# 5. The Structure of Social Networks

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## Abstract

Inspired by empirical studies of networked systems such as the Internet, social networks and biological networks, researchers have in recent years developed a variety of techniques and models to help us understand or predict the behaviour of these complex systems. In this chapter we will introduce some of the key concepts of complex networks and review some of the major findings from the field. Leading on from this background material we will show that analysing the patterns of interconnections between agents can be used to detect dominance relationships. Another important aspect of social order is the enforcement of social norms. By coupling network theory, game theory and evolutionary algorithms, we will examine the role that social networks (the structure of the interactions between agents) play in the emergence of social norms. Empirical and theoretical studies of networks are important stepping stones in gaining a deeper understanding of the dynamics and organisation of the complex systems that surround us.

## Introduction

Many of the systems that surround us, such as road traffic flow, communication networks, densely populated communities, ecosystems with competing species, or the human brain (with $10^{10}$ neurons), are large, complex, dynamic, and highly nonlinear in their global behaviour. However, over the past 10 years it has been shown that many complex systems exhibit similar topological features in the way their underlying elements are arranged (Albert and Barabási 2002).

Underlying much of the current research is the notion that, in some way, topology affects dynamics that take place on the network, and vice versa. In this chapter I will explore some of the properties of complex social networks.

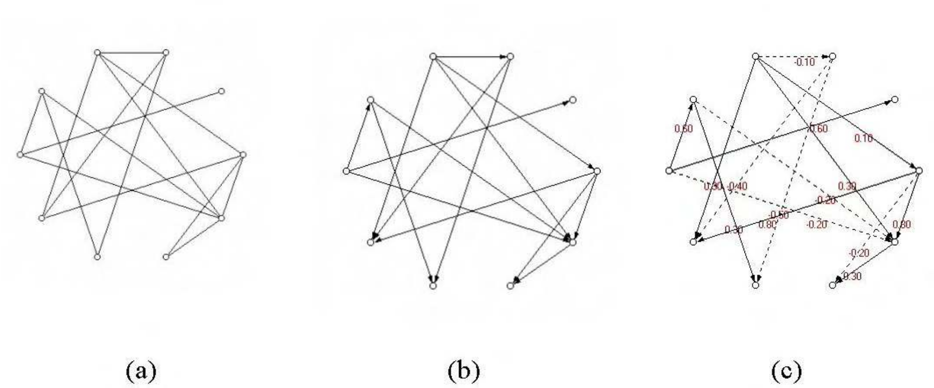
One of the most well known properties of social networks is the *small-world phenomenon*. This is something that most of us have experienced. Often we meet people with whom we have little in common, and unexpectedly find that we share a mutual acquaintance. The idea of '6 degrees of separation' is now firmly embedded in folklore, embracing everyone from Kevin Bacon to Monica Lewinsky (Watts 2003). Analysis of social networks has shown that the patterns of interactions that surround each of us, often determines our opportunities, level of influence, social circle, wealth, and even our mental well being.

In this chapter I explore some of the properties of complex social networks. The following section provides a number of examples of complex networks from a range of different contexts. Then, I provide an information theoretic-based approach to detect strong groups in social networks. It is followed by a detailed description of a game theoretic model, in which each of the agents is a decision making unit. This example shows how social network can influence the enforcement of certain behaviours within the system.

## Networks

In mathematical terms, a network is a graph in which the nodes and edges have values associated with them. A graph $G$ is defined as a pair of sets $G = \{V, E\}$, where $V$ is a set of $N$ nodes (vertices or points within the graph) labelled $v_0, v_1, \dots, v_N$ and $E \subseteq V \times V$ is a set of edges (links $(v_i, v_j)$ that connect pairs of elements $v_i, v_j$ within $V$). A set of vertices joined by edges is the simplest type of network. Networks can be more complex than this. For instance, there may be more than one type of vertex in a network, or more than one type of edge. Also, vertices may have certain properties. Likewise, edges may be directed. Such edges are known as arcs. Arcs and edges may also have weights. Figure 5.1, depicts networks with various types of properties. Taking the example of a social network of people, the vertices may represent men or women, people of different nationalities, locations, ages, incomes or any other attributes. Edges may represent relationships such as friendship, colleagues, sexual contact, geographical proximity or some other relationship. The edge may be directed such as supervisor and subordinate. Edges may represent the flow of information from one individual to another. Likewise the edges may carry a weight. This weight may represent a physical distance between 2 geographical proximity, frequency of interaction, degree to which a given person likes another person.

**Figure 5.1. Random graphs**




(a) Random graph. (b) Directed random graph. (c) Directed random graph with weights (random network)

## Network properties

### Connectivity

The degree to which the nodes of a network are directly connected is called connectivity. A network with high connectivity has a high ratio of edges to the number of nodes. To calculate a networks connectivity $C$, where $k$ is the number of edges and $N$ is the number of nodes in the network, the following equation is used;

$$C = \frac{k}{N(N-1)}$$

### Degree distribution

The degree of a node in a network is the number of edges or connections to that node (Newman 2003). The distribution function *P(k)* gives the probability that a node selected at random has exactly *k* edges (Albert and Barabási 2002). Plotting *P(k)* for a network forms a histogram of the degrees of the nodes, this represents the degree distribution (Newman 2003) or the number of nodes that has that number of edges for the network.

### Shortest average path length

The average path length, $(l)$ of a network is the average number of edges, or connections between nodes, that must be crossed in the shortest path between any 2 nodes (Watts 2003). It is calculated as:

$$l = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1}^{N} l_{\min}(i, j) ,$$

where $l_{min}(i, j)$ is the minimum distance between nodes $i$ and $j$.

## Diameter

The diameter of a network is the longest shortest path within a network. The diameter is defined as:

$$D = \max_{i,j} l_{min}(i, j)$$
.

## Clustering

A common property of many social networks is cliques. Cliques are groups of friends, where every member of the group knows every other member. The inherent tendency to cluster is quantified by the clustering coefficient (Watts and Strogatz 1998). For a given node $i$ within a network, with $k_i$ neighbours, the degree of clustering around node $i$ is defined to be the fraction of links the exist between the $k_i$ neighbours and the $k_i$ ($k_i$-1)/2potential links. Let $E_i$ , be the number of links that actually exist between the $k_i$ neighbours. The clustering coefficient is then:

$$CC = \frac{1}{N} \sum_{i=1}^{N} \frac{E_i}{k_i(k_i - 1)}$$
.

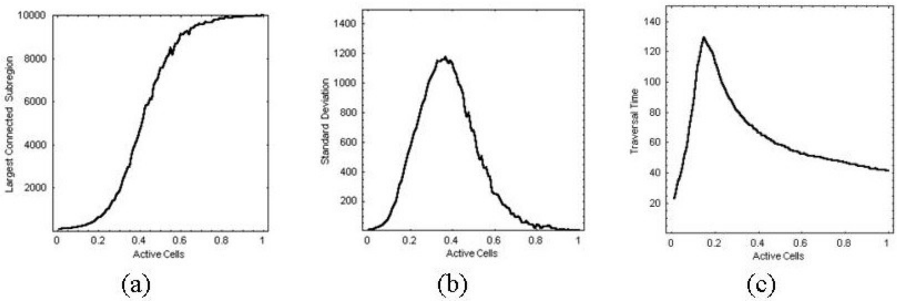
## Subgraphs

One of the first properties of random graphs that Erdös and Rényi studied was the appearance of subgraphs. A graph $G_1$ consisting of the set of nodes $P_1$ and the set $E_1$ of edges is said to be a subgraph of $G = \{P, E\}$ if $P_1 \subset P$ and $E_1 \subset E$. The simplest examples of subgraphs are *cycles*, *trees*, and *complete subgraphs*. One of the most fundamental concepts in graph theory is a walk. Given a graph $G$, a walk is a sequence ( $_0$, $_1$), ( $_1$, $_2$,), … ( $_{k\text{-}1}$, $_k$,) of vertices and edges from vertex $v_0$ to vertex $v_k$. The number of edges in the walk is its length. A walk in which all vertices are only visited once is known as a *path*. A walk in which all of the edges are distinct is a *trail*. A *cycle* is a closed loop of $k$ edges, such that every 2 consecutive edges only have common nodes. A *tree* is of order $k$ if it has $k$ nodes and $k-1$ edges, and none of its sub-graphs is a cycle. The average degree of a tree of order $k$ is $\langle k \rangle = 2 - \frac{2}{k}$, which approaches the limit 2 for large trees. A *complete subgraph* of order $k$ contains $k$ nodes and all the possible $k\frac{k-1}{2}$ edges, in other words, all the nodes in the subgraph are connected to all other nodes (Wilson 1994).

## Criticality

Probably the most important finding from random graph theory was the discovery of a critical threshold at which giant clusters form (in this context, cluster refers to a group of connected nodes) (Erdös and Rényi 1961). Below this threshold, the network exists as a series of disconnected subgraphs. Above this threshold, the graph is one all-encompassing cluster. Figure 5.2 shows this critical phase change using a simple graph model. Initially, all nodes within the graph are disconnected. At each time step, new edges are introduced. Figure 5.2 (a) shows that as edges are added, the nodes very quickly move from being disconnected to being fully connected. While Figure 5.2 (b) shows the standard deviation of the size of the largest connected subregion, the greatest deviation occurs at the critical threshold: at either side of this point the network exists as a series of small-disconnected subregions or as one giant cluster. Finally, Figure 5.2 (c) shows the time required to traverse the graph from one node to another. Again, the maximum time required to traverse the graph exists just below this critical level. This phase change is particularly important in percolation and epidemic processes.

**Figure 5.2. Example of criticality phenomena in the evolution of graphs**




Critical phase changes in connectivity of simple random lattice, as the proportion of active cells increases (x-axes). (a) Average size of the largest connected subregion (LCS). (b) Standard deviation in the size of LCS. (c) Traversal time for the LCS. Each point is the result of 1000 iterations of a simulation. Note that the location of the phase change (here $\approx 0.2$) varies according to the way we defined the connectivity within the model.

## Real world complex networks

In this section we look at what is known about the structure of networks of different types. Recent work on the mathematics of networks has been driven largely by observation of the properties of actual networks, and attempts to model the processes that generate their topology. An excellent example of the dual theoretical and observational approach is presented in the groundbreaking work by Watts and Strogatz (1998). The remainder of this section examines the statistical properties of a number of complex networks. Figure 5.3 illustrates a number of different complex networks.

## Ecological populations

Food webs are used regularly by ecologists to quantify the interactions between various species. In such systems, nodes represent species and the edges define predator–prey and other relationships that have positive and negative effects on the interacting species. Weights associated with the edges determines the magnitude of the relationship (May 2001). Solé and Montoya (2001) studied the topological properties of the Ythan Estuary (containing some 134 species with an average of 8.7 interactions per species), the Silwood Park ecosystem (Memmott et al. 2000) (with some 154 species with each species on average having 4.75 interactions) and the Little Rock Lake ecosystem (having a total of 182 species each of which interacts with an average of 26.05 other species). These studies found that these ecosystems were highly clustered and quite resilient against attack.

## Social systems

Studies of human and other animal groups have determined that the structure of many communities conforms to the small worlds model. For example, Liljeros et al. (2001) studied the sexual relationships of 2,810 individuals in Sweden during 1996. The result was a network in which the degrees of the vertices conformed to power–law distribution. Another commonly studied class of systems consists of association networks. Given a centroid, the task is to calculate the distance (number of steps removed) one individual is from another. Perhaps the best known example is the network of collaboration between movie actors (Newman 2000), which takes the popular form of Bacon numbers (Tjaden and Wasson 1997). Other popular examples include Lewinsky numbers and scientific collaboration networks (Newman 2003). The most notable of the scientific collaboration networks are the Erdös numbers (Hoffman 1998) that use the famous mathematician Paul Erdös as the centroid.

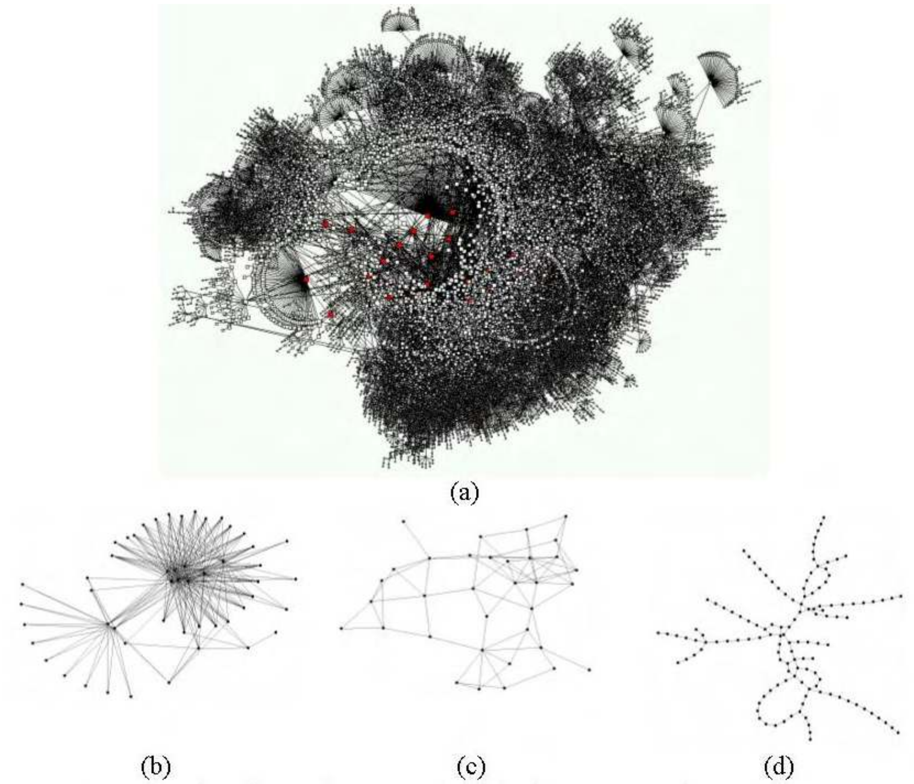
## World Wide Web and communications networks.

Computer networks and other communication networks have in recent times attracted a lot of attention. The World Wide Web (WWW), the largest information network, contained close to 1 billion nodes (pages) by the end of 1999 (Lawrence and Giles 1998). Albert et al. (2000) studied subsets of the WWW and found that they conformed to a scale free network. Other networks, such as the Internet and mobile phone networks, were also found to conform to the scale free network model (Faloutsos et al. 1999).

## Neural networks

Studies of the simple neural structure of earthworms (Watts and Strogatz 1998) were able to study the topological properties of underlying networks. In Watts and Strogatz's model of these brains, nodes represented neurons, and edges were used to depict the synaptic connections. The neural structure of the earthworm was found to conform to the small world network model, in which groups of neurons were highly clustered with random connections to other clusters.

**Figure 5.3. Examples of complex networks**




(a) the Internet, where nodes are routers and edges show physical network connections. (b) an ecosystem (c) professional collaboration networks between doctors; and (d) rail network of Barcelona, where nodes are subway stations and edges represent rail connections.

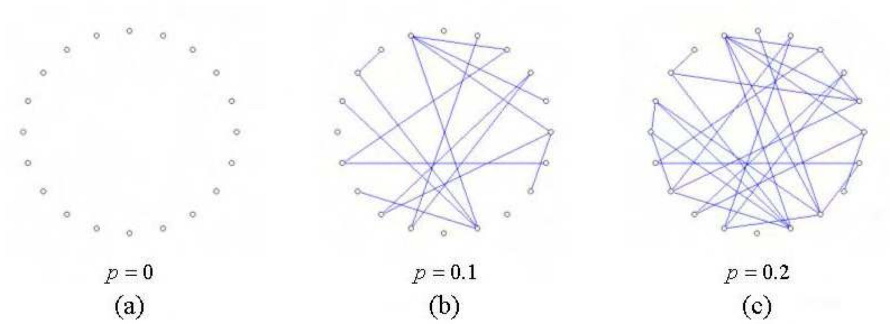
## Models of complex networks

Complex networks can be generally divided into two major classes based on their degree distribution $P_k$, i.e. the probability that a node in the network is connected to $k$ other nodes. This first class of graphs is referred to as exponential graphs, and the distribution of edges (i.e. the numbers per node) conforms to a Poisson distribution. This class of graphs includes the Erdös-Rényi type random graphs, and *small-world* networks.

## Random graphs

Paul Erdös and Alfred Rényi (1959, 1960, 1961) were the first to introduce the concept of random graphs in 1959. The simple model of a network involves taking some number of vertices, N and connecting nodes by selecting edges from the N(N-1)/2 possible edges at random (Albert and Barabási 2002; Newman 2003). Figure 5.4 shows three random graphs where the probability of an edge being selected is *p=0*, *p=0.1* and *p=0.2*.

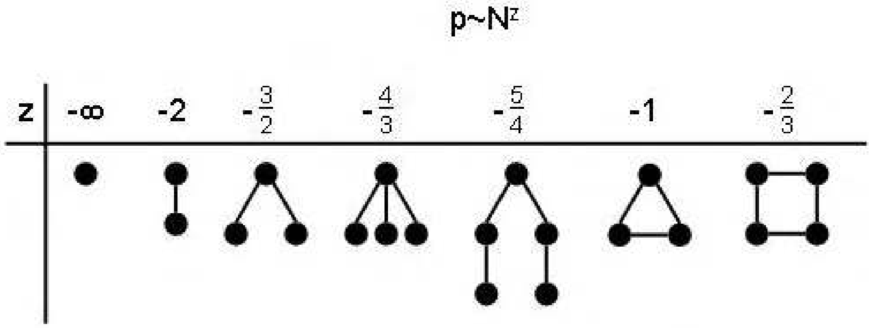
**Figure 5.4. Erdös-Rényi model of random graph evolution**




(a) Initially 20 nodes are isolated. (b) Pairs of nodes are connected with a probability of p of selecting an edge. In this case (b) *p=0.1* , (c) *p=0.2,* notice how the nodes become quickly connected

The Erdös and Rényi random graph studies explore how the expected topology of the random graph changes as a function of the number of links (Strogatz 2001). It has been shown that when the number of links is below $1/N$, the graph is fragmented into small isolated clusters. Above this threshold the network becomes connected as one single cluster or giant component (Figures 5.2 and 5.4). At the threshold the behaviour is indeterminate (Strogatz 2001). Random graphs also show the emergence of subgraphs. Erdös and Rényi (1959, 1960, 1961) explored the emergence of these structures, which form patterns such as trees, cycles and loops. Like the giant component, these subgraphs have distinct thresholds where they form (See Figure 5.5).

**Figure 5.5. Different subgraphs appear at varying threshold probabilities in a random graph (After Albert and Barabási 2002)**

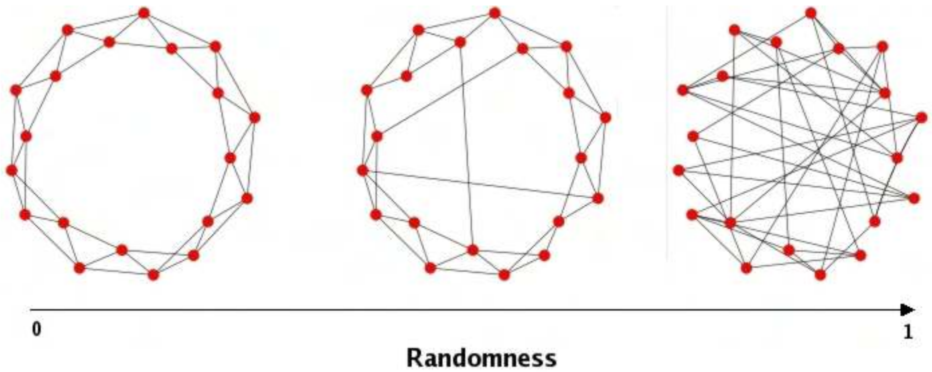



## Small-world networks

In the late 1960s, Stanley Milgram (1967) performed the famous *small-worlds* experiment. While no physical networks were constructed during this experiment, the results do provide valuable insights into the structure of social networks. Essentially, the experiment examined the distribution of paths lengths in an acquaintance network by asking participants to pass a letter to one of their first-name acquaintances in an attempt to get the letter to the designated target. While most of the letters were lost, about one quarter reached the target person. One average the letter passed through the hands of between 5 and 6 people. This experiment was the source of the popular concept of *6 degrees of separation*.

The ground breaking work of Watts and Strogatz (1998) showed that many complex networks display two key features: they possessed the 6 degrees of separation phenomenon, that Milgram discovered; but locally they had many properties similar to that of a regular lattice. In an attempt to model these systems Watts and Strogatz (1998) proposed a one-parameter model, which interpolates between an ordered finite dimensional lattice and a random graph. The algorithm behind the model is as follows: start with a regular ring lattice with $N$ nodes in which every node is connected to its first $K$ neighbours ($K/2$ neighbours on either side); and then randomly rewire each edge of the lattice with a probability $p$ such that self-connections and duplicate edges are excluded. This process introduces long-range edges which connect nodes that otherwise would be part of different neighbourhoods. Varying the value of $p$ moves the system from being fully ordered $(p = 0)$ to random $(p = 1)$. Figure 5.6 shows steps in this transition. Small world networks have been used to describe a wide variety of real world networks and processes; Newman (2000) and Albert and Barabási (2002) provide excellent review of the work done in this field.

**Figure 5.6. Progressive transition between regular and random graphs**




Source: Watts and Strogatz 1998

## Scale-free networks

The second class of graphs is referred to as *scale free networks*. Specifically, the frequency of nodes $P_k$ with $k$ connections follows a power-law distribution $P_k \approx k^{-\gamma}$, in which most nodes are connected with small proportion of other nodes, and a small proportion of nodes are highly connected (Albert and Barabási 2002). In exponential networks the probability that a node has a high number of connections is very low. In scale free networks, however, highly connected nodes (i.e. $k >> \langle k \rangle$) are statistically significant (Albert and Barabási 2002).

# Hierarchies and dominance

In competitive interactions between two individuals there is always a winner and a loser. If an individual A consistently defeats player B, then it is said that A dominates B. Such a relationship can be captured in a network in which the nodes represent players and arcs show which player is dominated. Animal behaviourists have frequently employed linear hierarchical ranking techniques to determine the dominant individuals within a community.

## Dominance and linear hierarchical ranking

There are many procedures, of varying complexity, for ranking the members of a social group into a dominance hierarchy (see de Vries 1998 for a review of these techniques). In general dominance hierarchy, techniques can be divided into 2 categories. The first class of methods attempts to determine the dominance ranking by maximising or minimising some numerical criteria. The second class aims to provide a measure of overall individual success from which the rank can be directly derived. One relatively simple ranking method belonging to the second class is David's score (David 1987, 1988).

Individual ranks calculated with David's Score are not disproportionately weighted by minor deviations from the main dominance direction within dyads, because win/loss asymmetries are taken into account by the use of dyadic dominance proportions in the calculations. The proportion of wins/losses by individual $i$ in his interactions with another individual $j$ ($P_{ij}$) is the number of times that $i$ defeats $j$ ($\alpha_{ij}$) divided by the total number of interactions between $i$ and $j$ ($n_{ij}$), i.e. $P_{ij} = \alpha_{ij}/n_{ij}$. The proportion of losses by $i$ in interactions with $j$, is $P_{ji} = 1 - P_{ij}$. If $n_{ij} = 0$ then $P_{ij} = 0$ and $P_{ji} = 0$ (David 1988; de Vries 1998). The David's Score for each member, $i$ of a group is calculated with the formula:

$$DS_i = w_i^1 + w_i^2 - l_i^1 - l_i^2$$

where $w_i^1$ represents the sum of $i$'s $P_{ij}$ values, $w_i^2$ represents the summed $w_i^1$ values weighted by the appropriate $P_{ij}$ values, (see the worked example in the following section) of those individuals with which $i$ interacted, $l_i^1$ represents the sum of $i$'s $P_{ji}$ values and $l_i^2$ represents the summed $l_i^1$ values (weighted by the appropriate $P_{ji}$ values) of those individuals with which $i$ interacted (David 1988: p. 108; de Vries 1998).

## A worked example

Table 5.1 shows a worked example with calculated w, w2, l, and l2 values. Specifically for individual $A$, $w_A^1$ represents the sum of $A$'s $P_{Aj}$ values (i.e. $w_A^1 = 0.75 + 0.8 + 1 + 0.5 = 3.05$), and $w_2^2$ represents the summed $w_i^1$ values (weighted by the appropriate $P_{ij}$ values) of those individuals with which A interacted (i.e. $w_i^2 = \left[(0.75 \times 2.05) + (0.8 \times 1.4) + (1 \times 0) + (0.5 \times 0.5)\right] = 2.90$. A's $l^1$ and $l^2$ values are calculated in a similar manner (David 1988; de Vries 1998).

**Table 5.1. Example of ranking between players according to David's score**

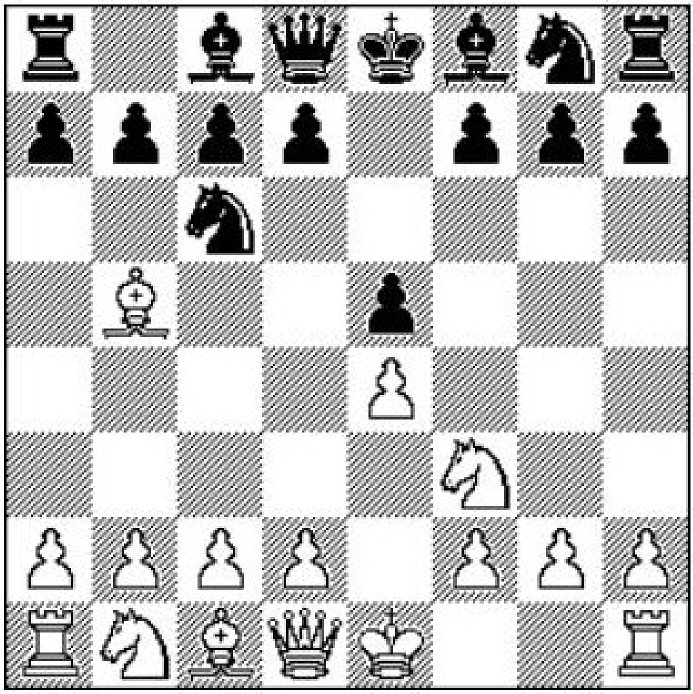
| | | *Player Losses* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | A | B | C | D | E | $w_i^1$ | $w_i^2$ | $DS$ |
| Player *Victories* | A | — | 4.5(0.75) | 4(0.8) | 7(1) | 2(0.5) | 3.05 | 2.90 | 3.20 |
| | B | 1.5(0.25) | — | 4(0.8) | 0(0) | 5(1) | 2.05 | 2.38 | 2.45 |
| | C | 1(0.2) | 1(0.2) | — | 0(0) | 4(1) | 1.40 | 1.52 | -0.20 |
| | D | 0(0) | 0(0.0) | 0(0) | — | 0(0) | 0.00 | 0.00 | -1.95 |
| | E | 2(0.5) | 0(0.0) | 0(0) | 0(0) | — | 0.50 | 1.52 | -3.5 |
| | $l_i^1$ | 0.95 | 0.95 | 1.60 | 1.00 | 2.50 | | | |
| | $l_i^2$ | 1.80 | 1.03 | 1.52 | 0.95 | 3.02 | | | |

## Bobby Fischer and the Ruy Lopez opening line

Bobby Fischer is probably the most famous chess player of all time and, in many peoples' view, the strongest. In the 1970s, Fischer achieved remarkable wins against top ranked grandmasters. His celebrated 1972 World Championship Match with Boris Spassky in Reykjavik made headline news all around the world. One of Fischer's favoured opening lines is the Ruy Lopez. The Ruy Lopez has been a potent weapon for Bobby throughout his career. Strategic play across the board suited Fischers talents. In his prime (and later in his career) Fischer was so proficient (dominant) in the main lines of the Ruy Lopez that many of his opponents chose irregular setups when attempting to defend their positions.

The Ruy Lopez was named after the Spanish clergyman, Ruy Lopez, of Safra, Estramadura. In the mid-sixteenth century, he published the first systematic analysis of the opening. The speed of development, flexibility and attacking nature, has seen the Ruy Lopez remain popular since its conception to the modern chess era. Figure 5.7 shows the board setup of the Ruy Lopez.

**Figure 5.7. The Ruy Lopez opening line**



All the games played by Fischer were converted into a network. Each player was represented as node or vertex within the system, with each game between 2 players shown as an arc. The arcs are drawn from loser to the winner, and given a weight of 1 for each victory of a given player. In the event of a draw/tie/stalemate, an arc was drawn in both directions and given a weight of ½. In many ways, the data presented here is limited, as the dataset does not contain all top level games played within the Ruy Lopez opening system (some notable players are missing from the database), nor does it capture all the Ruy Lopez games played between players. Also, some of the games are incomplete or the result of the game is unknown. While these constraints will limit the accuracy of the result, the dataset is complete enough to detect trends and regularities, and will not greatly influence the general findings of this chapter.

## Fischer's dominance

In this experiment, I examine the relationships between Fischer and his nearest neighbours. The network contains a total of 65 players. Figure 5.8 shows the network relationships between Fischer and his immediate opponents, the size of the arc represents the level of dominance of that player. Application of David's score to this network reveals that Bobby Fischer is the most successful player within his local neighbourhood. Table 5.2 lists the top 10 players in the neighbourhood.

**Figure 5.8. Bobby Fischer's network of immediate opponents**

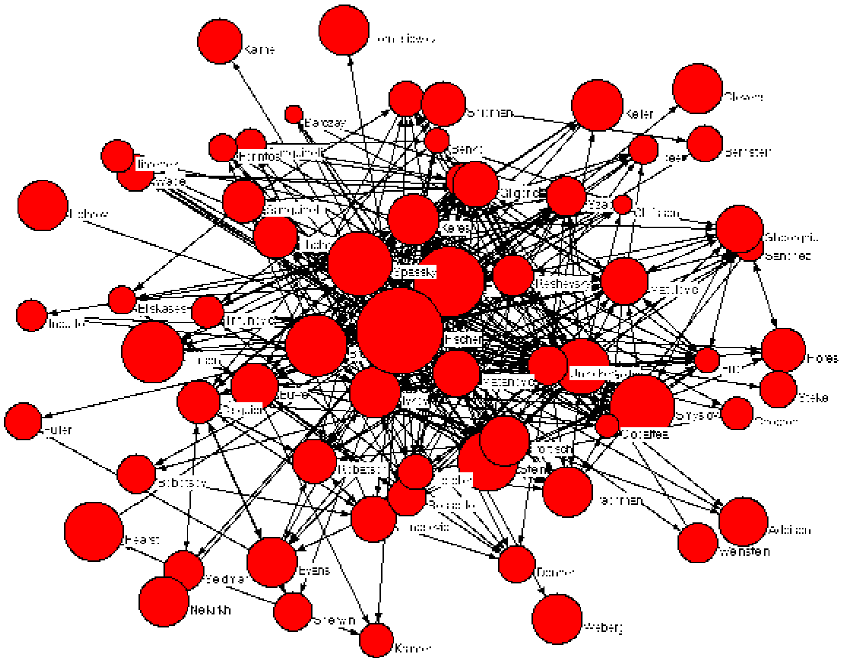



**Table 5.2. Top 10 players in Fischer's gaming network**

| *Rank* | *Player* | *Rank* | *Player* |
|---|---|---|---|
| 1. | Bobby Fischer | 6. | David Bronstein |
| 2. | Mikhail Tal | 7. | Leonid Stein |
| 3. | Vasily Smyslov | 8. | Eliot Hearst |
| 4. | Boris Spassky | 9. | Yefim Geller |
| 5. | Bent Larsen | 10. | Borislav Ivkov |

## Some closing comments on dominance hierarchies

In this section, I have been able to show that, from the network formed by the competitive interactions of individuals, key or dominant individuals can be identified. The proposed approach can also be used to determine a 'pecking order' between individuals within a group, in which individuals are ranked from most to least dominant. While the context of illustration is the world of chess, the framework can be applied to any network in which dominance between elements can be established. One should note, however, that hidden patterns, such as incorrect weightings and missing links or nodes, can distort the final result.

## Enforcement of social norms

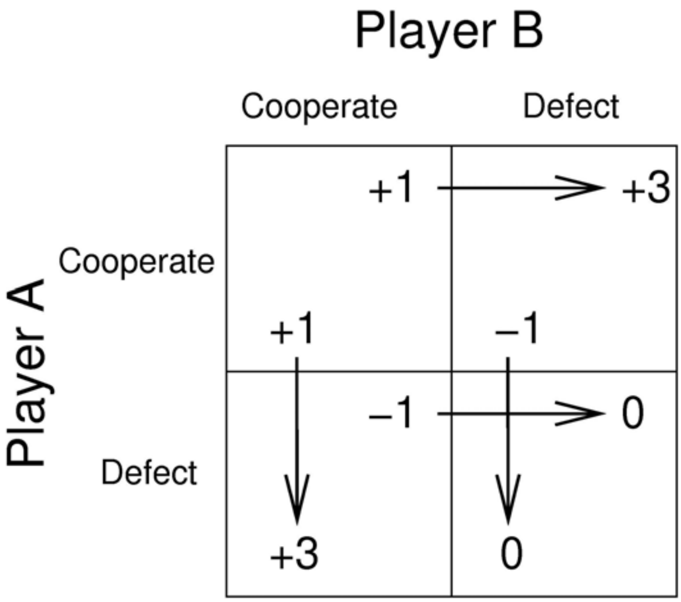
As individuals, we are each better off when we make use of a common resource without making a contribution to the maintenance of that resource. However, if every individual acted in this manner, the common resource would be depleted and all individuals would be worse off. Social groups often display a high degree of coordinated behaviour that serves to regulate such conflicts of interest. When this behaviour emerges without the intervention of a central authority, we tend to attribute this behaviour to the existence of social norms (Axelrod 1986). A social norm is said to exist within a given social setting when individuals act in a certain way and are punished when seen not to be acting in accordance with the norm. Dunbar (1996, 2003) suggests that social structure and group size play important roles in the emergence of social norms and cooperative group behaviour.

### Models of social dilemmas

All social dilemmas are marked by at least one deficient equilibrium (Luce and Raiffa 1957). It is deficient in that there is at least one other outcome in which everyone is better off. It is equilibrium in that no one has an incentive to change their behaviour. The Prisoners' Dilemma is the canonical example of such a social dilemma. The Prisoners' Dilemma is a $2\times 2$ non-zero sum, non-cooperative game, where *non-zero sum* indicates that the benefits obtained by a player are not necessarily the same as the penalties received by another player, and *non-cooperative* indicates that no per-play communication is permitted between players. In its most basic form, each player has 2 choices: cooperate or defect. Based on the adopted strategies, each player receives a payoff.

Figure 5.9 shows some typical values used to explore the behaviour of the Prisoners' Dilemma. The payoff matrix must satisfy the following conditions (Rapoport 1966): defection always pays more, mutual cooperation beats mutual defection, and alternating between strategies doesn't pay. Figure 5.9 also shows the dynamics of this game, the vertical arrows signify the row player's preferences and horizontal arrows the column player's preferences. As can be seen from this figure, the arrows converge on the mutual defection state, which defines a stable equilibrium.

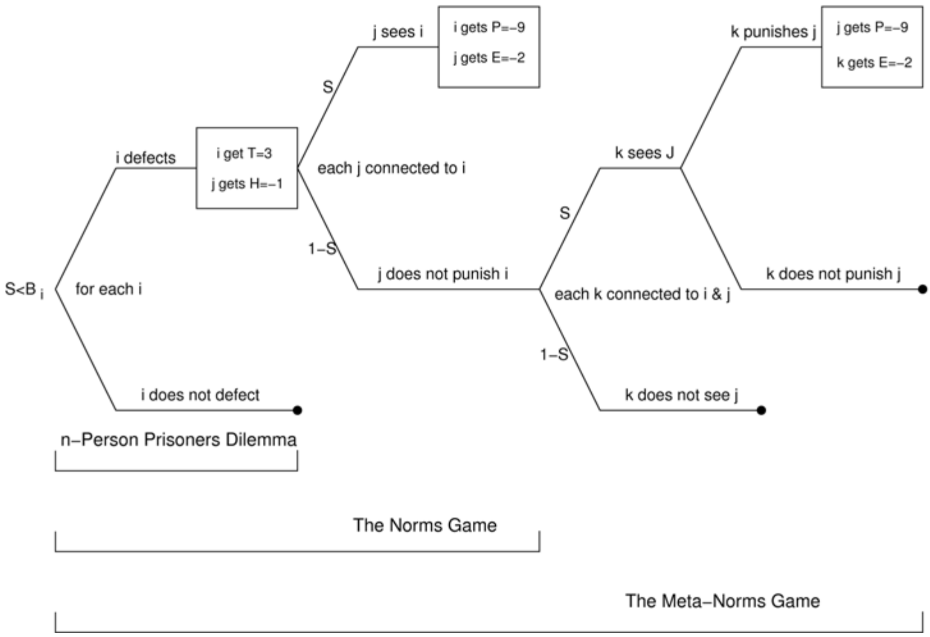
**Figure 5.9. Prisoners' Dilemma**




The payoff structure of Prisoners' Dilemma: the game has an unstable equilibrium of mutual cooperation, and a stable equilibrium of mutual defection, this is shown by the arrows, moving away from mutual cooperation to mutual defection

While the 2-person Prisoner's Dilemma has been applied to many real-world situations, there are a number of problems that cannot be modelled. The Tragedy of the Commons is the best known example of such a dilemma (Hardin 1968). While *n*-person games are commonly used to study such scenarios, they generally ignore social structure, as players are assumed to be in a well-mixed environment (Rapoport 1970). In real social systems, however, people interact with small tight cliques, with loose, long-distance connections to other groups. Also, traditional *n*-person games don't allow players to punish individuals that do not conform to acceptable group behaviour, which is another common feature of many social systems. To overcome these limitations, I will introduce the Norms and Meta-Norms games, which are variations on the *n*-person Prisoners' Dilemma. They can easily be played out on a network and allow players to punish other players for not cooperating. Figure 5.10 illustrates the structure of the n-person Prisoners' Dilemma, Norms and Meta-Norms games.

**Figure 5.10. The architecture of the Norms and Meta-norms games (After Axelrod 1986)**




Both games start with a variation on the n-person Prisoner's Dilemma. The Norms Game allows players to punish those players caught defecting. The Meta-Norms Game allows players to punish those players who do not punish defectors.

## The Norms game

The Norms game begins when an individual (*i*) has the opportunity to defect. This opportunity is accompanied by a known chance of being observed defecting (*S*) by one of *i*'s nearest neighbours. If *i* defects, he/she gets a payoff *T* (temptation to defect) of *3*, and each other player that is connected to *i*, receives a payoff *H* (hurt by the defection) of *-1*. If the player does not defect then each player receives a payoff of zero. To this point the game is equivalent to an *n*-person Prisoners' Dilemma played on a network (Rapoport 1970). However should *i* choose to defect, then one of his *n* neighbours may see the act (with probability *1-S*) and may choose to punish *i*. If *i* is punished he receives a payoff of *P=-9*, however the individual who elects to punish *i* also incurs an expense associated with dealing out the punishment of *E=-2*. Therefore the enforcement of a social norm *to cooperate* requires an altruistic sacrifice.

From the above description it can be seen that each player's strategy has 2 dimensions. The first dimension of player *i*'s strategy is *boldness* ($B_i$), which determines when the player will defect. Defection occurs when $S < B_i$. The second dimension of *i*'s strategy is *vengefulness* (*V*), which is the probability that a player will punish another player if caught defecting. The greater the vengefulness the more likely they are to punish another player.

## The Meta-Norms game

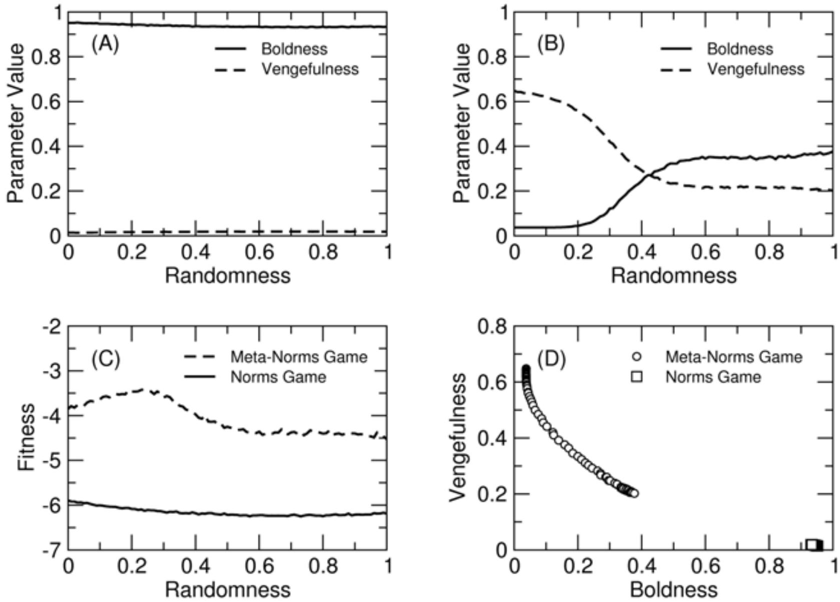
The Meta-Norms game is an extension of the Norms game. If player *i* chooses to defect, and player *j* elects not to punish *i*, and *i* and *j* have a common neighbour *k*, and *k* observes *j* not punishing *i*, then *k* has can punish *j*. Again, *j* receives the penalty $P=-9$, and, like the norm game, *k* receives an expenses $E=-2$.

Like the Prisoners' Dilemma, the Norms game and Meta-Norms game have unstable mutual cooperation equilibrium and a stable mutual defection equilibrium. The altruistic punishment is also an unstable strategy, as punishing an individual also requires a self-sacrifice. The stable strategy is mutual defection with no punishment for defectors. However, the global adoption of this strategy means that the population as a whole is worse off than if the unstable equilibrium strategy of mutual cooperation with punishment for defectors is adopted.

# Model of social structure

Let us imagine 2 variables (*B* and *V*) that make up a strategy are each allowed to take on a value between [0,1]. The variables represent the probability of defecting and punishing respectively. The variables are each encoded as a 16 bit binary number (as per Axelrod 1986). The evolution of players' strategies proceeds in the following fashion: (1) A small world network of 100 players with a degree of randomness *p* is created; (2) Each player is seeded with a random strategy; (3) The *score* or *fitness* of each play is determined from a given player's strategy and the strategies of the players in their immediate neighbourhood; (4) When the scores of all the players are determined, a weighted roulette wheel selection scheme is used to select the strategies of the players in the next generation; (5) A mutation operator is then applied. Each bit has a 1 per cent chance of being flipped; (6) Steps 3–5 are repeated 500 times, and the final results are recorded; (7) Steps 2–6 are repeated 10,000 times. (8) Steps 1–7 are repeated for *p* values between 0 and 1 in increments of 0.01. The above experimental configuration was repeated for both the Norms game and the Meta-Norms game. Figure 5.11 shows the results of these simulations.

**Figure 5.11. Simulation results**




(A) Average values for boldness and vengefulness over social networks with varying degrees of randomness for the Norms game. (B) Average values for boldness and vengefulness over social networks with varying degrees of randomness for the Meta-Norms game. (C) Comparison of the fitness values for the Norms game and Meta-Norms game. (D) Trade-off between Boldness and Vengefulness

From the simulation results we can see that, regardless of the social structure, the first order altruistic punishment isn't enough to enforce the social norm of mutual cooperation. Figure 5.11(A) shows that, regardless of the social structure, the vengefulness decreases to zero, and boldness increases toward one. Essentially all players are attempting to exploit the shared resource, with no fear of being punished. However, for the Meta-Norms game, with second-order punishment, there is a distinct set of circumstances when the population as a whole will not exploit the common resource. Figure 5.11(B) shows that, when the social structure is regular and highly clustered, players boldness decreases, but as the social structure becomes more random (and clustering breaks down), the boldness of a given player increases, and each individual attempts to exploit the common resource. However, the level of exploitation is lower than that observed in the Norms game. These differences in system behaviour are also seen in the average payoff received by a player (Figure 5.11(C)). The average payoff per player in the Meta-Norms game is always higher that that received in the Norms game. The average payoff for the Meta-Norms game maximises just before the transition to a state of global exploitation. Statistical analysis of the network structure reveals that this maximum payoff point coincides with the breakdown of clustering within the network. Finally, Figure 5.11(D) depicts the trade-off between vengefulness and boldness. The Norms game (squares) converges to a strategy of low vengefulness and high boldness. While the Meta-Norms game produces a range of behaviours (circles), from the plot it can be seen that there is a trade-off between boldness and vengefulness. The Meta-Norms game produces a wide variety of strategies. These strategies are governed by the topology of the underlying social network. The trade-off surface can be thought of as the set of viable strategies, as nonviable strategies (such as high boldness and vengefulness) are selected against.

## Discussion and implications

The results from the previous section provide a number of interesting insights into the emergence of social norms and group behaviour. Social structure and second order interactions seem to play an important role in the evolution of group behaviour. In the wider literature, there are many recorded instances where these 2 factors have been observed to influence group behaviour. Here I will explore 3 examples.

### Animal innovation

Japanese macaque were among the first primates observed by humans to display innovation and diffusion of new novel behaviours to other group members (Reader and Laland 2004). While many individual animals invent new behaviour patterns, most new behaviours (even if they are beneficial) are unlikely to become fixed within the community. Reader and Laland (2002) have shown that there is a link between the social structure of primates and the frequency with which new technologies are uptaken. Populations that tend to be more cliquish are more likely to adopt a new behaviour as member of the clique help to reinforce the novel behaviour.

### Social cohesion

Dunbar (1996) has shown that there is a correlation between neocortex size and the natural group size of primates. Also correlated with neocortex size is the cliquishness of the social structure. Dunbar (2003) conjectures that the increase in neocortex size may mean that individuals can manage and maintain more group relationships. The ability to maintain more complex relationships may allow individuals to locally enforce social behaviour. It has also been observed that, when primate groups grow too large, social order breaks down and the troop split into 2 or more smaller troops in which social order is re-established (Dunbar 2003).

### Control of social behaviour

The notion of Meta-Norms is widely used in denunciation in communist societies. When authorities accuse someone of doing something wrong, others are called upon to denounce the accused. Not participating in this form of punishment is itself taken as a defection against the group and offenders are punished.

### Some comments on the enforcement of social norms

In this section, I explored the emergence and enforcement of social norms through the use of 2 variations on the *n*-persons Prisoners' Dilemma. The simulation results suggest that a combination of second order interactions, altruistic punishment and social structure can produce coherent social behaviour. Such features have been observed to enforce norms in a number of social systems. The results from this study open a number of interesting future directions:

- As conjectured by Dunbar (2003), social order in primate troops breaks down when the troop becomes too large. This raises the question: What is the relationship between link density, number of nodes and other network statistics, and how do these statistics influence the behaviour of evolutionary games such as those described in this chapter?
- Coalitions and factions form and dissolve through time. How do the general results change if the underlying network is allowed to evolve?
- Several studies (Luce and Raiffa 1957) have shown that concepts such as the Nash equilibrium don’t hold when rational players are substituted for human players. Do the patterns and tradeoffs described previously hold when rational computer players are replaced by human decision makers?

All these questions require further experimentation but can be explored in the context of the framework proposed here.

## Closing comments

From social networks to large scale critical infrastructure, the systems that surround us are large and complex. Despite their obvious differences these systems share a number of common regularities—such as the small world properties. In this chapter we have reviewed some recent work on the structure and function of networked systems. Work in this area has been motivated to a high degree by empirical studies of real-world networks such as the Internet, the World Wide Web, social networks, collaboration networks, citation networks and a variety of biological networks. We have reviewed these empirical studies, focusing on a number of statistical properties of networks that have received particular attention, including path lengths, degree distributions, and clustering. Moving beyond these statistical regularities, the structure and nature of the interactions between the elements within a system can provide insights into the dynamics taking place in the network and as well as the way the network is being shaped by the dynamics.

In this chapter I have explored several notions. First, by understanding the nature of the interaction between players of chess we can extract dominance hierarchies. These hierarchies can be examined through time to gain an understanding as to how the system evolves, in this case, how the dynamics affect the topology. I also explored how the structure of a network affects the dynamics taking place upon it and showed that the regularities we see in social systems may be a consequence of the dynamics taking place. These regularities are also echoed across other types of social systems, suggesting universal laws of organisation.

In looking forward to future developments in this area, it is clear that there is much to be done. The study of complex networks is still in its infancy. Several general areas stand out as promising for future research. First, while we are beginning to understand some of the patterns and statistical regularities in the structure of real world networks, our techniques for analysing networks are, at present, no more than a grab-bag of miscellaneous and largely unrelated tools (Newman 2003). We do not yet, as in some other fields, have a systematic program for characterising network structure. We need a systematic framework by which we can analyse complex networks in order to identify key dynamical and structural properties. Second, there is much to be done in developing models of networks, both to help us understand network topology and to act as a substrate for the study of processes taking place on networks (Watts and Strogatz 1998; Newman 2003). Finally, and perhaps the most important direction for future study, is the behaviour of processes taking place on networks. The work describing the interplay between social structure and game theoretic decision making is only a timid first attempt at describing such processes, and yet this, in a sense, is the ultimate goal in the field: to understand the behaviour of the network systems that surround us.

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