of $|\Delta\eta| < \delta$, and with a cut on the two-particle invariant mass $\hat{s} > M^2$. The sum rule in eq. (4.19) is a characteristic of SILH. However, the signal from Higgs-pair production at the LHC is not so prominent. It was suggested that, for a light Higgs, this process is best studied in the channel $b\bar{b}\gamma\gamma$ [35], but the small branching ratio of $h \rightarrow \gamma\gamma$ makes the SILH rate unobservable. However, in SILH, one can take advantage of the growth of the cross section with energy. Although we do not perform here a detailed study, it may be possible that, with sufficient luminosity, the signal of $b\bar{b}b\bar{b}$ with high invariant masses could be distinguished from the SM background. Notice however that, because of the high boost of the Higgs boson, the b jets are often not well separated. The case in which the Higgs decays to two real W 's appears more promising for detection. The cleanest channel is the one with two like-sign leptons, where $hh \rightarrow \ell^\pm \ell^\pm \nu\nu$ jets, studied in refs. [35, 25].

The operator O_H is purely generated by the strongly-interacting sector, as indicated by its $\sim g_\rho^2/m_\rho^2$ coefficient. Also c_y and c_6 , even though they arise from the interplay between weak and strong couplings, are sensitive to g_ρ^2/m_ρ^2 and thus indirectly test the non-linearity of the Higgs sector. Therefore probing the effects of these couplings is crucial for testing SILH. The operators $O_{W,B}$, O_{BB} and O_g , on the other hand, only depend on the scale of new physics, not on its strength, as indicated by their $\propto 1/m_\rho^2$ coefficient. Because of this fact, in the strong coupling limit $g_\rho \gg g_{SM}$, their effects in Higgs decay rates are subleading with respect to those induced by c_H and c_y . Indeed the contribution of $O_{W,B}$, O_{BB} and O_g to amplitudes scales like s/m_ρ^2 times the SM contribution. While at electroweak energies this effect is very small, it can become a sizeable at higher energies.

As an example, the operator O_g contributes to processes like $gg \rightarrow hh, Z_L^0 Z_L^0, W_L^+ W_L^-$ with scattering amplitudes $\mathcal{A}(gg \rightarrow hh)_{\text{SILH}} \simeq \mathcal{A}(gg \rightarrow hh)_{\text{SM}} c_g s/m_\rho^2$ which are, at most,

of the same order of magnitude of the SM one, for the maximal energy $s \simeq m_\rho^2$, where the corresponding resonances can be directly produced. Double-Higgs production at the LHC from O_g was recently studied in ref. [25]. Also the operators $O_{W,B}$ can contribute to high-energy production of Higgs and longitudinal gauge bosons. Indeed, by using the equations of motion for the gauge bosons, these operators can be rewritten as the product of a fermionic current times a bilinear in H and can give new-physics effects in $\bar{q}q' \rightarrow hZ_L^0, hW_L^\pm, Z_L^0W_L^\pm, W_L^\pm W_L^\mp$.

The operators $O_{HW,HB}$, in spite of their overall 1-loop suppression, are sensitive to g_ρ^2/m_ρ^2 , indicating that in principle they probe the strong dynamics. Indeed, they induce corrections to the process $h \rightarrow Z\gamma$ and to the magnetic moment anomaly of the W that are $O(v^2/f^2)$ relative to the SM contribution. In practice, however, these quantities are experimentally not well accessible. Therefore c_{HW} and c_{HB} should be tested in vector boson production, where their contribution relative to the SM is quantitatively similar (indeed even $g_\rho^2/16\pi^2$ smaller) to that of $c_{W,B}$.

The effect of the operators $O_{W,B}, O_{HW,HB}$ can be tested through precise measurements of triple gauge vertices. At the LHC, the anomalous couplings in eq. (4.5) can be probed at the level of $10^{-1}-10^{-2}$ [36], but at a sub-TeV linear collider the precision can be improved up to $10^{-3}-10^{-4}$ [37, 38]. This is highly competitive with the \hat{S} determination at LEP and can probe values of m_ρ up to 6–8 TeV.

As discussed in section 1, the signals studied in this paper are important because they are model-independent tests of a strongly-interacting electroweak-breaking sector in presence of a light Higgs. However, the first evidence for this kind of new physics at the LHC may come from production of the resonances at the mass scale m_ρ . Therefore, it is useful to compare the indirect effects studied here with the direct resonance production. As an illustrative example, we consider the case of new spin-one charged resonances ρ_H^\pm . These particles can be interpreted as bound states analogous to the technirho, in composite models, or as the heavy gauge bosons W_H^\pm in Little-Higgs models. They have mass m_ρ , coupling to the strong sector (Higgs and longitudinal gauge bosons) equal to g_ρ and coupling to the weak sector (quarks, leptons and transverse gauge bosons) equal to g^2/g_ρ . Indeed, in the effective theory below m_ρ , they give rise to the operators $O_{W,B}$ with coefficients of order g^2/m_ρ^2 . The cross section for the resonant production of ρ_H^\pm is

$$\sigma(pp \rightarrow \rho_H^\pm + X) = \frac{\pi g^4}{12g_\rho^2} \left. \frac{\tau d\mathcal{L}}{\hat{s} d\tau} \right|_{\hat{s}=m_\rho^2}, \tag{4.20}$$

where $\tau/\hat{s}d\mathcal{L}/d\tau$ is the parton luminosity at an energy equal to the resonance mass. For $2 \text{ TeV} \lesssim m_\rho \lesssim 4 \text{ TeV}$, we find

$$\sigma(pp \rightarrow \rho_H^\pm + X) = \left(\frac{4\pi}{g_\rho}\right)^2 \left(\frac{3 \text{ TeV}}{m_\rho}\right)^6 0.5 \text{ fb.} \tag{4.21}$$

The ρ_H^\pm branching ratios are

$$BR(\rho_H^- \rightarrow \mu\bar{\nu}) = \frac{1}{3}BR(\rho_H^- \rightarrow b\bar{t}) = \frac{2g^4}{g_\rho^4} \left(1 + \frac{24g^4}{g_\rho^4}\right)^{-1}, \quad (4.22)$$

$$BR(\rho_H^- \rightarrow hW^-) = BR(\rho_H^- \rightarrow Z^0W^-) = \frac{1}{2} \left(1 + \frac{24g^4}{g_\rho^4}\right)^{-1}. \quad (4.23)$$

The resonances are most easily detected when they decay directly into leptons or top quarks. However, as shown in eqs. (4.22)–(4.23), for large g_ρ , these decay modes are suppressed and gauge and Higgs bosons then provide the dominant decay channels (in some specific models the coupling of ρ_H to the top can be larger than g^2/g_ρ — see section 3). Notice that, as g_ρ grows, the experimental identification of the resonance becomes increasingly hard, not only because the leptonic signal is suppressed, but also because the decay width becomes large. Detection of a broad resonance decaying into gauge and Higgs bosons is experimentally challenging and the study of indirect signals becomes more important in the region of large g_ρ .

For order-unity coefficients c_i , we have described the SILH in terms of the two parameters m_ρ and g_ρ . An alternative description can be done in terms of two mass scales. They can be chosen as $4\pi f$, the scale at which the σ -model would become fully strongly-interacting in the absence of new resonances, and m_ρ , the scale at which new states appear. An upper bound on m_ρ is obtained from the theoretical NDA requirement $m_\rho < 4\pi f$, while a lower bound on m_ρ comes from the experimental constraint on the \hat{S} parameter, see eq. (2.18).

Searches at the LHC, and possibly at the ILC, will probe unexplored regions of the $4\pi f$ – m_ρ space. Precise measurements of Higgs production and decay rates at the LHC will be able to explore values of $4\pi f$ up to 5–7 TeV, mostly testing the existence of c_H and c_Y . These measurements can be improved with a luminosity upgrading of the LHC. Higgs-physics studies at a linear collider could reach a sensitivity on $4\pi f$ up to about 30 TeV. Analyses of strong gauge-boson scattering and double-Higgs production at the LHC can be sensitive to values of $4\pi f$ up to about 4 TeV. These studies are complementary to Higgs precision measurements, as they test only the coefficient c_H and probe processes highly characteristic of a strong electroweak-breaking sector with a light Higgs boson.

On the other side, the parameter m_ρ can be probed at colliders by studying pair-production of longitudinal gauge bosons and Higgs, by testing triple gauge vertices or, more directly, by producing the new resonances. For fixed m_ρ , resonance production at the LHC will overwhelm the indirect signal of longitudinal gauge boson and Higgs production, at large $4\pi f$ (small g_ρ). However, at low $4\pi f$ (large g_ρ) resonance searches become less effective in constraining the parameter m_ρ and the indirect signal gains importance. While the search for new resonances is most favorable at the LHC, precise measurements of triple gauge vertices at the ILC can test m_ρ up to 6–8 TeV. With complementary information from collider data, we will explore a large portion of the interesting region of the $4\pi f$ – m_ρ plane, testing the composite nature of the Higgs.

5. Strongly-interacting top quark

In section 3 we have seen that, in some explicit realizations of the SILH, the top quark is required to be strongly coupled to the resonances of the electroweak-breaking sector. Here we want to study, in a model-independent way, the phenomenological implications of this strongly-coupled top quark, much in the same spirit of section 2 for the case of the Higgs boson.

Let us first consider the case in which, in addition to the Higgs, the right-handed top also belongs to the strongly-coupled sector. The low-energy effective Lagrangian can be written by generalizing the rules 1, 2 and 3 of section 2.2, noticing that each t_R leg added to leading interactions carries an extra factor $1/(fm_\rho^{1/2})$. We find three dimension-6 operators suppressed by $1/f^2$ and involving t_R :

$$\frac{c_t y_t}{f^2} H^\dagger H \bar{q}_L \tilde{H} t_R + h.c. + \frac{i c_R}{f^2} H^\dagger D_\mu H \bar{t}_R \gamma^\mu t_R + \frac{c_{4t}}{f^2} (\bar{t}_R \gamma^\mu t_R) (\bar{t}_R \gamma_\mu t_R). \quad (5.1)$$

We are not considering dimension-6 operators suppressed by $1/m_\rho^2$ since their effects are smaller than those in eq. (5.1) for large g_ρ . The first term of eq. (5.1) was already included in eq. (2.15). Nevertheless, here it is only present for the top quark and therefore it violates the universality of c_y . The difference $c_t - c_y$ can be viewed as originating from an insertion of $H^\dagger H/f^2$ on the t_R line. The second term of eq. (5.1) violates the custodial symmetry, and therefore it generates a contribution to \hat{T} at the one-loop level

$$\hat{T} \sim \frac{N_c c_R^2 v^2 \Lambda^2}{16\pi^2 f^4} = 0.02 c_R^2 \left(\frac{\Lambda}{f}\right)^2 \xi, \quad (5.2)$$

where Λ is the scale that cuts off the one-loop momentum divergence. In models in which $\Lambda \sim m_\rho$ the 95% CL bound $\hat{T} \lesssim 0.002$ translates, via eq. (5.2), into a severe upper bound on $c_R^2 \xi$. This bound on c_R can be easily satisfied in models in which the strong sector preserves a custodial symmetry under which t_R transforms as a singlet. This guarantees $c_R = 0$ at tree-level. Another possibility to evade the bound on c_R is to reduce the scale Λ in eq. (5.2). This can be achieved in models in which t_R transforms non-trivially under the custodial group as discussed in section 3. In this case $\Lambda \sim m_{cust}$ where m_{cust} is the mass of the custodial partners of the t_R . Assuming $m_{cust} \ll m_\rho$ we can satisfy the bound from \hat{T} even if $c_R \sim 1$.

Similarly, we can consider the case in which t_L and H are strongly coupled. We have now the following $1/f^2$ dimension-6 operators in the low-energy Lagrangian involving $q_L = (t_L, b_L)$:

$$\begin{aligned} & \frac{c_q y_b}{f^2} H^\dagger H \bar{q}_L H b_R + \frac{c_q y_t}{f^2} H^\dagger H \bar{q}_L \tilde{H} t_R + h.c. + \frac{i c_L^{(1)}}{f^2} H^\dagger D_\mu H \bar{q}_L \gamma^\mu q_L \\ & + \frac{i c_L^{(3)}}{f^2} H^\dagger \sigma^i D_\mu H \bar{q}_L \gamma^\mu \sigma^i q_L + \frac{c_{4q}}{f^2} (\bar{q}_L \gamma^\mu q_L) (\bar{q}_L \gamma_\mu q_L). \end{aligned} \quad (5.3)$$

The possibility of having a strongly-coupled q_L has, however, severe constraints from flavor physics due to b_L . For example, the operator proportional to c_{4q} in eq. (5.3) contributes

to Δm_B , the mass difference of neutral B mesons

$$\Delta m_B = \frac{2}{3} \xi c_{4q} m_B \frac{f_B^2}{v^2} \theta_{bd}^2, \quad (5.4)$$

where the angle θ_{bd} parametrizes the projection of b_L into the d mass eigenstate. From the requirement that the new contribution to Δm_B does not exceed 20% of the experimental value, we obtain

$$\xi c_{4q} \left(\frac{\theta_{bd}}{V_{ub}} \right)^2 < 2 \times 10^{-3}. \quad (5.5)$$

Therefore, unless there is a flavor-symmetry reason for having an alignment of b_L with a mass eigenstate more accurate than the corresponding CKM angle V_{ub} , the bound in eq. (5.5) disfavors a strongly-coupled q_L . Thus, we will not further consider this possibility.

Flavor constraints have been studied in detail, e.g. see [39], within the framework of warped extra dimensions and they apply to holographic Higgs models whose low energy effective description is described by our Lagrangian (2.15)

5.1 Phenomenology of a strongly-interacting t_R

The presence of the operators (5.1) gives non-universal modifications to the couplings of the top to the Higgs and gauge bosons. In particular, we find that the couplings $h\bar{t}t$ and $Z\bar{t}_R t_R$ are given by

$$\begin{aligned} g_{htt} &= \frac{gm_t}{2m_W} \left[1 - \xi \left(c_t + c_y + \frac{c_H}{2} \right) \right], \\ g_{Zt_R t_R} &= -\frac{2g \sin^2 \theta_W}{3 \cos \theta_W} \left(1 - \frac{3}{8 \sin^2 \theta_W} c_R \xi \right). \end{aligned} \quad (5.6)$$

At the LHC the coupling g_{htt} can be measured in the process $gg \rightarrow t\bar{t}h$, $h \rightarrow \gamma\gamma$. An accuracy on g_{htt} up to 5% can be reached at a linear collider with $\sqrt{s} = 800$ GeV and $L = 1000$ fb $^{-1}$ [29]. The coupling $Zt_R t_R$ can only be measured with accuracy at future e^+e^- colliders. For $\sqrt{s} = 500$ GeV and $L = 300$ fb $^{-1}$, one can reach a sensitivity up to $\xi c_R \sim 0.04$ [29]. Deviations on the SM vertex $Zt_R t_R$ can also be tested in flavor-violating processes. For example, one-loop penguin diagrams, mediated by the Z , generates the effective operator

$$\tilde{c}_{ij} \bar{d}_L^i \gamma_\mu d_L^j \bar{f} \gamma^\mu (Q_f \sin^2 \theta_W - T_f P_L) f, \quad (5.7)$$

where Q_f and T_f are the electric charge and the third isospin component of the generic fermion f , P_L is the left chiral projector, and

$$\tilde{c}_{ij} = \frac{c_R \xi \alpha G_F V_{ti}^* V_{tj} m_t^2}{4\sqrt{2}\pi \sin^2 \theta_W m_W^2} \ln \frac{m_\rho}{m_t}. \quad (5.8)$$

This operator contributes to many rare $\Delta F = 1$ processes like $B \rightarrow X_s \ell^+ \ell^-$, $B \rightarrow X_s \bar{\nu} \nu$, $B_s \rightarrow \ell^+ \ell^-$, $K^+ \rightarrow \pi^+ \bar{\nu} \nu$, ϵ'/ϵ , etc. The typical experimental sensitivities or theoretical uncertainties of these processes is no better than 10–20%. To estimate the new-physics contribution, it is useful to express \tilde{c}_{ij} in units of the SM contribution

$$\frac{\tilde{c}_{ij}}{\tilde{c}_{ij}^{\text{SM}}} = c_R \xi f(x_{Wt}) \ln \frac{m_\rho}{m_t}, \quad (5.9)$$

where $x_{Wt} = m_W^2/m_t^2$ and

$$f(x_{Wt}) = \frac{(1 - x_{Wt})^2}{2[1 - 7x_{Wt} + 6x_{Wt}^2 - x_{Wt}(3 + 2x_{Wt}) \ln x_{Wt}]} \simeq 0.34. \quad (5.10)$$

This shows that present limits from flavour physics give only a mild constraint on $c_R \xi$ and, with improved experimental accuracy, the coefficient c_R can potentially lead to observable signals. It is also interesting to consider effects of c_R in rare top decays. By defining θ_{ti} as the mixing angle between the current t_R state and the mass eigenstates, we find

$$\Gamma(t \rightarrow cZ) = \frac{m_t}{2\pi} \left(\frac{\xi c_R g \theta_{tc}}{16 \cos \theta_W} \right)^2 \frac{(1 + 2x_{Zt})(1 - x_{Zt})^2}{x_{Zt}}, \quad (5.11)$$

where $x_{Zt} = m_Z^2/m_t^2$. This gives a branching ratio

$$BR(t \rightarrow cZ) = 2 \times 10^{-4} (\xi c_R)^2 \left(\frac{\theta_{tc}}{V_{cb}} \right)^2. \quad (5.12)$$

Since the LHC is expected to reach a sensitivity on $BR(t \rightarrow cZ)$ of 2×10^{-4} with 100 fb^{-1} , a signal from c_R is possible, but requires a mixing angle θ_{tc} larger than the corresponding CKM element V_{cb} .

Let us finally comment on possible implications of the operator proportional to c_{4t} . Analogously to c_H for WW scattering, this operator induces a $t\bar{t}$ scattering that grows with energy. At the LHC this will give an enhancement of the cross-section $pp \rightarrow t\bar{t}t\bar{t}$ where a $t\bar{t}$ pair is produced by the new 4-top interaction. The coefficient c_{4t} gives also contributions to flavor processes. For example it contributes to Δm_D , the mass difference of neutral D mesons:

$$\Delta m_D = \frac{2}{3} \xi c_{4t} m_D \frac{f_D^2}{v^2} \theta_{tc}^2 \theta_{tu}^2 = \xi c_{4t} \left(\frac{\theta_{tc}}{V_{cb}} \right)^2 \left(\frac{\theta_{tu}}{V_{ub}} \right)^2 2 \times 10^{-11} \text{ MeV}. \quad (5.13)$$

For mixing angles of the order of the corresponding CKM elements, this prediction is not far from the present experimental bound, which is $\Delta m_D < 4 \times 10^{-11} \text{ MeV}$.

6. Conclusions

If the weak scale originates from dimensional transmutation in some new strong sector then the physics of the Higgs will manifest important deviations with respect to Standard Model expectations. Technicolor represents the simplest and perhaps most dramatic such possibility: no narrow state can be identified as the Higgs boson. Simple technicolor is however at odds with electroweak precision tests and largely because of the absence of a light Higgs resonance. Models where, in addition to the three eaten Goldstone bosons, a light pseudo-Goldstone Higgs appears in the low-energy theory can fare better in electroweak data for two reasons. On one hand, a light Higgs screens the infrared contribution to \hat{S} . On the other hand, the vacuum dynamics of the pseudo-Goldstone is determined by extra parameters (SM couplings among them) and therefore one can imagine obtaining v^2/f^2 a little bit below 1, which is enough to suppress the UV contribution $\hat{S} \sim (N_{TC} g^2 / 16\pi^2)(v^2/f^2)$

below the experimental bound. While work has been done in the past on the effective low-energy description of Higgsless theories like technicolor, less effort has been devoted to the construction of an effective theory for the pseudo-Goldstone Higgs. This is of course partly justified by the actual existence of specific models which allow for a quantitative description of the resonance sector. Still, we believe that the construction of such an effective theory is an important task, and this has been the primary goal of this paper.

After its proposal [7] and some work in the eighties [8], the idea of a pseudo-Goldstone Higgs was recently revived by its realizations in warped compactifications (Holographic Goldstones) and in Little Higgs models. As we emphasized in this paper, these new models represent weakly-coupled variants of the original QCD-like proposals. For what pertains low-energy phenomenology, we found it useful to characterize these theories in terms of a mass scale m_ρ and a coupling g_ρ . In 5D models, these are respectively the Kaluza-Klein mass and coupling. In Little-Higgs models these are the masses and self-couplings that regulate the quadratic divergence of the Higgs mass. In the limit $g_\rho \rightarrow 4\pi$, these models coincide with a generic strongly-coupled theory [7]. A crucial test of these theories will undoubtedly proceed through the search for new resonances at the LHC. This is certainly more true at small enough g_ρ , where the resonances are narrow. However, these theories also predict important deviations from the SM in the physics of the Higgs. These deviations are associated to non-renormalizable operators in the low-energy description. The study of these indirect effects should nicely complement the direct searches, especially for large g_ρ where the resonances become heavier and broader. Indeed, the leading dimension-6 operators have a coefficient $\sim (g_\rho/m_\rho)^2$, indicating the relevance of these effects even when the resonances are heavy provided g_ρ is large.

Using our simplified description in terms of (g_ρ, m_ρ) , we have derived the form of the leading dimension-6 effective Lagrangian. Our description thus encompasses all models with Goldstone Higgs, although the effective-Lagrangian approach is best motivated in the regime where g_ρ is large. Models based on 5D are favored to be in this regime, while in Little Higgs models there is more freedom (some couplings are favored to be large, others can be weak). Our effective Lagrangian is shown in eq. (2.15). One can distinguish two classes of effects, the “new couplings”, which are genuinely sensitive to the new strong force, and the “form factors” which are basically sensitive to the spectrum. The “new couplings”, described by c_H, c_T, c_6 and c_γ , are determined by an expansion in the Higgs field, and test its strong self interaction, characterized by $g_\rho/m_\rho = 1/f$. The rest of the terms in eq. (2.15), can be basically viewed as higher-derivative dressings of the quadratic (free) Higgs action: as such they do not test equally well the strongly-coupled nature of the Higgs. More precisely, the operators proportional to c_W, c_B, c_γ and c_g represent genuine “form factor” effects since they have a $1/m_\rho^2$ coefficient. Remarkably, the coefficients c_γ and c_g are associated to a suppression factor $\sim (g_{SM}^2/16\pi^2)(1/m_\rho^2)$ that can be seen as arising from the product of a strong loop $1/(16\pi^2 f^2)$ factor times g_{SM}^2/g_ρ^2 . This second factor, which determines the dependence on just $m_\rho = g_\rho f$, and suppresses these effects at large g_ρ , is dictated by the Goldstone symmetry and by its preservation by both the gluon and photon fields. The operators O_{HW} and O_{HB} are special in that they have a structure similar to the form

factors but a coefficient $g_\rho^2/(16\pi^2 m_\rho^2)$ that depends on g_ρ , like for the “new couplings”. In practice, however, they are experimentally less relevant than the “new couplings” at measuring g_ρ/m_ρ . Therefore c_{HW} and c_{HB} should be tested in vector boson production, where their contribution relative to the SM is quantitatively similar (indeed even $g_\rho^2/16\pi^2$ smaller) to that of $c_{W,B}$. In this respect these effects can be practically classified as form factors.

The form factors lead to corrections to SM amplitudes whose relative size scales with energy like E^2/m_ρ^2 . In particular their effects on the on-shell couplings of a light Higgs are of order $m_W^2/m_\rho^2 = (g_{SM}^2/g_\rho^2)v^2/f^2$. On the other hand the effects of the “new couplings” are of order v^2/f^2 and thus dominate for a strongly-coupled Higgs sector with $g_\rho^2 \gg g_{SM}^2$. Our conclusions differ from the widely held expectation that anomalous couplings between Higgs, photons and gluons should be the most important effect, given that they arise at one-loop level in the SM. We do find corrections to all on-shell couplings of the Higgs, including those to photons and gluons, but the origin of these corrections is the Higgs self-interaction (c_H) and Higgs coupling to fermions (c_y) and in particular to the top quark. The measurement of all possible Higgs production and decay channels at the LHC with 300 fb^{-1} should allow a test of these interactions with a sensitivity on v^2/f^2 of order 0.2. The detection of a deviation from the SM in this range of v^2/f^2 , in the absence of new light states, and in particular of additional light scalars, would be an indirect but clear signature of new strong dynamics involving the Higgs. A direct assessment of the strongly-coupled nature of the Higgs would only be obtained by observing the self interactions among the Higgs and the longitudinally polarized vector bosons. The way to study these interactions is the same as in ordinary Higgsless theories: through the scattering of vector bosons that were collinearly radiated from elementary fermions. In the Higgsless case, the scattering amplitude among longitudinal vector boson grows like E^2/v^2 . In our case, the light Higgs fails to fully moderate this growth and the amplitude behaves like E^2/f^2 . As the Higgs plays the role of a fourth Goldstone, in addition to the $V_L V_L \rightarrow V_L V_L$ channels we also have strong double-Higgs production $V_L V_L \rightarrow hh$ with a comparable cross section. It is well known that the study of high-energy scattering among longitudinal vector bosons is not a straightforward task at the LHC. With 300 fb^{-1} , the expected sensitivity on v^2/f^2 is between 0.5 and 0.7. Our study further motivates analyses of WW scattering, even in presence of a light Higgs. An interesting issue which may be studied is the possibility to detect the reaction of double-Higgs production. Given the high energy and p_T of the b -jets from Higgs decay and the presence of a rapidity gap, it may be possible to select these events over the QCD background. Of course the upgraded luminosity of the second phase of LHC would make a crucial difference in these searches.

Through precise measurements of Higgs physics, the ILC will largely improve the sensitivity on v^2/f^2 down to ~ 0.01 , corresponding to $4\pi f$ up to 30 TeV. If, at this level, no deviation from the SM is detected, it will be fair to say that the idea of a composite light Higgs is ruled out. Measurements of the anomalous triple gauge vertices at the ILC can test the value of m_ρ up to 6–8 TeV. This sensitivity is far superior to what has been reached at LEP, through the measurement of \hat{S} ($m_\rho \gtrsim 2 \text{ TeV}$), or what can be achieved at the LHC, through direct resonance production ($m_\rho \sim 3 \text{ TeV}$).

One aspect of theories with a composite Higgs is that the top sector tends to

couple with intermediate strength $\sim \sqrt{g_\rho y_t}$ to the strong sector. It is then natural to consider the possibility that one of the two helicities (t_L or t_R) goes all the way to being composite. Moreover, in most of the realistic constructions to generate Yukawa couplings, a composite t_R also has phenomenological advantages, in that it allows to relax some significant bounds from $\delta\rho$ and the $Z\bar{b}b$ vertex. In section 5, we have therefore extended our effective Lagrangian to the case of a fully composite t_R . Although model dependent, there are important implications in flavor physics. Remarkably, for $f \sim v$ and assuming a mixing pattern that follows the size entries of the CKM matrix, we predict flavor effects ($B \rightarrow X_s \ell^+ \ell^-$, $K^+ \rightarrow \pi^+ \nu \nu$, $t \rightarrow cZ$, Δm_D , ...) possibly within future experimental reach. However, the leading signature of top compositeness is associated to the reaction of four top-quark production. We plan to perform a detailed study of this and other implications of composite Higgs and top at the LHC in a future work.

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A. Integrating out vectors and scalars

Here we describe the low-energy action obtained by integrating out fields of mass m_ρ at tree level. We shall need the standard CCWZ notation [13] to write terms in the Goldstone action. The action of $g \in \mathcal{G}$ on the Goldstone operator Π , defined in eq. (2.4), is given by

$$gU(\Pi) \equiv U(g(\Pi))h(\Pi, g), \tag{A.1}$$

where

$$h = e^{i\xi^a T^a} \quad \xi^a \equiv \xi^a(\Pi, g) \tag{A.2}$$

is an element of the unbroken subgroup \mathcal{H} . Under the group action $\Pi \rightarrow g(\Pi)$ one has then $U \rightarrow gU h^\dagger$. If T^A and T^a are the broken and unbroken generators respectively, we define

$$U^\dagger \partial_\mu U = i\mathcal{D}_\mu^A T^A + i\mathcal{E}_\mu^a T^a \equiv i\mathcal{D}_\mu + i\mathcal{E}_\mu \tag{A.3}$$

with transformations under \mathcal{G}

$$\mathcal{D}_\mu(\Pi) \rightarrow \mathcal{D}_\mu(g(\Pi)) = h(\Pi, g)\mathcal{D}_\mu(\Pi)h(\Pi, g)^\dagger \tag{A.4}$$

$$\mathcal{E}_\mu(\Pi) \rightarrow \mathcal{E}_\mu(g(\Pi)) = h(\Pi, g)\mathcal{E}_\mu(\Pi)h(\Pi, g)^\dagger - ih(\Pi, g)\partial_\mu h(\Pi, g)^\dagger. \tag{A.5}$$

Notice that for space dependent Π configurations, \mathcal{D}_μ and \mathcal{E}_μ transform under a local \mathcal{H} symmetry, in particular \mathcal{E}_μ transforms like the associated gauge field. Massive multiplets

fill reducible representations Φ of the unbroken group \mathcal{H} . The action of the global group \mathcal{G} is realized through the “local” \mathcal{H} transformations

$$\Phi \rightarrow h(\Pi, g)\Phi. \tag{A.6}$$

Using the last 3 equations the most general Lagrangian for the non-linearly realized \mathcal{G} can be written, just by using the rules for a “local” \mathcal{H} gauge group [13]. In particular \mathcal{E}_μ defines the \mathcal{H} -covariant derivative $\partial_\mu + i\mathcal{E}_\mu$ on the massive fields Φ .

The weak gauging of G_{SM} is obtained by changing eq. (A.3) to $U^\dagger(\partial_\mu + iA_\mu)U \equiv i\bar{\mathcal{D}}_\mu(\pi, A) + i\bar{\mathcal{E}}_\mu(\pi, A)$. Indeed since we treat the gauge fields as spectators in what follows, we can, without loss of generality, gauge the full $\mathcal{H} \supset G_{SM}$. Thus $\bar{\mathcal{E}}_\mu^a$ transforms as the \mathcal{H} -gauge field both under the global \mathcal{G} -transformations and under the genuinely local $g \equiv h(x)$. It is useful to have the expressions of $\bar{\mathcal{E}}$ and $\bar{\mathcal{D}}$ at lowest order in Π

$$\bar{\mathcal{E}}_\mu = A_\mu + \mathcal{E}_\mu(\Pi, D_\nu) = A_\mu - \frac{i}{2}\Pi \overleftrightarrow{D}_\mu \Pi + \mathcal{O}(\Pi^4) \tag{A.7}$$

$$\bar{\mathcal{D}}_\mu = \mathcal{D}_\mu(\Pi, D_\nu) = D_\mu \Pi - \frac{1}{6} \left[\Pi, \Pi \overleftrightarrow{D}_\mu \Pi \right] + \mathcal{O}(\Pi^5), \tag{A.8}$$

where $D_\mu = \partial_\mu + iA_\mu$ is the \mathcal{H} covariant derivative. (In the last second equality of both equations we have specified to the interesting case in which \mathcal{G}/\mathcal{H} is a symmetric space.)

We want to classify the 4-derivative structures that can lead to couplings involving two Goldstones and two gauge fields. There are 2 relevant structures

$$O_1 = \text{Tr}[F_{\mu\nu}(\bar{\mathcal{E}})F^{\mu\nu}(\bar{\mathcal{E}})], \quad O_2 = \text{Tr}[\bar{\mathcal{D}}_\mu \bar{\mathcal{D}}_\nu F^{\mu\nu}(\bar{\mathcal{E}})], \tag{A.9}$$

where $F_{\mu\nu}(\bar{\mathcal{E}}) = \partial_\mu \bar{\mathcal{E}}_\nu - \partial_\nu \bar{\mathcal{E}}_\mu + i[\bar{\mathcal{E}}_\mu, \bar{\mathcal{E}}_\nu]$. Substituting eqs. (A.7)–(A.8) into eq. (A.9) we find that O_W and O_B emerge by expanding O_1 , while O_{HW} and O_{HB} emerge from O_2 . It is also evident that operators of the above form cannot involve two gluons and two Higgses. Indicating by $\bar{D}_\mu \equiv \partial_\mu + i\bar{\mathcal{E}}_\mu$ the full \mathcal{H} -covariant derivative, one could write down other structures like $\bar{\mathcal{D}}_\mu \bar{D}_\nu \bar{D}^\nu \bar{\mathcal{D}}^\mu$, $\bar{\mathcal{D}}_\mu \bar{D}^\mu \bar{D}_\nu \bar{\mathcal{D}}^\nu$ or $\bar{\mathcal{D}}_\mu \bar{D}^\mu \bar{\mathcal{D}}_\nu \bar{\mathcal{D}}^\nu$. These are however shown to give either the same effects at dimension 6 or terms involving at least four Goldstones.

The question remains onto which effects can be generated at tree level in minimally coupled theories, such as Holographic Goldstones or Little Higgses. One distinctive feature of O_{HW} and O_{HB} is that they give rise to interactions involving on-shell photons and electrically neutral states. This cannot occur at tree level in a minimally coupled theory where photon interactions are purely dictated by covariant derivatives. On the other hand O_W and O_B do not lead to any extra interactions for on-shell photons, so that one may expect them to arise at tree level by integrating out heavy states. This is indeed the case for both Holographic Goldstones and Little Higgses, which are known to give rise to a contribution to $\hat{S} \propto c_W + c_B$ through the exchange of heavy vector states. Let us briefly outline how this effects come about within our formalism by focussing on the case of a massive vector V transforming in the adjoint of \mathcal{H} . In the ungauged limit we have the option to choose V_μ to transform like \mathcal{E}_μ under \mathcal{G} . Then the most general two derivative \mathcal{G} -invariant action is given by

$$m_\rho^4 \mathcal{L}^0 = m_\rho^2 \mathcal{D}_\mu^A \mathcal{D}_A^\mu - \frac{1}{4} (F_{\mu\nu}^V)^2 + \frac{1}{2} m_\rho^2 (V_\mu - \mathcal{E}_\mu)^2, \tag{A.10}$$

where $F_{\mu\nu}^V = \partial_\mu V_\nu - \partial_\nu V_\mu + i[V_\mu, V_\nu]$. Notice that terms involving covariant derivatives acting on the homogeneously transforming combination $\hat{V}_\mu \equiv V_\mu - \mathcal{E}_\mu$ have more than two derivatives. A limitation to two derivatives, also automatically eliminates the term $(\partial_\mu \hat{V}^\mu - i[V^\mu, \hat{V}^\mu])^2$ which would imply the presence of a scalar ghost with mass $\sim m_\rho$. The gauging of \mathcal{H} then amounts to $\mathcal{E} \rightarrow \bar{\mathcal{E}}$ in the above equation. According to this change V will also have to transform under the genuinely local $g = h(x)$ precisely like A . Notice that the structure we have thus outlined is the same of Little Higgs model with a product group structure with V playing the role of the vector boson of the second $SU(2)'$, the one under which the SM fermions are uncharged. The mass term in eq. (A.10) mimics the effects of the Goldstone which breaks the gauge group to the, low energy, diagonal $SU(2)$, under which both A and V transform thus like gauge fields. By integrating out V we find the following correction to the low-energy effective action

$$\Delta\mathcal{L} = -\frac{1}{4g_\rho^2} F_{\mu\nu}(\bar{\mathcal{E}}) F^{\mu\nu}(\bar{\mathcal{E}}) - \frac{1}{2g_\rho^2} D^\mu F_{\mu\nu}(\bar{\mathcal{E}}) \frac{1}{\partial^2 + m_\rho^2} D^\rho F_\rho^\nu(\bar{\mathcal{E}}) + \dots \quad (\text{A.11})$$

where the dots indicate terms more than quadratic in the field strength and with at least 4-derivatives. As we already explained the first term gives rise to O_W and O_B . The second term, instead, gives rise to four derivative corrections to pure gauge kinetic terms

$$O_{2W} = \frac{1}{2g_\rho^2 m_\rho^2} (D^\mu W_{\mu\nu}^i)(D_\rho W^{i\rho\nu}) \quad O_{2B} = \frac{1}{2g_\rho^2 m_\rho^2} (\partial^\mu B_{\mu\nu})(\partial_\rho B^{\rho\nu}). \quad (\text{A.12})$$

Let us consider now the effect of integrating out massive scalars. At the two derivative level we can have mixings of the type $D_\mu \Phi D^\mu$ and $\Phi D_\mu D^\mu$, where we have suppressed the indices; we assume, of course that the \mathcal{H} -quantum numbers of Φ and the contractions ensure \mathcal{H} -invariance. Integrating out Φ , the leading operators involve then at least 4 derivatives. At dimension 6, one is then easily convinced that only one term is generated in general

$$\frac{1}{m_\rho^2} (D^2 H^\dagger)(D^2 H), \quad (\text{A.13})$$

which can be induced by the exchange of massive doublets. The other possible contraction at dimension 6 order, $(D_\mu D_\nu H^\dagger)(D^\nu D^\mu H)$, cannot arise from scalar exchange as it involves a $J = 2$ part. The more interesting terms arise at dimension 8, and are irrelevant to our analysis.

B. Effective Lagrangian in the canonical basis

The first term of the effective Lagrangian (4.1) involves Higgs derivative terms that makes difficult to read off physical effects. At order ξ , we can eliminate these derivative terms by performing the following non-linear redefinition of the Higgs:

$$h \rightarrow h - \frac{c_H \xi}{2} \left(h + \frac{h^2}{v} + \frac{h^3}{3v^2} \right). \quad (\text{B.1})$$

After this redefinition, the corrections to the SM Lagrangian are now given by

$$\mathcal{L}_h = \xi \left\{ -\frac{m_H^2}{2v} \left[\left(c_6 - \frac{3c_H}{2} \right) h^3 + \left(6c_6 - \frac{25c_H}{3} \right) \frac{h^4}{4v} + \dots \right] \right\}$$

$$\begin{aligned}
& +\frac{m_f}{v}\bar{f}f\left[\left(c_y+\frac{c_H}{2}\right)h+(3c_y+c_H)\frac{h^2}{2v}+\dots\right] \\
& -c_H m_W^2\left(\frac{h}{v}+\frac{2h^2}{v^2}+\dots\right)W_\mu^+W^{-\mu}-\frac{c_H m_Z^2}{2}\left(\frac{h}{v}+\frac{2h^2}{v^2}+\dots\right)Z_\mu Z^\mu \\
& +\left(\frac{h}{v}+\frac{h^2}{2v^2}\right)\left[\frac{g^2}{2g_\rho^2}(\hat{c}_W W_\mu^- \mathcal{D}^{\mu\nu} W_\nu^+ + \text{h.c.})+\frac{g^2}{2g_\rho^2}Z_\mu \mathcal{D}^{\mu\nu}\left[\hat{c}_Z Z_\nu+\left(\frac{2\hat{c}_W}{\sin 2\theta_W}-\frac{\hat{c}_Z}{\tan\theta_W}\right)A_\nu\right]\right. \\
& -\frac{g^2}{(4\pi)^2}\left(\frac{c_{HW}}{2}W^{+\mu\nu}W_{\mu\nu}^-+\frac{c_{HW}+\tan^2\theta_W c_{HB}}{4}Z^{\mu\nu}Z_{\mu\nu}-2\sin^2\theta_W c_{\gamma Z}F^{\mu\nu}Z_{\mu\nu}\right)+\dots \\
& \left.+\frac{\alpha g^2 c_\gamma}{4\pi g_\rho^2}F^{\mu\nu}F_{\mu\nu}+\frac{\alpha_s y_t^2 c_g}{4\pi g_\rho^2}G^{a\mu\nu}G_{\mu\nu}^a\right]\}. \tag{B.2}
\end{aligned}$$

Eq. (B.2) could also have been obtained from eq. (4.1) by using the Higgs equation of motion. The Lagrangian eq. (B.2) is in a more suitable basis to compare it with the effective Lagrangian arising from specific SILH models, and extract the predictions of these models for the coefficients c_i . For this purpose it will be useful to give the Higgs couplings corrected by eq. (B.2) at order ξ and at zero momentum (or, equivalently, neglecting $g^2\xi/g_\rho^2$ corrections). For the hWW , hff and h^3 coupling, we have

$$g_{hWW} = gm_W \left[1 - \frac{c_H}{2}\xi\right], \tag{B.3}$$

$$g_{hff} = \frac{gm_f}{2m_W} \left[1 - \xi\left(\frac{c_H}{2} + c_y\right)\right], \tag{B.4}$$

$$g_{hhh} = \frac{gm_H^2}{4m_W} \left[1 + \xi\left(c_6 - \frac{3c_H}{2}\right)\right]. \tag{B.5}$$

C. Loop functions for the Higgs radiative decays

Here we give the loop functions describing the Higgs radiative decays in eqs. (4.11)–(4.13), including the $\mathcal{O}(\alpha_S)$ corrections coming from matching the SM contribution to the operator

basis, in the limit of heavy top.

$$I_g = \frac{1}{2}F_{1/2}(x_{tH}) \left(1 + \frac{11\alpha_S}{4\pi}\right), \quad I_\gamma = \frac{4}{3}F_{1/2}(x_{tH}) \left(1 - \frac{\alpha_S}{\pi}\right), \quad J_\gamma = F_1(x_{WH}) \quad (\text{C.1})$$

$$I_Z = \frac{2(3 - 5t_W^2)}{3t_W} [I_1(x_{tH}, x_{tZ}) - I_2(x_{tH}, x_{tZ})] \left(1 - \frac{\alpha_S}{\pi}\right) \quad (\text{C.2})$$

$$J_Z = \left[\left(1 + \frac{2}{x_{WH}}\right)t_W - \left(5 + \frac{2}{x_{WH}}\right)\frac{1}{t_W} \right] I_1(x_{WH}, x_{WZ}) + 4 \left(\frac{3}{t_W} - t_W\right) I_2(x_{WH}, x_{WZ}) \quad (\text{C.3})$$

$$t_W \equiv \tan \theta_W, \quad x_{ij} \equiv \frac{4m_i^2}{m_j^2}, \quad i = t, W, \quad j = H, Z \quad (\text{C.4})$$

$$F_{1/2}(x) = -2x [1 + (1 - x)f(x)], \quad F_1(x) = 2 + 3x [1 + (2 - x)f(x)] \quad (\text{C.5})$$

$$I_1(x, y) = \frac{xy}{2(x - y)} \left\{ 1 + \frac{xy}{x - y} [f(x) - f(y)] + \frac{2}{x - y} [g(x) - g(y)] \right\} \quad (\text{C.6})$$

$$I_2(x, y) = -\frac{xy}{2(x - y)} [f(x) - f(y)] \quad (\text{C.7})$$

$$f(x) = \arcsin^2(x^{-1/2}), \quad g(x) = \sqrt{x - 1} \arcsin(x^{-1/2}). \quad (\text{C.8})$$

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