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THE STRUCTURE OF EQUIVALENCE CLASSES

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The structure of equivalence classes can be completely described by four parameters: class size, number of nodes, the distribution of "singles" among nodes, and directionality of training. Class size refers to the number of stimuli in a class. Nodes are stimuli linked by training to at least two other stimuli. Singles are stimuli linked by training to only one other stimulus. The distribution of singles refers to the number of singles linked by training to each node. Directionality of training refers to the use of stimuli as samples and as comparison stimuli in training. These four parameters define the different ways in which the stimuli in a class can be organized, and thus provide a basis for systematically characterizing the properties of stimuli in a given equivalence class. The four parameters can also be used to account for the development of individual differences that are commonly characterized in terms of "understanding" and connotative meaning.

Methods are described for generating all possible combinations of parameter values, and a formula is introduced which specifies all of the parameter values for an equivalence class. Its utility for interrelating experimental procedures is demonstrated by analyzing a number of representative experiments that have addressed equivalence-class formation.

Key words: equivalence class, distribution of singles, node, single, directionality of training, individual differences, derived relations, equivalence relation, training cluster (TC)

At first, a picture of a peach, the spoken word "peach," the written word PEACH, and the smell of a peach are stimuli that are unrelated in terms of physical properties and meaning. With exposure to appropriate contingencies, these stimuli become interrelated and form an equivalence class. Because the stimuli in the class do not necessarily share any physical properties, their relatedness is very likely to be the result of training, either in the laboratory or in a natural setting. In this article we consider how groups of physically disparate stimuli come to be functionally linked to form classes of equivalent stimuli, how different forms of linkage can be quantitatively described and summarized, and how each variation in linkage may influence the degree of relatedness exhibited by the stimuli in the class.

Formation of Equivalence Classes

The establishment of equivalence classes will be illustrated by considering the formation of two five-member classes. At least two classes of stimuli must be established concurrently; the stimuli in Class 1 are A1, B1, C1, D1, and E1, and those in Class 2 are A2, B2, C2, D2, and E2. Following the notation of Fields, Verhavė, and Fath (1984), numbers designate class and letters designate stimuli within a class. Relations between the stimuli in each class are established in pairwise fashion on a trial-by-trial basis, using a conditional discrimination procedure (Sidman, 1971). On each trial, at least three stimuli are presented: a sample (Sa) and a positive comparison stimulus (Co+) both drawn from the same class, and at least one negative comparison stimulus (Co−) drawn from the other class. By convention, the Sa is listed first, the Co+ second, and the Co− third. The two stimuli drawn from the same class become linked by reinforcing the selection of the Co+ and extinguishing or punishing the selection of the Co−.

N stimuli can become a class if $(N - 1)$ two-term relations are established by training and if each of the stimuli in the class is used in at least one of the two-term training relations (Fields et al., 1984). In the present example, four two-term relations must be established by conditional-discrimination training. Class 1 can be established by training with (A1 B1 A2), (B1 C1 A2), (C1 D1 A2), and

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Table 1

All two-term relations in a class of five stimuli after unidirectional training. The stimuli are indicated by the letters A, B, C, D, and E. Each two-term relation in the class is listed in the body of the matrix, with the sample first and the positive comparison stimulus next. The superscript affixed to each two-term relation denotes its function: Training relation (*), Transitive relation (T), Symmetrical relation (S), Reflexive relation (R), and Equivalence relation (E).

	Positive comparison stimuli						Positive comparison stimuli				
Samples	A	B	C	D	E	Samples	A	B	C	D	E
A. Unidirectional Training						B. Bidirectional Training					
A	AA^R	AB^*	AC^T	AD^T	AE^T	A	AA^R	AB^*	AC^T	AD^T	AE^T
B	BA^S	BB^R	BC^*	BD^T	BE^T	B	BA^*	BB^R	BC^*	BD^T	BE^T
C	CA^E	CB^S	CC^R	CD^*	CE^T	C	CA^T	CB^*	CC^R	CD^*	CE^T
D	DA^E	DB^E	DC^S	DD^R	DE^*	D	DA^T	DB^T	DC^*	DD^R	DE^*
E	EA^E	EB^E	EC^E	ED^S	EE^R	E	EA^T	EB^T	EC^T	ED^*	EE^R

(D1 E1 A2), and Class 2 can be established by training with (A2 B2 A1), (B2 C2 A1), (C2 D2 A1), and (D2 E2 A1). Because a given stimulus serves only one behavioral function in a given two-term relation, this training is said to be unidirectional.

FORMAL PROPERTIES OF STIMULI IN EQUIVALENCE CLASSES AFTER UNIDIRECTIONAL TRAINING

If such unidirectional training transforms the groups of stimuli into functional classes, all of the stimuli in a group will be interrelated (i.e., will function as members of that class) without additional training. Table 1A shows all 25 two-term stimulus relations in a class of five stimuli. Each two-term relation is designated by two letters, sample first and positive comparison second. The four two-term relations established by training according to the example above (AB, BC, CD, and DE) are indicated by superscript asterisks. All of the remaining 21 two-term relations could come into existence without training (Sidman & Tailby, 1982). As a group, they are called emergent relations. Each will be described briefly.

Reflexive Relations

Functional reflexivity is exemplified by identity matching. Given a sample stimulus from the class, the choice of a comparison stimulus that is physically identical to the sample will occur without additional training. In Table 1A, each reflexive relation is located on the diagonal beginning in the upper left hand corner of the matrix and is labeled with a superscript R.

Symmetrical Relations

For a pair of stimuli used in training, one may be used only as the Sa and the other as the Co+. If a class has been established, the stimuli in each training pair should also be related in the reverse order, without direct training; this is called symmetry. That is, stimuli remain related even though their discriminative behavioral functions change. In Table 1A, each symmetrical relation is labeled with a superscript S.

Transitive Relations

After training, some pairs of stimuli are related to each other by prior linkage to common stimuli that mediate their relation. For example, A and C are related because both are linked to the intervening stimulus, B, through the training pairs AB and BC. Likewise, A and E are related because they are linked through prior training with AB, BC, CD, and DE, and mediated by the stimuli B, C, and D. In Table 1A, each transitive relation is labeled with a superscript T.

Each stimulus in a transitive relation serves the same behavioral function in testing (Sa or Co+) as it served in training. In addition, all intervening stimuli that link a transitive pair must have functioned as both Sa and Co+ during prior training. The dual discriminative function served by each intervening stimulus acts as the "behavioral glue in a stimulus chain" that links transitively related stimuli. Thus, transitive relations can be accounted for in terms of behavioral functions acquired by each stimulus in the chain through prior training, and it is not necessary to assume symmetry to explain the linkage between transitively related stimuli. Indeed, D'Amato and Colombo (1986) and D'Amato, Salmon, Loukas, and Tomie (1985) reported the emergence of transitive control without symmetrical control. On the other hand, others have reported their joint emergence (Feniello, Sidman, & deRose, 1986). The conditions that produce separate or joint emergence must be clarified by further empirical investigation.

Equivalence Relations

Each of the remaining stimulus pairs listed in Table 1A, labeled with the superscript E, is called an equivalence relation (Sidman & Tailby, 1982). Like transitive relations, they are related to each other by prior linkage to common mediating stimuli. For example, the E sample and the A positive comparison stimulus are linked by the mediating stimuli D, C, and B. Unlike transitive relations, however, the linkage of two equivalent stimuli cannot be explained purely in terms of the specific functions served by all stimuli during prior training: In this particular example, all of the intervening stimuli served dual behavioral functions in prior training. In contrast, E and A serve stimulus control functions in the equivalence relation that differ from those established by training; although A functions as a sample in testing, it served only as a positive comparison stimulus in training, and vice versa for E. Logically, the functional equivalence of EA can exist only by assuming that E acquired the behavioral function served by D during DE training, and A acquired the behavioral function served by B during AB training; that is, by assuming that A and B are symmetrically related as are D and E. Thus, symmetry is logically necessary to account for the linkage of equivalent stimuli.

FORMAL PROPERTIES OF STIMULI IN EQUIVALENCE CLASSES AFTER BIDIRECTIONAL TRAINING

When bidirectional training is conducted, each stimulus is used as a sample in some training pairs and as a positive comparison stimulus in others. For example, in Table 1B, the E stimulus serves as Co+ when training is conducted with DE and as a sample when training is conducted with ED. After training, all of the stimulus pairs not used in training can be either transitively or reflexively related. It is not possible to determine whether the stimuli in the pairs of the resulting class are symmetrical or equivalent. These latter properties can be evaluated, however, by adding a new stimulus (F) to the class by unidirectional training (AF), and then testing for symmetry with FA and equivalence with FB.

INTERACTION BETWEEN EQUIVALENCE AND TRANSITIVE RELATIONS

Both transitive relations and equivalence relations are mediated by nodal stimuli. Taken together, transitive and equivalence relations will be referred to as derivative relations. An equivalence class containing N stimuli can have a maximum of $(N - 2)(N - 1)$ derivative relations. The proportion of derivative relations that are transitive or equivalent depends on the two-term relations used to establish the class. If unidirectional training is conducted using one stimulus as a sample and all other stimuli as positive comparisons, all of the derivative relations will be equivalents. When bidirectional training is conducted, all derivative relations will be transitive. Finally, the unidirectional training illustrated in Table 1 produces an equal proportion of transitive and equivalence relations. In general, although the maximum number of derivative relations is governed by class size, the mix of transitive and equivalence relations is governed by the specific set of two-term training relations used to establish the class.

CONTROL OF BEHAVIOR BY EMERGENT RELATIONS

Although the terms "reflexive," "symmetrical," "transitive," and "equivalence relations" are logically defined, empirical evidence is needed to determine whether these stimulus–stimulus relations actually control behavior. Direct empirical demonstrations of behavior controlled by such relations have been reported in a number of studies since 1971 (Devany, Hayes, & Nelson, 1986; Imam & Chase, 1986; Lazar, 1977; Lazar, Davis-Lang, & Sanchez, 1984; Lazar & Kotlarchyk, 1986; Mackay &

Sidman, 1984; Sidman, 1971; Sidman & Cresson, 1973; Sidman, Cresson, & Willson-Morris, 1974; Sidman, Kirk, & Willson-Morris, 1985; Sidman, Willson-Morris, & Kirk, 1986; Spradlin, Cotter, & Baxley, 1973; Spradlin & Dixon, 1976; Stromer, 1986; Stromer & Osborne, 1982; Wetherby, Karlan, & Spradlin, 1983).

FUNCTIONAL AND STRUCTURAL PARAMETERS THAT INFLUENCE EQUIVALENCE CLASS FORMATION

Since 1971, several studies have investigated the effects of functional parameters upon the establishment of equivalence classes. Sidman and Tailby (1982) demonstrated that gradual reduction and eventual elimination of reinforcement of responding on training trials enhanced the rate of emergence of control by transitive and equivalence relations. Sidman et al. (1985) demonstrated that the sequence in which new stimuli are added to a class, and the sequence in which various transitive and equivalence relations are introduced for evaluation, influence the formation of equivalence classes. Spradlin and Dixon (1976) and Lazar (1977) demonstrated that when training with the minimum number of two-term relations does not produce stimulus control by transitive and equivalence relations, additional "redundant" training with some of the transitive stimulus pairs can induce the emergence of control by other transitive and equivalent stimulus pairs. McIlvane and Stoddard (1981) demonstrated that classes can be formed by introducing new negative comparison stimuli within the context of samples and positive comparison stimuli drawn from extant classes. The negative comparisons form equivalence classes by "exclusion."

"Structural" parameters define the logical organization of stimuli within an equivalence class. These structural parameters are (a) defined in terms of a single presentation of a stimulus pair, (b) constant for every presentation, and (c) atemporal. The identification of the structural parameters of equivalence classes and the determination of their effects on the behavioral properties of stimuli within a class have, however, received little attention. In this article we systematically describe a set of structural parameters that specifies the logical organization of the stimuli in an equivalence class.

Table 2

Number of unique training clusters for classes containing from 3 to 11 stimuli. Effects of nodes, distribution of singles, and directionality of training are indicated. Details in text.

(1) Stim. per class	(2) Max. no. nodes	(3)* Max. no. TCs	(4) Directional variants/TC	(5) All directional TCs
3	1	1	9	9
4	2	2	27	54
5	3	3	81	243
6	6	6	243	1,458
7	5	10	729	7,290
8	6	20	2,187	43,740
9	7	36	6,561	236,196
10	8	72	19,683	1,417,176
11	9	136	59,049	8,030,664

* For N odd, maximum number of TCs = $2^{(N-4)} + 2^{(N-5)/2}$. For N even, maximum number of TCs = $2^{(N-4)} + 2^{(N-4)/2}$.

THE STRUCTURE OF EQUIVALENCE CLASSES

In a previous report, we discussed two independent variables that partly define the logical organization of the stimuli in an equivalence class (Fields et al., 1984). These were class size and the number of nodal stimuli in a class.

Class Size

Class size limits all other structural parameters of an equivalence class. It also determines the maximum number of derivative relations in a class. For a class of N stimuli, there are a maximum of $[(N - 2)(N - 1)]$ derived relations.

Nodes

Stimuli in an equivalence class can serve as "singles" or as "nodes." A node is a stimulus in a class that is linked directly through training to at least two other stimuli; a single is a stimulus that is linked directly through training to only one other stimulus. For example, if conditional-discrimination training is conducted with the two-term relations (AB, AC, CD, CE, and EF), A, C, and E are nodes, and B, D, and F are singles. An equivalence class, then, may contain from 1 to $(N - 2)$ nodes. Column 2 of Table 2 lists the maximum number of nodal variations that can be used for a class of N stimuli. Thus, the structure of a class can be differentiated partly in terms of its number of nodal stimuli. That formulation, however, is incomplete. Two additional parameters are needed to describe exhaustively the structure of equivalence classes. These are the distribution of single stimuli among nodes and the directionality of the training linkages.

The Distribution of Single Stimuli

For classes of up to five stimuli, single stimuli can be distributed among the nodes in only one way. However, for classes containing six or more stimuli, singles can be distributed among nodes in many ways (H. O'Mara, Jr., personal communication, 1984).[1] To illustrate, there are two ways of distributing the four single stimuli in a six-member class that contains two nodes. Both are shown in spider diagrams in the first row of Figure 1.

All of the ways in which a stimulus class can be established depend upon the specification of each unique distribution of single stimuli for every number of nodes. Each unique set of nodes and singles will be called a training cluster (TC). Column 3 of Table 2 indicates the total number of different TCs that can be used to establish a class of N stimuli. As class size increases, the number of unique TCs that can be used to establish an equivalence class also increases. Because the increase in nodes is linear, the more rapid expansion of TCs must be due to the growth in the number of distributions of singles. A detailed description of procedures needed to produce all TCs will be discussed below.

Directionality of Training

The TCs characterize the structure of equivalence classes without regard to the directionality of linking the stimuli in each two-term relation. Directionality of training must be considered, however, when exhaustively analyzing the structure of equivalence classes. The stimuli in a two-term relation can be linked unidirectionally or bidirectionally. In unidirectional training, one stimulus is an Sa only

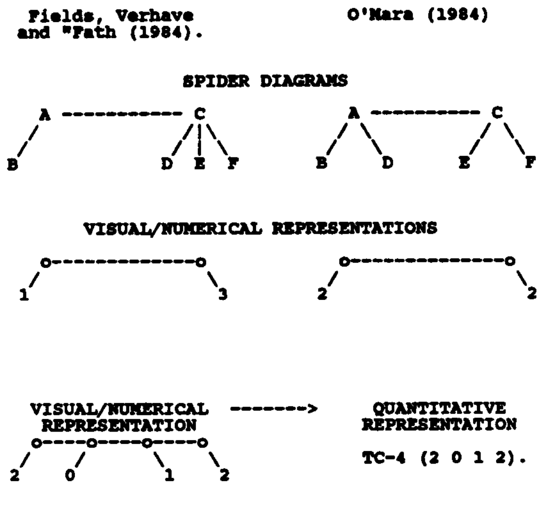

Fig. 1. Spider diagrams of two two-node training clusters (TCs), illustrated in the top row. Each letter designates a stimulus in the class. Stimuli in the top row of each spider diagram are nodes and those on the bottom row are singles. Each diagram illustrates a different distribution of singles for a two-node TC. The middle row illustrates a visual/numerical representation of each spider diagram. Each circle represents a node and the number beneath the circle represents the number of singles linked to the node by training. The bottom row illustrates the transformation of a visual/numerical representation to a quantitative formula representation of the same training cluster: TC means training cluster, the 4 in the formula refers to the number of nodes in the training cluster, and the numbers in parentheses refer to the number of singles linked to each node in the TC.

and the other is a Co+ only. Thus, a pair of stimuli, A and B, can be linked in two ways: A → B or B → A. In bidirectional training, each stimulus is an Sa in some trials and a Co+ in other trials, and is represented by A ↔ B. Therefore, three different directional variants can be used to link the stimuli in each two-term relation in a TC. Because $(N - 1)$ two-term relations must be established, and each can be established in one of three directional forms, the total number of directional training variants is $3^{(N-1)}$. As class size increases, the number of directional variants for each TC increases rapidly as shown by Column 4 of Table 2. Because each directional variant can be used with each TC, the total number of unique ways of interrelating the stimuli in a class is equal to the values in Column 4 of Table 2 multiplied by the number of TCs that can be used to establish a class (Column 3 of Table 2). The number of different TCs that can be used to establish a class

[1] In correspondence with us, Mr. O'Mara noted that there are more than $(N - 2)$ ways of establishing equivalence classes for classes containing at least six stimuli. In addition, the formulas that predict the total number of TCs when nodes and singles are considered were developed by Mr. O'Mara. His incisive observation alerted us to the limitation of our earlier work and served as a most effective catalyst for us to extend our analysis of equivalence class formation.

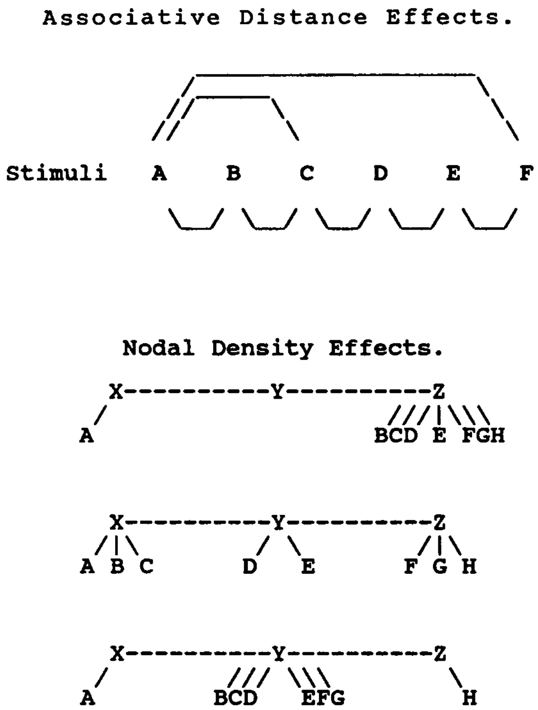

Fig. 2. The theoretical linkage of terms in two transitive or equivalence relations (upper half). Stimuli are represented by letters. The five two-term relations used for training are illustrated by loops presented beneath the terms. Two of the potential transitive/equivalence relations, A-C and A-F, are illustrated by loops drawn above the letters. The AF relations are linked by four nodes (B, C, D, and E), and the AC relation is linked by one node (B). The bottom portion of the figure illustrates nodal density. Three different TCs are illustrated for an 11-member class. Three nodes and eight singles are used in each TC. Each TC, however, contains a different distribution of singles. Each TC illustrates a different nodal density for the middle node.

of *N* stimuli becomes very large as class size increases (Column 5 of Table 2). A detailed description of procedures needed to produce each unique directional TC is presented below.

EFFECTS OF NODES, DISTRIBUTION OF SINGLES, AND DIRECTIONALITY OF TRAINING ON TRANSITIVE AND EQUIVALENCE RELATIONS

Intervening Nodes or "Associative Distance"

Stimuli in transitive or equivalence relations are linked by intervening cues that have served as nodes in prior training. We use the term "associative distance" to refer to the number of intervening nodes that separate the stimuli in a transitive or equivalence relation. For example, in the top half of Figure 2, the AC relation is mediated by only one node (B), whereas the AF relation is mediated by four nodes (B, C, D, and E). Earlier, we postulated that control exerted by transitive relations would be inversely related to associative distance (Fields et al., 1984). A similar functional relation should hold for equivalence relations. Fragmentary data presented by Lazar et al. (1984) and by Sidman et al. (1985) suggest that the point at which transitive or equivalence relationships begin to control responding is inversely related to associative distance. Using somewhat different procedures, D'Amato and Colombo (1986) studied the control exerted by transitively related stimuli using reaction time as the dependent variable and found it to be directly related to associative distance. Fields (1986) found that accuracy of responding to transitive relations is inversely related to associative distance. Each study differed parametrically and used different dependent variables. Although they corroborate each other's findings, a systematic study of the influence of associative distance upon the establishment of control by transitive and equivalence relations is needed.

Distribution of Singles or "Nodal Density"

Nodal stimuli, which mediate both equivalence and transitive relations, can have many singles linked to them by training. The number of singles linked to a node is termed "nodal density" and is determined by the distribution of singles, as illustrated in the lower half of Figure 2. In each TC, AH is mediated by the internal node, Y. As the distribution of singles changes, the number of singles linked to Y increases from 0 (in the upper TC), to 2 (in the middle TC), to 6 (in the bottom TC). For two reasons, it is likely that nodal density, and, thus, the distribution of singles, influences the formation and/or asymptotic performance maintained by transitive and equivalence relations. First, manipulation of the distribution of singles produces a vast number of unique conditions for establishing equivalence classes, each resulting in different nodal densities. It seems unlikely that each of these variations would all have the same effect. Second, equivalence-class formation involves the linkage of new stimuli to previously interrelated groups of stimuli. Similar relations also obtain in much of the research conducted on verbal learning, memory, and interference. The classical positive and negative transfer studies demonstrated that new stimuli linked to a target stimulus could inhibit or facilitate future learning in which the target stimulus was used, whereas proaction and retroaction studies demonstrated that new stimuli linked to a target stimulus could influence subsequent recall or recognition of previously learned stimulus–stimulus relations (Catania, 1984; McGeoch, 1952; Osgood, 1953; Woodworth, 1938). Therefore, increasing the number of singles linked to a nodal stimulus might inhibit or enhance the formation of emergent relations.

At present, we lack the empirical data needed to develop principles that could predict the directional effects of nodal density on the formation of emergent relations.

Directionality of Training

To date, many experiments have demonstrated the establishment of equivalence classes with the use of unidirectional training. There is only one published experiment, however, that provides evidence suggesting that directionality of training influences the formation of emergent relations. Spradlin and Dixon (1976) compared unidirectional and bidirectional training and found that although no transitive stimulus control occurred after unidirectional training, subsequent bidirectional training resulted in control by transitive relations. A series of empirical studies is needed to clarify the effects of training directionality on equivalence class formation.

INDIVIDUAL DIFFERENCES AND THE MULTITUDE OF TCS

A large number of TCs can be used to establish an equivalence class with a set of N stimuli. Because each TC is distinguished by number of nodes, distribution of singles, and directionality of training, it is plausible to assume that different TCs will influence the behavioral properties acquired by the stimuli in a class. This, in turn, may explain the induction of individual differences that are commonly characterized as differences in degree of understanding or connotative meaning. First, let us assume that understanding or meaning is inferred from the observed emergence of control by equivalence and transitive relations (Lazar et al., 1984; Sidman, 1971; Sidman et al., 1974; Spradlin & Dixon, 1976). Next let us consider three observations and their traditional explanations: (a) In natural settings, most people learn equivalences among a given set of stimuli. Most individuals' experiences relevant to the formation of equivalence classes are idiosyncratic, informal, and ambiguously specified, but all appear to produce a rough commonality of outcome. Thus, traditional educational and developmental theories have concluded that the emergence of equivalences can be attributed to inherent biological capabilities (Lenneberg, 1967) or to "acts of inventiveness" (Bruner, Goodnow, & Austin, 1956). (b) It then follows that those few individuals who do not form equivalences are thought to have biological deficits that account for their learning disabilities. (c) Of those individuals who do form equivalences, many have overlapping but somewhat different "understandings" of the same class. These distinctions reflect individual differences in ability to "recognize connections" or differences in IQ. None of these accounts is wholly satisfactory because the presumed causal agents are not observable and are inferred from the very data that are to be explained.

We propose a different explanation, based upon the large but finite number of TCs that can be used to establish equivalence classes. With regard to point (a), a large number of different TCs can be used to establish equivalence relations among a given set of stimuli. All of the individuals who learn the equivalences in a natural setting are exposed to at least one of the many different TCs that can induce equivalences. The very large number of different TCs that can establish an equivalence class can characterize the breadth of experiences all of which produce a common outcome. With regard to point (b), because the number of TCs that can produce a class is limited, those individuals who do not learn equivalence relations have not been exposed to one of these TCs. Inability to form a class would, thus, be attributed to methodological rather than to biological limitations. Subsequent training with an appropriate TC should result in class formation. With regard to point (c), of those people who learn a class of stimuli, many individuals will have somewhat different understandings of the meaning of the class. We interpret this to mean that for different individuals, different transitive and equivalence relations emerge, or that the same relations exert different degrees of stimulus control. The different TCs that prevail as different individuals learn an equivalence class predict just such outcomes. Indeed, different individuals have distinctly different "understandings" of a given class. Thus, the breadth of TCs can also account for differences in connotative meaning among many individuals, all of whom share a common "understanding" of a category. All three aspects of this formulation can be subjected to empirical confirmation or disconfirmation in either laboratory or natural settings.

THE PRODUCTION OF ALL TRAINING CLUSTERS

The production of all TCs is a two-fold process. First, for a class of N stimuli with a given number of nodes, all of the distributions of singles among the nodes must be generated. As this is done, however, directionality of training is disregarded. Second, all directional variants for a TC must be generated. The algorithms for generating all distributions of singles and all directional TC variants are described and illustrated in this section.

The Distribution of Singles

To determine all of the ways in which a stimulus class can be established, we replaced the spider diagrams with a quantitative representation of the distribution of single stimuli among nodes, as illustrated in Figure 1. The stimuli used as nodes or singles in the spider diagrams on the top row of Figure 1 are assigned arbitrarily. Therefore, the spider diagrams were abstracted by substituting a circle for each node, and a numeral corresponding to the number of singles linked to each node as shown in the second row of Figure 1. These partially quantified visual representations of TCs were then transformed into a purely numerical form, as illustrated in the bottom section of Figure 1 using a four-node TC. In its visual/numerical representation, the four nodes appear as a "string" running from left to right. The row of numbers beneath the nodal string designates the number of singles linked to each node for that particular TC. Because each numeral in the series represents one node, a TC can be fully designated as follows: TC-X(A A A . . .), where X designates the number of nodes, and each A in the parentheses designates the number of single stimuli linked to each adjacent node in the training cluster. Thus, TC-4(2 0 1 2) is the formula designation of the TC illustrated at the bottom of Figure 1.

Generating All Distributions of Singles for Nondirectional TCs

To generate all possible TCs for a class of N stimuli, the nodes in any TC are conceived of as a string. Those at the ends of the string are called "end nodes," and the remainder are called "internal nodes." End nodes and internal nodes must be treated separately when allocating singles.

End nodes. At least one single must be assigned to each end node in a TC. If more than two singles can be allocated to the end nodes, they must be assigned in all combinations that sum to the allocated number. For example, in the top panel of Figure 3, if six singles are to be allocated to the end nodes, they must be assigned to the left and right end nodes in the following combinations: 1 and 5, 2 and 4, and 3 and 3.

Internal nodes. The number of singles available for allocation to the internal nodes must be assigned in all possible combinations from the minimum to the maximum number available. The actual distribution of singles can be generated by counting in a "base" arithmetic fashion, in ascending order, as illustrated in the second panel of Figure 3. The internal nodes can be thought of as "places" analogous to units, tens, hundreds, and so on, moving from right to left. Because four singles are available for allocation in this example, they are assigned by counting incrementally in a "base 4" fashion. For a given base 4 value, the numeral assigned to each internal node designates the number of singles linked to that node. Thus, the assignment "1 2 1" means that 1, 2, and 1 single stimuli are linked, respectively, to the three adjacent internal nodes.

Summation rules for producing TCs. A specific assignment of single stimuli to the internal nodes must be used in combination with all of the end-node assignments such that the number of stimuli used as nodes plus the total number of singles equals the number of stimuli in the class.

The third panel of Figure 3 illustrates the method of assigning singles to nodes for an 11-member class. Starting with the assignment of the lowest base arithmetic number to the internal nodes, one single is assigned to the left-most end node, and all remaining singles are assigned to the right hand end node so that the sum of nodes and singles equals *N*. Next, the number of singles assigned to the end nodes is manipulated as indicated above, and the number of singles assigned to the internal nodes remains constant. Once this process is completed, the base number assigned to the internal nodes is increased by one, and the cycle is repeated.

Mirror image TCs. While generating TCs, mirror image sequences of single stimuli will be produced, such as TC-5(1 0 0 3 2) and TC-5(2 3 0 0 1), indicated by "#." They are also illustrated at the bottom of Figure 3 as spider diagrams. Because mirror image TCs are formally and functionally identical, one of these "twins" is excluded when enumerating all of the TCs for a given class.

Table 3 lists all individual TCs for classes of 3 to 11 stimuli. TCs are categorized within a class by the number of nodes used in training. Each string of numbers in a cell refers to the set of singles that appears in the parentheses of the TC formula.

Table 4 summarizes the number of different TCs for classes of 3 to 11 stimuli. TCs are categorized within a class by the number of nodes used in training. Cellular values in Table 4 are derived by adding the number of TCs within each "class size × nodal category" region in Table 3. The bottom line of Table 4 indicates the total number of different TCs that can be used to establish each stimulus class.

The Generation of All Directional TC Variants

As mentioned above, when directionality of training is considered, each two-term relation in a TC can be established in one of three ways: (A → B), (B → A), or (A ↔ B). Since there are ($N - 1$) training links and each can be established in three ways, there are $3^{(N-1)}$ directional variants for a given TC. The algorithm for generating each directional variant is illustrated in Figure 4 using a five-member class containing three nodes, TC-3(1 0 1). Each of the training links in the TC can be visu-

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      End-Node Allocation
o-----o-----o-----o-----o
1                       5
2                       4
3                       3

 Internal Node Allocation
o-----o-----o-----o-----o
      0     0     1
      0     0     2
      0     0     3
      0     0     4
      0     1     0
      0     1     1
      0     1     2
      0     1     3
      0     2     0
      0     2     1
      0     2     2
      0     3     0
      0     3     1
      1     0     0
      1     0     1
      1     0     2
      1     0     3
      1     1     0
      1     1     1
      1     1     2
      1     2     0
      1     2     1
      2     0     0
           etc.

      All Node Allocation
o-----o-----o-----o-----o
1     0     0     1     4
2     0     0     1     3
3     0     0     1     2
4     0     0     1     1
1     0     0     2     3
2     0     0     2     2
3     0     0     2     1
1     0     0     3     2   #
2     0     0     3     1
1     0     0     4     1
1     0     1     0     4
      :
2     3     0     0     1   #

        Mirror-Image TCs
       TC - 5 (1 0 0 3 2)
o-------o-------o-------o-------o
/                      /|\     / \

       TC - 5 (2 3 0 0 1)
o-------o-------o-------o-------o
/ \    /|\                      \
```

Fig. 3. Allocation of singles to the end nodes and internal nodes in a TC. The class of 11 stimuli contains five nodes and six singles. The top panel illustrates how the six singles can be distributed in three ways among the end nodes. The second panel illustrates the allocation of one to four single stimuli among the internal nodes in the TC. The third panel illustrates the allocation of all singles to all nodes in the TC. Two mirror image TCs are indicated by the symbol #. The spider diagrams at the bottom illustrate the mirror image TC indicated by #.

Table 3

Training clusters for classes of 3 to 11 stimuli. The number strings in each cell refer to * in TC-N(*).

Stimuli per class	Number of nodes 1	2	3	4	5	6	7	8	9
3	2								
4	3	1 1							
5	4	1 2	1 0 1						
6	5	1 3	1 0 2	1 0 0 1					
		2 2	1 1 1						
7	6	1 4	1 0 3	1 0 0 2	1 0 0 0 1				
		2 3	2 0 2	1 0 1 1					
			1 1 2						
			1 2 1						
8	7	1 5	1 0 4	1 0 0 3	1 0 0 0 2	1 0 0 0 0 1			
		2 4	2 0 3	2 0 0 2	1 0 0 1 1				
		3 3	1 1 3	1 0 1 2	1 0 1 0 1				
			2 1 2	2 0 1 1					
			1 2 2	1 0 2 1					
			1 3 1	1 1 1 1					
9	8	1 6	1 0 5	1 0 0 4	1 0 0 0 3	1 0 0 0 0 2	1 0 0 0 0 0 1		
		2 5	2 0 4	2 0 0 3	2 0 0 0 2	1 0 0 0 1 1			
		3 4	3 0 3	1 0 1 3	1 0 0 1 2	1 0 0 1 0 1			
			1 1 4	2 0 1 2	2 0 0 1 1				
			2 1 3	3 0 1 1	1 0 0 2 1				
			1 2 3	1 0 2 2	1 0 1 0 2				
			2 2 2	2 0 2 1	1 0 2 0 1				
			1 3 2	1 0 3 1	1 1 0 1 1				
			1 4 1	1 1 1 2	1 1 1 0 1				
				1 1 2 1					

alized as a position in a numerical string; in this case, there are four. Each link can be assigned a value of 0, 1, or 2, where 0 corresponds to →, 1 corresponds to ←, and 2 corresponds to ↔. A directional variant, then, can be represented quantitatively by a series of four zeros, ones, or twos (e.g., "0 2 0 1"). Because the training links can be established in three ways, the four numerals assigned to training links can be conceived of as a "base 3" number. All directional variants, then, can be produced by listing each base 3 numeral from 0 to $3^{(N-1)}$ in ascending order (as indicated in the right column), and then substituting the appropriate directionality arrows for each number in the adjacent spider diagrams, as illustrated in the left column of the figure. TCs that contain unidirectional links only are labeled with a "U." TCs that contain both unidirectional and bidirectional links are called mixed variants, and are labeled with an "M." The last variant, labeled with a "B," is the only one with all bidirectional links.

FORMULA REPRESENTATION OF DIRECTIONAL TC VARIANTS

Unidirectional TCs

A TC in which all two-term stimulus relations are established bidirectionally is represented completely by the formula, TC-X(A A . . .). If all of the stimuli in a class are linked unidirectionally that formula is not complete, because it does not distinguish between TCs that have different patterns of training directionality even though they have the same number of nodes and distribution of singles. Three such unidirectional TC variants are illustrated in Figure 5. Unidirectionality is reflected in the number of times a node serves as an Sa or a Co+; a node can serve as an Sa for all training links, as a Co+ for all training links, or as an Sa for some training links and as a Co+ for others. Adding such information to the bidirectional formula, therefore, provides a complete quantitative characterization of the structure of an equivalence class estab-

Table 3

(*Continued*)

Stimuli per class	Number of nodes								
	1	2	3	4	5	6	7	8	9
10	9	17	106	1005	10004	100003	1000002	10000001	
		26	205	2004	20003	200002	1000011		
		35	304	3003	10013	100012	1000101		
		44	115	1014	20012	200011	1001001		
			214	2013	30011	100021			
			313	3012	10022	100102			
			124	4011	20021	200101			
			223	1023	10031	100111			
			133	2022	10103	100201			
			232	3021	20102	101011			
			142	1032	10112	101101			
			151	2031	20111	110011			
				1041	10202				
				1113	10211				
				2112	11012				
				1212	11021				
				2211	11111				
				1221	10301				
				1311	12101				
11	10	18	107	1006	10005	100004	1000003	10000002	100000001
		27	206	2005	20004	200003	2000002	10000011	
		36	305	3004	30003	100013	1000012	10000101	
		45	404	1015	10014	200012	2000011	10001001	
			116	2014	20013	300011	1000021		
			215	3013	30012	100022	1000102		
			314	4012	40011	200021	2000101		
			125	5011	10023	100031	1000111		
			224	1024	20022	100103	1000201		
			323	2023	30021	200102	1001002		
			134	3022	10032	300101	1001011		
			233	4021	20031	100112	1001101		
			143	1033	10041	200111	1002001		
			242	2032	10104	100121	1010011		
			152	3031	20103	100202	1010101		
			161	1042	10113	200201	1100011		
				2041	20112	100211			
				1105	30111	100301			
				1114	10122	101012			
				2113	20121	201011			
				1123	10131	101021			
				2122	10203	101102			
				3121	20202	101111			
				1132	10212	101201			
				2131	20211	102011			
				1141	10221	110021			
				1222	10302	110201			
				1231	10311	111011			
					10401				
					21002				
					11013				
					21012				
					11022				
					21021				
					11031				
					11112				
					11121				
					12021				

```
                                   "Base 3"         Type
                                 Directionality      of
      Letter-Line Array          Representation   Training
----------------------------     --------------   --------
A ---> B ---> C ---> D ---> E      0  0  0  0        U
A ---> B ---> C ---> D <--- E      0  0  0  1        U
A ---> B ---> C ---> D <==> E      0  0  0  2        M
A ---> B ---> C <--- D ---> E      0  0  1  0        U
A ---> B ---> C <--- D <--- E      0  0  1  1        U
A ---> B ---> C <--- D <==> E      0  0  1  2        M
A ---> B ---> C <==> D ---> E      0  0  2  0        M
A ---> B ---> C <==> D <--- E      0  0  2  1        M
A ---> B ---> C <==> D <==> E      0  0  2  2        M
A ---> B <--- C ---> D ---> E      0  1  0  0        U
A ---> B <--- C ---> D <--- E      0  1  0  1        U
A ---> B <--- C ---> D <==> E      0  1  0  2        M
A ---> B <--- C <--- D ---> E      0  1  1  0        U
A ---> B <--- C <--- D <--- E      0  1  1  1        U
A ---> B <--- C <--- D <==> E      0  1  1  2        M
A ---> B <--- C <==> D ---> E      0  1  2  0        M
A ---> B <--- C <==> D <--- E      0  1  2  1        M
A ---> B <--- C <==> D <==> E      0  1  2  2        M
A ---> B <==> C ---> D ---> E      0  2  0  0        M
          :                           :
          :                           :
A <==> B <==> C <==> D <==> E      2  2  2  2        B
```

Fig. 4. The systematic production of directional TC variants for a class containing five stimuli, three of which serve as nodes. U means that all two-term relations are trained unidirectionally, M means that some two-term relations are trained unidirectionally and others are trained bidirectionally, and B means that all two-term relations are trained bidirectionally.

lished by unidirectional training. This information is added to the formula, TC-X(A A A . . .), on a line above the numbers that designate the singles linked to each node, as summarized at the bottom of Figure 5. For each node, information regarding training directionality will be represented by two numerals

Table 4

Number of training clusters for classes containing 3 to 11 stimuli. For each class, number of possible TCs is indicated as a function of number of nodes used in training.

Number of nodes	Stimuli per class								
	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1
2		1	1	2	2	3	3	4	4
3			1	2	4	6	9	12	16
4				1	2	6	10	19	28
5					1	3	9	19	38
6						1	3	12	28
7							1	4	16
8								1	4
9									1
Total	1	2	3	6	10	20	36	72	136

separated by a full colon (Sa:Co+). The numeral preceding the colon indicates the number of links in which the nodal stimulus serves as an Sa, whereas the numeral that follows the colon indicates the number of links for which it serves as a Co+. Each set of numbers in parentheses contains directionality information about a particular node, and is written above the A in the basic formula that designates the node in the TC. The two-line formula, then, provides a complete symbolic description of each unique unidirectional TC variant. Formula representations of each unidirectional TC in Figure 5 are listed to the right of their corresponding letter-line arrays. To illustrate the nomenclature, the formula for the third TC states that the node with two singles serves as an Sa for two links and as a Co+ for one.

```
Letter-Line Arrays                     TC Formulas
==============================================================

                                    Node ------> A

A    B    C    D    E                       (1:3)
---->                                  TC-1 ( 4 )
<--------
<------------
<----------------
==============================================================

                                    Node ------> A

A    B    C    D    E                       (2:2)
<----                                  TC-1 ( 4 )
-------->
------------>
<----------------
==============================================================

                                    Nodes -----> A    C

A    B    C    D    E                       (2:0 2:1)
---->                                  TC-2 ( 1    2 )
-------->
          ---->
          -------->
==============================================================

TC Formula for Unidirectional Variants Only.

     (Sa:Co+  Sa:Co+  Sa:Co+)         2.
TC-X (  A       A       A   )         1.

Row 1. Singles.
       Number of Singles attached to a Node.

       X = Number of Nodes.
       A = Number of Singles linked to a given Node.

Row 2. Unidirectional Links.
       Number of Sa & Co+ functions served by a Node.

       Sa:Co+ = Sample:Positive Comparison.
```

Fig. 5. Letter-line arrays illustrating representative TCs in which all two-term relations are established by unidirectional training. The class contains five stimuli. Each TC differs in terms of the number of nodes used and the directionality of training for each relation. The formula representation for each TC is listed to the right of each letter-line array. Information in the bottom portion of the Figure defines all of the terms in the formulas that represent unidirectional TCs.

```
Letter-Line Arrays                    TC Formulas
=========================================================

                                Nodes ---> A     C     F

A   B   C   D   E   F   G                 (1:0   2:1   0:0)
<--->                                TC-3 ( 1     2     1 )
-------->                                 (0:1   1:0   1:1)
        ---->
        -------->
        <----------->
                    <--->
=========================================================

                                Nodes ---> A     C     F

A   B   C   D   E   F   G                 (0:1   2:1   1:0)
<----                                TC-3 ( 1     2     1 )
<------->                                 (1:0   1:0   0:1)
        ---->
        -------->
        <------------
                    <--->
=========================================================

TC Formula for Mixed Variants.

     (Sa:Co+  Sa:Co+  Sa:Co+)          2.
TC-X (  A       A       A    )         1.
     ( N:S     N:S     N:S   )         3.

Row 1. Singles.
    Number of Singles attached to a Node.

    X = Number of Nodes.
    A = Number of Singles linked to a given Node.

Row 2. Unidirectional Links.
    Number of Sa & Co+ functions served by a Node.

    Sa:Co+ = Sample:Positive Comparison.

Row 3. Bidirectional Links.
    Number of Bidirectional links from a
    given Node to other Nodes and Singles.

    N:S = Nodes:Singles.
```

Fig. 6. Letter-line arrays illustrating representative mixed TCs in which some two-term relations are established by unidirectional training and others are established with bidirectional training. The class contains seven stimuli with the same three nodes, A, C, and F. Each TC differs in terms of the specific two-term relations that are linked bidirectionally and in the directionality of training when a given two-term relation is linked unidirectionally. The formula representation for each TC is listed to the right of each letter-line array. Information in the bottom portion defines all of the terms in the formula used to represent training with a mixed TC.

Mixed TCs

When mixed training is conducted, some links are unidirectional and some are bidirectional. Two mixed TCs are illustrated by the letter-line arrays in Figure 6. The unidirectional formula provides an incomplete specification of mixed TCs, because it does not provide any information about bidirectional training links. Adding such information to the unidirectional formula, therefore, provides a complete quantitative characterization of the structure of an equivalence class established by mixed training. This information is added to the unidirectional formula on a line beneath the numbers that designate the singles linked to each node, as summarized at the bottom of Figure 6. Each bidirectional two-term relation that involves a node links that node to another node or to a single, as illustrated in Figure 6. For each such node, that information is presented by a set of two numerals separated by a colon (N:S). The numeral preceding the colon indicates the number of bidirectional links that connect the particular node to other nodes. The numeral that follows the colon indicates the number of bidirectional links that connect the particular node to single stimuli. Each (N:S) in parentheses is written beneath the A in the basic formula that designates the node being characterized with bidirectionality information. The three-line formula, then, provides a complete symbolic description of each unique mixed TC variant. Formulas for mixed TCs are illustrated in the two upper panels of Figure 6 to the right of their respective letter-line arrays.

The Uniqueness of Each TC Formula

The formula that represents a given TC is unique for that TC. Thus, if the letter-line

```
Letter-Line Arrays                    TC Formulas
=========================================================

                                Nodes ---> A     C     F

A   B   C   D   E   F   G                 (0:1   2:1   1:0)
<---                                 TC-3 ( 1     2     1 )
<------->                                 (1:0   1:0   0:1)
        ---->
        -------->
        <------------
                    <--->
=========================================================

                                Nodes ---> C     E     G

A   B   C   D   E   F   G                 (1:0   2:1   0:1)
                    ---->            TC-3 ( 1     2     1 )
        -------->                         (0:1   1:0   1:0)
            <----
                <------->
    <--------------------
<------->
=========================================================

                                Nodes ---> G     E     C

G   B   E   D   F   C   A                 (0:1   2:1   1:0)
<----                                TC-3 ( 1     2     1 )
<------->                                 (1:0   1:0   0:1)
        ---->
        -------->
        <------------
                    <--->
=========================================================
```

Fig. 7. Formula transformations that demonstrate the identity of TCs that appear to be different when represented visually with letter-line arrays. Letter-line arrays of three TCs are presented on the left, with their corresponding TC formulas listed on the right. The TCs presented in the first two panels appear to be different. The transformation of the TC formula in the middle panel, as indicated in the bottom panel, demonstrates the identity of the two TCs.

```
     LETTER-LINE ARRAY        FORMULA DESIGNATION

3 STIMULI PER CLASS.
====================

              Stromer and Osborne (1982)
              --------------------------
Experiment 1.

<===============>                              (0:2)
A       B       C                         TC-1 ( 2 )
-------->
        <--------

Experiment 2.

        <=======>                              (2:0)
A       B       C                         TC-1 ( 2 )
-------->
---------------->

Experiment 3.

<===============>
A       B       C                              (1:1)
-------->                                 TC-1 ( 2 )
        -------->

4 STIMULI PER CLASS.
====================

               Sidman and Tailby (1982).
               -------------------------

        <===============>
        <=======>
B       A       C       D
<--------                                (2:0  0:2)
        -------->                   TC-2 ( 1    1 )
                <--------

          Spradlin, Cotter and Baxley (1973).
          -----------------------------------

       ---------------->
A      B       C       D                 (2:0  0:2)
--------------->                    TC-2 ( 1    1 )
----------------------->
       -------->

Transformation to the letter-line representation of
the procedure used by Sidman and Tailby.

D       A       C       B
<--------
        -------->
                <--------
```

```
     LETTER-LINE ARRAY        FORMULA DESIGNATION

        Wetherby, Karlin and Spradlin (1983).
        -------------------------------------

                <=======>
         <===============>
<========>
A        B      C       D                (2:0  0:2)
        -------->                   TC-2 ( 1    1 )
---------------->
------------------------>

Transformation to the letter-line representation of
the procedure used by Sidman and Tailby.

D       A       C       B
<--------
        -------->
                <--------

5 STIMULI PER CLASS.
====================

         Lazar, Davis-Lang, and Sanchez (1984).
         --------------------------------------

<===============================>
<===============>
<=======>
        <=======================>
        <===============>
                <===============>
A       B       C       D       E        (1:1 1:2)
------------------------>           TC-2 ( 1   2 )
        <--------
                <--------
                        <--------

6 STIMULI PER CLASS.
====================

         Sidman, Kirk and Willson-Morris (1985).
         ---------------------------------------

<==============================>
<========================>
<==================>
<============>
            <==================>
            <============>
            <======>
                  <============>
      ------------------------->
      ------------------->
      ------------->
                  ------->
                        <-------
B     A     C     E     D     F    (2:0 0:2 1:1 2:0)
<-----                        TC-4 ( 1   0   0   1 )
      ------>
            <------
                  <------
                        ------>
```

Fig. 8. The use of letter-line arrays and TC formula designations to depict representative experiments demonstrating equivalence class formation that have been reported in the experimental literature.

arrays or other visual depictions of two TCs appear to be different, but the formulas are the same, the two TCs must be the same. For example, the letter-line arrays depicted in the first two panels of Figure 7 appear to be different, even though they are identical TCs. This can be demonstrated by analyzing their formulas, which are presented to the right of each letter-line array. The formula and letter-line array in the top panel will be held constant as a reference. The formula for the TC in the middle panel lists the nodes alphabetically from left to right. By rearranging the left-to-right location of the columns of information for each node, as illustrated in the bottom panel, the formula becomes identical to that presented in the top panel. The identity of the two TCs can be confirmed by transforming the letter-line array in the middle panel, as illustrated in the bottom panel. This is accomplished by placing the nodal letters in the same positions as the nodes of the array used in the top panel (substituting G for A, E for C, and C for F), and then substituting the letter designation of the singles as needed. Differences in letter designation are not important because they represent an arbitrary assignment of stimuli (Fields et al., 1984). The letter-line array that is produced is the same as that in the first panel, which confirms the identity of the TCs presented in top and middle panels. Formula representations, then, facilitate the identification of TCs that are the same even though their visual representations appear to be different. More generally, formula designations of TCs facilitate the comparison of procedures used to establish equivalence classes.

Formula Designations for Some Representative Experiments

Figure 8 contains letter-line arrays that depict the training, transitivity, equivalence, and symmetry test pairs used in a number of representative experiments dealing with the formation of equivalence classes (Lazar et al., 1984; Sidman & Tailby, 1982; Sidman et al., 1985; Spradlin & Dixon, 1976; Spradlin et al., 1973; Stromer & Osborne, 1982; Wetherby et al., 1983). All of these procedures used unidirectional training only. Illustrations are supplied here for classes containing from three to six stimuli. For each experiment, the letter-line array is illustrated on the left and the formula representation of the training condition on the right. The directionality conventions used in the letter-line arrays are as described earlier. The two-term Sa/Co+ relations used in training are illustrated by the arrows drawn beneath the row of letters designating stimuli in the class. The arrows above the letters designate transitive, equivalence, or symmetrical relations used in testing. The formulas were derived using the conventions summarized in Figures 5 and 6. Stromer and Osborne (1982) conducted three experiments using unidirectional training with three stimuli per class. All procedures used one node, but the behavioral functions of the node differed in each experiment. Equivalence was studied with four stimuli per class by Spradlin et al. (1973), Sidman and Tailby (1982), and Wetherby et al. (1983). Although the letter-line arrays for each training procedure appear to be different, the same formula designation is obtained for each procedure. Therefore, the training procedures are formally identical. This is confirmed by the illustrated transformations of the letter-line arrays (Fields et al., 1984). Finally, formula representations of training procedures using five and six stimuli per class are presented for experiments conducted by Lazar et al. (1984) and by Sidman et al. (1985).

INTEGRATION AND GENERAL CONSIDERATIONS

The Hierarchical Organization of Structural Parameters

The four structural parameters that govern the organization of equivalence classes are related hierarchically:

1. Stimuli per class,
2. Number of training stimuli used as nodes,
3. Distribution of single stimuli among nodes, and
4. Directionality of training.

Thus, the values for a given parameter can be chosen only after values for higher order parameters have been set. Determining the effect of one value of a given parameter requires the exploration of many values of the lower order parameters. For example, to investigate the effects of a two-node TC, the class size must first be fixed. Determining the effects of a two-node TC cluster on equivalence and transitivity requires studying the effect of the distributions of singles and training directionality when the class contains two nodes.

The Domain of Variables That Define Equivalence Classes

A complete description of the structure of equivalence classes can be given by considering the four structural parameters—class size, number of nodes, the distribution of singles, and directionality of training. These form a quantifiable multidimensional domain that can be used to locate and interrelate systematically the vast range of TCs involved in the establishment of an equivalence class. Thus, the description can also be used to integrate the results of empirical research by presenting them as functions of the four parameters that define the experimental domain.

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